COL 780: Computer Vision
Assignment-3

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1 Camera Calibration

In this section, I am going to describe how to determine the intrinsic parameters of a camera using Zhang's Calibration technique.

Let K be our intrinsic parameter matrix defined as,

$$K = \begin{bmatrix} \alpha & \gamma & u_o \\ 0 & \beta & v_o \\ 0 & 0 & 1 \end{bmatrix}$$

1.1 Homography between the model plane and its image

Without loss of generality, we assume the model plane is on $Z = 0$ of the world coordinate system. Let's denote the i^{th} column of the rotation matrix R by r_i and let $m = [u, v]^T$ denote a 2D point and let $M = [X, Y, Z]^T$ denote a 3D point.

$$\begin{aligned} s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} &= K \begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} \\ &= K \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \end{aligned}$$

Let $\tilde{m}$ and $\tilde{M}$ denote the homogeneous coordinates of m and M respectively, then we have

$$s\tilde{m} = H\tilde{M} \;\text{ with }\; H = K \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

Given an image of the model plane (checkerboard pattern) 1, an homography can be estimated. Let it be denoted by $H = [h_1\ h_2\ h_3]$.

$$\begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \lambda K \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

where λ is an arbitrary scalar. Using the knowledge that r_1 and r_2 are orthonormal, we have

$$\begin{aligned} &h_1^T K^{-T} K^{-1} h_2 = 0 \\ &h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2 \end{aligned}$$

Let $\omega = K^{-T} K^{-1}$. Note that ω is a symmetric matrix. Therefore we only need five or more equations to solve for ω. Since each image gives 2 equations, with 3 images we can obtain an unique solution for ω.

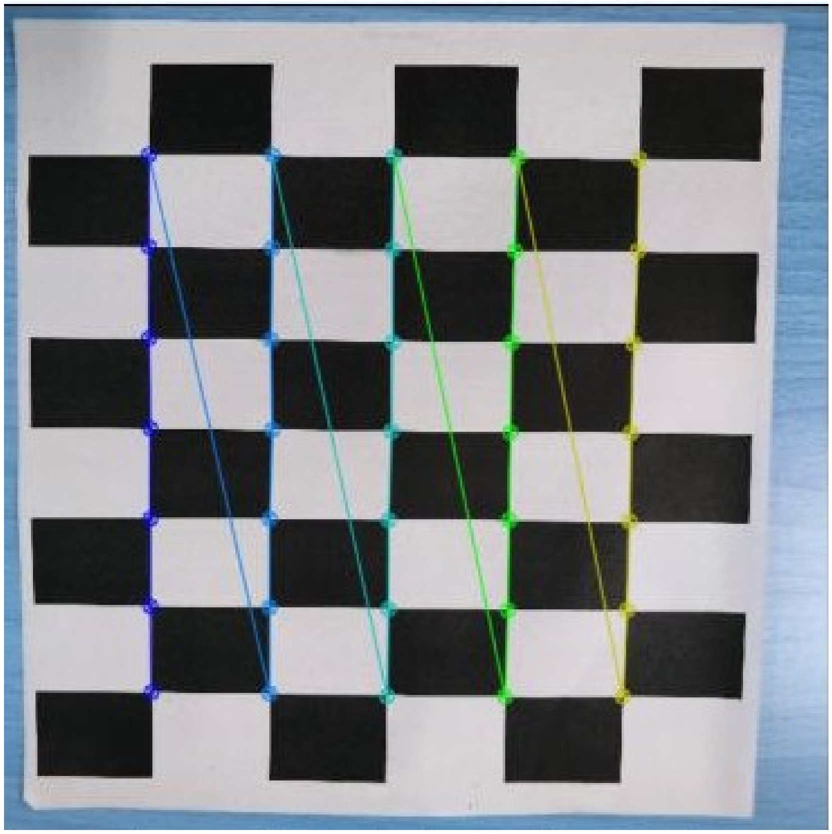

Figure 1: Checkerboard corner detection using opencv

1.2 Closed form solution

Let ω matrix take the form,

$$\omega = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{12} & \omega_{22} & \omega_{23} \\ \omega_{13} & \omega_{23} & \omega_{33} \end{bmatrix}$$

