

STAT 410 - Section 1 - Fall 2021 Homework #05

Sharvi Tomar

TOTAL POINTS

10 / 10

QUESTION 1

4 10 pts

1.1 4w 2.5 / 2.5

✓ - **0 pts** Correct

- **0.5 pts** Marked the wrong region in graph
- **1 pts** Have drawn a wrong graph
- **0.5 pts** Wrong Jacobian

1.2 4x 1 / 1

✓ - **0 pts** Correct

- **0.5 pts** Incorrect integral limit or answer for $0 < t < 600$
- **0.5 pts** Incorrect integral limit or answer for $600 < t < 1000$
- **0.5 pts** Wrong setting or not included support
- **1 pts** No valid work

1.3 4y 2.5 / 2.5

✓ - **0 pts** Correct

- **0.5 pts** Jacobian not correct
- **1 pts** Jacobian missing
- **0.5 pts** Answer not correct
- **1 pts** Graph missing
- **0.5 pts** Support in graph not correct

1.4 4z 1.5 / 1.5

✓ - **0 pts** Correct

- **0.5 pts** Incorrect integral limits or answer for $0 < v < 3$
- **0.5 pts** Incorrect integral limits or answer for $v > 3$
- **1 pts** Wrong initial setting or wrong support
- **1.5 pts** No valid work

1.5 4aa 2.5 / 2.5

✓ - **0 pts** Correct

- **1 pts** No works for Jacobian
- **0.5 pts** Used wrong Jacobian
- **1 pts** Wrong sketch for the graph
- **0.5 pts** Wrong final joint pdf
- **2.5 pts** No valid work

$$w) T = 100X + 200Y$$

$$0 \leq x \leq 6$$

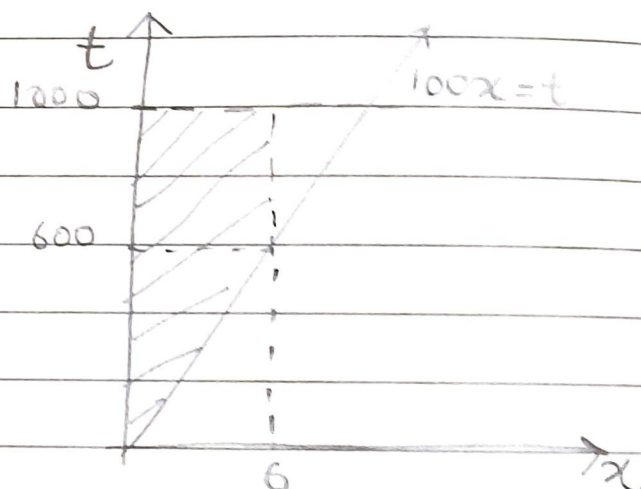
$$0 \leq y \leq 5 - x/2, \quad Y = \frac{T}{200} - \frac{X}{2}$$

$$0 \leq \frac{t}{200} - \frac{x}{2} \leq 5 - \frac{x}{2}$$

$$100x \leq t \leq 1000$$

$$J = \begin{vmatrix} \frac{\partial T}{\partial x} & \frac{\partial T}{\partial t} \\ \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial t} \end{vmatrix} = \begin{vmatrix} 100 & 1 \\ -1/2 & 1/200 \end{vmatrix}$$

$$|J| = 1/200$$



$$f_{X,T}(x,t) = f_{X,Y}\left(x, \frac{t}{200} - \frac{x}{2}\right) |J|$$

$$= \left[\frac{3x + 2\left(\frac{t}{200} - \frac{x}{2}\right)}{240} \right] \cdot \frac{1}{200}$$

$$= \frac{2x + 0.01t}{48000}$$

$$f_{X,T}(x,t) = \frac{2x + 0.01t}{48000}, \quad 0 \leq x \leq 6, \quad 100x \leq t \leq 1000$$

