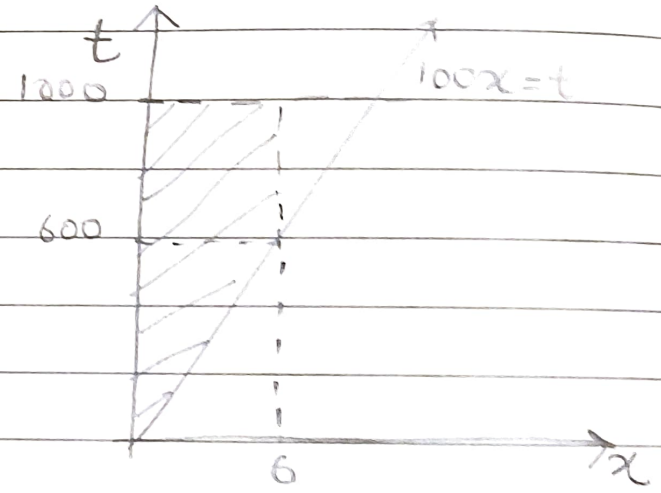
w) $T = 100X + 200Y$

$0 \le x \le 6$

$0 \le y \le 5 - x/2$, $Y = \frac{T}{200} - \frac{X}{2}$

$0 \le \frac{t}{200} - \frac{x}{2} \le 5 - \frac{x}{2}$



$100x \le t \le 1000$

$$J = \begin{matrix} & x & t \\ x & 1 & 0 \\ y & -1/2 & 1/200 \end{matrix}$$

$|J| = 1/200$

$$f_{X,T}(x,t) = f_{X,Y}\left(x, \frac{t}{200} - \frac{x}{2}\right)|J|$$

$$= \left[\frac{3x + 2\left(\frac{t}{200} - \frac{x}{2}\right)}{240}\right] \cdot \frac{1}{200}$$

$$= \frac{2x + 0.01t}{48000}$$

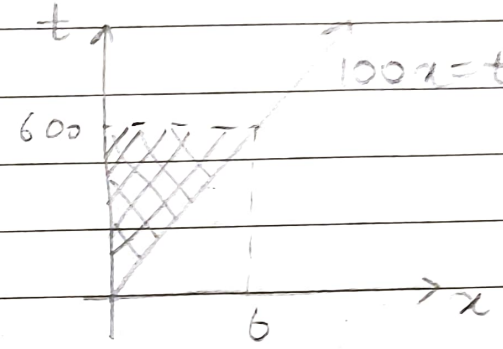
$$f_{X,T}(x,t) = \frac{2x + 0.01t}{48000}, \quad 0 \le x \le 6, \quad 100x \le t \le 1000$$

a) $f_T(t)$

Case-1 $0 \le t < 600$

$$f_T(t) = \int_0^{t/100} \frac{2x + 0.01t}{48000}\, dx$$

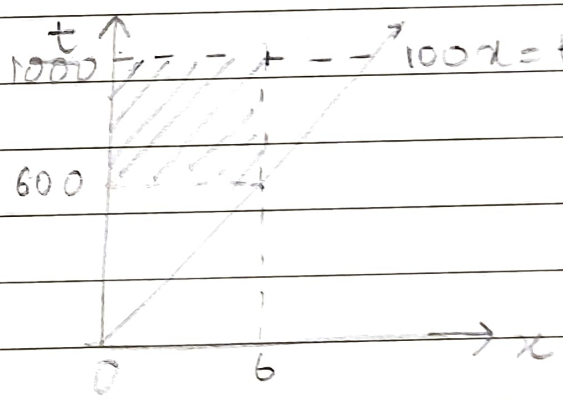
$$= t^2/240,000,000$$



Case-2 $600 \le t < 1000$

$$f_T(t) = \int_0^{6} \frac{2x + 0.01t}{48000}\, dx$$

$$= \frac{t + 600}{800000}$$

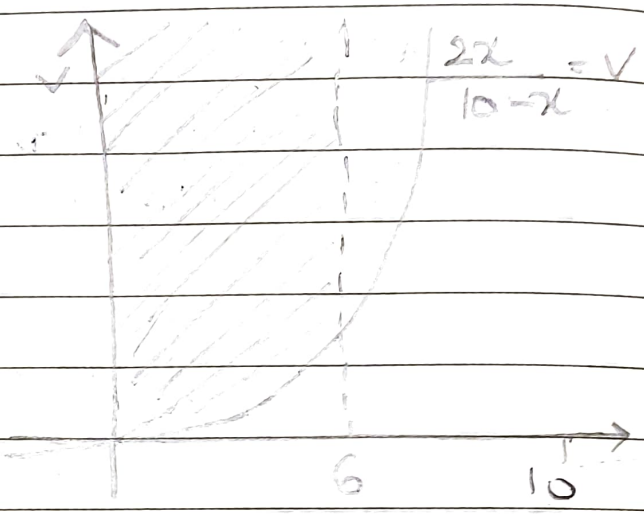


$$f_T(t) = \begin{cases} t^2/240000000 & 0 \le t < 600 \\ \frac{t + 600}{800000} & 600 \le t \le 1000 \\ 0 & \text{elsewhere} \end{cases}$$

y) $V = X/Y$

$0 \le x \le 6$

$0 \le y \le 5 - x/2$, $Y = X/V$



$0 \le \frac{x}{V} \le 5 - \frac{x}{2}$, $V \ge 0$

$\frac{x}{V} \le 5 - \frac{x}{2}$

$2x \le V(10-x)$

$\frac{2x}{10-x} \le V$ ~~$\le x$~~ ,

$$J = \begin{matrix} & x & v \\ x & 1 & 0 \\ y & 1/v & -x/v^2 \end{matrix} = -\frac{x}{v^2}, \quad |J| = \left|-\frac{x}{v^2}\right|$$