then the intrinsic parameters (or K) can be obtained by the following equations,

$$\begin{aligned} v_o &= \left(\omega_{12}\omega_{13} - \omega_{11}\omega_{23}\right) / \left(\omega_{11}\omega_{22} - \omega_{12}^2\right) \\ \lambda &= \omega_{33} - \left[\omega_{13}^2 + v_o\left(\omega_{12}\omega_{13} - \omega_{11}\omega_{23}\right)\right] / \omega_{11} \\ \alpha &= \sqrt{\lambda/\omega_{11}} \\ \beta &= \sqrt{\lambda\omega_{11}/\left(\omega_{11}\omega_{22} - \omega_{12}^2\right)} \\ \gamma &= -\omega_{12}\alpha^2\beta/\lambda \\ u_o &= \gamma v_o/\beta - \omega_{13}\alpha^2/\lambda \end{aligned}$$

2 Generating Augmented-Reality pictures

After obtaining the intrinsic parameters, extrinsic parameters for a given image can be obtained as following,

$$\begin{aligned} r_1 &= \lambda K^{-1} h_1 \\ r_2 &= \lambda K^{-1} h_2 \\ r_3 &= r_1 \times r_2 \\ t &= \lambda K^{-1} h_3 \end{aligned}$$

where $\lambda = 1/\|K^{-1}h_1\| = 1/\|K^{-1}h_2\|$. Now the world points can be projected using the equation,

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

The interesting thing about these world points is that they can even originate from a virtual model. To illustrate this point, I have employed a virtual cube which is loaded from a `cube.obj` file and then projected it onto the image with the assistance of the `cv2.projectPoints()` module in opencv. The resulting images are displayed below.

Figure 2: Top view

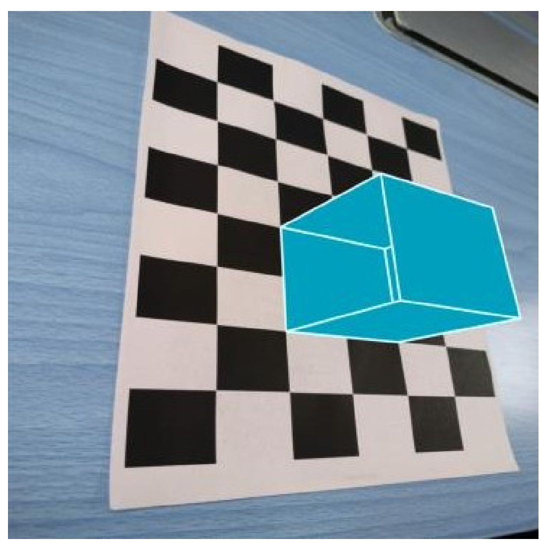

Figure 3: Left view

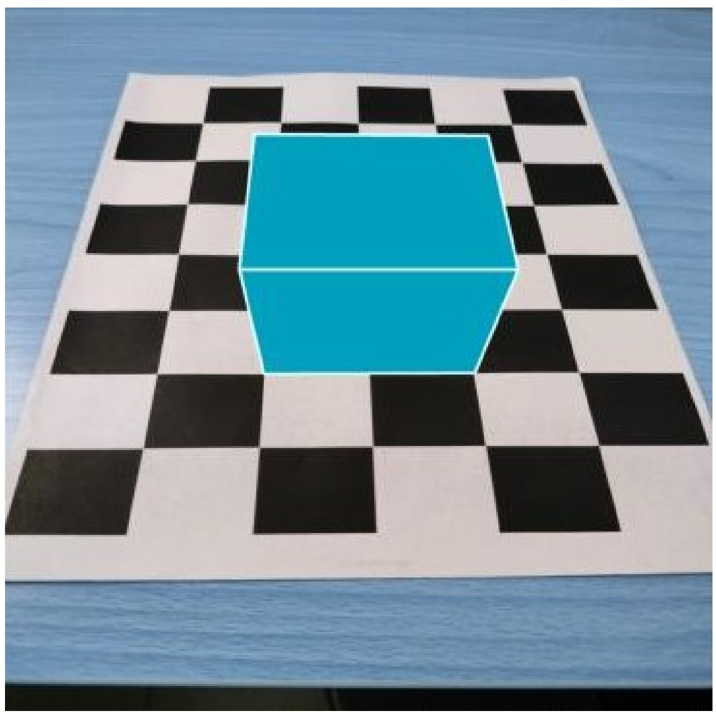

Figure 4: Front view