Chapter 1

Problem Solutions

1.1

(a) fcc: 8 corner atoms $\times$ 1/8 = 1 atom
6 face atoms $\times$ ½ = 3 atoms
Total of 4 atoms per unit cell

(b) bcc: 8 corner atoms $\times$ 1/8 = 1 atom
1 enclosed atom = 1 atom
Total of 2 atoms per unit cell

(c) Diamond: 8 corner atoms $\times$ 1/8 = 1 atom
6 face atoms $\times$ ½ = 3 atoms
4 enclosed atoms = 4 atoms
Total of 8 atoms per unit cell

1.2

(a) 4 Ga atoms per unit cell

$$\text{Density } = \frac{4}{\left(5.65x10^{-8}\right)^3} \Rightarrow$$

$$\text{Density of Ga } = 2.22x10^{22} \ cm^{-3}$$

4 As atoms per unit cell, so that

$$\text{Density of As } = 2.22x10^{22} \ cm^{-3}$$

(b)

8 Ge atoms per unit cell

$$\text{Density } = \frac{8}{\left(5.65x10^{-8}\right)^3} \Rightarrow$$

$$\text{Density of Ge } = 4.44x10^{22} \ cm^{-3}$$

1.3

(a) Simple cubic lattice; $a = 2r$

$$\text{Unit cell vol } = a^3 = (2r)^3 = 8r^3$$

$$\text{1 atom per cell, so atom vol. } = (1)\left(\frac{4\pi r^3}{3}\right)$$

Then

$$Ratio = \frac{\left(\frac{4\pi r^3}{3}\right)}{8r^3} \times 100\% \Rightarrow \quad Ratio = 52.4\%$$

(b) Face-centered cubic lattice

$$d = 4r = a\sqrt{2} \Rightarrow \quad a = \frac{d}{\sqrt{2}} = 2\sqrt{2}\, r$$

$$\text{Unit cell vol } = a^3 = \left(2\sqrt{2}\, r\right)^3 = 16\sqrt{2}\, r^3$$

$$\text{4 atoms per cell, so atom vol. } = 4\left(\frac{4\pi r^3}{3}\right)$$

Then

$$Ratio = \frac{4\left(\frac{4\pi r^3}{3}\right)}{16\sqrt{2}\, r^3} \times 100\% \Rightarrow \quad Ratio = 74\%$$

(c) Body-centered cubic lattice

$$d = 4r = a\sqrt{3} \Rightarrow a = \frac{4}{\sqrt{3}} r$$

$$\text{Unit cell vol. } = a^3 = \left(\frac{4}{\sqrt{3}} r\right)^3$$

$$\text{2 atoms per cell, so atom vol. } = 2\left(\frac{4\pi r^3}{3}\right)$$

Then

$$Ratio = \frac{2\left(\frac{4\pi r^3}{3}\right)}{\left(\frac{4r}{\sqrt{3}}\right)^3} \times 100\% \Rightarrow \quad Ratio = 68\%$$

(d) Diamond lattice

$$\text{Body diagonal } = d = 8r = a\sqrt{3} \Rightarrow a = \frac{8}{\sqrt{3}} r$$

$$\text{Unit cell vol. } = a^3 = \left(\frac{8r}{\sqrt{3}}\right)^3$$

$$\text{8 atoms per cell, so atom vol. } 8\left(\frac{4\pi r^3}{3}\right)$$

Then

$$Ratio = \frac{8\left(\frac{4\pi r^3}{3}\right)}{\left(\frac{8r}{\sqrt{3}}\right)^3} \times 100\% \Rightarrow \quad Ratio = 34\%$$

1.4

From Problem 1.3, percent volume of fcc atoms is 74%; Therefore after coffee is ground,

$$\text{Volume } = 0.74 \ cm^3$$

1.5

(a) $a = 5.43\ \overset{\circ}{A}$ From 1.3d, $a = \dfrac{8}{\sqrt{3}} r$

so that $r = \dfrac{a\sqrt{3}}{8} = \dfrac{(5.43)\sqrt{3}}{8} = 1.18\ \overset{\circ}{A}$

Center of one silicon atom to center of nearest neighbor $= 2r \Rightarrow \underline{2.36\ \overset{\circ}{A}}$

(b) Number density

$$= \frac{8}{\left(5.43x10^{-8}\right)^3} \Rightarrow \text{Density } = 5x10^{22}\ cm^{-3}$$

(c) Mass density

$$= \rho = \frac{N(At.Wt.)}{N_A} = \frac{\left(5x10^{22}\right)(28.09)}{6.02x10^{23}} \Rightarrow$$

$$\underline{\rho = 2.33\ grams / cm^3}$$

1.6

(a) $a = 2r_A = 2(1.02) = 2.04\ \overset{\circ}{A}$

Now

$$2r_A + 2r_B = a\sqrt{3} \Rightarrow 2r_B = 2.04\sqrt{3} - 2.04$$

so that $r_B = 0.747\ \overset{\circ}{A}$

(b) A-type; 1 atom per unit cell

$$\text{Density } = \frac{1}{\left(2.04x10^{-8}\right)^3} \Rightarrow$$

$$\text{Density(A)} = \underline{1.18x10^{23}\ cm^{-3}}$$

B-type: 1 atom per unit cell, so

$$\text{Density(B)} = \underline{1.18x10^{23}\ cm^{-3}}$$

1.7

(b)

$$a = 1.8 + 1.0 \Rightarrow \underline{a = 2.8\ \overset{\circ}{A}}$$

(c)

Na: Density $= \dfrac{1/2}{\left(2.8x10^{-8}\right)^3}$ $\underline{= 2.28x10^{22}\ cm^{-3}}$

Cl: Density (same as Na) $\underline{= 2.28x10^{22}\ cm^{-3}}$

(d)
Na: At.Wt. = 22.99
Cl: At. Wt. = 35.45
So, mass per unit cell

$$= \frac{\frac{1}{2}(22.99) + \frac{1}{2}(35.45)}{6.02x10^{23}} = 4.85x10^{-23}$$

Then mass density is

$$\rho = \frac{4.85x10^{-23}}{\left(2.8x10^{-8}\right)^3} \Rightarrow$$

$$\underline{\rho = 2.21\ gm / cm^3}$$

1.8

(a) $a\sqrt{3} = 2(2.2) + 2(1.8) = 8\ \overset{\circ}{A}$
so that

$$\underline{a = 4.62\ \overset{\circ}{A}}$$

Density of A $= \dfrac{1}{\left(4.62x10^{-8}\right)^3} \Rightarrow \underline{1.01x10^{22}\ cm^{-3}}$

Density of B $= \dfrac{1}{\left(4.62x10^{-8}\right)} \Rightarrow \underline{1.01x10^{22}\ cm^{-3}}$

(b) Same as (a)
(c) Same material

1.9

(a) Surface density

$$= \frac{1}{a^2\sqrt{2}} = \frac{1}{\left(4.62x10^{-8}\right)^2\sqrt{2}} \Rightarrow$$

$$\underline{3.31x10^{14}\ cm^{-2}}$$

Same for A atoms and B atoms
(b) Same as (a)
(c) Same material

1.10

(a) Vol density $= \dfrac{1}{a_o^3}$

Surface density $= \dfrac{1}{a_o^2\sqrt{2}}$

(b) Same as (a)

1.11
Sketch

1.12
(a)

$$\left(\frac{1}{1}, \frac{1}{3}, \frac{1}{1}\right) \Rightarrow \underline{(313)}$$

(b)

$$\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right) \Rightarrow \underline{(121)}$$

1.13

(a) Distance between nearest (100) planes is:

$$\underline{d = a = 5.63\ \mathring{A}}$$

(b)Distance between nearest (110) planes is:

$$d = \frac{1}{2}a\sqrt{2} = \frac{a}{\sqrt{2}} = \frac{5.63}{\sqrt{2}}$$

or

$$\underline{d = 3.98\ \mathring{A}}$$

(c) Distance between nearest (111) planes is:

$$d = \frac{1}{3}a\sqrt{3} = \frac{a}{\sqrt{3}} = \frac{5.63}{\sqrt{3}}$$

or

$$\underline{d = 3.25\ \mathring{A}}$$

1.14

(a)

Simple cubic: $a = 4.50\ \mathring{A}$

(i) (100) plane, surface density,

$$= \frac{1\ atom}{\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{4.94x10^{14}\ cm^{-2}}$$

(ii) (110) plane, surface density,

$$= \frac{1\ atom}{\sqrt{2}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{3.49x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

$$= \frac{3\left(\frac{1}{6}\right) atoms}{\frac{1}{2}\left(a\sqrt{2}\right)(x)} = \frac{\frac{1}{2}}{\frac{1}{2}\cdot a\sqrt{2}\cdot\frac{a\sqrt{3}}{\sqrt{2}}} = \frac{1}{\sqrt{3}a^2}$$

$$= \frac{1}{\sqrt{3}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{2.85x10^{14}\ cm^{-2}}$$

(b)

Body-centered cubic

(i) (100) plane, surface density,

Same as (a),(i); surface density $\underline{4.94x10^{14}\ cm^{-2}}$

(ii) (110) plane, surface density,

$$= \frac{2\ atoms}{\sqrt{2}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{6.99x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

Same as (a),(iii), surface density $\underline{2.85x10^{14}\ cm^{-2}}$

(c)

Face centered cubic

(i) (100) plane, surface density

$$= \frac{2\ atoms}{\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{9.88x10^{14}\ cm^{-2}}$$

(ii) (110) plane, surface density,

$$= \frac{2\ atoms}{\sqrt{2}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{6.99x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

$$= \frac{\left(3\cdot\frac{1}{6} + 3\cdot\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}a^2} = \frac{4}{\sqrt{3}\left(4.50x10^{-8}\right)^2}$$

or $\underline{1.14x10^{15}\ cm^{-2}}$

1.15

(a)

(100) plane of silicon – similar to a fcc,

$$\text{surface density} = \frac{2\ atoms}{\left(5.43x10^{-8}\right)^2} \Rightarrow$$

$$\underline{6.78x10^{14}\ cm^{-2}}$$

(b)

(110) plane, surface density,

$$= \frac{4\ atoms}{\sqrt{2}\left(5.43x10^{-8}\right)^2} \Rightarrow \underline{9.59x10^{14}\ cm^{-2}}$$

(c)

(111) plane, surface density,

$$= \frac{4\ atoms}{\sqrt{3}\left(5.43x10^{-8}\right)^2} \Rightarrow \underline{7.83x10^{14}\ cm^{-2}}$$

1.16

$$d = 4r = a\sqrt{2}$$

then

$$a = \frac{4r}{\sqrt{2}} = \frac{4(2.25)}{\sqrt{2}} = 6.364\ \mathring{A}$$

(a)

$$\text{Volume Density} = \frac{4\ atoms}{\left(6.364x10^{-8}\right)^3} \Rightarrow$$

$$\underline{1.55x10^{22}\ cm^{-3}}$$

(b)

Distance between (110) planes,

$$= \frac{1}{2}a\sqrt{2} = \frac{a}{\sqrt{2}} = \frac{6.364}{\sqrt{2}} \Rightarrow$$

or

$\underline{4.50\ \overset{\circ}{A}}$

(c)
Surface density

$$= \frac{2\ atoms}{\sqrt{2}a^2} = \frac{2}{\sqrt{2}\left(6.364x10^{-8}\right)^2}$$

or

$\underline{3.49x10^{14}\ cm^{-2}}$

1.17

Density of silicon atoms $= 5x10^{22}\ cm^{-3}$ and 4 valence electrons per atom, so
Density of valence electrons $\underline{2x10^{23}\ cm^{-3}}$

1.18

Density of GaAs atoms

$$= \frac{8\ atoms}{\left(5.65x10^{-8}\right)^3} = 4.44x10^{22}\ cm^{-3}$$

An average of 4 valence electrons per atom,
Density of valence electrons $\underline{1.77x10^{23}\ cm^{-3}}$

1.19

(a) Percentage $= \dfrac{2x10^{16}}{5x10^{22}} x100\% \Rightarrow$

$\underline{4x10^{-5}\%}$

(b) Percentage $= \dfrac{1x10^{15}}{5x10^{22}} x100\% \Rightarrow$

$\underline{2x10^{-6}\%}$

1.20

(a) Fraction by weight $\approx \dfrac{\left(5x10^{16}\right)(30.98)}{\left(5x10^{22}\right)(28.06)} \Rightarrow$

$\underline{1.10x10^{-6}}$

(b) Fraction by weight

$$\approx \frac{\left(10^{18}\right)(10.82)}{\left(5x10^{16}\right)(30.98) + \left(5x10^{22}\right)(28.06)} \Rightarrow$$

$\underline{7.71x10^{-6}}$

1.21

Volume density $= \dfrac{1}{d^3} = 2x10^{15}\ cm^{-3}$

So

$d = 7.94x10^{-6}\ cm = 794\ \overset{\circ}{A}$

We have $a_o = 5.43\ \overset{\circ}{A}$

So

$$\frac{d}{a_o} = \frac{794}{5.43} \Rightarrow \underline{\frac{d}{a_o} = 146}$$

Chapter 2

Problem Solutions

2.1 Computer plot

2.2 Computer plot

2.3 Computer plot

2.4

For problem 2.2; Phase $= \dfrac{2\pi x}{\lambda} - \omega t =$ constant

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} - \omega = 0 \text{ or } \underline{\frac{dx}{dt} = v_p = +\omega\left(\frac{\lambda}{2\pi}\right)}$$

For problem 2.3; Phase $= \dfrac{2\pi x}{\lambda} + \omega t =$ constant

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} + \omega = 0 \text{ or } \underline{\frac{dx}{dt} = v_p = -\omega\left(\frac{\lambda}{2\pi}\right)}$$

2.5

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

Gold: $E = 4.90\ eV = (4.90)\left(1.6x10^{-19}\right)\ J$

So

$$\lambda = \frac{\left(6.625x10^{-34}\right)\left(3x10^{10}\right)}{(4.90)\left(1.6x10^{-19}\right)} \Rightarrow 2.54x10^{-5}\ cm$$

or

$$\underline{\lambda = 0.254\ \mu m}$$

Cesium: $E = 1.90\ eV = (1.90)\left(1.6x10^{-19}\right)\ J$

So

$$\lambda = \frac{\left(6.625x10^{-34}\right)\left(3x10^{10}\right)}{(1.90)\left(1.6x10^{-19}\right)} \Rightarrow 6.54x10^{-5}\ cm$$

or

$$\underline{\lambda = 0.654\ \mu m}$$

2.6

(a) Electron: (i) K.E. $= T = 1\ eV = 1.6x10^{-19}\ J$

$$p = \sqrt{2mT} = \sqrt{2\left(9.11x10^{-31}\right)\left(1.6x10^{-19}\right)}$$

or

$$\underline{p = 5.4x10^{-25}\ kg-m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{5.4x10^{-25}} \Rightarrow$$

or

$$\underline{\lambda = 12.3\ \overset{\circ}{A}}$$

(ii) K.E. $= T = 100\ eV = 1.6x10^{-17}\ J$

$$p = \sqrt{2mT} \Rightarrow \underline{p = 5.4x10^{-24}\ kg-m/s}$$

$$\lambda = \frac{h}{p} \Rightarrow \underline{\lambda = 1.23\ \overset{\circ}{A}}$$

(b) Proton: K.E. $= T = 1\ eV = 1.6x10^{-19}\ J$

$$p = \sqrt{2mT} = \sqrt{2\left(1.67x10^{-27}\right)\left(1.6x10^{-19}\right)}$$

or

$$\underline{p = 2.31x10^{-23}\ kg-m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{2.31x10^{-23}} \Rightarrow$$

or

$$\underline{\lambda = 0.287\ \overset{\circ}{A}}$$

(c) Tungsten Atom: At. Wt. = 183.92

For $T = 1\ eV = 1.6x10^{-19}\ J$

$$p = \sqrt{2mT}$$

$$= \sqrt{2(183.92)\left(1.66x10^{-27}\right)\left(1.6x10^{-19}\right)}$$

or

$$\underline{p = 3.13x10^{-22}\ kg=m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{3.13x10^{-22}} \Rightarrow$$

or

$$\underline{\lambda = 0.0212\ \overset{\circ}{A}}$$

(d) A 2000 kg traveling at 20 m/s:

$$p = mv = (2000)(20) \Rightarrow$$

or

$$\underline{p = 4x10^{4}\ kg-m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{4x10^{4}} \Rightarrow$$

or

$$\underline{\lambda = 1.66x10^{-28}\ \overset{\circ}{A}}$$

2.7

$$E_{avg} = \frac{3}{2}kT = \frac{3}{2}(0.0259) \Rightarrow$$

or

$$\underline{E_{avg} = 0.01727\ eV}$$

Now

$$p_{avg} = \sqrt{2mE_{avg}}$$

$$= \sqrt{2\left(9.11x10^{-31}\right)(0.01727)\left(1.6x10^{-19}\right)}$$

or

$$\underline{p_{avg} = 7.1x10^{-26}\ kg - m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{7.1x10^{-26}} \Rightarrow$$

or

$$\underline{\lambda = 93.3\ \overset{\circ}{A}}$$

2.8

$$E_p = h\nu_p = \frac{hc}{\lambda_p}$$

Now

$$E_e = \frac{p_e^2}{2m} \text{ and } p_e = \frac{h}{\lambda_e} \Rightarrow E_e = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2$$

Set $E_p = E_e$ and $\lambda_p = 10\lambda_e$

Then

$$\frac{hc}{\lambda_p} = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2 = \frac{1}{2m}\left(\frac{10h}{\lambda_p}\right)^2$$

which yields

$$\lambda_p = \frac{100h}{2mc}$$

$$E_p = E = \frac{hc}{\lambda_p} = \frac{hc}{100h} \cdot 2mc = \frac{2mc^2}{100}$$

$$= \frac{2\left(9.11x10^{-31}\right)\left(3x10^8\right)^2}{100} \Rightarrow$$

So

$$\underline{E = 1.64x10^{-15}\ J = 10.3\ keV}$$

2.9

(a) $E = \frac{1}{2}mv^2 = \frac{1}{2}\left(9.11x10^{-31}\right)\left(2x10^4\right)^2$

or

$$E = 1.822x10^{-22}\ J \Rightarrow \underline{E = 1.14x10^{-3}\ eV}$$

Also

$$p = mv = \left(9.11x10^{-31}\right)\left(2x10^4\right) \Rightarrow$$

$$\underline{p = 1.822x10^{-26}\ kg - m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{1.822x10^{-26}} \Rightarrow$$

$$\underline{\lambda = 364\ \overset{\circ}{A}}$$

(b)

$$p = \frac{h}{\lambda} = \frac{6.625x10^{-34}}{125x10^{-10}} \Rightarrow$$

$$\underline{p = 5.3x10^{-26}\ kg - m/s}$$

Also

$$v = \frac{p}{m} = \frac{5.3x10^{-26}}{9.11x10^{-31}} = 5.82x10^4\ m/s$$

or

$$\underline{v = 5.82x10^6\ cm/s}$$

Now

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\left(9.11x10^{-31}\right)\left(5.82x10^4\right)^2$$

or

$$E = 1.54x10^{-21}\ J \Rightarrow \underline{E = 9.64x10^{-3}\ eV}$$

2.10

(a) $E = h\nu = \frac{hc}{\lambda} = \frac{\left(6.625x10^{-34}\right)\left(3x10^8\right)}{1x10^{-10}}$

or

$$E = 1.99x10^{-15}\ J$$

Now

$$E = e \cdot V \Rightarrow 1.99x10^{-15} = \left(1.6x10^{-19}\right)V$$

so

$$\underline{V = 12.4x10^3\ V = 12.4\ kV}$$

(b) $p = \sqrt{2mE} = \sqrt{2\left(9.11x10^{-31}\right)\left(1.99x10^{-15}\right)}$

$$= 6.02x10^{-23}\ kg - m/s$$

Then

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{6.02x10^{-23}} \Rightarrow \underline{\lambda = 0.11\ \overset{\circ}{A}}$$

2.11

(a) $$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{10^{-6}} \Rightarrow$$

$$\underline{\Delta p = 1.054x10^{-28}\ kg-m/s}$$

(b)

$$E = \frac{hc}{\lambda} = hc\left(\frac{p}{h}\right) = pc$$

So

$$\Delta E = c(\Delta p) = \left(3x10^{8}\right)\left(1.054x10^{-28}\right) \Rightarrow$$

or

$$\Delta E = 3.16x10^{-20}\ J \Rightarrow \underline{\Delta E = 0.198\ eV}$$

2.12

(a) $$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{12x10^{-10}} \Rightarrow$$

$$\underline{\Delta p = 8.78x10^{-26}\ kg-m/s}$$

(b)

$$\Delta E = \frac{1}{2}\cdot\frac{(\Delta p)^2}{m} = \frac{1}{2}\cdot\frac{\left(8.78x10^{-26}\right)^2}{5x10^{-29}} \Rightarrow$$

$$\Delta E = 7.71x10^{-23}\ J \Rightarrow \underline{\Delta E = 4.82x10^{-4}\ eV}$$

2.13

(a) Same as 2.12 (a), $\underline{\Delta p = 8.78x10^{-26}\ kg-m/s}$

(b)

$$\Delta E = \frac{1}{2}\cdot\frac{(\Delta p)^2}{m} = \frac{1}{2}\cdot\frac{\left(8.78x10^{-26}\right)^2}{5x10^{-26}} \Rightarrow$$

$$\Delta E = 7.71x10^{-26}\ J \Rightarrow \Delta E = 4.82x10^{-7}\ eV$$

2.14

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{10^{-2}} = 1.054x10^{-32}$$

$$p = mv \Rightarrow \Delta v = \frac{\Delta p}{m} = \frac{1.054x10^{-32}}{1500} \Rightarrow$$

or

$$\underline{\Delta v = 7x10^{-36}\ m/s}$$

2.15

(a) $$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{10^{-10}} \Rightarrow$$

$$\underline{\Delta p = 1.054x10^{-24}\ kg-m/s}$$

(b) $$\Delta t = \frac{1.054x10^{-34}}{(1)\left(1.6x10^{-19}\right)} \Rightarrow$$

or

$$\underline{\Delta t = 6.6x10^{-16}\ s}$$

2.16

(a) If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to Schrodinger's wave equation, then

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi_1(x,t)}{\partial x^2} + V(x)\Psi_1(x,t) = j\hbar\frac{\partial\Psi_1(x,t)}{\partial t}$$

and

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi_2(x,t)}{\partial x^2} + V(x)\Psi_2(x,t) = j\hbar\frac{\partial\Psi_2(x,t)}{\partial t}$$

Adding the two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}\left[\Psi_1(x,t)+\Psi_2(x,t)\right]$$
$$+V(x)\left[\Psi_1(x,t)+\Psi_2(x,t)\right]$$
$$= j\hbar\frac{\partial}{\partial t}\left[\Psi_1(x,t)+\Psi_2(x,t)\right]$$

which is Schrodinger's wave equation. So $\Psi_1(x,t)+\Psi_2(x,t)$ is also a solution.

(b)

If $\Psi_1\cdot\Psi_2$ were a solution to Schrodinger's wave equation, then we could write

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(\Psi_1\cdot\Psi_2\right) + V(x)\left(\Psi_1\cdot\Psi_2\right)$$
$$= j\hbar\frac{\partial}{\partial t}\left(\Psi_1\cdot\Psi_2\right)$$

which can be written as

$$\frac{-\hbar^2}{2m}\left[\Psi_1\frac{\partial^2\Psi_2}{\partial x^2} + \Psi_2\frac{\partial^2\Psi_1}{\partial x^2} + 2\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}\right]$$
$$+V(x)\Psi_1\cdot\Psi_2 = j\hbar\left[\Psi_1\frac{\partial\Psi_2}{\partial t} + \Psi_2\frac{\partial\Psi_1}{\partial t}\right]$$

Dividing by $\Psi_1\cdot\Psi_2$ we find

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2} + \frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2} + \frac{1}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$
$$+V(x) = j\hbar\left[\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t} + \frac{1}{\Psi_1}\frac{\partial\Psi_1}{\partial x}\right]$$

Since Ψ_1 is a solution, then

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2}+V(x)=j\hbar\cdot\frac{1}{\Psi_1}\cdot\frac{\partial\Psi_1}{\partial t}$$

Subtracting these last two equations, we are left with

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\frac{\partial^2\Psi_2}{\partial x^2}+\frac{2}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$

$$=j\hbar\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t}$$

Since Ψ_2 is also a solution, we may write

$$\frac{-\hbar^2}{2m}\frac{1}{\Psi_2}\frac{\partial^2\Psi_2}{\partial x^2}+V(x)=j\hbar\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t}$$

Subtracting these last two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{2}{\Psi_1\Psi_2}\cdot\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}-V(x)=0$$

This equation is not necessarily valid, which means that $\Psi_1\Psi_2$ is, in general, not a solution to Schrodinger's wave equation.

2.17

$$\Psi(x,t)=A[\sin(\pi x)]\exp(-j\omega t)$$

$$\int_{-1}^{+1}|\Psi(x,t)|^2dx=1=|A|^2\int_{-1}^{+1}\sin^2(\pi x)dx$$

or

$$\left|A^2\right|\cdot\left[\frac{1}{2}x-\frac{1}{4\pi}\sin(2\pi x)\right]_{-1}^{+1}=1$$

which yields

$$\underline{\left|A^2\right|=1} \text{ or } \underline{A=+1,-1,+j,-j}$$

2.18

$$\Psi(x,t)=A[\sin(n\pi x)]\exp(-j\omega t)$$

$$\int_{0}^{+1}|\Psi(x,t)|^2dx=1=|A|^2\int_{0}^{+1}\sin^2(n\pi x)dx$$

or

$$|A|^2\cdot\left[\frac{1}{2}x-\frac{1}{4n\pi}\sin(2n\pi x)\right]_{0}^{+1}=1$$

which yields

$$\underline{|A|^2=2} \text{ or}$$

$$\underline{A=+\sqrt{2},-\sqrt{2},+j\sqrt{2},-j\sqrt{2}}$$

2.19

Note that $\int_0^\infty \Psi\cdot\Psi^* dx=1$

Function has been normalized

(a) Now

$$P=\int_0^{a_o/4}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2dx$$

$$=\frac{2}{a_o}\int_0^{a_o/4}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o/4}$$

or

$$P=-1\left[\exp\left(\frac{-2a_o}{4a_o}\right)-1\right]=1-\exp\left(\frac{-1}{2}\right)$$

which yields

$$\underline{P=0.393}$$

(b)

$$P=\int_{a_o/4}^{a_o/2}\left(\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right)^2dx$$

$$=\frac{2}{a_o}\int_{a_o/4}^{a_o/2}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_{a_o/4}^{a_o/2}$$

or

$$P=-1\left[\exp(-1)-\exp\left(\frac{-1}{2}\right)\right]$$

which yields

$$\underline{P=0.239}$$

(c)

$$P=\int_0^{a_o}\left(\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right)^2dx$$

$$=\frac{2}{a_o}\int_0^{a_o}\exp\left(\frac{-2x}{a_o}\right)dx=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o}$$

or

$$P=-1[\exp(-2)-1]$$

which yields

$$\underline{P=0.865}$$

2.20

(a) $kx - \omega t = \text{constant}$

Then

$$k\frac{dx}{dt} - \omega = 0 \Rightarrow \frac{dx}{dt} = v_p = +\frac{\omega}{k}$$

or

$$v_p = \frac{1.5x10^{13}}{1.5x10^9} = 10^4 \ m/s$$

$$\underline{v_p = 10^6 \ cm/s}$$

(b)

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{1.5x10^9}$$

or

$$\underline{\lambda = 41.9 \ \overset{\circ}{A}}$$

Also

$$p = \frac{h}{\lambda} = \frac{6.625x10^{-34}}{41.9x10^{-10}} \Rightarrow$$

or

$$\underline{p = 1.58x10^{-25} \ kg-m/s}$$

Now

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.625x10^{-34})(3x10^8)}{41.9x10^{-10}}$$

or

$$E = 4.74x10^{-17} \ J \Rightarrow \underline{E = 2.96x10^2 \ eV}$$

2.21

$$\underline{\psi(x) = A\exp[-j(kx + \omega t)]}$$

where

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$= \frac{\sqrt{2(9.11x10^{-31})(0.015)(1.6x10^{-19})}}{1.054x10^{-34}}$$

or

$$\underline{k = 6.27x10^8 \ m^{-1}}$$

Now

$$\omega = \frac{E}{\hbar} = \frac{(0.015)(1.6x10^{-19})}{1.054x10^{-34}}$$

or

$$\underline{\omega = 2.28x10^{13} \ rad/s}$$

2.22

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{(1.054x10^{-34})^2 \pi^2 n^2}{2(9.11x10^{-31})(100x10^{-10})^2}$$

so

$$E = 6.018x10^{-22} n^2 \ (J)$$

or

$$E = 3.76x10^{-3} n^2 \ (eV)$$

Then

$$n = 1 \Rightarrow \underline{E_1 = 3.76x10^{-3} \ eV}$$

$$n = 2 \Rightarrow \underline{E_2 = 1.50x10^{-2} \ eV}$$

$$n = 3 \Rightarrow \underline{E_3 = 3.38x10^{-2} \ eV}$$

2.23

(a) $E = \dfrac{\hbar^2 n^2 \pi^2}{2ma^2}$

$$= \frac{(1.054x10^{-34})^2 \pi^2 n^2}{2(9.11x10^{-31})(12x10^{-10})^2}$$

$$= 4.81x10^{-20} n^2 \ (J)$$

So

$$E_1 = 4.18x10^{-20} \ J \Rightarrow \underline{E_1 = 0.261 \ eV}$$

$$E_2 = 1.67x10^{-19} \ J \Rightarrow \underline{E_2 = 1.04 \ eV}$$

(b)

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$$

or

$$\lambda = \frac{(6.625x10^{-34})(3x10^8)}{1.67x10^{-19} - 4.18x10^{-20}} \Rightarrow$$

$$\lambda = 1.59x10^{-6} \ m$$

or

$$\underline{\lambda = 1.59 \ \mu m}$$

2.24

(a) For the infinite potential well

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2} \Rightarrow n^2 = \frac{2ma^2 E}{\hbar^2 \pi^2}$$

so

$$n^2 = \frac{2(10^{-5})(10^{-2})^2(10^{-2})}{(1.054x10^{-34})^2 \pi^2} = 1.82x10^{56}$$

or

$$\underline{n = 1.35x10^{28}}$$

(b)

$$\Delta E = \frac{\hbar^2\pi^2}{2ma^2}\left[(n+1)^2 - n^2\right]$$

$$= \frac{\hbar^2\pi^2}{2ma^2}(2n+1)$$

or

$$\Delta E = \frac{(1.054x10^{-34})^2\pi^2(2)(1.35x10^{28})}{2(10^{-5})(10^{-2})^2}$$

$$\underline{\Delta E = 1.48x10^{-30}\ J}$$

Energy in the (n+1) state is $1.48x10^{-30}$ Joules larger than 10 mJ.

(c)

Quantum effects would not be observable.

2.25

For a neutron and n = 1:

$$E_1 = \frac{\hbar^2\pi^2}{2ma^2} = \frac{(1.054x10^{-34})\pi^2}{2(1.66x10^{-27})(10^{-14})^2}$$

or

$$\underline{E_1 = 2.06x10^6\ eV}$$

For an electron in the same potential well:

$$E_1 = \frac{(1.054x10^{-34})^2\pi^2}{2(9.11x10^{-31})(10^{-14})^2}$$

or

$$\underline{E_1 = 3.76x10^9\ eV}$$

2.26

Schrodinger's wave equation

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$$

We know that

$$\psi(x) = 0 \text{ for } x \ge \frac{a}{2} \text{ and } x \le \frac{-a}{2}$$

$$V(x) = 0 \text{ for } \frac{-a}{2} \le x \le \frac{+a}{2}$$

so in this region

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

Solution is of the form

$$\psi(x) = A\cos Kx + B\sin Kx$$

where $K = \sqrt{\frac{2mE}{\hbar^2}}$

Boundary conditions:

$$\psi(x) = 0 \text{ at } x = \frac{+a}{2},\ x = \frac{-a}{2}$$

So, first mode:

$$\underline{\psi_1(x) = A\cos Kx}$$

where $\underline{K = \frac{\pi}{a}}$ so $\underline{E_1 = \frac{\pi^2\hbar^2}{2ma^2}}$

Second mode:

$$\underline{\psi_2(x) = B\sin Kx}$$

where $\underline{K = \frac{2\pi}{a}}$ so $\underline{E_2 = \frac{4\pi^2\hbar^2}{2ma^2}}$

Third mode:

$$\underline{\psi_3(x) = A\cos Kx}$$

where $\underline{K = \frac{3\pi}{a}}$ so $\underline{E_3 = \frac{9\pi^2\hbar^2}{2ma^2}}$

Fourth mode:

$$\underline{\psi_4(x) = B\sin Kx}$$

where $\underline{K = \frac{4\pi}{a}}$ so $\underline{E_4 = \frac{16\pi^2\hbar^2}{2ma^2}}$

2.27

The 3-D wave equation in cartesian coordinates, for V(x,y,z) = 0

$$\frac{\partial^2\psi(x,y,z)}{\partial x^2} + \frac{\partial^2\psi(x,y,z)}{\partial y^2} + \frac{\partial^2\psi(x,y,z)}{\partial z^2} + \frac{2mE}{\hbar^2}\psi(x,y,z) = 0$$

Use separation of variables, so let

$$\psi(x,y,z) = X(x)Y(y)Z(z)$$

Substituting into the wave equation, we get

$$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2}XYZ = 0$$

Dividing by XYZ and letting $k^2 = \frac{2mE}{\hbar^2}$, we obtain

$$(1)\quad \frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

We may set

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} = -k_x^2 \text{ so } \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Solution is of the form

$$X(x) = A\sin(k_x x) + B\cos(k_x x)$$

Boundary conditions: $X(0) = 0 \Rightarrow B = 0$

and $X(x = a) = 0 \Rightarrow k_x = \frac{n_x \pi}{a}$

where $n_x = 1, 2, 3, \ldots$

Similarly, let

$$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} = -k_y^2 \text{ and } \frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2} = -k_z^2$$

Applying the boundary conditions, we find

$$k_y = \frac{n_y \pi}{a}, \; n_y = 1, 2, 3, \ldots$$

$$k_z = \frac{n_z \pi}{a}, \; n_z = 1, 2, 3, \ldots$$

From Equation (1) above, we have

$$-k_x^2 - k_y^2 - k_z^2 + k^2 = 0$$

or

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \frac{2mE}{\hbar^2}$$

so that

$$E \Rightarrow E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

2.28

For the 2-dimensional infinite potential well:

$$\frac{\partial^2 \psi(x,y)}{\partial x^2} + \frac{\partial^2 \psi(x,y)}{\partial y^2} + \frac{2mE}{\hbar^2}\psi(x,y) = 0$$

Let $\psi(x,y) = X(x)Y(y)$

Then substituting,

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + \frac{2mE}{\hbar^2}XY = 0$$

Divide by XY

So

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} + \frac{2mE}{\hbar^2} = 0$$

Let

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} = -k_x^2$$

or

$$\frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Solution is of the form:

$$X = A\sin(k_x x) + B\cos(k_x x)$$

But $X(x = 0) = 0 \Rightarrow B = 0$

So

$$X = A\sin(k_x x)$$

Also, $X(x = a) = 0 \Rightarrow k_x a = n_x \pi$

Where $n_x = 1, 2, 3, \ldots$

So that $k_x = \frac{n_x \pi}{a}$

We can also define

$$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} = -k_y^2$$

Solution is of the form

$$Y = C\sin(k_y y) + D\cos(k_y y)$$

But

$$Y(y = 0) = 0 \Rightarrow D = 0$$

and

$$Y(y = b) = 0 \Rightarrow k_y b = n_y \pi$$

so that

$$k_y = \frac{n_y \pi}{b}$$

Now

$$-k_x^2 - k_y^2 + \frac{2mE}{\hbar^2} = 0$$

which yields

$$E \Rightarrow E_{n_x n_y} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2}\right)$$

Similarities: energy is quantized

Difference: now a function of 2 integers

2.29

(a) Derivation of energy levels exactly the same as in the text.

(b) $\Delta E = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_2^2 - n_1^2\right)$

For $n_2 = 2$, $n_1 = 1$

Then

$$\Delta E = \frac{3\hbar^2 \pi^2}{2ma^2}$$

(i) $a = 4\ \mathring{A}$

$$\Delta E = \frac{3\left(1.054x10^{-34}\right)^2 \pi^2}{2\left(1.67x10^{-27}\right)\left(4x10^{-10}\right)^2} \Rightarrow$$

$$\underline{\Delta E = 3.85x10^{-3}\ eV}$$

(ii) $a = 0.5\ cm$

$$\Delta E = \frac{3\left(1.054x10^{-34}\right)^2 \pi^2}{2\left(1.67x10^{-27}\right)\left(0.5x10^{-2}\right)^2} \Rightarrow$$

$$\underline{\Delta E = 2.46x10^{-17}\ eV}$$

2.30

(a) For region II, $x > 0$

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_O)\psi_2(x) = 0$$

General form of the solution is

$$\psi_2(x) = A_2 \exp(jK_2x) + B_2 \exp(-jK_2x)$$

where

$$K_2 = \sqrt{\frac{2m}{\hbar^2}(E - V_O)}$$

Term with B_2 represents incident wave, and term with A_2 represents the reflected wave.

