# Probability Analysis of Age of Information in Multi-hop Networks

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# Introduction

•AGE OF information (AoI) is a metric that measures the information freshness from the perspective of the receiver monitoring a remote process. It is defined as the elapsed time since the generation of the freshest packet at the receiver.
•AoI increases until the arrival of a fresher status update at the receiver and drops upon its successful reception. Therefore, receiving status updates regularly plays a key role in sustaining information freshness.
•In addition, the staleness of a newly received update is characterized by the time it has spent in the network to reach the destination. As a result, a good AoI performance is achieved when status updates are delivered not only regularly but also timely.

# Key words

- Peak AoI
- preemptive Last Generated First Served (PLGFS)
- Sampling Event
- Sampling Period

Our main motive for this work is:

•we consider a N-hop network with time-invariant packet loss probabilities on each link.

•We derive closed form equations for the probability mass function of AoI.

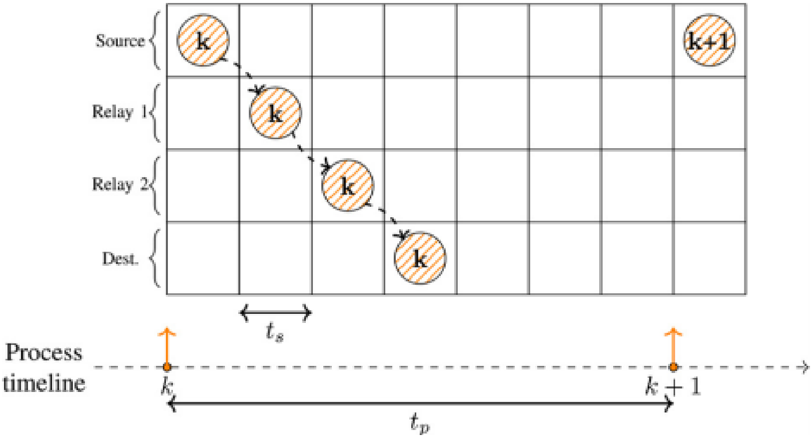
# system model

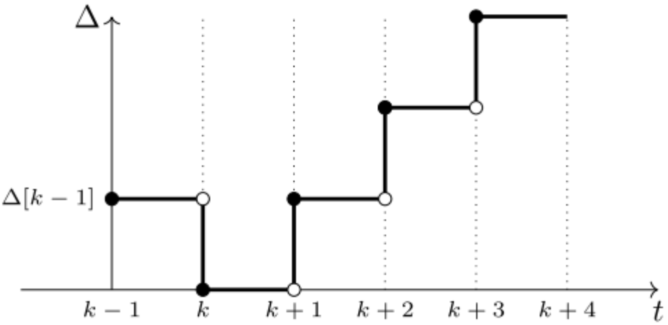
•We consider a physical process located at the source node generating status updates periodically and a monitor located at the receiver node. The source and receiver nodes are N-hops away from each other.

•Each transmitter over the path, i.e., the source and the intermediate relay nodes, discards any older packet in the transmission queue upon the arrival of a new update.

•Each link n is prone to packet loss with time-invariant failure probability, i.e.,$p_n(t) = p_n, \forall t$ .

•Time is divided into slots of length $t_s$ which is also the smallest time unit in our model. Each packet transmission starts at the beginning of a slot and completes within the same slot.


Source
Relay 1
Relay 2
Dest.
k
k+1
$t_s$
Process timeline
$k$
$k+1$
$t_p$



Δ
Δ[k − 1]
k − 1
k
k + 1
k + 2
k + 3
k + 4
t

# Analysis

Let $\gamma_n[k] \in \{0, 1\}, n \in \{1, 2, ..., N\}$, indicate the outcome of the transmission on the n-th link in sampling period k with $Pr[\gamma_n[k] = 1] = 1 - p_n, \forall k$. Furthermore, let $\Delta_n[k], n \in \{0, 1, 2, ..., N\}$, be the AoI at hop n after the transmission slot that is allocated to the previous link.

$$\Delta_n[k] = \begin{cases} \Delta_{n-1}[k] & \gamma_n[k] = 1 \\ \Delta_n[k-1] + 1 & \gamma_n[k] = 0 \end{cases} \tag{1}$$

• we begin with a simple single hop network.

The failure probability on the first link is denoted with $p_1 \in [0, 1]$. Thus, the probability of the AoI at the relay being $\delta_1 \in Z \geq 0$ can be written as:

$$Pr[\Delta_1[k] = \delta_1] = (1 - p_1)p_1^{\delta_1} \forall k \tag{2}$$

$$E[\Delta_1] = \sum_{\delta_1=0}^{\infty} Pr[\Delta_1[k] = \delta_1].\delta_1 \tag{3}$$

$$= \frac{p_1}{1 - p_1} \tag{4}$$

• Now we consider a 2 hop network

•The loss probabilities on two links are independent, we can treat each link independently.

