STAT 510: Homework 08

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Due: Monday, April 11, 11:59 PM

Exercise 1 (Using a Wald Test)

(**LW** 10.7) In 1861, 10 essays appeared in the *New Orleans Daily Crescent*. They were signed "Quintus Curtius Snodgrass" and some people suspected they were actually written by Mark Twain. To investigate this, we will consider the proportion of three letter words found in an author's work.

From eight Twain essays we have:

```
twain = c(0.225, 0.262, 0.217, 0.240, 0.230, 0.229, 0.235, 0.217)
```

From 10 Snodgrass essays we have:

```
snod = c(0.209, 0.205, 0.196, 0.210, 0.202, 0.207, 0.224, 0.223, 0.220, 0.201)
```

Perform a Wald test for equality of the means. Use the nonparametric plug-in estimator. Report the p-value and an approximate 95 percent confidence interval for the difference of means. What do you conclude?

Solution

Here, we are testing

$$H_0 : \delta = 0 \quad \text{versus} \quad H_1 : \delta \neq 0$$

where $\delta = \mu_1 - \mu_2$.

The nonparametric plug-in estimate of δ is

$$\hat{\delta} = \bar{x} - \bar{y}$$

with an estimated standard error

$$\hat{\text{se}}\left(\hat{\delta}\right) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$

Thus the Wald test statistic is

$$w = \frac{\hat{\delta} - 0}{\hat{\text{se}}\left(\hat{\delta}\right)} = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}.$$

With the given data, we calculate the value of the test statistic using R.

```
est = mean(twain) - mean(snod)
se = sqrt(var(twain) / length(twain) + var(snod) / length(snod))
(w = est / se)
```

```
## [1] 3.703554
```

As this is a two-sided test, the p-value is given by

$$2 \cdot P(Z > |w|)$$

With the given data, we calculate this p-value using R.

```
(p_val = pnorm(abs(w), lower.tail = FALSE) * 2)
```

```
## [1] 0.0002126003
```

Based on this p-value, we **reject the null hypothesis** assuming $\alpha = 0.05$.

We can also readily obtain an approximate 95% confidence interval for δ using

$$\hat{\delta} \ \pm \ 2 \cdot \hat{\text{se}}\left(\hat{\delta}\right)$$

```
est + c(-1, 1) * qnorm(0.975) * se
```

```
## [1] 0.01043973 0.03391027
```

Note that 0 is not contained in this interval, which verifies our decision to reject when $\alpha = 0.05$.

Exercise 2 (Using a Permutation Test)

(**LW** 10.7) In 1861, 10 essays appeared in the *New Orleans Daily Crescent*. They were signed "Quintus Curtius Snodgrass" and some people suspected they were actually written by Mark Twain. To investigate this, we will consider the proportion of three letter words found in an author's work.

From eight Twain essays we have:

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twain = c(0.225, 0.262, 0.217, 0.240, 0.230, 0.229, 0.235, 0.217)
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From 10 Snodgrass essays we have:

```
snod = c(0.209, 0.205, 0.196, 0.210, 0.202, 0.207, 0.224, 0.223, 0.220, 0.201)
```

Perform a permutation test to test for equality of the means. Report the p-value. What is your conclusion?

Solution

For this test, we will use the difference of means as the statistic that will be permuted. (You could also consider permuting the Wald statistic.)

$$\hat{\delta} = \bar{x} - \bar{y}$$

```
calc_perm_stat = function(sample_1, sample_2) {

  # get sample size information
  n_1 = length(sample_1)
  n_2 = length(sample_2)

  # create ungrouped data
  full_data = c(sample_1, sample_2)

  # get indexes for permuted sample_1
  perm_idx = sample(1:length(full_data), size = n_1)
```

```r
  # create "new" samples according to null
  perm_sample_1 = full_data[perm_idx]
  perm_sample_2 = full_data[-perm_idx]

  # calculate statistic on generated samples
  mean(perm_sample_1) - mean(perm_sample_2)

}
```

