STAT 431 Exam Prep — Sample Problems

Fall 2022

True or False: A discrete prior always leads to a discrete posterior (assuming the posterior exists).

Circle the ONE BEST answer:

An improper prior may lead to a posterior that is ...

proper   improper   either of these   neither of these

Fill in the blank:

To make inference about a new unobserved random variable from the same model as the data, a Bayesian uses a ________________________ distribution.

Give an example of an inferential statement that a Bayesian **could** make, but a (pure) frequentist **could not** make.

Fully specify the **conditional** distribution of X given $Y = y$ if their joint density is

$$f(x, y) \quad \propto \quad ye^{-2xy}, \qquad x > 0, \quad 0 < y < 1$$

Alice and Ben have the same data ($\boldsymbol{y}$) and model: one normal sample with *known* mean. However, Alice parameterizes the model using the variance σ^2, while Ben uses the precision τ^2.

Each of them uses a Bayesian analysis, but Alice uses **Jeffreys'** prior on σ^2, while Ben uses **Jeffreys'** prior on τ^2.

Alice finds that

$$\mathrm{Prob}_A\big(\sigma^2 > 2 \mid \boldsymbol{y}\big) \;=\; 0.1$$

What will Ben find for

$$\mathrm{Prob}_B\big(\tau^2 \geq 1/2 \mid \boldsymbol{y}\big)?$$

Scalar parameter θ has posterior density $p(\theta \mid \boldsymbol{y})$ defining a distribution from which this simulation sample was obtained:

$$\theta^{(1)}, \;\; \theta^{(2)}, \;\; \ldots, \;\; \theta^{(S)}$$

Which (if any) of these would be used to approximate

$$\mathrm{Prob}(\theta > 0 \mid \boldsymbol{y}) \quad ?$$

$$\frac{1}{S}\sum_{s=1}^{S} p(\theta^{(s)} \mid \boldsymbol{y}) \qquad\qquad \frac{1}{S}\sum_{s=1}^{S} I(\theta^{(s)} > 0)\, p(\theta^{(s)} \mid \boldsymbol{y})$$

$$\frac{1}{S}\sum_{s=1}^{S} I(\theta^{(s)} > 0) \qquad\qquad \frac{1}{S}\sum_{s=1}^{S} I(\theta^{(s)} > 0)\, \theta^{(s)}$$

$$\frac{1}{S}\sum_{s=1}^{S} \theta^{(s)} \qquad\qquad \frac{1}{S}\sum_{s=1}^{S} \theta^{(s)}\, p(\theta^{(s)} \mid \boldsymbol{y})$$