

Discrete-Time Sliding Mode Controller For Magnetic Levitation System Using Minima Operator

Vinay Pandey^{1,2}, Shyam Kamal¹ and Sandip Ghosh¹

Abstract—This paper presents the development of a discrete-time sliding mode controller for a magnetic levitation system, incorporating a minimum operator approach. The primary objective is to precisely stabilize the position of a ferromagnetic ball above the ground within a finite time frame, achieved by utilizing an electromagnet to counteract the gravitational force. In pursuit of this goal, we have devised two discrete-time sliding mode controllers incorporating discrete-time reaching laws. Moreover, a novel formulation for discrete-time sliding mode control with a minimum operator, specifically tailored for this application, has been proposed to ensure finite-time tracking. Simulation results indicate that the proposed methodology, employing the minimum operator approach, delivers a superior response compared to conventional discrete-time sliding mode controllers. Furthermore, the issue of chattering, commonly observed in traditional discrete-time sliding mode controllers used for magnetic levitation systems, is effectively mitigated in our proposed approach.

I. INTRODUCTION

Magnetic levitation systems (MLS) are versatile in their engineering applications, spanning a wide range of industries and technologies. Magnetic levitation offers an intriguing scenario for exploring nonlinear unstable dynamics and crafting corresponding control strategies. The system aims to elevate objects to a specific height by magnetizing the coil, employing the principle of non-contact. Previous studies have explored the control design using feedback linearization techniques [1] and [2], PID control [3], adaptive control [4], robust control [5] and using neural networks based control [6]. Additionally, continuous sliding mode control (CSMC) strategies, known for their robustness compared to other feedback techniques, have been implemented in MLS [7]-[10].

The levitation dynamics display inherent instability characterized by open-loop nonlinear behavior. The objective of the control law introduced in [11] was to minimize the power consumption of the magnetic suspension system. Research in [12] has investigated magnetically levitated conveyors. Additionally, [13] outlines the design of an estimator employing variable structure systems principles.

The majority of controllers developed thus far for MLS, including CSMCs, have been in continuous time. very few work [10], [14]- [15], have been reported for discrete control of MLS. As digital control presents numerous advantages

over analog counterparts, such as superior noise rejection and increased flexibility for software modifications. Consequently, the widespread availability of affordable microprocessors and digital computing technology, capable of rapid sampling rates, makes the adoption of a discrete-time (DT) sliding mode controller advantageous for magnetic levitation system control also CSMC controllers developed for MLS have historically suffered from the issue of chattering, which poses a significant drawback.

Recently discrete sliding mode control (DSMC) grew rapidly and gained widespread use in the field of control due to the increased use of digital computers [16]- [18]. The notable characteristic of DSMC, as opposed to CSMC, is that it offers limited switching frequency [19]- [20] leads to a popular research field in practical control engineering applications. Also, in [21] authors developed two reaching law (RL) based DSMC using the minimum operator. based on finite time stability of discrete autonomous system [22]-[23].

In this paper, we have developed a DSMC for MLS utilizing DTRL [21]. Subsequently, we compared the developed results with previous results. This approach proves more practical for MLS compared to previous methods, chattering associated with quasi sliding mode domain reduces its practical implementation. Our simulation results validate the effectiveness of the minimum operator-based DSMC for magnetic levitation system control.

The structure of this paper is as follows: Section II offers a system description of the MLS, and Section III provides a necessary overview of the minima-based reaching law. Sections IV and V detail the design process of discrete-time sliding mode controllers based on minima-based reaching laws for MLS and present numerical simulation results. Finally, in Section VI, we draw our conclusions.

