AI1103 : Assignment 2

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Download all python codes from

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https://github.com/shashank−anirudh−rachapalle/
AI1103/tree/main/Assignment2/codes
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and latex codes from

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https://github.com/shashank−anirudh−rachapalle/
AI1103/tree/main/Assignment2/Assignment2.
tex
```

Problem statement(GATE 68)

Let X and Y be random variables having the joining probability density function

$$f(x, y) = \begin{cases} \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2} & -\infty < x < \infty, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

The covariance between the random variables X and Y is

Solution(GATE 68)

Covariance between X and Y is E[XY]-E[X]E[Y]

$$\begin{aligned} E[XY] &= \int_0^1 \int_{-\infty}^{\infty} xy f_{XY}(x, y)\, dx\, dy \\ &= \int_0^1 \int_{-\infty}^{\infty} xy \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2}\, dx\, dy \\ &= \int_0^1 y^2\, dy \\ E[XY] &= \frac{1}{3} \end{aligned} \tag{1}$$

Y has marginal probability

$$\begin{aligned} f_Y(y) &= \int_{-\infty}^{\infty} f_{XY}(x, y)\, dx = 1 \\ \implies E[Y] &= \frac{1}{2} \end{aligned} \tag{2}$$

$$\begin{aligned} E[X] &= \int_0^1 \int_{-\infty}^{\infty} x f_{XY}(x, y)\, dx\, dy \\ &= \int_0^1 \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2}\, dx\, dy \\ &= \int_0^1 y\, dy \\ E[X] &= \frac{1}{2} \end{aligned} \tag{3}$$

From (1),(2) and (3)

$$\begin{aligned} Cov(x, y) &= E[XY] - E[X]E[Y] \\ &= \frac{1}{3} - \frac{1}{2} \times \frac{1}{2} \\ Cov(x, y) &= \frac{1}{12} \end{aligned}$$