## Homework #10

Fall 2021 A. Stepanov

(due Friday, November 12, by 5:00 p.m. CST)

## No credit will be given without supporting work.

7. Let  $\psi > 0$  and let  $X_1, X_2, \dots, X_n$  be a random sample from a probability distribution with probability density function

$$f(x; \psi) = \frac{\psi}{2\psi} \cdot (2-x)^{\psi-1}, \qquad 0 < x < 2,$$
 zero otherwise.

Recall:  $W = -\ln\left(1 - \frac{X}{2}\right)$  has an Exponential distribution with mean  $\theta = \frac{1}{\psi}$ 

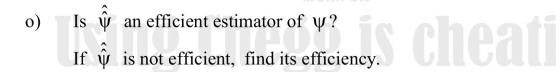
$$\hat{\hat{\psi}} = \frac{n-1}{\sum_{i=1}^{n} \left(-\ln\left(1 - \frac{X_i}{2}\right)\right)}$$
 is an unbiased estimator of  $\psi$ .

k) Suggest a confidence interval for  $\psi$  with  $(1 - \alpha) 100 \%$  confidence level.

① Use 
$$Y = \sum_{i=1}^{n} \left( -\ln\left(1 - \frac{X_i}{2}\right) \right)$$
.

- ② If T has a Gamma  $(\alpha, \theta = 1/\lambda)$  distribution, then  $2T/\theta = 2\lambda T$  has a  $\chi^2(2\alpha)$  distribution.
- 1) Suppose n = 3, and  $x_1 = 0.62$ ,  $x_2 = 1.54$ ,  $x_3 = 1.86$ . Use part (k) to construct a 90% confidence interval for  $\psi$ .
- m) Find a sufficient statistic  $u(X_1, X_2, ..., X_n)$  for  $\psi$ .

n) Find the Fisher information  $I(\psi)$ .



- ① Find  $Var(\hat{\psi})$ . ("Hint": Recall Homework #08 problem 7 part (g).)
- 2 Find the Rao-Cramér lower bound.
- Is  $\hat{\psi}$  an efficient estimator of  $\psi$ ? Does Var( $\hat{\psi}$ ) attain the R.C.L.B.? If  $\hat{\psi}$  is not efficient, find its efficiency.

8. Let  $\xi > 0$  and let  $X_1, X_2, \dots, X_n$  be a random sample from a probability distribution with probability density function

$$f(x;\xi) = \frac{1}{2} \xi^4 x^{11} e^{-\xi x^3}, \quad x > 0,$$
 zero elsewhere.

Recall:  $W = X^3$  has a Gamma ( $\alpha = 4, \theta = \frac{1}{\xi}$ ) distribution.

$$\hat{\xi} = \frac{4n-1}{\sum_{i=1}^{n} X_i^3}$$
 is an unbiased estimator of  $\xi$ .

- h) Suggest a confidence interval for  $\xi$  with  $(1-\alpha)100\%$  confidence level.
  - ① Use  $Y = \sum_{i=1}^{n} X_{i}^{3}$ .
  - ② If T has a Gamma  $(\alpha, \theta = 1/\lambda)$  distribution, then  $2T/\theta = 2\lambda T$  has a  $\chi^2(2\alpha)$  distribution.

- i) Suppose n = 5,  $x_1 = 0.3$ ,  $x_2 = 0.6$ ,  $x_3 = 1.2$ ,  $x_4 = 1.3$ ,  $x_5 = 1.8$ . Use part (h) to construct a 90% confidence interval for  $\xi$ .
- j) Find a sufficient statistic  $u(X_1, X_2, ..., X_n)$  for  $\xi$ .
- k) Find the Fisher information  $I(\xi)$ .

(After you are done with part (k), glance back at Homework #09 problem 8 part (g).)

1) Is  $\hat{\xi}$  an efficient estimator of  $\xi$ ?

If  $\hat{\xi}$  is not efficient, find its efficiency.

- ① Find  $Var(\hat{\xi})$ . ("Hint": Recall Homework #08 problem 8 part (d).)
- ② Find the Rao-Cramér lower bound.
- Is  $\hat{\xi}$  an efficient estimator of  $\xi$ ? Does  $Var(\hat{\xi})$  attain the R.C.L.B.? If  $\hat{\xi}$  is not efficient, find its efficiency.

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9. Let  $\lambda > 0$  and let  $X_1, X_2, ..., X_n$  be a random sample from a probability distribution with probability density function

$$f(x; \lambda) = \frac{\lambda}{x^2}$$
,  $x \ge \lambda$ , zero otherwise.

d) Find a sufficient statistic  $u(X_1, X_2, ..., X_n)$  for  $\lambda$ .