STAT 410 - Section 1 - Fall 2021 Homework #11

Sharvi Tomar

TOTAL POINTS

9.5 / 10

QUESTION 1

10 2 pts

1.1 10ab **1 / 1**

✓ **- 0 pts Correct**

- 0.5 pts Wrong final ans in 10a

- 0.5 pts Wrong final ans in 10b

1.2 10c **1 / 1**

✓ **- 0 pts Correct**

- 0.5 pts Incorrect power for mu=63.1

- 0.5 pts Incorrect power for mu=67.9

QUESTION 2

11 3 pts

2.1 11ab **1.5 / 1.5**

✓ **- 0 pts Correct**

- 0.5 pts (a) Wrong test statistic

- 0.5 pts (a) Wrong p value

- 0.5 pts (b) Wrong rejection region

2.2 11c **1.5 / 1.5**

✓ **- 0 pts Correct**

- 0.5 pts Wrong power for u = 63.2

- 0.5 pts Wrong power for u = 54.2

- 0.5 pts No final answer

QUESTION 3

12 2.5 pts

3.1 12ab **1 / 1**

✓ **- 0 pts Correct**

- 0.5 pts a) not correct

- 0.5 pts b) not correct

3.2 12cd **1.5 / 1.5**

✓ **- 0 pts Correct**

- 0.5 pts c) not correct

- 0.5 pts d) not correct

- 1.5 pts missing

QUESTION 4

13 2.5 pts

4.1 13abc **1.5 / 1.5**

✓ **- 0 pts Correct**

- 0.5 pts Mistake in 13a

- 0.5 pts Mistake in 13b

- 0.5 pts Mistake in 13c

4.2 13d **0.5 / 1**

- 0 pts Correct

✓ **- 0.5 pts not correct**

- 1 pts missing

10) $H_0: \mu \leq 60$

$H_1: \mu > 60$

Right-tailed test

a) $\bar{x} = 65.4$

Observed value of test-statistic is:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{65.4 - 60}{12/\sqrt{9}} = 1.35$$

Right-tailed test p-value $= P(z > 1.35)$

$= 1 - P(z \leq 1.35)$

$= 1 - 0.9115$

$= 0.0885$

b) Rejection region at $\alpha = 0.05$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} > Z_{\alpha}$$

$$\Rightarrow \bar{X} > \mu + Z_{\alpha}\frac{\sigma}{\sqrt{n}}$$

$$\Rightarrow \bar{X} > 60 + 1.645 \cdot \frac{12}{\sqrt{9}}$$

$$\Rightarrow \bar{X} > 66.58$$

c) i) Power $(\mu = 63.1) = P(\text{Reject } H_0 \mid H_0 \text{ is false})$

$$= P(\bar{X} > 66.58 \mid \mu = 63.1)$$

$$P\left(Z > \frac{66.58 - 63.1}{12/\sqrt{9}}\right)$$

$$= P(Z > 0.87)$$

$$= 1 - 0.8078 = \underline{0.1922}$$

ii) Power $(\mu = 67.9) = P(\text{Reject } H_0 \mid H_0 \text{ is false})$

$$= P(\bar{X} > 66.58 \mid \mu = 67.9)$$

$$= P\left(Z > \frac{66.58 - 67.9}{12/\sqrt{9}}\right)$$

$$= P(Z > -0.33)$$

$$= 1 - 0.3707 = \underline{0.6293}$$

1.1 10ab **1 / 1**

✓ **- 0 pts** **Correct**

- 0.5 pts Wrong final ans in 10a

- 0.5 pts Wrong final ans in 10b

b) Rejection region at $\alpha = 0.05$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} > Z_{\alpha}$$