Region I, $x < 0$

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0$$

The general solution is of the form

$$\psi_1(x) = A_1 \exp(jK_1x) + B_1 \exp(-jK_1x)$$

where

$$K_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Term involving B_1 represents the transmitted wave, and the term involving A_1 represents the reflected wave; but if a particle is transmitted into region I, it will not be reflected so that $A_1 = 0$.

Then

$$\underline{\psi_1(x) = B_1 \exp(-jK_1x)}$$

$$\underline{\psi_2(x) = A_2 \exp(jK_2x) + B_2 \exp(-jK_2x)}$$

(b)

Boundary conditions:

(1) $\psi_1(x = 0) = \psi_2(x = 0)$

(2) $\left.\frac{\partial \psi_1(x)}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_2(x)}{\partial x}\right|_{x=0}$

Applying the boundary conditions to the solutions, we find

$$B_1 = A_2 + B_2$$

$$K_2A_2 - K_2B_2 = -K_1B_1$$

Combining these two equations, we find

$$A_2 = \left(\frac{K_2 - K_1}{K_2 + K_1}\right)B_2 \quad \text{and} \quad B_1 = \left(\frac{2K_2}{K_2 + K_1}\right)B_2$$

The reflection coefficient is

$$R = \frac{A_2A_2^*}{B_2B_2^*} \Rightarrow \underline{R = \left(\frac{K_2 - K_1}{K_2 + K_1}\right)^2}$$

The transmission coefficient is

$$T = 1 - R \Rightarrow \underline{T = \frac{4K_1K_2}{(K_1 + K_2)^2}}$$

2.31

In region II, $x > 0$, we have

$$\psi_2(x) = A_2 \exp(-K_2x)$$

where

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

For $V_O = 2.4\ eV$ and $E = 2.1\ eV$

$$K_2 = \left\{\frac{2\left(9.11x10^{-31}\right)(2.4 - 2.1)\left(1.6x10^{-19}\right)}{\left(1.054x10^{-34}\right)^2}\right\}^{1/2}$$

or

$$K_2 = 2.81x10^9\ m^{-1}$$

Probability at x compared to x = 0, given by

$$P = \left|\frac{\psi_2(x)}{\psi_2(0)}\right|^2 = \exp(-2K_2x)$$

(a) For $x = 12\ \mathring{A}$

$$P = \exp\left[-2\left(2.81x10^9\right)\left(12x10^{-10}\right)\right] \Rightarrow$$

$$\underline{P = 1.18x10^{-3} = 0.118\,\%}$$

(b) For $x = 48\ A°$

$$P = \exp\left[-2\left(2.81x10^9\right)\left(48x10^{-10}\right)\right] \Rightarrow$$

$$\underline{P = 1.9x10^{-10}\ \%}$$

2.32

For $V_O = 6\ eV$, $E = 2.2\ eV$

We have that

$$T = 16\left(\frac{E}{V_O}\right)\left(1-\frac{E}{V_O}\right)\exp(-2K_2 a)$$

where

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(9.11x10^{-31})(6-2.2)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 9.98x10^9\ m^{-1}$$

For $a = 10^{-10}\ m$

$$T = 16\left(\frac{2.2}{6}\right)\left(1-\frac{2.2}{6}\right)\exp\left[-2(9.98x10^9)(10^{-10})\right]$$

or

$$\underline{T = 0.50}$$

For $a = 10^{-9}\ m$

$$\underline{T = 7.97x10^{-9}}$$

2.33

Assume that Equation [2.62] is valid:

$$T = 16\left(\frac{E}{V_O}\right)\left(1-\frac{E}{V_O}\right)\exp(-2K_2 a)$$

(a) For $m = (0.067)m_o$

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(0.067)(9.11x10^{-31})(0.8-0.2)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 1.027x10^9\ m^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1-\frac{0.2}{0.8}\right)\exp\left[-2(1.027x10^9)(15x10^{-10})\right]$$

or

$$\underline{T = 0.138}$$

(b) For $m = (1.08)m_o$

$$K_2 = \left\{\frac{2(1.08)(9.11x10^{-31})(0.8-0.2)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 4.124x10^9\ m^{-1}$$

Then

$$T = 3\exp\left[-2(4.124x10^9)(15x10^{-10})\right]$$

or

$$\underline{T = 1.27x10^{-5}}$$

2.34

$V_O = 10x10^6\ eV$, $E = 3x10^6\ eV$, $a = 10^{-14}\ m$

and $m = 1.67x10^{-27}\ kg$

Now

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(1.67x10^{-27})(10-3)(10^6)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 5.80x10^{14}\ m^{-1}$$

So

$$T = 16\left(\frac{3}{10}\right)\left(1-\frac{3}{10}\right)\exp\left[-2(5.80x10^{14})(10^{-14})\right]$$

or

$$\underline{T = 3.06x10^{-5}}$$

2.35

Region I, $V = 0$ $(x < 0)$; Region II, $V = V_O$ $(0 < x < a)$; Region III, $V = 0$ $(x > a)$.

(a) Region I;

$$\psi_1(x) = A_1\exp(jK_1 x) + B_1\exp(-jK_1 x)$$

(incident) (reflected)

Region II;

$$\psi_2(x) = A_2\exp(K_2 x) + B_2\exp(-K_2 x)$$

Region III;

$$\psi_3(x) = A_3\exp(jK_1 x) + B_3\exp(-jK_1 x)$$

(b)

In region III, the B_3 term represents a reflected wave. However, once a particle is transmitted into region III, there will not be a reflected wave which means that $B_3 = 0$.

(c)

Boundary conditions:

For $x = 0$: $\psi_1 = \psi_2 \Rightarrow A_1 + B_1 = A_2 + B_2$

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \Rightarrow jK_1A_1 - jK_1B_2 = K_2A_2 - K_2B_2$$

For $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2\exp(K_2 a) + B_2\exp(-K_2 a) = A_3\exp(jK_1 a)$$

And also

$$\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx} \Rightarrow$$

$$K_2 A_2 \exp(K_2 a) - K_2 B_2 \exp(-K_2 a)$$
$$= jK_1 A_3 \exp(jK_1 a)$$

Transmission coefficient is defined as

$$T = \frac{A_3 A_3^*}{A_1 A_1^*}$$

so from the boundary conditions, we want to solve for A_3 in terms of A_1. Solving for A_1 in terms of A_3, we find

$$A_1 = \frac{+jA_3}{4K_1 K_2}\{(K_2^2 - K_1^2)[\exp(K_2 a) - \exp(-K_2 a)]$$
$$-2jK_1 K_2[\exp(K_2 a) + \exp(-K_2 a)]\ \}\exp(jK_2 a)$$

We then find that

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4K_1 K_2)^2}\{(K_2^2 - K_1^2)[\exp(K_2 a)$$
$$-\exp(-K_2 a)]^2$$
$$+4K_1^2 K_2^2[\exp(K_2 a) + \exp(-K_2 a)]^2\ \}$$

We have

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

and since $V_O >> E$, then $K_2 a$ will be large so that

$$\exp(K_2 a) >> \exp(-K_2 a)$$

Then we can write

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4K_1 K_2)^2}\{(K_2^2 - K_1^2)[\exp(K_2 a)]^2$$
$$+4K_1^2 K_2^2[\exp(K_2 a)]^2\}$$

which becomes

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4K_1 K_2)^2}(K_2^2 + K_1^2)\exp(2K_2 a)$$

Substituting the expressions for K_1 and K_2, we find

$$K_1^2 + K_2^2 = \frac{2mV_O}{\hbar^2}$$

and

$$K_1^2 K_2^2 = \left[\frac{2m(V_O - E)}{\hbar^2}\right]\left[\frac{2mE}{\hbar^2}\right]$$

$$= \left(\frac{2m}{\hbar^2}\right)(V_O = E)(E)$$

or

$$K_1^2 K_2^2 = \left(\frac{2m}{\hbar^2}\right)^2 V_O\left(1 - \frac{E}{V_O}\right)(E)$$

Then

$$A_1 A_1^* = \frac{A_3 A_3^*\left(\frac{2mV_O}{\hbar^2}\right)^2 \exp(2K_2 a)}{16\left[\left(\frac{2m}{\hbar^2}\right)^2 V_O\left(1 - \frac{E}{V_O}\right)(E)\right]}$$

$$= \frac{A_3 A_3^*}{16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2K_2 a)}$$

or finally

$$\underline{T = \frac{A_3 A_3^*}{A_1 A_1^*} = 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2K_2 a)}$$

2.36

Region I: $V = 0$

$$\frac{\partial^2\psi_1}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1 = 0 \Rightarrow$$

$$\psi_1 = A_1 \exp(jK_1 x) + B_1 \exp(-jK_1 x)$$

(incident wave) (reflected wave)

where $K_1 = \sqrt{\frac{2mE}{\hbar^2}}$

Region II: $V = V_1$

$$\frac{\partial^2\psi_2}{\partial x^2} + \frac{2m(E - V_1)}{\hbar^2}\psi_2 = 0 \Rightarrow$$

$$\psi_2 = A_2 \exp(jK_2 x) + B_2 \exp(-jK_2 x)$$

(transmitted wave) (reflected wave)

where $K_2 = \sqrt{\frac{2m(E - V_1)}{\hbar^2}}$

Region III: $V = V_2$

$$\frac{\partial^2\psi_3}{\partial x^2} + \frac{2m(E - V_2)}{\hbar^2}\psi_3 = 0 \Rightarrow$$

$$\psi_3 = A_3 \exp(jK_3 x)$$

(transmitted wave)

where $K_3 = \sqrt{\frac{2m(E - V_2)}{\hbar^2}}$

There is no reflected wave in region III.

The transmission coefficient is defined as

$$T = \frac{v_3}{v_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*} = \frac{K_3}{K_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*}$$

From boundary conditions, solve for A_3 in terms of A_1. The boundary conditions are:

$x = 0$: $\psi_1 = \psi_2 \Rightarrow A_1 + B_1 = A_2 + B_2$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow K_1 A_1 - K_1 B_1 = K_2 A_2 - K_2 B_2$$

$x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \exp(jK_2 a) + B_2 \exp(-jK_2 a) = A_3 \exp(jK_3 a)$$

$$\frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_3}{\partial x} \Rightarrow$$

$$K_2 A_2 \exp(jK_2 a) - K_2 B_2 \exp(-jK_2 a) = K_3 A_3 \exp(jK_3 a)$$

But $K_2 a = 2n\pi \Rightarrow$

$$\exp(jK_2 a) = \exp(-jK_2 a) = 1$$

Then, eliminating B_1, A_2, B_2 from the above equations, we have

$$T = \frac{K_3}{K_1} \cdot \frac{4K_1^2}{(K_1 + K_3)^2} \Rightarrow \underline{T = \frac{4K_1 K_3}{(K_1 + K_3)^2}}$$

2.37

(a) Region I: Since $V_O > E$, we can write

$$\frac{\partial^2 \psi_1}{\partial x^2} - \frac{2m(V_O - E)}{\hbar^2} \psi_1 = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2 \psi_2}{\partial x^2} + \frac{2mE}{\hbar^2} \psi_2 = 0$$

Region III: $V \to \infty \Rightarrow \psi_3 = 0$

The general solutions can be written, keeping in mind that ψ_1 must remain finite for $x < 0$, as

$$\psi_1 = B_1 \exp(+K_1 x)$$

$$\psi_2 = A_2 \sin(K_2 x) + B_2 \cos(K_2 x)$$

$$\psi_3 = 0$$

where

$$K_1 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}} \quad \text{and} \quad K_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b)

Boundary conditions:

$x = 0$: $\psi_1 = \psi_2 \Rightarrow B_1 = B_2$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow K_1 B_1 = K_2 A_2$$

$x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \sin K_2 a + B_2 \cos K_2 a = 0$$

or

$$B_2 = -A_2 \tan K_2 a$$

(c)

$$K_1 B_1 = K_2 A_2 \Rightarrow A_2 = \left(\frac{K_1}{K_2}\right) B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{K_1}{K_2}\right) B_2$$

From $B_2 = -A_2 \tan K_2 a$, we can write

$$B_2 = -\left(\frac{K_1}{K_2}\right) B_2 \tan K_2 a$$

which gives

$$1 = -\left(\frac{K_1}{K_2}\right) \tan K_2 a$$

In turn, this equation can be written as

$$1 = -\sqrt{\frac{V_O - E}{E}} \tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

or

$$\underline{\sqrt{\frac{E}{V_O - E}} = -\tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]}$$

This last equation is valid only for specific values of the total energy E. The energy levels are quantized.

2.38

$$E_n = \frac{-m_o e^4}{(4\pi \in_o)^2 2\hbar^2 n^2} \ (J)$$

$$= \frac{m_o e^3}{(4\pi \in_o)^2 2\hbar^2 n^2} \ (eV)$$

$$= \frac{-(9.11x10^{-31})(1.6x10^{-19})^3}{\left[4\pi(8.85x10^{-12})\right]^2 2(1.054x10^{-34})^2 n^2} \Rightarrow$$

$$E_n = \frac{-13.58}{n^2} \ (eV)$$

Then

$$n = 1 \Rightarrow \quad \underline{E_1 = -13.58\ eV}$$
$$n = 2 \Rightarrow \quad \underline{E_2 = -3.395\ eV}$$
$$n = 3 \Rightarrow \quad \underline{E_3 = -1.51\ eV}$$
$$n = 4 \Rightarrow \quad \underline{E_4 = -0.849\ eV}$$

2.39

We have

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

and

$$P = 4\pi r^2 \psi_{100}\psi^*_{100} = 4\pi r^2 \cdot \frac{1}{\pi} \cdot \left(\frac{1}{a_o}\right)^3 \exp\left(\frac{-2r}{a_o}\right)$$

or

$$P = \frac{4}{(a_o)^3} \cdot r^2 \exp\left(\frac{-2r}{a_o}\right)$$

To find the maximum probability

$$\frac{dP(r)}{dr} = 0$$

$$= \frac{4}{(a_o)^3}\left\{r^2\left(\frac{-2}{a_o}\right)\exp\left(\frac{-2r}{a_o}\right) + 2r\exp\left(\frac{-2r}{a_o}\right)\right\}$$

which gives

$$0 = \frac{-r}{a_o} + 1 \Rightarrow \quad \underline{r = a_o}$$

or $r = a_o$ is the radius that gives the greatest probability.

2.40

ψ_{100} is independent of θ and ϕ, so the wave equation in spherical coordinates reduces to

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{2m_o}{\hbar^2}(E - V(r))\psi = 0$$

where

$$V(r) = \frac{-e^2}{4\pi \in_o r} = \frac{-\hbar^2}{m_o a_o r}$$

For

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right) \Rightarrow$$

$$\frac{d\psi_{100}}{dr} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2}\left(\frac{-1}{a_o}\right)\exp\left(\frac{-r}{a_o}\right)$$

Then

$$r^2 \frac{d\psi_{100}}{dr} = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} r^2 \exp\left(\frac{-r}{a_o}\right)$$

so that

$$\frac{d}{dr}\left(r^2 \frac{d\psi_{100}}{dr}\right)$$

$$= \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2}\left[2r\exp\left(\frac{-r}{a_o}\right) - \left(\frac{r^2}{a_o}\right)\exp\left(\frac{-r}{a_o}\right)\right]$$

Substituting into the wave equation, we have

$$\frac{-1}{r^2\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2}\left[2r\exp\left(\frac{-r}{a_o}\right) - \frac{r^2}{a_o}\exp\left(\frac{-r}{a_o}\right)\right]$$

$$+ \frac{2m_o}{\hbar^2}\left[E + \frac{\hbar^2}{m_o a_o r}\right] \cdot \left(\frac{1}{\sqrt{\pi}}\right) \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right) = 0$$

where

$$E = E_1 = \frac{-m_o e^4}{(4\pi \in_o)^2 \cdot 2\hbar^2} \Rightarrow \quad E_1 = \frac{-\hbar^2}{2m_o a_o^2}$$

Then the above equation becomes

$$\frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2}\left[\exp\left(\frac{-r}{a_o}\right)\right]\left\{\frac{-1}{r^2 a_o}\left[2r - \frac{r^2}{a_o}\right]\right.$$

$$\left. + \frac{2m_o}{\hbar^2}\left(\frac{-\hbar^2}{2m_o a_o} + \frac{\hbar^2}{m_o a_o r}\right)\right\} = 0$$

or

$$\frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2}\left[\exp\left(\frac{-r}{a_o}\right)\right]$$

$$\times\left\{\frac{-2}{a_o r} + \frac{1}{a_o^2} + \left(\frac{-1}{a_o^2} + \frac{2}{a_o r}\right)\right\} = 0$$

which gives 0 = 0, and shows that ψ_{100} is indeed a solution of the wave equation.

2.41

All elements from Group I column of the periodic table. All have one valence electron in the outer shell.

Chapter 3

Problem Solutions

3.1 If a_o were to increase, the bandgap energy would decrease and the material would begin to behave less like a semiconductor and more like a metal. If a_o were to decrease, the bandgap energy would increase and the material would begin to behave more like an insulator.

3.2
Schrodinger's wave equation
$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x)\cdot\Psi(x,t)=j\hbar\frac{\partial\Psi(x,t)}{\partial t}$$
Let the solution be of the form
$$\Psi(x,t)=u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$
Region I, $V(x)=0$, so substituting the proposed solution into the wave equation, we obtain
$$\frac{-\hbar^2}{2m}\cdot\frac{\partial}{\partial x}\left\{jku(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]+\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$
$$=j\hbar\left(\frac{-jE}{\hbar}\right)\cdot u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$
which becomes
$$\frac{-\hbar^2}{2m}\left\{(jk)^2u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]+2jk\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]+\frac{\partial^2u(x)}{\partial x^2}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$
$$=+Eu(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$
This equation can then be written as
$$-k^2u(x)+2jk\frac{\partial u(x)}{\partial x}+\frac{\partial^2u(x)}{\partial x^2}+\frac{2mE}{\hbar^2}\cdot u(x)=0$$
Setting $u(x)=u_1(x)$ for region I, this equation becomes
$$\frac{d^2u_1(x)}{dx^2}+2jk\frac{du_1(x)}{dx}-\left(k^2-\alpha^2\right)u_1(x)=0$$
where
$$\alpha^2=\frac{2mE}{\hbar^2}$$
Q.E.D.

In region II, $V(x)=V_O$. Assume the same form of the solution
$$\Psi(x,t)=u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$
Substituting into Schrodinger's wave equation, we obtain
$$\frac{-\hbar^2}{2m}\left\{(jk)^2u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]+2jk\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]+\frac{\partial^2u(x)}{\partial x^2}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$
$$+V_Ou(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$
$$=Eu(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$
This equation can be written as
$$-k^2u(x)+2jk\frac{\partial u(x)}{\partial x}+\frac{\partial^2u(x)}{\partial x^2}-\frac{2mV_O}{\hbar^2}u(x)+\frac{2mE}{\hbar^2}u(x)=0$$
Setting $u(x)=u_2(x)$ for region II, this equation becomes
$$\frac{d^2u_2(x)}{dx^2}+2jk\frac{du_2(x)}{dx}-\left(k^2-\alpha^2+\frac{2mV_O}{\hbar^2}\right)u_2(x)=0$$
where
$$\alpha^2=\frac{2mE}{\hbar^2}$$
Q.E.D.

3.3
We have

$$\frac{d^2u_1(x)}{dx^2}+2jk\frac{du_1(x)}{dx}-\left(k^2-\alpha^2\right)u_1(x)=0$$

The proposed solution is

$$u_1(x)=A\exp\left[j(\alpha-k)x\right]+B\exp\left[-j(\alpha+k)x\right]$$

The first derivative is

$$\frac{du_1(x)}{dx}=j(\alpha-k)A\exp\left[j(\alpha-k)x\right]$$
$$-j(\alpha+k)B\exp\left[-j(\alpha+k)x\right]$$

and the second derivative becomes

$$\frac{d^2u_1(x)}{dx^2}=\left[j(\alpha-k)\right]^2A\exp\left[j(\alpha-k)x\right]$$
$$+\left[j(\alpha+k)\right]^2B\exp\left[-j(\alpha+k)x\right]$$

Substituting these equations into the differential equation, we find

$$-(\alpha-k)^2A\exp\left[j(\alpha-k)x\right]$$
$$-(\alpha+k)^2B\exp\left[-j(\alpha+k)x\right]$$
$$+2jk\{j(\alpha-k)A\exp\left[j(\alpha-k)x\right]$$
$$-j(\alpha+k)B\exp\left[-j(\alpha+k)x\right]\}$$
$$-\left(k^2-\alpha^2\right)\{A\exp\left[j(\alpha-k)x\right]$$
$$+B\exp\left[-j(\alpha+k)x\right]\}=0$$

Combining terms, we have

$$\left\{-\left(\alpha^2-2\alpha k+k^2\right)-2k(\alpha-k)\right.$$
$$\left.-\left(k^2-\alpha^2\right)\right\}A\exp\left[j(\alpha-k)x\right]$$
$$+\left\{-\left(\alpha^2+2\alpha k+k^2\right)+2k(\alpha+k)\right.$$
$$\left.-\left(k^2-\alpha^2\right)\right\}B\exp\left[-j(\alpha+k)x\right]=0$$

We find that

$0=0$ Q.E.D.

For the differential equation in $u_2(x)$ and the proposed solution, the procedure is exactly the same as above.

3.4
We have the solutions

$$u_1(x)=A\exp\left[j(\alpha-k)x\right]+B\exp\left[-j(\alpha+k)x\right]$$

for $0<x<a$

$$u_2(x)=C\exp\left[j(\beta-k)x\right]+D\exp\left[-j(\beta+k)x\right]$$

for $-b<x<0$

The boundary conditions:

$$u_1(0)=u_2(0)$$

which yields

$$A+B-C-D=0$$

Also

$$\left.\frac{du_1}{dx}\right|_{x=0}=\left.\frac{du_2}{dx}\right|_{x=0}$$

which yields

$$(\alpha-k)A-(\alpha+k)B-(\beta-k)C+(\beta+k)D=0$$

The third boundary condition is

$$u_1(a)=u_2(-b)$$

which gives

$$A\exp\left[j(\alpha-k)a\right]+B\exp\left[-j(\alpha+k)a\right]$$
$$=C\exp\left[j(\beta-k)(-b)\right]+D\exp\left[-j(\beta+k)(-b)\right]$$

This becomes

$$A\exp\left[j(\alpha-k)a\right]+B\exp\left[-j(\alpha+k)a\right]$$
$$-C\exp\left[-j(\beta-k)b\right]-D\exp\left[j(\beta+k)b\right]=0$$

The last boundary condition is

$$\left.\frac{du_1}{dx}\right|_{x=a}=\left.\frac{du_2}{dx}\right|_{x=-b}$$

which gives

$$j(\alpha-k)A\exp\left[j(\alpha-k)a\right]$$
$$-j(\alpha+k)B\exp\left[-j(\alpha+k)a\right]$$
$$=j(\beta-k)C\exp\left[j(\beta-k)(-b)\right]$$
$$-j(\beta+k)D\exp\left[-j(\beta+k)(-b)\right]$$

This becomes

$$(\alpha-k)A\exp\left[j(\alpha-k)a\right]$$
$$-(\alpha+k)B\exp\left[-j(\alpha+k)a\right]$$
$$-(\beta-k)C\exp\left[-j(\beta-k)b\right]$$
$$+(\beta+k)D\exp\left[j(\beta+k)b\right]=0$$

3.5 Computer plot

3.6 Computer plot

3.7

$$P'\frac{\sin\alpha a}{\alpha a}+\cos\alpha a=\cos ka$$

Let $ka=y$, $\alpha a=x$

Then

$$P'\frac{\sin x}{x}+\cos x=\cos y$$

Consider $\frac{d}{dy}$ of this function

$$\frac{d}{dy}\left\{\left[P'\cdot(x)^{-1}\cdot\sin x\right]+\cos x\right\}=-\sin y$$

We obtain

$$P'\left\{(-1)(x)^{-2}\sin x\frac{dx}{dy}+(x)^{-1}\cos x\frac{dx}{dy}\right\}$$

$$-\sin x\frac{dx}{dy}=-\sin y$$

Then

$$\frac{dx}{dy}\left\{P'\left[\frac{-1}{x^2}\sin x+\frac{\cos x}{x}\right]-\sin x\right\}=-\sin y$$

For $y=ka=n\pi$, $n=0,1,2,\ldots$

$$\Rightarrow \sin y=0$$

So that, in general, then

$$\frac{dx}{dy}=0=\frac{d(\alpha a)}{d(ka)}=\frac{d\alpha}{dk}$$

And

$$\alpha=\sqrt{\frac{2mE}{\hbar^2}}\Rightarrow\frac{d\alpha}{dk}=\frac{1}{2}\left(\frac{2mE}{\hbar^2}\right)^{-1/2}\left(\frac{2m}{\hbar^2}\right)\frac{dE}{dk}$$

This implies that

$$\underline{\frac{d\alpha}{dk}=0=\frac{dE}{dk}}\text{ for }\underline{k=\frac{n\pi}{a}}$$

3.8

$$f(\alpha a)=9\frac{\sin\alpha a}{\alpha a}+\cos\alpha a=\cos ka$$

(a) $ka=\pi\Rightarrow\cos ka=-1$

1st point: $\alpha a=\pi$: 2nd point: $\alpha a=1.66\pi$

(2nd point by trial and error)

Now

$$\alpha a=a\sqrt{\frac{2mE}{\hbar^2}}\Rightarrow E=\left(\frac{\alpha a}{a}\right)^2\cdot\frac{\hbar^2}{2m}$$

So

$$E=\frac{(\alpha a)^2}{\left(5x10^{-10}\right)^2}\cdot\frac{\left(1.054x10^{-34}\right)^2}{2\left(9.11x10^{-31}\right)}\Rightarrow$$

$$E=(\alpha a)^2\left[2.439x10^{-20}\right]\ (J)$$

or

$$E=(\alpha a)^2(0.1524)\quad(eV)$$

So

$$\alpha a=\pi\Rightarrow E_1=1.504\ eV$$

$$\alpha a=1.66\pi\Rightarrow E_2=4.145\ eV$$

Then

$$\underline{\Delta E=2.64\ eV}$$

(b)
$ka=2\pi\Rightarrow\cos ka=+1$

1st point: $\alpha a=2\pi$

2nd point: $\alpha a=2.54\pi$

Then

$$E_3=6.0165\ eV$$

$$E_4=9.704\ eV$$

so

$$\underline{\Delta E=3.69\ eV}$$

(c)
$ka=3\pi\Rightarrow\cos ka=-1$

1st point: $\alpha a=3\pi$

2nd point: $\alpha a=3.44\pi$

Then

$$E_5=13.537\ eV$$

$$E_6=17.799\ eV$$

so

$$\underline{\Delta E=4.26\ eV}$$

(d)
$ka=4\pi\Rightarrow\cos ka=+1$

1st point: $\alpha a=4\pi$

2nd point: $\alpha a=4.37\pi$

Then

$$E_7=24.066\ eV$$

$$E_8=28.724\ eV$$

so

$$\underline{\Delta E=4.66\ eV}$$

3.9

(a) $0<ka<\pi$

For $ka=0\Rightarrow\cos ka=+1$

By trial and error: 1st point: $\alpha a=0.822\pi$

2nd point: $\alpha a=\pi$

From Problem 3.8, $E=(\alpha a)^2(0.1524)\quad(eV)$

Then

$$E_1=1.0163\ eV$$

$$E_2=1.5041\ eV$$

so

$$\underline{\Delta E=0.488\ eV}$$

(b)
$\pi<ka<2\pi$

Using results of Problem 3.8

1st point: $\alpha a=1.66\pi$

2nd point: $\alpha a=2\pi$

Then

$E_3 = 4.145\ eV$

$E_4 = 6.0165\ eV$

so

$\underline{\Delta E = 1.87\ eV}$

(c)
$2\pi < ka < 3\pi$

1st point: $\alpha a = 2.54\pi$

2nd point: $\alpha a = 3\pi$

Then

$E_5 = 9.704\ eV$

$E_6 = 13.537\ eV$

so

$\underline{\Delta E = 3.83\ eV}$

(d)
$3\pi < ka < 4\pi$

1st point: $\alpha a = 3.44\pi$

2nd point: $\alpha a = 4\pi$

Then

$E_7 = 17.799\ eV$

$E_8 = 24.066\ eV$

so

$\underline{\Delta E = 6.27\ eV}$

3.10

$$6\frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$$

Forbidden energy bands

(a) $ka = \pi \Rightarrow \cos ka = -1$

1st point: $\alpha a = \pi$

2nd point: $\alpha a = 1.56\pi$ (By trial and error)

From Problem 3.8, $E = (\alpha a)^2 (0.1524)\ eV$

Then

$E_1 = 1.504\ eV$

$E_2 = 3.660\ eV$

so

$\underline{\Delta E = 2.16\ eV}$

(b)
$ka = 2\pi \Rightarrow \cos ka = +1$

1st point: $\alpha a = 2\pi$

2nd point: $\alpha a = 2.42\pi$

Then

$E_3 = 6.0165\ eV$

$E_4 = 8.809\ eV$

so

$\underline{\Delta E = 2.79\ eV}$

(c)
$ka = 3\pi \Rightarrow \cos ka = -1$

1st point: $\alpha a = 3\pi$

2nd point: $\alpha a = 3.33\pi$

Then

$E_5 = 13.537\ eV$

$E_6 = 16.679\ eV$

so

$\underline{\Delta E = 3.14\ eV}$

(d)
$ka = 4\pi \Rightarrow \cos ka = +1$

1st point: $\alpha a = 4\pi$

2nd point: $\alpha a = 4.26\pi$

Then

$E_7 = 24.066\ eV$

$E_8 = 27.296\ eV$

so

$\underline{\Delta E = 3.23\ eV}$

3.11

Allowed energy bands

Use results from Problem 3.10.

(a)

$0 < ka < \pi$

1st point: $\alpha a = 0.759\pi$ (By trial and error)

2nd point: $\alpha a = \pi$

We have

$$E = (\alpha a)^2 (0.1524)\ eV$$

Then

$E_1 = 0.8665\ eV$

$E_2 = 1.504\ eV$

so

$\underline{\Delta E = 0.638\ eV}$

(b)

$\pi < ka < 2\pi$

1st point: $\alpha a = 1.56\pi$

2nd point: $\alpha a = 2\pi$

Then

$E_3 = 3.660\ eV$

$E_4 = 6.0165\ eV$

so

$\underline{\Delta E = 2.36\ eV}$

(c)

$2\pi < ka < 3\pi$

1st point: $\alpha a = 2.42\pi$

2nd point: $\alpha a = 3\pi$

Then

$E_5 = 8.809\ eV$

$E_6 = 13.537\ eV$

so

$$\underline{\Delta E = 4.73\ eV}$$

(d)

$3\pi < ka < 4\pi$

1st point: $\alpha a = 3.33\pi$

2nd point: $\alpha a = 4\pi$

Then

$E_7 = 16.679\ eV$

$E_8 = 24.066\ eV$

so

$$\underline{\Delta E = 7.39\ eV}$$

3.12

$$T = 100K\ ;\ \ E_g = 1.170 - \frac{\left(4.73x10^{-4}\right)(100)^2}{636+100} \Rightarrow$$

$$\underline{E_g = 1.164\ eV}$$

$$T = 200K \Rightarrow \underline{E_g = 1.147\ eV}$$

$$T = 300K \Rightarrow \underline{E_g = 1.125\ eV}$$

$$T = 400K \Rightarrow \underline{E_g = 1.097\ eV}$$

$$T = 500K \Rightarrow \underline{E_g = 1.066\ eV}$$

$$T = 600K \Rightarrow \underline{E_g = 1.032\ eV}$$

3.13

The effective mass is given by

$$m^* = \left(\frac{1}{\hbar^2}\cdot\frac{d^2E}{dk^2}\right)^{-1}$$

We have that

$$\frac{d^2E}{dk^2}(curve\ A) > \frac{d^2E}{dk^2}(curve\ B)$$

so that

$$\underline{m^*(curve\ A) < m^*(curve\ B)}$$

3.14

The effective mass for a hole is given by

$$m_p^* = \left(\frac{1}{\hbar^2}\cdot\left|\frac{d^2E}{dk^2}\right|\right)^{-1}$$

We have that

$$\left|\frac{d^2E}{dk^2}\right|(curve\ A) > \left|\frac{d^2E}{dk^2}\right|(curve\ B)$$

so that

$$\underline{m_p^*(curve\ A) < m_p^*(curve\ B)}$$

3.15

Points A, B: $\dfrac{\partial E}{\partial k} < 0 \Rightarrow$ velocity in –x direction;

Points C, D: $\dfrac{\partial E}{\partial x} > 0 \Rightarrow$ velocity in +x direction;

Points A, D; $\dfrac{\partial^2 E}{\partial k^2} < 0 \Rightarrow$ negative effective mass;

Points B, C; $\dfrac{\partial^2 E}{\partial k^2} > 0 \Rightarrow$ positive effective mass;

3.16

$$E - E_C = \frac{k^2\hbar^2}{2m}$$

At $k = 0.1\left(\overset{\circ}{A}\right)^{-1} \Rightarrow \dfrac{1}{k} = 10\ \overset{\circ}{A} = 10^{-9}\ m$

So

$k = 10^{+9}\ m^{-1}$

For A:

$$(0.07)\left(1.6x10^{-19}\right) = \frac{\left(10^9\right)^2\left(1.054x10^{-34}\right)^2}{2m}$$

which yields

$$m = 4.96x10^{-31}\ kg$$

so

$$\underline{curve\ A;\ \frac{m}{m_o} = 0.544}$$

For B:

$$(0.7)\left(1.6x10^{-19}\right) = \frac{\left(10^9\right)\left(1.054x10^{-34}\right)^2}{2m}$$

which yields

$m = 4.96x10^{-32}\ kg$

so

$$\underline{Curve\ B:\ \frac{m}{m_o} = 0.0544}$$

3.17

$$E_V - E = \frac{k^2\hbar^2}{2m}$$

$k = 0.1\left(A^{*}\right)^{-1} \Rightarrow 10^{9}\ m^{-1}$

For Curve A:

$$(0.08)\left(1.6x10^{-19}\right) = \frac{\left(10^{9}\right)^{2}\left(1.054x10^{-34}\right)^{2}}{2m}$$

which yields

$$m = 4.34x10^{-31}\ kg \Rightarrow \underline{\frac{m}{m_o} = 0.476}$$

For Curve B:

$$(0.4)\left(1.6x10^{-19}\right) = \frac{\left(10^{9}\right)^{2}\left(1.054x10^{-34}\right)^{2}}{2m}$$

which yields

$$m = 8.68x10^{-32}\ kg \Rightarrow \underline{\frac{m}{m_o} = 0.0953}$$

3.18

(a) $E = h\nu$

Then

$$\nu = \frac{E}{h} = \frac{(1.42)\left(1.6x10^{-19}\right)}{\left(6.625x10^{-34}\right)} \Rightarrow$$

$$\underline{\nu = 3.43x10^{14}\ Hz}$$

(b)

$$\lambda = \frac{c}{\nu} = \frac{3x10^{8}}{3.43x10^{14}} = 8.75x10^{-7}\ m$$

or

$$\underline{\lambda = 0.875\ \mu m}$$

3.19

(c) Curve A: Effective mass is a constant
Curve B: Effective mass is positive around $k = 0$, and is negative around $k = \pm\dfrac{\pi}{2}$.

3.20

$$E = E_O - E_1 \cos\left[\alpha\left(k - k_O\right)\right]$$

$$\frac{dE}{dk} = \left(-E_1\right)(-\alpha)\sin\left[\alpha\left(k - k_O\right)\right]$$

$$= +E_1\alpha\sin\left[\alpha\left(k - k_O\right)\right]$$

So

$$\frac{d^2E}{dk^2} = E_1\alpha^2\cos\left[\alpha\left(k - k_O\right)\right]$$

Then

$$\left.\frac{d^2E}{dk^2}\right|_{k=k_O} = E_1\alpha^2$$

We have

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\cdot\frac{d^2E}{dk^2} = \frac{E_1\alpha^2}{\hbar^2}$$

or

$$\underline{m^* = \frac{\hbar^2}{E_1\alpha^2}}$$

3.21

For the 3-dimensional infinite potential well, $V(x) = 0$ when $0 < x < a$, $0 < y < a$, and $0 < z < a$. In this region, the wave equation is

$$\frac{\partial^2\psi(x,y,z)}{\partial x^2} + \frac{\partial^2\psi(x,y,z)}{\partial y^2} + \frac{\partial^2\psi(x,y,z)}{\partial z^2} + \frac{2mE}{\hbar^2}\psi(x,y,z) = 0$$

Use separation of variables technique, so let

$$\psi(x,y,z) = X(x)Y(y)Z(z)$$

Substituting into the wave equation, we have

$$YZ\frac{\partial^2X}{\partial x^2} + XZ\frac{\partial^2Y}{\partial y^2} + XY\frac{\partial^2Z}{\partial z^2} + \frac{2mE}{\hbar^2}\cdot XYZ = 0$$

Dividing by XYZ, we obtain

$$\frac{1}{X}\cdot\frac{\partial^2X}{\partial x^2} + \frac{1}{Y}\cdot\frac{\partial^2Y}{\partial y^2} + \frac{1}{Z}\cdot\frac{\partial^2Z}{\partial z^2} + \frac{2mE}{\hbar^2} = 0$$

Let

$$\frac{1}{X}\cdot\frac{\partial^2X}{\partial x^2} = -k_x^2 \Rightarrow \frac{\partial^2X}{\partial x^2} + k_x^2X = 0$$

The solution is of the form

$$X(x) = A\sin k_xx + B\cos k_xx$$

Since $\psi(x,y,z) = 0$ at $x = 0$, then $X(0) = 0$ so that $B \equiv 0$.

