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Inductive Type-Directed Top-Down Program Synthesis from Input-Output Examples with Higher-Order Functions

Master Thesis

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Abstract

TODO: write me :)

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Chapter 1

Introduction

1.1 About program synthesis in general

put in some context, [link 2](#)

- What is program synthesis
- Problem too general, needs to be restricted. In Chapter 2 we will see how other restrict the search space. Restrict to components.
- Motivate why it is still interesting to have such a synthesiser (thin interfaces and so on?)
- List of contributions
- Explain the structure of the thesis. (maybe merge with the contributions?)

for motivation you could write something about the extremely restricted list library in ocaml :)

Consider you are writing code in a functional programming language with a smaller choice of library functions than you are used to. For example (true story), if you are using OCaml and you are surprised that `replicate` is missing even in the more complete core library [cite jane street core](#), you could spend a couple of minutes writing your recursive version of `replicate`. Or you can synthesize it with TAMANDU in less than one second from other components. **TODO:** rewrite without 'you' and maybe don't mention ocaml, core and all that thing?.

1.2 Problem definition

have many components, put them together into a program, no lambdas, no if-then-else, no recursion

1.3 Contributions

Evaluation, exploring the baseline algorithm, exploring the search space

Chapter 2

Related Work

Here discuss only synthesis of functional programs. Input-output examples are an intuitive and easy way to specify programs. Types are good to prune the search space, but simple types are not enough to specify a program. So the research went in two directions: on one side, there was the need to do something to the I/O-examples to make the search more efficient, on the other side more complex and expressive types that can act as a specification, for example the *refinement types* from [7]. Lately, there was the idea that one can automatically generate those ty[ypes from I/O-examples, to have both the advantages of an intuitive and easy specification and the research done on type systems, liquid types and something like that.

Let's start. My 5 papers, organize some information about them.

- Synquid (Nadia) put the replicate example if you can (should not be that difficult). Not that "user friendly". Lambda, conditionals, recursion, components. Synthesis starting from a partial program possible. Fast, good for sorting. It is possible to download the tool (put link), try it online and there is even an emacs-mode for it.
 1. What is the specification? The refinement type of the desired program, that is a type decorated with logical predicates. Can bring the replicate example.
 2. What is the target language? Haskell, I think. I saw lambda abstractions and recursion. Can generate and use higher-order components. You can specify own datatypes and own components. Pattern matching, structural recursion, ability to use and generate polymorphic higher-order functions, reasoning about universal and recursive properties of data structures.
The Synquid language has lambda expressions, pattern matching, conditional and fixpoint.

3. What can it do well and fast? How fast? Most advanced benchmark: generate various sorting algorithms for data structures with complex invariants (search trees, heaps, balanced trees). This is out of scope of my synthesis procedure. For the "easy" benchmarks (what I have) everything under 0.4 s. For more complicated stuff like sorting and tree insertion under 5 seconds. For most complicated stuff (red-black trees) under 20 s.
 4. What is the difficulty? What can they not generate (or take a long time to generate)? How much time do they need? It is not always easy for an inexperienced user to specify a program and to find the right 'measure' function for a datatype.
They need up to 20 s to generate balance over red-black trees. They usually don't give more than 5 components and 6 measure functions over the needed data types.
The specification is quite big compared to the synthesised program, half of the programs the specification is one third of the generated program or more.
 5. What do they do? In [7] SYNQUID is proposed. Refinement types (types decorated with logical predicates) are used to prune the search space. SMT-solvers are used to satisfy the logical predicates. The key is the new procedure for type inference (called modular refinement type reconstruction), which thank to its modularity scales better than other existing inference procedures for refinement types. Programs can therefore be type checked even before they are put together.
- Lambda square (Feser). How would you specify replicate? Give an idea of how it is generated. User friendly, as only I/O-examples need to be specified, type is reconstructed automatically.
 1. What is the specification? I/O-examples
 2. What is the target language? Their synthesis algorithm targets a functional programming language that permits higher-order functions, combinators like map, fold and filter, pattern-matching, recursion, and a flexible set of primitive operators and constants. Actually, lambda calculus with algebraic data types and recursion.
 3. What can it do well and fast? How fast? Generating programs over nested structures like trees of lists, lists of lists and lists of trees. They generate the half of the benchmarks in under 0.43 s.

-
4. What is the difficulty? What can they not generate (or take a long time)? How much time do they need? Only 7 higher-order combinators and not much more first-order components.
They use relatively many examples (between 3 and 12, mostly 4 to 6).
They need up to 320 s to generate droplast. Other benchmarks that take more than 100 s to synthesise are removing duplicates from a list, inserting a tree under each leaf of another tree and dropping the smallest number in a list of lists.
 5. What do they do? Inductive generalization, deduction, enumerative search. First, generate *hypotheses* (that is, programs with free variables like $\lambda x. \text{map } ?f \ x$ where $?f$ is a placeholder for an unknown program that needs to be generated) in a type-aware manner (the type is inferred from the I/O-examples). Then deduction based on the semantics of the higher-order combinators is used either to quickly refute a hypothesis or infer new I/O-examples to guide the synthesis of missing functions. Best-first enumerative search is used to enumerate candidate programs to fill in the missing parts of hypotheses.
The tool proposed in [3] is called λ^2 and generates its output in λ -calculus with algebraic types and recursion. The user specifies the desired program providing input-output examples. No particular knowledge is required from the user, as was demonstrated using random input-output examples. The examples are inductively generalized in a type-aware manner to a set of hypotheses (programs that possibly have free variables). The key idea are the hard-coded deduction rules used to prune the search space depending on the semantics of some of the higher-order combinators (map, fold, filter and a few others). Deduction is also used to infer new input-output examples in order to generate the programs needed to fill in the holes in the hypotheses. This tool is able to synthesize programs manipulating recursive data structures like lists, trees and nested data structures such as lists of lists and trees of lists. The tool is able to synthesise all benchmark programs in under 7 minutes.
- Escher (Kincaid). User friendly, as untyped and only I/O-examples need to be specified. Powerful (recursion, special if-then-else, components) Goal graph.
 1. What is the specification? I/O-examples. Oracle simulating interaction with the user.
 2. What is the target language? Recursion, special treatment for con-

ditionals, components. That's really all they have: either apply a component to its arguments (including component self referring to the program being generated for recursion), a constant, an input variable or a conditional. Programming language is untyped. Components may be higher-order.

