STAT 410 - Section 1 - Fall 2021 Homework #03

Sharvi Tomar

TOTAL POINTS

10 / 10

QUESTION 1

4 10 pts

1.1 4hi **1.5 / 1.5**

✓ **- 0 pts** **Correct**

- 0.5 pts h) Answer not correct

- 0.5 pts i) Answer not correct

1.2 4jk **1.5 / 1.5**

✓ **- 0 pts** **Correct**

- 0.5 pts j) Answer not correct

- 0.5 pts k) Answer not correct

1.3 4l **1 / 1**

✓ **- 0 pts** **Correct**

- 0.5 pts Wrong limits in the integral for P(Y>3.6, X>1.6)

- 0.5 pts Wrong limits in the integral for P(X>1.6)

1.4 4mn **2 / 2**

✓ **- 0 pts** **Correct**

- 0.5 pts Wrong limits of integral in 4n

- 0.5 pts Wrong limits of integral in 4m

1.5 4op **2 / 2**

✓ **- 0 pts** **Correct**

- 0.5 pts (o) Incorrect setup

- 0.5 pts (o) Incorrect answer

- 0.5 pts (p) Incorrect setup

- 0.5 pts (p) Incorrect answer

1.6 4q **2 / 2**

✓ **- 0 pts** **Correct**

- 0.5 pts 0<y<2 Incorrect setup

- 0.5 pts 0<y<2 Incorrect answer

- 0.5 pts 2<y<5 Incorrect setup

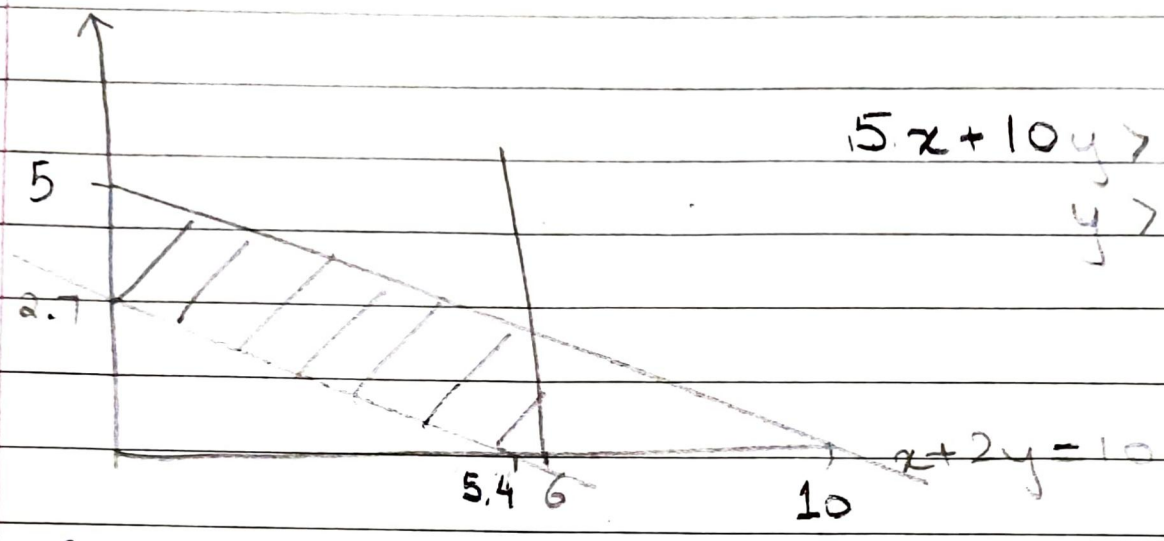
- 0.5 pts 2<y<5 Incorrect answer

- 1 pts Incorrect by summing up the two cases.

