STAT 410 **Homework #01** Fall 2021
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(due Friday, September 3, by 5:00 p.m. CDT)

No credit will be given without supporting work.

1. Grades on Fall 2021 STAT 410 Exam 1 were not very good *. Graphed, their distribution had a shape similar to the probability density function **.

$$f_X(x) = \frac{x+2}{C}, \qquad 16 \le x \le 80, \qquad \text{zero elsewhere.}$$

a) Find the value of C that makes $f_X(x)$ a valid probability density function.

b) Find the cumulative distribution function of X, $F_X(x) = P(X \le x)$.

"Hint": To double-check your answer: should be $F_X(16) = 0$, $F_X(80) = 1$.

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* The probability distribution is fictional, the exam has not happened yet. Hopefully, the actual grades will be slightly better than these.

** Exam scores should have a discrete (instead of continuous) nature. A continuous probability distribution is used as an approximation, since the alternative would have been dealing with a discrete random variable with 65 possible values (16, 17, 18, … , 79, 80), which is not nearly as much fun as I am describing it here.

1. (continued)

As a way of "curving" the results, the instructor announced that he would replace each person's grade, X, with a new grade, $Y = g(X)$, where $g(x) = 8\sqrt{x+20}$.

c) Find the support (the range of possible values) of the probability distribution of Y.

d) Use part (b) and the c.d.f. approach to find the c.d.f. of Y, $F_Y(y)$.

"Hint": $F_Y(y) = P(Y \le y) = P(g(X) \le y) = \ldots$.

e) Use the change-of-variable technique to find the p.d.f. of Y, $f_Y(y)$.

"Hint": $f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right|$.

"Hint": To double-check your answer: should be $f_Y(y) = F_Y'(y)$.

f) Is μ_Y equal to $g(\mu_X)$? $\mu_X = E(X)$, $\mu_Y = E(Y)$.

2. Consider a continuous random variable X with the probability density function

$$f_X(x) = \frac{x+2}{18}, \quad -2 \le x \le 4, \quad \text{zero elsewhere.}$$

Consider $Y = g(X) = \frac{16}{X^2}$. Find the probability distribution of Y.

You are welcome to use a computer and/or calculator on any problem to evaluate any integral. For the supporting work, you should include the full integral (with the function and the bounds) and the answer. For example,

$$\int_0^x u^2\,du = \frac{x^3}{3}, \qquad \int_0^4 \left(\int_0^{\sqrt{x}} x^2 y\,dy \right) dx = 32, \qquad \int_1^\infty \left(\int_0^y \frac{1}{(2x+y)^3}\,dx \right) dy = \frac{2}{9}.$$