Algorithms Analysis and Design Project Discrete Optimization

Team R: Ahana, Anandhini, Ananya, Anmol, Megha, Nikhil, Shashwat, Shivam, Tanvi

Advisors: Dr. Kannan Srinathan, Preet Thakkar

Recap (As of October 22nd)

- Divided into 3 sub-teams:
 - Constraint Programming: Ahana, Anandhini, Ananya, Tanvi
 - Local Search: Anmol, Shashwat, Shivam
 - Linear / Mixed Integer Programming: Megha, Nikhil
- Completed Pascal van Hatenryck Coursera Lectures
- LS team and Nikhil completed the assignment
- Made notes to be formalized into a guide later
- Read theory using external resources and made notes
- Presented progress in mid-discussion

Local Search Subteam - Progress Since

- Both Anmol and Shashwat implemented Euclidean Traveling Salesman Problem Assignment. Best score :50/60 Summary of results. (Codes can be found here and here)
- Approaches tried over 2-3 days and improved over several days for both
 - a. Swapping adjacent O(N) and all pairs $O(N^2)$
 - Initializations: Polar sort, Nearest neighbor, Random with restarts, spanning tree 2-approximation
 - c. 2-opt
 - d. Simulated Annealing with Reheats and Restarts
 - e. Guided Fast Local Search [Best]

Local Search Subteam - Progress Since

- 3. <u>Completed notes</u> (adding Guided Fast Local Search, Simulated annealing, 2-approx algorithm for TSP), converted notes to a formal guide
- 4. Wrote a report on Euclidean TSP(treating it as a running problem)
- 5. Our codes have been deployed here.

Input format is number of cities and their coordinates. Output is the ordering of cities. On dragging the output file to the second output, a visualization of the output ordering on 2D space can be obtained here.

Constraint Programming - Progress Since

- Made a guide on constraint programming and its applications with examples like 8 queens, map colouring, sudoku, cryptoarithmetic puzzles, magic series, stable marriage problem, car sequencing problem, perfect square problem, job shop scheduling, character case conversion.
- Added Appendix
- A sudoku solver in python (code is included with the notes)
- A webpage displaying all the information related to constraint programming and a tool within the website to input sudoku values and output the result

Link to the website: https://discreteoptim.herokuapp.com/

Ahana, Anandhini, Ananya, Tanvi

LP - Progress Since

<u>Link to LP guide</u> Made the notes (incl. illustrations and proofs) on Linear Programming for the guide. Read up on MIP as well. Topics covered:

- 1. Geometric and algebraic perspective of LP
- 2. Simplex algorithm with detailed proofs.
- 3. LP Duality and applications like: Max-flow min cut, Minimum Cost flow, Linear Regression, Linear Classifiers, Minimax theorem
- 4. Google OR Tools and LP Solvers
- 5. Analysis of methods to solve LP problems: Simplex, Ellipsoid, Integer Point
- 6. Read up on Karmarkar's algorithm, semi-definite programming, approximation algorithms

MIP - Progress Since

- Covered branching methods like subtour constraints used for TSP and bounding solution using cutting plane techniques (Goromy, Cover cuts).
- <u>Sensitivity Analysis</u> of Simplex -- extremely useful in practice to re-run <u>Dual Simplex</u> with addition of new constraints/variables.
- Solved Facility Location problem. <u>Received 74/80 score.</u>
 <u>Report on the same.</u>

Hashcode - Self-Driving Car Fleet Optimization

- 1. Attempted in a timed 4 hour-environment by Nikhil, Shashwat
- 2. We implemented a simple but interesting greedy simulation
- 3. Our score set us to achieve Global Rank 13th!
- 4. Had Local Search formulation ready, but didn't need to implement due to great performance by our greedy
- 5. Wrote a <u>detailed report</u> to be published on Medium soon! Will help IIIT and external students prepare too.

