HW-1

Friday, September 9, 2022   2:44 PM

1) Let $C = 1$ if there is a convention downtown
& $C = 0$ otherwise.

Let Y be the likelihood for the commute time

Given that,

$Y|C=0 \sim \text{Uniform}(15,20)$

$Y|C=1 \sim \text{Uniform}(15,30)$

$P(C=1) = 1/4$

$P(C=0) = 3/4$

a) Using Bayes' Rule,

$$P(C=1|Y=18) = \frac{P(C=1, Y=18)}{P(Y=18)}$$

$$= \frac{f(18|C=1) * \pi(C=1)}{f(18|C=1) * \pi(C=1) + f(18|C=0) * \pi(C=0)}$$

$$= \frac{\frac{1}{15} * \frac{1}{4}}{\frac{1}{15} * \frac{1}{4} + \frac{1}{5} * \frac{3}{4}} = 0.1$$

$$\boxed{P(C=1|Y=18) = 0.1}$$

b) Using Bayes' Rule,

$$P(C=1|Y=28) = \frac{P(C=1, Y=28)}{P(Y=28)}$$

$$= \frac{f(28|C=1) * \pi(C=1)}{f(28|C=1) * \pi(C=1) + \underbrace{\cancel{f(28|C=0) * \pi(C=0)}}_{=0}} \to 0$$

$=0$ as commute time lies b/w 15–20 in case there is no convention & hence $Y = 28$ is not possible.

$$= \frac{f(28|C=1) * \pi(C=1)}{f(28|C=1) * \pi(C=1)}$$

$$= 1$$

$$\boxed{P(C=1|Y=28) = 1}$$

2) a) i) $P(X_1=1)$

ii) $P(X_2=0|X_1=1)$

iii) $P(X_2=0)$

iv) $P(X_1=1 | X_2=0)$

b) i) $P(X_1=1) = \theta$

ii) $P(X_2=0|X_1=1) \sim \text{Binomial}(n, \lambda)$

$= {}^nC_n (1-\lambda)^n$

$= (1-\lambda)^n$

iii) $P(X_2=0)$

Summing the probability of X_2 $[P(X_2=0)]$ over all values of X_1 $[0,1]$

$= P(X_2=0|X_1=0) + P(X_2=0|X_1=1)$

$= (1-\theta) + \theta(1-\lambda)^n$

iv) $P(X_1|X_2) = \dfrac{P(X_2|X_1) * P(X_1)}{P(X_2)}$

$P(X_1=1|X_2=0) = \dfrac{P(X_2=0|X_1=1) * P(X_1=1)}{P(X_2=0)}$

$= \dfrac{(1-\lambda)^n \theta}{(1-\lambda)^n \theta + (1-\theta)}$

3. a) · Marginal density of X

$$f_X(x) = \begin{cases} e^{-x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

· Conditional density of Y given X=x

$$f(Y=y|X=x) = \frac{x^y e^{-x}}{y!} \quad , y=0,1,2\ldots$$

b) Marginal density of Y

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$f_{X,Y}(x,y) = f_{Y|X}(y|x) * f_X(x)$$

$$= \frac{x^y e^{-2x}}{y!}$$

$$f_Y(y) = \int_0^\infty \frac{x^y e^{-2x}}{y!} dx$$

$$= \frac{1}{2^{(y+1)}}$$

c) $f_Y(y) = \dfrac{1}{2^{(y+1)}}$

Geometric Distribution

$P(X=\lambda) = (1-p)^{\lambda-1} p$

$\text{Prob}(X = 1+\lambda) = \left(1-\frac{1}{2}\right)^{\lambda+1-1} \cdot \left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)^\lambda \left(\frac{1}{2}\right)$

$= \dfrac{1}{2^{1+\lambda}}$

$E(X) = 1/p = \dfrac{1}{1/2} = 2$

$\therefore$ Y $\sim$ follows Geometric Distribution.