- 1 pts Incorrect by taking wrong support for y

410 - HW3

h)



$5x + 10y > 27$

$y > 27 - 0.5x$

$$P(100x + 200y > 540)$$

$$= 1 - \int_0^{5.4} \int_0^{2.7-0.5x} \frac{3x+2y}{240} \, dy \, dx$$

$$= 1 - \int_0^{5.4} \frac{1}{240} \left[3xy + y^2\right]_0^{2.7-0.5x} dx$$

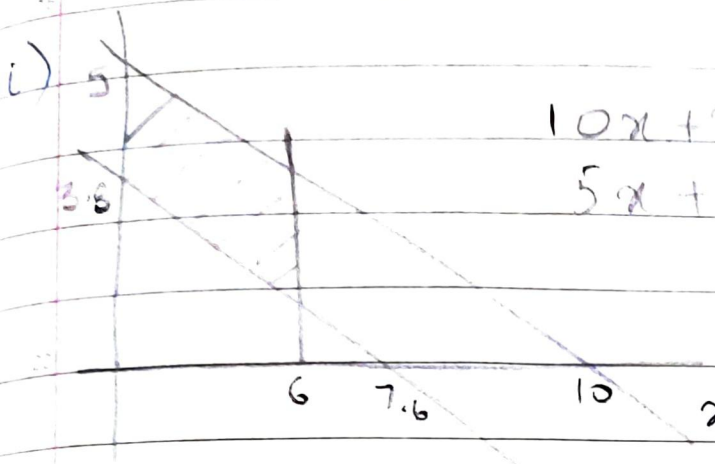
$$= 1 - \int_0^{5.4} \frac{(-1.25x^2 + 5.4x + 7.29)}{240} dx$$

$$= 1 - \frac{1}{240} \left[\frac{-1.25x^3}{3} + \frac{5.4x^2}{2} + 7.29x\right]_0^{5.4}$$

$$= 1 - 0.2187$$

$$= 0.7813$$

i)



$P(100x + 200Y > 760)$

$10x + 20y = 76$

$5x + 10y = 38$

$y = 3.8 - 0.5x$

$x + 2y < 10$

$y = 5 - 0.5x$

$P(100x + 200Y > 760)$

$$= \int_0^6 \int_{3.8-0.5x}^{5-0.5x} \left(\frac{3x+2y}{240}\right) dy \, dx$$

$$= \int_0^6 \left[\frac{3xy + y^2}{240} \right]_{3.8-0.5x}^{5-0.5x} dx$$

$$= \int_0^6 \left(\frac{2.4x + 10.56}{240}\right) dx = \left[\frac{1.2x^2 + 10.56x}{240} \right]_0^6$$

