CS217 - Data Structures & Algorithm Analysis (DSAA)

Lecture #8

▶Elementary Data Structures

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Reading: Part III Introduction & Chapter 10

Aims of this lecture

- To introduce **data structures** and their typical operations.
- **Stacks**, **queues**, **priority queues** and **linked lists**.
- To work out the **running time** for operations on these data structures.
- To identify pros and cons for data structures in terms of efficiency.

Data Structures

- **Dynamic sets** that can store and retrieve elements.
- Data structures are techniques for representing finite dynamic sets of elements
- Each element can contain:
  - a **key,** used to identify the element
  - **Satellite data,** carried around but unused by the data structure
  - **Attributes,** that are manipulated by the data structure eg., pointers to other objects
- Often keys stem from a **totally ordered set** (e. g. numbers)
  - Allows to define the minimum, successor and predecessor

Data Structure Operations

- Operations on a dynamic sets S can be grouped into **queries** and **modifying operations**:
- Typical operations:
  - **Search(S, k):** returns a pointer **x** to the element with **key k**, or NIL
  - **Insert(S, x):** given a pointer **x** to an element adds the element to S
  - **Delete(S, x):** given a pointer **x** to an element removes it from S
  - **Minimum(S), Maximum(S):** return pointer **x** resp. with smallest or largest key
  - **Successor(S, x), Predecessor(S, x):** next larger (smaller) than Key(x)
- **Time** often measured using n as the number of elements in S.

Data Structure Operations

- What's the runtime of each operation on an **array**?
- **Search(S, k):** returns a pointer **x** to the element with **key k**, or NIL $\boldsymbol{\Theta(n)}$
- **Insert(S, x):** given a pointer **x** to an element adds the element to S $\boldsymbol{\Theta(1)}$
- **Delete(S, x):** given a pointer **x** to an element removes it from S $\boldsymbol{\Theta(1)}$
- **Minimum(S), Maximum(S):** return pointer **x** resp. with smallest or largest key $\boldsymbol{\Theta(n)}$
- **Successor(S, x), Predecessor(S, x):** next larger (smaller) than Key(x) $\boldsymbol{\Theta(n)}$

▶Data Structure Operations

- What's the runtime of each operation on a **sorted array**? $\boldsymbol{\Theta}(\log \boldsymbol{n})$
- **Search(S, k):** returns a pointer **x** to the element with **key k**, or NIL $\boldsymbol{\Theta}(\boldsymbol{n})$
- **Insert(S, x):** given a pointer **x** to an element adds the element to S $\boldsymbol{\Theta}(\boldsymbol{n})$
- **Delete(S, x):** given a pointer **x** to an element removes it from S $\boldsymbol{\Theta}(\boldsymbol{1})$
- **Minimum(S), Maximum(S):** return pointer **x** resp. with smallest or largest key $\boldsymbol{\Theta}(\boldsymbol{1})$
- **Successor(S, x), Predecessor(S, x):** next larger (smaller) than Key(x)

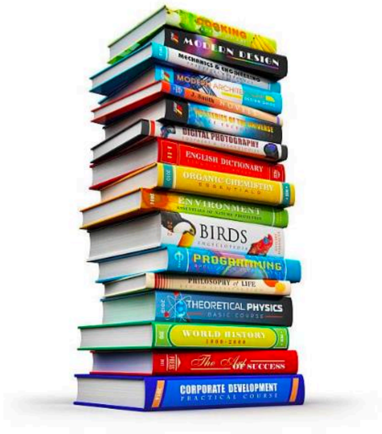
We'll now see some data structures that improve on the array implementation for many of the dynamic-set operations.

Roadmap for the next lectures

- Simple data structures
  - Stacks
  - Queues
  - Linked lists
  - Binary search trees
  - Graphs
- Advanced data structures
  - Balanced trees
  - Priority queues

▶ Stacks

3
6
8



- Only the **top element** is accessible in a stack.
  - Last-in, first-out policy (LIFO)
- Insert is usually called **Push**, and Delete is called **Pop**.