II. SYSTEM DESCRIPTION

The illustrated MLS, depicted in Fig 1, consists of a sphere exhibiting ferromagnetic properties, which is suspended within a magnetic field generated by an electromagnet. This configuration allows the electromagnet to exert an attractive force on the ferromagnetic sphere, counteracting the force of gravity and thus keeping the sphere at a predetermined elevation. The governing equation of this system is delineated

¹Authors are with the Department of Electrical Engineering, Indian Institute of Technology (BHU), Varanasi, 221005, India, ²Author is also with the Semi-Conductor Laboratory, Government of India, S.A.S Nagar, 140301. vinaypandey.rs.eee21@iitbhu.ac.in, shyamkamal.eee@iitbhu.ac.in, sghosh.eee@iitbhu.ac.in

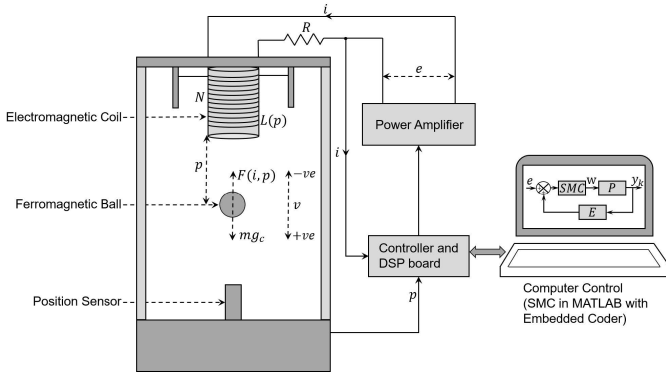


Fig. 1: Schematic of a MLS

below.

$$\begin{aligned} \frac{dp}{dt} &= v, Ri + \frac{d(L(p)i)}{dt} = e, m \frac{dv}{dt} = mg_c - F(i, p) \\ F(i, p) &= Q \left(\frac{i}{p} \right)^2, Q = \frac{\mu_0 AN^2}{4}, L(p) = L_1 + \frac{2Q}{p} \end{aligned} \quad (1)$$

The following variables are assigned as: $p = x_1$ (representing the position of the ball), $v = x_2$ (representing the velocity of the ball), $i = x_3$ (denoting the current in the coil), R (indicating the resistance of the coil), L (referring to the inductance of the coil), g_c = gravitational constant, Q = magnetic force constant, and m = mass of the ball also, L_1 denotes the parameter related to electromagnetic coil inductance.

Therefore, the dynamic formulation of the MLS model (1) can be expressed in the guise of nonlinear dynamics as follows:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \quad (2)$$

Considering the state vector x as $[x_1, x_2, x_3]^T$ where, $x_1 = p, x_2 = v, x_3 = i$, input as u and output y , the state-space model of the MLS will be expressed as :

$$\begin{aligned} \frac{dx_1}{dt} &= x_2, \frac{dx_2}{dt} = g - \frac{Q}{m} \left(\frac{x_3}{x_1} \right)^2 \\ \frac{dx_3}{dt} &= -\frac{R}{L} x_3 + \frac{2Q}{L} \left(x_2 \frac{x_3}{x_1^2} \right) + \frac{u}{L} \end{aligned} \quad (3)$$

The control strategies aim to guide the variables x_1, x_2 , and x_3 towards their target equilibrium values x_{1d}, x_{2d} , and x_{3d} , correspondingly where x_{3d} satisfies $x_{3d} = x_{1d} \sqrt{\frac{gm}{Q}}$.

A. Discrete linear model of the MLS

The nonlinear representation of the MLS provided in equation (2) undergoes precise linearization through a feedback linearizing transformation.

Consider the transformation $z = (z_1, z_2, z_3)^T$ as:

$$z = T(x) = \begin{bmatrix} x_1 - x_{1d} \\ x_2 \\ g - \frac{Q/m \cdot (x_3/x_1)^2}{1} \end{bmatrix} \quad (4)$$

TABLE I: Parameters of the MLS.

