Illinois Institute of Technology
Department of Computer Science

Homework Assignment 2

CS 535 Design and Analysis of Algorithms
Fall Semester, 2016

Due: Thursday, September 8, 2016

Remember the Honesty Pledge!

The first two exercises explore the activity selection problem of CLRS3 section 16.1.

1. There are alternatives to CLRS3's greedy choice of the earliest finishing time. For each of the following alternatives, prove or disprove that it constructs an optimal schedule (assume ties are broken arbitrarily) and briefly outline the implementation costs:

   (a) Choose the job that ends latest.

   (b) Choose the job that starts first.

   (c) Choose the job of shortest duration.

   (d) **PhD Qualifying Exam Section Problem 2.**
   Discard the job with the most conflicts with other jobs, stopping when there are no more conflicts.

2. Consider a weighted form of the activity selection problem in which each of the n activities is weighted by its importance (the weight is unrelated to the starting and finishing times and to the duration). We want to choose a subset of non-conflicting activities with the highest total weight.

   (a) Prove that the heuristic of choosing the earliest finishing time does not always result in an optimal schedule.

   (b) Find an $O(n^2)$ algorithm to compute an optimal schedule.

3. Your new hybrid car that uses fuel extremely efficiently, but can only travel 100 miles on a single battery. The cars fuel is stored in a single-use battery, which must be replaced after at most 100 miles. The actual fuel is virtually free, but the batteries are expensive and can only be installed by licensed battery-replacement technicians. Thus, even if you decide to replace your battery early, you must still pay full price for the new battery to be installed. Moreover, because these batteries are in high demand, no one can afford to own more than one battery at a time.

   You are trying to get from San Francisco to New York City on the new Inter-Continental Super-Highway, which runs in a direct line between these two cities. There are several fueling stations along the way; each station charges a different price for installing a new battery. Before you start your trip, you carefully print the Wikipedia page listing the locations and prices of every fueling station on the way. Given this information, how do you decide the best places to stop for fuel? More formally, suppose you are given two arrays $D[1 \ldots n]$ and $C[1 \ldots n]$, where $D[i]$ is the distance from the start of the highway to the ith station, and $C[i]$ is the cost to replace your battery at the ith station. Assume that your trip starts and ends at fueling stations (so $D[1] = 0$ and $D[n]$ is the total length of your trip), and that your car starts with an empty battery (so you must install a new battery at station 1).

(a) Describe, prove correct, and analyze a greedy algorithm to find the minimum number of refueling stops needed to complete your trip. Prove that your algorithm is correct

(b) Show that your greedy algorithm in part (a) does not minimize the total cost of travel.

(c) Describe an efficient algorithm to compute the locations of the fuel stations you should stop at that does minimize the total cost of travel.