

# CMPS 203 – HM5

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*Question1.*

Using Hoares rules, prove:

$$\{x = y\} x := x + 1; y := y + 1 \{x = y\}$$

*Proof.* We can get this by Sequence Rule and Assign Rule.

$$\frac{x = y \Rightarrow x + 1 = y + 1 \quad \frac{\{x + 1 = y + 1\} x := x + 1 \{x = y + 1\} \quad \{x = y + 1\} y := y + 1 \{x = y\}}{\{x + 1 = y + 1\} x := x + 1; y := y + 1 \{x = y\}}}{\{x = y\} x := x + 1; y := y + 1 \{x = y\}}$$

*Question2.*

Using Hoares rules, prove:

$$\{y = z\} \text{while } b \text{ do } y := y - x \{ \exists k. z = (y + k * x) \}$$

for an arbitrary boolean expression b.

*Proof.*

- (1) For the Precondition, It's easy to proof by Consequence Rule

$$\frac{\{y = z\} \Rightarrow \{z = y + 0 * x\} \quad \{z = y + 0 * x\} \Rightarrow \{\exists k. z = (y + k * x)\}}{\{y = z\} \Rightarrow \{\exists k. z = (y + k * x)\}}$$

- (2) Otherwise, we can get below by Assign Rule and Sequence Rule.

$$\frac{\{\exists k. z = y - x + k * x\} \Rightarrow \{\exists k. z = y + (k - 1) * x\} \quad \{(\exists k. z = y - x + k * x) \wedge b\} y := y - x \{ \exists k. z = y + k * x \}}{\{(\exists k. z = y + (k - 1) * x) \wedge b\} y := y - x \{ \exists k. z = (y + k * x) \}}$$

$$\frac{\{\exists k. z = y + (k - 1) * x\} \Rightarrow \{\exists k. z = y + k * x\} \quad \{(\exists k. z = y + (k - 1) * x) \wedge b\} y := y - x \{ \exists k. z = y + k * x \}}{\{(\exists k. z = (y + k * x)) \wedge b\} y := y - x \{ \exists k. z = (y + k * x) \}}$$

- (3) From (1) and (2) and While Loop Rule and Consequence Rule, we can get

$$\frac{\{y = z\} \Rightarrow \{\exists k. z = (y + k * x)\} \quad \{(\exists k. z = (y + k * x)) \wedge b\} y := y - x \{ \exists k. z = (y + k * x) \}}{\frac{\{y = z \wedge b\} y := y - x \{ \exists k. z = (y + k * x) \}}{\{y = z\} \text{while } b \text{ do } y := y - x \{ \exists k. z = (y + k * x) \}}}$$

Hence,  $\{y = z\} \text{while } b \text{ do } y := y - x \{ \exists k. z = (y + k * x) \}$ . for an arbitrary boolean expression b.