Illinois Institute of Technology
Department of Computer Science

Homework Assignment 7

CS 535 Design and Analysis of Algorithms
Fall Semester, 2016

Due: Thursday, October 13, 2016

Remember the Honesty Pledge!

1. Prove that $A_3(j) > \text{tower}(j)$, where

$$\text{tower}(n) = \begin{cases} 2^{\text{tower}(n-1)} & \text{if } n > 0, \\ 1 & \text{if } n = 0. \end{cases}$$

2. Calculate the amortized costs of `Make-Set`, `Find-Set`, and `Union` using the following potential functions:

   (a) $\Phi = \sum_{\text{nodes } x} \texttt{depth}(x)$.
   (b) $\Phi = \sum_{\text{nodes } x} \texttt{height}(x)$.

3. Do the amortized analysis of the Union-Find data structure with rank and *path-halving path compression* in which the grandparent of each node is made its parent during a find operation.

4. **PhD Qualifying Exam Section Problem 8.**Consider the following game. I choose a positive integer n and keep it secret; your goal is to discover this integer. We play the game in rounds. In each round, you write a list of at most n integers on the blackboard. If you write more than n numbers in a single round, you lose. (Thus, in the first round, you must write only the number 1; do you see why?) If n is one of the numbers you wrote, you win the game; otherwise, I announce which of the numbers you wrote is smaller or larger than n, and we proceed to the next round. For example:

You	Me
1	It's bigger than 1.
4, 42	It's between 4 and 42.
8, 15, 16, 23, 30	It's between 8 and 15.
9, 10, 11, 12, 13, 14	It's 11; you win!

   Describe a strategy that allows you to win in $O(\alpha(n))$ rounds.