STAT 410

Homework #04

Fall 2021
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(due Friday, September 24, by 5:00 p.m. CDT)

No credit will be given without supporting work.

4. Every week, Alex receives 1,000 rubles allowance from his parents [1 US dollar ≈ 73 Russian rubles]. He usually spends most of it buying candy. In Alex's favorite candy store, W&W's (a cheap imitation of M&M's) are sold in bulk at 100 rubles per kg, and Яeese's Pieces (knock off Reese's Pieces) are sold at 200 rubles per kg. Alex's Mom is very concerned about this unhealthy habit; she made Alex promise her that he would not buy more than 6 kg of W&W's (she does not know that he also buys Яeese's Pieces). Let X and Y denote the weight (in kg) of W&W's and Яeese's Pieces Alex buys, respectively. Let the joint probability density function for (X, Y) be

$$f(x,y) = \frac{3x+2y}{240}, \qquad x \ge 0, \quad y \ge 0, \quad x \le 6, \quad 100x + 200y \le 1000,$$

zero otherwise.

X – W&W's, Y – Яeese's Pieces.

Recall (Homework #3):

$$f_{Y|X}(y|x) = \frac{12x+8y}{5(2+x)(10-x)}, \qquad 0 < y < \frac{10-x}{2}, \qquad 0 < x < 6.$$

$$E(Y \mid X = x) = \frac{(20+7x)(10-x)}{30(2+x)}, \qquad 0 < x < 6.$$

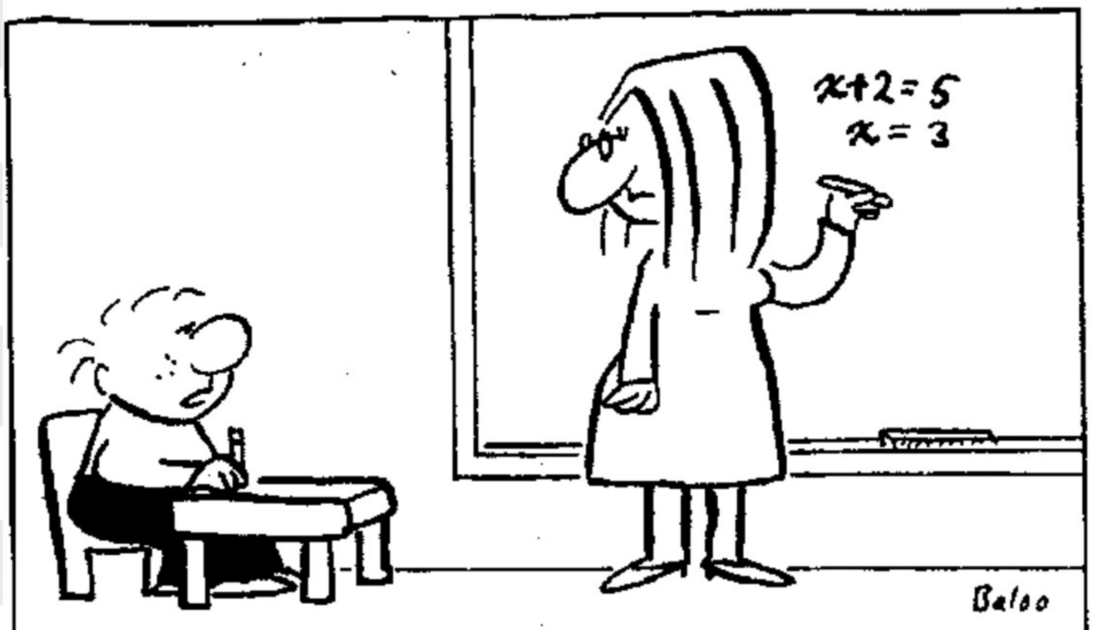
r) Find $\mathrm{Var}(Y \mid X = x)$. You do not have to simplify the final answer.

s) Find the probability distribution of the total weight of candy $W = X + Y$.

t) Let $T = 100\,X + 200\,Y$ denote the amount Alex spends on candy. Find the cumulative distribution function of T, $F_T(t)$.

u) Find $E(T)$, the average (expected) amount Alex spends on candy.

v) Let $V = \dfrac{X}{Y}$. Find the cumulative distribution function of V, $F_V(v)$.



"*Just a darn minute! Yesterday you said X equals two!*"

You are welcome to use a computer and/or calculator on any problem to evaluate any integral. For the supporting work, you should include the full integral (with the function and the bounds) and the answer. For example,

$$\int_0^x u^2\,du = \frac{x^3}{3}, \qquad \int_0^4 \left(\int_0^{\sqrt{x}} x^2\,y\,dy \right) dx = 32, \qquad \int_1^\infty \left(\int_0^y \frac{1}{(2x+y)^3}\,dx \right) dy = \frac{2}{9}.$$