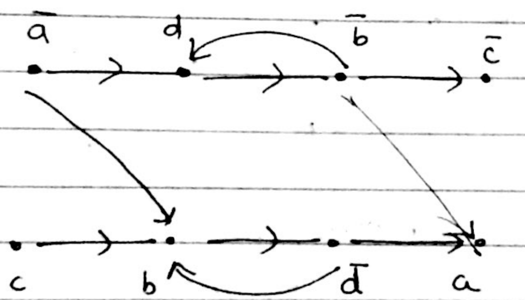
2. Consider a di-graph $D = (V, A)$ where each literal x can be represented either True or False. having vertices (V_x & $V_{\bar{x}}$).

For each clause $a \vee b$ we need to add two arcs to A having vertices $(V_{\bar{a}}, V_b)$, $(V_{\bar{b}}, V_a)$

x and $\bar{x}$ are in same strong connected component if there is a path from x to $\bar{x}$ & $\bar{x}$ to x. In this case 2-CNF-SAT is satisfiable if and only if there isn't any literal x such that $x = \text{TRUE} \wedge \bar{x} = \text{TRUE}$ in D.

* Algorithm,

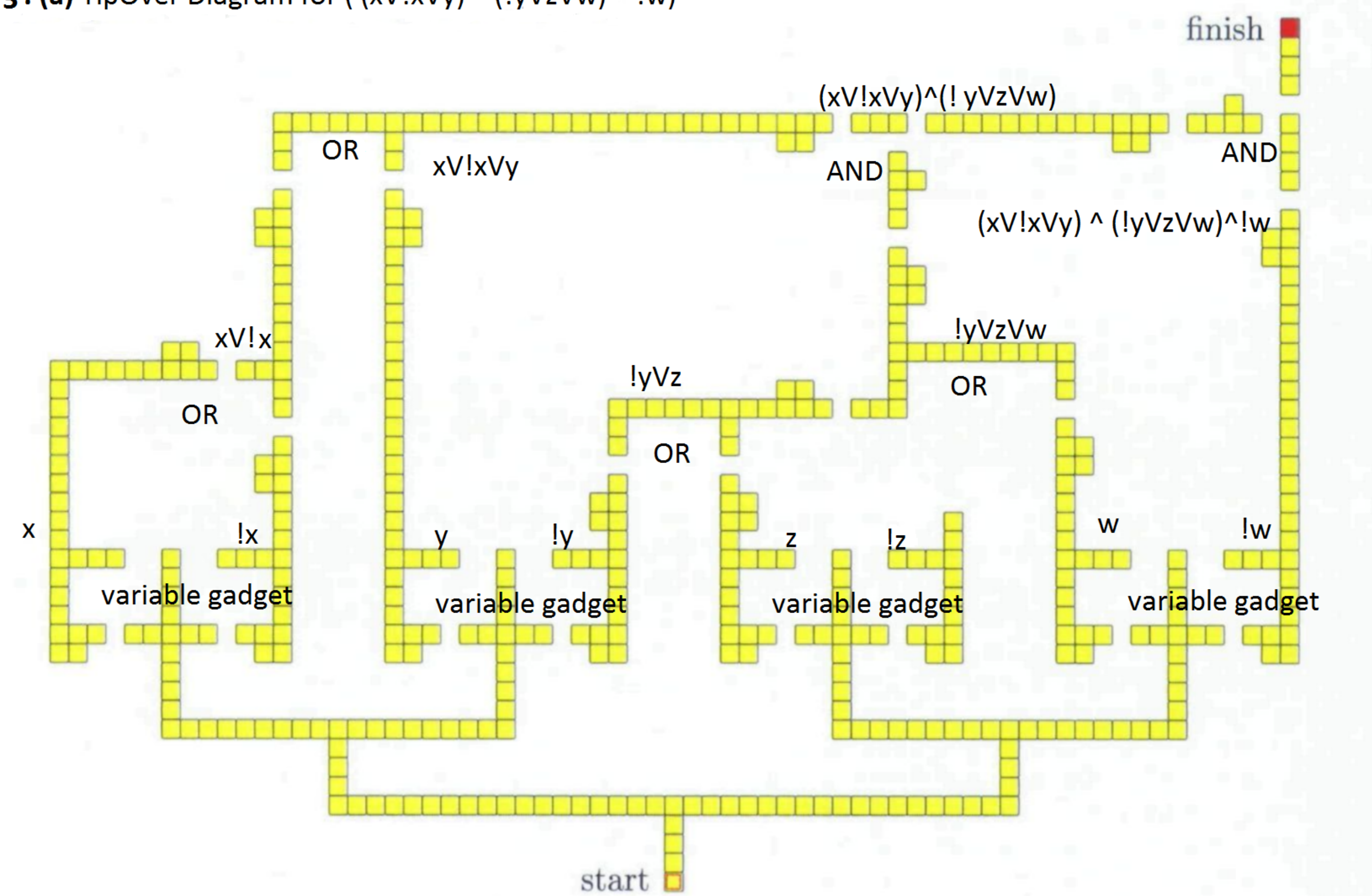
1) Construct graph D as below
2) Find strong connected component of graph D.
3) For each strong connected component, check that x and $\bar{x}$ are both in it
4) Return False if found some or else return TRUE.

