Homework 10 (Due December 5)

Grade Distribution (Total=8+12+8+8+8=44).

1. Suppose that X and Y are both Bernoulli random variables, say, $X \sim Bernoulli(p_1)$ and $Y \sim Bernoulli(p_2)$. Show that X and Y are independent if and only if $Cov(X, Y) = 0$.

2. The number of accidents that a person has in a given year is a Poisson random variable with mean λ. However, suppose that the value of λ changes from person to person, being equal to 2 for 60 percent of the population and 3 for the other 40 percent. If a person is chosen at random,

   (a) What is the probability that he will have 0 accidents?

   (b) What is the probability that he will have exactly 3 accidents in a certain year?

   (c) What is the conditional probability that he will have 3 accidents in a given year, given that he had no accidents the preceding year?

3. The time T it takes me to complete a certain task is exponentially distributed with parameter λ. If I finish the job in no more than one time unit I get to flip a fair coin: heads I am paid one dollar, tails I am paid two dollars. If it takes me more than one time unit to complete the job I am simply paid one dollar. Let X be the amount I earn from the job.

   (a) From the description above, write down the conditional probability mass function of X, given that $T = t$. Your answers will be a two-case formula that depends on whether $t > 1$ or $t \leq 1$.

   (b) Find EX.

4. For a real number z let $\{z\}$ denote the fractional part of z, in other words, $\{z\} = z - [z]$, the difference between z and $[z]$ where $[z]$ is the largest integer less than or equal to z.

   Now suppose that X and Y are independent random variables, $Y \sim Unif(0, 1)$, and X has probability density function $f_X(x)$. Show that the fractional part $\{X + Y\}$ of the random variable $X + Y$ has $Unif(0, 1)$ distribution. [Hint. Condition on X.]

5. There are n items in a box labeled H and m in a box labeled T. A coin that comes up heads with probability p and tails with probability $1 - p$ is flipped. Each time it comes up heads, an item is removed from the H box, and each time it comes up tails, an item is removed from the T box. (If a box is empty and its outcome occurs, then no items are removed.) Find the expected number of coin flips needed for both boxes to become empty. [Hint: Condition on the number of heads in the first $n + m$ flips.]