Program Structures and Algorithms – Assignment 4

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Tasks:

1. Implement a height-weighted Quick Union with Path Compression class (UF_HWQUPC) and ensure all unit tests are successful.
2. Develop a UF client that takes a number n and generates random pairs of integers between 0 and n-1, counting the number of connections until all sites are connected.
3. Investigate the relationship between the number of objects (n) and the number of pairs (m) required to reduce the number of components to 1, and provide justification based on observations.

Relationship Conclusion: $m = C \times n\ ln(n)$

Explanation:

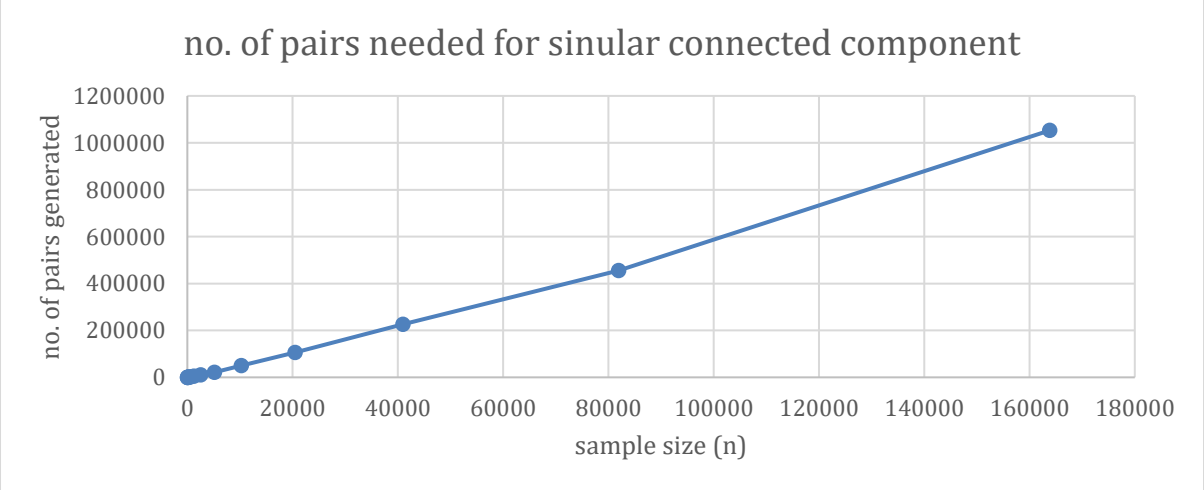
1. The number of connections required to connect all n objects is equal to $(n - 1)$. This is because each connection reduces the number of connected components by one.
2. The probability of selecting two unconnected objects in each iteration of the algorithm is given by $p = 1/n * (n - 1)/n = (n - 1)/n^2$. This is because there are (n - 1) unconnected objects, and the probability of selecting each object is 1/n.
3. The expected number of connections in each iteration of the algorithm is given by $m/n = p = (n - 1)/n^2$.
4. The expected number of connections required to connect all n objects is given by $m = n \times (n - 1)/n^2 = (n - 1)/n$.
5. Approximating $(n - 1)/n$ as $ln(n)$, we get $m = n * ln(n)$.
6. Finally, multiplying by a constant C account for the constant factors involved in the algorithm and can be estimated experimentally. Hence, the final formula is $m = C \times n\ ln(n)$.

Code:

Submitted to GitHub Repository: https://github.com/sharunkumar-ks/INFO6205/pull/3/files

Graphical Representation:

Complete data is available in the **Union Find with Path Compression.xlsx** file.



Unit Test Screenshots: