AI1103 : Assignment 2

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Problem statement(GATE 68)

Let X and Y be random variables having the joining probability density function

$$
f(x,y) = \begin{cases} \dfrac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2} & -\infty < x < \infty, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}
\tag{0.0.1}
$$

The covariance between the random variables X and Y is

Solution(GATE 68)

Covariance between X and Y is $E(XY) - E(X)E(Y)$

$$
E(XY) = \int_0^1 \int_{-\infty}^{\infty} xy f_{XY}(x,y)\, dx\, dy
\tag{0.0.2}
$$

$$
= \int_0^1 \int_{-\infty}^{\infty} xy \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2}\, dx\, dy
\tag{0.0.3}
$$

$$
= \int_0^1 y^2\, dy
\tag{0.0.4}
$$

$$
E(XY) = \frac{1}{3}
\tag{0.0.5}
$$

Y has marginal probability (y ∈ (0,1))

$$
f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dx = 1
\tag{0.0.6}
$$

$$
\implies E(Y) = \int_0^1 y f_Y(y)\, dy
\tag{0.0.7}
$$

$$
E(Y) = \frac{1}{2}
\tag{0.0.8}
$$

$$
E(X) = \int_0^1 \int_{-\infty}^{\infty} x f_{XY}(x,y)\, dx\, dy
\tag{0.0.9}
$$

$$
= \int_0^1 \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2}\, dx\, dy
\tag{0.0.10}
$$

$$
= \int_0^1 y\, dy
\tag{0.0.11}
$$

$$
E(X) = \frac{1}{2}
\tag{0.0.12}
$$

$$
\tag{0.0.13}
$$

From (0.0.5),(0.0.8),(0.0.12)

$$
Cov(X,Y) = E(XY) - E(X)E(Y)
\tag{0.0.14}
$$

$$
= \frac{1}{3} - \frac{1}{2} \times \frac{1}{2}
\tag{0.0.15}
$$

$$
Cov(X,Y) = \frac{1}{12}
\tag{0.0.16}
$$