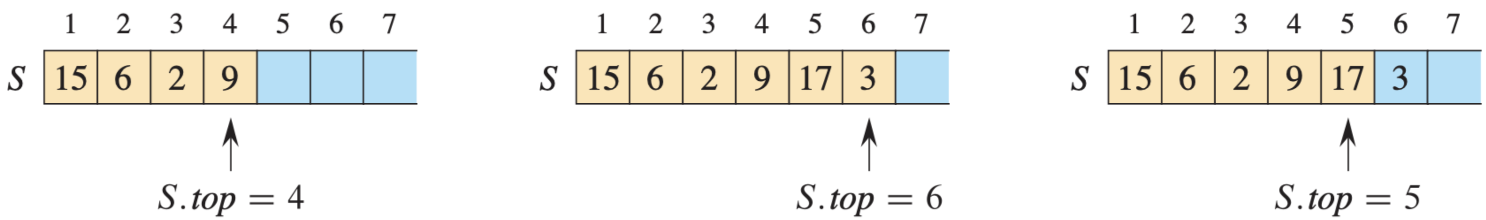
Stacks implemented using arrays

- Stacks can be implemented as an array S with attribute S.top.

```
PUSH(S, x)
if S.top == S.size
    error "overflow"
else S.top = S.top + 1
    S[S.top] = x
```

```
STACK-EMPTY(S)
if S.top == 0
    return TRUE
else return FALSE
```

```
POP(S)
if STACK-EMPTY(S)
    error "underflow"
else S.top = S.top − 1
    return S[S.top + 1]
```



(a) (b) (c)

- All stack operations take time $O(1)$.

Stacks Application (1): Bracket Balance Checking

- $1 + \{2 * [x + (4y - z)] * [5x - (5y + z)] - 5t\}$
- {[()][()]}
- Are the brackets correctly balanced or not?
- Read the expression: **Push** each opening bracket and **pop** for each closing bracket
- If the type of popped bracket always matches return **true,** else return **false**
- What's the runtime of the algorithm?

▶Stacks Application (2): Postfix expression

- $5 * ((9 + 3) * (4 * 2) + 7)$ (infix expression)
- 5 9 3 + 4 2 * * 7 + * (postfix expression)
- Parsing postfix expressions is somewhat easier than infix expressions. Why?
- Read the tokens one at a time:
  - If it is an operand, **push** it on the stack
  - If it is a binary operator **pop** twice, apply the operator, and **push** the result back on the stack
- What is the runtime of the algorithm?

Stacks Application (2): Postfix expression

- $5 * ((9 + 3) * (4 * 2) + 7)$ (infix expression)
- 5 9 3 + 4 2 * * 7 + * (postfix expression)

Stack operations	Stack elements
push(5)	5
push(9)	5 9
push(3)	5 9 3
push(pop() + pop())	5 12
push(4)	5 12 4
push(2)	5 12 4 2
push(pop() * pop())	5 12 8
push(pop() * pop())	5 96
push(7)	5 96 7
push(pop() + pop())	5 103
push(pop() * pop())	515

