

STAT 410 - Section 1 - Fall 2021 Homework #06

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TOTAL POINTS

10 / 10

QUESTION 1

- **0.5 pts** Wrong transformation of $5C+4K+3L$ in 5(h)

1 3.5 pts

1.1 1gh **1.5 / 1.5**

✓ - **0 pts** Correct

- **0.5 pts** (g) Wrong PDF or wrong derivation for max

X_i

- **0.5 pts** (h) Wrong PDF or wrong derivation for min

X_i

- **0.5 pts** Arithmetic miss

1.2 1ij **2 / 2**

✓ - **0 pts** Correct

- **0.75 pts** (i) Wrong PDF or wrong derivation of Y_2

- **0.75 pts** (j) Wrong PDF or wrong derivation of Y_4

- **0.5 pts** Arithmetic miss

- **1 pts** No probability calculation from pdf

QUESTION 2

5 6.5 pts

2.1 5abc **2 / 2**

✓ - **0 pts** Correct

- **0.5 pts** b) Should be (1-Your Answer)

- **0.5 pts** c) Should be (1-Your Answer)

- **0.5 pts** c) Not correct

2.2 5de **2 / 2**

✓ - **0 pts** Correct

- **0.5 pts** d) Answer not correct

- **0.5 pts** e) Answer not correct

2.3 5fgh **2.5 / 2.5**

✓ - **0 pts** Correct

- **0.5 pts** Wrong transformation of L in 5(g)

- **0.5 pts** Wrong value of variance in 5h

$$g) P(\max X_i > 68)$$

$$P(\max X_i \leq x) = [F_X(x)]^n$$

$$1 - P(\max X_i > x) = [F_X(x)]^n$$

$$P(\max X_i > x) = 1 - [F_X(x)]^n$$

$$P(\max X_i > 68) = 1 - \left[\frac{(68-16)(68+20)}{6400} \right]^6$$

$$= 0.86639$$

$$h) P(\min X_i < 36)$$

$$P(\min X_i > x) = [1 - F_X(x)]^n$$

$$1 - P(\min X_i < x) = [1 - F_X(x)]^n$$

$$P(\min X_i < x) = 1 - [1 - F_X(x)]^n$$

$$P(\min X_i < 36) = 1 - \left[1 - \frac{(36-16)(36+20)}{6400} \right]^6$$

$$= 0.6847$$

1.11gh 1.5 / 1.5

✓ - 0 pts Correct

- 0.5 pts (g) Wrong PDF or wrong derivation for max X_i
- 0.5 pts (h) Wrong PDF or wrong derivation for min X_i
- 0.5 pts Arithmetic miss

$$i) P(X_2 > 52)$$

$$= 1 - P(\text{at least 2 observations} \leq 52)$$

$$= 1 - \left[{}^6C_2 [F_X(52)]^2 [1 - F_X(52)]^4 + {}^6C_3 [F_X(52)]^3 [1 - F_X(52)]^3 + \right. \\ \left. {}^6C_4 [F_X(52)]^4 [1 - F_X(52)]^2 + {}^6C_5 [F_X(52)]^5 [1 - F_X(52)]^1 + \right. \\ \left. {}^6C_6 [F_X(52)]^6 \right]$$

$$= 0.22558$$

$$j) P(Y_4 > 60)$$

$$= 1 - P(\text{At least 4 obs} \leq 60)$$

$$= 1 - \left[{}^6C_4 [F_X(60)]^4 [1 - F_X(60)]^2 + {}^6C_5 [F_X(60)]^5 [1 - F_X(60)] + {}^6C_6 [F_X(60)]^6 \right]$$

$$= 0.55848$$

1.2 1ij 2 / 2

✓ - 0 pts Correct

- 0.75 pts (i) Wrong PDF or wrong derivation of Y2
- 0.75 pts (j) Wrong PDF or wrong derivation of Y4
- 0.5 pts Arithmetic miss
- 1 pts No probability calculation from pdf

$$\begin{aligned}
 5. \quad a) \quad & P(C > 430) \\
 & P\left(\frac{C - \mu_C}{\sigma_C} > \frac{430 - 434}{10}\right) \\
 & = P(Z > -0.4) \\
 & = 1 - P(Z \leq -0.4) \\
 & = 1 - 0.3446 \\
 & = 0.6554
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & P(K > 225) \\
 & P\left(\frac{K - \mu_K}{\sigma_K} > \frac{225 - 222}{3}\right) \\
 & = P(Z > 1) \\
 & = 1 - P(Z \leq 1) \\
 & = 1 - 0.8413 \\
 & = 0.1587
 \end{aligned}$$

