

# STAT 410 - Section 1 - Fall 2021 Homework #02

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TOTAL POINTS

**10 / 10**

QUESTION 1

**13 2.5 / 2.5**

✓ - **0 pts** Correct

- **0.5 pts** Wrong method for (a)
- **0.5 pts** Wrong final answer for (a)
- **0.5 pts** Wrong method for (b)
- **0.5 pts** Wrong final answer for (b)
- **0.5 pts** Wrong method for (c)
- **0.5 pts** Wrong final answer for (c)
- **1 pts** Some of the process was right but arithmetic miss

QUESTION 2

**4 7.5 pts**

**2.1 4ab 1.5 / 1.5**

✓ - **0 pts** Correct

- **0.5 pts** a) Region not correct
- **0.5 pts** b) Answer is incorrect.
- **1.5 pts** No submission

**2.2 4cd 3 / 3**

✓ - **0 pts** Correct

- **1 pts** d) Missing one part
- **0.5 pts** d) One part not correct

**2.3 4efg 3 / 3**

✓ - **0 pts** Correct

- **1 pts** Wrong limits of integral & wrong answer in 4e
- **1 pts** Wrong limits of integral & wrong answer in 4f
- **1 pts** Wrong limits of integral & wrong answer in 4g

STAT-410 HW-2

$$3.a) p(-2) + p(4) + \int_{-2}^4 f(x) dx = 1$$

$$C + .25 + \int_{-2}^4 \frac{x+2}{30} dx = 1$$

$$C + .25 + \frac{1}{30} \left[ \frac{x^2}{2} + 2x \right]_{-2}^4 = 1$$

$$C + \frac{1}{30} \left( \frac{4^2 - 2^2}{2} + 2(4+2) \right) = .75$$

$$C = 0.15$$

$$b) E(X) = -2 \cdot p(-2) + 4 \cdot p(4) + \int_{-2}^4 x \cdot f(x) dx$$

$$= -2(.15) + 4(.25) + \frac{1}{30} \int_{-2}^4 (x^2 + 2x) dx$$

$$= -.30 + 1 + \frac{1}{30} \left[ \frac{x^3}{3} + x^2 \right]_{-2}^4$$

$$= .7 + \frac{1}{30} \left[ \frac{4^3 - 2^3}{3} + 4^2 - 2^2 \right]$$

$$= 1.9$$

$$c) \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = (-2)^2 \cdot p(-2) + 4^2 \cdot p(4) + \int_{-2}^4 x^2 f(x) dx$$

$$= 4(.15) + 16(.25) + \frac{1}{30} \int_{-2}^4 (x^3 + 2x^2) dx$$

$$= .6 + 4 + \frac{1}{30} \left[ \frac{x^4}{4} + \frac{2x^3}{3} \right]_{-2}^4$$

$$= .6 + 4 + 3.6$$

$$= 8.2$$

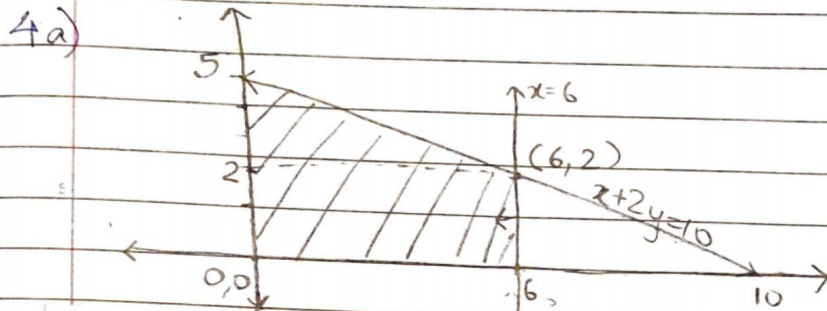
$$\text{Var}(X) = 8.2 - (1.9)^2$$

$$= 4.59$$

13 2.5 / 2.5

✓ - 0 pts Correct

- 0.5 pts Wrong method for (a)
- 0.5 pts Wrong final answer for (a)
- 0.5 pts Wrong method for (b)
- 0.5 pts Wrong final answer for (b)
- 0.5 pts Wrong method for (c)
- 0.5 pts Wrong final answer for (c)
- 1 pts Some of the process was right but arithmetic miss



b)

