



南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目: 概率论
考试时长: 120 分钟

开课单位: 数学系
命题教师: 孙景瑞、洪杰梁

题号	1	2	3	4	5	6
分值	14 分	18 分	18 分	10 分	16 分	24 分

本试卷共 (6) 大题, 满分 (100) 分。(考试结束后请将试卷、答题本、草稿纸一起交给监考老师)

1. Suppose each of three persons tosses a coin. If the outcome of one of the tosses differs from the other outcomes, then the game ends. If not, then all the three persons start over and retoss their coins.

- (1) (6 points) Assuming fair coins, what is the probability that the game will end with the first round of tosses?
(2) (8 points) If all three coins are biased and have probability $1/4$ of landing heads, what is the probability that the game will end at the first round?

2. A simplified model for the movement of the price of a stock supposes that on each day the stock's price either moves up 1 unit with probability p or moves down 1 unit with probability $1 - p$. The changes on different days are assumed to be independent.

- (1) (5 points) What is the probability that after 2 days the stock will be at its original price?
(2) (6 points) What is the probability that after 3 days the stock's price will have increased by 1 unit?
(3) (7 points) Given that after 3 days the stock's price has increased by 1 unit, what is the probability that it went up on the first day?

3. Let X be a random variable with probability density function given by

$$f(x) = \begin{cases} cx^{-4}, & \text{if } x \geq 1; \\ 0, & \text{if } x < 1. \end{cases}$$

where $c > 0$ is some constant.

- (1) (5 points) Find c .
(2) (6 points) Calculate the probability $P(0.5 < X \leq 2)$.
(3) (7 points) Find the cumulative distribution function $F_X(x)$.

4. (10 points) Let $X \sim \mathcal{N}(0, 1)$ be a standard normal random variable, that is, the probability density function of X is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, -\infty < x < \infty.$$

Find the probability density function of $Y = e^X$.

5. Let X, Y be two independent random variables where X is uniformly distributed on $(0, 1)$ and Y has a density function

$$f_Y(y) = \begin{cases} \frac{1}{2} e^{-y/2}, & y > 0, \\ 0, & y \leq 0. \end{cases}$$

- (1) (6 points) Find the joint density function of (X, Y) .
 (2) (10 points) Consider a quadratic equation $a^2 + 2Xa + Y = 0$ in a . Find the probability that a admits a real value.

6. A random vector (X, Y) has a joint density function

$$f(x, y) = \begin{cases} be^{-(x+y)}, & 0 < x < 1, 0 < y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

- (1) (6 points) Find the value of b .
 (2) (8 points) Find the marginal density functions, $f_X(x)$ and $f_Y(y)$.
 (3) (10 points) Find the cumulative distribution function of $U = \max\{X, Y\}$.