Also, $\psi(x,y,z) = 0$ at $x = a$, then $X(a) = 0$ so we must have $k_xa = n_x\pi$, where $n_x = 1, 2, 3, \ldots$

Similarly, we have

$$\frac{1}{Y}\cdot\frac{\partial^2Y}{\partial y^2} = -k_y^2 \quad\text{and}\quad \frac{1}{Z}\cdot\frac{\partial^2Z}{\partial z^2} = -k_z^2$$

From the boundary conditions, we find

$k_y a = n_y \pi$ and $k_z a = n_z \pi$

where $n_y = 1, 2, 3, \ldots$ and $n_z = 1, 2, 3, \ldots$

From the wave equation, we have

$$-k_x^2 - k_y^2 - k_z^2 + \frac{2mE}{\hbar^2} = 0$$

The energy can then be written as

$$\underline{E = \frac{\hbar^2}{2m}\left(n_x^2 + n_y^2 + n_z^2\right)\left(\frac{\pi}{a}\right)^2}$$

3.22

The total number of quantum states in the 3-dimensional potential well is given (in k-space) by

$$g_T(k)dk = \frac{\pi k^2 dk}{\pi^3} \cdot a^3$$

where

$$k^2 = \frac{2mE}{\hbar^2}$$

We can then write

$$k = \frac{1}{\hbar} \cdot \sqrt{2mE}$$

Taking the differential, we obtain

$$dk = \frac{1}{\hbar} \cdot \sqrt{2m} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{E}} \cdot dE = \frac{1}{\hbar} \cdot \sqrt{\frac{m}{2E}} \cdot dE$$

Substituting these expressions into the density of states function, we obtain

$$g_T(E)dE = \frac{\pi a^3}{\pi^3}\left(\frac{2mE}{\hbar^2}\right) \cdot \frac{1}{\hbar} \cdot \sqrt{\frac{m}{2E}} \cdot dE$$

Noting that

$$\hbar = \frac{h}{2m}$$

this density of states function can be simplified and written as

$$g_T(E)dE = \frac{4\pi a^3}{h^3}(2m)^{3/2} \cdot \sqrt{E} \cdot dE$$

Dividing by a^3 will yield the density of states, so that

$$\underline{g(E) = \frac{4\pi(2m)^{3/2}}{h^3} \cdot \sqrt{E}}$$

3.23

$$g_C(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}$$

Now

$$g_T = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\int_{E_C}^{E_C+kT}\sqrt{E - E_C} \cdot dE$$

$$= \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)\left(E - E_C\right)^{3/2}\Big|_{E_C}^{E_C+kT}$$

$$= \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)(kT)^{3/2}$$

Then

$$g_T = \frac{4\pi\left[2(0.067)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(\frac{2}{3}\right)$$

$$\times\left[(0.0259)\left(1.6x10^{-19}\right)\right]^{3/2}$$

or

$$\underline{g_T = 3.28x10^{23}\ m^{-3} = 3.28x10^{17}\ cm^{-3}}$$

3.24

$$g_V(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_V - E}$$

Now

$$g_T = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\int_{E_V-kT}^{E_V}\sqrt{E_V - E} \cdot dE$$

$$= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left(E_V - E\right)^{3/2}\Big|_{E_V-kT}^{E_V}$$

$$= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)(kT)^{3/2}$$

$$g_T = \frac{4\pi\left[2(0.48)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(\frac{2}{3}\right)$$

$$\times\left[(0.0259)\left(1.6x10^{-19}\right)\right]^{3/2}$$

or

$$\underline{g_T = 6.29x10^{24}\ m^{-3} = 6.29x10^{18}\ cm^{-3}}$$

3.25

(a) $g_C(E) = \dfrac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}$

$$= \frac{4\pi\left[2(1.08)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(1.6x10^{-19}\right)^{1/2}\sqrt{E - E_C}$$

$$= 4.77x10^{46}\sqrt{E - E_C}\quad m^{-3}J^{-1}$$

or

$$g_C(E) = 7.63x10^{21}\sqrt{E-E_C}\ cm^{-3}eV^{-1}$$

Then

E	g_C
$E_C + 0.05\ eV$	$1.71x10^{21}\ cm^{-3}eV^{-1}$
$E_C + 0.10\ eV$	$2.41x10^{21}$
$E_C + 0.15\ eV$	$2.96x10^{21}$
$E_C + 0.20\ eV$	$3.41x10^{21}$

(b) $g_V(E) = \dfrac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_V-E}$

$$= \frac{4\pi\left[2(0.56)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(1.6x10^{-19}\right)^{1/2}\sqrt{E_V-E}$$

$$= 1.78x10^{46}\sqrt{E_V-E}\quad m^{-3}J^{-1}$$

$$g_V(E) = 2.85x10^{21}\sqrt{E_V-E}\ cm^{-3}eV^{-1}$$

E	$g_V(E)$
$E_V - 0.05\ eV$	$0.637x10^{21}\ cm^{-3}\ eV^{-1}$
$E_V - 0.10\ eV$	$0.901x10^{21}$
$E_V - 0.15\ eV$	$1.10x10^{21}$
$E_V - 0.20\ eV$	$1.27x10^{21}$

3.26

$$\frac{g_C}{g_V} = \frac{\left(m_n^*\right)^{3/2}}{\left(m_p^*\right)^{3/2}} \Rightarrow \underline{\frac{g_C}{g_V} = \left(\frac{m_n^*}{m_p^*}\right)^{3/2}}$$

3.27

Computer Plot

3.28

$$\frac{g_i!}{N_i!\left(g_i-N_i\right)!} = \frac{10!}{8!(10-8)!}$$

$$= \frac{(10)(9)(8!)}{(8!)(2!)} = \frac{(10)(9)}{(2)(1)} \Rightarrow \underline{= 45}$$

3.29

(a) $f(E) = \dfrac{1}{1+\exp\left[\dfrac{\left(E_C+kT\right)-E_C}{kT}\right]}$

$$= \frac{1}{1+\exp(1)} \Rightarrow \underline{f(E) = 0.269}$$

(b)

$$1-f(E) = 1-\frac{1}{1+\exp\left[\dfrac{\left(E_V-kT\right)-E_V}{kT}\right]}$$

$$= 1-\frac{1}{1+\exp(-1)} \Rightarrow \underline{1-f(E) = 0.269}$$

3.30

$$f(E) = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$$

(a) $E-E_F = kT$, $f(E) = \dfrac{1}{1+\exp(1)} \Rightarrow$

$\underline{f(E) = 0.269}$

(b) $E-E_F = 5kT$, $f(E) = \dfrac{1}{1+\exp(5)} \Rightarrow$

$\underline{f(E) = 6.69x10^{-3}}$

(c) $E-E_F = 10kT$, $f(E) = \dfrac{1}{1+\exp(10)} \Rightarrow$

$\underline{f(E) = 4.54x10^{-5}}$

3.31

$$1-f(E) = 1-\frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$$

or

$$1-f(E) = \frac{1}{1+\exp\left(\dfrac{E_F-E}{kT}\right)}$$

(a) $E_F-E = kT$, $\underline{1-f(E) = 0.269}$

(b) $E_F-E = 5kT$, $\underline{1-f(E) = 6.69x10^{-3}}$

(c) $E_F-E = 10kT$, $\underline{1-f(E) = 4.54x10^{-5}}$

3.32

(a) $T = 300K \Rightarrow kT = 0.0259\ eV$

$$f(E) = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)} \approx \exp\left[\frac{-\left(E-E_F\right)}{kT}\right]$$

E	$f(E)$
E_C	$6.43x10^{-5}$
$E_C+(1/2)kT$	$3.90x10^{-5}$
E_C+kT	$2.36x10^{-5}$
$E_C+(3/2)kT$	$1.43x10^{-5}$
E_C+2kT	$0.87x10^{-5}$

(b) $T = 400K \Rightarrow kT = 0.03453$

E	$f(E)$
E_C	$7.17x10^{-4}$
$E_C+(1/2)kT$	$4.35x10^{-4}$
E_C+kT	$2.64x10^{-4}$
$E_C+(3/2)kT$	$1.60x10^{-4}$
E_C+2kT	$0.971x10^{-4}$

3.33

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{\left(1.054x10^{-34}\right)^2 n^2\pi^2}{2\left(9.11x10^{-31}\right)\left(10x10^{-10}\right)^2}$$

or

$$E_n = 6.018x10^{-20} n^2 \; J = 0.376 n^2 \; eV$$

For $n = 4 \Rightarrow E_4 = 6.02\ eV$,

For $n = 5 \Rightarrow E_5 = 9.40\ eV$.

As a 1st approximation for $T > 0$, assume the probability of $n = 5$ state being occupied is the same as the probability of $n = 4$ state being empty. Then

$$1 - \frac{1}{1+\exp\left(\dfrac{E_4 - E_F}{kT}\right)} = \frac{1}{1+\exp\left(\dfrac{E_5 - E_F}{kT}\right)}$$

$$\Rightarrow \frac{1}{1+\exp\left(\dfrac{E_F - E_4}{kT}\right)} = \frac{1}{1+\exp\left(\dfrac{E_5 - E_F}{kT}\right)}$$

or

$$E_F - E_4 = E_5 - E_F \Rightarrow E_F = \frac{E_4 + E_5}{2}$$

Then

$$E_F = \frac{6.02 + 9.40}{2} \Rightarrow \underline{E_F = 7.71\ eV}$$

3.34

(a) For 3-Dimensional infinite potential well,

$$E = \frac{\hbar^2\pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

$$= \frac{\left(1.054x10^{-34}\right)^2\pi^2}{2\left(9.11x10^{-31}\right)\left(10^{-9}\right)^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

$$= 0.376\left(n_x^2 + n_y^2 + n_z^2\right)\ eV$$

For 5 electrons, energy state corresponding to $n_x n_y n_z = 221 = 122$ contains both an electron and an empty state, so

$$E_F = (0.376)\left(2^2 + 2^2 + 1^2\right) \Rightarrow$$

$$\underline{E_F = 3.384\ eV}$$

(b) For 13 electrons, energy state corresponding to $n_x n_y n_z = 323 = 233$ contains both an electron and an empty state, so

$$E_F = (0.376)\left(2^2 + 3^2 + 3^2\right) \Rightarrow$$

$$\underline{E_F = 8.272\ eV}$$

3.35

The probability of a state at $E_1 = E_F + \Delta E$ being occupied is

$$f_1(E_1) = \frac{1}{1+\exp\left(\dfrac{E_1 - E_F}{kT}\right)} = \frac{1}{1+\exp\left(\dfrac{\Delta E}{kT}\right)}$$

The probability of a state at $E_2 = E_F - \Delta E$ being empty is

$$1 - f_2(E_2) = 1 - \frac{1}{1+\exp\left(\dfrac{E_2 - E_F}{kT}\right)}$$

$$= 1 - \frac{1}{1+\exp\left(\dfrac{-\Delta E}{kT}\right)} = \frac{\exp\left(\dfrac{-\Delta E}{kT}\right)}{1+\exp\left(\dfrac{-\Delta E}{kT}\right)}$$

or

$$1 - f_2(E_2) = \frac{1}{1+\exp\left(\dfrac{+\Delta E}{kT}\right)}$$

Hence, we have that

$$f_1(E_1) = 1 - f_2(E_2) \quad \text{Q.E.D.}$$

3.36

(a) At energy E_1, we want

$$\frac{\dfrac{1}{\exp\left(\dfrac{E_1-E_F}{kT}\right)}-\dfrac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}}{\dfrac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}}=0.01$$

This expression can be written as

$$\frac{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}{\exp\left(\dfrac{E_1-E_F}{kT}\right)}-1=0.01$$

$$\Rightarrow 1=(0.01)\exp\left(\frac{E_1-E_F}{kT}\right)$$

or

$$E_1=E_F+kT\ln(100)$$

Then

$$\underline{E_1=E_F+4.6kT}$$

(b)

At $E_1=E_F+4.6kT$,

$$f(E_1)=\frac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}=\frac{1}{1+\exp\left(\dfrac{4.6kT}{kT}\right)}$$

which yields

$$\underline{f(E_1)=0.00990\approx 0.01}$$

3.37

(a) $E_F=6.25\ eV$, $T=300K$, At $E=6.50\ eV$

$$f(E)=\frac{1}{1+\exp\left(\dfrac{6.50-6.25}{0.0259}\right)}=6.43x10^{-5}$$

or

$$\underline{6.43x10^{-3}\%}$$

(b)

$$T=950K\Rightarrow kT=(0.0259)\left(\frac{950}{300}\right)$$

or

$$kT=0.0820\ eV$$

Then

$$f(E)=\frac{1}{1+\exp\left(\dfrac{6.50-6.25}{0.0820}\right)}=0.0453$$

or $\underline{4.53\%}$

(c) $$1-0.01=\frac{1}{1+\exp\left(\dfrac{-0.30}{kT}\right)}=0.99$$

Then

$$1+\exp\left(\frac{-0.30}{kT}\right)=\frac{1}{0.99}=1.0101$$

which can be written as

$$\exp\left(\frac{+0.30}{kT}\right)=\frac{1}{0.0101}=99$$

Then

$$\frac{0.30}{kT}=\ln(99)\Rightarrow kT=\frac{0.30}{\ln(99)}=0.06529$$

So

$$\underline{T=756K}$$

3.38

(a)

$$f(E)=\frac{1}{1+\exp\left(\dfrac{7.15-7.0}{0.0259}\right)}=0.00304$$

or

$$\underline{0.304\%}$$

(b)

At $T=1000K\Rightarrow kT=0.08633\ eV$

Then

$$f(E)=\frac{1}{1+\exp\left(\dfrac{7.15-7.0}{0.08633}\right)}=0.1496$$

or $\underline{14.96\%}$

(c)

$$f(E)=\frac{1}{1+\exp\left(\dfrac{6.85-7.0}{0.0259}\right)}=0.997$$

or

$$\underline{99.7\%}$$

(d)

At $E=E_F$, $\underline{f(E)=\frac{1}{2}}$ for all temperatures.

3.39

For $E=E_1$,

$$f(E)=\frac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}\approx\exp\left[\frac{-(E_1-E_F)}{kT}\right]$$

Then

$$f(E_1) = \exp\left(\frac{-0.30}{0.0259}\right) \Rightarrow \underline{f(E_1) = 9.3x10^{-6}}$$

For $E = E_2$, $E_F - E_2 = 1.12 - 0.3 = 0.82\ eV$

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\frac{-0.82}{0.0259}\right)}$$

or

$$1 - f(E) \approx 1 - \left[1 - \exp\left(\frac{-0.82}{0.0259}\right)\right]$$

$$= \exp\left(\frac{-0.82}{0.0259}\right) \Rightarrow \underline{1 - f(E) = 1.78x10^{-14}}$$

(b)
For $E_F - E_2 = 0.4 \Rightarrow E_1 - E_F = 0.72\ eV$
At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-0.72}{0.0259}\right)$$

so

$$\underline{f(E) = 8.45x10^{-13}}$$

At $E = E_2$,

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right] = \exp\left(\frac{-0.4}{0.0259}\right)$$

so

$$\underline{1 - f(E) = 1.96x10^{-7}}$$

3.40
(a) At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E - E_F)}{kT}\right] = \exp\left(\frac{-0.30}{0.0259}\right)$$

or

$$\underline{f(E) = 9.3x10^{-6}}$$

At $E = E_2$, then
$E_F - E_2 = 1.42 - 0.3 = 1.12\ eV$,
So

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right] = \exp\left(\frac{-1.12}{0.0259}\right)$$

or

$$\underline{1 - f(E) = 1.66x10^{-19}}$$

(b)
For $E_F - E_2 = 0.4 \Rightarrow E_1 - E_F = 1.02\ eV$,
At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-1.02}{0.0259}\right)$$

or

$$\underline{f(E) = 7.88x10^{-18}}$$

At $E = E_2$,

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right] = \exp\left(\frac{-0.4}{0.0259}\right)$$

or

$$\underline{1 - f(E) = 1.96x10^{-7}}$$

3.41

$$f(E) = \left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^{-1}$$

so

$$\frac{df(E)}{dE} = (-1)\left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^{-2} \times \left(\frac{1}{kT}\right)\exp\left(\frac{E - E_F}{kT}\right)$$

or

$$\frac{df(E)}{dE} = \frac{\frac{-1}{kT}\exp\left(\frac{E - E_F}{kT}\right)}{\left[1 + \exp\left(\frac{E - E_F}{kT}\right)\right]^2}$$

(a) $T = 0$, For

$$E < E_F \Rightarrow \exp(-\infty) = 0 \Rightarrow \frac{df}{dE} = 0$$

$$E > E_F \Rightarrow \exp(+\infty) = +\infty \Rightarrow \frac{df}{dE} = 0$$

$$\text{At } E = E_F \Rightarrow \frac{df}{dE} \to -\infty$$

3.42
(a) At $E = E_{midgap}$,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_g}{2kT}\right)}$$

Si: $E_g = 1.12\ eV$,

$$f(E) = \frac{1}{1 + \exp\left[\frac{1.12}{2(0.0259)}\right]}$$

or

$$\underline{f(E) = 4.07x10^{-10}}$$

Ge: $E_g = 0.66\ eV$,

$$f(E) = \frac{1}{1 + \exp\left[\dfrac{0.66}{2(0.0259)}\right]}$$

or

$$\underline{f(E) = 2.93x10^{-6}}$$

GaAs: $E_g = 1.42\ eV$,

$$f(E) = \frac{1}{1 + \exp\left[\dfrac{1.42}{2(0.0259)}\right]}$$

or

$$\underline{f(E) = 1.24x10^{-12}}$$

(b)
Using results of Problem 3.35, the answers to part (b) are exactly the same as those given in part (a).

3.43

$$f(E) = 10^{-6} = \frac{1}{1 + \exp\left(\dfrac{0.55}{kT}\right)}$$

Then

$$1 + \exp\left(\frac{0.55}{kT}\right) = \frac{1}{10^{-6}} = 10^{+6} \Rightarrow$$

$$\exp\left(\frac{0.55}{kT}\right) \approx 10^{+6} \Rightarrow \left(\frac{0.55}{kT}\right) = \ln\left(10^{6}\right)$$

or

$$kT = \frac{0.55}{\ln\left(10^{6}\right)} \Rightarrow \underline{T = 461K}$$

3.44
At $E = E_2$, $f(E_2) = 0.05$
So

$$0.05 = \frac{1}{1 + \exp\left(\dfrac{E_2 - E_F}{kT}\right)}$$

Then

$$\frac{E_2 - E_F}{kT} = \ln(19)$$

By symmetry, at $E = E_1$, $1 - f(E_1) = 0.05$,
So

$$\frac{E_F - E_1}{kT} = \ln(19)$$

Then

$$\frac{E_2 - E_1}{kT} = 2\ln(19)$$

(a)
At $T = 300K$, $kT = 0.0259\ eV$

$$E_2 - E_1 = \Delta E = (0.0259)(2)\ln(19) \Rightarrow$$
$$\underline{\Delta E = 0.1525\ eV}$$

(b)
At $T = 500K$, $kT = 0.04317\ eV$

$$E_2 - E_1 = \Delta E = (0.04317)(2)\ln(19) \Rightarrow$$
$$\underline{\Delta E = 0.254\ eV}$$

Chapter 4

Problem Solutions

4.1

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

(a) Silicon

$T(°K)$	$kT \quad (eV)$	$n_i\left(cm^{-3}\right)$
200	0.01727	$7.68x10^4$
400	0.03453	$2.38x10^{12}$
600	0.0518	$9.74x10^{14}$

(b) Germanium (c) GaAs

$T(°K)$	$n_i\left(cm^{-3}\right)$	$n_i\left(cm^{-3}\right)$
200	$2.16x10^{10}$	1.38
400	$8.60x10^{14}$	$3.28x10^9$
600	$3.82x10^{16}$	$5.72x10^{12}$

4.2

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

$$\left(10^{12}\right)^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{T}{300}\right)^3 \exp\left(\frac{-1.12}{kT}\right)$$

Then

$$\exp\left(\frac{1.12}{kT}\right) = \left(2.912x10^{14}\right)\left(\frac{T}{300}\right)^3$$

By trial and error

$$\underline{T = 381K}$$

4.3

Computer Plot

4.4

$$n_i^2 = N_{CO} N_{VO} \cdot (T)^3 \cdot \exp\left(\frac{-E_g}{kT}\right)$$

So

$$\frac{n_i^2(T_2)}{n_i^2(T_1)} = \left(\frac{T_2}{T_1}\right)^3 \exp\left[-E_g\left(\frac{1}{kT_2} - \frac{1}{kT_1}\right)\right]$$

At $T_2 = 300K \Rightarrow kT = 0.0259\ eV$

At $T_1 = 200K \Rightarrow kT = 0.01727\ eV$

Then

$$\left(\frac{5.83x10^7}{1.82x10^2}\right)^2 = \left(\frac{300}{200}\right)^3 \exp\left[-E_g\left(\frac{1}{0.0259} - \frac{1}{0.01727}\right)\right]$$

or

$$1.026x10^{11} = 3.375\exp\left[(19.29)E_g\right]$$

which yields

$$\underline{E_g = 1.25\ eV}$$

For $T = 300K$,

$$\left(5.83x10^7\right)^2 = \left(N_{CO}N_{VO}\right)(300)^3 \exp\left(\frac{-1.25}{0.0259}\right)$$

or

$$\underline{N_{CO}N_{VO} = 1.15x10^{29}}$$

4.5

(a) $g_C f_F \propto \sqrt{E - E_C}\, \exp\left[\frac{-(E - E_F)}{kT}\right]$

$$\propto \sqrt{E - E_C}\, \exp\left[\frac{-(E - E_C)}{kT}\right] \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

Let $E - E_C \equiv x$

Then

$$g_C f_F \propto \sqrt{x}\, \exp\left(\frac{-x}{kT}\right)$$

Now, to find the maximum value

$$\frac{d\left(g_C f_F\right)}{dx} \propto \frac{1}{2} x^{-1/2} \exp\left(\frac{-x}{kT}\right) - \frac{1}{kT} \cdot x^{1/2} \exp\left(\frac{-x}{kT}\right) = 0$$

This yields

$$\frac{1}{2x^{1/2}} = \frac{x^{1/2}}{kT} \Rightarrow x = \frac{kT}{2}$$

Then the maximum value occurs at

$$\underline{E = E_C + \frac{kT}{2}}$$

(b)

$$g_V\left(1 - f_F\right) \propto \sqrt{E_V - E}\, \exp\left[\frac{-(E_F - E)}{kT}\right]$$

$$\propto \sqrt{E_V - E}\, \exp\left[\frac{-(E_F - E_V)}{kT}\right] \exp\left[\frac{-(E_V - E)}{kT}\right]$$

Let $E_V - E \equiv x$

Then

$$g_V(1-f_F) \propto \sqrt{x}\exp\left(\frac{-x}{kT}\right)$$

To find the maximum value

$$\frac{d\left[g_V(1-f_F)\right]}{dx} \propto \frac{d}{dx}\left[\sqrt{x}\exp\left(\frac{-x}{kT}\right)\right] = 0$$

Same as part (a). Maximum occurs at

$$x = \frac{kT}{2} = E_V - E$$

or

$$\underline{E = E_V - \frac{kT}{2}}$$

4.6

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{E_1 - E_C}\exp\left[\frac{-(E_1-E_C)}{kT}\right]}{\sqrt{E_2 - E_C}\exp\left[\frac{-(E_2-E_C)}{kT}\right]}$$

where

$$E_1 = E_C + 4kT \quad \text{and} \quad E_2 = E_C + \frac{kT}{2}$$

Then

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{4kT}}{\sqrt{\frac{kT}{2}}}\exp\left[\frac{-(E_1-E_2)}{kT}\right]$$

$$= 2\sqrt{2}\exp\left[-\left(4-\frac{1}{2}\right)\right] = 2\sqrt{2}\exp(-3.5)$$

or

$$\underline{\frac{n(E_1)}{n(E_2)} = 0.0854}$$

4.7
Computer Plot

4.8

$$\frac{n_i^2(A)}{n_i^2(B)} = \frac{\exp\left(\frac{-E_{gA}}{kT}\right)}{\exp\left(\frac{-E_{gB}}{kT}\right)} = \exp\left[\frac{-(E_{gA}-E_{gB})}{kT}\right]$$

or

$$\frac{n_i(A)}{n_i(B)} = \exp\left[\frac{-(E_{gA}-E_{gB})}{2kT}\right]$$

$$= \exp\left[\frac{-(1-1.2)}{2(0.0259)}\right] = \exp\left[\frac{+0.20}{2(0.0259)}\right]$$

or

$$\underline{\frac{n_i(A)}{n_i(B)} = 47.5}$$

4.9
Computer Plot

4.10

$$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Silicon: $m_p^* = 0.56m_O$, $m_n^* = 1.08m_O$

$$\underline{E_{Fi} - E_{midgap} = -0.0128\ eV}$$

Germanium: $m_p^* = 0.37m_O$, $m_n^* = 0.55m_O$

$$\underline{E_{Fi} - E_{midgap} = -0.0077\ eV}$$

Gallium Arsenide: $m_p^* = 0.48m_O$, $m_n^* = 0.067m_O$

$$\underline{E_{Fi} - E_{midgap} = +0.038\ eV}$$

4.11

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{3}{4}(0.0259)\ln\left(\frac{1.4}{0.62}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_{midgap} = +0.0158\ eV}$$

(b)

$$E_{Fi} - E_{midgap} = \frac{3}{4}(0.0259)\ln\left(\frac{0.25}{1.10}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_{midgap} = -0.0288\ eV}$$

4.12

$$E_{Fi} - E_{midgap} = \frac{1}{2}(kT)\ln\left(\frac{N_V}{N_C}\right)$$

$$= \frac{1}{2}(kT)\ln\left(\frac{1.04x10^{19}}{2.8x10^{19}}\right) = -0.495(kT)$$

$T(°K)$	$kT\ (eV)$	$E_{Fi} - E_{midgap}\ (eV)$
200	0.01727	−0.0085
400	0.03453	−0.017
600	0.0518	−0.0256

4.13
Computer Plot

4.14
Let $g_C(E) = K = \text{constant}$
Then,

$$n_O = \int_{E_C}^{\infty} g_C(E) f_F(E) dE$$

$$= K \int_{E_C}^{\infty} \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

$$\approx K \int_{E_C}^{\infty} \exp\left[\frac{-(E-E_F)}{kT}\right] dE$$

Let

$$\eta = \frac{E-E_F}{kT} \quad \text{so that} \quad dE = kT \cdot d\eta$$

We can write

$$E - E_F = (E_C - E_F) - (E_C - E)$$

so that

$$\exp\left[\frac{-(E-E_F)}{kT}\right] = \exp\left[\frac{-(E_C-E_F)}{kT}\right] \cdot \exp(-\eta)$$

The integral can then be written as

$$n_O = K \cdot kT \cdot \exp\left[\frac{-(E_C-E_F)}{kT}\right] \int_0^{\infty} \exp(-\eta) d\eta$$

which becomes

$$n_O = K \cdot kT \cdot \exp\left[\frac{-(E_C-E_F)}{kT}\right]$$

4.15
Let $g_C(E) = C_1(E - E_C)$ for $E \ge E_C$

$$n_O = \int_{E_C}^{\infty} g_C(E) f_F(E) dE$$

$$= C_1 \int_{E_C}^{\infty} \frac{(E-E_C)}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

or

$$n_O \approx C_1 \int_{E_C}^{\infty} (E-E_C) \exp\left[\frac{-(E-E_F)}{kT}\right] dE$$

Let

$$\eta = \frac{E-E_C}{kT} \quad \text{so that} \quad dE = kT \cdot d\eta$$

We can write

$$(E - E_F) = (E - E_C) + (E_C - E_F)$$

Then

$$n_O = C_1 \exp\left[\frac{-(E_C-E_F)}{kT}\right] \times \int_{E_C}^{\infty} (E-E_C) \exp\left[\frac{-(E-E_C)}{kT}\right] dE$$

or

$$= C_1 \exp\left[\frac{-(E_C-E_F)}{kT}\right] \times \int_0^{\infty} (kT)\eta[\exp(-\eta)](kT) d\eta$$

We find that

$$\int_0^{\infty} \eta \exp(-\eta) d\eta = \frac{e^{-\eta}}{1}(-\eta - 1)\Big|_0^{\infty} = +1$$

So

$$n_O = C_1 (kT)^2 \exp\left[\frac{-(E_C-E_F)}{kT}\right]$$

4.16
We have $\frac{r_1}{a_O} = \in_r \left(\frac{m_O}{m^*}\right)$

For Germanium, $\in_r = 16$, $m^* = 0.55 m_O$
Then

$$r_1 = (16)\left(\frac{1}{0.55}\right) a_O = 29(0.53)$$

so

$$r_1 = 15.4\ \mathring{A}$$

The ionization energy can be written as

$$E = \left(\frac{m^*}{m_O}\right)\left(\frac{\in_O}{\in_S}\right)^2 (13.6) \quad eV$$

$$= \frac{0.55}{(16)^2}(13.6) \Rightarrow \underline{E = 0.029\ eV}$$

4.17

We have $\frac{r_1}{a_O} = \in_r \left(\frac{m_O}{m^*}\right)$

For GaAs, $\in_r = 13.1$, $m^* = 0.067 m_O$

Then

$$r_1 = (13.1)\left(\frac{1}{0.067}\right)(0.53)$$

or

$$\underline{r_1 = 104\ \overset{\circ}{A}}$$

The ionization energy is

$$E = \left(\frac{m^*}{m_O}\right)\left(\frac{\in_O}{\in_S}\right)^2 (13.6) = \frac{0.067}{(13.1)^2}(13.6)$$

or

$$\underline{E = 0.0053\ eV}$$

4.18

(a) $p_O = \frac{n_i^2}{n_O} = \frac{\left(1.5x10^{10}\right)^2}{5x10^4} \Rightarrow$

$\underline{p_O = 4.5x10^{15}\ cm^{-3}}$, $p_O > n_O \Rightarrow$ p-type

(b)

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{4.5x10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.3266\ eV}$$

4.19

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$$= 1.04x10^{19} \exp\left(\frac{-0.22}{0.0259}\right)$$

so

$$\underline{p_O = 2.13x10^{15}\ cm^{-3}}$$

Assuming

$$E_C - E_F = 1.12 - 0.22 = 0.90\ eV$$

Then

$$n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$= 2.8x10^{18} \exp\left(\frac{-0.90}{0.0259}\right)$$

or

$$\underline{n_O = 2.27x10^4\ cm^{-3}}$$

4.20

(a) $T = 400K \Rightarrow kT = 0.03453\ eV$

$$N_C = 4.7x10^{17}\left(\frac{400}{300}\right)^{3/2} = 7.24x10^{17}\ cm^{-3}$$

Then

$$n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$= 7.24x10^{17} \exp\left(\frac{-0.25}{0.03453}\right)$$

or

$$\underline{n_O = 5.19x10^{14}\ cm^{-3}}$$

Also

$$N_V = 7x10^{18}\left(\frac{400}{300}\right)^{3/2} = 1.08x10^{19}\ cm^{-3}$$

and

$$E_F - E_V = 1.42 - 0.25 = 1.17\ eV$$

Then

$$p_O = 1.08x10^{19} \exp\left(\frac{-1.17}{0.03453}\right)$$

or

$$\underline{p_O = 2.08x10^4\ cm^{-3}}$$

(b)

$$E_C - E_F = kT \ln\left(\frac{N_C}{n_O}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{5.19x10^{14}}\right)$$

or $\underline{E_C - E_F = 0.176\ eV}$

Then

$$E_F - E_V = 1.42 - 0.176 = 1.244\ eV$$

and

$$p_O = \left(7x10^{18}\right)\exp\left(\frac{-1.244}{0.0259}\right)$$

or $\underline{p_O = 9.67x10^{-3}\ cm^{-3}}$

4.21

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

or

$$E_F - E_V = kT \ln\left(\frac{N_V}{p_O}\right)$$

$$= (0.0259)\ln\left(\frac{1.04x10^{19}}{10^{15}}\right) = 0.24\ eV$$

Then

$$E_C - E_F = 1.12 - 0.24 = 0.88\ eV$$

So

$$n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$= 2.8x10^{19} \exp\left(\frac{-0.88}{0.0259}\right)$$

or

$$\underline{n_O = 4.9x10^4\ cm^{-3}}$$

4.22

(a) $p_O = n_i \exp\left(\dfrac{E_{Fi} - E_F}{kT}\right)$

$$1.5x10^{10} \exp\left(\frac{0.35}{0.0259}\right)$$

or

$$\underline{p_O = 1.11x10^{16}\ cm^{-3}}$$

(b)

From Problem 4.1, $n_i(400K) = 2.38x10^{12}\ cm^{-3}$

$$kT = (0.0259)\left(\frac{400}{300}\right) = 0.03453\ eV$$

Then

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.03453)\ln\left(\frac{1.11x10^{16}}{2.38x10^{12}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.292\ eV}$$

(c)

From (a)

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{1.11x10^{16}}$$

or

$$\underline{n_O = 2.03x10^4\ cm^{-3}}$$

From (b)

$$n_O = \frac{n_i^2}{p_O} = \frac{(2.38x10^{12})^2}{1.11x10^{16}}$$

or

$$\underline{n_O = 5.10x10^8\ cm^{-3}}$$

4.23

(a) $p_O = n_i \exp\left(\dfrac{E_{Fi} - E_F}{kT}\right)$

$$= (1.8x10^6)\exp\left(\frac{0.35}{0.0259}\right)$$

or

$$\underline{p_O = 1.33x10^{12}\ cm^{-3}}$$

(b) From Problem 4.1,

$n_i(400K) = 3.28x10^9\ cm^{-3}$, $kT = 0.03453\ eV$

Then

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.03453)\ln\left(\frac{1.33x10^{12}}{3.28x10^9}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.207\ eV}$$

(c) From (a)

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.8x10^6)^2}{1.33x10^{12}}$$

or

$$\underline{n_O = 2.44\ cm^{-3}}$$

From (b)

$$n_O = \frac{(3.28x10^9)^2}{1.33x10^{12}}$$

or

$$\underline{n_O = 8.09x10^6\ cm^{-3}}$$

4.24

For silicon, $T = 300K$, $E_F = E_V$

$$\eta' = \frac{E_V - E_F}{kT} = 0 \Rightarrow F_{1/2}(\eta') = 0.60$$

We can write

$$p_O = \frac{2}{\sqrt{\pi}} N_V F_{1/2}(\eta') = \frac{2}{\sqrt{\pi}}\left(1.04x10^{19}\right)(0.60)$$

or

$$\underline{p_O = 7.04x10^{18}\ cm^{-3}}$$

4.25

Silicon, $T = 300K$, $n_O = 5x10^{19}\ cm^{-3}$

We have

$$n_O = \frac{2}{\sqrt{\pi}} N_C F_{1/2}(\eta_F)$$

or

$$5x10^{19} = \frac{2}{\sqrt{\pi}}\left(2.8x10^{19}\right)F_{1/2}(\eta_F)$$

which gives

$$F_{1/2}(\eta_F) = 1.58$$

Then

$$\eta_F = 1.3 = \frac{E_F - E_C}{kT}$$

or $E_F - E_C = (1.3)(0.0259) \Rightarrow$

$$\underline{E_C - E_F = -0.034\ eV}$$

4.26

For the electron concentration

$$n(E) = g_C(E) f_F(E)$$

The Boltzmann approximation applies so

$$n(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}\exp\left[\frac{-(E - E_F)}{kT}\right]$$

or

$$n(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$\times\sqrt{kT}\sqrt{\frac{E - E_C}{kT}}\exp\left[\frac{-(E - E_C)}{kT}\right]$$

Define

$$x = \frac{E - E_C}{kT}$$

Then

$$n(E) \to n(x) = K\sqrt{x}\exp(-x)$$

To find maximum $n(E) \to n(x)$, set

$$\frac{dn(x)}{dx} = 0 = K\left[\frac{1}{2}x^{-1/2}\exp(-x) + x^{1/2}(-1)\exp(-x)\right]$$

or

$$0 = Kx^{-1/2}\exp(-x)\left[\frac{1}{2} - x\right]$$

which yields

$$x = \frac{1}{2} = \frac{E - E_C}{kT} \Rightarrow \quad \underline{E = E_C + \frac{1}{2}kT}$$