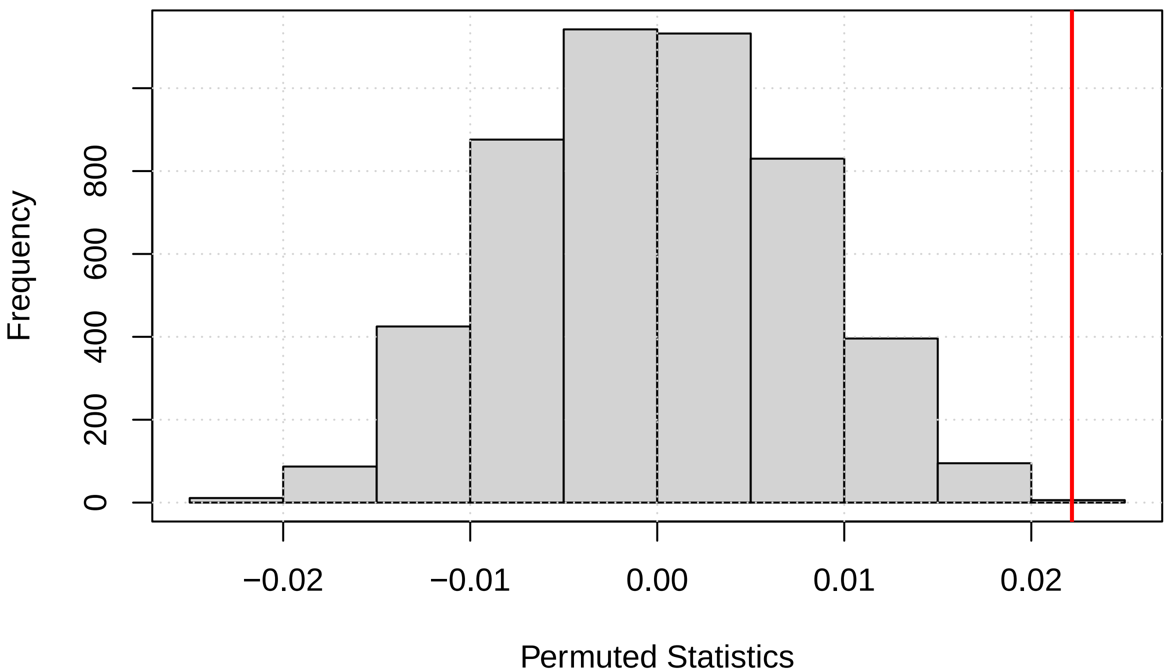
```r
# generate permutations and calculate test statistic
set.seed(42)
permuted_statistics = replicate(n = 5000, calc_perm_stat(twain, snod))
```

```r
# calculate statistic on observed data
observed_statistic = mean(twain) - mean(snod)
```

```r
# calculate p-value of permutation test
mean(permuted_statistics > observed_statistic)
```

```
## [1] 2e-04
```

```r
# visualize permutation test
hist(permuted_statistics, main = "", xlab = "Permuted Statistics")
box()
grid()
abline(v = observed_statistic, col = "red", lwd = 2)
```



At any reasonble α, we reject the null hypothesis.

Exercise 3 (Power and Size)

(**LW** 10.5) Let $X_1, \ldots, X_n \sim \text{Uniform}(0, \theta)$ and define $Y_n = \max\{X_1, \ldots, X_n\}$. We want to test

$$H_0 : \theta = 0.5 \quad \text{versus} \quad H_1 : \theta > 0.5.$$

The Wald test is not appropriate since Y does not converge to a Normal. So, suppose we decide to test this hypothesis by rejecting H_0 when $Y > c$.

- Find the power function.
- What choice of c will make the size of the test 0.05?
- In a sample of size $n = 20$ with $Y = 0.48$ what is the p-value? What conclusion about H_0 would you make?
- In a sample of size $n = 20$ with $Y = 0.52$ what is the p-value? What conclusion about H_0 would you make?

