Inductive Synthesis from Higher-Order Functions

Master Thesis

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Abstract

This example thesis briefly shows the main features of our thesis style, and how to use it for your purposes.

Contents

Chapter 1

Introduction

This is version `v1.4` of the template.

We assume that you found this template on our institute's website, so we do not repeat everything stated there. Consult the website again for pointers to further reading about LaTeX. This chapter only gives a brief overview of the files you are looking at.

1.1 Features

The rest of this document shows off a few features of the template files. Look at the source code to see which macros we used!

The template is divided into TeX files as follows:

1. `thesis.tex` is the main file.

2. `extrapackages.tex` holds extra package includes.

3. `layoutsetup.tex` defines the style used in this document.

4. `theoremsetup.tex` declares the theorem-like environments.

5. `macrosetup.tex` defines extra macros that you may find useful.

6. `introduction.tex` contains this text.

7. `sections.tex` is a quick demo of each sectioning level available.

8. `refs.bib` is an example bibliography file. You can use BibTeX to quote references. For example, read if you can get a hold of it.

1.1.1 Extra package includes

The file `extrapackages.tex` lists some packages that usually come in handy. Simply have a look at the source code. We have added the following comments based on our experiences:

REC This package is recommended.

OPT This package is optional. It usually solves a specific problem in a clever way.

ADV This package is for the advanced user, but solves a problem frequent enough that we mention it. Consult the package's documentation.

As a small example, here is a reference to the Section *Features* typeset with the recommended *varioref* package:

See Section **??**.

1.1.2 Layout setup

This defines the overall look of the document – for example, it changes the chapter and section heading appearance. We consider this a 'do not touch' area. Take a look at the excellent *Memoir* documentation before changing it.

In fact, take a look at the excellent *Memoir* documentation, full stop.

1.1.3 Theorem setup

This file defines a bunch of theorem-like environments.

Theorem 1.1 *An example theorem.*

Proof Proof text goes here. □

Note that the q.e.d. symbol moves to the correct place automatically if you end the proof with an `enumerate` or `displaymath`. You do not need to use `\qedhere` as with *amsthm*.

Theorem 1.2 (Some Famous Guy) *Another example theorem.*

Proof This proof

1. ends in an enumerate.

Proposition 1.3 *Note that all theorem-like environments are by default numbered on the same counter.*

Proof This proof ends in a display like so:

$$f(x) = x^2.$$

1.1.4 Macro setup

For now the macro setup only shows how to define some basic macros, and how to use a neat feature of the *mathtools* package:

$$|a|, \quad \left|\frac{a}{b}\right|, \quad \left|\frac{a}{b}\right|.$$

Chapter 2

Related Work

Try to answer the following three question for each paper read:

1. What is new in this approach? Or better, what is the approach. Describe technically the approach, so that you can answer technical questions.

2. What is the trick? (Why are they better than others?)

3. Which examples they can do really well? What kind of examples do they target? What is the most complicated thing they can generate?

Nadia Polikarpova 2015

here is a talk: http://research.microsoft.com/apps/video/default.aspx?id=255528&l=i

and here is the code: https://bitbucket.org/nadiapolikarpova/synquid
In [**?**] Synquid is proposed. Refinement types (types decorated with logical predicates) are used to prune the search space. SMT-solvers are used to satisfy the logical predicates. The key is the new procedure for type inference (called modular refinement type reconstruction), which thank to its modularity scales better than other existing inference procedures for refinement types. Programs can therefore be type checked even before they are put together. Examples that this tool is able to synthesize include several sorting algorithms, binary-search tree manipulations, red-black tree rotation as well as other benchmarks also used by other tools (**TODO:** read about these benchmarks and write if there is something interesting). The user specifies the desired program by providing a goal refinement type.

