STAT 410 **Homework #05** Fall 2021
A. Stepanov

(due Friday, October 1, by 5:00 p.m. CDT)

No credit will be given without supporting work.

4. Every week, Alex receives 1,000 rubles allowance from his parents [1 US dollar ≈ 73 Russian rubles]. He usually spends most of it buying candy. In Alex's favorite candy store, W&W's (a cheap imitation of M&M's) are sold in bulk at 100 rubles per kg, and Яeese's Pieces (knock off Reese's Pieces) are sold at 200 rubles per kg. Alex's Mom is very concerned about this unhealthy habit; she made Alex promise her that he would not buy more than 6 kg of W&W's (she does not know that he also buys Яeese's Pieces). Let X and Y denote the weight (in kg) of W&W's and Яeese's Pieces Alex buys, respectively. Let the joint probability density function for (X , Y) be

$$f(x,y) = \frac{3x+2y}{240}, \qquad x \ge 0,\ y \ge 0,\ x \le 6,\ 100x + 200y \le 1000,$$

zero otherwise.

X – W&W's, Y – Яeese's Pieces.

w) Let $T = 100\,X + 200\,Y$ denote the amount Alex spends on candy.

Find the joint probability density function of (X, T), $f_{X,T}(x,t)$.
Sketch the support of (X, T).

x) **Use part (w)** to find the p.d.f. of $T = 100\,X + 200\,Y$, $f_T(t)$.

"Hint": We already know the answer. So NO, you are NOT allowed to simply take the derivative of the c.d.f. of T, $F_T(t)$, from Homework #4. However, you are welcome to use this to double-check your answer. And **show your work**.

y) Let $V = \frac{X}{Y}$.

Find the joint probability density function of (X, V), $f_{X,V}(x, v)$.

Sketch the support of (X, V).

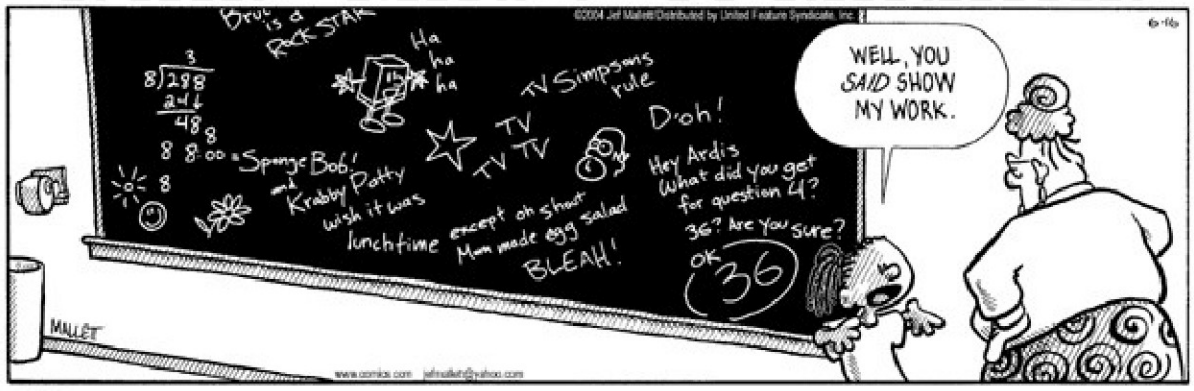
z) **Use part (y)** to find the p.d.f. of $V = \frac{X}{Y}$, $f_V(v)$.

"Hint": We already know the answer. So NO, you are NOT allowed to simply take the derivative of the c.d.f. of V, $F_V(v)$, from Homework #4. However, you are welcome to use this to double-check your answer. And **show your work**.

aa) Let $U = X \cdot Y$.

Find the joint probability density function of (X, U), $f_{X,U}(x, u)$.

Sketch the support of (X, U).



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You are welcome to use a computer and/or calculator on any problem to evaluate any integral. For the supporting work, you should include the full integral (with the function and the bounds) and the answer. For example,

$$\int_0^x u^2\,du = \frac{x^3}{3}, \qquad \int_0^4 \left(\int_0^{\sqrt{x}} x^2 y\,dy \right) dx = 32, \qquad \int_1^{\infty} \left(\int_0^{y} \frac{1}{(2x+y)^3}\,dx \right) dy = \frac{2}{9}.$$