

AI1103 : Assignment 2

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Download all python codes from

<https://github.com/shashank-anirudh-rachapalle/AI1103/tree/main/Assignment2/codes>

and latex codes from

<https://github.com/shashank-anirudh-rachapalle/AI1103/tree/main/Assignment2/Assignment2.tex>

$$E(X) = \int_0^1 \int_{-\infty}^{\infty} x f_{XY}(x, y) dx dy \quad (0.0.8)$$

$$= \int_0^1 \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2} dx dy \quad (0.0.9)$$

$$= \int_0^1 y dy \quad (0.0.10)$$

$$E(X) = \frac{1}{2} \quad (0.0.11)$$

$$(0.0.12)$$

PROBLEM STATEMENT(GATE 68)

Let X and Y be random variables having the joining probability density function

$$f(x, y) = \begin{cases} \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2} & -\infty < x < \infty, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.1)$$

The covariance between the random variables X and Y is

From (0.0.5),(0.0.7),(0.0.11)

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y) \quad (0.0.13)$$

$$= \frac{1}{3} - \frac{1}{2} \times \frac{1}{2} \quad (0.0.14)$$

$$\text{Cov}(X, Y) = \frac{1}{12} \quad (0.0.15)$$

SOLUTION(GATE 68)

Covariance between X and Y is $E(XY) - E(X)E(Y)$

$$E(XY) = \int_0^1 \int_{-\infty}^{\infty} xy f_{XY}(x, y) dx dy \quad (0.0.2)$$

$$= \int_0^1 \int_{-\infty}^{\infty} xy \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2} dx dy \quad (0.0.3)$$

$$= \int_0^1 y^2 dy \quad (0.0.4)$$

$$E(XY) = \frac{1}{3} \quad (0.0.5)$$

Y has marginal probability

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x, y) dx = 1 \quad (0.0.6)$$

$$\Rightarrow E(Y) = \frac{1}{2} \quad (0.0.7)$$