Feser 2015
The tool proposed in [**?**] is called λ^2 and generates its output in λ-calculus with algebraic types and recursion. The user specifies the desired program providing input-output examples. No particular knowledge is required from the user, as was demonstrated using random input-output examples

The examples are inductively generalized in a type-aware manner to a set of hypotheses (programs that possibly have free variables). The key idea are the hard-coded deduction rules used to prune the search space depending on the semantics of some of the higher-order combinators (map, fold, filter and a few others). Deduction is also used to infer new input-output examples in order to generate the programs needed to fill in the holes in the hypotheses. This tool is able to synthesize programs manipulating recursive data structures like lists, trees and nested data structures such as lists of lists and trees of lists. The examples that require much more time to be synthesized than the others are dedup (remove duplicate elements from a list), droplast (drop the last element in a list), tconcat (insert a tree under each leaf of another tree), cprod (return the Cartesian product of a list of lists), dropmins (drop the smallest number in a list of lists), but all of them are synthesized under 7 minutes.

Kincaid 2013

In [**?**] Escher is presented, an inductive synthesis algorithm that learns a recursive procedure from input-output examples provided by the user. The user must provide a "closed" set of examples, otherwise recursion cannot be handled properly The target language is untyped, first-order and purely functional. The algorithm is parametrized by components that can be instantiated differently to suit different domains. The approach combines enumerative search and conditional inference. The key idea is to use a special data structure, a *goal graph*, to infer conditional branches instead of treating `if-then-else` as a component. Observational equivalence is also used to prune the search space. Programs with the same value vectors (output of the program when applied to the inputs of the input-output examples) are considered equivalent and only one of them is synthesized. An implementation of the tool was tested on a benchmark consisting of recursive programs (including tail-recursive, divide-and-conquer and mutually recursive programs) drawn from functional programming assignments and standard list and tree manipulation programs. For all examples the same fixed set of components was used. The tool is able to synthesize all of them quickly. There is very little information on how many input-output examples were needed to synthesize the benchmarks and how difficult it is for a non-experienced user to come up with a "closed" set of examples.

Osera 2015

The tool in [**?**] is called Myth and uses not only type information but also input-output examples to restrict the search space. The special data structure used to hold this information is the *refinement tree*. This system can synthesize higher-order functions, programs that use higher order functions and work with large algebraic data types.

There is an ML-like type system that incorporates input-output examples. Two pieces: a *refinement tree* and an enumerative search.

Two major operations: refine the goal type and the examples and guess a term of the right type that matches the examples.

A small example to show what does the procedure. The user specifies a goal type incorporating input-output examples as well as the "background": the types and functions that can be used.

```
stutter :
```

Baseline

3.1 Syntax

it's the description of the syntax of the programs in the search space as well as how to derive programs from programs.

Rewrite this without mixing syntax with semantics and with the baseline algorithm In the context Γ we store the bindings from library components and input variables to their types.

In the context Δ we store the bindings from active and inactive holes to their types.

Terms are applications of library components or input variables (denoted by x in the grammar), values (denoted by v) and holes ($?x$ and $?\underline{v}$).

We distinguish between active holes $?x$, that can be substituted with other programs, from inactive holes $?\underline{v}$, that can only be replaced with values. To determine the exact value, a heuristic using a mix of symbolic execution and brute force is applied later. Every active hole having a value type (denoted by V in the grammar) can be turned into an inactive hole.

Programs without active holes are considered *closed*. Those are the programs in which we plug in the input and execute as far as possible to see whether it agrees with the expected output.

Values are integer constants (denoted by n for brevity), lists of values and tuples of values. *Does it make sense to have a special treatment for list values and tuples? Wouldn't it be better to use library components to construct them?*

We use a standard type system featuring functional and universal types, as well as integers, lists and tuples. We distinguish between two kinds of type variables: $?X$ are the type variables that can be instantiated with other types

when it comes to unification, X are the type variables that are already fixed by the goal type and have to remain like that. Universal types are assumed to be used like in Haskell only as outer wrapping of the types of library components and the goal type specified by the user.