1.1 4w 2.5 / 2.5

✓ - 0 pts Correct

- 0.5 pts Marked the wrong region in graph

- 1 pts Have drawn a wrong graph

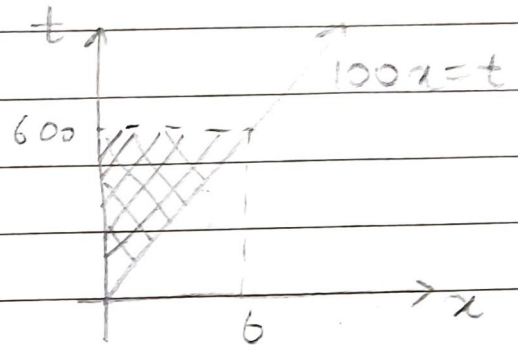
- 0.5 pts Wrong Jacobian

$$2) f_T(t)$$

Case-1 $0 \leq t \leq 600$

$$f_T(t) = \int_0^{t/100} \frac{2x + 0.01t}{48000} dx$$

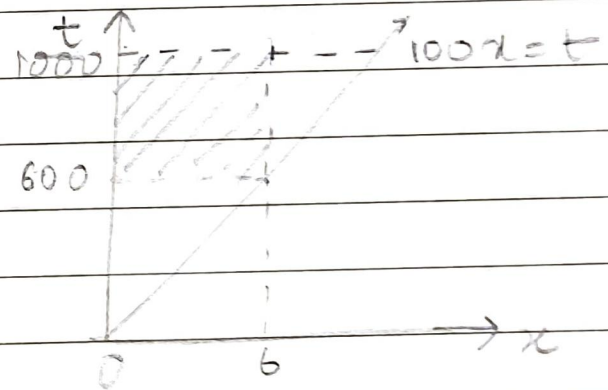
$$= \frac{t^2}{240000000}$$



Case-2 $600 \leq t < 1000$

$$f_T(t) = \int_0^6 \frac{2x + 0.01t}{48000} dx$$

$$= \frac{t + 600}{800000}$$



$$f_T(t) = \begin{cases} \frac{t^2}{240000000} \\ \frac{t + 600}{800000} \\ 0 \end{cases}$$

$$0 \leq t < 600$$

$$600 \leq t < 1000$$

elsewhere

1.2 4x 1 / 1

✓ - 0 pts Correct

- 0.5 pts Incorrect integral limit or answer for $0 < t < 600$
- 0.5 pts Incorrect integral limit or answer for $600 < t < 1000$
- 0.5 pts Wrong setting or not included support
- 1 pts No valid work

$$y) \quad V = X/Y$$

$$0 \leq x \leq 6$$

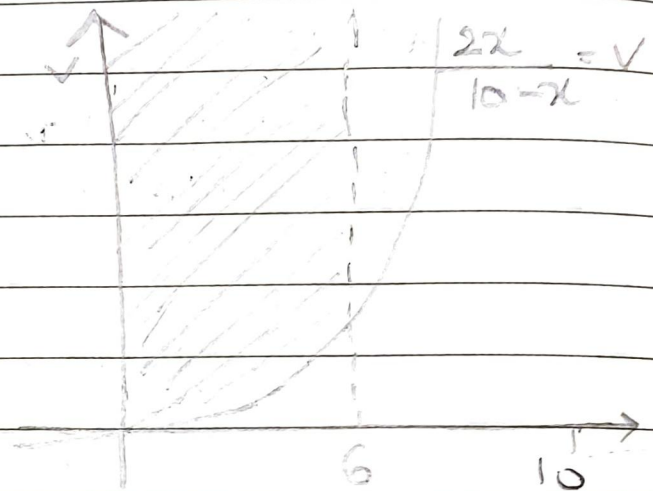
$$0 \leq y \leq 5 - x/2, \quad Y = X/V$$

$$0 \leq \frac{x}{v} \leq 5 - \frac{x}{2}, \quad v \geq 0$$

$$\frac{x}{v} \leq 5 - \frac{x}{2}$$

$$2x \leq v(10 - x)$$

$$\frac{2x}{10 - x} \leq v, \quad \cancel{v \leq 10}$$



$$J = \begin{vmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 1/v & -x/v^2 \end{vmatrix} = -\frac{x}{v^2}, \quad |J| = \left| -\frac{x}{v^2} \right|$$