Queues

head	3	6	8	tail

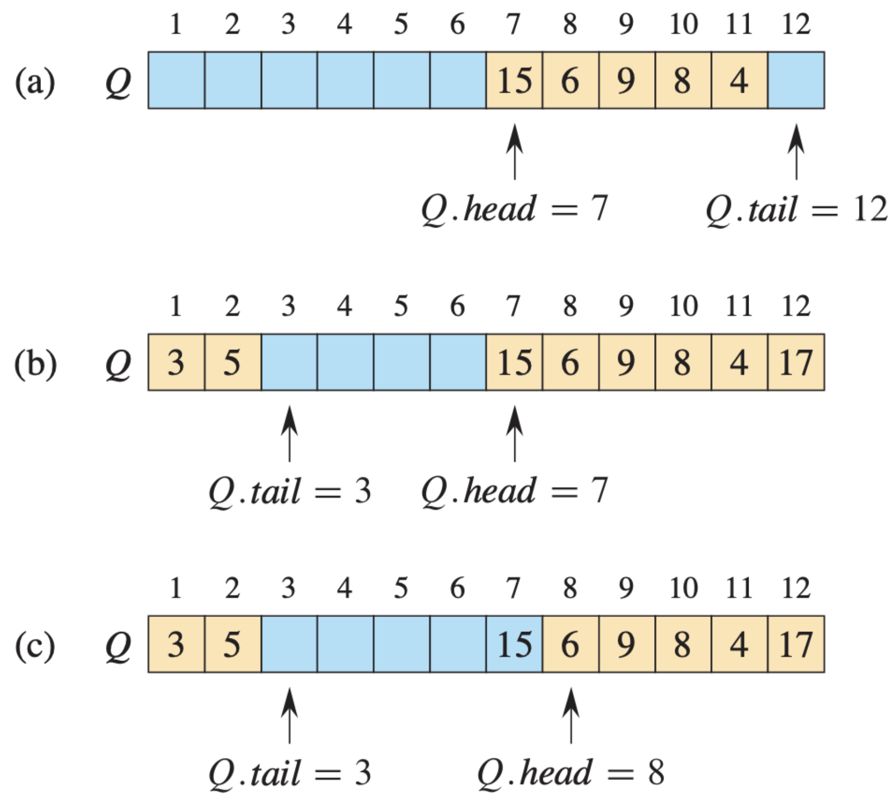
- The British love them ☺
- The first element in a queue is accessible.
  - First-in, first-out policy (FIFO)
- Insert is called **Enqueue**, Delete is called **Dequeue**.
- Queues have a **head** and a **tail**, like in a supermarket
  - Elements are added to the tail
  - Elements are extracted from the head

Queues implemented using arrays

- Queues can be stored in an array "**wrapped around**".

```
ENQUEUE(Q, x)
    Q[Q.tail] = x
    if Q.tail == Q.size
        Q.tail = 1
    else Q.tail = Q.tail + 1
```

```
DEQUEUE(Q)
    x = Q[Q.head]
    if Q.head == Q.size
        Q.head = 1
    else Q.head = Q.head + 1
    return x
```



- All queue operations take time $O(1)$.

Queues: Applications

- Playlists (eg., iTunes)
- Dispensing requests on a shared resource (eg., a printer, a server, a processor etc.,)
- Data buffers (eg., streaming services)
- What if I have priorities on the use of the resource?

Priority Queues: Motivation

- Schedule jobs on a computer shared among multiple users
- A max-priority queue keeps track of the jobs to be performed and their relative priorities
- When a job is finished the scheduler selects the job with highest priority from those pending
- Jobs can be added to the scheduler at any time

Job	Owner	Priority (key)
Job 1	Yao Xin	35
Job 12	Oliveto Pietro	2
Job 24	Hao Qi	22
Job 25	Yu Shiqi	18
Job 72	Yao Xin	30