Solution

Using results from previous homework about the distribution of the maximum when sampling from a uniform distribution, we can obtain the power function.

$$\beta(\theta) = P(Y > c) = 1 - P(Y \le c) = \boxed{1 - \left(\frac{c}{\theta}\right)^n}$$

Setting the power function at $\theta = 0.50$ equal to 0.05 and solving gives us the value of c that makes this a level 0.05 test.

$$\boxed{c = \frac{0.95^{1/n}}{2}}$$

With $n = 20$ and $Y = 0.52$ we calculate the p-value.

$$\text{p-value} = P(Y > 0.48 \mid \theta = 0.50) = 1 - \left(\frac{0.48}{0.50}\right)^{20} = \boxed{0.5579976}$$

```
1 - punif(0.48, min = 0, max = 0.50) ^ 20
```

```
## [1] 0.5579976
```

With any reasonable α, we would fail to reject the null.

With $Y = 0.52$, we immediately reject the null, as the p-value is 0 since a maximum of 0.52 cannot be observed when $\theta = 0.50$.

$$\text{p-value} = P(Y > 0.52 \mid \theta = 0.50) = \boxed{0}$$

Exercise 4 (Two Simple Hypotheses)

(**LW** 10.8) Let $X_1, \ldots, X_n \sim N(\theta, 1)$. Consider testing

$$H_0 : \theta = 0 \quad \text{versus} \quad H_1 : \theta = 1.$$

Let the rejection region be $R = \{x^n : T(x^n) > c\}$ where $T(x^n) = \frac{1}{n}\sum_{i=1}^n X_i$.

- Find c so that the test has size α.
- Find the power under H_1, that is, find, $\beta(1)$.
- Show that, $\beta(1) \to 1$ as $n \to \infty$.

Solution

We first find the power function.

$$\beta(\theta) = P\left(\bar{X} > c\right) = P\left(Z > \frac{c-\theta}{1/\sqrt{n}}\right) = 1 - \Phi\left(\sqrt{n}(c-\theta)\right)$$

Next, we set $\theta = 0$ in the power function, and set the power equal to α.

$$\beta(0) = 1 - \Phi\left(\sqrt{n}(c)\right)$$

Solving gives us the value of c that makes this a level α test.

$$\boxed{c = \frac{\Phi(1-\alpha)}{\sqrt{n}} = \frac{z_\alpha}{\sqrt{n}}}$$

Now that we have c, it is trivial to obtain the power under the alternative.

$$\boxed{\beta(1) = 1 - \Phi(\sqrt{n}(c-1))}$$

As $n \to \infty$ we see that $c \to 0$, so $\sqrt{n}(c-1)) \to -\infty$, and $\Phi(\sqrt{n}(c-1)) \to 0$.

Thus, finally we have

$$\boxed{\beta(1) \to 1}.$$

Exercise 5 (Testing for a Normal Mean)

(**LW** 10.13) Let $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$. Construct the likelihood ratio test for

$$H_0 : \mu = \mu_0 \quad \text{versus} \quad H_1 : \mu \neq \mu_0.$$

Compare to the Wald test.

Solution

First, define

$$\theta_0 = (\mu_0, \sigma_0^2)$$

and

$$\theta_1 = (\mu_1, \sigma_1^2).$$

While μ_0 is assumed to be known, the remaining quantities must be estimated using maximum likelihood.

$$\hat{\mu_1} = \bar{x}$$

$$\hat{\sigma}_1^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\hat{\sigma}_0^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_0)^2$$

We can then plug these known and estimates quantities into their respective likelihoods.

$$\mathcal{L}\left(\hat{\theta}_1\right) = \left(\frac{1}{2\pi\hat{\sigma}_1^2}\right)^{n/2} \exp\left[-\frac{1}{2\cdot\hat{\sigma}_1^2}\sum_{i=1}^{n}(x_i - \bar{x})^2\right] = \left(\frac{ne^{-1}}{2\pi\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)^{n/2}$$

The second equality hold as a result of "algebra" which is suppressed here.

$$\mathcal{L}\left(\hat{\theta}_0\right) = \left(\frac{1}{2\pi\hat{\sigma}_0^2}\right)^{n/2} \exp\left[-\frac{1}{2\cdot\hat{\sigma}_0^2}\sum_{i=1}^{n}(x_i - \mu_0)^2\right] = \left(\frac{ne^{-1}}{2\pi\sum_{i=1}^{n}(x_i - \mu_0)^2}\right)^{n/2}$$

Now we can calculate the likelihood ratio.

