STAT 410 - Section 1 - Fall 2021 Homework #04

Sharvi Tomar

TOTAL POINTS

10 / 10

QUESTION 1

4 10 pts

1.1 4r 1 / 1

✓ **- 0 pts Correct**

- 0.5 pts Incorrect setup for E(Y^2 | X = x)

- 0.5 pts Incorrect setup or answer for V(Y^2 | X = x)

1.2 4s 3.5 / 3.5

✓ **- 0 pts Correct**

- 0.5 pts Mistake in splitting the support of W

(0<w<5, 5<w<6, 6<w<8)

- 0.5 pts One wrong integral limit in 0<w<5

- 0.5 pts One wrong integral limit in 5<w<6

- 0.5 pts One wrong integral limit in 6<w<8

1.3 4tu 3 / 3

✓ **- 0 pts Correct**

- 1 pts t) Missing one part

- 0.5 pts t) Answer not correct

- 0.5 pts u) Should add them together

- 0.5 pts u) Not correct

1.4 4v 2.5 / 2.5

✓ **- 0 pts Correct**

- 1 pts Incorrect initial setting(support of V..)

- 0.5 pts Incorrect setup when 0<v<3

- 0.5 pts Incorrect answer when 0<v<3

- 0.5 pts Incorrect setup when v>3

- 0.5 pts Incorrect answer when v>3

- 2.5 pts No valid submission

e) $\text{Var}(Y|X) = E(Y^2|X) - [E(Y|X)]^2$

$$E(Y^2|X) = \int_{-\infty}^{5-x/2} y^2 f_{Y|X}(y|x)\,dy$$

$$= \int_{0}^{} y^2 \frac{(12x+8y)}{5(2+x)(10-x)}\,dy$$

$$= \frac{(10-x)^2(3x+10)}{40(x+2)} \qquad 0<x<6$$

$$E(Y|X) = \frac{(20+7x)(10-x)}{30(2+x)}$$

$0<x<6$

$$\text{Var}(Y|X) = \frac{(10-x)^2(3x+10)}{40(x+2)} - \left[\frac{(20+7x)(10-x)}{30(2+x)}\right]^2$$

1.1 4r **1 / 1**

✓ **- 0 pts** **Correct**

- 0.5 pts Incorrect setup for E(Y^2 | X = x)

- 0.5 pts Incorrect setup or answer for V(Y^2 | X = x)

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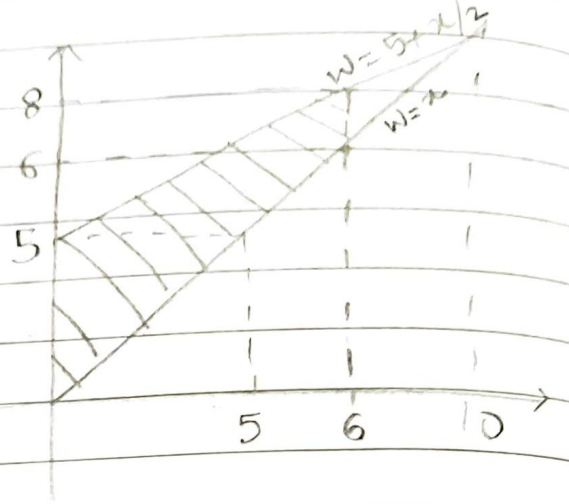
5) $W = X + Y$

$Y = W - X$

$0 \le W - X \le 5 - X/2$

$X \le W \le 5 + X/2$



Case-1 $0 \le W < 5$

$$f_W(w) = \int_0^w \frac{3x + 2(w-x)}{240}\,dx$$

$$= w^2/96$$

Case-2 $5 \le W < 6$

$$f_W(w) = \int_{2w-10}^{w} \frac{3x + 2(w-x)}{240}\,dx$$

$$= \frac{-7w^2 + 80w - 100}{480}$$

Case-3 $6 \le W \le 8$

$$f_W(w) = \int_{2w-10}^{b} \frac{3x + 2(w-x)}{240}\,dx$$

$$= \frac{-3w^2 + 26w - 16}{120}$$

$$f_W(w) = \begin{cases} w^2/96 & 0 \le w < 5 \\ \dfrac{-7w^2 + 80w - 100}{480} & 5 \le w < 6 \\ \dfrac{-3w^2 + 26w - 16}{120} & 6 \le w \le 8 \end{cases}$$

