

General Algorithm Backtracking

Algorithm Backtrack ($X[1 \dots i]$)

// Given a template of a generic backtrack algorithm

// Input: $X[1 \dots i]$ specifies first 'i' promising component of a solution

// Output: All the tuples representing the problem's solution

if ($X[1 \dots i]$) is a solution

work ($X[1 \dots i]$)

else

for each element $x \in S_i$ consistent with $X[1 \dots i]$ and the constraints do

$X[i+1] \leftarrow x$

Backtrack($X[1 \dots i+1]$)

}

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Hamiltonian

Algorithm Hamiltonian(k)

```
{ repeat
  { nextvalue(k)
    if (X[k] == 0) then
      return
    if (k == n)
      write (X[1:n])
    else
      Hamiltonian(k+1)
  } until (false)
}
```

$$TC = O(n^n)$$

Algorithm Next value(k)

```
{ repeat
  { X[k] = (X[k] + 1) % (n + 1)
    if (X[k] == 0)
      return
    if ((G[X[k-1], X[k]] != 0))
      { for (j = 1 to k-1)
        { if (X[k] == X[j])
          break;
        if (j == k) then
          if ((k < n) or (k == n) and G[X[n], X[1]] != 0)
            return;
        }
      } until (false)
  }
}
```

Graph Coloring

At

Algorithm mColoring(k)

// this algorithm was formed using Recursive backtracking

// the graph is represented as boolean adjacency $G[1:n, 1:n]$

// k is the index of the next vertex to color

{ repeat

{ Nextvalue(k);

if ($X[k] = 0$)

return

if ($k == n$) then

work($x[1:n]$);

else
mcoloring(k+1);

} until (false);

}

Algorithm Nextvalue(k)

{ repeat {

$X[k] = (X[k] + 1) \bmod(n+1)$

if ($X[k] = 0$)

return
for ($j = 1$ to n) do {

{ if ($(G[k, j] \neq 0)$ and ($x[k] = x[j]$))

then break

} if ($j = n+1$) then

return

} until (false);

}

$$TC = O(nm^n)$$

Sum of Subsets

$$\begin{array}{c} \boxed{\sum W_i x_i, k, \sum W_i} \\ \swarrow x=1 \quad \searrow x=0 \end{array}$$

$$\boxed{\sum W_i x_i + W_k, k+1, \sum W_i - W_k}$$

$$\boxed{\sum W_i x_i, k+1, \sum W_i - W_k}$$

Algorithm SumSubst (s, k, r)

{ $x[k] = 1$

if ($s + w[k] = m$)

then work ($x[1:k]$);

an if ($s + w[k] + w[k+1] \leq m$) then

SumSubst ($s + w[k], k+1, r - w[k]$);

if (($s + r - w[k] \geq m$) and ($s + w[k+1] \leq m$)) then

← $x[k] = 0$

SumSubst ($s, k+1, r - w[k]$);

}

}

Time Complexity

$$TC = O(2^n)$$

KnapSack - backtracking.

Algorithm BKnap(k, cp, cw)

```
{ //left child
  if ( $cw + w[k] \leq m$ ) then
  {
     $y[k] = 1$ 
    if ( $k < n$ ) then
      BKnap( $k+1, cp + p[k], cw + w[k]$ );
    if ( $(cp + p[k] > fp)$  and  $(k = n)$ ) then
    {
       $fp = cp + p[k]$ 
       $fw = cw + w[k]$ 
      for ( $j = 1$  to  $k$ ) do
         $x[j] = y[j]$ 
    }
  }
```

```
  //Right child
  if (Bound( $cp, cw, k$ )  $\geq fp$ ) then
  {
     $y[k] = 0$ 
    if ( $k < n$ ) then
      BKnap( $k+1, cp, cw$ )
    if ( $(cp > fp)$  and  $(k = n)$ ) then
    {
       $fp = cp$ 
       $fw = cw$ 
      for  $j = 1$  to  $k$  do
         $x[j] = y[j]$ 
    }
  }
```


Algorithm Bound (cp, am, k)

```
if  $b = cp$  ;  
   $c = cw$   
  for ( $i = kn$  to  $ndo$ )  
  {  
     $c = c + w[i]$   
    if ( $c < m$ ) then  
       $b = b + p[i]$   
    else  
      return  $b + (1 - (c - m) / w[i]) * p[i]$  ;  
  }  
  return  $k$  ;  
}
```

Time Complexity $TC = O(2^n)$

N Queens

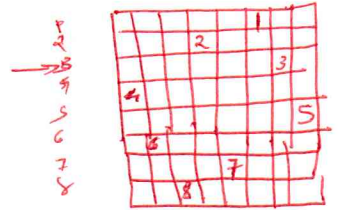
4 queens: $(2, 4, 1, 3)$
 $(3, 1, 4, 2)$

(7)

8 queens: $(4, 6, 8, 2, 7, 1, 3, 5)$

Algorithm NQueens(K, n)

```
{ for i = 1 to n do
  { if place(K, i) then
    { x[K] = i
      if (K == n)
        write (X[1:n]);
      Else NQueens(K+1, n);
    }
  }
}
```



Algorithm Place(K, i)

```
{ for (j = 1 to K-1)
  { if  $(X[j] = i)$  or  $(Abs(X[j] - i) = Abs(j - K))$ 
    then return false
  }
  return true
}
```

same column (pointing to $X[j] = i$)
same diagonal (pointing to $Abs(X[j] - i) = Abs(j - K)$)

TC = Best case $O(n!)$