•The contribution of the source-to-relay link to $\Delta_1$, is analogous to the single-hop case. However, in contrast to the source node, the information at the relay is now $\delta_1$ periods old.

$$\Delta_{n-1}[k_n] = \delta_{n-1} \tag{5}$$

$$Pr[\Delta_2[k] = \delta_2 | \Delta_1[k_2] = \delta_1] = \begin{cases} 0 & \delta_2 < \delta_1 \\ (1-p_2)p_2^{\delta_2-\delta_1} & \delta_2 \geq \delta_1 \end{cases} \tag{6}$$

$$Pr[\Delta_2[k] = \delta_2] = \sum_{\delta_1=0}^{\delta_2} Pr[\Delta_2[k] = \delta_2 | \Delta_1[k_2] = \delta_1] \times Pr[\Delta_1[k_2] = \delta_1] \tag{7}$$

$$Pr[\Delta_2[k] = \delta_2] = (1-p_1)(1-p_2).\frac{p_2^{\delta_2+1} - p_1^{\delta_2+1}}{p_2 - p_1} \tag{8}$$

$$E[\Delta_2] = \frac{p_1}{1-p_1} + \frac{p_2}{1-p_2} \tag{9}$$

# Age at nth link

- we extend our results to n-hop

$$
\begin{aligned}
&Pr[\Delta_n[k] = \delta_n] \\
&= \sum_{\delta_{n-1}=0}^{\delta_n} Pr[\Delta_n[k] = \delta_n | \Delta_{n-1}[k_n] = \delta_{n-1}] \times Pr[\Delta_{n-1}[k_n] = \delta_{n-1}], \forall k
\end{aligned}
\tag{10}
$$

•Closed form for higher number of hops can also be obtained using a similar analysis. For instance for 3-hops we obtain:

$$
Pr[\Delta_3[k] = \delta_3] = \frac{\prod_{i=1}^{3}(1 - p_i)}{p_2 - p_1} \sum_{j=1}^{2} (-1)^j . p_j . \frac{p_3^{\delta_3+1} - p_j^{\delta_3+1}}{p_3 - p_j} \tag{11}
$$

# Algorithm

**Algorithm 1** Recursive Age Function: $f(\delta_n, n, \mathbf{p}^n) = o$

**Input:** $\delta_n$ age, $n$ number of hops, $\mathbf{p}^n$ vector of loss probabilities for n hops

**Output:** $o$ the probability of age $\delta_n$ with n hops

Initialize: $o \leftarrow 0$

**if** $n = 1$ **then**

  **return** $(1 - p_n) \cdot {p_n}^{\delta_n}$

**else**

  **for** $\delta_{n-1}, \in [0\ \delta_n]$ **do**

    $o \leftarrow o + \left((1 - p_n) \cdot {p_n}^{\delta_n - \delta_{n-1}}\right) \cdot f(\delta_{n-1}, n-1, \mathbf{p}^{n-1})$

  **end for**

  **return** $o$

**end if**

# Simulation

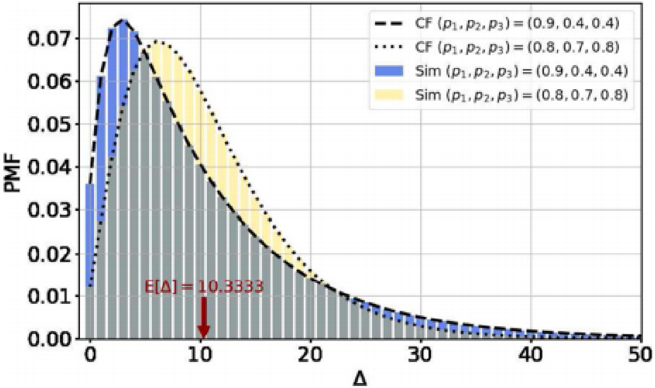
•. Each scenario is simulated for $T_{sim} = 100000$ sampling periods and repeated 100 times

•We measure the expected AoI as:

$$E[\Delta_3] = \frac{1}{T_{sim}} \sum_{k=1}^{T_{sim}} \Delta_3[k] \tag{12}$$

• we can also use

$$E[\Delta_3] = \sum_{i=1}^{3} \frac{p_i}{1 - p_i} \tag{13}$$



CF $(p_1, p_2, p_3) = (0.9, 0.4, 0.4)$
CF $(p_1, p_2, p_3) = (0.8, 0.7, 0.8)$
Sim $(p_1, p_2, p_3) = (0.9, 0.4, 0.4)$
Sim $(p_1, p_2, p_3) = (0.8, 0.7, 0.8)$
$E[\Delta] = 10.3333$
PMF
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0
10
20
30
40
50
$\Delta$