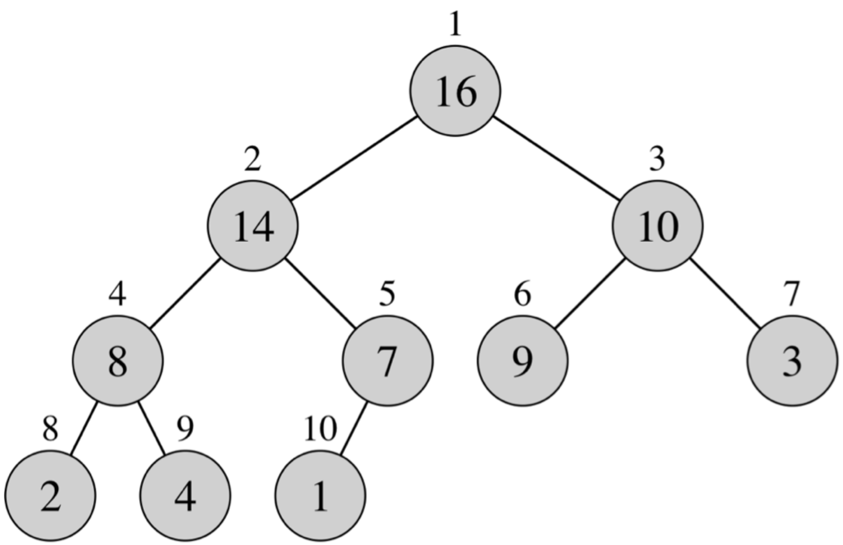
- **Use a heap!**

➢ Heap Properties

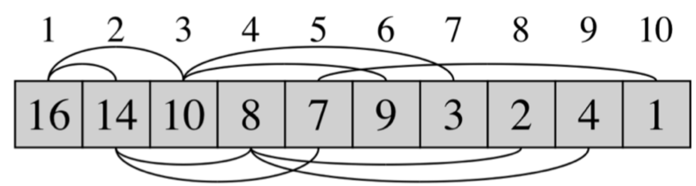
- **Max-heap property**: for every node other than the root, the parent is no smaller than the node, $A[\mathit{Parent}(i)] \geq A[i]$.

- In a max-heap, the **root** always stores a **largest** element.

  → this is what we want!



(a)



(b)

- **Min-heap property**: for every node other than the root, the parent is no larger than the node, $A[\mathit{Parent}(i)] \leq A[i]$.

Priority Queue based on max-heap

- A data structure for maintaining a set S of elements with an associated element called key (the priority).

Operation	Time
Insert(S, x, k) – inserts *x* with key *k* into *S*	
Maximum (S) – returns the element in *S* with the largest key	
Extract-Max(S) – removes and returns element in *S* with the largest key	
Increase-Key(S, x, k) – increases they key of *x* to a larger value *k* (element may float up in the heap)	

➢ Priority Queue based on max-heap

- A data structure for maintaining a set S of elements with an associated element called key (the priority).

Operation	Time
Insert(S, x, k) – inserts *x* with key *k* into *S*	$O(\log n)$
Maximum (S) – returns the element in *S* with the largest key	$O(1)$
Extract-Max(S) – removes and returns element in *S* with the largest key	$O(\log n)$
Increase-Key(S, x, k) – increases they key of *x* to a larger value *k* (element may float up in the heap)	$O(\log n)$

Job x: x.satellite_data; x.job_address x.priority (key)

To increase priorities: we need a way to **map** the position of job x in the heap (and update it as it moves in the heap) to the position in our set of jobs)

Min-priority queue based on min-heap also exist: we will use them in graph algorithms (eg., Djikstra, Prim)

Find and extract next job

```
MAX-HEAP-MAXIMUM(A)
if A.heap-size < 1
    error "heap underflow"
return A[1]

MAX-HEAP-EXTRACT-MAX(A)
max = MAX-HEAP-MAXIMUM(A)
A[1] = A[A.heap-size]
A.heap-size = A.heap-size - 1
MAX-HEAPIFY(A, 1)
return max
```

