STAT 410 | Fall 2021
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Homework #06

(due Friday, October 15, by 5:00 p.m. CDT)

No credit will be given without supporting work.

1. Grades on Fall 2021 STAT 410 Exam 1 were not very good *. Graphed, their distribution had a shape similar to the probability density function **.

$$f_X(x) = \frac{x+2}{3,200}, \qquad 16 \le x \le 80, \qquad \text{zero elsewhere.}$$

Recall (Homework #1):

$$F_X(x) = \frac{x^2 + 4x - 320}{6,400} = \frac{(x-16)(x+20)}{6,400}, \qquad 16 \le x < 80.$$

Six exam papers were selected at random. That is, let $X_1, X_2, X_3, X_4, X_4, X_6$ be a random sample (i.i.d.) of size $n = 6$ from the above probability distribution. Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5 < Y_6$ be the corresponding order statistics.

g) Find the probability that the largest score of these 6 papers is above 68. That is, find $P(Y_6 > 68) = P(\max X_i > 68)$.

h) Find the probability that the lowest score of these 6 papers is below 36. That is, find $P(Y_1 < 36) = P(\min X_i < 36)$.

i) Find the probability that the second lowest score of these 6 papers is above 52. That is, find $P(Y_2 > 52)$.

j) Find the probability that the third largest score of these 6 papers (the fourth lowest score) is above 60. That is, find $P(Y_4 > 60)$.

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* The probability distribution is fictional, the exam has not happened yet. Hopefully, the actual grades will be slightly better than these.

** Exam scores should have a discrete (instead of continuous) nature. A continuous probability distribution is used as an approximation, since the alternative would have been dealing with a discrete random variable with 65 possible values (16, 17, 18, … , 79, 80), which is not nearly as much fun as I am describing it here.

5. Mary had a little lamb (we all know the rest). Realizing that she has more love to give, Mary decided to purchase more livestock. At a local market, the prices for a calf (baby cow), C, and a kid (baby goat), K, vary from day to day and jointly follow a bivariate normal distribution with

$$\mu_C = \$434, \quad \sigma_C = \$10, \quad \mu_K = \$222, \quad \sigma_K = \$3, \quad \rho_{CK} = 0.60.$$

a) Find the probability that on a given day, a calf costs more than \$430. That is, find $P(C > 430)$.

b) Find the probability that on a given day, a kid costs more than \$225. That is, find $P(K > 225)$.

c) Suppose that on a given day, a calf costs \$430. Find the probability that a kid costs more than \$225. That is, find $P(K > 225 \mid C = 430)$.

d) Find the probability that on a given day, a calf costs more than two kids. That is, find $P(C > 2K)$.

e) Mary buys 5 calves and 4 kids. Find the probability that she paid more than \$3,000. That is, find $P(5C + 4K > 3{,}000)$.

5. (continued)

Suppose that the price of a calf, C, the price of a kid K, and the price of a lamb (baby sheep), L, [in dollars] jointly follow $N_3(\mu, \Sigma)$ distribution with

$$\mu = \begin{pmatrix} \mu_C \\ \mu_K \\ \mu_L \end{pmatrix} = \begin{pmatrix} 434 \\ 222 \\ 331 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 100 & 18 & 24 \\ 18 & 9 & 9 \\ 24 & 9 & 36 \end{pmatrix}.$$

f) What is the value of ρ_{CL}?

g) Find the probability that on a given day, a lamb costs more than \$334. That is, find $P(L > 334)$.

h) Mary buys 5 calves, 4 kids, and 3 lambs. Find the probability that she paid more than \$4,000. That is, find $P(5C + 4K + 3L > 4{,}000)$.