$f_{X,V}(x,v) = f_{X,Y}(x, x/v) \cdot |J|$

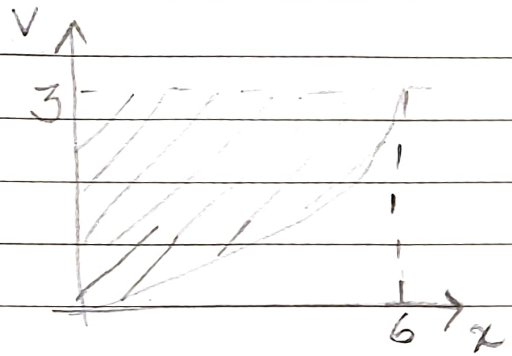
$= \frac{3x + 2x/v}{240} \left|-\frac{x}{v^2}\right|$ $\quad x>0, v>0$ so $\frac{x}{v^2} > 0$

$= \frac{x^2(3v+2)}{240 v^3}$, $0 \le x \le 6$ $\quad 0 \le V \le \frac{2x}{10-x}$

2) Case-1 $0 \le v < 3$

$$f_V(v) = \int_0^{\frac{10v}{2+v}} \frac{x^2(3v+2)}{240v^3}\,dx$$

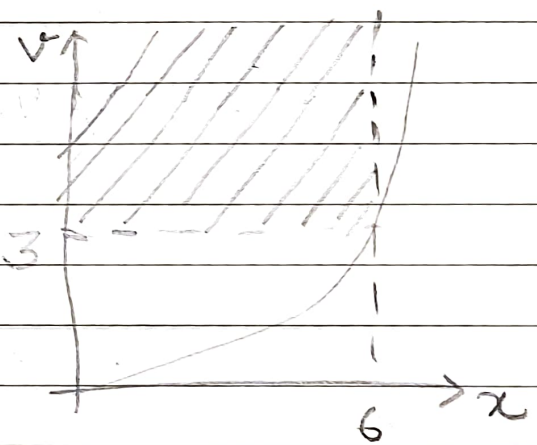
$$= \frac{25(3v+2)}{18(v+2)^3}$$



Case-2 $v > 3$

$$f_V(v) = \int_0^{6} \frac{x^2(3v+2)}{240v^3}\,dx$$

$$= \frac{3(3v+2)}{10v^3}$$



$$f_V(v) = \begin{cases} \frac{25(3v+2)}{18(v+2)^3} & 0 \le v < 3 \\ \frac{3(3v+2)}{10v^3} & v \ge 3 \\ 0 & \text{elsewhere} \end{cases}$$

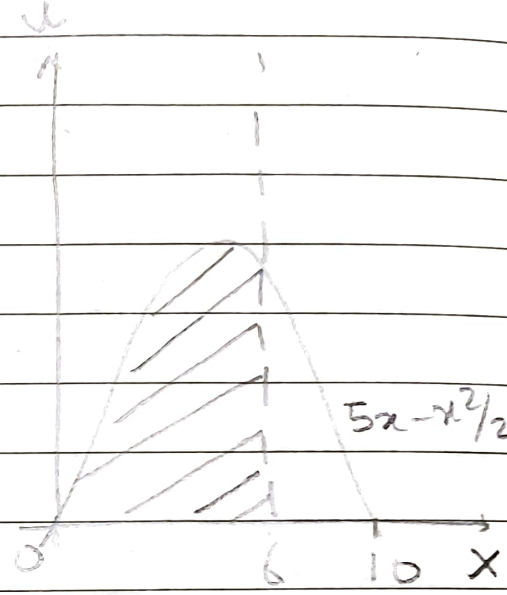
aa) $U = X \cdot Y$

$x = X, \quad 0 \le x \le 6$

$Y = U/X, \quad 0 \le \frac{U}{x} \le 5 - \frac{x}{2}$

$0 \le \frac{u}{x} \Rightarrow u \ge 0$

$u \le 5x - x^2/2$



$$J = \begin{vmatrix} x & 1 & 0 \\ y & -\frac{u}{x^2} & \frac{1}{x} \end{vmatrix} = 1/x$$

$$f_{X,U}(x,u) = f_{X,Y}\left(x, \frac{u}{x}\right) \cdot |J|$$

$$= \frac{3x + 2(u/x)}{240} \cdot \frac{1}{x}$$

$$= \frac{3x^2 + 2u}{240x^2}, \quad 0 \le x \le 6, \quad 0 \le u \le 5x - x^2/2$$