$$\Rightarrow \bar{X} > \mu + Z_{\alpha}\frac{\sigma}{\sqrt{n}}$$

$$\Rightarrow \bar{X} > 60 + 1.645 \cdot \frac{12}{\sqrt{9}}$$

$$\Rightarrow \bar{X} > 66.58$$

c) i) Power $(\mu = 63.1) = P(\text{Reject } H_0 \mid H_0 \text{ is false})$

$$= P(\bar{X} > 66.58 \mid \mu = 63.1)$$

$$P\left(Z > \frac{66.58 - 63.1}{12/\sqrt{9}}\right)$$

$$= P(Z > 0.87)$$

$$= 1 - 0.8078 = \underline{0.1922}$$

ii) Power $(\mu = 67.9) = P(\text{Reject } H_0 \mid H_0 \text{ is false})$

$$= P(\bar{X} > 66.58 \mid \mu = 67.9)$$

$$= P\left(Z > \frac{66.58 - 67.9}{12/\sqrt{9}}\right)$$

$$= P(Z > -0.33)$$

$$= 1 - 0.3707 = \underline{0.6293}$$

1.2 10c **1 / 1**

✓ **- 0 pts** **Correct**

- 0.5 pts Incorrect power for mu=63.1

- 0.5 pts Incorrect power for mu=67.9

11. $H_0 : \mu = 60$
$H_1 : \mu \neq 60$
Two-tailed test

a) $\bar{x} = 65.4$
Observed value of test statistic is
$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{65.4 - 60}{12/\sqrt{9}} = 1.35$$
$$Z = 1.35 > 0$$

Right-tailed test p-value: $P(Z > 1.35)$
$= 1 - P(Z \leq 1.35)$
$= 1 - 0.9115$
$= 0.0885$
p-value for 2-tailed test = 2 * p-value for right-tailed test
$= 2 * 0.0885$
$= 0.177$

b) Rejection region at $\alpha = 0.05$
$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} < -Z_{\alpha/2} \quad \text{or} \quad Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} > Z_{\alpha/2}$$
$$\Rightarrow \bar{X} < \mu_0 - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \quad \text{or} \quad \bar{X} > \mu_0 + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$
$$\Rightarrow \bar{X} < 60 - 1.96 \cdot \frac{12}{\sqrt{9}} \quad \text{or} \quad \bar{X} > 60 + 1.96 \cdot \frac{12}{\sqrt{9}}$$
$$\Rightarrow \bar{X} < 52.16 \quad \text{or} \quad \bar{X} > 67.84$$

2.1 11ab **1.5 / 1.5**

✓ **- 0 pts Correct**

- 0.5 pts (a) Wrong test statistic

- 0.5 pts (a) Wrong p value

- 0.5 pts (b) Wrong rejection region

c) i) Power $(\mu = 63.2) = P(\text{Reject } H_0 \mid H_0 \text{ is false})$

$= P(\bar{X} < 52.16 \mid \mu = 63.2) + P(\bar{X} > 67.84 \mid \mu = 63.2)$

$= P\left(Z < \frac{52.16 - 63.2}{12/\sqrt{9}}\right) + P\left(Z > \frac{67.84 - 63.2}{12/\sqrt{9}}\right)$

$= P(Z < -2.76) + P(Z > 1.16)$

$= 0.00289 + 1 - P(Z \leq 1.16)$

$= 0.00289 + 1 - 0.123024$

$= 0.12591$

ii) Power $(\mu = 54.2) = P(\text{Reject } H_0 \mid H_0 \text{ is false})$

$= P(\bar{X} < 52.16 \mid \mu = 54.2) + P(\bar{X} > 67.84 \mid \mu = 54.2)$

$= P\left(Z < \frac{52.16 - 54.2}{12/\sqrt{9}}\right) + P\left(Z > \frac{67.84 - 54.2}{12/\sqrt{9}}\right)$

$= P(Z < -0.51) + P(Z > 3.41)$

$= 0.305025 + 1 - P(Z \leq 3.41)$

$= 0.305025 + 1 - 0.999675$

$= 0.30535$

2.2 11c 1.5 / 1.5

✓ **- 0 pts** Correct

- 0.5 pts Wrong power for u = 63.2

- 0.5 pts Wrong power for u = 54.2

- 0.5 pts No final answer

12 a) $H_0: X_7 > c$ $\quad X_7 \sim \text{Poisson}(7\lambda)$

$H_a: X_7 \leq c$ $\quad \lambda = 1.5$

$\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true})$

$0.05 = P(X_7 \leq c \mid \lambda = 1.5)$

$0.05 = P(\text{Poisson}(10.5) \leq c)$

$0.05 = P(\text{Poisson}((0.5) \leq 5)$

$\Rightarrow \boxed{c = 5}$

b) $\text{Power}(\lambda = 1) = P(\text{Reject } H_0 \mid H_0 \text{ is Not true})$