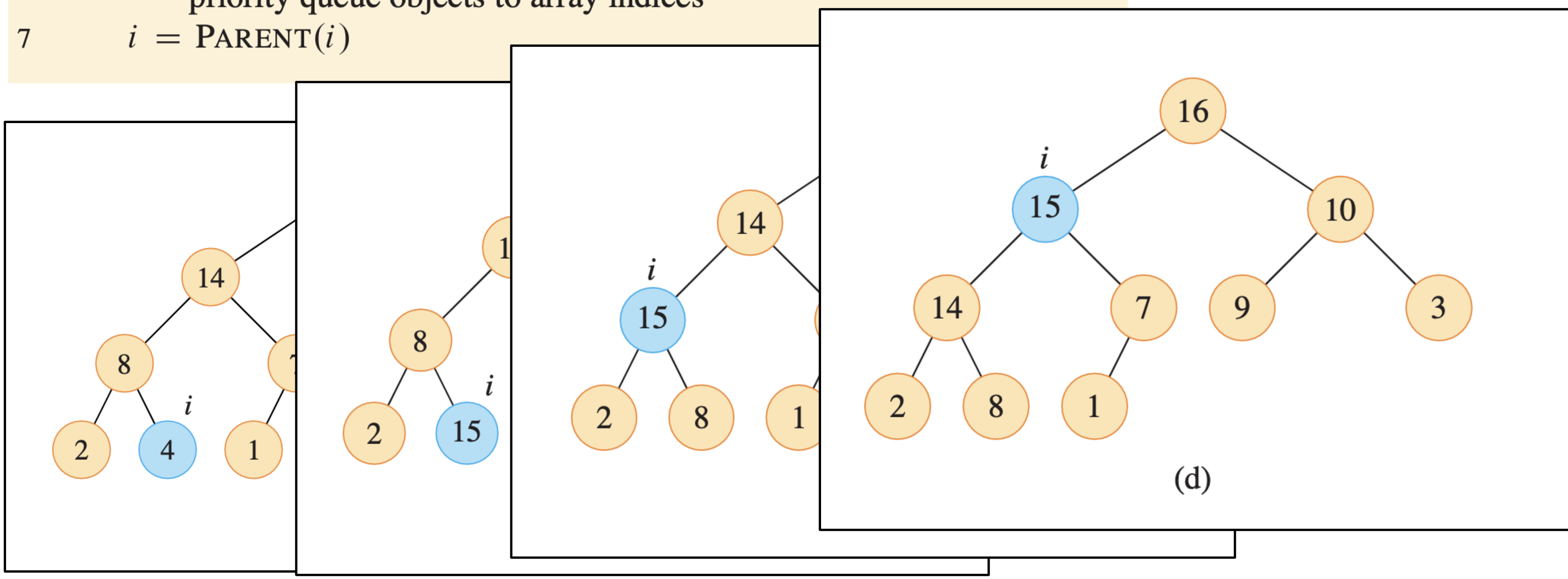
Job	Owner	Priority (key)	Handle	Handle after Build-Heap
1	Yao	35	1	1
12	Oliveto	2	2	5
24	Hao	22	3	3
25	Yu	18	4	4
72	Yao	30	5	2

1	2	3	4	5	
35	30	22	18	2	
1	72	24	25	12	

mapping

Increase job priority

```
MAX-HEAP-INCREASE-KEY(A, x, k)
if k < x.key
    error "new key is smaller than current key"
x.key = k
find the index i in array A where object x occurs
while i > 1 and A[PARENT(i)].key < A[i].key
    exchange A[i] with A[PARENT(i)], updating the information that maps
        priority queue objects to array indices
    i = PARENT(i)
```



Insert new job

```
MAX-HEAP-INSERT(A, x, n)
if A.heap-size == n
    error "heap overflow"
A.heap-size = A.heap-size + 1
k = x.key
x.key = -∞
A[A.heap-size] = x
map x to index heap-size in the array
MAX-HEAP-INCREASE-KEY(A, x, k)
```

- Put the new object at the bottom of the heap (end of array)
- Save priority k
- Set its priority to minimum $(-\infty)$ to preserve heap properties
- Call Max-Increase-Heap-Key (A, x, k)