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) dx dy = 1$$

$$\int_0^6 \left( \int_0^{5-x/2} \left( \frac{3x+2y}{c} \right) dy \right) dx = 1$$

$$\int_0^6 \frac{1}{c} \left[ 3xy + y^2 \right]_0^{5-x/2} dx = 1$$

$$\int_0^6 3x \left( 5 - \frac{x}{2} \right) + 25 + \frac{x^2}{4} - 5x dx = c$$

$$\int_0^6 -\frac{5x^2}{4} + 10x + 25 = c$$

$$-\frac{5 \times 6^3}{4 \times 3} + \frac{10}{2} (6^2) + 25(6) = c = 240$$

2.14ab 1.5 / 1.5

✓ - 0 pts Correct

- 0.5 pts a) Region not correct
- 0.5 pts b) Answer is incorrect.
- 1.5 pts No submission

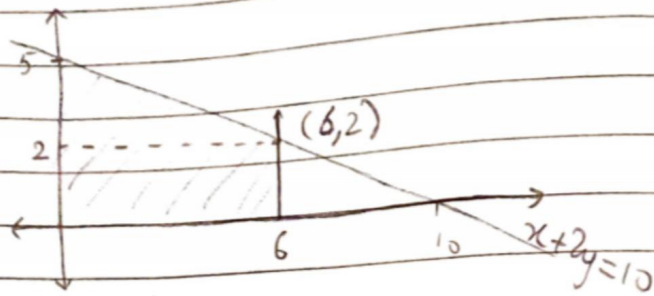
$$c) f_x(x) = \int_0^{5-x/2} \frac{3x+2y}{240} dy, \quad 0 \leq x \leq 6$$

$$= \frac{1}{240} \left[ 3xy + y^2 \right]_0^{5-x/2}, \quad 0 \leq x \leq 6$$

$$= \frac{1}{240} \left[ 3x\left(5-\frac{x}{2}\right) + 25 + \frac{x^2}{4} - 5x \right]$$

$$f_x(x) = \frac{1}{48} \left( 2x - \frac{x^2}{4} + 5 \right), \quad 0 \leq x \leq 6$$

d)



$$f_x(y) = \int_0^6 \frac{3x+2y}{240} dx \quad 0 \leq y < 2 \quad - (A)$$

$$\int_0^{10-2y} \frac{3x+2y}{240} dx \quad 2 \leq y \leq 5 \quad - (B)$$

Calculating (A)

$$(A) = \frac{1}{240} \left[ \frac{3x^2}{2} + 2yx \right]_0^6 \quad 0 \leq y < 2$$

$$A = (2y+9)/40$$

$$0 \leq y < 2$$

Calculating (B)

$$(B) = \frac{1}{240} \left[ \frac{3x^2}{2} + 2yx \right]_0^{10-2y} \quad 2 \leq y \leq 5$$

$$B = (y^2 - 20y + 75)/120$$

$$2 \leq y \leq 5$$

$$f_x(y) = \begin{cases} (2y+9)/40 \\ (y^2 - 20y + 75)/120 \end{cases}$$

$$0 \leq y < 2$$

$$2 \leq y \leq 5$$

2.2 4cd 3 / 3

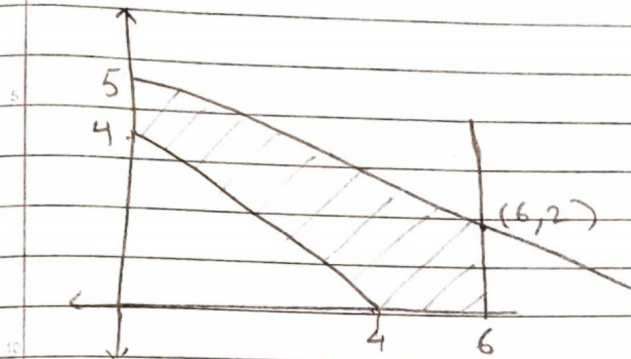
✓ - 0 pts Correct

- 1 pts d) Missing one part

- 0.5 pts d) One part not correct



c)  $P(X+Y > 4)$



$$P(X+Y > 4) = 1 - \int_0^4 \int_0^{4-x} \frac{3x+2y}{240} dy dx$$

$$= 1 - \int_0^4 \frac{1}{240} \left[ 3xy + y^2 \right]_0^{4-x} dx$$

$$= 1 - \frac{1}{240} \int_0^4 (3x(4-x) + 16 + x^2 - 8x) dx$$

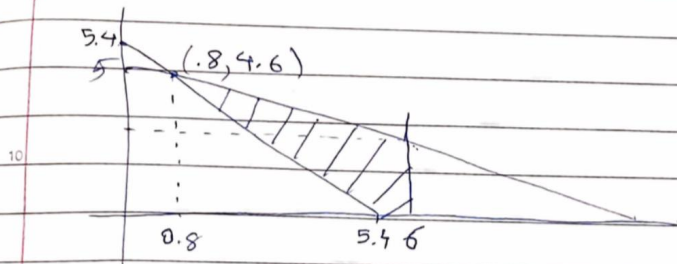
$$= 1 - \frac{2}{240} \int_0^4 (-x^2 + 2x + 8) dx$$