$$= 0.444$$

1.1 4hi **1.5 / 1.5**

✓ **- 0 pts** **Correct**

- 0.5 pts h) Answer not correct

- 0.5 pts i) Answer not correct

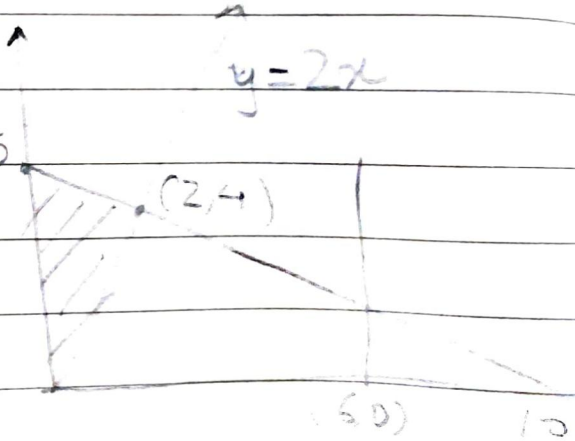
j) $P(X < 0.5Y)$

$$= \int_0^2 \int_{2x}^{5-x/2} \left(\frac{3x+2y}{240}\right) dy\, dx$$

$$= \int_0^2 \left[\frac{3xy + y^2}{240}\right]_{2x}^{5-x/2} dx$$

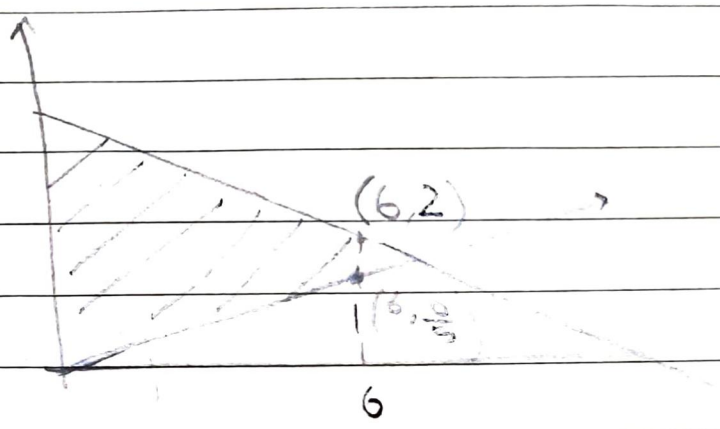
$$= \int_0^2 \frac{-11.25x^2 + 10x + 25}{240} dx$$

$$= \left[\frac{-11.25x^3}{3} + 5x^2 + 25x\right]_0^2 \frac{1}{240} = 0.1666.$$



k) $P(X<5 \cap Y)$

$$= 1 - \int_6^6 \int_0^{2/5} \left(\frac{3x+2y}{240}\right) dy\, dx$$



$$= 1 - \int_0^6 \left[\frac{3x^2y + y^2}{240}\right]_0^{2/5} dx$$

$$= 1 - \int_0^6 \frac{x^2}{375}\, dx$$

$$= 1 - \left[\frac{x^3}{3 \times 375}\right]_0^6 = 1 - 0.192 = 0.808$$

1.2 4jk **1.5 / 1.5**

✓ **- 0 pts** **Correct**

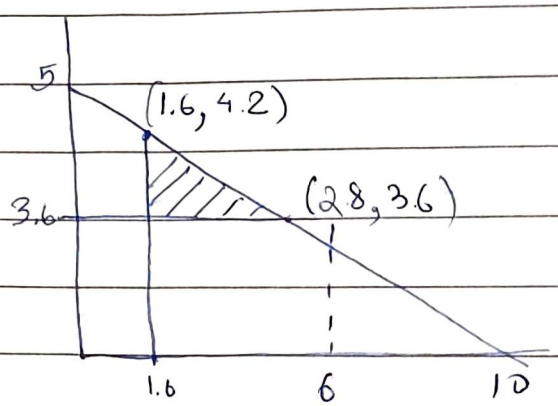
- 0.5 pts j) Answer not correct

- 0.5 pts k) Answer not correct

l) $P(Y > 3.6 \mid X > 1.6)$

$$= \int_{3.6}^{4.2} \int_{1.6}^{10-2y} \frac{(3x+2y)}{240} \, dx \, dy$$

$$\frac{\int_{1.6}^{6} \frac{20+8x-x^2}{192} \, dx}{0.78711}$$



$$= \int_{3.6}^{4.2} \frac{1}{240} \left[\frac{3x^2}{2} + 2xy \right]_{1.6}^{10-2y} dy$$

$$\frac{\frac{1}{192} \left[20x + \frac{8x^2}{2} - \frac{x^3}{3} \right]_{1.6}^{6}}{0.78711}$$

$$= \int_{3.6}^{4.2} \frac{(2y^2 - 43.2y + 146.16)}{240} \, dy$$

$$= \frac{\frac{1}{240} \left[\frac{2y^3}{3} - \frac{43.2y^2}{2} + 146.16y \right]_{3.6}^{4.2}}{0.78711}$$