Parameter	Symbol	Value
Coil resistance	R	28.7 Ω
Coil inductance	L1	0.65 H
Gravitational constant	g	9.81 m/sec ²
Magnetic force constant	Q	1.8×10^{-4}
Levitated object mass	m	11.87×10^{-3} Kg
Plant initial condition	x1	0.0255 m
	x2	0 m/sec
	x3	1.1 A
Desired steady state values	x1d	0.01 m
	x2d	0 m/sec
	x3d	0.2884 A

The system in the form of state space will be as follows:

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} z_2 \\ z_3 \\ \alpha(z) + \theta(z)u \end{bmatrix} \quad (5)$$

where,

$$\begin{aligned} \alpha(z) &= 2(g - z_3) \left(\left(1 - \frac{2Q}{L(z_1 + x_{1d})} \right) \frac{z_2}{z_1 + x_{1d}} + \frac{R}{L} \right) \\ \theta(z) &= -\frac{2Q}{L(z_1 + x_{1d})} \left(\sqrt{\frac{Q}{m}} (g - z_3) \right). \end{aligned} \quad (6)$$

To nullify the nonlinear elements in equation (5), the design of the outer-loop feedback controller u takes the following form:

$$u = -\theta^{-1}(z)(\alpha(z) + v) \quad (7)$$

where v represents the inner-loop feedback controller. Therefore, the magnetic levitation system's nonlinear model (2) can be expressed as after the application of control input (7),

$$\dot{z} = Az(k) + Bu(k) \quad y(k) = Cz(k); \quad (8)$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \quad C = (1 \quad 0 \quad 0).$$

The linear model undergoes discretization with a sampling time of $\tau = 0.1$ seconds. Consequently, the continuous-time system described in (8) is transformed into a discrete-time system (DTS) is as follows:

$$z(k+1) = \Phi z(k) + \Gamma u(k) \quad y(k) = C\Phi z(k); \quad (9)$$

where the state $z(k) \in \mathbb{R}^3$, the input $u(k) \in \mathbb{R}$, and the output $y(k) \in \mathbb{R}$. Moreover, $\Phi = e^{A\tau} \in \mathbb{R}^{3 \times 3}$, $\Gamma = A^{-1}(e^{A\tau} - I) \in \mathbb{R}^{3 \times 1}$ and $C \in \mathbb{R}^{1 \times 3}$. the DTS matrices are as follows:

$$\Phi = \begin{pmatrix} 1 & 0.1 & 0.005 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{pmatrix}; \quad \Gamma = \begin{pmatrix} 0.0002 \\ 0.005 \\ 0.1 \end{pmatrix}.$$

After the discretization of the system model.

III. MINIMA BASED REACHING LAW FOR DSMC

In this section, we introduce two minima-based reaching laws for DSMC [21] applicable to both unperturbed and perturbed DTS. This law is based on a difference equation formulation with minima for finite-time stability of discrete dynamical system [22].

A. RL1 for DTS

Considering the following system:

$$s(k+1) = s(k) - \text{sign}[s(k)] \min(|s(k)|, \omega) \quad (10)$$

where $\omega \in \mathbb{R}^+$ is user selected gain.

B. RL2 for DTS

Consider also the following system:

$$s(k+1) = s(k) - \gamma_1 \text{sign}[s(k)] \min\left(\frac{|s(k)|}{\gamma_1}, |s(k)|^\beta\right). \quad (11)$$

Where $\gamma_1 \in \mathbb{R}^+$ and $\beta \in (0, 1)$.

IV. DSMC FOR MLS

The switching function is defined as $s(k) := \rho(z(k))$, and the corresponding sliding manifold is defined by the level set $\rho^{-1}(0) := \{z \in D : \rho(z) = 0\}$. For analytical purposes, the switching function is defined as follows:

$$s(k) = c^T z(k), \quad (12)$$

where $c \in \mathbb{R}^{n \times 1}$ with $c^T \Gamma \neq 0$.