3. What can they do well and fast? How fast? Synthesise recursive programs (tail recursive, divide-and-conquer, mutually recursive). In the evaluation, they give 23 components to all benchmarks, none of them is higher-order. Can generate all benchmarks in under 11 s, most of them in under 1 s.
4. What is the difficulty? What can they not generate (or take a long time)? How much time do they need? The user must give a *closed* example set, that is for each example, there must be another example that can be used for a recursive call.
5. What do they do? Forward search: inductive enumeration, add a new component to an already synthesised program. Conditional inference: use the goal graph to see if you can join to synthesised programs by a conditional statement. A heuristic guides the alternation between forward search and conditional inference. Programs are associated with value vectors. Search space is also pruned based on *observational equivalence*, that is programs with equivalent value vectors are treated as equivalent programs. Can be formalised as a non-deterministic transition system over configurations (triples consisting of a set of synthesised programs, a goal graph and a list of input-output examples) with six transition rules: an initial rule to start the search, a terminate rule to terminate the search and four synthesis rules (one for forward search, two for conditional split and one to ask for new input-output examples to evaluate a recursive call). The recursive calls are answered by the oracle. Rule scheduling is added to turn this into a practical system (different heuristics).
In [2] ESCHER is presented, an inductive synthesis algorithm that learns a recursive procedure from input-output examples provided by the user. *The user must provide a "closed" set of examples, otherwise recursion cannot be handled properly* The target language is untyped, first-order and purely functional. The algorithm is parametrized by components that can be instantiated differently to suit different domains. The approach combines enumerative search and conditional inference. The key idea is to use a special data structure, a *goal graph*, to infer conditional branches

instead of treating if-then-else as a component. Observational equivalence is also used to prune the search space. Programs with the same value vectors (output of the program when applied to the inputs of the input-output examples) are considered equivalent and only one of them is synthesized. An implementation of the tool was tested on a benchmark consisting of recursive programs (including tail-recursive, divide-and-conquer and mutually recursive programs) drawn from functional programming assignments and standard list and tree manipulation programs. For all examples the same fixed set of components was used. The tool is able to synthesize all of them quickly. There is very little information on how many input-output examples were needed to synthesize the benchmarks and how difficult it is for a non-experienced user to come up with a "closed" set of examples.

- Myth (Osera). Like mine, requires type and I/O-examples. Refinement tree.
 1. What is the specification? A type signature, the components and a list of input-output examples.
 2. What is the target language? pattern matching, recursion, higher-order functions in typed programming languages. Can synthesise higher-order functions, programs using higher-order functions and work with large algebraic datatypes. ML-like language with algebraic data types, match, top-level function definitions and explicitly recursive functions.
 3. What can they do well and fast? How fast? Recursive programs with pattern matching. It's very fast, many programs are synthesised in around 0.1 s. It can also generate larger programs (75 AST nodes) in reasonable time (3 s for calculating the set of free variables in an untyped lambda-calculus).
 4. What is the difficulty? What can they not generate (or take a long time)? How much time do they need? To generate recursive functions, they also need a closed set of examples, so that a recursive call to the function being synthesised can be answered by an input-output example. They also require relatively many examples. They use a relatively large context, but they do not say how big it is. Lacks support richer types like products and polymorphic types.
 5. What do they do? The tool in [5] is called MYTH and uses not only type information but also input-output examples to restrict

2. RELATED WORK

the search space. The special data structure used to hold this information is the *refinement tree*. This system can synthesize higher-order functions, programs that use higher order functions and work with large algebraic data types.

There is an ML-like type system that incorporates input-output examples. Two pieces: a *refinement tree* and an enumerative search. Two major operations: refine the goal type and the examples (push them down in the refinement tree) and guess a term of the right type that matches the examples (for one of the nodes of the refinement tree).

- Leon (can bring it, it's deductive, so it's different than the others. It also targets functional programs.) [4]

Chapter 3

Top down type driven synthesis

In this chapter we will formally define our type-directed top-down synthesis procedure. We will start with an intuitive description of the problem and move on to the formal definitions of the target programming language and the search space. Finally, we will define the synthesis procedure and present some enhancements.

3.1 Problem

Recall the example from Chapter 1 where we wanted to synthesise `replicate`. As our synthesis procedure is type-driven and example-based, the user specifies a program by providing its type along with a few I/O-examples.

Let us specify `replicate` as follows.

```
replicate :: ∀X. Int → X → List X
replicate 3 1 = [1,1,1]
replicate 2 [] = [[]], [[]]
```

We also need a library of components, from which we are going to compose our program. Let us assume we have the standard list combinators `map`, `foldr`, `foldl` with `enumTo`, the function that returns a list from 1 up to its argument, and `const`, that always returns its first argument along with the list constructors `cons` and `[]` and the integer constructors `succ` and `0`.

