AI1103 : Assignment 2

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Download all python codes from

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https://github.com/shashank−anirudh−rachapalle/
    AI1103/tree/main/Assignment2/codes
```

and latex codes from

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https://github.com/shashank−anirudh−rachapalle/
    AI1103/tree/main/Assignment2/Assignment2.
    tex
```

Problem statement(GATE 68)

Let X and Y be random variables having the joining probability density function

$$f(x,y) = \begin{cases} \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2} & -\infty < x < \infty, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases} \tag{0.0.1}$$

The covariance between the random variables X and Y is

Solution(GATE 68)

Covariance between X and Y is $E(XY) - E(X)E(Y)$

$$\int_{-\infty}^{\infty} xe^{-\frac{1}{2y}(x-y)^2}\, dx \tag{0.0.2}$$

$$= \int_{-\infty}^{\infty} (x-y)e^{-\frac{1}{2y}(x-y)^2}\, dx + y\int_{-\infty}^{\infty} e^{-\frac{1}{2y}(x-y)^2}\, dx \tag{0.0.3}$$

$$= 0 + [\frac{\sqrt{\pi y^3}}{\sqrt{2}} erf(\frac{x-y}{\sqrt{2y}})]_{-\infty}^{\infty} \tag{0.0.4}$$

$$= \sqrt{2\pi} y^{\frac{3}{2}} \tag{0.0.5}$$

$$E(XY) = \int_0^1 \int_{-\infty}^{\infty} xy f_{XY}(x,y)\, dx\, dy \tag{0.0.6}$$

$$= \int_0^1 \int_{-\infty}^{\infty} xy \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2}\, dx\, dy \tag{0.0.7}$$

$$= \int_0^1 \frac{y}{\sqrt{2\pi y}} (\int_{-\infty}^{\infty} xe^{-\frac{1}{2y}(x-y)^2}\, dx)\, dy \tag{0.0.8}$$

$$= \int_0^1 y^2\, dy \tag{0.0.9}$$

$$E(XY) = \frac{1}{3} \tag{0.0.10}$$

Y has marginal probability (y ∈ (0,1))

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dx = 1 \tag{0.0.11}$$

$$\implies E(Y) = \int_0^1 y f_Y(y)\, dy \tag{0.0.12}$$

$$E(Y) = \frac{1}{2} \tag{0.0.13}$$

$$E(X) = \int_0^1 \int_{-\infty}^{\infty} x f_{XY}(x,y)\, dx\, dy \tag{0.0.14}$$

$$= \int_0^1 \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2}\, dx\, dy \tag{0.0.15}$$

$$= \int_0^1 \frac{1}{\sqrt{2\pi y}} (\int_{-\infty}^{\infty} xe^{-\frac{1}{2y}(x-y)^2}\, dx)\, dy \tag{0.0.16}$$

$$= \int_0^1 y\, dy \tag{0.0.17}$$

$$E(X) = \frac{1}{2} \tag{0.0.18}$$

From (0.0.10),(0.0.13),(0.0.18)

$$Cov(X,Y) = E(XY) - E(X)E(Y) \tag{0.0.19}$$

$$= \frac{1}{3} - \frac{1}{2} \times \frac{1}{2} \tag{0.0.20}$$

$$Cov(X,Y) = \frac{1}{12} \tag{0.0.21}$$