For the hole concentration

$$p(E) = g_V(E)\left[1 - f_F(E)\right]$$

From the text, using the Maxwell-Boltzmann approximation, we can write

$$p(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_V - E}\exp\left[\frac{-(E_F - E)}{kT}\right]$$

or

$$p(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$$\times\sqrt{kT}\sqrt{\frac{E_V - E}{kT}}\exp\left[\frac{-(E_V - E)}{kT}\right]$$

Define $x' = \dfrac{E_V - E}{kT}$

Then

$$p(x') = K'\sqrt{x'}\exp(-x')$$

To find the maximum of $p(E) \to p(x')$, set $\dfrac{dp(x')}{dx'} = 0$. Using the results from above, we find the maximum at

$$\underline{E = E_V - \frac{1}{2}kT}$$

4.27

(a) Silicon: We have

$$n_O = N_C\exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

We can write

$$E_C - E_F = (E_C - E_d) + (E_d - E_F)$$

For

$$E_C - E_d = 0.045\ eV,\ E_d - E_F = 3kT$$

$$n_O = \left(2.8x10^{19}\right)\exp\left[\frac{-0.045}{0.0259} - 3\right]$$

$$= \left(2.8x10^{19}\right)\exp(-4.737)$$

or

$$\underline{n_O = 2.45x10^{17}\ cm^{-3}}$$

We also have

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

Again, we can write

$$E_F - E_V = (E_F - E_a) + (E_a - E_V)$$

For

$$E_F - E_a = 3kT\,,\ E_a - E_V = 0.045\ eV$$

Then

$$p_O = (1.04x10^{19})\exp\left[-3 - \frac{0.045}{0.0259}\right]$$

$$= (1.04x10^{19})\exp(-4.737)$$

or

$$\underline{p_O = 9.12x10^{16}\ cm^{-3}}$$

(b)
GaAs: Assume $E_C - E_d = 0.0058\ eV$
Then

$$n_O = (4.7x10^{17})\exp\left[\frac{-0.0058}{0.0259} - 3\right]$$

$$= (4.7x10^{17})\exp(-3.224)$$

or

$$\underline{n_O = 1.87x10^{16}\ cm^{-3}}$$

Assume $E_a - E_V = 0.0345\ eV$
Then

$$p_O = (7x10^{18})\exp\left[\frac{-0.0345}{0.0259} - 3\right]$$

$$= (7x10^{18})\exp(-4.332)$$

or

$$\underline{p_O = 9.20x10^{16}\ cm^{-3}}$$

4.28
Computer Plot

4.29
(a) Ge:

$$p_O = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

Then

$$p_O = \frac{10^{13}}{2} + \sqrt{\left(\frac{10^{13}}{2}\right)^2 + (2.4x10^{13})^2}$$

or

$$\underline{p_O = 2.95x10^{13}\ cm^{-3}}$$

and

$$n_O = \frac{n_i^2}{p_O} = \frac{(2.4x10^{13})^2}{2.95x10^{13}} \Rightarrow$$

$$\underline{n_O = 1.95x10^{13}\ cm^{-3}}$$

(b)

$$n_O = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

Then

$$n_O = \frac{5x10^{15}}{2} + \sqrt{\left(\frac{5x10^{15}}{2}\right)^2 + (2.4x10^{13})^2}$$

or

$$\underline{n_O \approx 5x10^{15}\ cm^{-3}}$$

and

$$p_O = \frac{n_i^2}{n_O} = \frac{(2.4x10^{13})^2}{5x10^{15}} \Rightarrow$$

$$\underline{p_O = 1.15x10^{11}\ cm^{-3}}$$

4.30
For the donor level

$$\frac{n_d}{N_d} = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$= \frac{1}{1 + \frac{1}{2}\exp\left(\frac{0.20}{0.0259}\right)}$$

or

$$\underline{\frac{n_d}{N_d} = 8.85x10^{-4}}$$

And

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Now

$$E - E_F = (E - E_C) + (E_C - E_F)$$

or

$$E - E_F = kT + 0.245$$

Then

$$f_F(E) = \frac{1}{1 + \exp\left(1 + \frac{0.245}{0.0259}\right)} \Rightarrow$$

$$\underline{f_F(E) = 2.87x10^{-5}}$$

4.31

(a) $n_O = N_d = 2x10^{15}\ cm^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.5x10^{10}\right)^2}{2x10^{15}} \Rightarrow$$

$p_O = 1.125x10^5\ cm^{-3}$

(b)

$p_O = N_a = 10^{16}\ cm^{-3}$

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} \Rightarrow$$

$n_O = 2.25x10^4\ cm^{-3}$

(c)

$n_O = p_O = n_i = 1.5x10^{10}\ cm^{-3}$

(d)

$T = 400K \Rightarrow kT = 0.03453\ eV$

$$n_i^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{400}{300}\right)^3 \exp\left(\frac{-1.12}{0.03453}\right)$$

or

$n_i = 2.38x10^{12}\ cm^{-3}$

$$p_O = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$= 5x10^{13} + \sqrt{\left(5x10^{13}\right)^2 + \left(2.38x10^{12}\right)^2}$$

or

$p_O = 1.0x10^{14}\ cm^{-3}$

Also

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(2.38x10^{12}\right)^2}{10^{14}} \Rightarrow$$

$n_O = 5.66x10^{10}\ cm^{-3}$

(e)

$T = 500K \Rightarrow kT = 0.04317\ eV$

$$n_i^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{500}{300}\right)^3 \exp\left(\frac{-1.12}{0.04317}\right)$$

or

$n_i = 8.54x10^{13}\ cm^{-3}$

Now

$$n_O = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= 5x10^{13} + \sqrt{\left(5x10^{13}\right)^2 + \left(8.54x10^{13}\right)^2}$$

or

$n_O = 1.49x10^{14}\ cm^{-3}$

Also

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(8.54x10^{13}\right)^2}{1.49x10^{14}} \Rightarrow$$

$p_O = 4.89x10^{13}\ cm^{-3}$

4.32

(a) $n_O = N_d = 2x10^{15}\ cm^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.8x10^{6}\right)^2}{2x10^{15}} \Rightarrow$$

$p_O = 1.62x10^{-3}\ cm^{-3}$

(b)

$p_O = N_a = 10^{16}\ cm^{-3}$

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.8x10^{6}\right)^2}{10^{16}} \Rightarrow$$

$n_O = 3.24x10^{-4}\ cm^{-3}$

(c)

$n_O = p_O = n_i = 1.8x10^6\ cm^{-3}$

(d)

$kT = 0.03453\ eV$

$$n_i^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{400}{300}\right)^3 \exp\left(\frac{-1.42}{0.03453}\right)$$

or

$n_i = 3.28x10^9\ cm^{-3}$

Now

$p_O = N_a = 10^{14}\ cm^{-3}$

and

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(3.28x10^{9}\right)^2}{10^{14}} \Rightarrow$$

$n_O = 1.08x10^5\ cm^{-3}$

(e)

$kT = 0.04317\ eV$

$$n_i^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{500}{300}\right)^3 \exp\left(\frac{-1.42}{0.04317}\right)$$

or

$n_i = 2.81x10^{11}\ cm^{-3}$

Now

$$\underline{n_O = N_d = 10^{14}\ cm^{-3}}$$

Also

$$p_O = \frac{n_i^2}{n_O} = \frac{(2.81x10^{11})^2}{10^{14}} \Rightarrow$$

$$\underline{p_O = 7.90x10^8\ cm^{-3}}$$

4.33

(a) $N_a > N_d \Rightarrow$ p-type

(b) Si:

$$p_O = N_a - N_d = 2.5x10^{13} - 1x10^{13}$$

or

$$\underline{p_O = 1.5x10^{13}\ cm^{-3}}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{1.5x10^{13}} \Rightarrow$$

$$\underline{n_O = 1.5x10^7\ cm^{-3}}$$

Ge:

$$p_O = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \left(\frac{1.5x10^{13}}{2}\right) + \sqrt{\left(\frac{1.5x10^{13}}{2}\right)^2 + (2.4x10^{13})^2}$$

or

$$\underline{p_O = 3.26x10^{13}\ cm^{-3}}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{(2.4x10^{13})^2}{3.26x10^{13}} \Rightarrow$$

$$\underline{n_O = 1.77x10^{13}\ cm^{-3}}$$

GaAs:

$$\underline{p_O = 1.5x10^{13}\ cm^{-3}}$$

And

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.8x10^6)^2}{1.5x10^{13}} \Rightarrow$$

$$\underline{n_O = 0.216\ cm^{-3}}$$

4.34

For $T = 450K$

$$n_i^2 = (2.8x10^{19})(1.04x10^{19})\left(\frac{450}{300}\right)^3 \times \exp\left[\frac{-1.12}{(0.0259)(450/300)}\right]$$

or

$$\underline{n_i = 1.72x10^{13}\ cm^{-3}}$$

(a)

$N_a > N_d \Rightarrow$ p-type

(b)

$$p_O = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \frac{1.5x10^{15} - 8x10^{14}}{2} + \sqrt{\left(\frac{1.5x10^{15} - 8x10^{14}}{2}\right)^2 + (1.72x10^{13})^2}$$

or

$$\underline{p_O \approx N_a - N_d = 7x10^{14}\ cm^{-3}}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.72x10^{13})^2}{7x10^{14}} \Rightarrow$$

$$\underline{n_O = 4.23x10^{11}\ cm^{-3}}$$

(c)

Total ionized impurity concentration

$$N_I = N_a + N_d = 1.5x10^{15} + 8x10^{14}$$

or

$$\underline{N_I = 2.3x10^{15}\ cm^{-3}}$$

4.35

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{2x10^5} \Rightarrow$$

$$\underline{n_O = 1.125x10^{15}\ cm^{-3}}$$

$n_O > p_O \Rightarrow$ <u>n-type</u>

4.36

$$kT = (0.0259)\left(\frac{200}{300}\right) = 0.01727\ eV$$

$$n_i^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{200}{300}\right)^3 \times \exp\left[\frac{-1.42}{0.01727}\right]$$

or

$$\underline{n_i = 1.38\ cm^{-3}}$$

Now

$$n_O p_O = n_i^2 \Rightarrow 5p_O^2 = n_i^2$$

or\

$$p_O = \frac{n_i}{\sqrt{5}} \Rightarrow \underline{p_O = 0.617\ cm^{-3}}$$

And

$$n_O = 5p_O \Rightarrow \underline{n_O = 3.09\ cm^{-3}}$$

4.37
Computer Plot

4.38
Computer Plot

4.39
Computer Plot

4.40
n-type, so majority carrier = electrons

$$n_O = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= 10^{13} + \sqrt{\left(10^{13}\right)^2 + \left(2x10^{13}\right)^2}$$

or

$$\underline{n_O = 3.24x10^{13}\ cm^{-3}}$$

Then

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(2x10^{13}\right)^2}{3.24x10^{13}} \Rightarrow$$

$$\underline{p_O = 1.23x10^{13}\ cm^{-3}}$$

4.41

(a) $N_d > N_a \Rightarrow$ n-type

$$n_O = N_d - N_a = 2x10^{16} - 1x10^{16}$$

or

$$\underline{n_O = 1x10^{16}\ cm^{-3}}$$

Then

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} \Rightarrow$$

$$\underline{p_O = 2.25x10^4\ cm^{-3}}$$

(b)

$N_a > N_d \Rightarrow$ p-type

$$p_O = N_a - N_d = 3x10^{16} - 2x10^{15}$$

or

$$\underline{p_O = 2.8x10^{16}\ cm^{-3}}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.5x10^{10}\right)^2}{2.8x10^{16}} \Rightarrow$$

$$\underline{n_O = 8.04x10^3\ cm^{-3}}$$

4.42

(a) $n_O < n_i \Rightarrow$ p-type

(b) $\underline{n_O = 4.5x10^4\ cm^{-3}} \Rightarrow$ electrons: minority carrier

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.5x10^{10}\right)^2}{4.5x10^4} \Rightarrow$$

$\underline{p_O = 5x10^{15}\ cm^{-3}} \Rightarrow$ holes: majority carrier

(c)

$$p_O = N_a - N_d$$

so

$$5x10^{15} = N_a - 5x10^{15} \Rightarrow \underline{N_a = 10^{16}\ cm^{-3}}$$

Acceptor impurity concentration,

$\underline{N_d = 5x10^{15}\ cm^{-3}}$ Donor impurity concentration

4.43

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

For Germanium:

$T(°K)$	$kT(eV)$	$n_i(cm^{-3})$
200	0.01727	$2.16x10^{10}$
400	0.03454	$8.6x10^{14}$
600	0.0518	$3.82x10^{16}$

$$p_O = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2} \text{ and } N_a = 10^{15}\ cm^{-3}$$

$T(°K)$	$p_O(cm^{-3})$	$E_{Fi} - E_F\ (eV)$
200	$1.0x10^{15}$	0.1855
400	$1.49x10^{15}$	0.01898
600	$3.87x10^{16}$	0.000674

4.44

$$E_F - E_{Fi} = kT \ln\left(\frac{n_O}{n_i}\right)$$

For Germanium,

$$T = 300K \Rightarrow n_i = 2.4x10^{13}\ cm^{-3}$$

$$n_O = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$N_d(cm^{-3})$	$n_O(cm^{-3})$	$E_F - E_{Fi}\ (eV)$
10^{14}	$1.05x10^{14}$	0.0382
10^{16}	10^{16}	0.156
10^{18}	10^{18}	0.2755

4.45

$$n_O = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

Now

$$n_i = 0.05 n_O$$

so

$$n_O = 1.5x10^{15} + \sqrt{\left(1.5x10^{15}\right)^2 + \left[(0.05)n_O\right]^2}$$

which yields

$$n_O = 3.0075x10^{15}\ cm^{-3}$$

Then

$$n_i = 1.504x10^{14}\ cm^{-3}$$

We have

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

so

$$\left(1.504x10^{14}\right)^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{T}{300}\right)^3 \times \exp\left[\frac{-1.42}{(0.0259)(T/300)}\right]$$

By trial and error

$$T \approx 762K$$

4.46
Computer Plot

4.47
Computer Plot

4.48

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4}(0.0259)\ln(10) \Rightarrow$$

$$E_{Fi} - E_{midgap} = +0.0447\ eV$$

(b)
Impurity atoms to be added so

$$E_{midgap} - E_F = 0.45\ eV$$

(i) p-type, so add acceptor impurities
(ii) $E_{Fi} - E_F = 0.0447 + 0.45 = 0.4947\ eV$

$$p_O = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right) = 10^5 \exp\left(\frac{0.4947}{0.0259}\right)$$

or

$$p_O = N_a = 1.97x10^{13}\ cm^{-3}$$

4.49

$$n_O = N_d - N_a = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

so

$$N_d = 5x10^{15} + 2.8x10^{19} \exp\left(\frac{-0.215}{0.0259}\right) = 5x10^{15} + 6.95x10^{15}$$

so

$$N_d = 1.2x10^{16}\ cm^{-3}$$

4.50

(a) $$p_O = N_a = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

or

$$\exp\left[\frac{+(E_F - E_V)}{kT}\right] = \frac{N_V}{N_a} = \frac{1.04x10^{19}}{7x10^{15}}$$

$$= 1.49x10^3$$

Then

$$E_F - E_V = (0.0259)\ln(1.49x10^3)$$

or

$$\underline{E_F - E_V = 0.189\ eV}$$

(b)

If $E_F - E_V = 0.1892 - 0.0259 = 0.1633\ eV$

Then

$$N_a = 1.04x10^{19}\exp\left(\frac{-0.1633}{0.0259}\right)$$

$$= 1.90x10^{16}\ cm^{-3}$$

so that

$$\Delta N_a = 1.90x10^{16} - 7x10^{15} \Rightarrow$$

$$\underline{\Delta N_a = 1.2x10^{16}\ cm^{-3}}$$

Acceptor impurities to be added

4.51

(a) $E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right) = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right)$

or

$$\underline{E_F - E_{Fi} = 0.2877\ eV}$$

(b)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right) = 0.2877\ eV$$

(c)

For (a), $\underline{n_O = N_d = 10^{15}\ cm^{-3}}$

For (b)

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{10^{15}} \Rightarrow$$

$$\underline{n_O = 2.25x10^5\ cm^{-3}}$$

4.52

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{p_O}{n_i}\right) = 0.45\ eV$$

Then

$$p_O = (1.8x10^6)\exp\left(\frac{0.45}{0.0259}\right) \Rightarrow$$

$$\underline{p_O = 6.32x10^{13}\ cm^{-3}}$$

Now

$p_O < N_a$, Donors must be added

$$p_O = N_a - N_d \Rightarrow N_d = N_a - p_O$$

so

$$N_d = 10^{15} - 6.32x10^{13} \Rightarrow$$

$$\underline{N_d = 9.368x10^{14}\ cm^{-3}}$$

4.53

(a) $E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{2x10^{15}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.3056\ eV}$$

(b)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.3473\ eV}$$

(c)

$$\underline{E_F = E_{Fi}}$$

(d)

$kT = 0.03453\ eV$, $n_i = 2.38x10^{12}\ cm^{-3}$

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.03453)\ln\left(\frac{10^{14}}{2.38x10^{12}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.1291\ eV}$$

(e)

$kT = 0.04317\ eV$, $n_i = 8.54x10^{13}\ cm^{-3}$

$$E_F - E_{Fi} = kT\ln\left(\frac{n_O}{n_i}\right)$$

$$= (0.04317)\ln\left(\frac{1.49x10^{14}}{8.54x10^{13}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.0024\ eV}$$

4.54

(a) $E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{2x10^{15}}{1.8x10^6}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.5395\ eV}$$

(b)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.8x10^6}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.5811\ eV}$$

(c)

$$\underline{E_F = E_{Fi}\ E_{Fi} -}$$

(d)

$kT = 0.03453\ eV$, $n_i = 3.28x10^9\ cm^{-3}$

$$E_{Fi} - E_F = (0.03453)\ln\left(\frac{10^{14}}{3.28x10^9}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.3565\ eV}$$

(e)

$kT = 0.04317\ eV$, $n_i = 2.81x10^{11}\ cm^{-3}$

$$E_F - E_{Fi} = kT\ln\left(\frac{n_O}{n_i}\right)$$

$$= (0.04317)\ln\left(\frac{10^{14}}{2.81x10^{11}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.2536\ eV}$$

4.55

p-type

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.3294\ eV}$$

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Chapter 5

Problem Solutions

5.1

(a) $\underline{n_O = 10^{16}\ cm^{-3}}$

and

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.8x10^6\right)^2}{10^{16}} \Rightarrow$$

$$\underline{p_O = 3.24x10^{-4}\ cm^{-3}}$$

(b)

$J = e\mu_n n_O \mathrm{E}$

For GaAs doped at $N_d = 10^{16}\ cm^{-3}$,

$\mu_n \approx 7500\ cm^2 / V - s$

Then

$J = \left(1.6x10^{-19}\right)(7500)\left(10^{16}\right)(10)$

or

$\underline{J = 120\ A / cm^2}$

(b) (i) $\underline{p_O = 10^{16}\ cm^{-3}}$, $\underline{n_O = 3.24x10^{-4}\ cm^{-3}}$

(ii) For GaAs doped at $N_a = 10^{16}\ cm^{-3}$,

$\mu_p \approx 310\ cm^2 / V - s$

$J = e\mu_p p_O \mathrm{E}$

$= \left(1.6x10^{-19}\right)(310)\left(10^{16}\right)(10) \Rightarrow$

$\underline{J = 4.96\ A / cm^2}$

5.2

(a) $V = IR \Rightarrow 10 = (0.1R) \Rightarrow$

$\underline{R = 100\ \Omega}$

(b)

$$R = \frac{L}{\sigma A} \Rightarrow \sigma = \frac{L}{RA} \Rightarrow$$

$$\sigma = \frac{10^{-3}}{(100)\left(10^{-3}\right)} \Rightarrow$$

$\underline{\sigma = 0.01\ (\Omega - cm)^{-1}}$

(c)

$\sigma \approx e\mu_n N_d$

or

$0.01 = \left(1.6x10^{-19}\right)(1350) N_d$

or

$\underline{N_d = 4.63x10^{13}\ cm^{-3}}$

(d)

$\sigma \approx e\mu_p p_O \Rightarrow$

$0.01 = \left(1.6x10^{-19}\right)(480) p_O$

or

$p_O = 1.30x10^{14}\ cm^{-3} = N_a - N_d = N_a - 10^{15}$

or

$\underline{N_a = 1.13x10^{15}\ cm^{-3}}$

Note: For the doping concentrations obtained, the assumed mobility values are valid.

5.3

(a) $R = \dfrac{\rho L}{A} = \dfrac{L}{\sigma A}$ and $\sigma \approx e\mu_n N_d$

For $N_d = 5x10^{16}\ cm^{-3}$, $\mu_n \approx 1100\ cm^2 / V - s$

Then

$$R = \frac{0.1}{\left(1.6x10^{-19}\right)(1100)\left(5x10^{16}\right)(100)\left(10^{-4}\right)^2}$$

or

$\underline{R = 1.136x10^4\ \Omega}$

Then

$$I = \frac{V}{R} = \frac{5}{1.136x10^4} \Rightarrow \underline{I = 0.44\ mA}$$

(b)

In this case

$\underline{R = 1.136x10^3\ \Omega}$

Then

$$I = \frac{V}{R} = \frac{5}{1.136x10^3} \Rightarrow \underline{I = 4.4\ mA}$$

(c)

$$\mathrm{E} = \frac{V}{L}$$

For (a), $\mathrm{E} = \dfrac{5}{0.10} = 50\ V / cm$

And

$v_d = \mu_n \mathrm{E} = (1100)(50)$ or $\underline{v_d = 5.5x10^4\ cm / s}$

For (b), $\mathrm{E} = \dfrac{V}{L} = \dfrac{5}{0.01} = 500\ V / cm$

And

$v_d = (1100)(500) \Rightarrow \underline{v_d = 5.5x10^5\ cm / s}$

5.4

(a) GaAs:

$$R = \frac{\rho L}{A} = \frac{V}{I} = \frac{10}{20} = 0.5\ k\Omega = \frac{L}{\sigma A}$$

Now

$$\sigma \approx e\mu_p N_a$$

For $N_a = 10^{17}\ cm^{-3}$, $\mu_p \approx 210\ cm^2/V-s$

Then

$$\sigma = \left(1.6x10^{-19}\right)(210)\left(10^{17}\right) = 3.36\ (\Omega - cm)^{-1}$$

So

$$L = R\sigma A = (500)(3.36)\left(85x10^{-8}\right)$$

or

$$\underline{L = 14.3\ \mu m}$$

(b) Silicon

For $N_a = 10^{17}\ cm^{-3}$, $\mu_p \approx 310\ cm^2/V-s$

Then

$$\sigma = \left(1.6x10^{-19}\right)(310)\left(10^{17}\right) = 4.96\ (\Omega - cm)^{-1}$$

So

$$L = R\sigma A = (500)(4.96)\left(85x10^{-8}\right)$$

or

$$\underline{L = 21.1\ \mu m}$$

5.5

(a) $\mathrm{E} = \frac{V}{L} = \frac{3}{1} = 3\ V/cm$

$$v_d = \mu_n \mathrm{E} \Rightarrow \mu_n = \frac{v_d}{\mathrm{E}} = \frac{10^4}{3}$$

or

$$\underline{\mu_n = 3333\ cm^2/V-s}$$

(b)

$$v_d = \mu_n \mathrm{E} = (800)(3)$$

or

$$\underline{v_d = 2.4x10^3\ cm/s}$$

5.6

(a) Silicon: For $\mathrm{E} = 1\ kV/cm$,

$$v_d = 1.2x10^6\ cm/s$$

Then

$$t_t = \frac{d}{v_d} = \frac{10^{-4}}{1.2x10^6} \Rightarrow \underline{t_t = 8.33x10^{-11}\ s}$$

For GaAs, $v_d = 7.5x10^6\ cm/s$

Then

$$t_t = \frac{d}{v_d} = \frac{10^{-4}}{7.5x10^6} \Rightarrow \underline{t_t = 1.33x10^{-11}\ s}$$

(b)

Silicon: For $\mathrm{E} = 50\ kV/cm$,

$$v_d = 9.5x10^6\ cm/s$$

Then

$$t_t = \frac{d}{v_d} = \frac{10^{-4}}{9.5x10^6} \Rightarrow \underline{t_t = 1.05x10^{-11}\ s}$$

GaAs, $v_d = 7x10^6\ cm/s$

Then

$$t_t = \frac{d}{v_d} = \frac{10^{-4}}{7x10^6} \Rightarrow \underline{t_t = 1.43x10^{-11}\ s}$$

5.7

For an intrinsic semiconductor,

$$\sigma_i = en_i\left(\mu_n + \mu_p\right)$$

(a)

For $N_d = N_a = 10^{14}\ cm^{-3}$,

$$\mu_n = 1350\ cm^2/V-s,\ \mu_p = 480\ cm^2/V-s$$

Then

$$\sigma_i = \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)(1350 + 480)$$

or

$$\underline{\sigma_i = 4.39x10^{-6}\ (\Omega - cm)^{-1}}$$

(b)

For $N_d = N_a = 10^{18}\ cm^{-3}$,

$$\mu_n \approx 300\ cm^2/V-s,\ \mu_p \approx 130\ cm^2/V-s$$

Then

$$\sigma_i = \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)(300 + 130)$$

or

$$\underline{\sigma_i = 1.03x10^{-6}\ (\Omega - cm)^{-1}}$$

5.8

(a) GaAs

$$\sigma \approx e\mu_p p_O \Rightarrow 5 = \left(1.6x10^{-19}\right)\mu_p p_O$$

From Figure 5.3, and using trial and error, we find

$$\underline{p_O \approx 1.3x10^{17}\ cm^{-3},\ \mu_p \approx 240\ cm^2/V-s}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.8x10^6)^2}{1.3x10^{17}} \text{ or } \underline{n_O = 2.49x10^{-5}\ cm^{-3}}$$

(b) Silicon:

$$\sigma = \frac{1}{\rho} \approx e\mu_n n_O$$

or

$$n_O = \frac{1}{\rho e\mu_n} = \frac{1}{(8)(1.6x10^{-19})(1350)}$$

or

$$\underline{n_O = 5.79x10^{14}\ cm^{-3}}$$

and

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.5x10^{10})^2}{5.79x10^{14}} \Rightarrow \underline{p_O = 3.89x10^5\ cm^{-3}}$$

Note: For the doping concentrations obtained in part (b), the assumed mobility values are valid.

5.9

$$\sigma_i = en_i(\mu_n + \mu_p)$$

Then

$$10^{-6} = (1.6x10^{-19})(1000+600)n_i$$

or

$$\underline{n_i(300K) = 3.91x10^9\ cm^{-3}}$$

Now

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

or

$$E_g = kT\ln\left(\frac{N_C N_V}{n_i^2}\right) = (0.0259)\ln\left[\frac{(10^{19})^2}{(3.91x10^9)^2}\right]$$

or

$$\underline{E_g = 1.122\ eV}$$

Now

$$n_i^2(500K) = (10^{19})^2 \exp\left[\frac{-1.122}{(0.0259)(500/300)}\right]$$

$$= 5.15x10^{26}$$

or

$$\underline{n_i(500K) = 2.27x10^{13}\ cm^{-3}}$$

Then

$$\sigma_i = (1.6x10^{-19})(2.27x10^{13})(1000+600)$$

so

$$\underline{\sigma_i(500K) = 5.81x10^{-3}\ (\Omega - cm)^{-1}}$$

5.10

(a) (i) Silicon: $\sigma_i = en_i(\mu_n + \mu_p)$

$$\sigma_i = (1.6x10^{-19})(1.5x10^{10})(1350+480)$$

or

$$\underline{\sigma_i = 4.39x10^{-6}\ (\Omega - cm)^{-1}}$$

(ii) Ge:

$$\sigma_i = (1.6x10^{-19})(2.4x10^{13})(3900+1900)$$

or

$$\underline{\sigma_i = 2.23x10^{-2}\ (\Omega - cm)^{-1}}$$

(iii) GaAs:

$$\sigma_i = (1.6x10^{-19})(1.8x10^6)(8500+400)$$

or

$$\underline{\sigma_i = 2.56x10^{-9}\ (\Omega - cm)^{-1}}$$

(b) $R = \dfrac{L}{\sigma A}$

(i) $$R = \frac{200x10^{-4}}{(4.39x10^{-6})(85x10^{-8})} \Rightarrow$$

$$\underline{R = 5.36x10^9\ \Omega}$$

(ii) $$R = \frac{200x10^{-4}}{(2.23x10^{-2})(85x10^{-8})} \Rightarrow$$

$$\underline{R = 1.06x10^6\ \Omega}$$

(iii) $$R = \frac{200x10^{-4}}{(2.56x10^{-9})(85x10^{-8})} \Rightarrow$$

$$\underline{R = 9.19x10^{12}\ \Omega}$$

5.11

(a) $\rho = 5 = \dfrac{1}{e\mu_n N_d}$

Assume $\mu_n = 1350\ cm^2/V-s$

Then

$$N_d = \frac{1}{(1.6x10^{-19})(1350)(5)} \Rightarrow$$

$$\underline{N_d = 9.26x10^{14}\ cm^{-3}}$$

(b)

$T = 200K \rightarrow T = -75C$

$T = 400K \rightarrow T = 125C$

From Figure 5.2,

$$T = -75C,\ N_d = 10^{15}\ cm^{-3} \Rightarrow$$

$\mu_n \approx 2500\ cm^2/V-s$

$T = 125C$, $N_d = 10^{15}\ cm^{-3} \Rightarrow$

$\mu_n \approx 700\ cm^2/V-s$

Assuming $n_O = N_d = 9.26x10^{14}\ cm^{-3}$ over the temperature range,

For $T = 200K$,

$$\rho = \frac{1}{(1.6x10^{-19})(2500)(9.26x10^{14})} \Rightarrow$$

$$\underline{\rho = 2.7\ \Omega - cm}$$

For $T = 400K$,

$$\rho = \frac{1}{(1.6x10^{-19})(700)(9.26x10^{14})} \Rightarrow$$

$$\underline{\rho = 9.64\ \Omega - cm}$$

5.12

Computer plot

5.13

(a) $\mathrm{E} = 10\ V/cm \Rightarrow |v_d| = \mu_n \mathrm{E}$

$v_d = (1350)(10) \Rightarrow \quad v_d = 1.35x10^4\ cm/s$

so

$$T = \frac{1}{2}m_n^* v_d^2 = \frac{1}{2}(1.08)(9.11x10^{-31})(1.35x10^2)^2$$

or

$$\underline{T = 8.97x10^{-27}\ J \Rightarrow 5.6x10^{-8}\ eV}$$

(b)

$\mathrm{E} = 1\ kV/cm$,

$v_d = (1350)(1000) = 1.35x10^6\ cm/s$

Then

$$T = \frac{1}{2}(1.08)(9.11x10^{-31})(1.35x10^4)^2$$

or

$$\underline{T = 8.97x10^{-23}\ J \Rightarrow 5.6x10^{-4}\ eV}$$

5.14

(a) $n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$

$$= (2x10^{19})(1x10^{19})\exp\left(\frac{-1.10}{0.0259}\right)$$

$$= 7.18x10^{19} \Rightarrow n_i = 8.47x10^9\ cm^{-3}$$

For $N_d = 10^{14}\ cm^{-3} >> n_i \Rightarrow n_O = 10^{14}\ cm^{-3}$

Then

$$J = \sigma\mathrm{E} = e\mu_n n_O \mathrm{E}$$

$$= (1.6x10^{-19})(1000)(10^{14})(100)$$

or

$$\underline{J = 1.60\ A/cm^2}$$

(b)

A 5% increase is due to a 5% increase in electron concentration. So

$$n_O = 1.05x10^{14} = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

We can write

$$(1.05x10^{14} - 5x10^{13})^2 = (5x10^{13})^2 + n_i^2$$

so

$$n_i^2 = 5.25x10^{26}$$

$$= (2x10^{19})(1x10^{19})\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

which yields

$$2.625x10^{-12} = \left(\frac{T}{300}\right)^3 \exp\left(\frac{-1.10}{kT}\right)$$

By trial and error, we find

$$\underline{T = 456K}$$

5.15

(a) $\sigma = e\mu_n n_O + e\mu_p p_O$ and $n_O = \frac{n_i^2}{p_O}$

Then

$$\sigma = \frac{e\mu_n n_i^2}{p_O} + e\mu_p p_O$$

To find the minimum conductivity,

$$\frac{d\sigma}{dp_O} = 0 = \frac{(-1)e\mu_n n_i^2}{p_O^2} + e\mu_p \Rightarrow$$

which yields

$$\underline{p_O = n_i\left(\frac{\mu_n}{\mu_p}\right)^{1/2}} \quad \text{(Answer to part (b))}$$

Substituting into the conductivity expression

$$\sigma = \sigma_{min} = \frac{e\mu_n n_i^2}{\left[n_i(\mu_n/\mu_p)^{1/2}\right]} + e\mu_p\left[n_i(\mu_n/\mu_p)^{1/2}\right]$$

which simplifies to

$$\sigma_{min} = 2en_i\sqrt{\mu_n \mu_p}$$

The intrinsic conductivity is defined as

$$\sigma_i = en_i(\mu_n + \mu_p) \Rightarrow en_i = \frac{\sigma_i}{\mu_n + \mu_p}$$

The minimum conductivity can then be written as

$$\sigma_{\min} = \frac{2\sigma_i\sqrt{\mu_n\mu_p}}{\mu_n + \mu_p}$$

5.16

$$\sigma = e\mu n_i = \frac{1}{\rho}$$

Now

$$\frac{1/\rho_1}{1/\rho_2} = \frac{1/50}{1/5} = \frac{5}{50} = 0.10 = \frac{\exp\left(\frac{-E_g}{2kT_1}\right)}{\exp\left(\frac{-E_g}{2kT_2}\right)}$$

or

$$0.10 = \exp\left[-E_g\left(\frac{1}{2kT_1} - \frac{1}{2kT_2}\right)\right]$$

$$kT_1 = 0.0259$$

$$kT_2 = (0.0259)\left(\frac{330}{300}\right) = 0.02849$$

$$\frac{1}{2kT_1} = 19.305 \quad , \quad \frac{1}{2kT_2} = 17.550$$

Then

$$E_g(19.305 - 17.550) = \ln(10)$$

or

$$E_g = 1.312\ eV$$

5.17

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}$$

$$= \frac{1}{2000} + \frac{1}{1500} + \frac{1}{500}$$

$$= 0.00050 + 0.000667 + 0.0020$$

or

$$\mu = 316\ cm^2/V-s$$

5.18

$$\mu_n = (1300)\left(\frac{T}{300}\right)^{-3/2} = (1300)\left(\frac{300}{T}\right)^{+3/2}$$

(a)
At $T = 200K$, $\mu_n = (1300)(1.837) \Rightarrow$

$$\mu_n = 2388\ cm^2/V-s$$

(b)
At $T = 400K$, $\mu_n = (1300)(0.65) \Rightarrow$

$$\mu_n = 844\ cm^2/V-s$$

5.19

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{1}{250} + \frac{1}{500} = 0.006$$

Then

$$\mu = 167\ cm^2/V-s$$

5.20
Computer plot

5.21
Computer plot

5.22

$$J_n = eD_n\frac{dn}{dx} = eD_n\left(\frac{5x10^{14} - n(0)}{0.01 - 0}\right)$$

$$0.19 = (1.6x10^{-19})(25)\left(\frac{5x10^{14} - n(0)}{0.010}\right)$$

Then

$$\frac{(0.19)(0.010)}{(1.6x10^{-19})(25)} = 5x10^{14} - n(0)$$

which yields

$$n(0) = 0.25x10^{14}\ cm^{-3}$$

5.23

$$J = eD_n\frac{dn}{dx} = eD_n\frac{\Delta n}{\Delta x}$$

$$= (1.6x10^{-19})(25)\left(\frac{10^{16} - 10^{15}}{0 - 0.10}\right)$$

or

$$|J| = 0.36\ A/cm^2$$

For $A = 0.05\ cm^2$

$$I = AJ = (0.05)(0.36) \Rightarrow \quad I = 18\ mA$$

5.24

$$J_n = eD_n \frac{dn}{dx} = eD_n \frac{\Delta n}{\Delta x}$$

so

$$-400 = \left(1.6x10^{-19}\right) D_n \left(\frac{10^{17} - 6x10^{16}}{0 - 4x10^{-4}}\right)$$

or

$$-400 = D_n(-16)$$

Then

$$\underline{D_n = 25\ cm^2 / s}$$

5.25

$$J = -eD_p \frac{dp}{dx}$$

$$= -eD_p \frac{d}{dx}\left[10^{16}\left(1 - \frac{x}{L}\right)\right] = -eD_p\left(\frac{-10^{16}}{L}\right)$$

$$= \frac{\left(1.6x10^{-19}\right)(10)\left(10^{16}\right)}{10x10^{-4}}$$

or

$$\underline{J = 16\ A / cm^2} = \underline{\text{constant at all three points}}$$