- Try to put components together in order to get a list. Fix X as a type variable and fix n to be an `Int` and x to be an X . Try to synthesise a `List X`. Which components can you use in order to get a `List X`? You cannot use `enumTo`, because it only produces a list of integers, but all other possibilities are open. We could either fold with an interesting function, or map some function, or even use `const` with a smart first argument. But the first and easiest thing that has the type `List X` is `[]`.

```
replicate n x = []
```

Cool, we can evaluate this program straight away... and see that it does not satisfy the I/O-examples. It means, we must try further.

```
replicate n x = cons ?x ?xs
?x :: X
?xs :: List X
```

```
replicate n x = map ?f ?xs
?f :: ?Y → X
?xs :: List ?Y
```

?Y is a fresh variable. We omit foldl for brevity.

```
replicate n x = foldr ?f ?init ?xs
?f :: ?Y → List X → List X
?init :: List X
?xs :: List ?Y
```

```
replicate n x = const ?xs ?s
?xs :: List X
?s :: ?Y
```

Now we take the most promising program and try to fill in one of its *holes* (the fresh variables starting with ?. Let us decide that the first one is the most promising. We now have two holes to fill in, a function that can take something and return an X (note that it has to be possible for all possible instantiations of X and note that we have only one value of type X: x, the second argument to replicate).

What are the possibilities to instantiate ?f? Obviously, we cannot use map or enumTo, because they return lists. But all other possibilities are still open (again, we leave out foldl for brevity).

```
replicate n x = map (foldr ?g ?init) ?xs
?f = foldr ?g ?init :: List ?Z → X
?g :: ?Z → X → X
?init :: X
?xs :: List (List ?Z)
```

Note that we instantiated ?Y with List ?Z, because foldr takes a list as its last argument.

```
replicate n x = map (const ?x) ?xs
?f = const ?x :: ?Y → X
?x :: X
?xs :: List ?Y
```


Again, from all programs generated so far (6 in total) we choose the most promising one. Let us decide that it is the last one. It has two holes to fill in. For the first hole, $?x$, we have only one possibility. As this hole must be of type X , the only thing we can take is the second argument of `replicate`, x .

```
replicate n x = map (const x) ?xs
?f = const ?x :: ?Y → X
?x = x :: X
?xs :: List ?Y
```

Let us directly decide this is the most promising program so far (how to decide which program are we going to expand next is explained in Section 3.4). Like at the beginning, we have to generate a list. But this time we can choose what the elements of the list are, therefore we cannot rule out `enumTo`. As you see, we have a lot of possibilities, starting with `replicate n x = map (const x) []`, where we instantiate $?xs$ with `[]`, and ending with

```
replicate n x = map (const x) (enumTo ?n)
?f = const ?x :: ?Y → X
?x = x :: X
?xs = enumTo ?n :: List Int
?n :: Int
```

First we are going to evaluate the *closed* program (that is, without holes) `replicate n x = map (const x) []...` and see that it doesn't satisfy the I/O-examples. Then, as always, we take the most promising program and expand one of its holes. Let us decide that the most promising program is the last one. That is, we are searching for an integer. An integer can either be the constructor `0`, or the constructor `succ` applied to some integer hole, the first argument of `replicate`, n , or `const` with some clever first argument applied to something. This time, we got two closed programs that we are going to evaluate, `replicate n x = map (const x) (enumTo 0)` and `replicate n x = map (const x) (enumTo n)`. Evaluation shows that only the second one satisfies the I/O-examples.

- Important difference between X and $?X$. The first one is fixed and cannot be instantiated with anything, the second one is a free variable that can be instantiated with anything we like.

This example show following concepts

- Hole: unknown part of a program, for which we only know the type and that we can expand
- we maintain a frontier of programs with holes

- from this frontier, we always chose the most promising one. What program is the most promising can be determined by a cost function (Section 3.4)
- some programs are not worth expanding. For example, we see straight away that it does not make much sense to instantiate a hole with `const ?x ?y`, because we could as well write `?x` for it, which is always a shorter and preferable program. That is, some programs can be ruled out semantically (Section 3.5)
- The search space is quite big even with so few components. Exponential.

```
replicate n x = map Int X (const X Int x) (enumTo n)
```

3.2 Calculus

In this section we will introduce three different calculus.

- System F as in the excellent book of Pierce [6]
- The system we derive from System F and in which the components are defined (includes holes, input variables and recursion for terms and types)
- A subset of the second system, used for generation (only application of components, holes or input variables is allowed).

The exposition will closely follow Pierce [6].

Let's start with the standard System F. Our calculus is based on System F. A subset of our calculus is used for generation.

- generate programs from restricted calculus
- calculus itself is still powerful, because we use it to define the library components as well.

3.2.1 Terms and Types

- similar to Pierce
- except holes, components and free variables
- recursion
- recursive types that can take parameters
- the search space is not the whole language but only a subset of it

Real System F

$$t ::= x \mid \lambda x : T. t \mid t t \mid \Lambda X. t \mid t [T] \quad (\text{terms})$$

$$T ::= X \mid T \rightarrow T \mid \forall X. T \quad (\text{types})$$

$$\Gamma ::= \emptyset \mid \Gamma \cup \{x : T\} \mid \Gamma \cup \{X\} \quad (\text{variable bindings})$$

We use an extension of System F featuring holes $?x$, input variables i as well as named library components c and named types C . The use of the names enables recursion in the definition of library components and types. Named types also support type parameters. The number of type parameters supported by a named type is denoted as K in its definition.

