CMPS 203 – HM5

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*Quetiion*1.

Using Hoares rules, prove:

$$\{x = y\}\ x := x + 1;\ y := y + 1\ \{x = y\}$$

Proof. We can get this by Sequence Rule and Assign Rule.

$$\dfrac{x = y \Rightarrow x + 1 = y + 1 \qquad \dfrac{\{x + 1 = y + 1\}x := x + 1\{x = y + 1\} \qquad \{x = y + 1\}y := y + 1\{x = y\}}{\{x + 1 = y + 1\}\ x := x + 1;\ y := y + 1\ \{x = y\}}}{\{x = y\}\ x := x + 1;\ y := y + 1\ \{x = y\}}$$

*Quetiion*2.

Using Hoares rules, prove:

$$\{y = z\}while\ b\ do\ y := y - x\ \{\exists k.z = (y + k * x)\}$$

for an arbitrary boolean expression b.

Proof.

- (1) For the Precondition, It's easy to proof by Consequence Rule

$$\dfrac{\{y = z\} \Rightarrow \{z = y + 0 * x\} \qquad \{z = y + 0 * x\} \Rightarrow \{\exists k.z = (y + k * x)\}}{\{y = z\} \Rightarrow \{\exists k.z = (y + k * x)\}}$$

- (2) Otherwise, we can get below by Assign Rule and Sequence Rule.

$$\dfrac{\{\exists k.z = y - x + k * x\} \Rightarrow \{\exists k.z = y + (k - 1) * x\} \qquad \{\exists k.z = y - x + k * x\}y := y - x\{\exists k.z = y + k * x\}}{\{\exists k.z = y + (k - 1) * x\}y := y - x\{\exists k.z = (y + k * x)\}}$$

$$\dfrac{\{\exists k.z = y + (k - 1) * x\} \Rightarrow \{\exists k.z = y + k * x\} \qquad \{\exists k.z = y + (k - 1) * x\}y := y - x\{\exists k.z = y + k * x\}}{\{\exists k.z = (y + k * x)\}y := y - x\{\exists k.z = (y + k * x)\}}$$

- (3) From (1) and (2) and While Loop Rule and Consequence Rule, we can get

$$\dfrac{\dfrac{\{y = z\} \Rightarrow \{\exists k.z = (y + k * x)\} \qquad \{\exists k.z = (y + k * x)\}y := y - x\{\exists k.z = (y + k * x)\}}{\{y = z\}y := y - x\{\exists k.z = (y + k * x)\}}}{\{y = z\}while\ b\ do\ y := y - x\ \{\exists k.z = (y + k * x)\}}$$

Hence, $\{y = z\}while\ b\ do\ y := y - x\ \{\exists k.z = (y + k * x)\}$. for an arbitrary boolean expression b.