Definition 1: The DTS (9) satisfies the reaching condition if, for certain $k \geq 0$, the following conditions are fulfilled:

$$\begin{cases} |s(k)| > \omega \Rightarrow |s(k+1)| < |s(k)| - \epsilon, \\ |s(k)| \leq \omega \Rightarrow s(k+1) = 0, \end{cases}$$

where $\omega \in \mathbb{R}^+$ denotes a threshold value and $\epsilon \in \mathbb{R}^+$ is arbitrarily small.

A. Contrtoller deisgn based on RL1

Using switching function (12),

$$s(k+1) = c^T z(k+1),$$

Consider the system described by (9) and RL1 (10).

$$c^T (\Phi z(k) + \Gamma u(k)) = s(k) - \text{sign}[s(k)] \min(|s(k)|, \omega)$$

Now, the control input will be expressed as

$$u(k) = -(c^T \Gamma)^{-1} (c^T \Phi z(k) - s(k) + \text{sign}[s(k)] \min(|s(k)|, \omega)) \quad (13)$$

Theorem 1: Consider system (9) and RL (10), using control input (13) the absolute value of the switching function of the system goes to zero for some $t \geq I(s_0)$ where $I(s_0) = \frac{|s_0|}{\omega}$ is the settling time function (STF).

Proof When the control input obtained from equation (13) is implemented within the system (9), the system adheres to the RL1.

$$s(k+1) = s(k) - \text{sign}[s(k)] \min(|s(k)|, \omega)$$

Consider the scenario where $|s(0)| \leq \omega$, then $s(k+1) = 0$ implies in the next step $|s(k+1)| = 0$. Consequently, $|s(k+1)| = 0$, (satisfying Definition 1), with $I(s_0) = 1$. Moreover, for all $k \geq I(s_0)$, $|s(k)| = 0$, demonstrating the existence of QSM (as per Definition 1). For another scenario, if $|s(0)| > \omega$, then $|s(k+1)| = |s(k)| - \omega \text{sign}[s(k)]$. As $|s(k)| > \omega$, it follows that $|s(k+1)| \leq |s(k)| - \omega$. Additionally, it can be deduced as:

$$\begin{aligned} |s(k)| &\leq |s(k-1)| - \omega \\ &\leq |s(k-2)| - 2\omega \\ &\leq \vdots \\ &\leq |s(0)| - t\omega \end{aligned}$$

If $(|s(0)| - t\omega) \leq \omega$, then $|s(k)| \leq \omega$, implying $|s(k+1)| = 0$ for all $t \geq \frac{|s_0|}{\omega}$. The STF is $I(s_0) = \frac{|s_0|}{\omega}$. Therefore, $|s(k)| = 0$ assures within finite time steps.

B. Contrtoller design based on RL2

Using switching function (12),

$$s(k+1) = c^T z(k+1),$$

Consider the system described by (9) and RL2 (11).

$$\begin{aligned} c^T (\Phi z(k) + \Gamma u(k)) &= s(k) - \gamma_1 \text{sign}[s(k)] \\ &\quad \min\left(\frac{|s(k)|}{\gamma_1}, |s(k)|^\beta\right) \end{aligned}$$

Now, the control input will be expressed as

$$u(k) = -(c^T \Gamma)^{-1} (c^T \Phi z(k) - s(k) + \gamma_1 \text{sign}[s(k)] \min\left(\frac{|s(k)|}{\gamma_1}, |s(k)|^\beta\right)) \quad (14)$$

Definition 2: The DTS (9) satisfies the reaching condition if, for certain $t \geq 0$, the conditions hold: $\frac{|s(k)|}{\gamma_1} > |s(k)|^\beta \Rightarrow |s(k+1)| < |s(k)| - \epsilon$; $\frac{|s(k)|}{\gamma_1} \leq |s(k)|^\beta \Rightarrow |s(k+1)| = 0$, where $\epsilon \in \mathbb{R}^+$ is arbitrarily small.