$$\Lambda = \frac{\mathcal{L}\left(\hat{\theta}_1\right)}{\mathcal{L}\left(\hat{\theta}_0\right)} = \left(\frac{\sum_{i=1}^{n}(x_i - \mu_0)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)^{n/2}$$

Taking a log and multiplying by 2, we obtain the "likelihood ratio statistic" as defined in Wasserman, which has a limiting χ^2 distribution.

$$\lambda = 2\log\Lambda = n\log\left(\sum_{i=1}^{n}(x_i - \mu_0)^2\right) - n\log\left(\sum_{i=1}^{n}(x_i - \bar{x})^2\right)$$

To obtain a p-value, calculate

$$P\left(\chi_1^2 > \lambda\right).$$

Below here, we riff on the comparison between the LRT and Wald Test.

First, note that the Wald test statistic is

$$W = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.$$

Here we assume that s^2 is the usual unbiased estimator of σ^2, but note that we could use the MLE as well, but it will produce less interesting results.

We have seen many times that $W \approx N(0,1)$, but, if we assume that $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$, we actually know the **exact** distribution of W.

$$W \sim t_{n-1}$$

That is, W follows a t distribution with $n-1$ degrees of freedom. Note that asymptotically, this distribution is essentially a standard normal. Hence a "t-test" is more or less identical to the Wald test for large samples.

Note that we would reject the wald test if

$$|W| > t_{n-1,\alpha/2}$$

Now, recall the likelihood ratio.

$$\Lambda = \frac{\mathcal{L}\left(\hat{\theta}_1\right)}{\mathcal{L}\left(\hat{\theta}_0\right)} = \left(\frac{\sum_{i=1}^n (x_i - \mu_0)^2}{\sum_{i=1}^n (x_i - \bar{x})^2}\right)^{n/2}$$

Instead of leveraging the asymptotic behavior of the likelihood ratio statistic, instead, we note that we simply want to reject the null when Λ is large. That is, reject when

$$\Lambda > k$$

for some k that gives the appropriate level test. We can manipulate both the LHS and RHS of this equation as we please.

Recall that

$$\sum_{i=1}^n (x_i - \mu_0)^2 = \sum_{i=1}^n (x_i - \bar{x})^2 + n(\bar{x} - \mu_0)^2.$$

$$\Lambda = \left(\frac{\sum_{i=1}^n (x_i - \mu_0)^2}{\sum_{i=1}^n (x_i - \bar{x})^2}\right)^{n/2} = \left(\frac{\sum_{i=1}^n (x_i - \bar{x})^2 + n(\bar{x} - \mu_0)^2}{\sum_{i=1}^n (x_i - \bar{x})^2}\right)^{n/2} = \left(1 + \frac{n(\bar{x} - \mu_0)^2}{\sum_{i=1}^n (x_i - \bar{x})}\right)^{2/n}$$

So reject when $\Lambda > k$ is the same as rejecting when

$$\Lambda' = \frac{n(\bar{x} - \mu_0)^2}{\sum_{i=1}^n (x_i - \bar{x})} > k'$$

where

$$k' = k^{n/2} - 1$$

Via a similar argument, we reject if

$$\Lambda'' = \sqrt{\frac{n(\bar{x} - \mu_0)^2}{\sum_{i=1}^n (x_i - \bar{x})/(n-1)}} > k''.$$

Note that

$$\Lambda'' = \sqrt{\frac{n(\bar{x} - \mu_0)^2}{\sum_{i=1}^n (x_i - \bar{x})/(n-1)}} = \left|\frac{\bar{x} - \mu_0}{s/\sqrt{n}}\right| = |W|$$

Thus, necessarily, Λ'' has the same distribution as $|W|$, and thus the LRT and Wald test are the same, not just asymptotically, but exactly.

Exercise 6 (Testing for a Binomial Proportion)

(**LW** 10.13) Let $X \sim \text{Binomial}(n, p)$. Construct the likelihood ratio test for

$$H_0 : p = p_0 \quad \text{versus} \quad H_1 : p \neq p_0.$$

Compare to the Wald test.