5.26

$$J_p(x=0) = -eD_p \frac{dp}{dx}\bigg|_{x=0}$$

$$= -eD_p \frac{10^{15}}{\left(-L_p\right)} = \frac{\left(1.6x10^{-19}\right)(10)\left(10^{15}\right)}{5x10^{-4}}$$

or

$$J_p(x=0) = 3.2\ A / cm^2$$

Now

$$J_n(x=0) = eD_n \frac{dn}{dx}\bigg|_{x=0}$$

$$= eD_n\left(\frac{5x10^{14}}{L_n}\right) = \frac{\left(1.6x10^{-19}\right)(25)\left(5x10^{14}\right)}{10^{-3}}$$

or

$$J_n(x=0) = 2\ A / cm^2$$

Then

$$J = J_p(x=0) + J_n(x=0) = 3.2 + 2$$

or

$$\underline{J = 5.2\ A / cm^2}$$

5.27

$$J_p = -eD_p \frac{dp}{dx} = -eD_p \frac{d}{dp}\left[10^{15}\exp\left(\frac{-x}{22.5}\right)\right]$$

Distance x is in μm, so $22.5 \to 22.5x10^{-4}\ cm$.
Then

$$J_p = -eD_p\left(10^{15}\right)\left(\frac{-1}{22.5x10^{-4}}\right)\exp\left(\frac{-x}{22.5}\right)$$

$$= \frac{+\left(1.6x10^{-19}\right)(48)\left(10^{15}\right)}{22.5x10^{-4}}\exp\left(\frac{-x}{22.5}\right)$$

or

$$\underline{J_p = 3.41\exp\left(\frac{-x}{22.5}\right)\quad A / cm^2}$$

5.28

$$J_n = e\mu_n n\mathrm{E} + eD_n \frac{dn}{dx}$$

or

$$-40 = \left(1.6x10^{-19}\right)(960)\left[10^{16}\exp\left(\frac{-x}{18}\right)\right]\mathrm{E}$$

$$+\left(1.6x10^{-19}\right)(25)\left(10^{16}\right)\left(\frac{-1}{18x10^{-4}}\right)\exp\left(\frac{-x}{18}\right)$$

Then

$$-40 = 1.536\left[\exp\left(\frac{-x}{18}\right)\right]\mathrm{E} - 22.2\exp\left(\frac{-x}{18}\right)$$

Then

$$\mathrm{E} = \frac{22.2\exp\left(\frac{-x}{18}\right) - 40}{1.536\exp\left(\frac{-x}{18}\right)} \Rightarrow$$

$$\underline{\mathrm{E} = 14.5 - 26\exp\left(\frac{+x}{18}\right)}$$

5.29

$$J_T = J_{n,drf} + J_{p,dif}$$

(a) $J_{p,dif} = -eD_p \dfrac{dp}{dx}$ and

$$p(x) = 10^{15}\exp\left(\frac{-x}{L}\right) \quad \text{where} \quad L = 12\ \mu m$$

so

$$J_{p,dif} = -eD_p\left(10^{15}\right)\left(\frac{-1}{L}\right)\exp\left(\frac{-x}{L}\right)$$

or

$$J_{p,dif} = \frac{(1.6x10^{-19})(12)(10^{15})}{12x10^{-4}} \exp\left(\frac{-x}{12}\right)$$
or
$$\underline{J_{p,dif} = +1.6\exp\left(\frac{-x}{L}\right) \quad A/cm^2}$$
(b)
$$J_{n,drf} = J_T - J_{p,dif}$$
or
$$\underline{J_{n,drf} = 4.8 - 1.6\exp\left(\frac{-x}{L}\right)}$$
(c)
$$J_{n,drf} = e\mu_n n_O \mathrm{E}$$
Then
$$(1.6x10^{-19})(1000)(10^{16})\mathrm{E} = 4.8 - 1.6\exp\left(\frac{-x}{L}\right)$$
which yields
$$\underline{\mathrm{E} = \left[3 - 1\times\exp\left(\frac{-x}{L}\right)\right] \quad V/cm}$$

5.30

(a) $J = e\mu_n n(x)\mathrm{E} + eD_n \dfrac{dn(x)}{dx}$

Now $\mu_n = 8000\ cm^2/V-s$ so that
$$D_n = (0.0259)(8000) = 207\ cm^2/s$$
Then
$$100 = (1.6x10^{-19})(8000)(12)n(x) + (1.6x10^{-19})(207)\frac{dn(x)}{dx}$$
which yields
$$100 = 1.54x10^{-14}n(x) + 3.31x10^{-17}\frac{dn(x)}{dx}$$
Solution is of the form
$$n(x) = A + B\exp\left(\frac{-x}{d}\right)$$
so that
$$\frac{dn(x)}{dx} = \frac{-B}{d}\exp\left(\frac{-x}{d}\right)$$
Substituting into the differential equation, we have
$$100 = (1.54x10^{-14})\left[A + B\exp\left(\frac{-x}{d}\right)\right] - \frac{(3.31x10^{-17})}{d}B\exp\left(\frac{-x}{d}\right)$$
This equation is valid for all x, so
$$100 = 1.54x10^{-14}A$$
or
$$A = 6.5x10^{15}$$
Also
$$1.54x10^{-14}B\exp\left(\frac{-x}{d}\right) - \frac{(3.31x10^{-17})}{d}B\exp\left(\frac{-x}{d}\right) = 0$$
which yields
$$\underline{d = 2.15x10^{-3}\ cm}$$
At $x = 0$, $e\mu_n n(0)\mathrm{E} = 50$
so that
$$50 = (1.6x10^{-19})(8000)(12)(A+B)$$
which yields $B = -3.24x10^{15}$
Then
$$\underline{n(x) = 6.5x10^{15} - 3.24x10^{15}\exp\left(\frac{-x}{d}\right) \quad cm^{-3}}$$
(b)
At $x = 0$, $n(0) = 6.5x10^{15} - 3.24x10^{15}$
Or
$$\underline{n(0) = 3.26x10^{15}\ cm^{-3}}$$
At $x = 50\ \mu m$,
$$n(50) = 6.5x10^{15} - 3.24x10^{15}\exp\left(\frac{-50}{21.5}\right)$$
or
$$\underline{n(50) = 6.18x10^{15}\ cm^{-3}}$$
(c)
At $x = 50\ \mu m$, $J_{drt} = e\mu_n n(50)\mathrm{E}$
$$= (1.6x10^{-19})(8000)(6.18x10^{15})(12)$$
or
$$\underline{J_{drf}(x=50) = 94.9\ A/cm^2}$$
Then
$$J_{dif}(x=50) = 100 - 94.9 \Rightarrow$$
$$\underline{J_{dif}(x=50) = 5.1\ A/cm^2}$$

5.31

$$n = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

(a) $E_F - E_{Fi} = ax + b$, $b = 0.4$

$0.15 = a\left(10^{-3}\right) + 0.4$ so that $a = -2.5x10^2$

Then

$$E_F - E_{Fi} = 0.4 - 2.5x10^2 x$$

So

$$\underline{n = n_i \exp\left(\frac{0.4 - 2.5x10^2 x}{kT}\right)}$$

(b)

$$J_n = eD_n \frac{dn}{dx}$$

$$= eD_n n_i \left(\frac{-2.5x10^2}{kT}\right)\exp\left(\frac{0.4 - 2.5x10^2 x}{kT}\right)$$

Assume $T = 300K$, $kT = 0.0259\ eV$, and

$n_i = 1.5x10^{10}\ cm^{-3}$

Then

$$J_n = \frac{-\left(1.6x10^{-19}\right)(25)\left(1.5x10^{10}\right)\left(2.5x10^2\right)}{(0.0259)} \times \exp\left(\frac{0.4 - 2.5x10^2 x}{0.0259}\right)$$

or

$$J_n = -5.79x10^{-4} \exp\left(\frac{0.4 - 2.5x10^2 x}{0.0259}\right)$$

(i) At $x = 0$, $\underline{J_n = -2.95x10^3\ A/cm^2}$

(ii) At $x = 5\ \mu m$, $\underline{J_n = -23.7\ A/cm^2}$

5.32

(a) $J_n = e\mu_n n \mathrm{E} + eD_n \dfrac{dn}{dx}$

$$-80 = \left(1.6x10^{-19}\right)(1000)\left(10^{16}\right)\left(1 - \frac{x}{L}\right)\mathrm{E} + \left(1.6x10^{-19}\right)(25.9)\left(\frac{-10^{16}}{L}\right)$$

where $L = 10x10^{-4} = 10^{-3}\ cm$

We find

$$-80 = 1.6\mathrm{E} - 1.6\left(\frac{x}{10^{-3}}\right)\mathrm{E} - 41.44$$

or

$$80 = 1.6\left(\frac{x}{L} - 1\right)\mathrm{E} + 41.44$$

Solving for the electric field, we find

$$\underline{\mathrm{E} = \frac{38.56}{\left(\frac{x}{L} - 1\right)}}$$

(b)

For $J_n = -20\ A/cm^2$

$$20 = 1.6\left(\frac{x}{L} - 1\right)\mathrm{E} + 41.44$$

Then

$$\underline{\mathrm{E} = \frac{21.44}{\left(1 - \frac{x}{L}\right)}}$$

5.33

(a) $J = e\mu_n n \mathrm{E} + eD_n \dfrac{dn}{dx}$

Let $n = N_d = N_{do} \exp(-\alpha x)$, $J = 0$

Then

$$0 = \mu_n N_{do}\left[\exp(-\alpha x)\right]\mathrm{E} + D_n N_{do}(-\alpha)\exp(-\alpha x)$$

or

$$0 = \mathrm{E} + \frac{D_n}{\mu_n}(-\alpha)$$

Since $\dfrac{D_n}{\mu_n} = \dfrac{kT}{e}$

So

$$\underline{\mathrm{E} = \alpha\left(\frac{kT}{e}\right)}$$

(b)

$$V = -\int_0^{1/\alpha} \mathrm{E}dx = -\alpha\left(\frac{kT}{e}\right)\int_0^{1/\alpha} dx$$

$$= -\left[\alpha\left(\frac{kT}{e}\right)\right]\cdot\left(\frac{1}{\alpha}\right) \text{ so that } \underline{V = -\left(\frac{kT}{e}\right)}$$

5.34

From Example 5.5

$$\mathrm{E}_x = \frac{(0.0259)\left(10^{19}\right)}{\left(10^{16} - 10^{19}x\right)} = \frac{(0.0259)\left(10^3\right)}{\left(1 - 10^3 x\right)}$$

$$V = -\int_0^{10^{-4}} \mathrm{E}_x dx = -(0.0259)\left(10^3\right)\int_0^{10^{-4}} \frac{dx}{\left(1 - 10^3 x\right)}$$

$$= -(0.0259)\left(10^3\right)\left(\frac{-1}{10^3}\right)\ln\left[1-10^3 x\right]_0^{10^{-4}}$$

$$= (0.0259)\left[\ln(1-0.1)-\ln(1)\right]$$

or

$$\underline{V = -2.73\ mV}$$

5.35

From Equation [5.40]

$$E_x = -\left(\frac{kT}{e}\right)\left(\frac{1}{N_d(x)}\right)\cdot\frac{dN_d(x)}{dx}$$

Now

$$1000 = -(0.0259)\left(\frac{1}{N_d(x)}\right)\cdot\frac{dN_d(x)}{dx}$$

or

$$\frac{dN_d(x)}{dx} + 3.86x10^4\, N_d(x) = 0$$

Solution is of the form

$$N_d(x) = A\exp(-\alpha x)$$

and

$$\frac{dN_d(x)}{dx} = -A\alpha\exp(-\alpha x)$$

Substituting into the differential equation

$$-A\alpha\exp(-\alpha x) + 3.86x10^4 A\exp(-\alpha x) = 0$$

which yields

$$\underline{\alpha = 3.86x10^4\ cm^{-1}}$$

At $x = 0$, the actual value of $N_d(0)$ is arbitrary.

5.36

(a) $J_n = J_{drf} + J_{dif} = 0$

$$J_{dif} = eD_n\frac{dn}{dx} = eD_n\frac{dN_d(x)}{dx}$$

$$= \frac{eD_n}{(-L)}\cdot N_{do}\exp\left(\frac{-x}{L}\right)$$

We have

$$D_n = \mu_n\left(\frac{kT}{e}\right) = (6000)(0.0259) = 155.4\ cm^2/s$$

Then

$$J_{dif} = \frac{-\left(1.6x10^{-19}\right)(155.4)\left(5x10^{16}\right)}{\left(0.1x10^{-4}\right)}\exp\left(\frac{-x}{L}\right)$$

or

$$\underline{J_{dif} = -1.24x10^5\exp\left(\frac{-x}{L}\right)\quad A/cm^2}$$

(b)

$$0 = J_{drf} + J_{dif}$$

Now

$$J_{drf} = e\mu_n n\mathrm{E}$$

$$= \left(1.6x10^{-19}\right)(6000)\left(5x10^{16}\right)\left[\exp\left(\frac{-x}{L}\right)\right]\mathrm{E}$$

$$= 48\mathrm{E}\exp\left(\frac{-x}{L}\right)$$

We have

$$J_{drf} = -J_{dif}$$

so

$$48\mathrm{E}\exp\left(\frac{-x}{L}\right) = 1.24x10^5\exp\left(\frac{-x}{L}\right)$$

which yields

$$\underline{\mathrm{E} = 2.58x10^3\ V/cm}$$

5.37

Computer Plot

5.38

(a) $D = \mu\left(\frac{kT}{e}\right) = (925)(0.0259)$

so

$$\underline{D = 23.96\ cm^2/s}$$

(b)

For $D = 28.3\ cm^2/s$

$$\mu = \frac{28.3}{0.0259} \Rightarrow \underline{\mu = 1093\ cm^2/V-s}$$

5.39

We have $L = 10^{-1}\ cm = 10^{-3}\ m$,

$W = 10^{-2}\ cm = 10^{-4}\ m$, $d = 10^{-3}\ cm = 10^{-5}\ m$

(a)

We have

$p = 10^{16}\ cm^{-3} = 10^{22}\ m^{-3}$, $I_x = 1\ mA = 10^{-3}\ A$

Then

$$V_H = \frac{I_x B_z}{epd} = \frac{\left(10^{-3}\right)\left(3.5x10^{-2}\right)}{\left(1.6x10^{-19}\right)\left(10^{22}\right)\left(10^{-5}\right)}$$

or

$$\underline{V_H = 2.19\ mV}$$

(b)

$$E_H = \frac{V_H}{W} = \frac{2.19x10^{-3}}{10^{-2}}$$

or

$$\underline{E_H = 0.219\ V/cm}$$

5.40

(a) $V_H = \dfrac{-I_x B_z}{ned} = \dfrac{-(250x10^{-6})(5x10^{-2})}{(5x10^{21})(1.6x10^{-19})(5x10^{-5})}$

or

$$\underline{V_H = -0.3125\ mV}$$

(b)

$$E_H = \frac{V_H}{W} = \frac{-0.3125x10^{-3}}{2x10^{-2}} \Rightarrow$$

$$\underline{E_H = -1.56x10^{-2}\ V/cm}$$

(c)

$$\mu_n = \frac{I_x L}{enV_x Wd}$$

$$= \frac{(250x10^{-6})(10^{-3})}{(1.6x10^{-19})(5x10^{21})(0.1)(2x10^{-4})(5x10^{-5})}$$

or

$$\underline{\mu_n = 0.3125\ m^2/V-s = 3125\ cm^2/V-s}$$

5.41

(a) V_H = positive $\Rightarrow$ p-type

(b)

$$V_H = \frac{I_x B_z}{epd} \Rightarrow p = \frac{I_x B_z}{eV_H d}$$

$$= \frac{(0.75x10^{-3})(10^{-1})}{(1.6x10^{-19})(5.8x10^{-3})(10^{-5})}$$

or

$$\underline{p = 8.08x10^{21}\ m^{-3} = 8.08x10^{15}\ cm^{-3}}$$

(c)

$$\mu_p = \frac{I_x L}{epV_x Wd}$$

$$= \frac{(0.75x10^{-3})(10^{-3})}{(1.6x10^{-19})(8.08x10^{21})(15)(10^{-4})(10^{-5})}$$

or

$$\underline{\mu_p = 3.87x10^{-2}\ m^2/V-s = 387\ cm^2/V-s}$$

5.42

(a) $V_H = E_H W = -(16.5x10^{-3})(5x10^{-2})$

or

$$\underline{V_H = -0.825\ mV}$$

(b)

V_H = negative $\Rightarrow$ <u>n-type</u>

(c)

$$n = \frac{-I_x B_z}{edV_H}$$

$$= \frac{-(0.5x10^{-3})(6.5x10^{-2})}{(1.6x10^{-19})(5x10^{-5})(-0.825x10^{-3})}$$

or

$$\underline{n = 4.92x10^{21}\ m^{-3} = 4.92x10^{15}\ cm^{-3}}$$

(d)

$$\mu_n = \frac{I_x L}{enV_x Wd}$$

$$= \frac{(0.5x10^{-3})(0.5x10^{-2})}{(1.6x10^{-19})(4.92x10^{21})(1.25)(5x10^{-4})(5x10^{-5})}$$

or

$$\underline{\mu_n = 0.102\ m^2/V-s = 1020\ cm^2/V-s}$$

5.43

(a) V_H = negative $\Rightarrow$ <u>n-type</u>

(b) $n = \dfrac{-I_x B_z}{edV_H} \Rightarrow \underline{n = 8.68x10^{14}\ cm^{-3}}$

(c) $\mu_n = \dfrac{I_x L}{enV_x Wd} \Rightarrow \underline{\mu_n = 8182\ cm^2/V-s}$

(d) $\sigma = \dfrac{1}{\rho} = e\mu_n n = (1.6x10^{-19})(8182)(8.68x10^{14})$

or $\underline{\rho = 0.88\ (\Omega - cm)}$

Chapter 6

Problem Solutions

6.1
n-type semiconductor, low-injection so that

$$R' = \frac{\delta p}{\tau_{pO}} = \frac{5x10^{13}}{10^{-6}}$$

or

$$\underline{R' = 5x10^{19}\ cm^{-3}s^{-1}}$$

6.2

(a) $R_{nO} = \dfrac{n_O}{\tau_{nO}}$

and

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(10^{10}\right)^2}{10^{16}} = 10^4\ cm^{-3}$$

Then

$$R_{nO} = \frac{10^4}{2x10^{-7}} \Rightarrow \underline{R_{nO} = 5x10^{10}\ cm^{-3}s^{-1}}$$

(b)

$$R_n = \frac{\delta n}{\tau_{nO}} = \frac{10^{12}}{2x10^{-7}} \text{ or } R_n = 5x10^{18}\ cm^{-3}s^{-1}$$

so

$$\Delta R_n = R_n - R_{nO} = 5x10^{18} - 5x10^{10} \Rightarrow$$

$$\underline{\Delta R_n \approx 5x10^{18}\ cm^{-3}s^{-1}}$$

6.3
(a) Recombination rates are equal

$$\frac{n_O}{\tau_{nO}} = \frac{p_O}{\tau_{pO}}$$

$$n_O = N_d = 10^{16}\ cm^{-3}$$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

So

$$\frac{10^{16}}{\tau_{nO}} = \frac{2.25x10^4}{20x10^{-6}}$$

or

$$\underline{\tau_{nO} = 8.89x10^{+6}\ s}$$

(b) Generation Rate = Recombination Rate
So

$$G = \frac{2.25x10^4}{20x10^{-6}} \Rightarrow \underline{G = 1.125x10^9\ cm^{-3}s^{-1}}$$

(c)

$$\underline{R = G = 1.125x10^9\ cm^{-3}s^{-1}}$$

6.4

(a) $E = h\nu = \dfrac{hc}{\lambda} = \dfrac{\left(6.625x10^{-34}\right)\left(3x10^8\right)}{6300x10^{-10}}$

or

$E = 3.15x10^{-19}\ J$ This is the energy of 1 photon.
Now

$1\,W = 1\ J/s \Rightarrow$ $3.17x10^{18}$ photons/s

Volume = $(1)(0.1) = 0.1\ cm^{+3}$

Then

$$g = \frac{3.17x10^{18}}{0.1} \Rightarrow$$

$$\underline{g = 3.17x10^{19}\ e-h\ pairs/cm^3-s}$$

(b)

$$\delta n = \delta p = g\tau = \left(3.17x10^{19}\right)\left(10x10^{-6}\right)$$

or

$$\underline{\delta n = \delta p = 3.17x10^{14}\ cm^{-3}}$$

6.5
We have

$$\frac{\partial p}{\partial t} = -\nabla \bullet F_p^+ + g_p - \frac{p}{\tau_p}$$

and

$$J_p = e\mu_p p\mathrm{E} - eD_p \nabla p$$

The hole particle current density is

$$F_p^+ = \frac{J_p}{(+e)} = \mu_p p\mathrm{E} - D_p \nabla p$$

Now

$$\nabla \bullet F_p^+ = \mu_p \nabla \bullet (p\mathrm{E}) - D_p \nabla \bullet \nabla p$$

We can write

$$\nabla \bullet (p\mathrm{E}) = \mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}$$

and

$$\nabla \bullet \nabla p = \nabla^2 p$$

so

$$\nabla \bullet F_p^+ = \mu_p(\mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}) - D_p \nabla^2 p$$

Then

$$\frac{\partial p}{\partial t} = -\mu_p(\mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}) + D_p \nabla^2 p + g_p - \frac{p}{\tau_p}$$

We can then write

$$D_p \nabla^2 p - \mu_p(\mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}) + g_p - \frac{p}{\tau_p} = \frac{\partial p}{\partial t}$$

6.6

From Equation [6.18]

$$\frac{\partial p}{\partial t} = -\nabla \bullet F_p^+ + g_p - \frac{p}{\tau_p}$$

For steady-state, $\frac{\partial p}{\partial t} = 0$

Then

$$0 = -\nabla \bullet F_p^+ + g_p - R_p$$

and for a one-dimensional case,

$$\frac{dF_p^+}{dx} = g_p - R_p = 10^{20} - 2x10^{19} \Rightarrow$$

$$\frac{dF_p^+}{dx} = 8x10^{19}\ cm^{-3}s^{-1}$$

6.7

From Equation [6.18],

$$0 = -\frac{dF_p^+}{dx} + 0 - 2x10^{19}$$

or

$$\frac{dF_p^+}{dx} = -2x10^{19}\ cm^{-3}s^{-1}$$

6.8

We have the continuity equations

(1) $$D_p \nabla^2(\delta p) - \mu_p[\mathrm{E} \bullet \nabla(\delta p) + p\nabla \bullet \mathrm{E}] + g_p - \frac{p}{\tau_p} = \frac{\partial(\delta p)}{\partial t}$$

and

(2) $$D_n \nabla^2(\delta n) + \mu_n[\mathrm{E} \bullet \nabla(\delta n) + n\nabla \bullet \mathrm{E}] + g_n - \frac{n}{\tau_n} = \frac{\partial(\delta n)}{\partial t}$$

By charge neutrality

$$\delta n = \delta p \equiv \delta n \Rightarrow \nabla(\delta n) = \nabla(\delta p)$$

and $\nabla^2(\delta n) = \nabla^2(\delta p)$ and $\frac{\partial(\delta n)}{\partial t} = \frac{\partial(\delta p)}{\partial t}$

Also

$$g_n = g_p \equiv g,\ \frac{p}{\tau_p} = \frac{n}{\tau_n} \equiv R$$

Then we can write

(1) $$D_p \nabla^2(\delta n) - \mu_p[\mathrm{E} \bullet \nabla(\delta n) + p\nabla \bullet \mathrm{E}] + g - R = \frac{\partial(\delta n)}{\partial t}$$

and

(2) $$D_n \nabla^2(\delta n) + \mu_n[\mathrm{E} \bullet \nabla(\delta n) + n\nabla \bullet \mathrm{E}] + g - R = \frac{\partial(\delta n)}{\partial t}$$

Multiply Equation (1) by $\mu_n n$ and Equation (2) by $\mu_p p$, and then add the two equations.

We find

$$\left(\mu_n n D_p + \mu_p p D_n\right)\nabla^2(\delta n) + \mu_n \mu_p (p - n)\mathrm{E} \bullet \nabla(\delta n) + \left(\mu_n n + \mu_p p\right)(g - R) = \left(\mu_n n + \mu_p p\right)\frac{\partial(\delta n)}{\partial t}$$

Divide by $\left(\mu_n n + \mu_p p\right)$, then

$$\left(\frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p}\right)\nabla^2(\delta n) + \left[\frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p}\right]\mathrm{E} \bullet \nabla(\delta n) + (g - R) = \frac{\partial(\delta n)}{\partial t}$$

Define

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

and $$\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

Then we have

$$D'\nabla^2(\delta n) + \mu' \mathrm{E} \bullet \nabla(\delta n) + (g - R) = \frac{\partial(\delta n)}{\partial t}$$

Q.E.D.

6.9

For Ge: $T = 300K$, $n_i = 2.4x10^{13}\ cm^{-3}$

$$n = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= 10^{13} + \sqrt{\left(10^{13}\right)^2 + \left(2.4x10^{13}\right)^2}$$

or

$$n = 3.6x10^{13}\ cm^{-3}$$

Also

$$p = \frac{n_i^2}{n} = \frac{\left(2.4x10^{13}\right)^2}{3.6x10^{13}} = 1.6x10^{13}\ cm^{-3}$$

We have

$$\mu_n = 3900,\ \mu_p = 1900$$

$$D_n = 101,\ D_p = 49.2$$

Now

$$D' = \frac{D_n D_p (n+p)}{D_n n + D_p p}$$

$$= \frac{(101)(49.2)\left(3.6x10^{13} + 1.6x10^{13}\right)}{(101)\left(3.6x10^{13}\right) + (49.2)\left(1.6x10^{13}\right)}$$

or

$$\underline{D' = 58.4\ cm^2/s}$$

Also

$$\mu' = \frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p}$$

$$= \frac{(3900)(1900)\left(1.6x10^{13} - 3.6x10^{13}\right)}{(3900)\left(3.6x10^{13}\right) + (1900)\left(1.6x10^{13}\right)}$$

or

$$\underline{\mu' = -868\ cm^2/V-s}$$

Now

$$\frac{n}{\tau_n} = \frac{p}{\tau_p} \Rightarrow \quad \frac{3.6x10^{13}}{\tau_n} = \frac{1.6x10^{13}}{24\ \mu s}$$

which yields

$$\underline{\tau_n = 54\ \mu s}$$

6.10

$$\sigma = e\mu_n n + e\mu_p p$$

With excess carriers present

$$n = n_O + \delta n \quad \text{and} \quad p = p_O + \delta p$$

For an n-type semiconductor, we can write

$$\delta n = \delta p \equiv \delta p$$

Then

$$\sigma = e\mu_n\left(n_O + \delta p\right) + e\mu_p\left(p_O + \delta p\right)$$

or

$$\sigma = e\mu_n n_O + e\mu_p p_O + e\left(\mu_n + \mu_p\right)(\delta p)$$

so

$$\underline{\Delta\sigma = e\left(\mu_n + \mu_p\right)(\delta p)}$$

In steady-state, $\delta p = g'\tau$

So that

$$\underline{\Delta\sigma = e\left(\mu_n + \mu_p\right)\left(g'\tau_{pO}\right)}$$

6.11

n-type, so that minority carriers are holes. Uniform generation throughout the sample means we have

$$g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

Homogeneous solution is of the form

$$(\delta p)_H = A\exp\left(\frac{-t}{\tau_{pO}}\right)$$

and the particular solution is

$$(\delta p)_P = g'\tau_{pO}$$

so that the total solution is

$$(\delta p) = g'\tau_{pO} + A\exp\left(\frac{-t}{\tau_{pO}}\right)$$

At $t = 0$, $\delta p = 0$ so that

$$0 = g'\tau_{pO} + A \Rightarrow A = -g'\tau_{pO}$$

Then

$$\delta p = g'\tau_{pO}\left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right)\right]$$

The conductivity is

$$\sigma = e\mu_n n_O + e\mu_p p_O + e\left(\mu_n + \mu_p\right)(\delta p)$$

$$\approx e\mu_n n_O + e\left(\mu_n + \mu_p\right)(\delta p)$$

so

$$\sigma = \left(1.6x10^{-19}\right)(1000)\left(5x10^{16}\right)$$

$$+\left(1.6x10^{-19}\right)(1000+420)\left(5x10^{21}\right)\left(10^{-7}\right)$$

$$\times\left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right)\right]$$

Then

$$\sigma = 8 + 0.114\left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right)\right]$$

where $\tau_{pO} = 10^{-7}\ s$

6.12
n-type GaAs:

$$\Delta\sigma = e(\mu_n + \mu_p)(\delta p)$$

In steady-state, $\delta p = g'\tau_{pO}$. Then

$$\Delta\sigma = (1.6x10^{-19})(8500 + 400)(2x10^{21})(2x10^{-7})$$

or

$$\Delta\sigma = 0.57\ (\Omega - cm)^{-1}$$

The steady-state excess carrier recombination rate

$$R' = g' = 2x10^{21}\ cm^{-3}s^{-1}$$

6.13
For $t < 0$, steady-state, so

$$\delta p(0) = g'\tau_{pO} = (5x10^{21})(3x10^{-7}) \Rightarrow$$

$$\delta p(0) = 1.5x10^{15}\ cm^{-3}$$

Now

$$\sigma = e\mu_n n_O + e(\mu_n + \mu_p)(\delta p)$$

For $t \ge 0$, $\delta p = \delta p(0)\exp(-t/\tau_{pO})$

Then

$$\sigma = (1.6x10^{-19})(1350)(5x10^{16})$$
$$+(1.6x10^{-19})(1350 + 480)(1.5x10^{15})\exp(-t/\tau_{pO})$$

or

$$\sigma = 10.8 + 0.439\exp(-t/\tau_{pO})$$

We have that

$$I = AJ = A\sigma E = \frac{A\sigma V}{L}$$

so

$$I = \frac{(10^{-4})(5)}{(0.10)}\left[10.8 + 0.439\exp(-t/\tau_{pO})\right]$$

or

$$I = \left[54 + 2.20\exp(-t/\tau_{pO})\right]\ mA$$

where

$$\tau_{pO} = 3x10^{-7}\ s$$

6.14
(a) p-type GaAs,

$$D_n\nabla^2(\delta n) + \mu_n E \bullet \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

Uniform generation rate, so that

$\nabla(\delta n) = \nabla^2(\delta n) = 0$, then

$$g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

The solution is of the form

$$\delta n = g'\tau_{nO}\left[1 - \exp(-t/\tau_{nO})\right]$$

Now

$$R'_n = \frac{\delta n}{\tau_{nO}} = g'\left[1 - \exp(-t/\tau_{nO})\right]$$

(b)
Maximum value at steady-state, $n_O = 10^{14}\ cm^{-3}$
So

$$(\delta n)_O = g'\tau_{nO} \Rightarrow \tau_{nO} = \frac{(\delta n)_O}{g'} = \frac{10^{14}}{10^{20}}$$

or

$$\tau_{nO} = 10^{-6}\ s$$

(c)
Determine t at which
(i) $\delta n = (0.75)x10^{14}\ cm^{-3}$
We have

$$0.75x10^{14} = 10^{14}\left[1 - \exp(-t/\tau_{nO})\right]$$

which yields

$$t = \tau_{nO}\ln\left(\frac{1}{1 - 0.75}\right) \Rightarrow t = 1.39\ \mu s$$

(ii) $\delta n = 0.5x10^{14}\ cm^{-3}$
We find

$$t = \tau_{nO}\ln\left(\frac{1}{1 - 0.5}\right) \Rightarrow t = 0.693\ \mu s$$

(iii) $\delta n = 0.25x10^{14}\ cm^{-3}$
We find

$$t = \tau_{nO}\ln\left(\frac{1}{1 - 0.25}\right) \Rightarrow t = 0.288\ \mu s$$

6.15
(a)

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.5x10^{10})^2}{10^{15}} 2.25x10^4\ cm^{-3}$$

Then

$$R_{pO} = \frac{p_O}{\tau_{pO}} \Rightarrow \tau_{pO} = \frac{p_O}{R_{pO}} = \frac{2.25x10^4}{10^{11}}$$

or

$$\tau_{pO} = 2.25x10^{-7}\ s$$

Now

$$R'_p = \frac{\delta p}{\tau_{pO}} = \frac{10^{14}}{2.25x10^{-7}} \Rightarrow$$

or

$$R'_p = 4.44x10^{20}\ cm^{-3}s^{-1}$$

Recombination rate increases by the factor

$$\frac{R'_p}{R_{pO}} = \frac{4.44x10^{20}}{10^{11}} \Rightarrow \frac{R'_p}{R_{pO}} = 4.44x10^9$$

(b)

From part (a), $\tau_{pO} = 2.25x10^{-7}\ s$

6.16

Silicon, n-type. For $0 \le t \le 10^{-7}\ s$

$$\delta p = g'\tau_{pO}\left[1 - \exp(-t/\tau_{pO})\right]$$

$$= (2x10^{20})(10^{-7})\left[1 - \exp(-t/\tau_{pO})\right]$$

or

$$\delta p = 2x10^{13}\left[1 - \exp(-t/\tau_{pO})\right]$$

At $t = 10^{-7}\ s$,

$$\delta p(10^{-7}) = 2x10^{13}\left[1 - \exp(-1)\right]$$

or

$$\delta p(10^{-7}) = 1.26x10^{13}\ cm^{-3}$$

For $t > 10^{-7}\ s$,

$$\delta p = (1.26x10^{13})\exp\left[\frac{-(t-10^{-7})}{\tau_{pO}}\right]$$

where

$$\tau_{pO} = 10^{-7}\ s$$

6.17

(a) For $0 < t < 2x10^{-6}\ s$

$$\delta n = g'\tau_{nO}\left[1 - \exp(-t/\tau_{nO})\right]$$

$$= (10^{20})(10^{-6})\left[1 - \exp[-t/\tau_{nO}]\right]$$

or

$$\delta n = 10^{14}\left[1 - \exp(-t/\tau_{nO})\right]$$

where $\tau_{nO} = 10^{-6}\ s$

At $t = 2x10^{-6}\ s$

$$\delta n(2\ \mu s) = (10^{14})\left[1 - \exp(-2/1)\right]$$

or

$$\delta n(2\ \mu s) = 0.865x10^{14}\ cm^{-3}$$

For $t > 2x10^{-6}\ s$

$$\delta n = 0.865x10^{14}\exp\left[\frac{-(t-2x10^{-6})}{\tau_{nO}}\right]$$

(b) (i) At $t = 0$, $\delta n = 0$

(ii) At $t = 2x10^{-6}\ s$, $\delta n = 0.865x10^{14}\ cm^{-3}$

(iii) At $t \to \infty$, $\delta n = 0$

6.18

p-type, minority carriers are electrons

In steady-state, $\dfrac{\partial(\delta n)}{\partial t} = 0$, then

(a)

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

Solution is of the form

$$\delta n = A\exp(-x/L_n) + B\exp(+x/L_n)$$

But $\delta n = 0$ as $x \to \infty$ so that $B \equiv 0$.