TODO: Find a way to format t t as t t and not a tt, same for VV and TT
TODO: Find a way to format it vertically and adding comments to each line. For example, write "inactive hole" on the right of ?$\underline{v}$. Maybe it's better to write Γ and Δ as sets and not as grammars, as you are using them as sets. If you leave it like this, $\Delta \setminus \{?x : T\}$ would be undefined.

$$\Gamma ::= \varnothing \mid \Gamma \cup x : T \qquad\qquad\qquad\qquad\qquad \textit{(library components)}$$

$$\Delta ::= \varnothing \mid \Delta \cup ?x : T \mid \Delta \cup ?\underline{v} : T \qquad\qquad\qquad\qquad \textit{(holes)}$$

$$t ::= v \mid ?\underline{v} \mid ?x \mid x \mid tt \qquad\qquad\qquad\qquad\qquad\qquad \textit{(terms)}$$

$$v ::= n \mid v : v \mid [] \mid (v, v) \qquad\qquad\qquad\qquad\qquad\qquad \textit{(values)}$$

$$T ::= \text{Int} \mid \text{List } T \mid \text{Tuple } TT \mid T \to T \mid \forall X.T \mid X \mid ?X \qquad \textit{(types)}$$

$$V ::= \text{Int} \mid \text{List } V \mid \text{Tuple } VV \qquad\qquad\qquad\qquad\qquad \textit{(value types)}$$

It may appropriate to move this to the next section We assume that the goal program is normalized in the following sense. If the goal type specified by the user is a universal type, then we substitute each universally quantified type variable with a fresh fixed type variable X. If the goal type is a function type, then we abstract the type as much as possible and add input variables to the context Γ. For example, if the user specifies the goal program as

```
length :: ∀X. List X -> Int
length [1,2,3] == 3
length [2,2,2] == 3
length [] == 0
length [5] == 1
```

then we start our search from the partial program $\Gamma \cup \{xs : \text{List } X\} \vdash ?x :: \text{Int}$ where Γ already contains all bindings from library components to their types. Note that it must be possible to instantiate the type variable X with every possible type, therefore we do not have the right to prefer one instantiation over another and must treat X as an uninterpreted type.

TODO: Add the rules of the typing judgement The typing judgement is standard. To define the search graph we are going to explore in order to find the goal program, we also need the *derive* judgement, which says between which nodes of the graph (programs of the form $\Gamma, \Delta \vdash t :: T$) there is an

edge. To express the rules in a more compact form, we introduce *evaluation contexts*. An context is an expression with exactly one syntactic hole [] in which we can plug in any term. For example, if we have the context $\mathcal{E}$ we can place the term t into its hole and denote this new term by $\mathcal{E}[t]$.

TODO: Make a figure of all rules We can turn an active hole into an inactive hole if the active hole has a value type. do we need a premise? Wouldn't it be enough to write ?x :: V below the line?

$$\frac{\Gamma, \Delta \vdash ?x :: V}{\Gamma, \Delta \vdash ?x :: T \Mapsto \Gamma, \Delta \setminus \{?x : T\} \cup \{?\underline{v} : V\} \vdash ?\underline{v} :: V} \text{ D-VarVal}$$

An active hole $?x$ can be turned into a library component or an input variable x from the context Γ. The procedure fresh(T) transforms universally quantified type variables into fresh type variables $?X$ not used in Δ. The notation $\sigma(\Delta)$ denotes the application of the substitution σ to all types contained in the context Δ.

$$\frac{x : T_x \in \Gamma \qquad \sigma \text{ unifies } T \text{ with fresh}(T_x)}{\Gamma, \Delta \vdash ?x :: T \Mapsto \Gamma, \sigma(\Delta \setminus \{?x : T\}) \vdash x :: \sigma(T)} \text{ D-VarLib}$$

An active hole can also be turned into a function application of two new active holes.

$$\frac{?X \text{ is a fresh type variable}}{\Gamma, \Delta \vdash ?x :: T \Mapsto \Gamma, \Delta \setminus \{?x : T\} \cup \{?x_1 :?X \to T, ?x_2 :?X\} \vdash ?x_1\ ?x_2 :: T} \text{ D-VarApp}$$

In all other cases we just choose an active hole and expand it according to the three rules above.

$$\frac{\Gamma, \Delta \vdash ?x :: T_1 \Mapsto \Gamma, \Delta' \vdash t_1' :: T_1'}{\Gamma, \Delta \vdash t[?x] :: T \Mapsto \Gamma, \Delta' \vdash t[t_1] :: T[T_1/T_1']} \text{ D-App}$$

3.2 Baseline Algorithm

The breadth (or best) first search of the search graph, nothing fancy.