Question: where is the output? Can you define input and output pairs? Our system

$$t ::= x \mid \lambda x : T. t \mid t t \mid \Lambda X. t \mid t [T] \mid c \mid ?x \mid i \quad (\text{terms})$$

$$T ::= X \mid T \rightarrow T \mid \forall X. T \mid ?X \mid I \mid C [T, \dots, T] \quad (\text{types})$$

$$\Gamma ::= \emptyset \mid \Gamma \cup \{x : T\} \mid \Gamma \cup \{X\} \quad (\text{variable bindings})$$

$$\Xi ::= \emptyset \mid \Xi \cup \{?x : T\} \mid \Xi \cup \{?X\} \quad (\text{hole bindings})$$

$$\Phi ::= \emptyset \mid \Phi \cup \{i = t : T\} \mid \Phi \cup \{I = T : K\} \quad (\text{input variable bindings})$$

$$\Delta ::= \emptyset \mid \Delta \cup \{c = t : T\} \mid \Delta \cup \{C = T : K\} \quad (\text{library components})$$

Question: do we need the definitions of library components for synthesis? We use them only for evaluation, but we always evaluate programs during synthesis. Same for the input variables. Actually, for each I/O-example we get the pair (Φ, o) , where Φ instantiates all input variables and input types of a program and o is the expected output. Subset of our system that builds the search space

$$t ::= t t \mid t [T] \mid c \mid ?x \mid i \quad (\text{terms})$$

$$T ::= X \mid T \rightarrow T \mid \forall X. T \mid ?X \mid I \mid C [T, \dots, T] \quad (\text{types})$$

$$\Xi ::= \emptyset \mid \Xi \cup \{?x : T\} \mid \Xi \cup \{?X\} \quad (\text{hole bindings})$$

$$\Phi ::= \emptyset \mid \Phi \cup \{i = t : T\} \mid \Phi \cup \{I = T : K\} \quad (\text{input variable bindings})$$

$$\Delta ::= \emptyset \mid \Delta \cup \{c = t : T\} \mid \Delta \cup \{C = T : K\} \quad (\text{library components})$$

A program is the quadruplet $\{\Xi, \Phi, \Delta \vdash t :: T\}$. A term is called *closed* if it does not contain holes. A program is closed, if Ξ is empty and t and T do not contain holes.

3.2.2 Encodings

Note that in the definition of types we do not see familiar types such as booleans, integers, lists or trees. All these types can be encoded in System F using either the Church's or the Scott's encoding [1]. We opted for the Scott's encoding because it's more efficient in our case. Scott's booleans coincide with Church's booleans and are encoded as follows.

```

Bool  =  $\forall R. R \rightarrow R \rightarrow R$ 
true  =  $\Lambda R. \lambda x_1:R. \lambda x_2:R. x_1$ 
      : Bool
false =  $\Lambda R. \lambda x_1:R. \lambda x_2:R. x_2$ 
      : Bool
if-then-else =  $\Lambda X. \lambda b:Bool. \lambda t:X. \lambda f:X. b [X] t f$ 
              :  $\forall X. Bool \rightarrow X \rightarrow X \rightarrow X$ 

```

Scott's integers differ from Church's integers as they are more suitable for pattern matching. *because they don't unwrap the whole integer every time.*

```

Int =  $\forall R. R \rightarrow (Int \rightarrow R) \rightarrow R$ 
zero =  $\Lambda R. \lambda z:R. \lambda s:Int \rightarrow R. z$ 
      : Int
succ =  $\lambda n:Int. \Lambda R. \lambda z:R. \lambda s:Int \rightarrow R. s n$ 
      :  $Int \rightarrow Int$ 
case =  $\Lambda R. \lambda n:Int. \lambda a:R. \lambda f:Int \rightarrow R. n [R] a f$ 
      :  $\forall R. Int \rightarrow R \rightarrow (Int \rightarrow R) \rightarrow R$ 

```

Analogously, Scott's lists are a recursive type and naturally support pattern matching.

```

List X =  $\forall R. R \rightarrow (X \rightarrow List X \rightarrow R) \rightarrow R$ 
nil =  $\Lambda X. \Lambda R. \lambda n:R. \lambda c:X \rightarrow List X \rightarrow R. n$ 
      :  $\forall X. List X$ 
con =  $\Lambda X. \lambda x:X. \lambda xs:List X. \Lambda R. \lambda n:R. \lambda c:X \rightarrow List X \rightarrow R. c x xs$ 
      :  $\forall X. X \rightarrow List X \rightarrow List X$ 
case =  $\Lambda X. \Lambda Y. \lambda l:List X. \lambda n:Y. \lambda c:X \rightarrow List X \rightarrow Y. l [Y] n c$ 
      :  $\forall X. \forall Y. List X \rightarrow Y \rightarrow (X \rightarrow List X \rightarrow Y) \rightarrow Y$ 

```

3.2.3 Evaluation semantics

We usually evaluate only closed programs. Eager evaluation. Rules. Judgement $\Phi, \Delta \vdash t \longrightarrow t'$.