At $x = 0$, $\delta n = 10^{13}\ cm^{-3}$

Then

$$\delta n = 10^{13}\exp(-x/L_n)$$

Now

$$L_n = \sqrt{D_n\tau_{nO}}\ , \text{ where } D_n = \mu_n\left(\frac{kT}{e}\right)$$

or

$$D_n = (0.0259)(1200) = 31.1\ cm^2/s$$

Then

$$L_n = \sqrt{(31.1)(5x10^{-7})} \Rightarrow$$

or

$$L_n = 39.4\ \mu m$$

(b)

$$J_n = eD_n \frac{d(\delta n)}{dx} = \frac{eD_n(10^{13})}{(-L_n)} \exp(-x/L_n)$$

$$= \frac{-(1.6x10^{-19})(31.1)(10^{13})}{39.4x10^{-4}} \exp(-x/L_n)$$

or

$$\underline{J_n = -12.6 \exp(-x/L_n) \quad mA/cm^2}$$

6.19

(a) p-type silicon, $p_{pO} = 10^{14}\ cm^{-3}$ and

$$n_{pO} = \frac{n_i^2}{p_{pO}} = \frac{(1.5x10^{10})^2}{10^{14}} = 2.25x10^6\ cm^{-3}$$

(b) Excess minority carrier concentration

$$\delta n = n_p - n_{pO}$$

At $x = 0$, $n_p = 0$ so that

$$\delta n(0) = 0 - n_{pO} = -2.25x10^6\ cm^{-3}$$

(c) For the one-dimensional case,

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0 \quad \text{where} \quad L_n^2 = D_n \tau_{nO}$$

The general solution is of the form

$$\delta n = A\exp(-x/L_n) + B\exp(+x/L_n)$$

For $x \to \infty$, δn remains finite, so that $B = 0$.
Then the solution is

$$\underline{\delta n = -n_{pO} \exp(-x/L_n)}$$

6.20

p-type so electrons are the minority carriers

$$D_n \nabla^2(\delta n) + \mu_n \mathrm{E} \bullet \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

For steady state, $\frac{\partial(\delta n)}{\partial t} = 0$ and for $x>0$,
$g' = 0$, $\mathrm{E} = 0$, so we have

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0 \quad \text{or} \quad \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

where $L_n^2 = D_n \tau_{nO}$

The solution is of the form

$$\delta n = A\exp(-x/L_n) + B\exp(+x/L_n)$$

The excess concentration δn must remain finite, so that $B = 0$. At $x = 0$, $\delta n(0) = 10^{15}\ cm^{-3}$, so the solution is

$$\delta n = 10^{15} \exp(-x/L_n)$$

We have that $\mu_n = 1050\ cm^2/V-s$, then

$$D_n = \mu_n\left(\frac{kT}{e}\right) = (1050)(0.0259) = 27.2\ cm^2/s$$

Then

$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(27.2)(8x10^{-7})} \Rightarrow$$

$$L_n = 46.6\ \mu m$$

(a)

Electron diffusion current density at $x = 0$

$$J_n = eD_n \frac{d(\delta n)}{dx}\Big|_{x=0}$$

$$= eD_n \frac{d}{dx}\left[10^{15}\exp(-x/L_n)\right]_{x=0}$$

$$= \frac{-eD_n(10^{15})}{L_n} = \frac{-(1.6x10^{-19})(27.2)(10^{15})}{46.6x10^{-4}}$$

or

$$\underline{J_n = -0.934\ A/cm^2}$$

Since $\delta p = \delta n$, excess holes diffuse at the same rate as excess electrons, then

$$\underline{J_p(x=0) = +0.934\ A/cm^2}$$

(b)

At $x = L_n$,

$$J_n = eD_n \frac{d(\delta n)}{dx}\Big|_{x=L_n} = \frac{eD_n(10^{15})}{(-L_n)}\exp(-1)$$

$$= \frac{-(1.6x10^{-19})(27.2)(10^{15})}{46.6x10^{-4}}\exp(-1)$$

or

$$\underline{J_n = -0.344\ A/cm^2}$$

Then

$$\underline{J_p = +0.344\ A/cm^2}$$

6.21

n-type, so we have

$$D_p \frac{d^2(\delta p)}{dx^2} - \mu_p \mathrm{E}_O \frac{d(\delta p)}{dx} - \frac{\delta p}{\tau_{pO}} = 0$$

Assume the solution is of the form

$$\delta p = A\exp(sx)$$

Then

$$\frac{d(\delta p)}{dx} = As\exp(sx)\,,\quad \frac{d^2(\delta p)}{dx^2} = As^2\exp(sx)$$

Substituting into the differential equation

$$D_p As^2\exp(sx) - \mu_p \mathrm{E}_O As\exp(sx) - \frac{A\exp(sx)}{\tau_{pO}} = 0$$

or

$$D_p s^2 - \mu_p \mathrm{E}_O s - \frac{1}{\tau_{pO}} = 0$$

Dividing by D_p

$$s^2 - \frac{\mu_p}{D_p}\mathrm{E}_O s - \frac{1}{L_p^2} = 0$$

The solution for s is

$$s = \frac{1}{2}\left[\frac{\mu_p}{D_p}\mathrm{E}_O \pm \sqrt{\left(\frac{\mu_p}{D_p}\mathrm{E}_O\right)^2 + \frac{4}{L_p^2}}\right]$$

This can be rewritten as

$$s = \frac{1}{L_p}\left[\frac{\mu_p L_p \mathrm{E}_O}{2D_p} \pm \sqrt{\left(\frac{\mu_p L_p \mathrm{E}_O}{2D_p}\right)^2 + 1}\right]$$

We may define

$$\beta \equiv \frac{\mu_p L_p \mathrm{E}_O}{2D_p}$$

Then

$$s = \frac{1}{L_p}\left[\beta \pm \sqrt{1+\beta^2}\right]$$

In order that $\delta p = 0$ for $x > 0$, use the minus sign for $x > 0$ and the plus sign for $x < 0$. Then the solution is

$$\underline{\delta p(x) = A\exp(s_{-}x)} \text{ for } x > 0$$

$$\underline{\delta p(x) = A\exp(s_{+}x)} \text{ for } x < 0$$

where

$$\underline{s_{\pm} = \frac{1}{L_p}\left[\beta \pm \sqrt{1+\beta^2}\right]}$$

6.22
Computer Plot

6.23
(a) From Equation [6.55],

$$D_n \frac{d^2(\delta n)}{dx^2} + \mu_n \mathrm{E}_O \frac{d(\delta n)}{dx} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} + \frac{\mu_n}{D_n}\mathrm{E}_O \frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

We have that

$$\frac{D_n}{\mu_n} = \left(\frac{kT}{e}\right) \text{ so we can define}$$

$$\frac{\mu_n}{D_n}\mathrm{E}_O = \frac{\mathrm{E}_O}{(kT/e)} \equiv \frac{1}{L'}$$

Then we can write

$$\frac{d^2(\delta n)}{dx^2} + \frac{1}{L'}\cdot\frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

Solution will be of the form

$$\delta n = \delta n(0)\exp(-\alpha x) \quad \text{where} \quad \alpha > 0$$

Then

$$\frac{d(\delta n)}{dx} = -\alpha(\delta n) \quad \text{and} \quad \frac{d^2(\delta n)}{dx^2} = \alpha^2(\delta n)$$

Substituting into the differential equation, we have

$$\alpha^2(\delta n) + \frac{1}{L'}\cdot\left[-\alpha(\delta n)\right] - \frac{\delta n}{L_n^2} = 0$$

or

$$\alpha^2 - \frac{\alpha}{L'} - \frac{1}{L_n^2} = 0$$

which yields

$$\alpha = \frac{1}{L_n}\left\{\frac{L_n}{2L'} + \sqrt{\left(\frac{L_n}{2L'}\right)^2 + 1}\right\}$$

Note that if $\mathrm{E}_O = 0$, $L' \to \infty$, then $\alpha = \frac{1}{L_n}$

(b)

$$L_n = \sqrt{D_n \tau_{nO}} \quad \text{where} \quad D_n = \mu_n\left(\frac{dT}{e}\right)$$

or

$$D_n = (1200)(0.0259) = 31.1\ cm^2/s$$

Then

$$L_n = \sqrt{(31.1)(5x10^{-7})} = 39.4\ \mu m$$

For $\mathrm{E}_O = 12\ V/cm$, then

$$L' = \frac{(kT/e)}{\mathrm{E}_O} = \frac{0.0259}{12} = 21.6x10^{-4}\ cm$$

Then

$$\alpha = 5.75x10^{2}\ cm^{-1}$$

(c)
Force on the electrons due to the electric field is in the negative x-direction. Therefore, the effective diffusion of the electrons is reduced and the concentration drops off faster with the applied electric field.

6.24
p-type so the minority carriers are electrons, then

$$D_n\nabla^2(\delta n) + \mu_n\mathrm{E}\bullet\nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

Uniform illumination means that

$\nabla(\delta n) = \nabla^2(\delta n) = 0$. For $\tau_{nO} = \infty$, we are left with

$$\frac{d(\delta n)}{dt} = g' \quad \text{which gives} \quad \delta n = g't + C_1$$

For $t < 0$, $\delta n = 0$ which means that $C_1 = 0$. Then

$$\underline{\delta n = G_O' t} \quad \text{for} \quad 0 \le t \le T$$

For $t > T$, $g' = 0$ so we have $\dfrac{d(\delta n)}{dt} = 0$

Or

$$\underline{\delta n = G_O' T} \quad \text{(No recombination)}$$

6.25
n-type so minority carriers are holes, then

$$D_p\nabla^2(\delta p) - \mu_p\mathrm{E}\bullet\nabla(\delta p) + g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

We have $\tau_{pO} = \infty$, $\mathrm{E} = 0$, $\dfrac{\partial(\delta p)}{\partial t} = 0$ (steady state). Then we have

$$D_p\frac{d^2(\delta p)}{dx^2} + g' = 0 \quad \text{or} \quad \frac{d^2(\delta p)}{dx^2} = -\frac{g'}{D_p}$$

For $-L < x < +L$, $g' = G_O' =$ constant. Then

$$\frac{d(\delta p)}{dx} = -\frac{G_O'}{D_p}x + C_1 \quad \text{and}$$

$$\delta p = -\frac{G_O'}{2D_p}x^2 + C_1x + C_2$$

For $L < x < 3L$, $g' = 0$ so we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \text{ so that } \frac{d(\delta p)}{dx} = C_3 \quad \text{and}$$

$$\delta p = C_3x + C_4$$

For $-3L < x < -L$, $g' = 0$ so that

$$\frac{d^2(\delta p)}{dx^2} = 0,\ \frac{d(\delta p)}{dx} = C_5,\ \text{and}$$

$$\delta p = C_5x + C_6$$

The boundary conditions are
(1) $\delta p = 0$ at $x = +3L$; (2) $\delta p = 0$ at $x = -3L$;
(3) δp continuous at $x = +L$; (4) δp continuous at $x = -L$; The flux must be continuous so that
(5) $\dfrac{d(\delta p)}{dx}$ continuous at $x = +L$; (6) $\dfrac{d(\delta p)}{dx}$ continuous at $x = -L$.

Applying these boundary conditions, we find

$$\delta p = \frac{G_O'}{2D_p}(5L^2 - x^2) \quad \text{for} \quad -L < x < +L$$

$$\delta p = \frac{G_O'L}{D_p}(3L - x) \quad \text{for} \quad L < x < 3L$$

$$\delta p = \frac{G_O'L}{D_p}(3L + x) \quad \text{for} \quad -3L < x < -L$$

6.26

$$\mu_p = \frac{d}{\mathrm{E}_O t} = \frac{0.75}{\left(\frac{2.5}{1}\right)(160x10^{-6})} = 1875\ cm^2/V-s$$

Then

$$D_p = \frac{(\mu_p\mathrm{E}_O)^2(\Delta t)^2}{16t_O}$$

$$= \frac{\left[(1875)\left(\frac{2.5}{1}\right)\right]^2(75.5x10^{-6})^2}{16(160x10^{-6})}$$

which gives

$$D_p = 48.9\ cm^2/s$$

From the Einstein relation,

$$\frac{D_p}{\mu_p} = \frac{kT}{e} = \frac{48.9}{1875} = 0.02608\ V$$

6.27

Assume that $f(x,t) = (4\pi Dt)^{-1/2} \exp\left(\frac{-x^2}{4Dt}\right)$

is the solution to the differential equation

$$D\left(\frac{\partial^2 f}{\partial x^2}\right) = \frac{\partial f}{\partial t}$$

To prove: we can write

$$\frac{\partial f}{\partial x} = (4\pi Dt)^{-1/2}\left(\frac{-2x}{4Dt}\right)\exp\left(\frac{-x^2}{4Dt}\right)$$

and

$$\frac{\partial^2 f}{\partial x^2} = (4\pi Dt)^{-1/2}\left(\frac{-2x}{4Dt}\right)^2 \exp\left(\frac{-x^2}{4Dt}\right) + (4\pi Dt)^{-1/2}\left(\frac{-2}{4Dt}\right)\exp\left(\frac{-x^2}{4Dt}\right)$$

Also

$$\frac{\partial f}{\partial t} = (4\pi Dt)^{-1/2}\left(\frac{-x^2}{4D}\right)\left(\frac{-1}{t^2}\right)\exp\left(\frac{-x^2}{4Dt}\right) + (4\pi D)^{-1/2}\left(\frac{-1}{2}\right)t^{-3/2}\exp\left(\frac{-x^2}{4Dt}\right)$$

Substituting the expressions for $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial f}{\partial t}$ into the differential equation, we find $0 = 0$, Q.E.D.

6.28

Computer Plot

6.29

n-type

$$\delta n = \delta p = g'\tau_{pO} = \left(10^{21}\right)\left(10^{-6}\right) = 10^{15}\ cm^{-3}$$

We have $n_O = 10^{16}\ cm^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

Now

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_O + \delta n}{n_i}\right) = (0.0259)\ln\left(\frac{10^{16} + 10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.3498\ eV}$$

and

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right) = (0.0259)\ln\left(\frac{2.25x10^4 + 10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.2877\ eV}$$

6.30

(a) p-type

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right) = (0.0259)\ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.3294\ eV}$$

(b)

$$\delta n = \delta p = 5x10^{14}\ cm^{-3}$$

and

$$n_O = \frac{\left(1.5x10^{10}\right)^2}{5x10^{15}} = 4.5x10^4\ cm^{-3}$$

Then

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_O + \delta n}{n_i}\right) = (0.0259)\ln\left(\frac{4.5x10^4 + 5x10^{14}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.2697\ eV}$$

and

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right) = (0.0259)\ln\left(\frac{5x10^{15} + 5x10^{14}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.3318\ eV}$$

6.31

n-type GaAs; $n_O = 5x10^{16}\ cm^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.8x10^6\right)^2}{5x10^{16}} = 6.48x10^{-5}\ cm^{-3}$$

We have

$$\delta n = \delta p = (0.1)N_d = 5x10^{15}\ cm^{-3}$$

(a)

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_O + \delta n}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{16} + 5x10^{15}}{1.8x10^6}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.6253\ eV}$$

We have

$$E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{16}}{1.8x10^6}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.6228\ eV}$$

Now

$$E_{Fn} - E_F = \left(E_{Fn} - E_{Fi}\right) - \left(E_F - E_{Fi}\right)$$
$$= 0.6253 - 0.6228$$

so

$$\underline{E_{Fn} - E_F = 0.0025\ eV}$$

(b)

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{15}}{1.8x10^6}\right)$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.5632\ eV}$$

6.32

Quasi-Fermi level for minority carrier electrons

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_O + \delta n}{n_i}\right)$$

We have

$$\delta n = \left(10^{14}\right)\left(\frac{x}{50\ \mu m}\right)$$

Neglecting the minority carrier electron concentration

$$E_{Fn} - E_{Fi} = kT\ln\left[\frac{\left(10^{14}\right)(x)}{(50\ \mu m)\left(1.8x10^6\right)}\right]$$

We find

$x(\mu m)$	$E_{Fn} - E_{Fi}$ (eV)
0	–0.581
1	+0.361
2	+0.379
10	+0.420
20	+0.438
50	+0.462

Quasi-Fermi level for holes: we have

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right)$$

We have $p_O = 10^{16}\ cm^{-3}$, $\delta p = \delta n$

We find

$x(\mu m)$	$E_{Fi} - E_{Fp}$ (eV)
0	+0.58115
50	+0.58140

6.33

(a) We can write

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$

and

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right)$$

so that

$$\left(E_{Fi} - E_{Fp}\right) - \left(E_{Fi} - E_F\right) = E_F - E_{Fp}$$

$$= kT\ln\left(\frac{p_O + \delta p}{n_i}\right) - kT\ln\left(\frac{p_O}{n_i}\right)$$

or

$$E_F - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{p_O}\right) = (0.01)kT$$

Then

$$\frac{p_O + \delta p}{p_O} = \exp(0.01) = 1.010 \Rightarrow$$

$$\frac{\delta p}{p_O} = 0.010 \Rightarrow \text{ low-injection, so that}$$

$$\underline{\delta p = 5x10^{12}\ cm^{-3}}$$

(b)

$$E_{Fn} - E_{Fi} \approx kT \ln\left(\frac{\delta p}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{12}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.1505\ eV}$$

6.34
Computer Plot

6.35
Computer Plot

6.36
(a)

$$R = \frac{C_n C_p N_t\left(np - n_i^2\right)}{C_n(n+n') + C_p(p+p')}$$

$$= \frac{\left(np - n_i^2\right)}{\tau_{pO}(n+n') + \tau_{nO}(p+p')}$$

For $n = p = 0$

$$R = \frac{-n_i^2}{\tau_{pO}n_i + \tau_{nO}n_i} \Rightarrow \quad \underline{R = \frac{-n_i}{\tau_{pO} + \tau_{nO}}}$$

(b)
We had defined the net generation rate as $g - R = g_O + g' - (R_O + R')$ where $g_O = R_O$ since these are the thermal equilibrium generation and recombination rates. If $g' = 0$, then $g - R = -R'$ and $R' = \frac{-n_i}{\tau_{pO} + \tau_{nO}}$ so that $g - R = +\frac{n_i}{\tau_{pO} + \tau_{nO}}$. Thus a negative recombination rate implies a net positive generation rate.

6.37
We have that

$$R = \frac{C_n C_p N_t\left(np - n_i^2\right)}{C_n(n+n') + C_p(p+p')}$$

$$= \frac{\left(np - n_i^2\right)}{\tau_{pO}(n+n_i) + \tau_{nO}(p+n_i)}$$

If $n = n_O + \delta n$ and $p = p_O + \delta n$, then

$$R = \frac{(n_O + \delta n)(p_O + \delta n) - n_i^2}{\tau_{pO}(n_O + \delta n + n_i) + \tau_{nO}(p_O + \delta n + n_i)}$$

$$= \frac{n_O p_O + \delta n(n_O + p_O) + (\delta n)^2 - n_i^2}{\tau_{pO}(n_O + \delta n + n_i) + \tau_{nO}(p_O + \delta n + n_i)}$$

If $\delta n << n_i$, we can neglect the $(\delta n)^2$; also $n_O p_O = n_i^2$
Then

$$R = \frac{\delta n(n_O + p_O)}{\tau_{pO}(n_O + n_i) + \tau_{nO}(p_O + n_i)}$$

(a)
For n-type, $n_O >> p_O$, $n_O >> n_i$
Then

$$\underline{\frac{R}{\delta n} = \frac{1}{\tau_{pO}} = 10^{+7}\ s^{-1}}$$

(b)
Intrinsic, $n_O = p_O = n_i$
Then

$$\frac{R}{\delta n} = \frac{2n_i}{\tau_{pO}(2n_i) + \tau_{nO}(2n_i)}$$

or

$$\frac{R}{\delta n} = \frac{1}{\tau_{pO} + \tau_{nO}} = \frac{1}{10^{-7} + 5x10^{-7}} \Rightarrow$$

$$\underline{\frac{R}{\delta n} = 1.67x10^{+6}\ s^{-1}}$$

(c)
p-type, $p_O >> n_O$, $p_O >> n_i$
Then

$$\underline{\frac{R}{\delta n} = \frac{1}{\tau_{nO}} = \frac{1}{5x10^{-7}} = 2x10^{+6}\ s^{-1}}$$

6.38

(a) From Equation [6.56],

$$D_p \frac{d^2(\delta p)}{dx^2} + g' - \frac{\delta p}{\tau_{pO}} = 0$$

Solution is of the form

$$\delta p = g'\tau_{pO} + A\exp(-x/L_p) + B\exp(+x/L_p)$$

At $x = \infty$, $\delta p = g'\tau_{pO}$, so that $B \equiv 0$,

Then

$$\delta p = g'\tau_{pO} + A\exp(-x/L_p)$$

We have

$$D_p \frac{d(\delta p)}{dx}\Big|_{x=0} = s(\delta p)\Big|_{x=0}$$

We can write

$$\frac{d(\delta p)}{dx}\Big|_{x=0} = \frac{-A}{L_p} \quad \text{and} \quad (\delta p)\Big|_{x=0} = g'\tau_{pO} + A$$

Then

$$\frac{-AD_p}{L_p} = s(g'\tau_{pO} + A)$$

Solving for A we find

$$A = \frac{-sg'\tau_{pO}}{\frac{D_p}{L_p} + s}$$

The excess concentration is then

$$\delta p = g'\tau_{pO}\left[1 - \frac{s}{(D_p/L_p) + s} \cdot \exp\left(\frac{-x}{L_p}\right)\right]$$

where

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(10)(10^{-7})} = 10^{-3}\ cm$$

Now

$$\delta p = (10^{21})(10^{-7})\left[1 - \frac{s}{(10/10^{-3}) + s}\exp\left(\frac{-x}{L_p}\right)\right]$$

or

$$\underline{\delta p = 10^{14}\left[1 - \frac{s}{10^4 + s}\exp\left(\frac{-x}{L_p}\right)\right]}$$

(i) $s = 0$, $\underline{\delta p = 10^{14}\ cm^{-3}}$

(ii) $s = 2000\ cm/s$,

$$\underline{\delta p = 10^{14}\left[1 - 0.167\exp\left(\frac{-x}{L_p}\right)\right]}$$

(iii) $s = \infty$, $\underline{\delta p = 10^{14}\left[1 - \exp\left(\frac{-x}{L_p}\right)\right]}$

(b) (i) $s = 0$, $\underline{\delta p(0) = 10^{14}\ cm^{-3}}$

(ii) $s = 2000\ cm/s$, $\underline{\delta p(0) = 0.833x10^{14}\ cm^{-3}}$

(iii) $s = \infty$, $\underline{\delta p(0) = 0}$

6.39

$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(25)(5x10^{-7})} = 35.4x10^{-4}\ cm$$

(a)

At $x = 0$, $g'\tau_{nO} = (2x10^{21})(5x10^{-7}) = 10^{15}\ cm^{-3}$

Or $\delta n_O = g'\tau_{nO} = 10^{15}\ cm^{-3}$

For $x > 0$

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0 \Rightarrow \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

Solution is of the form

$$\delta n = A\exp(-x/L_n) + B\exp(+x/L_n)$$

At $x = 0$, $\delta n = \delta n_O = A + B$

At $x = W$,

$$\delta n = 0 = A\exp(-W/L_n) + B\exp(+W/L_n)$$

Solving these two equations, we find

$$A = \frac{-\delta n_O \exp(+2W/L_n)}{1 - \exp(2W/L_n)}$$

$$B = \frac{\delta n_O}{1 - \exp(2W/L_n)}$$

Substituting into the general solution, we find

$$\delta n = \frac{\delta n_O}{[\exp(+W/L_n) - \exp(-W/L_n)]} \times \{\exp[+(W-x)/L_n] - \exp[-(W-x)/L_n]\}$$

or

$$\underline{\delta n = \frac{\delta n_O \sinh[(W-x)/L_n]}{\sinh[W/L_n]}}$$

where

$\underline{\delta n_O = 10^{15}\ cm^{-3}}$ and $\underline{L_n = 35.4\ \mu m}$

(b)

If $\tau_{nO} = \infty$, we have

$$\frac{d^2(\delta n)}{dx^2} = 0$$

so the solution is of the form

$$\delta n = Cx + D$$

Applying the boundary conditions, we find

$$\delta n = \delta n_O \left(1 - \frac{x}{W}\right)$$

6.40

For $\tau_{pO} = \infty$, we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \quad \text{so that} \quad \frac{d(\delta p)}{dx} = A \quad \text{and}$$

$$\delta p = Ax + B$$

At $x = W$

$$-D_p \frac{d(\delta p)}{dx}\Big|_{x=W} = s \cdot (\delta p)\big|_{x=W}$$

or

$$-D_p A = s(AW + B)$$

which yields

$$B = \frac{-A}{s}\left(D_p + sW\right)$$

At $x = 0$, the flux of excess holes is

$$10^{19} = -D_p \frac{d(\delta p)}{dx}\Big|_{x=0} = -D_p A$$

so that

$$A = \frac{-10^{19}}{10} = -10^{18} \ cm^{-4}$$

and

$$B = \frac{10^{18}}{s}(10 + sW) = 10^{18}\left(\frac{10}{s} + W\right)$$

The solution is now

$$\delta p = 10^{18}\left(W - x + \frac{10}{s}\right)$$

(a)
For $s = \infty$,

$$\delta p = 10^{18}\left(20x10^{-4} - x\right) cm^{-3}$$

(b)
For $s = 2x10^3 \ cm/s$

$$\delta p = 10^{18}\left(70x10^{-4} - x\right) cm^{-3}$$

6.41

For $-W < x < 0$,

$$D_n \frac{d^2(\delta n)}{dx^2} + G_O' = 0$$

so that

$$\frac{d(\delta n)}{dx} = -\frac{G_O'}{D_p} x + C_1$$

and

$$\delta n = -\frac{G_O'}{2D_n} x^2 + C_1 x + C_2$$

For $0 < x < W$,

$$\frac{d^2(\delta n)}{dx^2} = 0, \text{ so } \frac{d(\delta n)}{dx} = C_3, \ \delta n = C_3 x + C_4$$

The boundary conditions are:

(1) $s = 0$ at $x = -W$, so that $\frac{d(\delta n)}{dx}\Big|_{x=-W} = 0$

(2) $s = \infty$ at $x = +W$, so that $\delta n(W) = 0$

(3) δn continuous at $x = 0$

(4) $\frac{d(\delta n)}{dx}$ continuous at $x = 0$

Applying the boundary conditions, we find

$$C_1 = C_3 = -\frac{G_O' W}{D_n}, \quad C_2 = C_4 = +\frac{G_O' W^2}{D_n}$$

Then, for $-W < x < 0$

$$\delta n = \frac{G_O'}{2D_n}\left(-x^2 - 2Wx + 2W^2\right)$$

and for $0 < x < +W$

$$\delta n = \frac{G_O' W}{D_n}(W - x)$$

6.42

Computer Plot

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Chapter 7

Problem Solutions

7.1

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

where $V_t = 0.0259\ V$ and $n_i = 1.5x10^{10}\ cm^{-3}$

We find

(a)

For $N_d = 10^{15}\ cm^{-3}$	$V_{bi}(V)$
$(i)\quad N_a = 10^{15}\ cm^{-3}$	$0.575\ V$
$(ii)\quad N_a = 10^{16}\ cm^{-3}$	0.635
$(iii)\quad N_a = 10^{17}\ cm^{-3}$	0.695
$(iv)\quad N_a = 10^{18}\ cm^{-3}$	0.754

(b)

For $N_d = 10^{18}\ cm^{-3}$	$V_{bi}(V)$
$(i)\quad N_a = 10^{15}\ cm^{-3}$	$0.754\ V$
$(ii)\quad N_a = 10^{16}\ cm^{-3}$	0.814
$(iii)\quad N_a = 10^{17}\ cm^{-3}$	0.874
$(iv)\quad N_a = 10^{18}\ cm^{-3}$	0.933

7.2

Si: $n_i = 1.5x10^{10}\ cm^{-3}$

Ge: $n_i = 2.4x10^{13}\ cm^{-3}$

GaAs: $n_i = 1.8x10^{6}\ cm^{-3}$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \quad \text{and} \quad V_t = 0.0259\ V$$

(a)

$N_d = 10^{14}\ cm^{-3}$, $N_a = 10^{17}\ cm^{-3}$

Then

$\underline{Si: V_{bi} = 0.635\ V}$, $\underline{Ge: V_{bi} = 0.253\ V}$,

$\underline{GaAs: V_{bi} = 1.10\ V}$

(b)

$N_d = 5x10^{16}\ cm^{-3}$, $N_a = 5x10^{16}\ cm^{-3}$

Then

$\underline{Si: V_{bi} = 0.778\ V}$, $\underline{Ge: V_{bi} = 0.396\ V}$,

$\underline{GaAs: V_{bi} = 1.25\ V}$

(c)

$N_d = 10^{17}\ cm^{-3}$, $N_a = 10^{17}\ cm^{-3}$

Then

$\underline{Si: V_{bi} = 0.814\ V}$, $\underline{Ge: V_{bi} = 0.432\ V}$,

$\underline{GaAs: V_{bi} = 1.28\ V}$

7.3

Computer Plot

7.4

Computer Plot

7.5

(a) n-side:

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.3294\ eV}$$

p-side:

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{10^{17}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.4070\ eV}$$

(b)

$$V_{bi} = 0.3294 + 0.4070$$

or

$$\underline{V_{bi} = 0.7364\ V}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259) \ln\left[\frac{(10^{17})(5x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.7363\ V}$$

(d)

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.736)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{17}}{5x10^{15}}\right)\left(\frac{1}{10^{17}+5x10^{15}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 0.426\ \mu m}$$

Now

$$x_p = \left[\frac{2(11.7)(8.85x10^{-14})(0.736)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{5x10^{15}}{10^{17}}\right)\left(\frac{1}{10^{17}+5x10^{15}}\right)\right]^{1/2}$$

or

$$\underline{x_p = 0.0213\ \mu m}$$

We have

$$|E_{max}| = \frac{eN_d x_n}{\in}$$

$$= \frac{(1.6x10^{-19})(5x10^{15})(0.426x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 3.29x10^4\ V/cm}$$

7.6

(a) n-side

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{2x10^{16}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_F - E_{Fi} = 0.3653\ eV}$$

p-side

$$E_{Fi} - E_F = (0.0259)\ln\left(\frac{2x10^{16}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_F = 0.3653\ eV}$$

(b)

$$V_{bi} = 0.3653 + 0.3653 \Rightarrow$$

$$\underline{V_{bi} = 0.7306\ V}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(2x10^{16})(2x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.7305\ V}$$

(d)

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})(0.7305)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{2x10^{16}}{2x10^{16}}\right)\left(\frac{1}{2x10^{16}+2x10^{16}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 0.154\ \mu m}$$

By symmetry

$$\underline{x_p = 0.154\ \mu m}$$

Now

$$|E_{max}| = \frac{eN_d x_n}{\in}$$

$$= \frac{(1.6x10^{-19})(2x10^{16})(0.154x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 4.76x10^4\ V/cm}$$

7.7

(b) $n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$

$$= 2.8x10^{19}\exp\left(\frac{-0.21}{0.0259}\right)$$

or

$$\underline{n_O = N_d = 8.43x10^{15}\ cm^{-3}} \quad \text{(n-region)}$$

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$$= 1.04x10^{19}\exp\left(\frac{-0.18}{0.0259}\right)$$

or

$$\underline{p_O = N_a = 9.97x10^{15}\ cm^{-3}} \quad \text{(p-region)}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(9.97x10^{15})(8.43x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.690\ V}$$

7.8

(a) GaAs: $V_{bi} = 1.20\ V$, $n_i = 1.8x10^6\ cm^{-3}$

$$x_p = 0.2W = 0.2(x_n + x_p)$$

or

$$\frac{x_p}{x_n} = 0.25$$

Also

$$N_d x_n = N_a x_p \Rightarrow \frac{x_p}{x_n} = \frac{N_d}{N_a} = 0.25$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

or

$$1.20 = (0.0259)\ln\left(\frac{0.25N_a^2}{n_i^2}\right)$$

Then

$$\frac{0.25N_a^2}{n_i^2} = \exp\left(\frac{1.20}{0.0259}\right)$$

or

$$N_a = 2n_i \exp\left[\frac{1.20}{2(0.0259)}\right]$$

or

$$\underline{N_a = 4.14x10^{16}\ cm^{-3}}$$

(b)

$$N_d = 0.25N_a \Rightarrow \underline{N_d = 1.04x10^{16}\ cm^{-3}}$$

(c)

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(1.20)}{1.6x10^{-19}} \times\left(\frac{4}{1}\right)\left(\frac{1}{4.14x10^{16} + 1.04x10^{16}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 0.366\ \mu m}$$

(d)

$$x_p = 0.25x_n \Rightarrow \underline{x_p = 0.0916\ \mu m}$$

(e)

$$\mathrm{E}_{max} = \frac{eN_d x_n}{\in} = \frac{eN_a x_p}{\in}$$

$$= \frac{(1.6x10^{-19})(1.04x10^{16})(0.366x10^{-4})}{(13.1)(8.85x10^{-14})}$$

or

$$\underline{\mathrm{E}_{max} = 5.25x10^4\ V/cm}$$

7.9

(a)

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.635\ V}$$

(b)

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.635)}{1.6x10^{-19}} \times\left(\frac{10^{16}}{10^{15}}\right)\left(\frac{1}{10^{16} + 10^{15}}\right)\right]^{1/2}$$

or $\underline{x_n = 0.864\ \mu m}$

Now

$$x_p = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.635)}{1.6x10^{-19}} \times\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{16} + 10^{15}}\right)\right]^{1/2}$$

or $\underline{x_p = 0.0864\ \mu m}$

(c)

$$\mathrm{E}_{max} = \frac{eN_d x_n}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{15})(0.864x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{\mathrm{E}_{max} = 1.34x10^4\ V/cm}$$

7.10

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \quad \text{and}$$

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

We can write

$$N_C N_V = N_{CO} N_{VO}\left(\frac{T}{300}\right)^3$$

Now

$$V_{bi} = V_t\left[\ln(N_a N_d) - \ln(n_i^2)\right]$$

$$= V_t\left[\ln(N_a N_d) - \ln(N_{CO}N_{VO}) - \ln\left(\frac{T}{300}\right)^3 + \frac{E_g}{kT}\right]$$

or

$$V_{bi} = V_t\left[\ln\left(\frac{N_a N_d}{N_{CO}N_{VO}}\right) - 3\ln\left(\frac{T}{300}\right) + \frac{E_g}{kT}\right]$$

or

$$0.40 = (0.0250)\left(\frac{T}{300}\right) \times \left[\ln\left[\frac{(5x10^{15})(10^{16})}{(2.8x10^{19})(1.04x10^{19})}\right] - 3\ln\left(\frac{T}{300}\right) + \frac{1.12}{(0.0259)(T/300)}\right]$$

Then

$$15.44 = \left(\frac{T}{300}\right)\left[-15.58 - 3\ln\left(\frac{T}{300}\right) + \frac{43.24}{(T/300)}\right]$$

By trial and error

$$\underline{T = 490K}$$

7.11

(a) $$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(5x10^{17})(10^{17})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.8556\,V}$$

(b)
For a 1% change in V_{bi}, assume that the change is due to n_i^2, where the major dependence on temperature is given by

$$n_i^2 \propto \exp\left(\frac{-E_g}{kT}\right)$$

Now

$$\frac{V_{bi}(T_2)}{V_{bi}(T_1)} = \frac{\ln\left[\dfrac{N_a N_d}{n_i^2(T_2)}\right]}{\ln\left[\dfrac{N_a N_d}{n_i^2(T_1)}\right]}$$

$$= \frac{\ln(N_a N_d) - \ln[n_i^2(T_2)]}{\ln(N_a N_d) - \ln[n_i^2(T_1)]}$$

$$= \frac{\ln(N_a N_d) - \ln(N_C N_V) - \left(\dfrac{-E_g}{kT_2}\right)}{\ln(N_a N_d) - \ln(N_C N_V) - \left(\dfrac{-E_g}{kT_1}\right)}$$

$$= \left\{\ln\left[(5x10^{17})(10^{17})\right] - \ln\left[(2.8x10^{19})(1.04x10^{19})\right] + \frac{E_g}{kT_2}\right\}$$

$$/\left\{\ln\left[(5x10^{17})(10^{17})\right] - \ln\left[(2.8x10^{19})(1.04x10^{19})\right] + \frac{E_g}{kT_1}\right\}$$

or

$$\frac{V_{bi}(T_2)}{V_{bi}(T_1)} = \frac{79.897 - 88.567 + \dfrac{E_g}{kT_2}}{79.897 - 88.567 + \dfrac{E_g}{kT_1}}$$

We can write

$$0.990 = \frac{-8.67 + \dfrac{E_g}{kT_2}}{-8.67 + \dfrac{1.12}{0.0259}} = \frac{-8.67 + \dfrac{E_g}{kT_2}}{34.57}$$

so that

$$\frac{E_g}{kT_2} = 42.90 = \frac{1.12}{(0.0259)\left(\dfrac{T_2}{300}\right)}$$

We then find

$$\underline{T_2 = 302.4K}$$

7.12

(b) For $N_d = 10^{16}\ cm^{-3}$,

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.3473\ eV}$$

For $N_d = 10^{15}\ cm^{-3}$,

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.2877\ eV}$$

Then

$$V_{bi} = 0.3473 - 0.2877$$

or

$$\underline{V_{bi} = 0.0596\ V}$$

7.13

(a) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(10^{16})(10^{12})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.456\ V}$$

(b)

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})(0.456)}{1.6x10^{-19}} \times \left(\frac{10^{12}}{10^{16}}\right)\left(\frac{1}{10^{16}+10^{12}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 2.43x10^{-7}\ cm}$$

(c)

$$x_p = \left[\frac{2(11.7)(8.85x10^{-14})(0.456)}{1.6x10^{-19}} \times \left(\frac{10^{16}}{10^{12}}\right)\left(\frac{1}{10^{16}+10^{12}}\right)\right]^{1/2}$$

or

$$\underline{x_p = 2.43x10^{-3}\ cm}$$

(d)

$$|E_{max}| = \frac{eN_d x_n}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(2.43x10^{-7})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 3.75x10^2\ V/cm}$$

7.14

Assume Silicon, so

$$L_D = \left(\frac{\in kT}{e^2 N_d}\right)^{1/2}$$

$$= \left[\frac{(11.7)(8.85x10^{-14})(0.0259)(1.6x10^{-19})}{(1.6x10^{-19})^2 N_d}\right]^{1/2}$$

or

$$L_D = \left(\frac{1.676x10^5}{N_d}\right)^{1/2}$$

(a) $N_d = 8x10^{14}\ cm^{-3}$, $\underline{L_D = 0.1447\ \mu m}$

(b) $N_d = 2.2x10^{16}\ cm^{-3}$, $\underline{L_D = 0.02760\ \mu m}$

(c) $N_d = 8x10^{17}\ cm^{-3}$, $\underline{L_D = 0.004577\ \mu m}$

Now

(a) $V_{bi} = 0.7427\ V$

(b) $V_{bi} = 0.8286\ V$

(c) $V_{bi} = 0.9216\ V$

Also

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi})}{1.6x10^{-19}} \times \left(\frac{8x10^{17}}{N_d}\right)\left(\frac{1}{8x10^{17}+N_d}\right)\right]^{1/2}$$

Then

(a) $\underline{x_n = 1.096\ \mu m}$

(b) $\underline{x_n = 0.2178\ \mu m}$

(c) $\underline{x_n = 0.02731\ \mu m}$

Now

(a) $\underline{\frac{L_D}{x_n} = 0.1320}$

(b) $\dfrac{L_D}{x_n} = 0.1267$

(c) $\dfrac{L_D}{x_n} = 0.1677$

7.15
Computer Plot

7.16

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(2x10^{16})(2x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$\underline{V_{bi} = 0.671\ V}$

(b)

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_R)}{1.6x10^{-19}} \times \left[\frac{2x10^{16}+2x10^{15}}{(2x10^{16})(2x10^{15})}\right]\right\}^{1/2}$$

or

$$W = \left[7.12x10^{-9}(V_{bi}+V_R)\right]^{1/2}$$

For $V_R = 0$, $\underline{W = 0.691x10^{-4}\ cm}$

For $V_R = 8\ V$, $\underline{W = 2.48x10^{-4}\ cm}$

(c)

$$E_{max} = \frac{2(V_{bi}+V_R)}{W}$$

For $V_R = 0$, $\underline{E_{max} = 1.94x10^4\ V/cm}$

For $V_R = 8\ V$, $\underline{E_{max} = 7.0x10^4\ V/cm}$

7.17

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(5x10^{17})(10^{17})}{(1.5x10^{10})^2}\right]$$

or

$\underline{V_{bi} = 0.856\ V}$

(b)

$$x_n = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(5.856)}{1.6x10^{-19}} \times \left(\frac{5x10^{17}}{1x10^{17}}\right)\left(\frac{1}{5x10^{17}+1x10^{17}}\right)\right]^{1/2}$$

or

$\underline{x_n = 0.251\ \mu m}$

Also

$$x_p = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(5.856)}{1.6x10^{-19}} \times \left(\frac{1x10^{17}}{5x10^{17}}\right)\left(\frac{1}{5x10^{17}+1x10^{17}}\right)\right]^{1/2}$$

or

$\underline{x_p = 0.0503\ \mu m}$

Also

$$W = x_n + x_p$$

or

$\underline{W = 0.301\ \mu m}$

(c)

$$E_{max} = \frac{2(V_{bi}+V_R)}{W} = \frac{2(5.856)}{0.301x10^{-4}}$$

or

$\underline{E_{max} = 3.89x10^5\ V/cm}$

(d)

$$C_T = \frac{\in A}{W} = \frac{(11.7)(8.85x10^{-14})(10^{-4})}{0.301x10^{-4}}$$

or

$\underline{C_T = 3.44\ pF}$

7.18

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{50N_a^2}{(1.5x10^{10})^2}\right]$$

We can write

$$\exp\left(\frac{0.752}{0.0259}\right) = \frac{50N_a^2}{\left(1.5x10^{10}\right)^2}$$

or

$$N_a = \frac{1.5x10^{10}}{\sqrt{50}}\exp\left[\frac{0.752}{2(0.0259)}\right]$$

and

$$\underline{N_a = 4.28x10^{15}\ cm^{-3}}$$

Then

$$\underline{N_d = 2.14x10^{17}\ cm^{-3}}$$

(b)

$$x_p \approx W \approx \left[\frac{2\in\left(V_{bi}+V_R\right)}{e}\cdot\left(\frac{1}{N_a}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(10.752)}{\left(1.6x10^{-19}\right)\left(4.28x10^{15}\right)}\right]^{1/2}$$

or

$$\underline{x_p = 1.80\ \mu m}$$

(c)

$$C' \approx \left[\frac{e\in N_a}{2\left(V_{bi}+V_R\right)}\right]^{1/2}$$

$$= \left[\frac{\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(4.28x10^{15}\right)}{2(10.752)}\right]^{1/2}$$

or

$$\underline{C' = 5.74x10^{-9}\ F/cm^2}$$

7.19

(a) Neglecting change in V_{bi}

$$\frac{C'(2N_a)}{C'(N_a)} = \left\{\frac{\left[\dfrac{2}{(2N_a+N_d)}\right]}{\left(\dfrac{1}{N_a+N_d}\right)}\right\}^{1/2}$$

For a $n^+p \Rightarrow N_d >> N_a$

Then

$$\frac{C'(2N_a)}{C'(N_a)} = \sqrt{2} = 1.414$$

so a <u>41.4% change</u>.