Theorem 2: Consider system (9) and RL (11). If $\frac{|s(0)|}{\gamma_1} \leq |s(0)|^\beta$ with $\gamma_1 \in (0, 1)$ and $\beta > 0$, using control input (14), the absolute value of the switching function of the system goes to zero for some $t \geq I(s_0)$, where $I(s_0) \leq \left\lceil \log_{[1-\gamma_1|s_0|^{\beta-1}]} \frac{\gamma_1^{\frac{1}{1-\beta}}}{|s_0|} \right\rceil + 1$.

Proof When the control input obtained from equation (14) is implemented within the system (9), the system adheres to the RL2:

$$s(k+1) = s(k) - \gamma_1 \text{sign}[s(k)] \min\left(\frac{|s(k)|}{\gamma_1}, |s(k)|^\beta\right).$$

Given the simplistic scenario, if $\frac{|s(0)|}{\gamma_1} \leq |s(0)|^\beta$, then

$$\begin{aligned} s(k+1) &= s(k) - \text{sign}[s(k)] |s(k)| \\ &\Rightarrow s(k+1) = 0 \end{aligned}$$

In the subsequent time step. Hence, we obtain $|s(k+1)| = 0$, (Definition 2 is met), with $I(s_0) = 1$. Also,

for all $t \geq I(s_0)$, $|s(k+1)| = 0$, showcasing the sliding reaching existence (as per Definition 2). Taking into account the following scenario, if $\frac{|s(0)|}{\gamma_1} > |s(0)|^\beta$, then

$$s(k+1) = s(k) - \gamma_1 \text{sign}[s(k)]|s(k)|^\beta.$$

We can express it as,

$$|s(k+1)| = |s(k) - \gamma_1 \text{sign}[s(k)]|s(k)|^\beta|.$$

Since $|s(k)| > |s(k)|^\beta$, it follows that,

$$|s(k+1)| \leq |s(k)| - \gamma_1 |s(k)|^\beta.$$

Further, it can be written as

$$\begin{aligned} |s(k)| &\leq |s(k-1)|(1 - \gamma_1 |s(k-1)|^{\beta-1}) \\ &\vdots \\ &\leq |s(0)|(1 - \gamma_1 |s(0)|^{\beta-1}) \dots \\ &\quad (1 - \gamma_1 |s(k-1)|^{\beta-1}) \\ &\vdots \\ |s(k)| &\leq |s(0)|(1 - \gamma_1 |s(0)|^{\beta-1})^k \end{aligned} \quad (15)$$

If $|s(0)|(1 - \gamma_1 |s(0)|^{\beta-1}) \leq \gamma_1 |s(k)|^\beta$, then from (15), $|s(k)| \leq \gamma_1 |s(k)|^\beta \Rightarrow |s(k+1)| = 0, \forall k \geq \left\lceil \log_{[1-\gamma_1 |s_0|^{\beta-1}]} \frac{\gamma_1 |s_0|^{\frac{1}{1-\beta}}}{|s_0|} \right\rceil + 1$. Moreover, the settling time

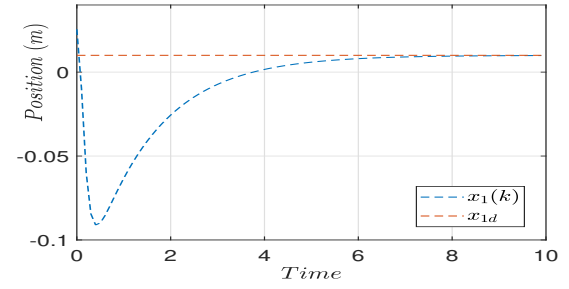
expression is given by $I(s_0) \leq \left\lceil \log_{[1-\gamma_1 |s_0|^{\beta-1}]} \frac{\gamma_1 |s_0|^{\frac{1}{1-\beta}}}{|s_0|} \right\rceil + 1$.

Thus, it can be inferred that the absolute value of the switching function approaches zero within a finite number of time steps.