Solution

First, note that under the alternative, we have seen the MLE for p.

$$\hat{p} = \frac{x}{n}$$

Next we write the likelihood under the null.

$$\mathcal{L}(p_0) = \binom{n}{x} p_0^x (1 - p_0)^{n-x}$$

Similarly, under the alternative we have

$$\mathcal{L}(\hat{p}) = \binom{n}{x} \hat{p}^x (1 - \hat{p})^{n-x}.$$

Next, we write the likelihood ratio. We would like to reject when this value is large.

$$\Lambda = \frac{\mathcal{L}(\hat{p})}{\mathcal{L}(p_0)} = \left(\frac{\hat{p}}{p_0}\right)^x \left(\frac{1-\hat{p}}{1-p_0}\right)^{n-x}$$

Taking a log and multiplying by 2, we obtain the "likelihood ratio statistic" as defined in Wasserman, which has a limiting χ^2 distribution.

$$\lambda = 2 \log \Lambda = 2x \log \left(\frac{\hat{p}}{p_0}\right) + 2(n - x) \log \left(\frac{1-\hat{p}}{1-p_0}\right)$$

Note that

$$\lambda \sim \chi^2_1$$

The Wald test here is

$$W = \frac{\hat{p} - p_0}{\sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}}} \sim N(0, 1).$$

These tests are similar, but not exactly equivalent. We will check two quick examples. The first example has a "small" sample size. Notice that it produces different p-values, but would result in the same decision.

```r
# setup
x = 8
n = 10
p_0 = 0.7
p_hat = x / n

# calculate statistics
lrt_stat = 2 * x * log(p_hat / p_0) + 2 * (n - x) * log ((1 - p_hat) / (1 - p_0))
wald_stat = (p_hat - p_0) / sqrt( p_hat * (1 - p_hat) / n )

# calculate p-values
pchisq(lrt_stat, df = 1, lower.tail = FALSE)
```

```
## [1] 0.4731363
```

```r
2 * pnorm(abs(wald_stat), lower.tail = FALSE)
```

```
## [1] 0.4291953
```

Our next example has a "large" sample size. Again, notice that the p-values are different, but in this case, more similar, and for most α values, we would make the same decision.

```r
# setup
x = 120
n = 200
p_0 = 0.52
p_hat = x / n

# calculate statistics
lrt_stat = 2 * x * log(p_hat / p_0) + 2 * (n - x) * log ((1 - p_hat) / (1 - p_0))
wald_stat = (p_hat - p_0) / sqrt( p_hat * (1 - p_hat) / n )

# calculate p-values
pchisq(lrt_stat, df = 1, lower.tail = FALSE)
```

```
## [1] 0.02294382
```

```r
2 * pnorm(abs(wald_stat), lower.tail = FALSE)
```

```
## [1] 0.02092134
```

Exercise 7 (Comparing Power)

(**CB** 8.13) Let $X_1, X_2 \sim \text{Uniform}(\theta, \theta + 1)$. Consider two tests for

$$H_0 : \theta = 0 \quad \text{versus} \quad H_1 : \theta > 0.$$

1. $\phi_1(X_1)$: Reject H_0 is $X_1 > 0.95$.
2. $\phi_2(X_1, X_2)$: Reject H_0 is $X_1 + X_2 > c$.

With these two tests…

- Find the value of c that makes ϕ_2 have the same size as ϕ_1.
- Calculate and plot the power of each test.

Solution

First, the size of ϕ_1.

$$\alpha_1 = P(X_1 > 0.95 \mid \theta = 0) = 0.05$$

Next, the size of ϕ_2.

$$\alpha_2 = P(X_1 + X_2 > c \mid \theta = 0) = \begin{cases} 1 & c < 0 \\ 1 - \frac{1}{2}c^2 & 0 \le c < 1 \\ \frac{1}{2}(2 - c)^2 & 1 \le c < 2 \\ 0 & c \ge 2 \end{cases}$$

To make ϕ_2 have the same level as ϕ_1 we solve the following for c.

$$0.05 = \frac{1}{2}(2-c)^2$$

We obtain $c = 2 - \sqrt{0.1} \approx 1.68$.