(b)

$$\frac{V_{bi}(2N_a)}{V_{bi}(N_a)} = \frac{kT\ln\left(\dfrac{2N_aN_d}{n_i^2}\right)}{kT\ln\left(\dfrac{N_aN_d}{n_i^2}\right)}$$

$$= \frac{kT\ln 2 + kT\ln\left(\dfrac{N_aN_d}{n_i^2}\right)}{kT\ln\left(\dfrac{N_aN_d}{n_i^2}\right)}$$

So we can write this as

$$\frac{V_{bi}(2N_a)}{V_{bi}(N_a)} = \frac{kT\ln 2 + V_{bi}(N_a)}{V_{bi}(N_a)}$$

so

$$\Delta V_{bi} = kT\ln 2 = (0.0259)\ln 2$$

or

$$\underline{\Delta V_{bi} = 17.95\ mV}$$

17.20

(a)

$$\frac{W(A)}{W(B)} = \frac{\left[\dfrac{2\in\left(V_{biA}+V_R\right)}{e}\left(\dfrac{N_a+N_{dA}}{N_aN_{dA}}\right)\right]^{1/2}}{\left[\dfrac{2\in\left(V_{biB}+V_R\right)}{e}\left(\dfrac{N_a+N_{dB}}{N_aN_{dB}}\right)\right]^{1/2}}$$

or

$$\frac{W(A)}{W(B)} = \left[\frac{\left(V_{biA}+V_R\right)}{\left(V_{biB}+V_R\right)}\cdot\frac{\left(N_a+N_{dA}\right)}{\left(N_a+N_{dB}\right)}\cdot\left(\frac{N_{dB}}{N_{dA}}\right)\right]^{1/2}$$

We find

$$V_{biA} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.7543\ V$$

$$V_{biB} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.8139\ V$$

So we find

$$\frac{W(A)}{W(B)} = \left[\left(\frac{5.7543}{5.8139}\right)\left(\frac{10^{18}+10^{15}}{10^{18}+10^{16}}\right)\left(\frac{10^{16}}{10^{15}}\right)\right]^{1/2}$$

or

$$\underline{\frac{W(A)}{W(B)} = 3.13}$$

(b)

$$\frac{E(A)}{E(B)} = \frac{\dfrac{2(V_{biA}+V_R)}{W(A)}}{\dfrac{2(V_{biB}+V_R)}{W(B)}} = \frac{W(B)}{W(A)}\cdot\frac{(V_{biA}+V_R)}{(V_{biB}+V_R)}$$

$$= \left(\frac{1}{3.13}\right)\left(\frac{5.7543}{5.8139}\right)$$

or

$$\frac{E(A)}{E(B)} = 0.316$$

(c)

$$\frac{C'_j(A)}{C'_j(B)} = \frac{\left[\dfrac{\in N_a N_{dA}}{2(V_{biA}+V_R)(N_a+N_{dA})}\right]^{1/2}}{\left[\dfrac{\in N_a N_{dB}}{2(V_{biB}+V_R)(N_a+N_{dB})}\right]^{1/2}}$$

$$= \left[\left(\frac{N_{dA}}{N_{dB}}\right)\left(\frac{V_{biB}+V_R}{V_{biA}+V_R}\right)\left(\frac{N_a+N_{dB}}{N_a+N_{dA}}\right)\right]^{1/2}$$

$$= \left[\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{5.8139}{5.7543}\right)\left(\frac{10^{18}+10^{16}}{10^{18}+10^{15}}\right)\right]^{1/2}$$

or

$$\frac{C'_j(A)}{C'_j(B)} = 0.319$$

17.21

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(4x10^{15})(4x10^{17})}{(1.5x10^{10})^2}\right] \Rightarrow$$

$$V_{bi} = 0.766\ V$$

Now

$$|E_{max}| = \left[\frac{2e(V_{bi}+V_R)}{\in}\left(\frac{N_a N_d}{N_a+N_d}\right)\right]^{1/2}$$

so

$$(3x10^5)^2 = \left[\frac{2(1.6x10^{-19})}{(11.7)(8.85x10^{-14})}\right](V_{bi}+V_R) \times\left[\frac{(4x10^{15})(4x10^{17})}{4x10^{15}+4x10^{17}}\right]$$

or

$$9x10^{10} = 1.22x10^9(V_{bi}+V_R) \Rightarrow$$

$$V_{bi}+V_R = 73.77\ V$$

and

$$V_R = 73\ V$$

(b)

$$V_{bi} = (0.0259)\ln\left[\frac{(4x10^{16})(4x10^{17})}{(1.5x10^{10})^2}\right] \Rightarrow$$

$$V_{bi} = 0.826\ V$$

$$(3x10^5)^2 = \left[\frac{2(1.6x10^{-19})}{(11.7)(8.85x10^{-14})}\right](V_{bi}+V_R) \times\left[\frac{(4x10^{16})(4x10^{17})}{4x10^{16}+4x10^{17}}\right]$$

which yields

$$V_{bi}+V_R = 8.007\ V$$

and

$$V_R = 7.18\ V$$

(c)

$$V_{bi} = (0.0259)\ln\left[\frac{(4x10^{17})(4x10^{17})}{(1.5x10^{10})^2}\right] \Rightarrow$$

$$V_{bi} = 0.886\ V$$

$$(3x10^5)^2 = \left[\frac{2(1.6x10^{-19})}{(11.7)(8.85x10^{-14})}\right](V_{bi}+V_R) \times\left[\frac{(4x10^{17})(4x10^{17})}{4x10^{17}+4x10^{17}}\right]$$

which yields

$$V_{bi}+V_R = 1.456\ V$$

and

$$V_R = 0.570\ V$$

17.22

(a) We have

$$\frac{C_j(0)}{C_j(10)} = \frac{\left[\dfrac{\in N_a N_d}{2V_{bi}(N_a+N_d)}\right]^{1/2}}{\left[\dfrac{\in N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}}$$

or

$$\frac{C_j(0)}{C_j(10)} = 3.13 = \left(\frac{V_{bi}+V_R}{V_{bi}}\right)^{1/2}$$

For $V_R = 10\,V$, we find

$$(3.13)^2 V_{bi} = V_{bi} + 10$$

or

$$\underline{V_{bi} = 1.14\,V}$$

(b)

$$x_p = 0.2W = 0.2(x_p + x_n)$$

Then

$$\frac{x_p}{x_n} = 0.25 = \frac{N_d}{N_a}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \Rightarrow$$

so

$$1.14 = (0.0259)\ln\left[\frac{0.25N_a^2}{(1.8x10^6)^2}\right]$$

We can then write

$$N_a = \frac{1.8x10^6}{\sqrt{0.25}}\exp\left[\frac{1.14}{2(0.0259)}\right]$$

or

$$\underline{N_a = 1.3x10^{16}\ cm^{-3}}$$

and

$$\underline{N_d = 3.25x10^{15}\ cm^{-3}}$$

7.23

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(5x10^{16})}{(1.8x10^6)^2}\right]$$

or

$$V_{bi} = 1.20\,V$$

Now

$$\frac{C_j'(V_{R1})}{C_j'(V_{R2})} = \frac{\left[\dfrac{1}{V_{bi}+V_{R1}}\right]^{1/2}}{\left[\dfrac{1}{V_{bi}+V_{R2}}\right]^{1/2}} = \left[\frac{V_{bi}+V_{R2}}{V_{bi}+V_{R1}}\right]^{1/2}$$

So

$$(3)^2 = \frac{1.20 + V_{R2}}{1.20 + 1} \Rightarrow$$

$$\underline{V_{R2} = 18.6\,V}$$

7.24

$$C' = \left[\frac{e \in N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{18})(10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.754\,V$$

For $N_a >> N_d$, we have

$$C' = \left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})(10^{15})}{2(V_{bi}+V_R)}\right]^{1/2}$$

or

$$C' = \left[\frac{8.28x10^{-17}}{V_{bi}+V_R}\right]^{1/2}$$

For $V_R = 1\,V$, $C' = 6.87x10^{-9}\ F/cm^2$

For $V_R = 10\,V$, $C' = 2.77x10^{-9}\ F/cm^2$

If $A = 6x10^{-4}\ cm^2$, then

For $V_R = 1\,V$, $C = 4.12\ pF$

For $V_R = 10\,V$, $C = 1.66\ pF$

The resonant frequency is given by

$$f_O = \frac{1}{2\pi\sqrt{LC}}$$

so that

For $V_R = 1\,V$, $\underline{f_O = 1.67\ MHz}$

For $V_R = 10\,V$, $\underline{f_O = 2.63\ MHz}$

7.25

$$|\mathrm{E}_{max}| = \frac{eN_d x_n}{\in}$$

For a p^+n junction,

$$x_n \approx \left[\frac{2\in(V_{bi}+V_R)}{eN_d}\right]^{1/2}$$

so that

$$|\mathrm{E}_{max}| = \left[\frac{2eN_d}{\in}(V_{bi}+V_R)\right]^{1/2}$$

Assuming that $V_{bi} << V_R$, then

$$N_d = \frac{\in E_{max}^2}{2eV_R} = \frac{(11.7)(8.85x10^{-14})(10^6)^2}{2(1.6x10^{-19})(10)}$$

or

$$\underline{N_d = 3.24x10^{17}\ cm^{-3}}$$

7.26

$$x_n = 0.1W = 0.1(x_n + x_p)$$

which yields

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} = 9$$

We can write

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{9N_a^2}{(1.5x10^{10})^2}\right]$$

We also have

$$C_j' = \frac{C_T}{A} = \frac{3.5x10^{-12}}{5.5x10^{-4}} = 6.36x10^{-9}\ F/cm^2$$

so

$$6.36x10^{-9} = \left[\frac{e \in N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}$$

Which becomes

$$4.05x10^{-17} = \frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})N_a(9N_a)}{2(V_{bi}+V_R)(N_a+9N_a)}$$

or

$$4.05x10^{-17} = \frac{7.46x10^{-32}N_a}{(V_{bi}+V_R)}$$

If $V_R = 1.2\ V$, then by iteration we find

$$\underline{N_a = 9.92x10^{14}\ cm^{-3}}$$

$$\underline{V_{bi} = 0.632\ V}$$

$$\underline{N_d = 8.93x10^{15}\ cm^{-3}}$$

7.27

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(5x10^{15})(10^{14})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.557\ V}$$

(b)

$$x_p = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.557)}{1.6x10^{-19}} \times \left(\frac{10^{14}}{5x10^{15}}\right)\left(\frac{1}{10^{14}+5x10^{15}}\right)\right]^{1/2}$$

or

$$\underline{x_p = 5.32x10^{-6}\ cm}$$

Also

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.557)}{1.6x10^{-19}} \times \left(\frac{5x10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5x10^{15}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 2.66x10^{-4}\ cm}$$

(c)

For $x_n = 30\ \mu m$, we have

$$30x10^{-4} = \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_R)}{1.6x10^{-19}} \times \left(\frac{5x10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5x10^{15}}\right)\right]^{1/2}$$

which becomes

$$9x10^{-6} = 1.27x10^{-7}(V_{bi}+V_R)$$

We find

$$\underline{V_R = 70.3\ V}$$

7.28

An n^+p junction with $N_a = 10^{14}\ cm^{-3}$,

(a)

A one-sided junction and assume $V_R >> V_{bi}$, then

$$x_p = \left[\frac{2\in V_R}{eN_a}\right]^{1/2}$$

so

$$\left(50x10^{-4}\right)^2 = \frac{2(11.7)\left(8.85x10^{-14}\right)V_R}{\left(1.6x10^{-19}\right)\left(10^{14}\right)}$$

which yields

$$\underline{V_R = 193\ V}$$

(b)

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} \Rightarrow x_n = x_p\left(\frac{N_a}{N_d}\right)$$

so

$$x_n = \left(50x10^{-4}\right)\left(\frac{10^{14}}{10^{16}}\right) \Rightarrow$$

or

$$\underline{x_n = 0.5\ \mu m}$$

(c)

$$\mathrm{E}_{\max} = \frac{2\left(V_{bi}+V_R\right)}{W} = \frac{2(193)}{50.5x10^{-4}}$$

or

$$\underline{\mathrm{E}_{\max} = 7.72x10^4\ V/cm}$$

7.29

(a) $$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(5x10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.796\ V$$

$$C = AC' = A\left[\frac{e\in N_a N_d}{2\left(V_{bi}+V_R\right)\left(N_a+N_d\right)}\right]^{1/2}$$

$$= \left(5x10^{-5}\right)\left[\frac{\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)}{2\left(V_{bi}+V_R\right)} \times \frac{\left(10^{18}\right)\left(5x10^{15}\right)}{\left(10^{18}+5x10^{15}\right)}\right]^{1/2}$$

or

$$C = \left(5x10^{-5}\right)\left[\frac{4.121x10^{-16}}{\left(V_{bi}+V_R\right)}\right]^{1/2}$$

For $V_R = 0$, $\underline{C = 1.14\ pF}$

For $V_R = 3\ V$, $\underline{C = 0.521\ pF}$

For $V_R = 6\ V$, $\underline{C = 0.389\ pF}$

We can write

$$\left(\frac{1}{C}\right)^2 = \frac{1}{A^2}\left[\frac{2\left(V_{bi}+V_R\right)\left(N_a+N_d\right)}{e\in N_a N_d}\right]$$

For the p^+n junction

$$\left(\frac{1}{C}\right)^2 \approx \frac{1}{A^2}\left[\frac{2\left(V_{bi}+V_R\right)}{e\in N_d}\right]$$

so that

$$\frac{\Delta(1/C)^2}{\Delta V_R} = \frac{1}{A^2}\cdot\frac{2}{e\in N_d}$$

We have

For $V_R = 0$, $\left(\frac{1}{C}\right)^2 = 7.69x10^{23}$

For $V_R = 6\ V$, $\left(\frac{1}{C}\right)^2 = 6.61x10^{24}$

Then, for $\Delta V_R = 6\ V$,

$$\Delta(1/C)^2 = 5.84x10^{24}$$

We find

$$N_d = \frac{2}{A^2 e\in}\cdot\frac{1}{\left(\frac{\Delta(1/C)^2}{\Delta V_R}\right)}$$

$$= \frac{2}{\left(5x10^{-5}\right)^2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)} \times \frac{1}{\left(\frac{5.84x10^{24}}{6}\right)}$$

so that

$$\underline{N_d = 4.96x10^{15} \approx 5x10^{15}\ cm^{-3}}$$

Now, for a straight line

$$y = mx + b$$

$$m = \frac{\Delta(1/C)^2}{\Delta V_R} = \frac{5.84x10^{24}}{6}$$

At $V_R = 0$, $\left(\frac{1}{C}\right)^2 = 7.69x10^{23} = b$

Then

$$\left(\frac{1}{C}\right)^2 = \left(\frac{5.84x10^{24}}{6}\right)\cdot V_R + 7.69x10^{23}$$

Now, at $\left(\frac{1}{C}\right)^2 = 0$,

$$0 = \left(\frac{5.84x10^{24}}{6}\right)\cdot V_R + 7.69x10^{23}$$

which yields

$$\underline{V_R = -V_{bi} = -0.790\ V}$$

or

$$V_{bi} \approx 0.796\ V$$

(b)

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{18})(6x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.860\ V$$

$$C = (5x10^{-5})\left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})}{2(V_{bi}+V_R)} \times \frac{(10^{18})(6x10^{16})}{(10^{18}+6x10^{16})}\right]^{1/2}$$

or

$$C = (5x10^{-5})\left[\frac{4.689x10^{-15}}{V_{bi}+V_R}\right]^{1/2}$$

Then

For $V_R = 0$, $\underline{C = 3.69\ pF}$

For $V_R = 3\ V$, $\underline{C = 1.74\ pF}$

For $V_R = 6\ V$, $\underline{C = 1.31\ pF}$

7.30

$$C' = \frac{C}{A} = \frac{1.3x10^{-12}}{10^{-5}} = 1.3x10^{-7}\ F/cm^2$$

(a) For a one-sided junction

$$C' = \left[\frac{e \in N_L}{2(V_{bi}+V_R)}\right]^{1/2}$$

where N_L is the doping concentration in the low-doped region.

We have $V_{bi} + V_R = 0.95 + 0.05 = 1.00\ V$

Then

$$(1.3x10^{-7})^2 = \frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})N_L}{2(1)}$$

which yields

$$\underline{N_L = 2.04x10^{17}\ cm^{-3}}$$

(b)

$$V_{bi} = V_t \ln\left(\frac{N_L N_H}{n_i^2}\right)$$

where N_H is the doping concentration in the high-doped region.

So

$$0.95 = (0.0259)\ln\left[\frac{(2.04x10^{17})N_H}{(1.5x10^{10})^2}\right]$$

which yields

$$\underline{N_H = 9.38x10^{18}\ cm^{-3}}$$

7.31

Computer Plot

7.32

(a) $V_{bi} = V_t \ln\left(\frac{N_{aO} N_{dO}}{n_i^2}\right)$

(c) p-region

$$\frac{d\mathrm{E}}{dx} = \frac{\rho(x)}{\in} = \frac{-eN_{aO}}{\in}$$

or

$$\mathrm{E} = \frac{-eN_{aO}x}{\in} + C_1$$

We have

$$\mathrm{E} = 0 \text{ at } x = -x_p \Rightarrow C_1 = \frac{-eN_{aO}x_p}{\in}$$

Then for $\underline{-x_p < x < 0}$

$$\underline{\mathrm{E} = \frac{-eN_{aO}}{\in}(x + x_p)}$$

n-region, $0 < x < x_O$

$$\frac{d\mathrm{E}_1}{dx} = \frac{\rho(x)}{\in} = \frac{eN_{dO}}{2\in}$$

or

$$\mathrm{E}_1 = \frac{eN_{dO}x}{2\in} + C_2$$

n-region, $x_O < x < x_n$

$$\frac{d\mathrm{E}_2}{dx} = \frac{\rho(x)}{\in} = \frac{eN_{dO}}{\in}$$

or

$$\mathrm{E}_2 = \frac{eN_{dO}x}{\in} + C_3$$

We have $\mathrm{E}_2 = 0$ at $x = x_n$, then

$$C_3 = \frac{-eN_{dO}x_n}{\in}$$

so that for $\underline{x_O < x < x_n}$

$$\underline{\mathrm{E}_2 = \frac{-eN_{dO}}{\in}(x_n - x)}$$

We also have

$\mathrm{E}_2 = \mathrm{E}_1$ at $x = x_O$

Then

$$\frac{eN_{dO}x_O}{2\in} + C_2 = \frac{-eN_{dO}}{\in}\left(x_n - x_O\right)$$

or

$$C_2 = \frac{-eN_{dO}}{\in}\left(x_n - \frac{x_O}{2}\right)$$

Then, for $0 < x < x_O$,

$$\underline{\mathrm{E}_1 = \frac{eN_{dO}x}{2\in} - \frac{eN_{dO}}{\in}\left(x_n - \frac{x_O}{2}\right)}$$

7.33

(a) $\dfrac{d^2\phi(x)}{dx^2} = \dfrac{-\rho(x)}{\in} = \dfrac{-d\mathrm{E}(x)}{dx}$

For $-2 < x < -1\ \mu m$, $\rho(x) = +eN_d$

So

$$\frac{d\mathrm{E}}{dx} = \frac{eN_d}{\in} \Rightarrow \mathrm{E} = \frac{eN_d x}{\in} + C_1$$

At $x = -2\ \mu m \equiv -x_O$, $\mathrm{E} = 0$

So

$$0 = \frac{-eN_d x_O}{\in} + C_1 \Rightarrow C_1 = \frac{eN_d x_O}{\in}$$

Then

$$\mathrm{E} = \frac{eN_d}{\in}\left(x + x_O\right)$$

At $x = 0$, $\mathrm{E}(0) = \mathrm{E}(x = -1\ \mu m)$, so

$$\mathrm{E}(0) = \frac{eN_d}{\in}(-1+2)x10^{-4}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(5x10^{15}\right)}{(11.7)\left(8.85x10^{-14}\right)}\left(1x10^{-4}\right)$$

which yields

$$\underline{\mathrm{E}(0) = 7.73x10^4\ V/cm}$$

(c)

Magnitude of potential difference is

$$|\phi| = \int \mathrm{E}dx = \frac{eN_d}{\in}\int\left(x + x_O\right)dx$$

$$= \frac{eN_d}{\in}\left(\frac{x^2}{2} + x_O \cdot x\right) + C_2$$

Let $\phi = 0$ at $x = -x_O$, then

$$0 = \frac{eN_d}{\in}\left(\frac{x_O^2}{2} - x_O^2\right) + C_2 \Rightarrow C_2 = \frac{eN_d x_O^2}{2\in}$$

Then we can write

$$|\phi| = \frac{eN_d}{2\in}\left(x + x_O\right)^2$$

At $x = -1\ \mu m$

$$|\phi_1| = \frac{\left(1.6x10^{-19}\right)\left(5x10^{15}\right)}{2(11.7)\left(8.85x10^{-14}\right)}\left[(-1+2)x10^{-4}\right]$$

or

$$|\phi_1| = 3.86\ V$$

Potential difference across the intrinsic region

$$|\phi_i| = \mathrm{E}(0)\cdot d = \left(7.73x10^4\right)\left(2x10^{-4}\right)$$

or

$$|\phi_i| = 15.5\ V$$

By symmetry, potential difference across the p-region space charge region is also $3.86\ V$. The total reverse-bias voltage is then

$$V_R = 2(3.86) + 15.5 \Rightarrow \underline{V_R = 23.2\ V}$$

7.34

(a) For the linearly graded junction,

$\rho(x) = eax$,

Then

$$\frac{d\mathrm{E}}{dx} = \frac{\rho(x)}{\in} = \frac{eax}{\in}$$

Now

$$\mathrm{E} = \int \frac{eax}{\in}dx = \frac{ea}{\in}\cdot\frac{x^2}{2} + C_1$$

At $x = +x_O$ and $x = -x_O$, $\mathrm{E} = 0$

So

$$0 = \frac{ea}{\in}\left(\frac{x_O^2}{2}\right) + C_1 \Rightarrow C_1 = \frac{-ea}{\in}\left(\frac{x_O^2}{2}\right)$$

Then

$$\underline{\mathrm{E} = \frac{ea}{2\in}\left(x^2 - x_O^2\right)}$$

(b)

$$\phi(x) = -\int \mathrm{E}dx = \frac{-ea}{2\in}\left[\frac{x^3}{3} - x_O^2 \cdot x\right] + C_2$$

Set $\phi = 0$ at $x = -x_O$, then

$$0 = \frac{-ea}{2\in}\left[\frac{-x_O^3}{3} + x_O^3\right] + C_2 \Rightarrow C_2 = \frac{eax_O^3}{3\in}$$

Then

$$\underline{\phi(x) = \frac{-ea}{2\in}\left(\frac{x^3}{3} - x_O^2 \cdot x\right) + \frac{eax_O^3}{3\in}}$$

7.35

We have that

$$C' = \left[\frac{ea \in^2}{12\left(V_{bi} + V_R\right)} \right]^{1/3}$$

then

$$\left(7.2x10^{-9}\right)^3$$

$$= \left[\frac{a\left(1.6x10^{-19}\right)\left[(11.7)\left(8.85x10^{-14}\right)\right]^2}{12(0.7 + 3.5)} \right]$$

which yields

$$\underline{a = 1.1x10^{20}\ cm^{-4}}$$

Chapter 8

Problem Solutions

8.1
In the forward bias
$$I_f \approx I_S \exp\left(\frac{eV}{kT}\right)$$
Then
$$\frac{I_{f1}}{I_{f2}} = \frac{I_S}{I_S}\cdot\frac{\exp\left(\frac{eV_1}{kT}\right)}{\exp\left(\frac{eV_2}{kT}\right)} = \exp\left[\frac{e}{kT}(V_1 - V_2)\right]$$
or
$$V_1 - V_2 = \left(\frac{kT}{e}\right)\ln\left(\frac{I_{f1}}{I_{f2}}\right)$$
(a)
$$\text{For } \frac{I_{f1}}{I_{f2}} = 10 \Rightarrow \underline{V_1 - V_2 = 59.9\ mV \approx 60\ mV}$$
(b)
$$\text{For } \frac{I_{f1}}{I_{f2}} = 100 \Rightarrow V_1 - V_2 = 119.3\ mV \approx 120 mV$$

8.2
$$I = I_S\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$
or we can write this as
$$\frac{I}{I_S} + 1 = \exp\left(\frac{eV}{kT}\right)$$
so that
$$V = \left(\frac{kT}{e}\right)\ln\left(\frac{I}{I_S} + 1\right)$$
In reverse bias, I is negative, so at
$\frac{I}{I_S} = -0.90$, we have
$$V = (0.0259)\ln(1 - 0.90) \Rightarrow$$
or
$$V = -59.6\ mV$$

8.3
Computer Plot

8.4
The cross-sectional area is
$$A = \frac{I}{J} = \frac{10x10^{-3}}{20} = 5x10^{-4}\ cm^2$$
We have
$$J \approx J_S \exp\left(\frac{V_D}{V_t}\right) \Rightarrow\ 20 = J_S \exp\left(\frac{0.65}{0.0259}\right)$$
so that
$$J_S = 2.52x10^{-10}\ A/cm^2$$
We can write
$$J_S = en_i^2\left[\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\cdot\sqrt{\frac{D_p}{\tau_{pO}}}\right] \backslash$$
We want
$$\frac{\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_{nO}}}}{\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\cdot\sqrt{\frac{D_p}{\tau_{pO}}}} = 0.10$$
or
$$\frac{\frac{1}{N_a}\cdot\sqrt{\frac{25}{5x10^{-7}}}}{\frac{1}{N_a}\cdot\sqrt{\frac{25}{5x10^{-7}}} + \frac{1}{N_d}\cdot\sqrt{\frac{10}{5x10^{-7}}}}$$
$$= \frac{7.07x10^3}{7.07x10^3 + \frac{N_a}{N_d}(4.47x10^3)} = 0.10$$
which yields
$$\frac{N_a}{N_d} = 14.24$$
Now
$$J_S = 2.52x10^{-10} = (1.6x10^{-19})(1.5x10^{10})^2$$
$$\times\left[\frac{1}{(14.24)N_d}\cdot\sqrt{\frac{25}{5x10^{-7}}} + \frac{1}{N_d}\cdot\sqrt{\frac{10}{5x10^{-7}}}\right]$$
We find
$$N_d = 7.1x10^{14}\ cm^{-3}$$
and
$$\underline{N_a = 1.01x10^{16}\ cm^{-3}}$$

8.5

(a)

$$\frac{J_n}{J_n+J_p}=\frac{\dfrac{eD_n n_{pO}}{L_n}}{\dfrac{eD_n n_{pO}}{L_n}+\dfrac{eD_p p_{nO}}{L_p}}$$

$$=\frac{\sqrt{\dfrac{D_n}{\tau_{nO}}}\cdot\dfrac{n_i^2}{N_a}}{\sqrt{\dfrac{D_n}{\tau_{nO}}}\cdot\dfrac{n_i^2}{N_a}+\sqrt{\dfrac{D_p}{\tau_{pO}}}\cdot\dfrac{n_i^2}{N_d}}$$

$$=\frac{1}{1+\sqrt{\dfrac{D_p\tau_{nO}}{D_n\tau_{pO}}}\cdot\left(\dfrac{N_a}{N_d}\right)}$$

We have

$$\frac{D_p}{D_n}=\frac{\mu_p}{\mu_n}=\frac{1}{2.4}\quad\text{and}\quad\frac{\tau_{nO}}{\tau_{pO}}=\frac{1}{0.1}$$

so

$$\frac{J_n}{J_n+J_p}=\frac{1}{1+\sqrt{\dfrac{1}{2.4}\cdot\dfrac{1}{0.1}}\left(\dfrac{N_a}{N_d}\right)}$$

or

$$\underline{\frac{J_n}{J_n+J_p}=\frac{1}{1+(2.04)\left(\dfrac{N_a}{N_d}\right)}}$$

(b)

Using Einstein's relation, we can write

$$\frac{J_n}{J_n+J_p}=\frac{\dfrac{e\mu_n}{L_n}\cdot\dfrac{n_i^2}{N_a}}{\dfrac{e\mu_n}{L_n}\cdot\dfrac{n_i^2}{N_a}+\dfrac{e\mu_p}{L_p}\cdot\dfrac{n_i^2}{N_d}}$$

$$=\frac{e\mu_n N_d}{e\mu_n N_d+\dfrac{L_n}{L_p}\cdot e\mu_p N_a}$$

We have

$$\sigma_n=e\mu_n N_d\quad\text{and}\quad\sigma_p=e\mu_p N_a$$

Also

$$\frac{L_n}{L_p}=\sqrt{\frac{D_n\tau_{nO}}{D_p\tau_{pO}}}=\sqrt{\frac{2.4}{0.1}}=4.90$$

Then

$$\underline{\frac{J_n}{J_n+J_p}=\frac{(\sigma_n/\sigma_p)}{(\sigma_n/\sigma_p)+4.90}}$$

8.6

For a silicon p^+n junction,

$$I_S=Aen_i^2\cdot\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}$$

$$=\left(10^{-4}\right)\left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2\cdot\frac{1}{10^{16}}\sqrt{\frac{12}{10^{-7}}}$$

or

$$\underline{I_S=3.94x10^{-15}\ A}$$

Then

$$I_D=I_S\exp\left(\frac{V_D}{V_t}\right)=\left(3.94x10^{-15}\right)\exp\left(\frac{0.50}{0.0259}\right)$$

or

$$\underline{I_D=9.54x10^{-7}\ A}$$

8.7

We want

$$\frac{J_n}{J_n+J_p}=0.95$$

$$=\frac{\dfrac{eD_n n_{pO}}{L_n}}{\dfrac{eD_n n_{pO}}{L_n}+\dfrac{eD_p p_{nO}}{L_p}}=\frac{\dfrac{D_n}{L_n N_a}}{\dfrac{D_n}{L_n N_a}+\dfrac{D_p}{L_p N_d}}$$

$$=\frac{\dfrac{D_n}{L_n}}{\dfrac{D_n}{L_n}+\dfrac{D_p}{L_p}\cdot\dfrac{N_a}{N_d}}$$

We obtain

$$L_n=\sqrt{D_n\tau_{nO}}=\sqrt{(25)\left(0.1x10^{-6}\right)}\Rightarrow$$

$$L_n=15.8\ \mu m$$

$$L_p=\sqrt{D_p\tau_{pO}}=\sqrt{(10)\left(0.1x10^{-6}\right)}\Rightarrow$$

$$L_p=10\ \mu m$$

Then

$$0.95 = \frac{\frac{25}{15.8}}{\frac{25}{15.8} + \frac{10}{10}\cdot\left(\frac{N_a}{N_d}\right)}$$

which yields

$$\frac{N_a}{N_d} = 0.083$$

8.8

(a) p-side: $E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right) \Rightarrow$$

$$E_{Fi} - E_F = 0.329\ eV$$

Also

n-side: $E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{10^{17}}{1.5x10^{10}}\right) \Rightarrow$$

$$E_F - E_{Fi} = 0.407\ eV$$

(b)
We can find

$$D_n = (1250)(0.0259) = 32.4\ cm^2/s$$

$$D_p = (420)(0.0259) = 10.9\ cm^2/s$$

Now

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2 \times\left[\frac{1}{5x10^{15}}\sqrt{\frac{32.4}{10^{-6}}} + \frac{1}{10^{17}}\sqrt{\frac{10.9}{10^{-7}}}\right]$$

or

$$J_S = 4.48x10^{-11}\ A/cm^2$$

Then

$$I_S = AJ_S = \left(10^{-4}\right)\left(4.48x10^{-11}\right)$$

or

$$I_S = 4.48x10^{-15}\ A$$

We find

$$I = I_S\exp\left(\frac{V_D}{V_t}\right)$$

$$= \left(4.48x10^{-15}\right)\exp\left(\frac{0.5}{0.0259}\right)$$

or

$$I = 1.08\ \mu A$$

(c)
The hole current is proportional to

$$I_P \propto en_i^2\cdot A\cdot\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}$$

$$= \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2\left(10^{-4}\right)\left(\frac{1}{10^{17}}\right)\sqrt{\frac{10.9}{10^{-7}}}$$

or

$$I_P \propto 3.76x10^{-16}\ A$$

Then

$$\frac{I_P}{I} = \frac{3.76x10^{-16}}{4.48x10^{-15}} \Rightarrow \frac{I_P}{I} = 0.0839$$

