

(due Friday, September 10, by 5:00 p.m. CDT)

No credit will be given without supporting work.

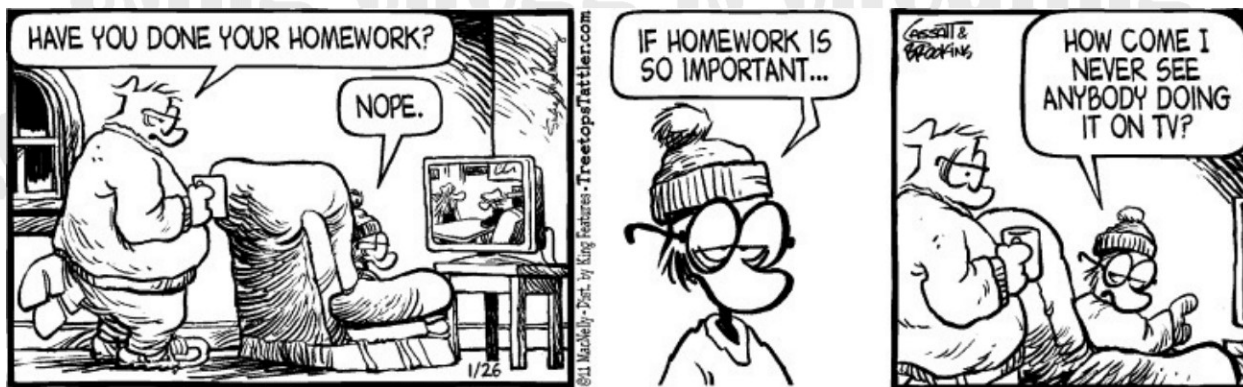
3. Consider a mixed random variable X with the p.m.f. of the discrete portion of the probability distribution

$$p(-2) = c, \quad p(4) = 0.25, \quad \text{zero otherwise,}$$

and the p.d.f. of the continuous portion of the probability distribution

$$f(x) = \frac{x+2}{30}, \quad -2 \leq x \leq 4, \quad \text{zero elsewhere.}$$

- a) Find the value of c that would make this a valid probability distribution.
- b) Find $E(X)$.
- c) Find $\text{Var}(X)$.



You are welcome to use a computer and/or calculator on any problem to evaluate any integral. For the supporting work, you should include the full integral (with the function and the bounds) and the answer.¹⁰ For example,

$$\int_0^x u^2 du = \frac{x^3}{3}, \quad \int_0^4 \left(\int_0^{\sqrt{x}} x^2 y dy \right) dx = 32, \quad \int_1^\infty \left(\int_0^y \frac{1}{(2x+y)^3} dx \right) dy = \frac{2}{9}.$$

4. Every week, Alex receives 1,000 rubles allowance from his parents [1 US dollar \approx 73 Russian rubles]. He usually spends most of it buying candy. In Alex's favorite candy store, W&W's (a cheap imitation of M&M's) are sold in bulk at 100 rubles per kg, and Reese's Pieces (knock off Reese's Pieces) are sold at 200 rubles per kg. Alex's Mom is very concerned about this unhealthy habit; she made Alex promise her that he would not buy more than 6 kg of W&W's (she does not know that he also buys Reese's Pieces). Let X and Y denote the weight (in kg) of W&W's and Reese's Pieces Alex buys, respectively. Let the joint probability density function for (X, Y) be

$$f(x, y) = \frac{3x+2y}{C}, \quad x \geq 0, \quad y \geq 0, \quad x \leq 6, \quad 100x + 200y \leq 1000, \\ \text{zero otherwise.}$$

X – W&W's, Y – Reese's Pieces.

- Sketch the support of (X, Y) .
That is, sketch $\{ (x, y) : x \geq 0, y \geq 0, x \leq 6, 100x + 200y \leq 1000 \}$.
- Find the value of C so that $f(x, y)$ is a valid joint probability density function.
- Find the marginal probability density function of X , $f_X(x)$.
Be sure to include its support.
- Find the marginal probability density function of Y , $f_Y(y)$.
Be sure to include its support.
“Hint”: It would be **wise** to break this problem into **pieces**.
- Find the probability that the total weight of candy that Alex buys is over 4.0 kg. That is, find $P(X + Y > 4.0)$.
- Find the probability that the total weight of candy that Alex buys is over 5.4 kg. That is, find $P(X + Y > 5.4)$.
- Find the probability that the total weight of candy that Alex buys is over 7.0 kg. That is, find $P(X + Y > 7.0)$.