The baseline algorithm is a simple best first search implemented using a priority queue. One possibility is to order the enqueued elements according to the number of library components and active holes.

The root node has the form $\Gamma, \{?x : T\} \vdash ?x :: T$ where T is neither functional nor universal type and Γ contains the input variables along with the library components. The successors of a node are the nodes reachable in one step of the derive judgement. That is, $\Gamma, \Delta' \vdash t' :: T'$ is a successor of $\Gamma, \Delta \vdash t :: T$ if it holds $\Gamma, \Delta \vdash t :: T \mapsto \Gamma, \Delta' \vdash t' :: T'$.

We don't need to explore nodes that are equivalent up to alpha conversion to already visited nodes.

A term is considered *closed* if it does not contain active holes. Closed terms are tested on the input-output examples. Programs with inactive holes are symbolically executed on the input-output examples and, if possible, the concrete value of the inactive hole is determined. Otherwise we try to solve it by brute force with a small timeout. Does it make sense in terms of performance? Usually the values we need are really simple like `[]`, `0`, `1`. Wouldn't it be more efficient to try some simple values first?

The input-output examples are given as a vector of inputs `I` and the vector of corresponding expected outputs `O`.

TODO: Typeset in a nicer way, for example with a vertical line instead of all those 'end' and nicer keyword formatting.

```
BFS(root, I, O)
  queue ← {root}
  visited ← {}
  while(timeout not reached)
    current ← queue.dequeue
    if (current.closed)
      if (current.test(I) == O)
        return current
      end
    else
      for (s in current.successors)
        if (!visited.alphacontains(s))
          queue ← queue.push(s)
        end
      end
    end
  end
end
```

Benchmarks

Some programs over numbers, some over lists, some over lists of lists and some over trees (What kind of trees?). For every program, try to get a sample implementation.

Types needed: `Int`, `[a]`, `Tree a`

Basic components needed: arithmetic (`+`, `-`, `*`, `/`), relation (`<`, `<=`, `==`, `/=`, `>=`, `>`), (maybe we do not need relations)

1. max of two numbers
   (hopefully) the easiest program

   ```
   max :: Int -> Int -> Int
   max 0 0 == 0
   max 1 0 == 1
   max 0 1 == 1
   max x y = if x > y then x else y
   ```

   We don't care about conditionals, we cannot synthesize this.
   This is the only function that requires a conditional branch.

2. square a number

   ```
   square :: Int -> Int -> Int
   square 0 == 0
   square 1 == 1
   square 2 == 4
   square 3 == 9
   square x = x * x
   ```

   That is, basic arithmetic operations like + - * / should be provided

3. tetrahedral numbers

```
tetrahedral :: Int -> Int
tetrahedral 1 == 1
tetrahedral 2 == 4
tetrahedral 3 == 10
```

closed form solution

```
tetrahedral n = n * (n+1) * (n+2) / 6
```

iterative solution

```
tetrahedral n = scanl1 (+) (scanl1 (+) [0..]) !! n
```

Another iterative solution (without infinite lists)

```
tetrahedral n = foldl1 (+) (scanl1 (+) (enumFromTo 1 n))
```

Components needed: `scanl1`, `!!`
Interestingly the iterative version is much faster than the closed form
solution

4. prime test
   I think this is too difficult

```
prime :: Int -> Int
prime 1 == 0
prime 2 == 1
prime 3 == 1
prime 4 == 0
prime 25 == 0
prime 29 == 1
prime n = minimum (1 : (map (mod n) (enumFromTo 2 (subtract 1 n
```

Components needed: `map, mod, minimum, enumFromTo, subtract`

5. average

```
average :: [Int] -> Int
average [1] == 1
average [1,3] == 2
average [1,2,3,6] == 6
average xs = (sum xs) `div` (length xs)
```

6. movingAverage (forward)

```
movingAverage :: Int -> [Int] -> [Int]
movingAverage 1 [1,2,3] == [1,2,3]
movingAverage 2 [1,2,3] == [2,2,3]
movingAverage 3 [3,2,4,1,5,2] == [3,2,3,2,3,2]
movingAverage n xs = map (average . take n) (init $ tails xs)
```

Components needed: `tails` from `Data.List` and `average` (one of the benchmarks), as well as `map`, `take` and `init` from `Prelude`.