Linked Lists: Array Disadvantages

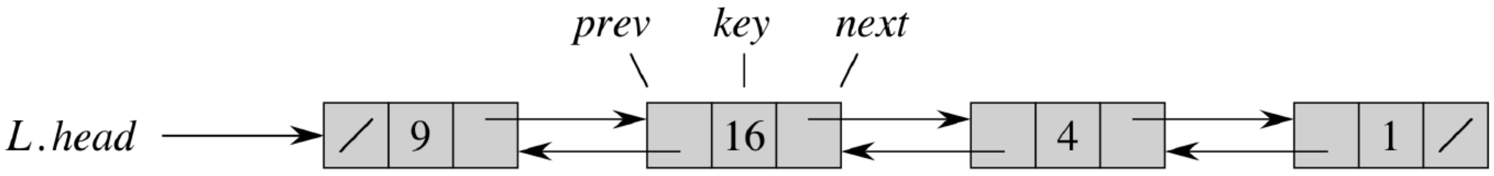
- You need to specify an initial size
- Changing the size of an array is troublesome
- Inserting and deleting elements in specific positions is difficult
- Let’s say we want to delete 10 and keep the order of the rest:

A	5	8	10	13	16	19	27	46	51	86
A	5	8		13	16	19	27	46	51	86
A	5	8	13	16	19	27	46	51	86	

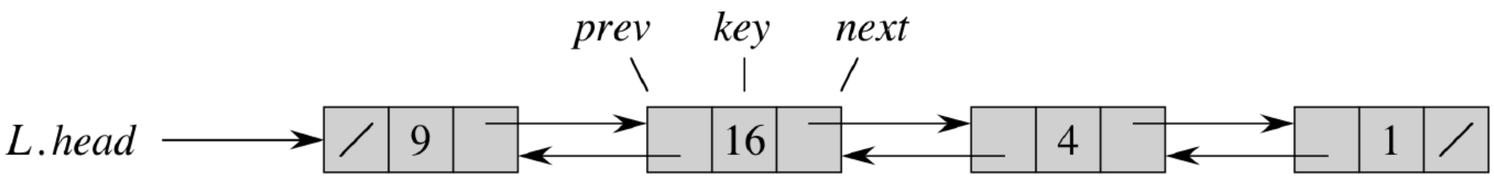
- What’s the time complexity?

Linked Lists

- Objects are linked using **pointers to the next element**.
- Linked lists can be **singly linked** or **doubly linked**: pointers to next and previous elements.
- Each element x has attributes
  - x.key – the key used to identify the element
  - x.next – a pointer to the next element
  - x.prev – a pointer to the previous element
  - Optional: further satellite data



Linked Lists: Searching



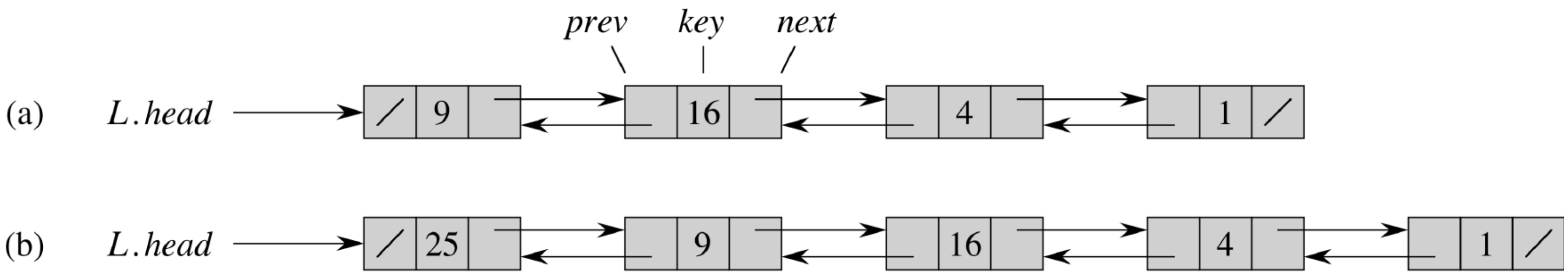
- Search inspects all elements in sequence and stops when the key has been found or the end of the list is reached.

