Homework 1 (Due September 19 during class)

Grade Distribution (Total=4+4+6+2+4+10=30).

Please simply answer as much as possible.

1. There is an urn with 4 green and 5 yellow balls. We choose 2 balls without replacement. Count the number of possible outcomes if
   (a) all the balls with the same colors are the same.
   (b) all the balls are numbered so that they are all different.
2. A person has 11 friends, of whom 7 will be invited to a party.
   (a) How many choices are there if 2 of the friends will not attend together?
   (b) How many choices if 3 of the friends will only attend together?
3. In how many ways can 5 novels, 3 mathematics books, and 2 chemistry books (they are all different) be arranged on a bookshelf if
   (a) the books can be arranged in any order?
   (b) the mathematics books must be together and the novels must be together?
   (c) the novels must be together, but the other books can be arranged in any order?
4. There are 7 different balls to be placed into 4 different boxes. How many possible ways if we require that every box contains at least 1 ball?
5. Choose 3 different numbers from $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$. How many ways are there if the sum of the three numbers is odd and strictly greater than 9?
6. There are n objects numbered $1, 2, 3, \cdots, n$. Let $1 \leq r \leq n-1$.
   (a) How many possible outcomes if we choose r objects?
   (b) How many possible outcomes if we choose r objects that do not contain object 1?
   (c) How many possible outcomes if we choose r objects that contain object 1?
   (d) Use the above to explain why the following holds (no computations are needed):
   $$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}.$$
   (e) Apply the definitions of $\binom{n}{r}$, $\binom{n-1}{r}$ and $\binom{n-1}{r-1}$ to prove the above with computations, that is, verify that
   $$\frac{n!}{(n-r)!r!} = \frac{(n-1)!}{(n-1-r)!r!} + \frac{(n-1)!}{(n-r)!(r-1)!}.$$