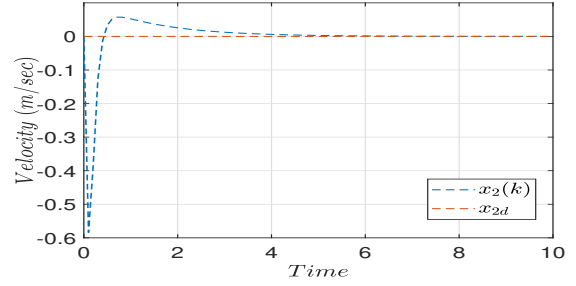
V. RESULTS AND DISCUSSION

The simulation outcomes demonstrate the efficacy of two devised DSMC controllers. Employing parameters $\omega = 0.5$, $\gamma_1 = 0.5$, $\beta = 0.1$ and $c = [0.66; 1; 0.12]^T$, along with initial conditions and simulation parameters detailed in Table I. Fig 2 portrays the cumulative results for the RL1-based DSMC controller, while Fig 3 illustrates the cumulative results for the RL2-based DSMC controller. Additionally, Fig 4 presents a comparison between RL1, RL2, and prior work by Bandal et al. [10]. Notably, Bandal et al. using Gao's reaching law utilized quasi-sliding mode (QSM) with a bandwidth of 0.0625, depicted in Fig 4(a), whereas both RL1 and RL2 controllers exhibit an ideal sliding mode akin to CSMC. Fig 2a and Fig 3(a) represent the position versus time plot, showcasing precise attainment of the desired state x_{1d} (0.01m) for RL1 and RL2 within a finite time frame respectively.

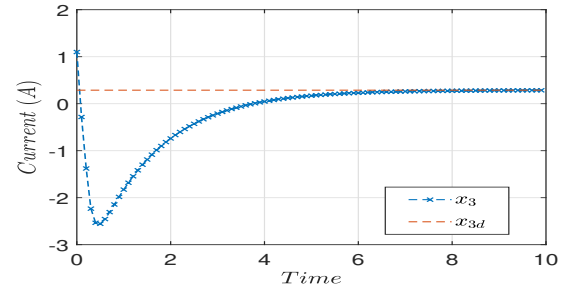
Furthermore, Fig 2(b), (c), (d), and (e) respectively demonstrate velocity, switching surface and current control input versus time plots for a RL1 based discrete-time sliding mode controller. Also Fig 3(b), (c), (d), and (e) respectively demonstrate velocity, switching surface and current control input versus time plots for an RL2 based discrete-time sliding mode controller. The ferromagnetic ball achieves its desired