Next, we obtain the two power functions.

$$\beta_1(\theta) = P(X_1 > c \mid \theta) = \begin{cases} 0 & \theta < -0.05 \\ \theta + 0.05 & -0.05 \leq \theta < 0.95 \\ 1 & \theta \geq 0.95 \end{cases}$$

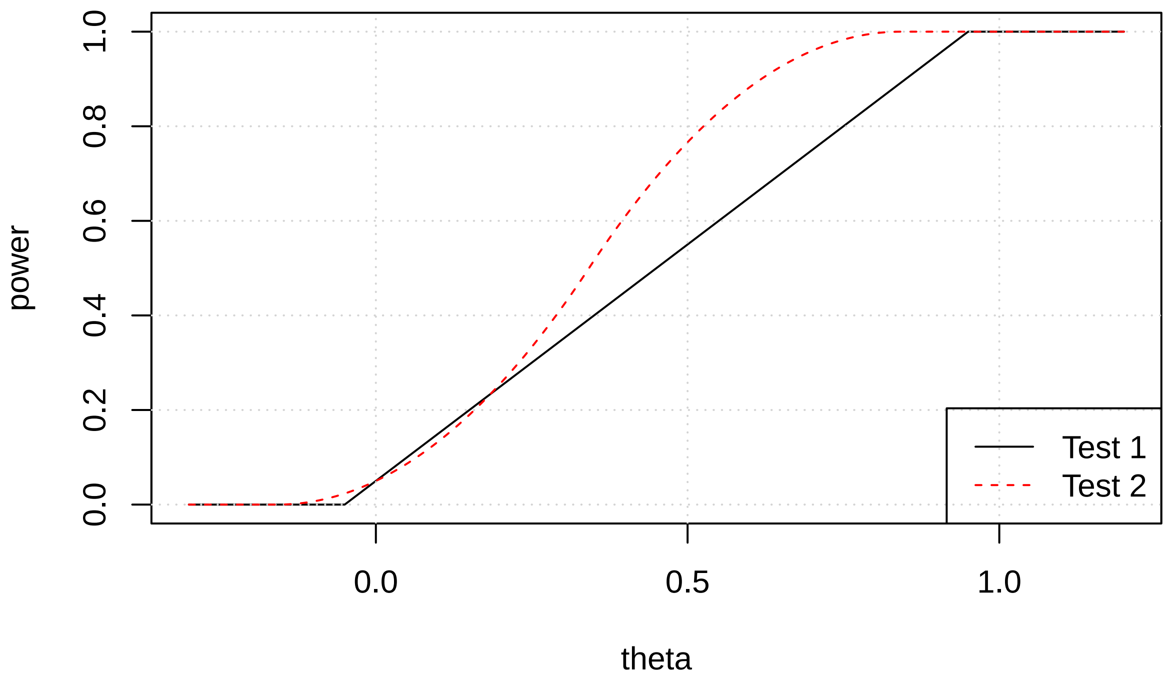
$$\beta_2(\theta) = P(X_1 + X_2 > c \mid \theta) = \begin{cases} 0 & \theta < \frac{c-2}{2} \\ \frac{1}{2}(2\theta + 2 - c)^2 & \frac{c-2}{2} \leq \theta < \frac{c-1}{2} \\ 1 - \frac{1}{2}(c - 2\theta)^2 & \frac{c-1}{2} \leq \theta < \frac{c}{2} \\ 1 & \theta \geq \frac{c}{2} \end{cases}$$

```r
beta_1 = function(theta) {
  if (theta < -0.05) {
    return(0)
  } else if (theta < 0.95) {
    return(theta + 0.05)
  } else {
    return(1)
  }
}
```

```r
beta_2 = function(theta) {
  c = 2 - sqrt(0.1)
  if (theta < (c - 2) / 2) {
    return(0)
  } else if (theta < (c - 1) / 2) {
    return(0.5 * (2 * theta + 2 - c) ^ 2)
  } else if (theta < c / 2) {
    return(1 - 0.5 * (c - 2 * theta) ^ 2)
  } else {
    return(1)
  }
}
```

```r
theta = seq(from = -0.3, to = 1.2, by = 0.001)
```

```r
plot(theta, sapply(theta, beta_1), type = "l",
     xlab = "theta", ylab = "power")
grid()
lines(theta, sapply(theta, beta_2), col = "red", lty = 2)
legend("bottomright",
       legend = c("Test 1", "Test 2"),
       col = c("black", "red"),
       lty = 1:2)
```