$$= \frac{0.0204}{0.78711} = 0.025917$$

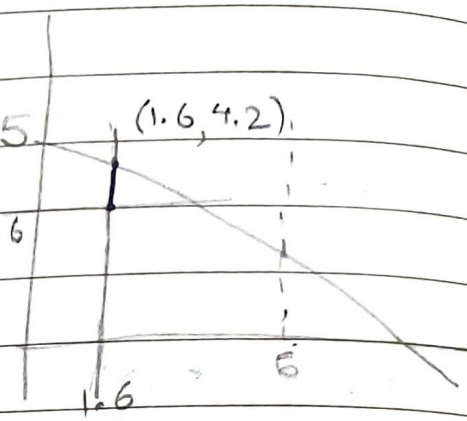
1.3 4l **1 / 1**

✓ **- 0 pts** **Correct**

- 0.5 pts Wrong limits in the integral for P(Y>3.6, X>1.6)

- 0.5 pts Wrong limits in the integral for P(X>1.6)

m) $P(Y>3.6 \mid X=1.6)$



$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} \rightarrow \text{from HW-2}$$

$$= \left(\frac{3x+2y}{240}\right)\left(\frac{192}{20+8x-x^2}\right) \quad ; \quad 0 \le y \le 5 - x/2$$

$$= \frac{4}{5}\left(\frac{3x+2y}{20+8x-x^2}\right)$$

$$P(Y>3.6 \mid X=1.6) = \int_{3.6}^{4.2} \frac{4}{5}\left(\frac{3x+2y}{20+8x-x^2}\right) dy \quad @\, x=1.6$$

$$= \frac{4}{5(20+8x-x^2)} \left[3xy+y^2\right]_{3.6}^{4.2}$$

$$= \frac{4}{5}\left(\frac{4.68+1.8x}{20-x^2+8x}\right) \quad \text{put } x=1.6$$

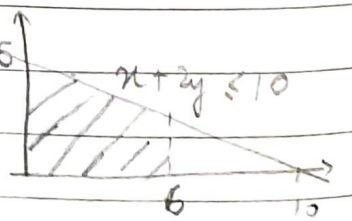
$$= \frac{4}{5}\left(\frac{4.68+1.8(1.6)}{20-(1.6)^2+8(1.6)}\right)$$

$$= 0.2$$

n) $E(Y \mid X=x)$

$$= \int y\, f_{Y|X}(y|x)\, dy \qquad [f_{Y|X}(y|x) \rightarrow \text{from Q4 m})]$$

$$= \int_0^{5-0.5x} y \frac{4}{5}\left(\frac{3x+2y}{20+8x-x^2}\right) dy$$



$$= \frac{0.8}{20+8x-x^2}\left[\frac{3xy^2}{2}+\frac{2}{3}y^3\right]_0^{5-0.5x}$$

$$= \frac{0.8}{(20+8x-x^2)}\left[1.5x(-0.5x+5)^2 + 0.666(-0.5x+5)^3\right], \; 0<x<6$$

$$= \frac{0.4(-0.5x+5)}{(x+2)}\left[1.5x + (0.666(-0.5x+5))\right], \; 0<x<6$$

1.4 4mn **2 / 2**

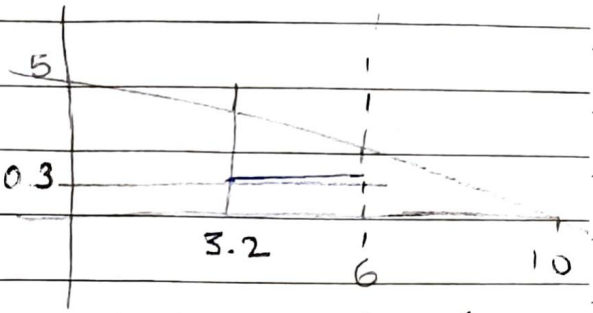
✓ **- 0 pts** **Correct**

- 0.5 pts Wrong limits of integral in 4n

- 0.5 pts Wrong limits of integral in 4m

Date / /

o) $P(X > 3.2 \mid Y = 0.3)$



$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} \rightarrow \text{from HW2}$$

$$= \left(\frac{3x+2y}{24\cancel{0}}\right)\left(\frac{4\cancel{0}}{9+2y}\right), \quad xy < 2 \;\&\; 0 < x < 6$$

$$= \frac{3x+2y}{6(9+2y)}$$

$$P(X > 3.2 \mid Y = 0.3) = \int_{3.2}^{6} \frac{3x+2y}{6(9+2y)}\,dx \quad @\ y = 0.3$$