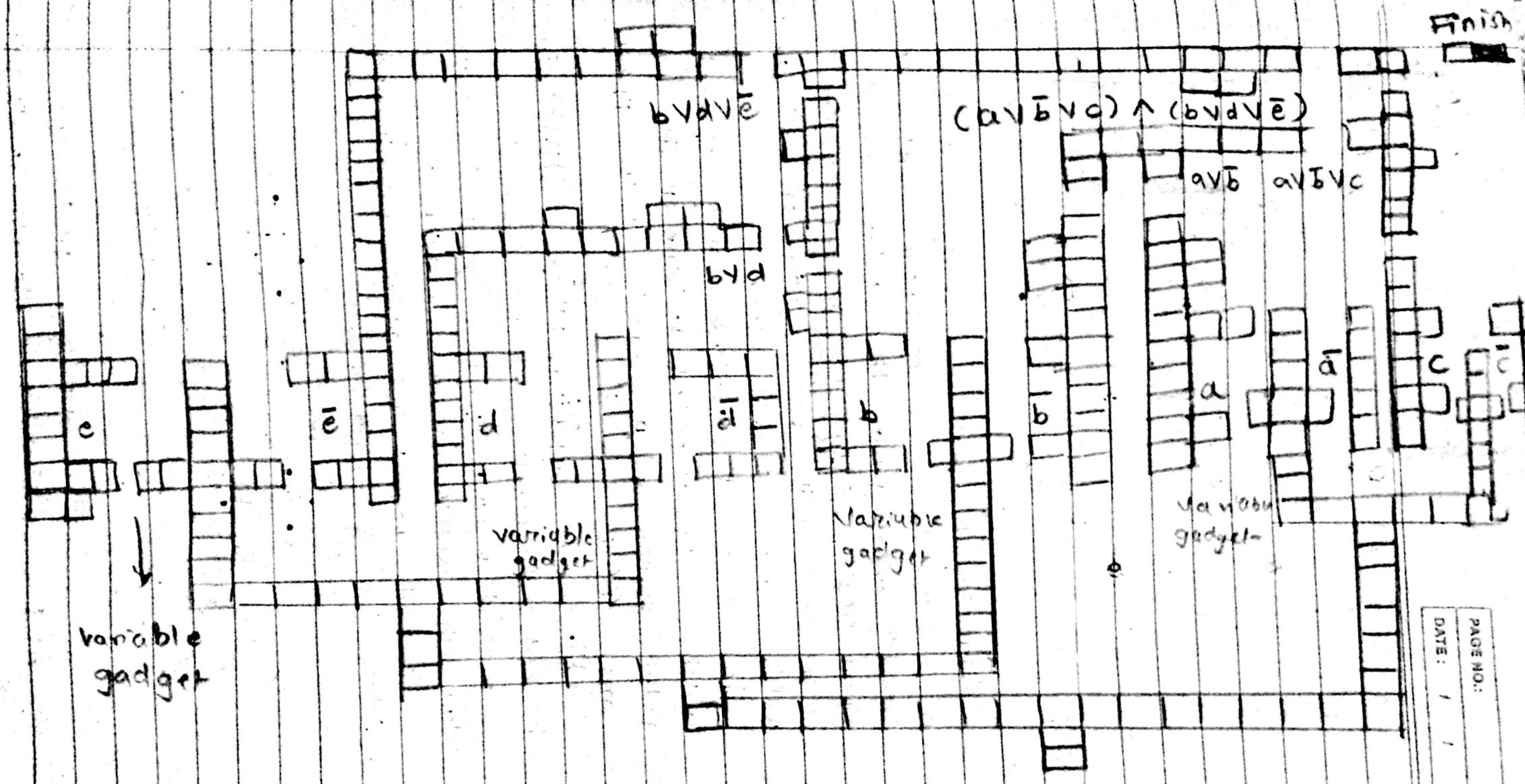


Graph constructed for set of clauses ⟸
$C = \{a \vee b, b \vee \bar{c}, \bar{b} \vee \bar{d}, b \vee d, d \vee a\}$