7. movingSum (backward)

```
movingSum :: Int -> [Int] -> [Int]
movingSum 1 [1,2,3] == [1,2,3]
movingSum 2 [1,2,3] == [1,3,5]
movingSum 3 [4,8,6,-1,-2,-3,-1,3,4,5] == [4,12,18,13,3,-6,-6,-1,6,12]
movingSum n xs = scanl1 (+) (zipWith (-) xs (replicate n 0 ++ xs))
```

8. waterflow problem
Given an array of "wall" heights, determine the volume of the puddles that can form if it rains.

```
water :: [Int] -> Int
water [1,2,3] == 0
water [5,2,5] == 3
water [2,3,1,6,1] == 2
water h = sum $
      zipWith (-)
        (zipWith min (scanl1 max h) (scanr1 max h))
        h
```

9. horner schema to evaluate polynomials

```
horner :: [Int] -> Int -> Int
horner [1,2,3] 1 == 6
horner [1,2,3] 2 == 11
horner [4,3,2] 3 == 47
horner p x = foldl1 ((+) . (x *)) p
```

Problem: we do not generate lambda's. Do we generate functions like `(x *)`?

10. sum-under, sum all integers up to the argument

```
sum_under :: Int -> Int
sum_under 0 == 0
sum_under 1 == 1
sum_under 2 == 3
sum_under 3 == 6
sum_under 4 == 10
sum_under n = sum [1..n]
```

Components needed: `sum, enumFromTo`

11. factorial

```
factorial :: Int -> Int
factorial 0 == 0
factorial 1 == 1
factorial 3 == 6
factorial 5 == 120
factorial n = product [1..n]
```

interesting for intermediate states

12. maximum of a list
I don't know (yet) how to specify a "global property" like greater or
smaller than all other elements in a list in Synquid. Moreover, it seems
a difficult property to extract from input-output examples.

```
maximum :: [Int] -> Int
maximum [1,3,2] == 3
maximum [4,2,1] == 4
maximum [1,3,5] == 5
maximum xs = foldr max (head xs) xs
```

Or just use the maximum function from Prelude, if it is given as a com-
ponent

13. append two lists
The specification given by Nadia does not synthesize the usual append
function. Maybe it's better to let her know...
Although it's possible to synthesize append in Synquid.

```
append :: [a] -> [a] -> [a]
append [1,2,3] [4,5,6] == [1,2,3,4,5,6]
append [1,2] [6,2,3] == [1,2,6,2,3]
append xs ys = foldr (:) ys xs
```

Or use ++ from Prelude, if we decide to provide it for this example
too.

14. length of a list
Can be also interesting for intermediate states

```
length :: [a] -> Int
length [1,2,3] == 3
length [2,2,2] == 3
length [] == 0
length [5] == 1
length xs = sum $ map (const 1) xs
```

15. list reversal

```
reverse :: [a] -> [a]
reverse [1,2,3] == [3,2,1]
reverse [5,2,3] == [3,2,5]
reverse [6,2,3,1] == [1,3,2,6]
reverse xs = foldl (flip (:)) [] xs
```

16. bagsum: [far,bar,gar,bar,bar,far] -> [(bar,3),(far,2),(gar,1)]
    Seems difficult and maybe intermediate states can be helpful. Should
    I take Int instead of a? I mean, a should belong to the typeclass Ord,
    otherwise we cannot yield a sorted output list. And we do not have
    any other base types anyway. But actually lists of integers are also
    ordable. Hence also lists of lists of integers and lists of lists of lists of
    integers and so on.

```
bagsum :: [a] -> [(a, Int)]
bagsum [1,1,1] == [(1,3)]
bagsum [4,4,2,1,2,1] == [(1,2),(2,2),(4,2)]
bagsum [3,2,1,3,2,3] == [(1,1),(2,2),(3,3)]
bagsum xs = map (head &&& length) (group (sort xs))
```

Components needed: &&& from Control.Arrow. And we also need
Tuples for this one.