1.2 4s **3.5 / 3.5**

✓ **- 0 pts** **Correct**

- 0.5 pts Mistake in splitting the support of W (0<w<5, 5<w<6, 6<w<8)

- 0.5 pts One wrong integral limit in 0<w<5

- 0.5 pts One wrong integral limit in 5<w<6

- 0.5 pts One wrong integral limit in 6<w<8

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t) $T = 100x + 200y$

$y = \frac{t}{200} - \frac{x}{2}$

$0 \le x \le 6 \qquad 0 \le y \le 5 - x/2$

$0 \le \frac{t}{200} - \frac{x}{2} \le 5 - \frac{x}{2}$

$100x \le t \le 1000$

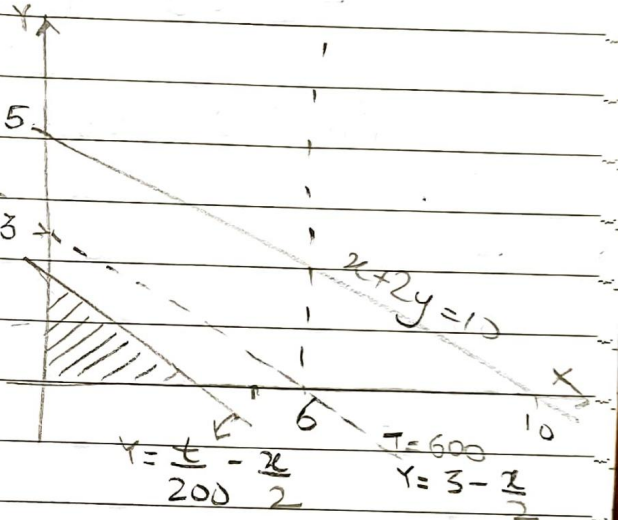
Case-1 $0 \le T \le 600$

$$F_T(t) = \int_0^{t/100} \int_0^{\frac{t}{200} - \frac{x}{2}} \frac{3x+2y}{240} \, dy \, dx$$

$$= \frac{t^3}{10^9} \times 1.38889$$

$$f_t(t) = F_T'(t)$$

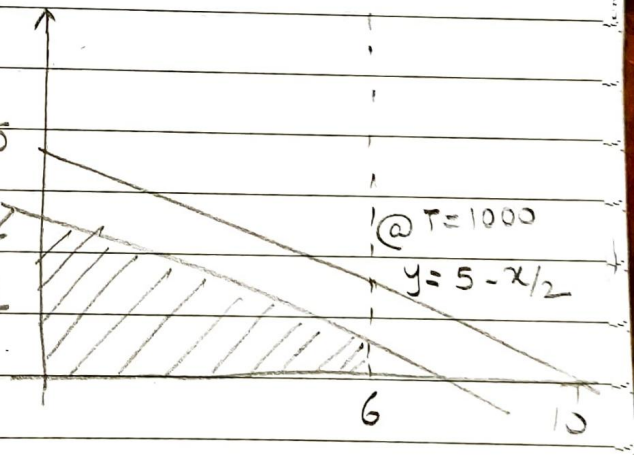
$$= \frac{t^2}{24 \times 10^7}$$



Case-2 $600 \le T \le 1000$

$$F_T(t) = \int_0^6 \int_0^{\frac{t}{200} - x/2} \frac{3x+2y}{240} \, dy \, dx$$

$$= \frac{t^2}{16 \times 10^5} + \frac{3t}{4000} - \frac{3}{8}$$



$$f_t(t) = F_T'(t)$$

$$= \frac{t}{8 \times 10^5} + \frac{3}{4000}$$

$$F_T(t) = \begin{cases} 0 & t \le 0 \\ \frac{t^3}{72 \times 10^7} & 0 \le t \le 600 \\ \frac{t^2}{16 \times 10^5} + \frac{3t}{4000} - \frac{3}{8} & 600 < t \le 1000 \\ 1 & t > 1000 \end{cases}$$