Define *value* v to be a term to which no evaluation rule apply.

$$\frac{c = t : T \in \Delta}{\Phi, \Delta \vdash c \longrightarrow t} \text{E-Lib}$$

$$\frac{i = t : T \in \Phi}{\Phi, \Delta \vdash i \longrightarrow t} \text{E-Inp}$$

$$\frac{\Phi, \Delta \vdash t_1 \longrightarrow t'_1}{\Phi, \Delta \vdash t_1 t_2 \longrightarrow t'_1 t_2} \text{E-App1}$$

$$\frac{\Phi, \Delta \vdash t_2 \longrightarrow t'_2}{\Phi, \Delta \vdash v_1 t_2 \longrightarrow v_1 t'_2} \text{E-App2}$$

$$\frac{}{\Phi, \Delta \vdash (\lambda x : T_{11}. t_{12}) v_2 \longrightarrow [x \mapsto v_2] t_{12}} \text{E-AppAbs}$$

$$\frac{}{\Phi, \Delta \vdash (\Lambda X. t_2) [T_2] \longrightarrow [X \mapsto T_2] t_2} \text{E-APPABS}$$

3.2.4 Type checking

Question: why do I mathematically need that many contexts? How can I summarize T-Var, T-Hol, T-Inp, T-Lib in one rule?

Question: Should I type System F or only "programs"? Answer: System F, of course. In the code you type library components as well.

I moved this section because type checking does not need unification.

The typing judgement $\Gamma, \Xi, \Phi, \Delta \vdash t : T$. Based on the book of Pierce.

$$\frac{x : T \in \Gamma}{\Gamma, \Xi, \Phi, \Delta \vdash x : T} \text{T-Var}$$

$$\frac{?x : T \in \Xi}{\Gamma, \Xi, \Phi, \Delta \vdash ?x : T} \text{T-Hol}$$

$$\frac{i = t : T \in \Phi}{\Gamma, \Xi, \Phi, \Delta \vdash i : T} \text{T-Inp}$$

$$\frac{c = t : T \in \Delta}{\Gamma, \Xi, \Phi, \Delta \vdash c : T} \text{T-Lib}$$

$$\frac{\Gamma \cup \{x : T_1\}, \Xi, \Phi, \Delta \vdash t_2 : T_2}{\Gamma, \Xi, \Phi, \Delta \vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2} \text{T-Abs}$$

$$\frac{\Gamma, \Xi, \Phi, \Delta \vdash t_1 : T_1 \rightarrow T_2 \quad \Gamma, \Xi, \Phi, \Delta \vdash t_2 : T_1}{\Gamma, \Xi, \Phi, \Delta \vdash t_1 t_2 : T_2} \text{ T-App}$$

$$\frac{\Gamma \cup \{X\}, \Xi, \Phi, \Delta \vdash t_2 : T_2}{\Gamma, \Xi, \Phi, \Delta \vdash \Lambda X. t_2 : \forall X. T_2} \text{ T-ABS}$$

$$\frac{\Gamma, \Xi, \Phi, \Delta \vdash t_1 : \forall X. T_{12}}{\Gamma, \Xi, \Phi, \Delta \vdash t_1 [T_2] : [X \mapsto T_2] T_{12}} \text{ T-APP}$$

3.2.5 Type unification

The unification algorithm is based on the unification algorithm for typed lambda calculus from the book of Pierce **TODO: cite the book properly!!!** and slightly modified to fit our needs.

How did you do type unification? Unification of universal types is not implemented. If you need to unify to universal types, you should transform all bound variables into holes and remove the universal quantifier.

A set of constraints is a set of types that should be equal under a substitution. The unification algorithm is supposed to output a substitution σ so that $\sigma(S) = \sigma(T)$ for every constraint $S = T$ in \mathcal{C} .

3.3 Search

how do we explore the search space (best-first search). (only priority queue)

3.3.1 Search space

Use only third system presented in Section System F.

Formal problem definition. Given a library Δ , a goal type T and a list of I/O-examples $[(\Phi_1, o_1), \dots, (\Phi_N, o_N)]$, find a closed term t so that $\emptyset, \emptyset, \Phi_1, \Delta \vdash t : T$ (that is, t has the goal type under an empty variable binding context and an empty hole binding context) and t satisfies all I/O-examples, that is $\Phi_n, \Delta \vdash t \longrightarrow^* \vdash t'$ and $\Phi_n, \Delta \vdash o_n \longrightarrow^* \vdash t'$ for all $n = 1, \dots, N$.

We see the search space as a graph of programs with holes (see third syntax presented in Section System F) where there is an edge between two terms t_1 and t_2 if and only if the judgement *derive* defined below $\Xi, \Phi, \Delta \vdash t_1 :: T_1 \Rightarrow \Xi', \Phi, \Delta \vdash t_2 :: T_2$ holds between the two.

To express the rules in a more compact form, we introduce *evaluation contexts*. An evaluation context is an expression with exactly one syntactic hole \square in which we can plug in any term. For example, if we have the context \mathcal{E} we can place the term t into its hole and denote this new term by $\mathcal{E}[t]$.

Input: Set of constraints $\mathcal{C} = \{T_{11} = T_{12}, T_{21} = T_{22}, \dots\}$

Output: Substitution σ so that $\sigma(T_{i1}) = \sigma(T_{i2})$ for every constraint $T_{i1} = T_{i2}$ in \mathcal{C}

Function *unify*(\mathcal{C}) **is**

```

  if  $\mathcal{C} = \emptyset$  then []
  else
    let  $\{T_1 = T_2\} \cup \mathcal{C}' = \mathcal{C}$  in
    if  $T_1 = T_2$  then
      | unify( $\mathcal{C}'$ )
    else if  $T_1 = ?X$  and  $?X$  does not occur in  $T_2$  then
      | unify( $[X \mapsto T_2]\mathcal{C}' \circ [X \mapsto T_2]$ )
    else if  $T_2 = ?X$  and  $?X$  does not occur in  $T_1$  then
      | unify( $[X \mapsto T_1]\mathcal{C}' \circ [X \mapsto T_1]$ )
    else if  $T_1 = T_{11} \rightarrow T_{12}$  and  $T_2 = T_{21} \rightarrow T_{22}$  then
      | unify( $\mathcal{C}' \cup \{T_{11} = T_{21}, T_{12} = T_{22}\}$ )
    else if  $T_1 = C [T_{11}, T_{12}, \dots, T_{1k}]$  and  $T_2 = C [T_{21}, T_{22}, \dots, T_{2k}]$ 
      then
        | unify( $\mathcal{C}' \cup \{T_{11} = T_{21}, T_{12} = T_{22}, \dots, T_{1k} = T_{2k}\}$ )
      else
        | fail
    end
  end
end
end