17. stutter
    Repeat every list element twice.

```
stutter :: [a] -> [a]
stutter [] == []
stutter [1,2,3] == [1,1,2,2,3,3]
stutter xs = concatMap (replicate 2) xs
```

18. map
    Isn't it a higher order function? I thought we synthesize only first or-
    der functions.
    How can we provide examples? I mean, we have to write functions as
    well.

```
map :: (a -> b) -> [a] -> [b]
map f xs = foldr ((:) . f) [] xs
```

19. zipWith
    it's a higher order function as well. We need Tuples.
    How do we provide the examples?

```
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]
zipWith f xs ys = map (uncurry f) (zip xs ys)
```

Components needed: `map`, `uncurry`, `zip`.

20. list drop

```
drop :: Int -> [a] -> [a]
drop 0 [1,2,3] == [1,2,3]
drop 1 [1,2,3] == [2,3]
drop 5 [5,4,2,5] == []
drop 3 [4,2,3,1] == [1]
drop n xs = snd (splitAt n xs)
```

21. droplast, drop the last element of a list

```
droplast :: [a] -> [a]
droplast [] == []
droplast [1] == []
droplast [1,2,3] == [1,2]
droplast [3,2,1]== [3,2]
droplst = init
droplast' xs = map fst (zip xs (enumFromTo 1 (subtract 1 (lengt
droplast'' xs = take (subtract 1 (length xs)) xs
```

22. dropmax, drop the greatest element of a list
    λ^2 takes much more time to synthesize droplast than dropmax. Why?

```
dropmax :: [Int] -> [Int]
dropmax [1,2] == [1]
dropmax [2,1] == [1]
dropmax [1,2,3] == [1,2]
dropmax [3,2,1] == [2,1]
dropmax [2,3,1] == [2,1]
dropmax xs = filter (/= (maximum xs)) xs
```

23. dedup, remove duplicates from a list
    λ^2 requires more time
    TODO: Find a non-recursive implementation that preserves the order
    of the elements Either implement it with `groupBy` or say you cannot im-
    plement it. Say it's not possible to get the usual semantics of deleting
    all but the first occurrence of some program.

24. sort by length (on lists of lists)

```
sortByLength :: [[a]] -> [[a]]
sortByLength [[1,2],[1,2,3],[1]] == [[1],[1,2],[1,2,3]]
sortByLength = sortBy (curry ((uncurry compare) . (length *** l
```

    TODO: Is there a shorter implementation? What's wrong with this
    one?
    Components needed: *(***)* from `Control.Arrow`

25. dropmins
λ^2 required more time to synthesize it

```
dropmins :: [[Int]] -> [[Int]]
dropmins [[1,2,3],[2,3,1],[2,3],[1]] == [[2,3],[2,3],[3],[]]
dropmins = map dropmin
```

TODO: Is there an implementation without the auxiliary function `dropmin` and without lambda expressions? Yes, there is one, but you don't want to see it. With `ap` and `join`.

26. lasts, last element of every list
another program on nested lists

```
lasts :: [[a]] -> [a]
lasts [[1,2,3],[3,2],[4,2,4,1],[2]] == [3,2,1,2]
lasts = map last
```

27. member of the tree
Something with trees. Membership seems a difficult thing to learn from input-output examples.

28. count leaves it a tree

29. nodes at level The standard Haskell tree is a rose tree. Defined in `Data.Tree`.

Nadia has more complicated examples with Red-Black-Trees, AVL-trees and different sorting algorithms

Appendix A

Dummy Appendix

You can defer lengthy calculations that would otherwise only interrupt the flow of your thesis to an appendix.

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