1. (18points) For $p = 2, 3, 5$ find $n_p(A_5)$ and $n_p(S_5)$

2. (20points) Find all conjugacy classes in the following groups:
(a). D_8;
(b). Q_8;
(c). A_4;
(d). $Z_2 \times S_3$.

3. (16points) (a). Identify all abelian groups with order 72.
(b). Let $|G| = pqr$, where p, q and r are primes with $p < q < r$. Prove that G has a normal Sylow subgroup for either p, q or r.

4. (20points) Let G be a group.
(a). If $|G| = 2n$ and n is odd, then G has a normal subgroup of index 2. Furthermore, G is solvable. (Hint: It is known that every finite group with odd order is solvable.)
(b). If each Sylow 2-subgroup of G is cyclic, then G is solvable. (Hint: prove that G has the unique normal subgroup of order m, where $G = 2^n m$ and m is odd.)

5. (12points) The set of all 3×3 real matrices A of the form:

$$H = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix},$$

where a, b and c are arbitrary real numbers is called the **Heisenberg Group**.
Determine the center $Z(H)$ of the Heisenberg group H. Show that the quotient group $H/Z(H)$ is commutative.

6. (14points) Prove **Zassenhaus Lemma**:
Let $H_1 \trianglelefteq H \leq G$, $K_1 \trianglelefteq K \leq G$, then:

$$H_1 (H \cap K) / H_1 (H \cap K_1) \cong K_1 (H \cap K) / K_1 (H_1 \cap K)$$