$$f_{x,v}(x, v) = f_{x,y}(x, x/v) \cdot |J|$$

$$= \frac{3x + 2x/v}{240} \cdot \left| -\frac{x}{v^2} \right|$$

$$x > 0, v > 0$$

$$\text{so } \frac{x}{v^2} > 0$$

$$= \frac{x^2(3v + 2)}{240 v^3}, \quad 0 \leq x \leq 6$$

$$0 \leq v \leq \frac{2x}{10 - x}$$

1.3 4y 2.5 / 2.5

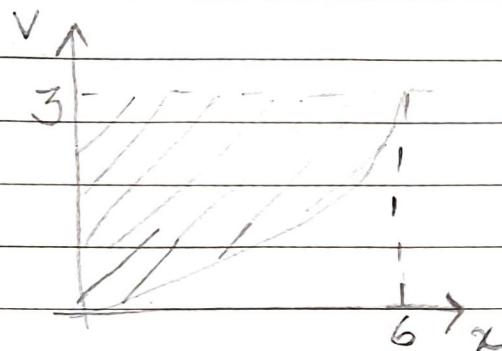
✓ - 0 pts Correct

- 0.5 pts Jacobian not correct
- 1 pts Jacobian missing
- 0.5 pts Answer not correct
- 1 pts Graph missing
- 0.5 pts Support in graph not correct

2) Case - 1 $0 \leq v < 3$

$$f_v(v) = \int_0^{\frac{10v}{2+v}} \frac{x^2(3v+2)}{240v^3} dx$$

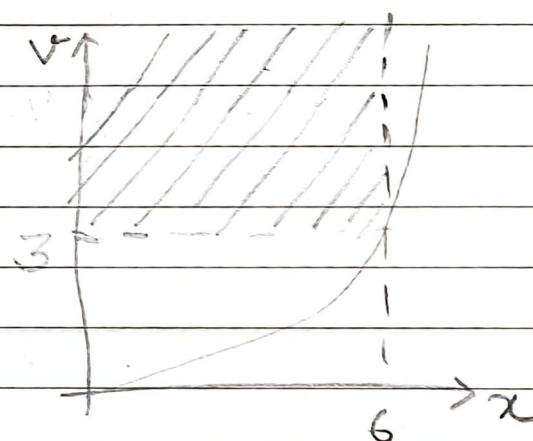
$$= \frac{25(3v+2)}{18(v+2)^3}$$



Case - 2 $v > 3$

$$f_v(v) = \int_0^6 \frac{x^2(3v+2)}{240v^3} dx$$

$$= \frac{3(3v+2)}{10v^3}$$



$$f_v(v) = \frac{25(3v+2)}{18(v+2)^3}$$

$$0 \leq v < 3$$

$$\frac{3(3v+2)}{10v^3}$$

$$v \geq 3$$

0

elsewhere

1.4 4z 1.5 / 1.5

✓ - 0 pts Correct

- 0.5 pts Incorrect integral limits or answer for $0 < v < 3$
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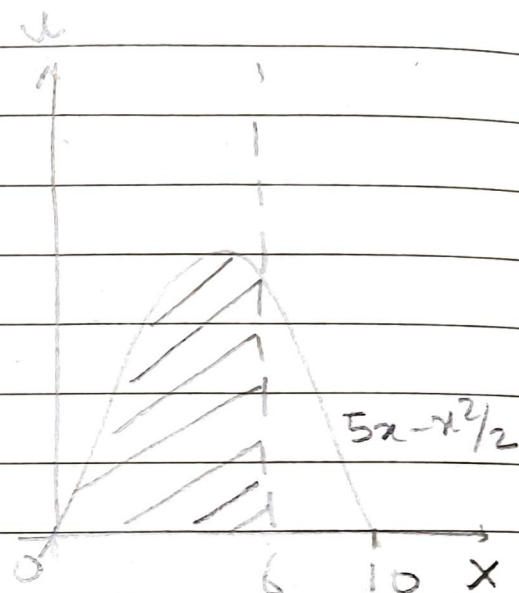
aa) $U = X \cdot Y$

$X = X, \quad 0 \leq x \leq 6$

$Y = U/X, \quad 0 \leq \frac{U}{x} \leq 5 - \frac{x}{2}$

$0 \leq \frac{u}{x} \Rightarrow u \geq 0$

$u \leq 5x - \frac{x^2}{2}$



$J = \begin{vmatrix} x & 1 & 0 \\ y & -\frac{u}{x^2} & \frac{1}{x} \end{vmatrix} = \frac{1}{x}$

$f_{X,U}(x,u) = f_{X,Y}(x, \frac{u}{x}) \cdot |J|$

$= \frac{3x + 2(u/x)}{240} \cdot \frac{1}{x}$

$= \frac{3x^2 + 2u}{240x^2}, \quad \begin{matrix} 0 \leq x \leq 6 \\ 0 \leq u \leq 5x - \frac{x^2}{2} \end{matrix}$

1.5 4aa 2.5 / 2.5

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