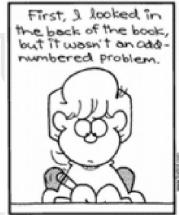
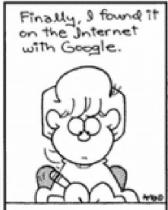
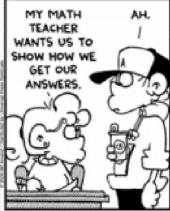
(due Friday, September 3, by 5:00 p.m. CDT)

No credit will be given without supporting work.









1. Grades on Fall 2021 STAT 410 Exam 1 were not very good*. Graphed, their distribution had a shape similar to the probability density function **.

$$f_X(x) = \frac{x+2}{C},$$
 $16 \le x \le 80,$

zero elsewhere.

- a) Find the value of C that makes $f_{X}(x)$ a valid probability density function.
- b) Find the cumulative distribution function of X, $F_X(x) = P(X \le x)$.

"Hint": To double-check your answer: should be $F_X(16) = 0$, $F_X(80) = 1$.

- * The probability distribution is fictional, the exam has not happened yet. Hopefully, the actual grades will be slightly better than these.
- ** Exam scores should have a discrete (instead of continuous) nature. A continuous probability distribution is used as an approximation, since the alternative would have been dealing with a discrete random variable with 65 possible values (16, 17, 18, ..., 79, 80), which is not nearly as much fun as I am describing it here.

(continued)

As a way of "curving" the results, the instructor announced that he would replace each person's grade, X, with a new grade, Y = g(X), where $g(x) = 8\sqrt{x+20}$.

- Find the support (the range of possible values) of the probability distribution of Y. c)
- Use part (b) and the c.d.f. approach to find the c.d.f. of Y, $F_{Y}(y)$. d)

"Hint":
$$F_Y(y) = P(Y \le y) = P(g(X) \le y) = ...$$

Use the change-of-variable technique to find the p.d.f. of Y, $f_{Y}(y)$. e)

"Hint":
$$f_{\mathbf{Y}}(y) = f_{\mathbf{X}}(g^{-1}(y)) \left| \frac{dx}{dy} \right|$$
.

To double-check your answer: should be $f_Y(y) = F_Y'(y)$. "Hint":

f) Is
$$\mu_Y$$
 equal to $g(\mu_X)$?

$$\mu_X = E(X), \quad \mu_Y = E(Y).$$

Consider a continuous random variable X with the probability density function

$$f_{\rm X}(x) = \frac{x+2}{18}, \qquad -2 \le x \le 4$$

zero elsewhere.

Consider
$$Y = g(X) = \frac{16}{X^2}$$
. Find the probability distribution of Y.

You are welcome to use a computer and/or calculator on any problem to evaluate any integral. For the supporting work, you should include the full integral (with the function and the bounds) and the answer. For example,

$$\int_{0}^{x} u^{2} du = \frac{x^{3}}{3}, \qquad \int_{0}^{4} \left(\int_{0}^{\sqrt{x}} x^{2} y dy \right) dx = 32, \qquad \int_{1}^{\infty} \left(\int_{0}^{y} \frac{1}{(2x+y)^{3}} dx \right) dy = \frac{2}{9}.$$