r) $\mathrm{Var}(Y|X) = E(Y^2|X) - [E(Y|X)]^2$

$$E(Y^2|x) = \int_{-\infty}^{\infty} y^2 f_{Y|X}(y|x)\,dy$$

$$= \int_{0}^{5-x/2} y^2 \frac{(12x+8y)}{5(2+x)(10-x)}\,dy$$

$$= \frac{(10-x)^2(3x+10)}{40(x+2)} \qquad 0<x<6$$

$$E(Y|x) = \frac{(20+7x)(10-x)}{30(2+x)}$$

$0<x<$

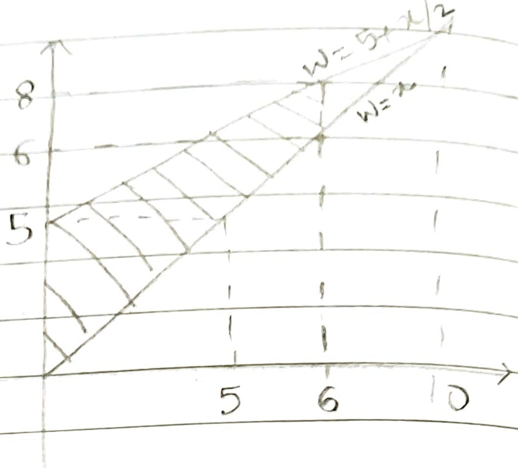
$$\mathrm{Var}(Y|x) = \frac{(10-x)^2(3x+10)}{40(x+2)} - \left[\frac{(20+7x)(10-x)}{30(2+x)}\right]^2$$

5) $W = X + Y$

$Y = W - X$

$0 \le w - x \le 5 - x/2$

$x \le w \le 5 + x/2$




Case-1 $0 \le w < 5$

$$f_W(w) = \int_0^w \frac{3x + 2(w-x)\,dx}{240}$$

$$= w^2/96$$

Case-2 $5 \le w < 6$

$$f_W(w) = \int_{2w-10}^{w} \frac{3x + 2(w-x)\,dx}{240}$$

$$= \frac{-7w^2 + 80w - 100}{480}$$

Case-3 $6 \le w \le 8$

$$f_W(w) = \int_{2w-10}^{6} \frac{3x + 2(w-x)\,dx}{240}$$

$$= \frac{-3w^2 + 26w - 16}{120}$$

$$f_W(w) = \begin{cases} w^2/96 & 0 \le w < 5 \\ \dfrac{-7w^2 + 80w - 100}{480} & 5 \le w < 6 \\ \dfrac{-3w^2 + 26w - 16}{120} & 6 \le w \le 8 \end{cases}$$

t) $T = 100X + 200Y$

$0 \le x \le 6 \qquad 0 \le y \le 5 - x/2$

$y = \frac{t}{200} - \frac{x}{2}$

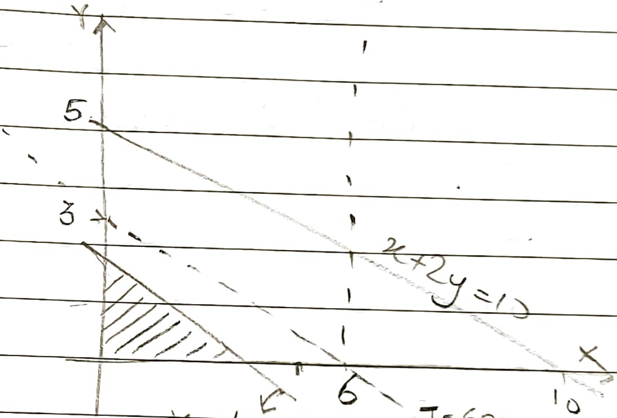
$0 \le \frac{t}{200} - \frac{x}{2} \le 5 - \frac{x}{2}$

$100x \le t \le 1000$

Case-1. $0 \le T \le 600$

$$F_T(t) = \int_0^{t/100} \int_0^{\frac{t}{200} - \frac{x}{2}} \frac{3x + 2y}{240} \, dy \, dx$$