 Nikhil, Shashwat

Hashcode - Photo Slideshow Optimization

- 1. Attempted in timed 4 hour-environment by Anmol, Nikhil, Shashwat
- 2. We dissected the problem into Weighted maximum bipartite matching on general graphs and ordering (as in TSP).
- 3. Coded 4 main approaches:
- a. Data exploitation using DFS sufficed for good score on file B
 b. Greedy + Local search for matching
 c. Greedy + Local search for ordering
 d. Combining b) and c)
- 4. Not as great results. Should have kept it simple, stupid! :')

 Anmol, Nikhil, Shashwat

Judging System - Minimizing Disagreements using MFAS

- Wrote algorithms for a judging system for subjective contests/grading based on resolving pairwise comparisons by judges into a final ranking. Compiled detailed report.
- Theory work explained in report:
 - Background and applications
 - Reduction to Minimum Feedback Arc Set
 - Proof of NP-Hardness
 - Survey of approx/greedy algorithms
 - MIP Formulation
 - Discrete optimization techniques applied: KL2, Chanas, 2-opt
 - Read Papers

Judging System - Minimizing Disagreements using MFAS

- Introduction

- Humans are bad at regression and numerical scoring, thus the need for pairwise comparisons. Ranking must minimize disagreements.
- Generally formulated as the Minimum Feedback Arc Set (MFAS) problem or the Linear Ordering problem (LOP).
- Proof of NP-hardness and survey of approximate/greedy results.

- Use cases (different data sets):

- Judging AAD projects ~ 40 teams, 5 judges (small but slow)
- Hackathons ~ 100s of teams, 10-12 judges (large but fast)
- Digital Art ~ 1000s of submissions, 15-20 judges (medium + medium)

Judging System - Minimizing Disagreements using MFAS (2)

- Synthetic Data Generation.
 - Generated by making a model of how judgements happen based on:
 - Varying judge preferences
 - Varying scores of teams on different criteria
 - Planted dream solution

- Approaches

- Exact approaches like Dynamic Programming, MIP for small cases.
- Local search heuristics like naive insertion, K-OPT, Lin-Kernighan,
 Chanas for handling large cases.

- Benchmarks

Lolib dataset with normalized matrices (IO, RandA1, RandA2, SGB)
 (Performance of our algos can be found here)
 Anmol, Nikhil, Shashwat

Branch and Bound

Introduced guide using **Branch and Bound**Notes on search strategies and here

Application to the knapsack problem

This topic serves as an introduction to the concepts of discrete optimization. The goal of a branch-and-bound algorithm is to find a value x that maximizes or minimizes the value of a real-valued function f(x), called an objective function, among some set S of admissible, or candidate solutions.

- Defines notion of a state
- Uninformed search strategies
- Informed search strategies

Ahana, Anmol, Megha, Shashwat, Tanvi

Final Deployments and Conclusion

- Deployed by Shivam, with individual attempts from Ahana, Tanvi and Megha
- Link to guide
- Link to deployments/codes
 - MFAS Judging System
 - Branch and Bound Knapsack (Plug-and-play search strategies)
 - Traveling Salesman (Multiple approaches using LS)
 - Facility Location Problem (LS + MIP)
 - Hashcode Codes
- Takeaway: Learnt different discrete optimization techniques for NP-Hard problems. Designed and deployed solutions for multiple problems, also dabbling with proofs and approximations.

Literature Reviewed

- Some of the many Research Paper/Articles read:
 - Survey of weighted MFAS approaches along with their own approach(MIP)
 - <u>Calculating MFAS on unweighted graphs with clever usage of SCCs</u>
 - Heuristics for LOP using local search techniques
 - Comprehensive report on guided local search and fast local search
 - On the important details for implementing Simulated Annealing
 - Comprehensive list of different variants of TSP including benchmarks
 - CPAIOR conference on the integration of CP, AI, OR techniques
 - <u>Library of resources for Operations Research</u>
 - <u>Helpful course in Linear Programming with low-level details</u>
 - Designing a better judging system
 - Local Search Techniques for Constrained Portfolio Selection Problems
 - Lectures on Linear Programming
 - Theory and Applications of Linear Programming
 - Guided Fast Local Search Detailed Guide
 - <u>Algorithms Book Dasgupta, Papadimitriou, Vazirani</u>
 - A Real-world Application of LP Walkthrough
 - CrowdBT Technique for crowdsourcing judging

Special Thanks to

- 1. Dr. Kannan Srinathan (Professor)
- 2. Preet Thakkar (TA)
- 3. Coursera course by Dr. Pascal Van Haternyck
- 4. Gavel by Anish Athalye
- 5. LOLIB dataset and Dr. Gerhard Reinelt, Dr. Rafael Marti
- 6. PicoJr GitHub for Hashcode Checkers used
- 7. MIT OCW Algorithms Analysis and Design
- 8. Stanford CS261 on LP & Duality by Tim Roughgarden