Illinois Institute of Technology
Department of Computer Science

Homework Assignment 13

CS 535 Design and Analysis of Algorithms
Fall Semester, 2016

Due: Wednesday, November 30, 2016

Remember the Honesty Pledge!

1. Prove inequality (3) on page 1 of the notes with the analysis of the nearest-neighbor heuristic for the TSP. (*Hint*: For n even it is a special case of inequality (2). What do you do for odd n?)

2. One criticism of the nearest-neighbor TSP heuristic is that the last edge added (from the last city back to the starting city) can be very long. How much effect can that last edge have on the ratio $HEUR/OPT$?

3. (a) Prove that given an arbitrary symmetric cost matrix C of inter-city distances, it is possible to force the triangle inequality to hold by adding a sufficiently large value to every element of C.

   (b) Does this mean that the bound on the quality of the closest-insertion heuristic holds for symmetric cost matrices C for which the triangle inequality *does not* hold?

4. The *cheapest insertion* algorithm for the TSP starts with an arbitrary city and repeatedly inserts another city into the existing tour, just like the closest-insertion algorithm analyzed in class on November 19. However, instead of selecting the closest city to the tour to be inserted, it inserts the city whose insertion is cheapest—that is, city k (not on the current tour) is inserted between adjacent cities i and j (on the current tour) with i, j, and k chosen to minimize $C_{ik} + C_{kj} - C_{ij}$.

   (a) Prove that if C is a symmetric cost matrix that satisfies the triangle inequality,

   $$\frac{\text{cost of cheapest insertion tour}}{\text{cost of optimal tour}} < 2.$$

   (b) How many operations, as a function of number of cities, are needed to implement this algorithm?

   (c) How far from optimal can the cheapest insertion tour actually be?