a) $E(T) = \int_{-\infty}^{\infty} t f_T(t)\, dt$

Using $f_T(t)$ from t)

$$= \int_{0}^{600} t\left(\frac{t^2}{24\times10^7}\right) dt + \int_{600}^{1000} t\left(\frac{t}{8\times10^5} + \frac{3}{4000}\right) dt$$

$$= 135 + \frac{1700}{3}$$

$E(T) = 701.6667$

1.3 4tu 3 / 3

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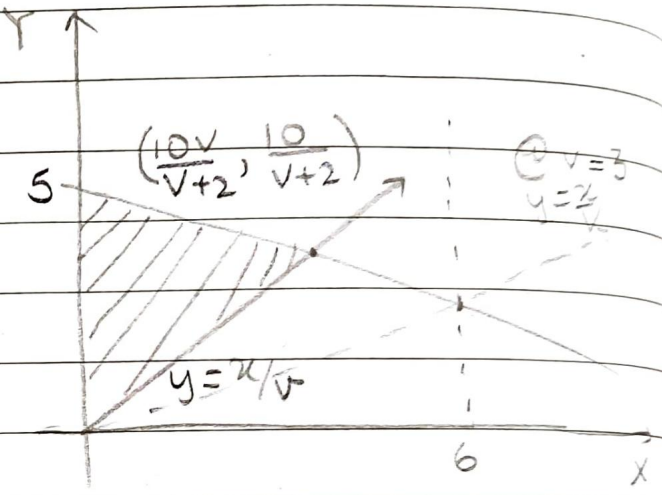
v) $V = \frac{X}{Y}$

$x>0,\ y>0$
$\Rightarrow v>0$

$F_V(v) = P\left(\frac{X}{Y} \le v\right)$
$= P(Y \ge X/v)$

Case-1 $0 < v \le 3$

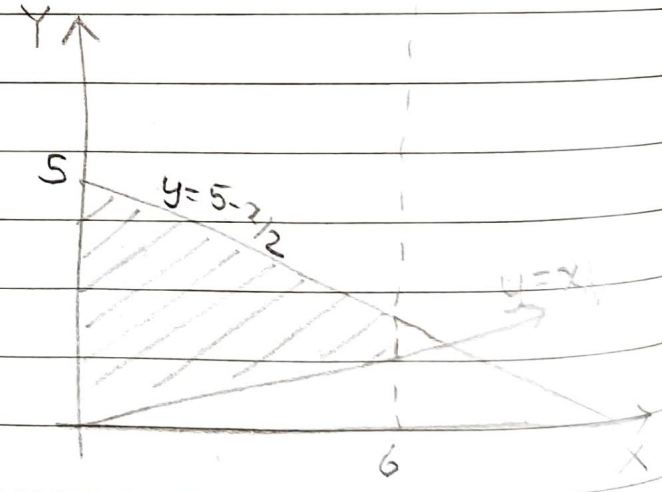
$$f_V(v) = \int_0^{\frac{10v}{v+2}} \int_{x/v}^{5-x/2} \frac{3x+2y}{240}\, dy\, dx$$



$$= \frac{25(v^2+v)}{18(v+2)^2}$$

Case 2 $3 < v < \infty$

$$F_V(v) = 1 - \int_0^6 \int_0^{x/v} \frac{3x+2y}{240}\, dy\, dx$$



$$= 1 - \left(\frac{9v+3}{10v^2}\right)$$

$$F_V(v) = \begin{cases} \frac{25(v^2+v)}{18(v+2)^2} & 0 < v \le 3 \\ 1 - \left(\frac{9v+3}{10v^2}\right) & 3 < v < \infty \end{cases}$$

1.4 4v **2.5 / 2.5**

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