15 marks 1. (a) Define precisely the covariance of random variables X and Y.

(b) Define precisely the correlation coefficient of random variables X and Y.

(c) Define precisely what it means for events A, B, C to be independent.

$\boxed{\text{10 marks}}$ 2. A fair coin is tossed 4 times.

(a) What is the probability of getting exactly 3 heads?

(b) What is the probability of getting exactly 3 heads conditioned on the event that the first two tosses came out the same?

5 marks 3. Let A, B be events so that $\mathbb{P}(A) = 0.5$, $\mathbb{P}(B) = 0.4$ and $\mathbb{P}(A \cup B) = 0.7$. What is $\mathbb{P}(A|B)$?

5 marks 4. If $X = \mathrm{Exp}(1)$ and $Y = \mathrm{Bin}(n, p)$ are independent, what is $\mathbb{P}(X > Y)$?

$\boxed{\text{20 marks}}$ 5. Consider variables (X, Y) which are uniformly distributed with density a over the triangle with corners $(0,0)$, $(6,0)$ and $(6,3)$.

(a) Find a

(b) find the marginal densities of X and Y

(c) Find $\mathbb{E}XY$.

(d) Find $\mathbb{P}(X > 6Y)$.

15 marks 6. (a) Precisely state the central limit theorem.

(b) Suppose the weight of a person has mean 75 (Kg) and variance $\sigma^2 = 100$. An airline has 400 passengers on a flight. Assume their weights are independent, and use the CLT to estimate the probability that their total weight exceeds 30500.

(c) Use Chebyshev's inequality to give a bound on the probability that the total weight exceeds 30500.

10 marks 7. If Z_1, Z_2 are independent $N(0,1)$ random variables, what is the distribution of each of the following:

(a) $2Z_1 + Z_2$

(b) $2Z_1 - Z_2$

$\boxed{\text{10 marks}}$ 8. (a) Let $X = \text{Poi}(\lambda)$ for some $\lambda > 0$. For which values of t is $\mathbb{E}e^{tX}$ finite? When it is finite, what is $\mathbb{E}e^{tX}$?

(b) Repeat the same for $Y = \text{Exp}(\lambda)$.

$\boxed{\text{15 marks}}$ 9. Alice and Bob arrange the digits $1 \ldots 9$ in independent random orders, and compare the resulting numbers digit by digit. Let Q be the number of digits in agreement. For example, if the numbers happen to be 475619283 and 374956182, then $Q = 2$ (the 7 and 8 are in the same position).

(a) what approximation rule gives an estimate for the distribution of Q?

(b) Find $\mathbb{E}Q$ (exactly!)

(c) Find $\mathrm{Var}(Q)$ (exactly!)