Illinois Institute of Technology
Department of Computer Science

Homework Assignment 9

CS 535 Design and Analysis of Algorithms
Fall Semester, 2016

Due: Thursday, October 27, 2016

Remember the Honesty Pledge!

1. What would happen in RECURSIVE-FFT (page 911) if line 4 was changed to "$\omega_n = e^{2\pi qi/n}$"? That is, find a simple relation between the output of RECURSIVE-FFT obtained with this change and the results obtained with the original procedure (that is, when $q = 1$).

2. Problem 30.3-3 on page 920.

3. This problem continues the subject of Problem 30.3-3. That problem examines how to economize in the computation of the twiddle factors, but not how to improve round-off errors—a significant subject completely ignored by the text except for a brief footnote on page 902. The round-off error in a twiddle factor depends on how many multiplications are used to compute it. Algorithms RECURSIVE-FFT (page 911) and ITERATIVE-FFT (page 917) compute the twiddle factors "on the fly".

   (a) In ITERATIVE-FFT, which twiddle factor(s) are computed with the most multiplications? How many multiplications is that?

   (b) By precomputing a table of all needed twiddle factors, we can reduce the number of multiplcations needed for any twiddle factor to $O(\log n)$. As in the text, assume that n is a power of 2, $n = 2^k$, and let ω_n be the principle nth root of unity, $\omega_n = e^{2\pi i/n}$. Define

   $$\alpha_r = e^{2\pi i/2^r},$$

   so that $\alpha_1 = -1$, $\alpha_2 = i$, $\alpha_3 = (1+i)/\sqrt{2}$, $\ldots$, $\alpha_k = \omega_n$. Then, if $\alpha_r = x_r + iy_r$, show that $\alpha_{r+1} = x_{r+1} + iy_{r+1}$, where

   $$x_{r+1} = \sqrt{\frac{1+x_r}{2}}, \qquad y_{r+1} = \frac{y_r}{2x_{r+1}}.$$

   (c) Show how, using the binary representation of i, ω_n^i can be computed by a product of at most k of the α_rs. Explain why this scheme uses $O(\log n)$ multiplications for each twiddle factor.

   (d) Give modified versions of algorithms RECURSIVE-FFT and ITERATIVE-FFT that use the twiddle factors as precomputed in part (b) instead of computing them "on the fly".

4. **PhD Qualifying Exam Section Problem 10.** Problem 30.2-8 on pages 914–915.