(**due Friday, December 3, by 5:00 p.m. CST**)

No credit will be given without supporting work.

10 – 11. A jackleg Discord user uses Discord mainly just to "collect" the homework answers (that is, not your average Discord user, since the majority of Discord users are, of course, on Discord to learn, allegedly). Suppose that the time the jackleg Discord user needs to spend on Discord to "collect" the homework answers varies from assignment to assignment according to a normal distribution with the overall standard deviation $\sigma = 12$ minutes. We wish to use (a random sample of) $n = 9$ homework assignments to test …

10. … $H_0 : \mu \le 60$ minutes   versus $H_1 : \mu > 60$ minutes.

a) If the sample mean is $\bar{x} = 65.4$ minutes, find the p-value of the test.

b) Find the Rejection Region for the test at $\alpha = 0.05$.   That is, for which values of the sample mean $\bar{X}$ should we reject H_0, if a 5% level of significance is used?

c) Find the power of the test if a 5% level of significance is used, and the true value of the average amount of time the jackleg Discord user needs to spend on Discord to "collect" the homework answers is   (i) $\mu = 63.1$ minutes,   (ii) $\mu = 67.9$ minutes.

11. … $H_0 : \mu = 60$ minutes   versus $H_1 : \mu \ne 60$ minutes.

a) If the sample mean is $\bar{x} = 65.4$ minutes, find the p-value of the test.

b) Find the Rejection Region for the test at $\alpha = 0.05$.   That is, for which values of the sample mean $\bar{X}$ should we reject H_0, if a 5% level of significance is used?

c) Find the power of the test if a 5% level of significance is used, and the true value of the average amount of time the jackleg Discord user needs to spend on Discord to "collect" the homework answers is   (i) $\mu = 63.2$ minutes,   (ii) $\mu = 54.2$ minutes.

12 – 13.   Recall Homework #07:

6.   After a homework assignment is posted on Compass2g, the weak-minded lazy Chegg enthusiasts flock to Chegg according to a Poisson process with the average rate of 1.5 weak-minded lazy Chegg enthusiasts per minute. (The pudding-brain lazy CourseHero worshipers show similar behavior.)

Let

X_t = the number of weak-minded lazy Chegg enthusiasts who show up in t minutes,

T_k = the time when the kth weak-minded lazy Chegg enthusiast shows up.

Then

X_t has a Poisson (λt) distribution,

T_k has a Gamma $(\alpha = k, \theta = \dfrac{1}{\lambda})$ distribution.

A weak-minded lazy Chegg enthusiast teams up with a pudding-brain lazy CourseHero worshiper in an attempt to show that the average rate of weak- minded lazy Chegg enthusiasts flocking to Chegg after a homework assignment is posted on Compass2g is less than 1.5 weak-minded lazy Chegg enthusiasts per minute. They wish to test

$$H_0 : \lambda = 1.5 \qquad \text{versus} \qquad H_1 : \lambda < 1.5.$$

However, they are "a little confused" about how to do it and cannot agree on the plan.

12.   The weak-minded lazy Chegg enthusiast wants to consider the number of weak-minded lazy Chegg enthusiasts who show up in 7 minutes, X_7, and then Reject H_0 if X_7 is too small.

a)   Help the weak-minded lazy Chegg enthusiast find the best rejection region with the significance level α closest to 0.05.

Hint 1:   Reject H_0 if $X_7 \le c.$ Hint 2: $X_7 \sim$ Poisson (7λ).

Hint 3:   Want c such that $0.05 = \alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true}) = P(X_7 \le c \mid \lambda = 1.5).$

b)   Find the power of the rejection region from part (a) if $\lambda = 1.$

Hint: Power $(\lambda = 1) = P(\text{Reject } H_0 \mid H_0 \text{ is NOT true}) = P(X_7 \le c \mid \lambda = 1).$

c) Consider the rejection region

Reject H_0 if at most 6 weak-minded lazy Chegg enthusiasts show up in 7 minutes.

That is, Reject H_0 if $X_7 \le 6$. Find the significance level α for this rejection region.

Hint: $\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true}) = P(X_7 \le 6 \mid \lambda = 1.5)$.

d) Suppose that 4 weak-minded lazy Chegg enthusiasts show up in 7 minutes.
Find the p-value of the test.

13. The pudding-brain lazy CourseHero worshiper wants to consider the time when the 4th weak-minded lazy Chegg enthusiast shows up, T_4, and then Reject H_0 if T_4 is too large.

a) Help the pudding-brain lazy CourseHero worshiper find the best Rejection Region with the significance level $\alpha = 0.05$.

Hint 1: Reject H_0 if $T_4 \ge c$. Hint 2: $T_4 \sim \text{Gamma}\left(\alpha = 4, \theta = \dfrac{1}{\lambda}\right)$.

Hint 3: Want c such that $0.05 = \alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true}) = P(T_4 \ge c \mid \lambda = 1.5)$.

Hint 4 (suggestion): If T has a $\text{Gamma}\left(\alpha, \theta = \dfrac{1}{\lambda}\right)$ distribution, then

$$\dfrac{2T}{\theta} = 2\lambda T \text{ has a } \chi^2(2\alpha) \text{ distribution.}$$

b) Consider the rejection region

Reject H_0 if we wait more than 5 minutes for the 4th weak-minded lazy Chegg enthusiast to show up.

That is, Reject H_0 if $T_4 > 5$. Find the significance level α for this rejection region.

Hint 1: $\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true}) = P(T_4 > 5 \mid \lambda = 1.5)$.
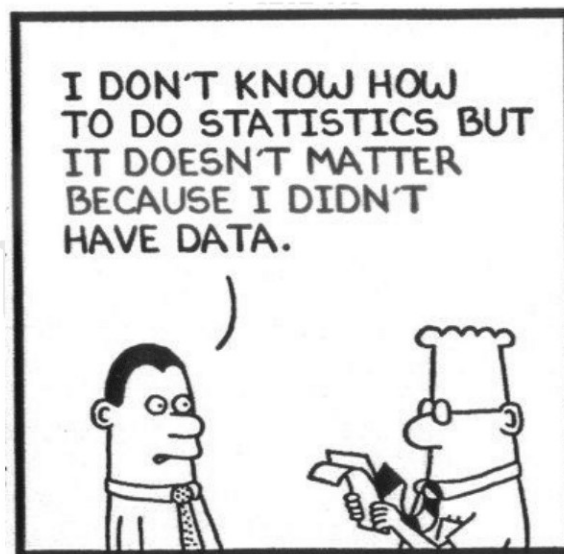
Hint 2 (suggestion): If T has a $\text{Gamma}\left(\alpha, \theta = \dfrac{1}{\lambda}\right)$ distribution, where α is an integer,

then $F_T(t) = P(T \le t) = P(X_t \ge \alpha)$ and $P(T > t) = P(X_t \le \alpha - 1)$,

where X_t has a $\text{Poisson}(\lambda t)$ distribution.

c)   Find the power of the rejection region from part (b) if $\lambda = 1$.

Hint: Power $(\lambda = 1) = P (\text{Reject } H_0 \mid H_0 \text{ is NOT true}) = P (T_4 \geq 5 \mid \lambda = 1)$.

d)   Suppose that the 4th weak-minded lazy Chegg enthusiast shows up at the very end of the 6th minute. Find the p-value of the test.