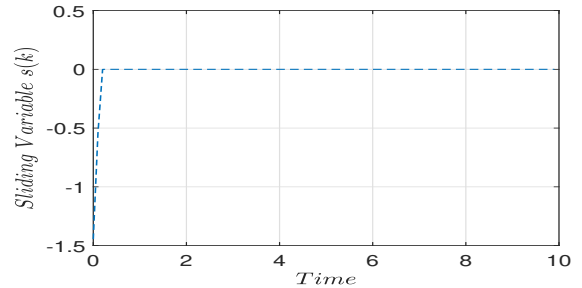
(a) RL1 state variable $x_1(k)$



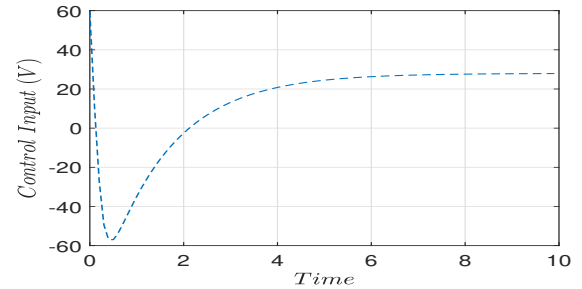
(b) RL1 state variable $x_2(k)$



(c) RL1 state variable $x_3(k)$

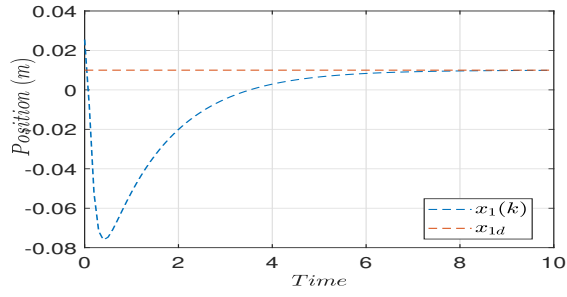


(d) RL1 sliding variable

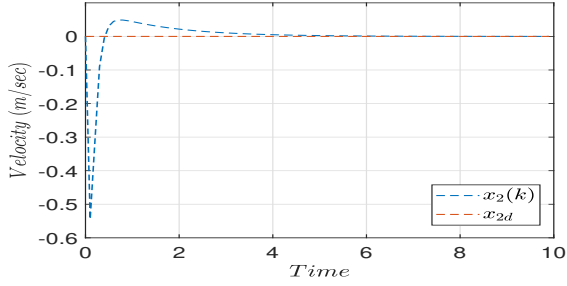


(e) RL1 control input

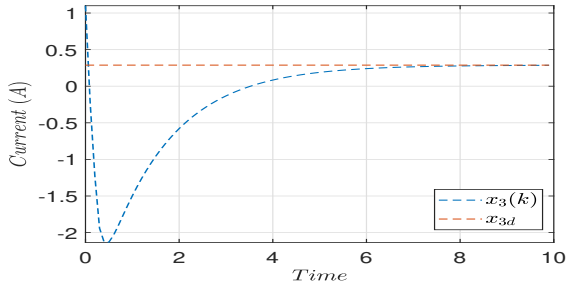
Fig. 2: RL1 State Variables, Sliding variable, and Control Input