Exercise 8 (Using Neyman-Pearson)

(**CB** 8.20) Let X be a random variable whose pmf under H_0 and H_1 is given by

x	1	2	3	4	5	6	7
$f(x \mid H_0)$	0.01	0.01	0.01	0.01	0.01	0.01	0.94
$f(x \mid H_1)$	0.06	0.05	0.04	0.03	0.02	0.01	0.79

Use the Neyman-Pearson Lemma to find the most powerful test for H_0 versus H_1 with size $\alpha = 0.04$. Compute the probability of a Type II Error for this test.

Solution

First, note the likelihood ratio.

x	1	2	3	4	5	6	7
$f(x \mid H_1)/f(x \mid H_0)$	6	5	4	3	2	1	0.84

Now, note that as x increases, the likelihood ratio decreases. So, because we would like to reject when the likelihood ratio is large, that corresponds to rejecting when x is small.

Then, to obtain a level α test, find c such that $P(X \leq c \mid H_0) = \alpha$. In this case, we accomplish that with $c = 4$.

$$P(\text{Type II Error}) = P(\text{Retain } H_0 \mid H_1) = P(X \geq 5 \mid H_1) = 0.82$$

Exercise 9 (Multiple Testing)

(**LW** 10.11) A randomized, double-blind experiment was conducted to assess the effectiveness of several drugs for reducing postoperative nausea. The data are as follows.

Treatment	Number of Patients	Incidence of Nausea
Placebo	80	45
Chlorpromazine	75	26
Dimenhydrinate	85	52
Pentobarbital (100 mg)	67	35
Pentobarbital (150 mg)	85	37

Test each drug versus the placebo at the 5% level with a two-sided test. Report the p-value for each test. Also report Bonferroni corrected p-values.

Solution

```r
prop_diff_test = function(x, n) {
  p = x / n
  z = diff(p) / sqrt(sum(p * (1 - p) / n))
  2 * pnorm(abs(z), lower.tail = FALSE)
}
```

```r
p_vals = c(
  prop_diff_test(x = c(26, 45), n = c(75, 80)),
  prop_diff_test(x = c(52, 45), n = c(85, 80)),
  prop_diff_test(x = c(35, 45), n = c(67, 80)),
  prop_diff_test(x = c(37, 45), n = c(85, 80))
)
```

```r
results = tibble::tibble(
  Drug = c("Chlorpromazine",
           "Dimenhydrinate",
           "Pentobarbital (100 mg)",
           "Pentobarbital (150 mg)"),
  "P-Value" = p_vals,
  "Corrected P-Value" = pmin(p_vals * 4, 1)
)
```

```r
knitr::kable(results, digits = 3)
```

Drug	P-Value	Corrected P-Value
Chlorpromazine	0.006	0.023
Dimenhydrinate	0.520	1.000
Pentobarbital (100 mg)	0.627	1.000
Pentobarbital (150 mg)	0.100	0.399

Note that, rather than changing the α for each test based on the number of tests, we can instead hold α constant, and change the p-value of the test. The two approaches will give the same decisions. Also note that p-values cannot be greater than 1.

Our final decisions here are: reject, accept, accept, accept.

Exercise 10 (Verifying Size)

(**LW** 10.12) Let $X_1, \ldots, X_n \sim \text{Poisson}(\lambda)$. Let $\lambda_0 > 0$. Find the size α Wald test for

$$H_0 : \lambda = \lambda_0 \quad \text{versus} \quad H_1 : \lambda \neq \lambda_0.$$

Set $\lambda_0 = 1$, $n = 20$, and $\alpha = 0.05$. Simulate from the null distribution and perform the Wald test. Repeat many times and count how often you reject the null. How close is this result to $\alpha = 0.05$.