$$= \frac{t^3}{10^9} \times 1.38889$$

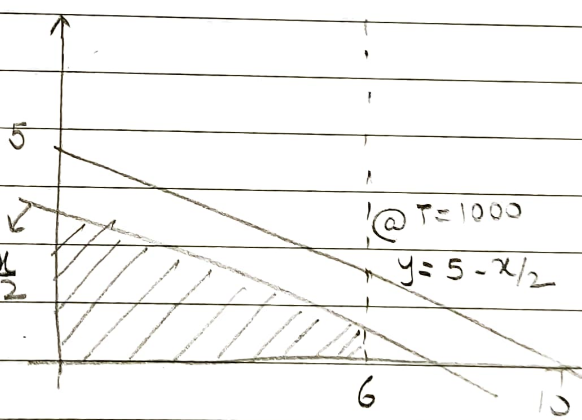



$$f_T(t) = F_T'(t)$$
$$= \frac{t^2}{24 \times 10^7}$$

Case-2 $600 \le T \le 1000$

$$F_T(t) = \int_0^6 \int_0^{\frac{t}{200} - x/2} \frac{3x + 2y}{240} \, dy \, dx$$

$$= \frac{t^2}{16 \times 10^5} + \frac{3t}{4000} - \frac{3}{8}$$




$$f_T(t) = F_T'(t)$$
$$= \frac{t}{8 \times 10^5} + \frac{3}{4000}$$

$$F_T(t) = \begin{cases} 0 & t \le 0 \\ \frac{t^3}{72 \times 10^7} & 0 \le t \le 600 \\ \frac{t^2}{16 \times 10^5} + \frac{3t}{4000} - \frac{3}{8} & 600 < t \le 1000 \\ 1 & t > 1000 \end{cases}$$

u) $E(T) = \int_{-\infty}^{\infty} t \, f_T(t)\,dt$

Using $f_T(t)$ from t)

$$= \int_{0}^{600} t\left(\frac{t^2}{24\times10^7}\right)dt + \int_{600}^{1000} t\left(\frac{t}{8\times10^5} + \frac{3}{4000}\right)dt$$

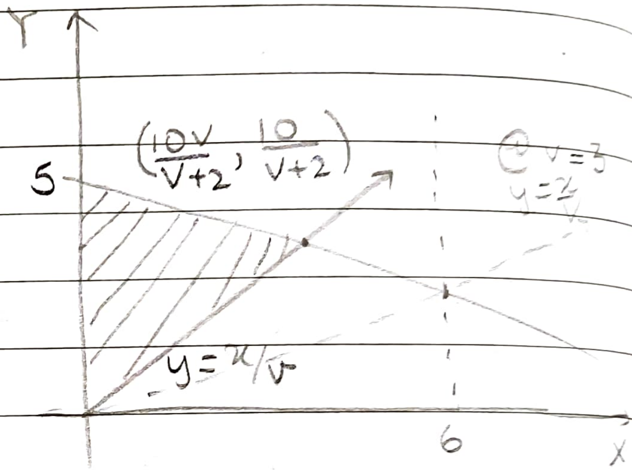
$$= 135 + \frac{1700}{3}$$

$E(T) = 701.6667$

v) $V = \frac{X}{Y}$ $\qquad$ $x>0,\ y>0 \Rightarrow v>0$

$$F_V(v) = P\left(X/Y \le v\right)$$
$$= P\left(Y \ge X/v\right)$$

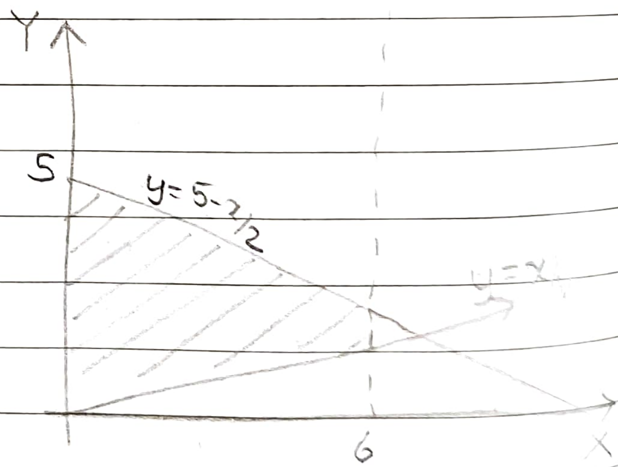
Case-1 $\quad 0 < v \le 3$




$$F_V(v) = \int_0^{\frac{10v}{v+2}} \int_{x/v}^{5-x/2} \frac{3x+2y}{240}\, dy\, dx$$

$$= \frac{25(v^2+v)}{18(v+2)^2}$$

Case 2 $\quad 3 < v < \infty$




$$F_V(v) = 1 - \int_0^6 \int_0^{x/v} \frac{3x+2y}{240}\, dy\, dx$$

$$= 1 - \left(\frac{9v+3}{10v^2}\right)$$

$$F_V(v) = \begin{cases} \frac{25(v^2+v)}{18(v+2)^2} & 0 < v \le 3 \\ 1 - \left(\frac{9v+3}{10v^2}\right) & 3 < v < \infty \end{cases}$$