```

Algorithm 1: Type unification

A hole $?x$ can be turned into a library component c from the context Δ or an input variable i from the context Φ . The procedure *fresh*(T) transforms universally quantified type variables into fresh type variables $?X$ not used in Ξ . The notation $\sigma(\Delta)$ denotes the application of the substitution σ to all types contained in the context Δ .

$$\frac{c : T_c \in \Delta \quad \sigma \text{ unifies } T \text{ with } \text{fresh}(T_c)}{\Xi, \Phi, \Delta \vdash ?x :: T \Rightarrow \sigma(\Xi \setminus \{?x : T\}), \Phi, \Delta \vdash c :: \sigma(T)} \text{D-VarLib}$$

$$\frac{i : T_i \in \Phi \quad \sigma \text{ unifies } T \text{ with } \text{fresh}(T_i)}{\Xi, \Phi, \Delta \vdash ?x :: T \Rightarrow \sigma(\Xi \setminus \{?x : T\}), \Phi, \Delta \vdash i :: \sigma(T)} \text{D-VarInp}$$

A hole can also be turned into a function application of two new active holes.

$$\frac{?X \text{ is a fresh type variable} \quad \Xi' = \Xi \setminus \{?x : T\} \cup \{?x_1 : ?X \rightarrow T, ?x_2 : ?X, ?X\}}{\Xi, \Phi, \Delta \vdash ?x :: T \Rightarrow \Xi', \Phi, \Delta \vdash ?x_1 ?x_2 :: T} \text{D-VarApp}$$

In all other cases we just choose a hole and expand it according to the three rules above.

$$\frac{\Xi, \Phi, \Delta \vdash ?x :: T_1 \Rightarrow \Xi', \Phi, \Delta \vdash t'_1 :: T'_1}{\Xi, \Phi, \Delta \vdash t[?x] :: T \Rightarrow \Xi', \Phi, \Delta \vdash t[t_1] :: [T_1 \mapsto T'_1]T} \text{D-App}$$

Note that the types of all successor programs unify with the types of their ancestors. Thus, the search is type directed. Only programs of the right type are generated.

3.3.2 Exploration

Write also about stack vs queue of open holes

We explore the search graph using a best first search. We can play with the algorithm in two points (marked in blue): first, we can define which hole to expand first, second, we can choose the compare function of the priority queue. Different approaches to define the compare function are discussed in the next session. Concerning the order of expansion of the holes, we tried two strategies: the first one was maintaining a stack of holes (Ξ implemented as a stack), which leads to the expansion of the deepest hole first, the second one was maintaining a queue of holes (Ξ implemented as a queue), which leads to the expansion of the oldest hole first.

Input: goal type T , library components Δ , list of input-output examples $[(\Phi_1, o_1), \dots, (\Phi_N, o_N)]$
Output: closed program $\{\Xi, \Phi_1, \Delta \vdash t :: T\}$ that satisfies all I/O-examples
`queue` \leftarrow PriorityQueue.empty **compare**
`queue` \leftarrow PriorityQueue.push `queue` $\{\Xi, \Phi_1, \Delta \vdash ?x :: T\}$
while not ((PriorityQueue.top `queue`) satisfies all I/O-examples) **do**
 `successors` \leftarrow **successor** (PriorityQueue.top `queue`)
 `queue` \leftarrow PriorityQueue.pop `queue`
 for all s in `successors` **do**
 `queue` \leftarrow PriorityQueue.push `queue` s
 end
end
return PriorityQueue.top `queue`
Algorithm 2: Best first search

3.4 Cost functions

The compare function in the best first search algorithm is $\text{cost } \$p_1\$ - \text{cost } \$p_2\$$. There are different possibilities to implement this cost function. We will present four alternatives.

number of nodes The first cost function is based only on the number of nodes of the term. Longer and more complicated terms are disadvantaged.

Input: term t

Output: weighted number of nodes in t

$\text{nof-nodes}(c) = 1$

$\text{nof-nodes}(?x) = 2$

$\text{nof-nodes}(i) = 0$

$\text{nof-nodes}(x) = 1$

$\text{nof-nodes}(\lambda x : T. t) = 1 + \text{nof-nodes}(t)$

$\text{nof-nodes}(t_1 t_2) = 1 + \text{nof-nodes}(t_1) + \text{nof-nodes}(t_2)$

$\text{nof-nodes}(\Lambda X. t) = 1 + \text{nof-nodes}(t)$

$\text{nof-nodes}(t [T]) = 1 + \text{nof-nodes}(t)$

Algorithm 3: Cost function based on the number of nodes

number of nodes and types The second cost function also adds a factor based on the types appearing in the term, thus penalizing terms with type application and very complicated types.

no same component In the third function we additionally penalize terms using the same component more than once.

length of the string The simplest and most imprecise method to take both the number of nodes and the complexity of the types appearing in the term is taking the length of the string of that term.

3.5 Black list

automatic generation of black list discussed in evaluation A black list is a list of terms. Programs containing a black term as a subterm are not allowed to have successors. Thus, the algorithm above is modified as follows.