$$c) P(K > 225 \mid C = 430)$$

Given $C=430$, K has a Normal Distribution with

$$\text{Mean} = 222 + 0.6 \left(\frac{3}{10} \right) (430 - 434) = 221.28$$

$$\text{Variance} = (1 - 0.6^2) (3^2) = 5.76$$

$$\text{Standard deviation} = \sqrt{\text{Var}} = \sqrt{5.76} = 2.4$$

$$P \left(Z > \frac{225 - 221.28}{2.4} \right)$$

$$= P(Z > 1.55)$$

$$= 1 - P(Z \leq 1.55)$$

$$= 1 - 0.9394 = 0.0606$$

2.15abc 2 / 2

✓ - 0 pts Correct

- 0.5 pts b) Should be (1-Your Answer)

- 0.5 pts c) Should be (1-Your Answer)

- 0.5 pts c) Not correct

$$d) P(C > 2K) \\ = P(C - 2K > 0)$$

'C - 2K' follows a Normal Distribution with

$$\text{Mean} = \mu_C - 2\mu_K = -10$$

$$\text{Variance} = \sigma_C^2 + 4\sigma_K^2 - 4\rho\sigma_C\sigma_K \\ = 100 + 4(9) - 4(.6)(10)(3) = 64$$

$$\text{Std dev} = \sqrt{64} = 8$$

$$P\left(Z > \frac{0 - (-10)}{8}\right)$$

$$= P(Z > 1.25)$$

$$= 1 - P(Z \leq 1.25)$$

$$= 1 - 0.8944 = 0.1056$$

$$e) P(5C + 4K > 3000)$$

'5C + 4K' follows a Normal Distribution with

$$\text{Mean} = 5\mu_C + 4\mu_K = 3058$$

$$\text{Var} = 25\sigma_C^2 + 16\sigma_K^2 + 40\rho\sigma_C\sigma_K = 3364$$

$$\text{Std. dev} = \sqrt{3364} = 58$$

$$P\left(Z > \frac{3000 - 3058}{58}\right)$$

$$= P(Z > -1)$$

$$= 1 - P(Z \leq -1)$$

$$= 1 - 0.1587 = 0.8413$$

2.2 5de 2 / 2

✓ - 0 pts Correct

- 0.5 pts d) Answer not correct

- 0.5 pts e) Answer not correct

$$\begin{aligned}
 f) \rho_{CL} &= \text{corr}(C, L) \\
 &= \frac{\text{cov}(C, L)}{\sqrt{\text{var}(C) \text{var}(L)}} \\
 &= \frac{24}{\sqrt{100 \times 36}} = 0.4
 \end{aligned}$$

$$\begin{aligned}
 g) & P(L > 334) \\
 & P\left(Z > \frac{334 - \mu_L}{\sigma_L}\right) \\
 &= P\left(Z > \frac{334 - 331}{6}\right) \\
 &= P(Z > 0.5) \\
 &= 1 - P(Z \leq 0.5) \\
 &= 1 - 0.6915 \\
 &= 0.3085
 \end{aligned}$$

$$h) P(5C + 4K + 3L > 4000)$$

' $5C + 4K + 3L$ ' follows a Normal Distribution with

$$\text{Mean} = \begin{pmatrix} 5 & 4 & 3 \end{pmatrix} \begin{pmatrix} 434 \\ 222 \\ 331 \end{pmatrix} = 4051$$

$$\text{Var} = \begin{pmatrix} 5 & 4 & 3 \end{pmatrix} \begin{pmatrix} 100 & 18 & 24 \\ 18 & 9 & 9 \\ 24 & 9 & 36 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix} = 4624$$

$$\text{Std dev} = \sqrt{4624} = 68$$

$$P(Z > \frac{4000 - 4051}{68})$$

$$\begin{aligned} &= P(Z > -0.75) \\ &= 1 - P(Z \leq -0.75) \\ &= 1 - 0.2266 \\ &= 0.7734 \end{aligned}$$

2.3 5fgh 2.5 / 2.5

✓ - 0 pts Correct

- 0.5 pts Wrong transformation of L in 5(g)
- 0.5 pts Wrong value of variance in 5h
- 0.5 pts Wrong transformation of $5C+4K+3L$ in 5(h)