AI1103 : Assignment 2

Shashank Anirudh - CS20BTECH11040

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PROBLEM STATEMENT(GATE 68)

Let X and Y be random variables having the joining probability density function

$$f(x, y) = \begin{cases} \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2} & -\infty < x < \infty, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

The covariance between the random variables X and Y is

SOLUTION(GATE 68)

Covariance between X and Y is $E(XY) - E(X)E(Y)$

$$E(XY) = \int_0^1 \int_{-\infty}^{\infty} xy f_{XY}(x, y) \, dx \, dy$$

$$= \int_0^1 \int_{-\infty}^{\infty} xy \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2} \, dx \, dy$$

$$= \int_0^1 y^2 \, dy$$

$$E(XY) = \frac{1}{3} \tag{1}$$

Y has marginal probability

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x, y) \, dx = 1$$

$$\implies E(Y) = \frac{1}{2} \tag{2}$$

$$E(X) = \int_0^1 \int_{-\infty}^{\infty} x f_{XY}(x, y) \, dx \, dy$$

$$= \int_0^1 \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2} \, dx \, dy$$

$$= \int_0^1 y \, dy$$

$$E(X) = \frac{1}{2} \tag{3}$$

From (1),(2) and (3)

$$Cov(X, Y) = E(XY) - E(X)E(Y)$$

$$= \frac{1}{3} - \frac{1}{2} \times \frac{1}{2}$$

$$Cov(X, Y) = \frac{1}{12}$$