$$= 1 - \frac{1}{120} \left[ -\frac{x^3}{3} + 4x^2 + 8(4) \right]$$

$$= 1 - \frac{16}{120} \left[ -\frac{4}{3} + 1 + 2 \right] = 1 - \frac{2}{15} \left( \frac{8}{3} \right) = 1 - \frac{2}{9}$$

$$P(X+Y > 4) = 7/9$$

f)  $P(X+Y > 5.4)$



$$P(X+Y > 5.4) = \iint_{0.54-y}^6 f(x, y) dx dy + \iint_{4.6}^{10-2y} f(x, y) dx dy$$

$$= \frac{1}{240} \int_0^2 \int_{5.4-y}^6 (3x+2y) dx dy + \frac{1}{240} \int_{4.6}^{10-2y} (3x+2y) dx dy$$

$$= \frac{1}{240} \int_0^2 \left[ \frac{3x^2}{2} + 2xy \right]_{5.4-y}^6 dy + \frac{1}{240} \int_2^{4.6} \left[ \frac{3x^2}{2} + 2xy \right]_{5.4-y}^{10-2y} dy$$

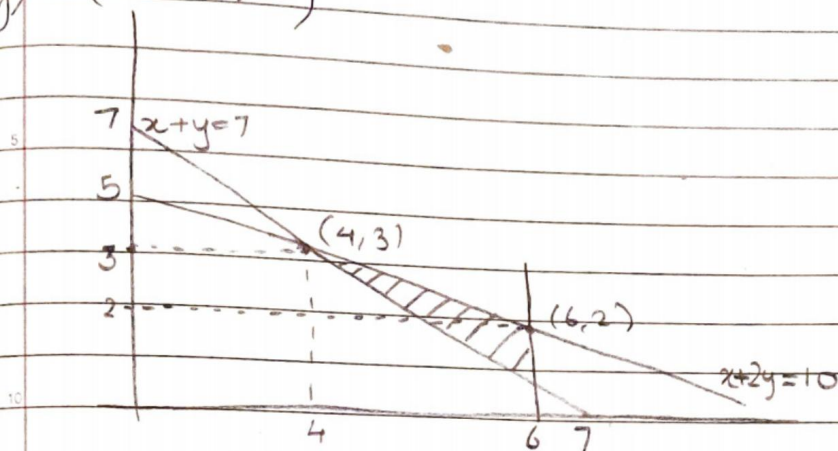
$$= \frac{1}{240} \int_0^2 (0.5y^2 + 17.4y + 10.26) dy + \frac{1}{240} \int_2^{4.6} (2.5y^2 - 34.6y + 106.26) dy$$

$$= \frac{1}{240} \left[ \frac{0.5y^3}{3} + \frac{17.4y^2}{2} + 10.26y \right]_0^2 + \frac{1}{240} \left[ \frac{2.5y^3}{3} - \frac{34.6y^2}{2} + 106.26y \right]_2^{4.6}$$

$$= 0.23605 + 0.22439$$

$$= 0.460445$$

g)  $P(X+Y > 7)$



$$P(X+Y > 7) = \int_4^6 \int_{7-x}^{5-x/2} \left( \frac{3x+2y}{240} \right) dy dx$$

$$= \frac{1}{240} \int_4^6 \left[ 3xy + y^2 \right]_{7-x}^{5-x/2} dx$$

$$= \frac{1}{240} \int_4^6 \left( 3x(5-x/2-7+x) + 25 + x^2/4 - 5x - 49 - x^2 + 14x \right) dx$$

$$= \frac{1}{240} \int_4^6 (0.75x^2 + 15x - 24) dx$$

$$= \frac{1}{240} \left[ \frac{0.75}{3} (6^3 - 4^3) + \frac{15}{2} (6^2 - 4^2) - 24(2) \right]$$

$$= \frac{1}{240} \left[ \frac{266}{3} - \frac{150}{5} - 48 \right]$$

$$= \frac{1}{240} [38 + 30 - 48] = \frac{1}{12} = 0.0833$$

### 2.3 4efg 3 / 3

✓ - 0 pts Correct

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