12 Week 8.1-MT Review

1. For two events E, F such that $P(E) > 0$, the conditional probability that F occurs given that E has occurred is denoted by

$$P(F|E) = \frac{P(F \cap E)}{P(E)}.$$

2. For two events E, F, we say E and F are independent if

$$P(E \cap F) = P(E) \cdot P(F).$$

3. **Total Probability Formula**: Let $A_1, \cdots, A_n$ be mutually exclusive so that $S = A_1 \cup \cdots \cup A_n$. Then for any event B,

$$P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i).$$

   Bayes' Theorem: Let $A_1, \cdots, A_n$ be mutually exclusive so that $S = A_1 \cup \cdots \cup A_n$. Then for any event B,

$$P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}.$$

4. For a discrete random variable X, we define the probability mass function (abbreviated as pmf) of X by

$$p(m) = P(X = m).$$

5. A nonnegative function $f : (-\infty, \infty) \to [0, \infty)$ is called a probability density function (pdf) if

$$\int_{-\infty}^{\infty} f(x)dx = 1.$$

   Then a random variable X is called a continuous random variable if there is a probability density function f such that for all $a \leq b$,

$$P(a \leq X \leq b) = \int_{a}^{b} f(x)dx.$$

6. Steps for finding the pdf of a function of a random variable:

   Step 1. Find the cdf of the transformed variable;
   Step 2. Differentiate to find the density;
   Step 3. Specify in what region the result holds.

7. For a continuous random variable X, the expectation of X is defined by

$$EX = E(X) = E[X] = \int_{-\infty}^{\infty} x \times f(x)dx,$$

where $f(x)$ is the pdf of X. For any function g, we have

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) \times f(x)dx.$$

8. The joint pdf $f(x, y)$ satisfies that

$$1 = \int\int f(x, y)dxdy.$$

9. The marginal density functions of X and Y are given by

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy, \quad f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx.$$

Two continuous random variables X and Y are independent if and only if

$$f(x, y) = f_X(x)f_Y(y).$$

10. For any jointly pdf $f(x, y)$, we have for any function $g : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$,

$$E[g(X, Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x, y)f(x, y)dxdy$$

11. The Bivariate Normal Distribution. We say that the random variables X, Y have a bivariate normal distribution if, for constants μ_x, μ_y, $\sigma_x > 0$, $\sigma_y > 0$, $-1 < \rho < 1$, their joint density function is given, for all $-\infty < x, y < \infty$, by

$$\begin{aligned} f(x, y) =& \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \\ & \exp\Big\{ -\frac{1}{2(1-\rho^2)}\Big[\Big(\frac{x-\mu_x}{\sigma_x}\Big)^2 + \Big(\frac{y-\mu_y}{\sigma_y}\Big)^2 - 2\rho\frac{(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y}\Big]\Big\}. \end{aligned}$$

We say $(X, Y) \sim \mathcal{N}(\mu_x, \mu_y; \sigma_x^2, \sigma_y^2; \rho)$.

12. The marginal pdfs $f_X(x)$ and $f_Y(y)$ of the Bivariate Normal Distribution is $X \sim \mathcal{N}(\mu_x, \sigma_x^2)$ and $Y \sim \mathcal{N}(\mu_y, \sigma_y^2)$.

$$Cov(X, Y) = E[(X-\mu_x)(Y-\mu_y)] = \rho\sigma_x\sigma_y, \quad \text{and } \rho = \frac{Cov(X, Y)}{\sqrt{Var(X)Var(Y)}}.$$

We call ρ is the correlation of X, Y.