LIST-SEARCH(L, k)

$x = L$.head
while $x \neq$ NIL and x.key $\neq k$ **do**
    $x = x$.next
return x

- The worst-case time is $\Theta(n)$, since it may have to search the entire list.

▶ Linked Lists: Inserting at the front

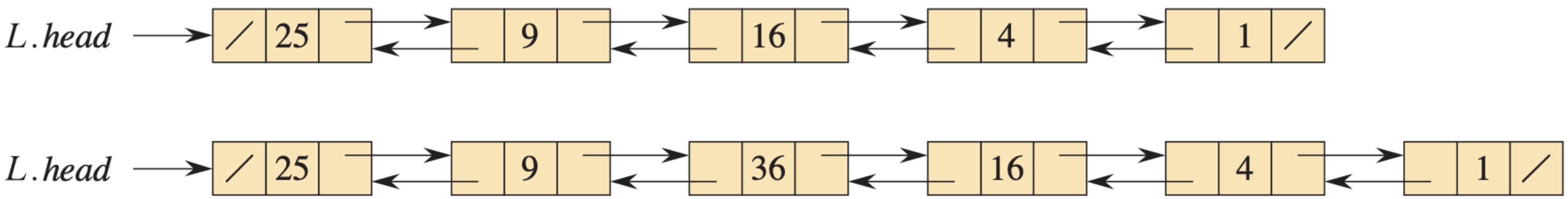


- New elements are added to the front of the list.

```
LIST-PREPEND(L, x)
x.next = L.head
x.prev = NIL
if L.head ≠ NIL
        L.head.prev = x
L.head = x
```

- The time for an insertion is $O(1)$.

Linked Lists: Inserting after element x

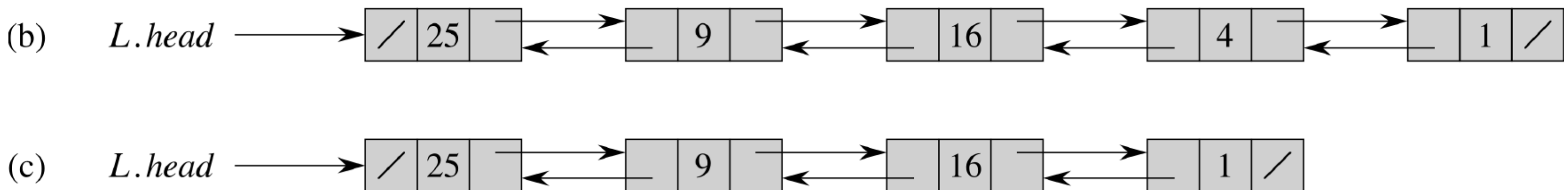


- New element added after element y.

```
LIST-INSERT(x, y)
    x.next = y.next
    x.prev = y
    if y.next ≠ NIL
        y.next.prev = x
    y.next = x
```

- The time for an insertion is $O(1)$ if you know the pointer to y

Linked Lists: Deleting



- If element x is known, update pointers to take it out.

List-Delete(L, x)

```
if x.prev ≠ NIL then
    x.prev.next = x.next
else
    L.head = x.next
if x.next ≠ NIL then
    x.next.prev = x.prev
```

- The time for a deletion is $O(1)$.
  But if we only have the key and need to search the element x, it's time $\Theta(n)$ in the worst case.

Summary

- **Stacks** and **Queues** are simple data structures that can
  - be implemented efficiently in arrays (modulo space issues)
  - Have a restricted set of operations, but these run in time $O(1)$.
- **Priority Queues**: all operations in at most $O(\log n)$ time
- Linked lists form an **unordered list** of elements
  - **Insertion** is fast if not important where it occurs: time $O(1)$.
  - **Searching** takes worst-case time $\Theta(n)$.
  - **Deletion** runs in time $O(1)$ if the element is known, otherwise we need to run a search beforehand and incur time $\Theta(n)$.