1. Let $C \subset M_n(F)$ be the scalar matrices over F. Prove that C is the center of $GL_n(F)$, denoted by $Z\left(GL_n(F)\right)$.

2. Let G be a finite group, $H, K \leq G$. Prove: $|HK| = \frac{|H||K|}{|H \cap K|}$. (Hint: HK may not be a group).

3. Let N, H be two different maximal normal subgroup of G, then $N \cap H$ is a maximal normal subgroup of H(also N).

4. Let G be a finite group, $\varphi \in \mathrm{Aut}(G)$, let

$$I = \left\{ g \in G \mid \varphi(g) = g^{-1} \right\}$$

(1). Suppose $|I| > \frac{3}{4}|G|$, prove: G is abelian;
(2). Suppose $|I| = \frac{3}{4}|G|$, prove: $\exists H \leq G$ s.t. H is abelian and $[G : H] = 2$.

5. Prove that $\mathrm{Aut}\left(Z_2 \oplus Z_2\right) \simeq S_3$.

6. Let $H, K \trianglelefteq G, G/H, G/K$ are all soluble groups. Prove: $G/H \cap K$ is also a soluble group. (Hint: you can use the lemma from the last lesson.)