- Time complexity for graph D is $O(m+n)$ where m are clauses and n are literals.
- In order to find strong connected components it costs $O(m+n)$ & to check whether x and $\bar{x}$ are in one strong connected component it costs $O(n)$. Therefore total cost needed is $O(m+n)$

3. (a) TipOver Diagram for ((xV!xVy) ^ (!yVzVw) ^ !w)

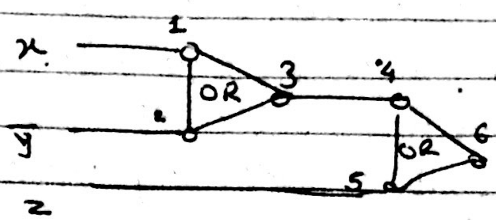




3 (b) Tipover diagram for 3 SAT expression $(a \vee \bar{b} \vee c) \wedge (b \vee d \vee \bar{e})$

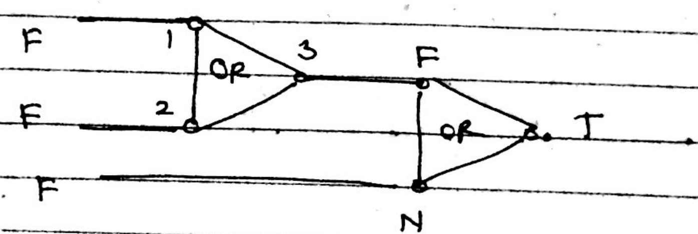
4. The claim states that nodes present in OR logic are colorable "T" if and only if at least one one of the input is colored "T":

Consider below OR graph. as in notes;



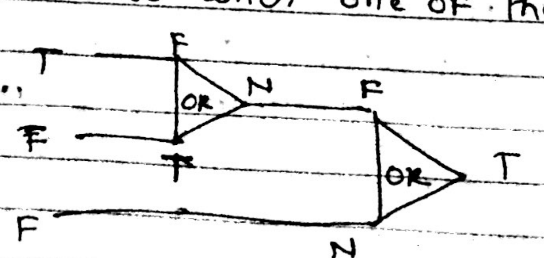
To prove the claim is right, we have to prove output at node 6 is "T", atleast one of input $x, \bar{y}, z$ should be "T".

Consider a case when all 3 inputs $x, \bar{y}, z$ be "F", then we get following colors.



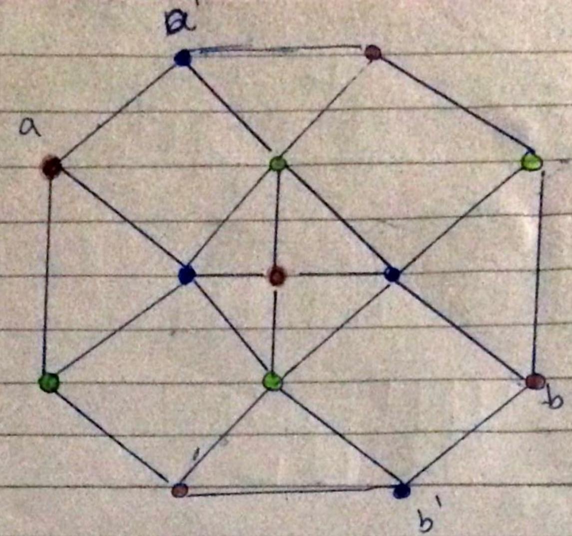
In the above scenario, nodes: 1, 2, 3 should be of different colors, but that's not possible as all inputs are "F". So nodes, 1, 2 can't be "T" or "N". Therefore, their isn't any coloring. Likewise considering remaining possiblities we can have different coloring combinations. (Total 2^3 coloring combinations considering 3 inputs $x, \bar{y}, z$) either "T", "F".

In case when one of the inputs be "T"



4: The output generated will be "T". Similarly will be the case in different coloring combinations. Hence the claim is true.

5.

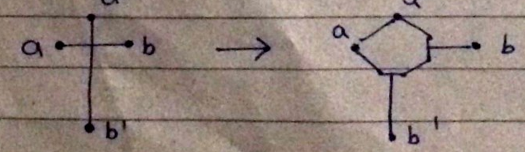


To determine whether a planar graph is 3-colorable is an NP complete problem consider above graph. This gadget has a symmetric property, as you can see the cross points (like clock position of 2:00 color will be same as 8:00 color).

Thus a crossed edge (a,b) can be replaced within as a embedded to one of the gadget's corner and b connected to the opposite corner of a.

The whole computing takes $O(|E|)^2$ time. Also a & b cannot be assigned with the same color based on symmetric property of the gadget. Hence the resulting graph is planar.

With the help of this gadget we can show reduction of 3-colorability for a graph.



Replacement of cross with gadget

As 3-colorable is NP complete for general graphs, we can say/prove 3-colorable is NP complete for planar graphs.