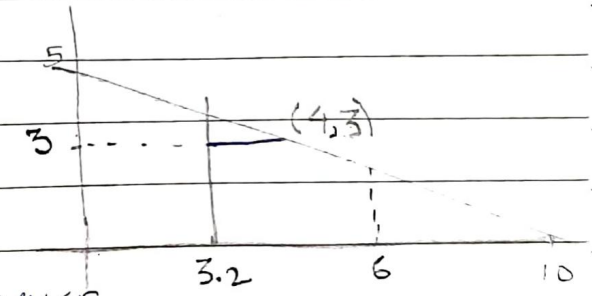
$$= \frac{1}{6(9+2y)}\left[\frac{3x^2}{2} + 2xy\right]_{3.2}^{6}$$

$$= \frac{5.6y + 38.64}{6(9+2y)} \quad \text{put } y = 0.3$$

$$= \frac{5.6(0.3) + 38.64}{6(9 + 2(0.3))}$$

$$= 0.7$$

p) $f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} \rightarrow \text{from HW2}$



$$= \left(\frac{3x+2y}{240}\right)\left(\frac{120}{75-20y+y^2}\right)$$

$2y < 5, \; x < 6$

$$P(X > 3.2 \mid Y = 3) = \int_{3.2}^{4} \frac{3x+2y}{2(75-20y+y^2)}\,dx$$

$$= \frac{1}{2(75-20y+y^2)}\left[\frac{3x^2}{2} + 2xy\right]_{3.2}^{4} \quad @\ y = 3$$

$$= \frac{1.6y + 8.64}{2(75-20y+y^2)} \quad \text{put } y = 3$$

$$= 0.28$$

1.5 4op **2 / 2**

✓ **- 0 pts** **Correct**

- 0.5 pts (o) Incorrect setup

- 0.5 pts (o) Incorrect answer

- 0.5 pts (p) Incorrect setup

- 0.5 pts (p) Incorrect answer

q) $E(X|Y=y)$

$= \int x f_{X|Y}(x|y)\,dx$

$$E(X|Y=y) = \begin{cases} \int_0^6 \frac{x(3x+2y)}{6(9+2y)}\,dx & 0<y<2 \\ \int_0^{10-2y} \frac{x}{2}\frac{(3x+2y)}{(75-20y+y^2)}\,dx & 2<y<5 \end{cases}$$

[$f_{X|Y}(x|y)$ → from Q4(b)]

[$f_{X|Y}(x|y)$ → from Q4(b)]

$$E(X|Y=y) = \begin{cases} \frac{1}{6(9+2y)}\left[\frac{3x^3}{3}+\frac{8x^2y}{2}\right]_0^6, & 0<y<2 \\ \frac{1}{2(75-20y+y^2}\left[\frac{3x^3}{3}+\frac{2x^2y}{2}\right], & 2<y<5 \end{cases}$$

$$E(X|Y=y) = \begin{cases} \frac{36+6y}{9+2y} & 0<y<2 \\ \frac{2(y-5)(10-y)}{(y-15)} & 2<y<5 \end{cases}$$

1.6 4q **2 / 2**

✓ **- 0 pts** **Correct**

- 0.5 pts 0<y<2 Incorrect setup

- 0.5 pts 0<y<2 Incorrect answer

- 0.5 pts 2<y<5 Incorrect setup

- 0.5 pts 2<y<5 Incorrect answer

- 1 pts Incorrect by summing up the two cases.

- 1 pts Incorrect by taking wrong support for y