

## Homework 9 (Due November 28)

Grade Distribution (Total=8+12+8+8+8+12=56).

1. The county hospital is located at the center of a square whose sides are 520 meters wide. If an accident occurs within this square, then the hospital sends out an ambulance. The road network is rectangular, so the travel distance from the hospital, whose coordinates are  $(0, 0)$ , to the point  $(x, y)$  is  $|x| + |y|$ . If an accident occurs at a point that is uniformly distributed in the square, find the expected travel distance of the ambulance.
2. Suppose that A and B each randomly and independently choose 4 of 11 objects. Find the expected number of objects
  - (a) chosen by both A and B;
  - (b) not chosen by either A or B;
  - (c) chosen by exactly one of A and B.
3. A total of  $n$  balls, numbered 1 through  $n$ , are put into  $n$  urns, also numbered 1 through  $n$  in such a way that ball  $i$  is equally likely to go into any of the urns  $1, 2, \dots, i$ . Find
  - (a) the expected number of urns that are empty;
  - (b) the probability that none of the urns is empty.
4. Suppose that  $X$  is a continuous random variable with density function  $f$ . Show that  $E(|X - a|)$  is minimized when  $a \in \mathbb{R}$  satisfies  $F(a) = 1/2$ .
5. Twenty individuals consisting of 10 married couples are to be seated at 8 different tables, 2 of which have 4 seats and 6 of which have 2 seats. If the seating is done “at random,” what is the expected number of married couples that are seated at the same table?
6. Individuals 1 through  $n$ ,  $n > 1$ , are to be recruited into a firm in the following manner: Individual 1 starts the firm and recruits individual 2. Individuals 1 and 2 will then compete to recruit individual 3. Once individual 3 is recruited, individuals 1, 2, and 3 will compete to recruit individual 4, and so on. Suppose that when individuals  $1, 2, \dots, i$  compete to recruit individual  $i + 1$ , each of them is equally likely to be the successful recruiter.
  - (a) Find the expected number of the individuals  $1, \dots, n$  who did not recruit anyone else.
  - (b) Derive an expression for the variance of the number of individuals who did not recruit anyone else, and evaluate it for  $n = 5$ .