One could use the synthesis algorithm presented in Section 3.3.2 to automatically synthesise black lists. For example, one could synthesise many programs corresponding to the identity function or to the empty list or to any other term, and add all those programs but one to the black list.

Input: term t

Output: sum of weighted number of nodes in term t and weighted number of nodes in the types appearing in t

$$\text{nof-nodes-type}(X) = 1$$

$$\text{nof-nodes-type}(\text{?}X) = 0$$

$$\text{nof-nodes-type}(I) = 0$$

$$\text{nof-nodes-type}(C [T_1, \dots, T_k]) = 0$$

$$\text{nof-nodes-type}(T_1 \rightarrow T_2) = 3 + \text{nof-nodes-type}(T_1) + \text{nof-nodes-type}(T_2)$$

$$\text{nof-nodes-type}(\forall X. T) = 1 + \text{nof-nodes-type}(T)$$

$$\text{nof-nodes-term}(c) = 1$$

$$\text{nof-nodes-term}(\text{?}x) = 2$$

$$\text{nof-nodes-term}(i) = 0$$

$$\text{nof-nodes-term}(x) = 1$$

$$\text{nof-nodes-term}(\lambda x : T. t) = 1 + \text{nof-nodes-term}(t) + \text{nof-nodes-type}(T)$$

$$\text{nof-nodes-term}(t_1 t_2) = 1 + \text{nof-nodes-term}(t_1) + \text{nof-nodes-term}(t_2)$$

$$\text{nof-nodes-term}(\Lambda X. t) = 1 + \text{nof-nodes-term}(t)$$

$$\text{nof-nodes-term}(t [T]) = 1 + \text{nof-nodes-term}(t) + \text{nof-nodes-type}(T)$$

$$\text{nof-nodes-and-types}(t) = \text{nof-nodes-term}(t)$$

Algorithm 4: Cost function based on the number of nodes and types

3.6 Templates

Top-down type-driven synthesis.

A template is a program with holes. We are interested in templates where all higher-order components are fixed and there are holes for the first-order components. The search space is thus similar to the search space described in 3.3.1, with the exception that Δ contains only the higher-order components. One of the new things are *closed holes* $\text{?}x$. Those are holes that are supposed to be filled in later with first-order components.

The idea behind the templates is that once the higher-order components are fixed, it should be easy and fast to find a first-order assignment to get the right program. So we could do a limited search from a template and if we do not find a program satisfying all of the I/O-examples we can move quickly to the next template.

We additionally restrict the space by requiring a template to have no more than M higher-order components and no more than P closed holes.

3.6.1 Successor rules

The successor rules are very similar to the ones defined in 3.3.1, apart from little modifications. That is, now we have a successor rule to close a hole,

Input: term t

Output: sum of the weighted number of nodes in term t , the weighted number of nodes in the types appearing in t and the weighted number of library components appearing more than once in t .

$$\text{nof-nodes-type}(X) = 5$$

$$\text{nof-nodes-type}(\text{?}X) = 1$$

$$\text{nof-nodes-type}(I) = 0$$

$$\text{nof-nodes-type}(C [T_1, \dots, T_k]) =$$

$$1 + \text{nof-nodes-type}(T_1) + \dots + \text{nof-nodes-type}(T_k)$$

$$\text{nof-nodes-type}(T_1 \rightarrow T_2) = 3 + \text{nof-nodes-type}(T_1) + \text{nof-nodes-type}(T_2)$$

$$\text{nof-nodes-type}(\forall X. T) = 5 + \text{nof-nodes-type}(T)$$

$$\text{nof-nodes-term}(c) = 1$$

$$\text{nof-nodes-term}(\text{?}x) = 1$$

$$\text{nof-nodes-term}(i) = 0$$

$$\text{nof-nodes-term}(x) = 5$$

$$\text{nof-nodes-term}(\lambda x : T. t) = 2 + \text{nof-nodes-term}(t) + \text{nof-nodes-type}(T)$$

$$\text{nof-nodes-term}(t_1 t_2) = 1 + \text{nof-nodes-term}(t_1) + \text{nof-nodes-term}(t_2)$$

$$\text{nof-nodes-term}(\Lambda X. t) = 5 + \text{nof-nodes-term}(t)$$

$$\text{nof-nodes-term}(t [T]) = 1 + \text{nof-nodes-term}(t) + \text{nof-nodes-type}(T)$$

$$\text{count}(t) = \sum_{c_i \text{ appears in } t} (\text{occurrences of } c_i \text{ in } t) - 1$$

$$\text{no-same-component}(t) = \text{nof-nodes-term}(t) + 2 \text{count}(t)$$

Algorithm 5: Cost function based on the number of nodes and types penalizing the use of a library component more than once

and we can not instantiate a hole with an input variable any more, because that is supposed to be done in the next step. All the rules are modified to take into account the restriction on the number of components and the number of closed holes. In order to do this, we need to pass along m , the number of higher-order components in the term whose subterms we are traversing.

So we can *close* a hole.

$$\frac{\begin{array}{c} |\Xi| \leq P \text{ and } m \leq M \\ T \text{ is a type a first-order component can have} \end{array}}{\Xi, \Phi, \Delta, m \vdash ?x :: T \Rightarrow \Xi, \Phi, \Delta, m \vdash \underline{?x} :: T} \text{G-VarClose}$$

We can instantiate a hole with a (higher-order) library component.

