STAT 410 **Homework #03** Fall 2021
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**(due Friday, September 17, by 5:00 p.m. CDT)**

*No credit will be given without supporting work.*

**4.** Every week, Alex receives 1,000 rubles allowance from his parents [1 US dollar ≈ 73 Russian rubles]. He usually spends most of it buying candy. In Alex's favorite candy store, W&W's (a cheap imitation of M&M's) are sold in bulk at 100 rubles per kg, and Яeese's Pieces (knock off Reese's Pieces) are sold at 200 rubles per kg. Alex's Mom is very concerned about this unhealthy habit; she made Alex promise her that he would not buy more than 6 kg of W&W's (she does not know that he also buys Яeese's Pieces). Let X and Y denote the weight (in kg) of W&W's and Яeese's Pieces Alex buys, respectively. Let the joint probability density function for (X, Y) be

$$f(x,y) = \frac{3x+2y}{240}, \quad x \ge 0,\ y \ge 0,\ x \le 6,\ 100x + 200y \le 1000,$$
zero otherwise.

X – W&W's, Y – Яeese's Pieces.

Recall (Homework #2):

$$f_X(x) = \frac{20 + 8x - x^2}{192}, \quad 0 < x < 6. \qquad f_Y(y) = \begin{cases} \frac{9 + 2y}{40} & 0 < y < 2 \\ \frac{75 - 20y + y^2}{120} & 2 < y < 5 \end{cases}$$

h) Find the probability that Alex would spend over 540 rubles buying candy. That is, find $P(100X + 200Y > 540)$.

i) Find the probability that Alex would spend over 760 rubles buying candy. That is, find $P(100X + 200Y > 760)$.

j) Find $P(X < 0.5Y)$.

k) Find $P(X < 5Y)$.

l) Suppose we know that Alex bought more than 1.6 kg of W&W's. What is the probability that he bought more than 3.6 kg of Яeese's Pieces? That is, find $P(Y > 3.6 \mid X > 1.6)$.

m) Suppose we know that Alex bought exactly 1.6 kg of W&W's. What is the probability that he bought more than 3.6 kg of Яeese's Pieces? That is, find $P(Y > 3.6 \mid X = 1.6)$.

n) Find $E(Y \mid X = x)$, the expected weight of Яeese's Pieces that Alex bought, given that he bought exactly $x$ kg of W&W's.

o) Suppose we know that Alex bought exactly 0.3 kg of Яeese's Pieces. What is the probability that he bought more than 3.2 kg of W&W's? That is, find $P(X > 3.2 \mid Y = 0.3)$.

p) Suppose we know that Alex bought exactly 3.0 kg of Яeese's Pieces. What is the probability that he bought more than 3.2 kg of W&W's? That is, find $P(X > 3.2 \mid Y = 3.0)$.

q) Find $E(X \mid Y = y)$, the expected weight of W&W's that Alex bought, given that he bought exactly $y$ kg of Яeese's Pieces.