STAT 431 — Applied Bayesian Analysis — Fall 2022

Homework 5

Due: November 11, 11:59 PM (US Central)

In your submission, please include any computer code and output that you used to answer the following problems. Any numerical values should be accurate to at least three significant digits, unless otherwise stated.

1. The data in file `airlinerdata.txt` are the same as used in R Example 3.5 from lecture. Consider this extended Bayesian Poisson hierarchical model for passenger-fatal event rates of different airliner types:

$$\begin{aligned} Y_i \mid \lambda_i &\sim \textit{indep} \text{ Poisson}(N_i \lambda_i) \\ \lambda_i \mid \alpha, \beta &\sim \textit{iid} \text{ Gamma}(\alpha, \beta) \\ \alpha, \beta &\sim \textit{iid} \text{ Gamma}(0.001, 0.001) \end{aligned}$$

   Note that this model uses a relatively "vague" choice for the hyperpriors.

   (a) [3 pts] Draw a directed acyclic graph (DAG) appropriate for this model. (Use the notation introduced in lecture, including "plates".)

   (b) [3 pts] Examine the provided JAGS code file `prob1model.bug`, which describes this model. Carefully note the names of the variables. Then set up and perform a preliminary run of this model using `rjags`, to check for convergence of the sampler. Use three chains, separately initialized. Report the following:
   - a list of the initial values you used (at least for the hyperparameters)
   - the plots you used to monitor and confirm convergence
   - an explicit choice of the number of iterations you will burn (i.e., exclude from using for inference)

   (c) [4 pts] Perform at least 100000 iterations per chain (after burn-in), and produce a summary of your inference results for α and β. Report the following:
   - a copy of the summary output given by R from which you can obtain approximations of posterior expected values, posterior standard deviations, Monte Carlo errors, and 95% posterior credible intervals
   - graphical approximations of the posterior densities

2. Consider random data $Y_1, \ldots, Y_n$ sampled from a Normal(μ, σ^2) distribution, where both μ and σ^2 are unknown. For observed values $y_1, \ldots, y_n$, recall the sample mean $\bar{y}$ and the usual sample variance s^2. Recall also the MLE

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2$$

   (which is different than s^2). Recall the "standard" noninformative prior and Jeffreys' prior, specified, respectively, by

$$\pi(\mu, \sigma^2) \propto \frac{1}{\sigma^2} \quad (\sigma^2 > 0) \qquad \text{and} \qquad \pi(\mu, \sigma^2) \propto \frac{1}{(\sigma^2)^{3/2}} \quad (\sigma^2 > 0)$$

(a) [2 pts] In terms of n, $\bar{y}$, s^2, and an appropriate quantile of the standard t-distribution, express the formula for the (two-sided) 95% equal-tailed posterior credible interval for μ when using each of the following:

(i) the "standard" noninformative prior

(ii) Jeffreys' prior

You may assume $s^2 > 0$. Use notation of the form $t_{\alpha,\nu}$ for the α upper quantile of the standard t-distribution with ν degrees of freedom.

(b) [3 pts] Assume for simplicity that $\bar{y} = 0$ and $s^2 = 1$. On the same graph, plot the *half-width* of the 95% equal-tailed posterior credible interval for μ versus $n = 2, 3, \ldots, 10$, both for the "standard" noninformative prior and for Jeffreys' prior. Use different plot characters, colors, and/or line types to distinguish between the two priors, clearly labeling which results are from which prior. Which prior appears to produce wider intervals?

[Note: Quantiles of standard t-distributions are available in R using function `qt`.]

3. GRADUATE SECTION ONLY

Consider the situation and data of Problem 1. Suppose a new airliner is under consideration, which will have some passenger-fatal event rate λ_{new} following the same model as the λ_is, and conditionally independent of them.

Modify the JAGS model code (*not* the data) and analysis to answer the following:

(a) [2 pts] Fully list your JAGS code (after modification).

(b) [3 pts] Approximate the posterior mean and *standard deviation*, and form an approximate 95% posterior (predictive) interval for λ_{new}. (Use JAGS, basing your answer on at least 100000 iterations per chain, after burn-in.)

Note: You must *explicitly* give these approximations. Just listing output from R is not enough!