Solution

Given previous results for the MLE of λ, $\hat{\lambda} = \bar{x}$, and its standard error, we can quickly obtain the Wald test.

$$W = \frac{\hat{\lambda} - \lambda_0}{\sqrt{\hat{\lambda}/n}}$$

For a level α test, we reject when

$$|W| > z_{\alpha/2}.$$

```
calc_wald_lambda = function(data, null) {
  (mean(data) - null) / sqrt(mean(data) / length(data))
}
```

```
set.seed(42)
test_stats = replicate(
  n = 100000,
  calc_wald_lambda(rpois(n = 20, lambda = 1), null = 1))
mean(abs(test_stats) > qnorm(0.975))
```

```
## [1] 0.05131
```

Simulating from the null and carrying out the test, we find that we do indeed reject close to 5% of the time.

Exercise 11 (Simulating Power)

Use the setup from Exercise 10 and perform a simulation study that is informative about the power of the test.

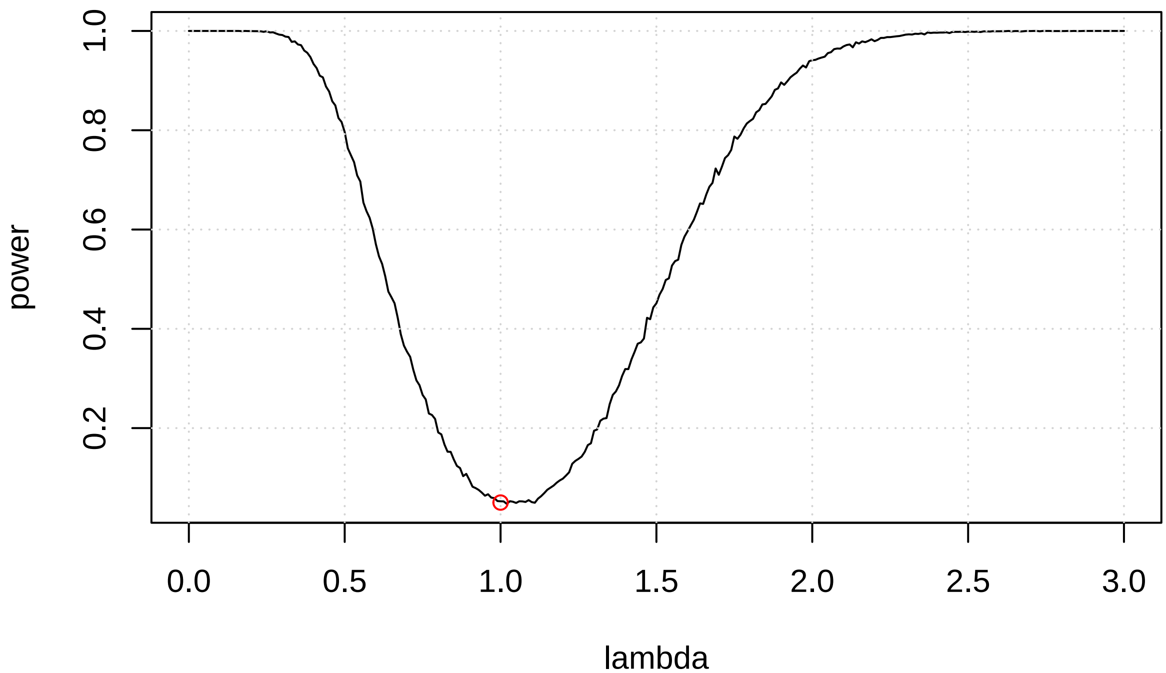
Solution

```
# lambda values at which the power will be calculated
lambda = seq(from = 0, to = 3, by = 0.01)
```

```
# function to calculate power given lambda
calc_power_lambda = function(lambda) {
  test_stats = replicate(
    n = 5000,
    calc_wald_lambda(rpois(n = 20, lambda = lambda), null = 1))
  mean(abs(test_stats) > qnorm(0.975))
}
```

```
# calculate power for each specified value of lambda
set.seed(42)
power = sapply(lambda, calc_power_lambda)
```

```
plot(lambda, power, type = "l")
grid()
points(x = 1, y = 0.05, col = "red")
```



As expected, we see the power increase as we move away from $\lambda = 1$, in either direction. Also note that the power increases faster as λ decreases. This makes sense, as smaller λ values will also produce smaller estimated standard errors, thus test statistics of greater magnitude. This also helps explain why the power briefly decreases before increasing as λ increases beyond the assumed null value of $\lambda = 1$.