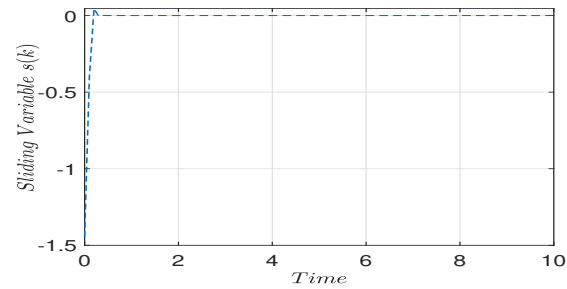
(a) RL2 state variable $x_1(k)$



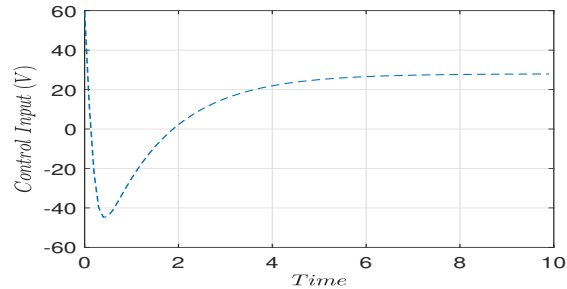
(b) RL2 state variable $x_2(k)$



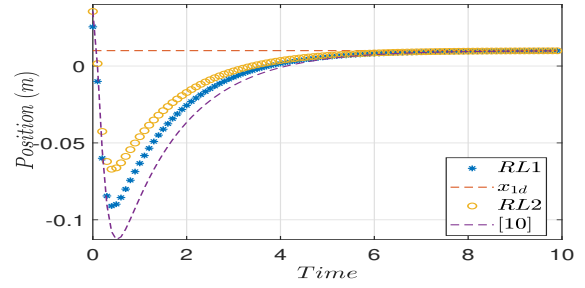
(c) RL2 state variable $x_3(k)$



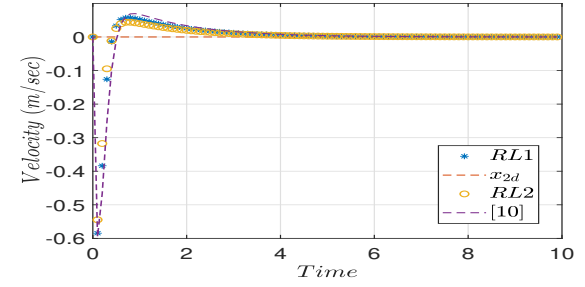
(d) RL2 sliding variable



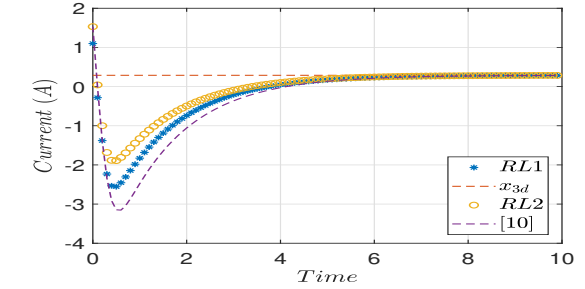
(e) RL2 control input



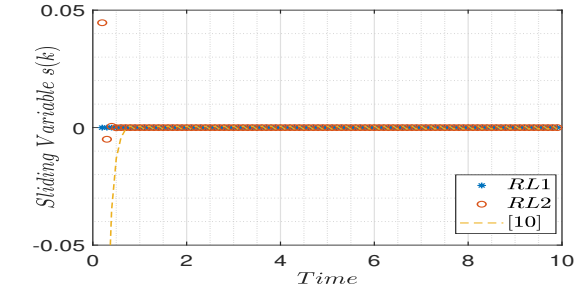
(a) comparison state variable $x_1(k)$



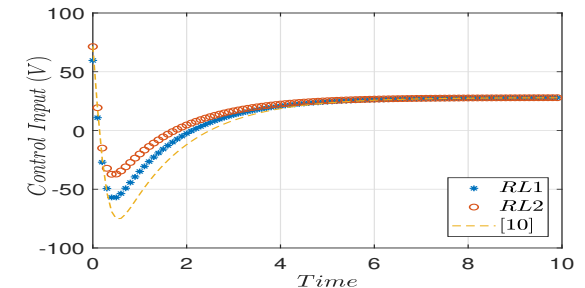
(b) comparison state variable $x_2(k)$



(c) comparison state variable $x_3(k)$



(d) comparison sliding variable



(e) comparison control input

Fig. 3: RL2 State Variables, Sliding variable and Control Input

Fig. 4: RL1, RL2 and [10] Comparison

steady-state position of 0.01 m in approximately 6.5 seconds, with the current stabilizing around 0.285 Amperes, near the targeted value of 0.2884 A with an RL1-based controller. Also with RL2-based controller ensures the desired steady-state position of 0.01 m in approximately 6.2 seconds, with the current stabilizing around 0.281 Amperes. The previous work shows minor oscillations around the desired values attributed to the quasi-sliding motion of the state trajectory, whereas RL1 and RL2 based controller shows ideal sliding motion along the surface.

VI. CONCLUSIONS

This paper introduces two DSMC control strategies for a magnetic levitation system. Its fast convergence rate characterizes DSMC based on RL1 and RL2 as compared to Gao's based control. These regulations tackle the constraints linked with Gao's reaching technique and Utkin's equivalent control method. With the proposed DSMC, the trajectory reaches a sliding surface, closely resembling its continuous-time counterpart without disturbances, thereby enhancing robustness. Moreover, DSMC proves more practical for magnetic levitation systems compared to previous DSMC-based control methods. The proposed laws for MLS successfully eliminate chattering while ensuring minimal control effort. Their effectiveness is supported by simulation results. Moving forward, the prospect of experimental validation using real-world setups presents an exciting opportunity for future research.

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