$= P(X_7 \leq c \mid \lambda = 1)$

$= P(\text{Poisson}(7) \leq 5)$

$= 0.301$

c) $\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true})$

$= P(X_7 \leq 6 \mid \lambda = 1.5)$

$= P(\text{Poisson}(10.5) \leq 6)$

$= 0.102$

3.1 12ab **1 / 1**

✓ **- 0 pts Correct**

- 0.5 pts a) not correct

- 0.5 pts b) not correct

12 a) $H_0: X_7 > c$ $\quad X_7 \sim \text{Poisson}(7\lambda)$

$H_a: X_7 \leq c$ $\quad \lambda = 1.5$

$\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true})$

$0.05 = P(X_7 \leq c \mid \lambda = 1.5)$

$0.05 = P(\text{Poisson}(10.5) \leq c)$

$0.05 = P(\text{Poisson}((0.5) \leq 5)$

$\Rightarrow \boxed{c = 5}$

b) $\text{Power}(\lambda = 1) = P(\text{Reject } H_0 \mid H_0 \text{ is Not true})$

$= P(X_7 \leq c \mid \lambda = 1)$

$= P(\text{Poisson}(7) \leq 5)$

$= 0.301$

c) $\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true})$

$= P(X_7 \leq 6 \mid \lambda = 1.5)$

$= P(\text{Poisson}(10.5) \leq 6)$

$= 0.102$

d) $P(X_7 \leq 4)$

$= P(\text{Poisson}(10.5) \leq 4)$

$= 0.021$

13 a) $P(T_4 \geq c \mid \lambda = 1.5) = 0.05$

$2T_4(1.5) \sim \chi^2(8)$

$\Rightarrow P(2\lambda T_4 \geq 2\lambda c) = 0.05$

$= P(\chi^2(8) \geq 3c) = 0.05$

$= 1 - P(\chi^2(8) < 3c) = 0.05$

$P(\chi^2(8) < 3c) = 0.95$

$3c = 15.51$

$c = 5.17$

Rejection Region $T_4 \geq 5.17$

3.2 12cd **1.5 / 1.5**

✓ **- 0 pts Correct**

- 0.5 pts c) not correct

- 0.5 pts d) not correct

- 1.5 pts missing

d) $P(X_7 \leq 4)$

$= P(\text{Poisson}(10.5) \leq 4)$

$= 0.021$

13 a) $P(T_4 \geq c \mid \lambda = 1.5) = 0.05$

$2T_4(1.5) \sim \chi^2(8)$

$\Rightarrow P(2\lambda T_4 \geq 2\lambda c) = 0.05$

$= P(\chi^2(8) \geq 3c) = 0.05$

$= 1 - P(\chi^2(8) < 3c) = 0.05$

$P(\chi^2(8) < 3c) = 0.95$

$3c = 15.51$

$c = 5.17$

Rejection Region $T_4 \geq 5.17$

13. b) $\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true})$

$= P(T_4 > 5 \mid \lambda = 1.5)$

$= P(X_5 \leq 3 \mid \lambda = 1.5)$

$= P(\text{Poisson}(7.5) \leq 3)$

$\alpha = 0.059$

c) Power $(\lambda=1)$ $= P(\text{Reject } H_0 \mid H_0 \text{ is not true})$

$= P(T_4 \geq 5 \mid \lambda=1)$

$= P(X_5 \leq 3 \mid \lambda=1)$

$= P(\text{Poisson}(5*1) \leq 3 \mid \lambda=1)$

$= P(\text{Poisson}(5) \leq 3 \mid \lambda=1)$

$= 0.265$

4.1 13abc **1.5 / 1.5**

✓ **- 0 pts** **Correct**

- 0.5 pts Mistake in 13a

- 0.5 pts Mistake in 13b

- 0.5 pts Mistake in 13c

d) $P(5 < T_4 < 6 \mid \lambda = 1.5)$

$P(X_5 \leq 3 \mid \lambda = 1.5) - P(X_6 \leq 3 \mid \lambda = 1.5)$

$= P(\text{Poisson}(5 * 1.5) \leq 3) - P(\text{Poisson}(6 * 1.5) \leq 3)$

$= P(\text{Poisson}(7.5) \leq 3) - P(\text{Poisson}(9) \leq 3)$

$= 0.059 - 0.021$

$= 0.038$

p-value = 0.038

4.2 13d **0.5 / 1**

- **0 pts** Correct

✓ - **0.5 pts** not correct

- **1 pts** missing