MCMC

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Markov chain

- Definition: A random process that undergoes transitions from one state to another on a state space

https://setosa.io/blog/2014/07/26/markov-chains/

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One way to simulate this weather would be to just say "Half of the days are rainy. Therefore, every day in our simulation will have a fifty percent chance of rain." This rule would generate the following sequence in simulation:

S R R S R S R R S R S S S R S R S S R S R S S R S S S R R S S S R

Markov chain

- Markov property: A Markov chain possesses a property that is characterised as "memoryless": the probability of the next state depends only on the current state and not on the sequence of events that preceded it.
- Markov property defined as:

$$\Pr(X_{n+1} = x | X_1 = x_1, X_2 = x_2, \cdots, X_n = x_n) \\ = Pr(X_{n+1} = x | X_n = x_n)$$

Ergodicity

- This means that during numerous iterations, the chain will explore every point (or possible state) and will do so proportionally to its probability.
- To be considered ergodic, the Markov chain must be
  - irreducible: for every state there is a positive probability of moving to any other state
  - aperiodic: the chain must not get trapped in cycles

Reversibility: reversible chains

- For a stationary distribution π and a transition matrix P, the reversibility condition can be written as
  $\pi(x)P(x,y) = \pi(y)P(y,x)$, for all $(x,y) \in S$

Link to Metropolis-Hastings and Gibbs

In his famous paper from 1953, Metropolis showed how to construct a Markov chain with stationary distribution π such that

$$\pi(x) = p_x, x \in S$$

Here the first step to obtaining the stationary distribution of a Markov chain is to prove that the probabilities of a distribution satisfy the reversibility condition.