Input: goal type T , library components Δ , list of input-output examples $[(\Phi_1, o_1), \dots, (\Phi_N, o_N)]$, black list $[b_1, \dots, b_M]$

queue \leftarrow PriorityQueue.empty compare
queue \leftarrow PriorityQueue.push queue $\{\Xi, \Phi_1, \Delta \vdash ?x :: T\}$
while not ((PriorityQueue.top queue) satisfies all I/O-examples) **do**
 if not ((PriorityQueue.top queue) contains subterm from black list) **then**
 successors \leftarrow successor (PriorityQueue.top queue)
 queue \leftarrow PriorityQueue.pop queue
 for all s in successors **do**
 queue \leftarrow PriorityQueue.push queue s
 end
 else
 queue \leftarrow PriorityQueue.pop queue
 end
end
Output: PriorityQueue.top queue

Algorithm 6: Best first search with black list

$$\frac{\begin{array}{c} |\Xi| \leq P \text{ and } m < M \\ c = t_c : T_c \in \Delta \\ \sigma \text{ unifies } T \text{ with fresh}(T_c) \end{array}}{\Xi, \Phi, \Delta, m \vdash ?x :: T \Rightarrow \sigma(\Xi \setminus \{?x : T\}), \Phi, \Delta, m + 1 \vdash c :: \sigma(T)} \text{ G-VarLib}$$

We can instantiate a hole with a function application of two fresh holes.

$$\frac{\begin{array}{c} |\Xi| < P \text{ and } m \leq M \\ ?X \text{ is a fresh type hole, } ?x_1 \text{ and } ?x_2 \text{ are fresh term holes} \\ \Xi' = \Xi \setminus \{?x : T\} \cup \{?x_1 : ?X \rightarrow T, ?x_2 : ?X, ?X\} \end{array}}{\Xi, \Phi, \Delta \vdash ?x :: T \Rightarrow \Xi', \Phi, \Delta \vdash ?x_1 ?x_2 :: T} \text{ G-VarApp}$$

We can expand one of the holes of the program according to one of the three rules above.

$$\frac{\Xi, \Phi, \Delta, m \vdash ?x :: T_1 \Rightarrow \Xi', \Phi, \Delta, m' \vdash t'_1 :: T'_1}{\Xi, \Phi, \Delta, m \vdash t[?x] :: T \Rightarrow \Xi', \Phi, \Delta, m' \vdash t[t_1] :: [T_1 \mapsto T'_1]T} \text{ G-App}$$

Chapter 4

Implementation

What could I talk about in this chapter?

- Programming language and compiler version
- put the type definitions and explain them (What are Fun, FUN and BuiltinFun) (built-in integers for speed)
- Library syntax and the type-checking when added to the library?
- eager evaluation, describe evaluator

Chapter 5

Evaluation

What I want to see in this chapter:

- Table of all components
- Table of synthesized programs with synthesis times for the different algorithms
- Figure of the synthesis times of synthesized programs
- Comparison to Feser
- Explain why automatic black list and templates perform so poorly
- Show advantages of using black lists (I know it's trivial)
- Talk about the trivial thing that if you have more functions with the same time you will need much more time to find the program you are looking for.
- Talk about the constants in the cost functions and how they affect the search space???
- Stack versus queue expanding → mention it in the definitions

TODO: size of specification and size of synthesised program!!!

- sensible to the choice of examples: for speed (enumTo prod enumTo prod...), examples must be small and there must be only a few of them. But then it is very likely, that another program is generated. It is very tricky to choose the right examples for enumFromTo and for member.
- runtime highly dependent on the number of components used in the solution **TODO: make a graph relating number of components to runtime**
- runtime also dependent on the number of components given. The less components are given, the faster the synthesiser can find the right

program (search space grows exponentially also in the components, higher branching factor).

5.1 Set up

Describe the machine and the testing set up, which components were used, what information did you give to the synthesizer, how many examples were given. What else?

5.2 Cost functions

Compare the different cost functions with each other, explain why are they good for some programs and bad for other. Table.

5.3 Black lists

In the presence of "useless" functions with a high branching factor like flip, const or uncurry black list pruning is a must. But it can also be useful in other cases as well. The important thing is to find a reasonable trade off between the length of the black list (don't forget that every subterm of every open program is matched against every item of the black list) and the degree of pruning. A longer black list prunes more of the search space, but the synthesis procedure also takes longer.

5.3.1 Benefits of black lists

Table of your super manual black list. Figure that for each cost function compares time with and without black list. Trivial words about pruning search space and useless branches.

5.3.2 Shortcomes of automatically generated black lists

Maybe really bring the automatically generated table. Point at the repetitions. Say that automatically generated I/O-examples are bad and we need a lot of them. Say that you tried only identity pruning, but one could also try to generate, say, the empty list or whatever.

5.4 Templates

Short section explaining that the templates you generate are not the templates you expect and why I found one example, where templates help! For dropmax I had a run out of memory exception with plain enumeration and I could synthesise it in 7 seconds with templates! (Ok, I modified templates to

put in only really higher-order components and close every hole, no matter the type).

It is very important to choose the examples well, because many generated programs are bad and will take forever to evaluate. And as we evaluate each closed program on the examples... We should really try to keep them small and simple. For example, I had a problem when I tried to generate `dropmax` using the example `[1,4,3]`, because one of the generated programs was `enumTo (prod (enumTo (prod _0)))`. It tried to construct a list with 479001600 elements and ran out of memory. At the same time, they should give enough information, otherwise a simpler program will be generated that, although it satisfies all given I/O-examples, it not what the user had in mind when writing the specification.

Conclusions

6.1 Conclusions

The baseline is not that bad. Gathered some data about the search space.

6.2 Future Work

Templates done well, augmented examples?

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