8.9

$$I = I_S\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

For a p^+n diode,

$$I_S = A\left(\frac{eD_p p_{nO}}{L_p}\right) = A\left(e\sqrt{\frac{D_p}{\tau_{pO}}}\cdot\frac{n_i^2}{N_d}\right)$$

$$= \left(10^{-4}\right)\left[\left(1.6x10^{-19}\right)\sqrt{\frac{10}{10^{-6}}}\cdot\frac{\left(2.4x10^{13}\right)^2}{10^{16}}\right]$$

or

$$I_S = 2.91x10^{-9}\ A$$

(a)
For $V_a = +0.2\ V$,

$$I = \left(2.91x10^{-9}\right)\left[\exp\left(\frac{0.2}{0.0259}\right) - 1\right]$$

or

$$I = 6.55\ \mu A$$

(b)
For $V_a = -0.2\ V$,

$$I = \left(2.91x10^{-9}\right)\left[\exp\left(\frac{-0.2}{0.0259}\right) - 1\right]$$

$$\approx -2.91x10^{-9}\ A$$

or $I = -I_S = -2.91\ nA$

8.10

For an n^+p silicon diode

$$I_S = Aen_i^2 \cdot \frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}}$$

$$= \frac{(10^{-4})(1.6x10^{-19})(1.5x10^{10})^2}{10^{16}}\sqrt{\frac{25}{10^{-6}}}$$

or

$$\underline{I_S = 1.8x10^{-15}\ A}$$

(a)

For $V_a = 0.5\,V$

$$I_D = I_S \exp\left(\frac{V_a}{V_t}\right) = (1.8x10^{-15})\exp\left(\frac{0.5}{0.0259}\right)$$

or

$$\underline{I_D = 4.36x10^{-7}\ A}$$

(b)

For $V_a = -0.5\,V$

$$\underline{I_D = -I_S = -1.8x10^{-15}\ A}$$

8.11

(a) We find

$$D_p = \mu_p\left(\frac{kT}{e}\right) = (480)(0.0259) = 12.4\ cm^2/s$$

and

$$L_p = \sqrt{D_p\tau_{pO}} = \sqrt{(12.4)(0.1x10^{-6})} \Rightarrow$$

$$L_p = 11.1\ \mu m$$

Also

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{(1.5x10^{10})^2}{10^{15}} = 2.25x10^5\ cm^{-3}$$

Then

$$J_{pO} = \frac{eD_p p_{nO}}{L_p} = \frac{(1.6x10^{-19})(12.4)(2.25x10^5)}{(11.1x10^{-4})}$$

or

$$J_{pO} = 4.02x10^{-10}\ A/cm^2$$

For $A = 10^{-4}\ cm^2$, then

$$\underline{I_{pO} = 4.02x10^{-14}\ A}$$

(b)

We have

$$D_n = \mu_n\left(\frac{kT}{e}\right) = (1350)(0.0259) = 35\ cm^2/s$$

and

$$L_n = \sqrt{D_n\tau_{nO}} = \sqrt{(35)(0.4x10^{-6})} \Rightarrow$$

$$L_n = 37.4\ \mu m$$

Also

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5x10^{10})^2}{5x10^{15}} = 4.5x10^4\ cm^{-3}$$

Then

$$J_{nO} = \frac{eD_n n_{pO}}{L_n} = \frac{(1.6x10^{-19})(35)(4.5x10^4)}{(37.4x10^{-4})}$$

or

$$J_{nO} = 6.74x10^{-11}\ A/cm^2$$

For $A = 10^{-4}\ cm^2$, then

$$\underline{I_{nO} = 6.74x10^{-15}\ A}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(5x10^{15})(10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.617\ V$$

Then for

$$V_a = \frac{1}{2}V_{bi} = 0.309\ V$$

We find

$$p_n = p_{nO}\exp\left(\frac{eV_a}{kT}\right)$$

$$= (2.25x10^5)\exp\left(\frac{0.309}{0.0259}\right)$$

or

$$\underline{p_n = 3.42x10^{10}\ cm^{-3}}$$

(d)

The total current is

$$I = (I_{pO} + I_{nO})\exp\left(\frac{eV_a}{kT}\right)$$

$$= (4.02x10^{-14} + 6.74x10^{-15})\exp\left(\frac{0.309}{0.0259}\right)$$

or

$$\underline{I = 7.13x10^{-9}\ A}$$

The hole current is

$$I_p = I_{pO}\exp\left(\frac{eV_a}{kT}\right)\exp\left[\frac{-(x-x_n)}{L_p}\right]$$

The electron current is given by

$$I_n = I - I_p$$

$$= 7.13x10^{-9} - \left(4.02x10^{-14}\right) \times\exp\left(\frac{0.309}{0.0259}\right)\exp\left[\frac{-(x-x_n)}{L_p}\right]$$

At $x = x_n + \frac{1}{2}L_p$

$$I_n = 7.13x10^{-9} - \left(6.10x10^{-9}\right)\exp\left(\frac{-1}{2}\right)$$

or

$$\underline{I_n = 3.43x10^{-9}\ A}$$

8.12

(a) The excess hole concentration is given by

$$\delta p_n = p_n - p_{nO}$$

$$= p_{nO}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]\exp\left(\frac{-x}{L_p}\right)$$

We find

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

and

$$L_p = \sqrt{D_p\tau_{pO}} = \sqrt{(8)\left(0.01x10^{-6}\right)} \Rightarrow$$

$$L_p = 2.83\ \mu m$$

Then

$$\delta p_n = \left(2.25x10^4\right) \times\left[\exp\left(\frac{0.610}{0.0259}\right)-1\right]\exp\left(\frac{-x}{2.83x10^{-4}}\right)$$

or

$$\underline{\delta p_n = 3.81x10^{14}\exp\left(\frac{-x}{2.83x10^{-4}}\right)\ cm^{-3}}$$

(b)

We have

$$J_p = -eD_p\frac{d(\delta p_n)}{dx}$$

$$= \frac{eD_p\left(3.81x10^{14}\right)}{\left(2.83x10^{-4}\right)}\exp\left(\frac{-x}{2.83x10^{-4}}\right)$$

At $x = 3x10^{-4}\ cm$,

$$J_p = \frac{\left(1.6x10^{-19}\right)(8)\left(3.81x10^{14}\right)}{2.83x10^{-4}}\exp\left(\frac{-3}{2.83}\right)$$

or

$$\underline{J_p = 0.597\ A/cm^2}$$

(c)

We have

$$J_{nO} = \frac{eD_n n_{pO}}{L_n}\exp\left(\frac{eV_a}{kT}\right)$$

We can determine that

$$n_{pO} = 4.5x10^3\ cm^{-3} \text{ and } L_n = 10.7\ \mu m$$

Then

$$J_{nO} = \frac{\left(1.6x10^{-19}\right)(23)\left(4.5x10^3\right)}{10.7x10^{-4}}\exp\left(\frac{0.610}{0.0259}\right)$$

or

$$J_{nO} = 0.262\ A/cm^2$$

We can also find

$$\underline{J_{pO} = 1.72\ A/cm^2}$$

Then, at $x = 3\ \mu m$,

$$J_n(3\ \mu m) = J_{nO} + J_{pO} - J_p(3\ \mu m)$$

$$= 0.262 + 1.72 - 0.597$$

or

$$\underline{J_n(3\ \mu m) = 1.39\ A/cm^2}$$

8.13

(a) From Problem 8.9 (Ge diode)

Low injection means

$$p_n(0) = (0.1)N_d = 10^{15}\ cm^{-3}$$

Now

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{\left(2.4x10^{13}\right)^2}{10^{16}} = 5.76x10^{10}\ cm^{-3}$$

We have

$$p_n(0) = p_{nO}\exp\left(\frac{V_a}{V_t}\right)$$

or

$$V_a = V_t\ln\left[\frac{p_n(0)}{p_{nO}}\right]$$

$$= (0.0259)\ln\left(\frac{10^{15}}{5.76x10^{10}}\right)$$

or

$$\underline{V_a = 0.253\ V}$$

(b)

For Problem 8.10 (Si diode)

$$n_p(0) = (0.1)N_a = 10^{15}\ cm^{-3}$$

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

Then

$$V_a = V_t \ln\left[\frac{n_p(0)}{n_{pO}}\right]$$

$$= (0.0259)\ln\left(\frac{10^{15}}{2.25x10^4}\right)$$

or

$$\underline{V_a = 0.635\ V}$$

8.14

The excess electron concentration is given by

$$\delta n_p = n_p - n_{pO}$$

$$= n_{pO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{-x}{L_n}\right)$$

The total number of excess electrons is

$$N_p = A\int_0^\infty \delta n_p\, dx$$

We may note that

$$\int_0^\infty \exp\left(\frac{-x}{L_n}\right)dx = -L_n \exp\left(\frac{-x}{L_n}\right)\Bigg|_0^\infty = L_n$$

Then

$$N_p = AL_n n_{pO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

We can find

$$D_n = 35\ cm^2/s \quad \text{and} \quad L_n = 59.2\ \mu m$$

Also

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5x10^{10})^2}{8x10^{15}} = 2.81x10^4\ cm^{-3}$$

Then

$$N_p = (10^{-3})(59.2x10^{-4})(2.81x10^4) \times\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

or

$$N_p = 0.166\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Then we find the total number of excess electrons in the p-region to be:

(a) $V_a = 0.3\ V$, $\underline{N_p = 1.78x10^4}$

(b) $V_a = 0.4\ V$, $\underline{N_p = 8.46x10^5}$

(c) $V_a = 0.5\ V$, $\underline{N_p = 4.02x10^7}$

Similarly, the total number of excess holes in the n-region is found to be:

$$N_n = AL_p p_{nO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

We find that

$$D_p = 12.4\ cm^2/s \quad \text{and} \quad L_p = 11.1\ \mu m$$

Also

$$p_{nO} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

Then

$$N_n = (2.50x10^{-2})\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

So

(a) $V_a = 0.3\ V$, $\underline{N_n = 2.68x10^3}$

(b) $V_a = 0.4\ V$, $\underline{N_n = 1.27x10^5}$

(c) $V_a = 0.5\ V$, $\underline{N_n = 6.05x10^6}$

8.15

$$I \propto n_i^2 \exp\left(\frac{eV_a}{kT}\right) \propto \exp\left(\frac{-E_g}{kT}\right)\exp\left(\frac{eV_a}{kT}\right)$$

Then

$$I \propto \exp\left(\frac{eV_a - E_g}{kT}\right)$$

so

$$\frac{I_1}{I_2} = \frac{\exp\left(\frac{eV_{a1} - E_{g1}}{kT}\right)}{\exp\left(\frac{eV_{a2} - E_{g2}}{kT}\right)}$$

or

$$\frac{I_1}{I_2} = \exp\left(\frac{eV_{a1} - eV_{a2} - E_{g1} + E_{g2}}{kT}\right)$$

We have

$$\frac{10x10^{-3}}{10x10^{-6}} = \exp\left(\frac{0.255 - 0.32 - 0.525 + E_{g2}}{0.0259}\right)$$

or

$$10^3 = \exp\left(\frac{E_{g2} - 0.59}{0.0259}\right)$$

Then

$$E_{g2} = 0.59 + (0.0259)\ln\left(10^3\right)$$

which yields

$$\underline{E_{g2} = 0.769\ eV}$$

8.16

(a) We have

$$I_S = Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

which can be written in the form

$$I_S = C'n_i^2$$

$$= C'N_{CO}N_{VO}\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

or

$$\underline{I_S = CT^3 \exp\left(\frac{-E_g}{kT}\right)}$$

(b)

Taking the ratio

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{T_2}{T_1}\right)^3 \cdot \frac{\exp\left(\dfrac{-E_g}{kT_2}\right)}{\exp\left(\dfrac{-E_g}{kT_1}\right)}$$

$$= \left(\frac{T_2}{T_1}\right)^3 \cdot \exp\left[+E_g\left(\frac{1}{kT_1} - \frac{1}{kT_2}\right)\right]$$

For $T_1 = 300K$, $kT_1 = 0.0259$, $\dfrac{1}{kT_1} = 38.61$

For $T_2 = 400K$, $kT_2 = 0.03453$, $\dfrac{1}{kT_2} = 28.96$

(i) Germanium, $E_g = 0.66\ eV$

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{400}{300}\right)^3 \exp\left[(0.66)(38.61 - 28.96)\right]$$

or

$$\underline{\frac{I_{S2}}{I_{S1}} = 1383}$$

(ii) Silicon, $E_g = 1.12\ eV$

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{400}{300}\right)^3 \cdot \exp\left[(1.12)(38.61 - 28.96)\right]$$

or

$$\underline{\frac{I_{S2}}{I_{S1}} = 1.17x10^5}$$

8.17

Computer Plot

8.18

One condition:

$$\left|\frac{I_f}{I_r}\right| = \frac{J_S \exp\left(\dfrac{eV_a}{kT}\right)}{J_S} = \exp\left(\frac{eV_a}{kT}\right) = 10^4$$

or

$$\frac{kT}{e} = \frac{V_a}{\ln\left(10^4\right)} = \frac{0.5}{\ln\left(10^4\right)}$$

or

$$\frac{kT}{e} = 0.05429 = (0.0259)\left(\frac{T}{300}\right)$$

which yields

$$T = 629K$$

Second conditon:

$$I_S = A\left(\frac{eD_n n_{pO}}{L_n} + \frac{eD_p p_{nO}}{L_p}\right)$$

$$= Aen_i^2\left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}\right)$$

$$= AeN_C N_V\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]\exp\left(\frac{-E_g}{kT}\right)$$

which becomes

$$10^{-6} = \left(10^{-4}\right)\left(1.6x10^{-19}\right)\left(2.8x10^{19}\right)\left(1.04x10^{19}\right)$$

$$\times\left(\frac{1}{5x10^{18}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{10^{15}}\sqrt{\frac{10}{10^{-7}}}\right)\exp\left(\frac{-E_g}{kT}\right)$$

or

$$\exp\left(\frac{+E_g}{kT}\right) = 4.66x10^{10}$$

For $E_g = 1.10\ eV$,

$$kT = \frac{E_g}{\ln\left(4.66x10^{10}\right)} = \frac{1.10}{\ln\left(4.66x10^{10}\right)}$$

or

$$kT = 0.04478\ eV = (0.0259)\left(\frac{T}{300}\right)$$

Then

$$T = 519K$$

This second condition yields a smaller temperature, so the maximum temperature is

$$\underline{T = 519K}$$

8.19

(a) We can write for the n-region

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0$$

The general solution is

$$\delta p_n = A\exp(+x/L_p) + B\exp(-x/L_p)$$

The boundary condition at $x = x_n$ gives

$$\delta p_n(x_n) = p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

$$= A\exp(+x_n/L_p) + B\exp(-x_n/L_p)$$

and the boundary condition at $x = x_n + W_n$ gives

$$\delta p_n(x_n + W_n) = 0$$

$$= A\exp[(x_n + W_n)/L_p] + B\exp[-(x_n + W_n)/L_p]$$

From this equation, we have

$$A = -B\exp[-2(x_n + W_n)/L_p]$$

Then, from the first boundary condition, we obtain

$$p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

$$= B\exp[-(x_n + 2W_n)/L_p] + B\exp(-x_n/L_p)$$

$$= B\exp(-x_n/L_p)[1 - \exp(-2W_n/L_p)]$$

We then obtain

$$B = \frac{p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{\exp(-x_n/L_p)[1 - \exp(-2W_n/L_p)]}$$

which can be written in the form

$$B = \frac{p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right] \cdot \exp[(x_n + W_n)/L_p]}{\exp(W_n/L_p) - \exp(-W_n/L_p)}$$

Also

$$A = \frac{-p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right] \cdot \exp[-(x_n + W_n)/L_p]}{\exp(W_n/L_p) - \exp(-W_n/L_p)}$$

The solution can now be written as

$$\delta p_n = \frac{p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{2\sinh\left(\frac{W_n}{L_p}\right)}$$

$$\times\left\{\exp\left[\frac{(x_n + W_n - x)}{L_p}\right] - \exp\left[\frac{-(x_n + W_n - x)}{L_p}\right]\right\}$$

or finally,

$$\delta p_n = p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right] \cdot \frac{\sinh\left(\frac{x_n + W_n - x}{L_p}\right)}{\sinh\left(\frac{W_n}{L_p}\right)}$$

(b)

$$J_p = -eD_p \frac{d(\delta p_n)}{dx}\bigg|_{x=x_n}$$

$$= \frac{-eD_p p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{\sinh\left(\frac{W_n}{L_p}\right)}$$

$$\times\left(\frac{-1}{L_p}\right)\cosh\left(\frac{x_n + W_n - x}{L_p}\right)\bigg|_{x=x_n}$$

Then

$$J_p = \frac{eD_p p_{nO}}{L_p}\coth\left(\frac{W_n}{L_p}\right) \cdot \left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

8.20

$$I_D \propto n_i^2 \exp\left(\frac{V_D}{V_t}\right)$$

For the temperature range $300 \le T \le 320K$, neglect the change in N_C and N_V

So

$$I_D \propto \exp\left(\frac{-E_g}{kT}\right) \cdot \exp\left(\frac{eV_D}{kT}\right)$$

$$\propto \exp\left[\frac{-(E_g - eV_D)}{kT}\right]$$

Taking the ratio of currents, but maintaining I_D a constant, we have

$$1 = \frac{\exp\left[\dfrac{-\left(E_g - eV_{D1}\right)}{kT_1}\right]}{\exp\left[\dfrac{-\left(E_g - eV_{D2}\right)}{kT_2}\right]} \Rightarrow$$

$$\frac{E_g - eV_{D1}}{kT_1} = \frac{E_g - eV_{D2}}{kT_2}$$

We have

$T = 300K$, $V_{D1} = 0.60\ V$ and

$$kT_1 = 0.0259\ eV\ ,\ \frac{kT_1}{e} = 0.0259\ V$$

$T = 310K$

$$kT_2 = 0.02676\ eV\ ,\ \frac{kT_2}{e} = 0.02676\ V$$

$T = 320K$

$$kT_3 = 0.02763\ eV\ ,\ \frac{kT_3}{e} = 0.02763\ V$$

So, for $\underline{T = 310K}$,

$$\frac{1.12 - 0.60}{0.0259} = \frac{1.12 - V_{D2}}{0.02676}$$

which yields

$$\underline{V_{D2} = 0.5827\ V}$$

For $\underline{T = 320K}$,

$$\frac{1.12 - 0.60}{0.0259} = \frac{1.12 - V_{D3}}{0.02763}$$

which yields

$$\underline{V_{D3} = 0.5653\ V}$$

8.21

Computer Plot

8.22

$$g_d = \frac{e}{kT} \cdot I_D = \frac{2x10^{-3}}{0.0259}$$

or

$$\underline{g_d = 0.0772\ S}$$

Also

$$C_d = \frac{1}{2}\left(\frac{e}{kT}\right)\left(I_{pO}\tau_{pO} + I_{nO}\tau_{nO}\right)$$

We have

$$\tau_{pO} = \tau_{nO} = 10^{-6}\ s$$

$$I_{pO} + I_{nO} = 2x10^{-3}\ A$$

Then

$$C_d = \frac{\left(2x10^{-3}\right)\left(10^{-6}\right)}{2(0.0259)} \Rightarrow$$

$$\underline{C_d = 3.86x10^{-8}\ F}$$

Then

$$Y = g_d + j\omega C_d$$

or

$$\underline{Y = 0.0772 + j\omega\left(3.86x10^{-8}\right)}$$

8.23 For a p^+n diode

$$g_d = \frac{I_{DQ}}{V_t}\quad ,\quad C_d = \frac{I_{DQ}\tau_{pO}}{2V_t}$$

Now

$$g_d = \frac{10^{-3}}{0.0259} = 3.86x10^{-2}\ S$$

and

$$C_d = \frac{\left(10^{-3}\right)\left(10^{-7}\right)}{2(0.0259)} = 1.93x10^{-9}\ F$$

Now

$$Z = \frac{1}{Y} = \frac{1}{g_d + j\omega C_d} = \frac{g_d - j\omega C_d}{g_d^2 + \omega^2 C_d^2}$$

We have $\omega = 2\pi f$,

We find:

$f = 10\ kHz$: $\underline{Z = 25.9 - j0.0814}$

$f = 100\ kHz$: $\underline{Z = 25.9 - j0.814}$

$f = 1\ MHz$: $\underline{Z = 23.6 - j7.41}$

$f = 10\ MHz$: $\underline{Z = 2.38 - j7.49}$

8.24

(b)

Two capacitances will be equal at some forward-bias voltage.

For a forward-bias voltage, the junction capacitance is

$$C_j = A\left[\frac{e \in N_a N_d}{2\left(V_{bi} - V_a\right)\left(N_a + N_d\right)}\right]^{1/2}$$

The diffusion capacitance is

$$C_d = \left(\frac{1}{2V_t}\right)\left(I_{pO}\tau_{pO} + I_{nO}\tau_{nO}\right)$$

where

$$I_{pO} = \frac{Aen_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

and

$$I_{nO} = \frac{Aen_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}}\cdot\exp\left(\frac{V_a}{V_t}\right)$$

We find

$$D_p = (320)(0.0259) = 8.29\ cm^2/s$$

$$D_n = (850)(0.0259) = 22.0\ cm^2/s$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{17})(5x10^{15})}{(1.5x10^{10})^2}\right] = 0.7363\ V$$

Now, we obtain

$$C_j = (10^{-4})\left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})}{2(V_{bi}-V_a)} \times \frac{(5x10^{15})(10^{17})}{(5x10^{15}+10^{17})}\right]^{1/2}$$

or

$$C_j = (10^{-4})\left[\frac{3.945x10^{-16}}{(V_{bi}-V_a)}\right]^{1/2}$$

We also obtain

$$I_{pO} = \frac{(10^{-4})(1.6x10^{-19})(1.5x10^{10})^2}{10^{17}}\sqrt{\frac{8.29}{10^{-7}}} \times\exp\left(\frac{V_a}{V_t}\right)$$

or

$$I_{pO} = 3.278x10^{-16}\exp\left(\frac{V_a}{V_t}\right)$$

Also

$$I_{nO} = \frac{(10^{-4})(1.6x10^{-19})(1.5x10^{10})^2}{5x10^{15}}\sqrt{\frac{22}{10^{-6}}} \times\exp\left(\frac{V_a}{V_t}\right)$$

or

$$I_{nO} = 3.377x10^{-15}\exp\left(\frac{V_a}{V_t}\right)$$

We can now write

$$C_d = \frac{1}{2(0.0259)}\left[(3.278x10^{-16})(10^{-7}) + (3.377x10^{-15})(10^{-6})\right]\cdot\exp\left(\frac{V_a}{V_t}\right)$$

or

$$C_d = 6.583x10^{-20}\cdot\exp\left(\frac{V_a}{V_t}\right)$$

We want to set $C_j = C_d$

So

$$(10^{-4})\left[\frac{3.945x10^{-16}}{0.7363-V_a}\right]^{1/2} = 6.583x10^{-20}\cdot\exp\left(\frac{V_a}{0.0259}\right)$$

By trial and error, we find

$$\underline{V_a = 0.463\ V}$$

At this voltage,

$$\underline{C_j = C_d \approx 3.8\ pF}$$

8.25

For a p^+n diode, $I_{pO} >> I_{nO}$, then

$$C_d = \left(\frac{1}{2V_t}\right)(I_{pO}\tau_{pO})$$

Now

$$\frac{\tau_{pO}}{2V_t} = 2.5x10^{-6}\ F/A$$

Then

$$\tau_{pO} = 2(0.0259)(2.5x10^{-6})$$

or

$$\underline{\tau_{pO} = 1.3x10^{-7}\ s}$$

At $1\ mA$,

$$C_d = (2.5x10^{-6})(10^{-3}) \Rightarrow$$

$$\underline{C_d = 2.5x10^{-9}\ F}$$

8.26

(a) $$C_d = \frac{1}{2}\left(\frac{e}{kT}\right)A(I_{pO}\tau_{pO} + I_{nO}\tau_{nO})$$

For a one-sided n^+p diode, $I_{nO} >> I_{pO}$, then

$$C_d = \frac{1}{2}\left(\frac{e}{kT}\right)A(I_{nO}\tau_{nO})$$

so

$$10^{-12} = \frac{1}{2}\left(\frac{1}{0.0259}\right)\left(10^{-3}\right)\left(I_{nO}\right)\left(10^{-7}\right)$$

or

$$\underline{I_{nO} = I_D = 0.518\ mA}$$

(b)

$$I_{nO} = A\frac{eD_n n_{pO}}{L_n}\exp\left(\frac{V_a}{V_t}\right)$$

We find

$$L_n = \sqrt{D_n \tau_{nO}} = 15.8\ \mu m \text{ and}$$

$$n_{pO} = \frac{n_i^2}{N_a} = 2.25x10^4\ cm^{-3}$$

Then

$$0.518x10^{-3} = \frac{\left(1.6x10^{-19}\right)(25)\left(2.25x10^4\right)\left(10^{-3}\right)}{15.8x10^{-4}}\exp\left(\frac{V_a}{V_t}\right)$$

or

$$0.518x10^{-3} = 5.70x10^{-14}\exp\left(\frac{V_a}{0.0259}\right)$$

We find

$$\underline{V_a = 0.594\ V}$$

(c)

$$g_d = \left(\frac{e}{kT}\right)I_D = \frac{1}{r_d} \Rightarrow$$

$$r_d = \frac{0.0259}{0.518x10^{-3}}$$

or

$$\underline{r_d = 50\ \Omega}$$

8.27

(a) p-region

$$R_p = \frac{\rho_p L}{A} = \frac{L}{\sigma_p A} = \frac{L}{A\left(e\mu_p N_a\right)}$$

so

$$R_p = \frac{0.2}{\left(10^{-2}\right)\left(1.6x10^{-19}\right)(480)\left(10^{16}\right)}$$

or

$$R_p = 26\ \Omega$$

n-region

$$R_n = \frac{\rho_n L}{A} = \frac{L}{\sigma_n A} = \frac{L}{A\left(e\mu_n N_d\right)}$$

so

$$R_n = \frac{0.10}{\left(10^{-2}\right)\left(1.6x10^{-19}\right)(1350)\left(10^{15}\right)}$$

or

$$R_n = 46.3\ \Omega$$

The total series resistance is

$$R = R_p + R_n = 26 + 46.3 \Rightarrow$$

$$\underline{R = 72.3\ \Omega}$$

(b)

$$V = IR \Rightarrow 0.1 = I(72.3)$$

or

$$\underline{I = 1.38\ mA}$$

8.28

$$R = \frac{\rho_n L(n)}{A(n)} + \frac{\rho_p L(p)}{A(p)}$$

$$= \frac{(0.2)\left(10^{-2}\right)}{2x10^{-5}} + \frac{(0.1)\left(10^{-2}\right)}{2x10^{-5}}$$

or

$$\underline{R = 150\ \Omega}$$

We can write

$$V = I_D R + V_t \ln\left(\frac{I_D}{I_S}\right)$$

(a) (i) $I_D = 1\ mA$

$$V = \left(10^{-3}\right)(150) + (0.0259)\ln\left(\frac{10^{-3}}{10^{-10}}\right)$$

or

$$\underline{V = 0.567\ V}$$

(ii) $I_D = 10\ mA$

$$V = \left(10x10^{-3}\right)(150) + (0.0259)\ln\left(\frac{10x10^{-3}}{10^{-10}}\right)$$

or $\underline{V = 1.98\ V}$

(b)

For $R = 0$

(i) $I_D = 1\ mA$

$$V = (0.0259)\ln\left(\frac{10^{-3}}{10^{-10}}\right) \Rightarrow$$

$$\underline{V = 0.417\ V}$$

(ii) $I_D = 10\ mA$

$$V = (0.0259)\ln\left(\frac{10x10^{-3}}{10^{-10}}\right) \Rightarrow$$

$$\underline{V = 0.477\ V}$$

8.29

$$r_d = 48\ \Omega = \frac{1}{g_d} \Rightarrow g_d = 0.0208$$

We have

$$g_d = \frac{e}{kT}\cdot I_D \Rightarrow I_D = (0.0208)(0.0259)$$

or

$$\underline{I_D = 0.539\ mA}$$

Also

$$I_D = I_S \exp\left(\frac{V_a}{V_t}\right) \Rightarrow V_a = V_t \ln\left(\frac{I_D}{I_S}\right)$$

so

$$V_a = (0.0259)\ln\left(\frac{0.539x10^{-3}}{2x10^{-11}}\right) \Rightarrow$$

$$\underline{V_a = 0.443\ V}$$

8.30

(a) $$\frac{1}{r_d} = \frac{dI_D}{dV_a} = I_S\left(\frac{1}{V_t}\right)\exp\left(\frac{V_a}{V_t}\right)$$

or

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259}\cdot\exp\left(\frac{0.020}{0.0259}\right)$$

which yields

$$\underline{r_d = 1.2x10^{11}\ \Omega}$$

(b)

For $V_a = -0.020\ V$,

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259}\cdot\exp\left(\frac{-0.020}{0.0259}\right)$$

or

$$\underline{r_d = 5.6x10^{11}\ \Omega}$$

8.31

Ideal reverse-saturation current density

$$J_S = \frac{eD_n n_{pO}}{L_n} + \frac{eD_p p_{nO}}{L_p}$$

We find

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{\left(1.8x10^6\right)^2}{10^{16}} = 3.24x10^{-4}\ cm^{-3}$$

and

$$p_{nO} = \frac{\left(1.8x10^6\right)^2}{10^{16}} = 3.24x10^{-4}\ cm^{-3}$$

Also

$$L_n = \sqrt{D_n\tau_{nO}} = \sqrt{(200)\left(10^{-8}\right)} = 14.2\ \mu m$$

$$L_p = \sqrt{D_p\tau_{pO}} = \sqrt{(6)\left(10^{-8}\right)} = 2.45\ \mu m$$

Then

$$J_S = \frac{\left(1.6x10^{-19}\right)(200)\left(3.24x10^{-4}\right)}{14.2x10^{-4}} + \frac{\left(1.6x10^{-19}\right)(6)\left(3.24x10^{-4}\right)}{2.45x10^{-4}}$$

so

$$\underline{J_S = 8.57x10^{-18}\ A/cm^2}$$

Reverse-biased generation current density

$$J_{gen} = \frac{en_iW}{2\tau_O}$$

We have

$$V_{bi} = V_t \ln\left(\frac{N_aN_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{\left(10^{16}\right)\left(10^{16}\right)}{\left(1.8x10^6\right)^2}\right]$$

or

$$V_{bi} = 1.16\ V$$

And

$$W = \left[\frac{2\in\left(V_{bi}+V_R\right)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(13.1)\left(8.85x10^{-14}\right)(1.16+5)}{1.6x10^{-19}} \times\left[\frac{10^{16}+10^{16}}{\left(10^{16}\right)\left(10^{16}\right)}\right]\right]^{1/2}$$

or

$$W = 1.34x10^{-4}\ cm$$

Then

$$J_{gen} = \frac{\left(1.6x10^{-19}\right)\left(1.8x10^6\right)\left(1.34x10^{-4}\right)}{2\left(10^{-8}\right)}$$

or

$$\underline{J_{gen} = 1.93x10^{-9}\ A/cm^2}$$

Generation current dominates in GaAs reverse-biased junctions.

8.32
(a) We can write

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= n_i^2\left(1.6x10^{19}\right)\left[\frac{1}{10^{16}}\sqrt{\frac{25}{5x10^{-7}}} + \frac{1}{10^{16}}\sqrt{\frac{10}{5x10^{-7}}}\right]$$

or

$$J_S = n_i^2\left(1.85x10^{-31}\right)$$

We also have

$$J_{gen} = \frac{en_iW}{2\tau_O}$$

For $V_{bi} + V_R = 5\ V$, we find $W = 1.14x10^{-4}\ cm$
So

$$J_{gen} = \frac{\left(1.6x10^{-19}\right)\left(1.14x10^{-4}\right)n_i}{2\left(5x10^{-7}\right)}$$

or

$$J_{gen} = n_i\left(1.82x10^{-17}\right)$$

When $J_S = J_{gen}$,

$$1.85x10^{-31}n_i = 1.82x10^{-17}$$

which yields

$$n_i = 9.88x10^{13}\ cm^{-3}$$

We have

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

Then

$$\left(9.88x10^{13}\right)^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{T}{300}\right)^3 \times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

By trial and error, we find

$$\underline{T = 505K}$$

At this temperature

$$J_S = J_{gen} = \left(1.82x10^{-17}\right)\left(9.88x10^{13}\right) \Rightarrow$$

$$\underline{J_S = J_{gen} = 1.8x10^{-3}\ A/cm^2}$$

(b)

$$J_S \exp\left(\frac{V_a}{V_t}\right) = J_{gen}\exp\left(\frac{V_a}{2V_t}\right)$$

At $T = 300K$

$$J_S = \left(1.5x10^{10}\right)^2\left(1.85x10^{-31}\right)$$

or

$$J_S = 4.16x10^{-11}\ A/cm^2$$

and

$$J_{gen} = \left(1.5x10^{10}\right)\left(1.82x10^{-17}\right) \Rightarrow$$

or

$$J_{gen} = 2.73x10^{-7}\ A/cm^2$$

Then we can write

$$\exp\left(\frac{V_a}{2V_t}\right) = \frac{J_{gen}}{J_S} = \frac{2.73x10^{-7}}{4.16x10^{-11}} = 6.56x10^3$$

so that

$$V_a = 2(0.0259)\ln\left(6.56x10^3\right) \Rightarrow$$

$$\underline{V_a = 0.455\ V}$$

8.33
(a) We can write

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

We find

$$D_n = (3000)(0.0259) = 77.7\ cm^2/s$$

$$D_p = (200)(0.0259) = 5.18\ cm^2/s$$

Then

$$J_S = \left(1.6x10^{-19}\right)\left(1.8x10^{6}\right)^2\left[\frac{1}{10^{17}}\sqrt{\frac{77.7}{10^{-8}}} + \frac{1}{10^{17}}\sqrt{\frac{5.18}{10^{-8}}}\right]$$

or

$$J_S = 5.75x10^{-19}\ A/cm^2$$

so

$$I_S = AJ_S = \left(10^{-3}\right)\left(5.75x10^{-19}\right)$$

or

$$\underline{I_S = 5.75x10^{-22}\ A}$$

We also have

$$I_{gen} = \frac{en_iWA}{2\tau_O}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(10^{17})(10^{17})}{(1.8x10^6)^2}\right]$$

or

$$V_{bi} = 1.28\ V$$

Also

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(1.28+5)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{17}+10^{17}}{(10^{17})(10^{17})}\right)\right]^{1/2}$$

or

$$W = 0.427x10^{-4}\ cm$$

so

$$I_{gen} = \frac{(1.6x10^{-19})(1.8x10^6)(0.427x10^{-4})(10^{-3})}{2(10^{-8})}$$

or

$$\underline{I_{gen} = 6.15x10^{-13}\ A}$$

The total reverse-bias current

$$I_R = I_S + I_{gen} = 5.75x10^{-22} + 6.15x10^{-13}$$

or

$$\underline{I_R \approx 6.15x10^{-13}\ A}$$

Forward Bias: Ideal diffusion current

For $V_a = 0.3\ V$

$$I_D = I_S \exp\left(\frac{V_a}{V_t}\right) = (5.75x10^{-22})\exp\left(\frac{0.3}{0.0259}\right)$$

or

$$\underline{I_D = 6.17x10^{-17}\ A}$$

For $V_a = 0.5\ V$

$$I_D = (5.75x10^{-22})\exp\left(\frac{0.5}{0.0259}\right)$$

or

$$\underline{I_D = 1.39x10^{-13}\ A}$$

Recombination current

For $V_a = 0.3\ V$:

$$W = \left[\frac{2(13.1)(8.85x10^{-14})(1.28-0.3)}{1.6x10^{-19}}\left(\frac{2x10^{17}}{10^{34}}\right)\right]^{1/2}$$

or

$$W = 0.169x10^{-4}\ cm$$

Then

$$I_{rec} = \frac{en_i WA}{2\tau_O}\exp\left(\frac{V_a}{2V_t}\right)$$

$$= \frac{(1.6x10^{-19})(1.8x10^6)(0.169x10^{-4})(10^{-3})}{2(10^{-8})}$$

$$\times\exp\left[\frac{0.3}{2(0.0259)}\right]$$

or

$$\underline{I_{rec} = 7.96x10^{-11}\ A}$$

For $V_a = 0.5\ V$

$$W = \left[\frac{2(13.1)(8.85x10^{-14})(1.28-0.5)}{1.6x10^{-19}}\left(\frac{2x10^{17}}{10^{34}}\right)\right]^{1/2}$$

or

$$W = 0.150x10^{-4}\ cm$$

Then

$$I_{rec} = \frac{(1.6x10^{-19})(1.8x10^6)(0.15x10^{-4})(10^{-3})}{2(10^{-8})}$$

$$\times\exp\left[\frac{0.5}{2(0.0259)}\right]$$

or

$$\underline{I_{rec} = 3.36x10^{-9}\ A}$$

Total forward-bias current:

For $V_a = 0.3\ V$;

$$I_D = 6.17x10^{-17} + 7.96x10^{-11}$$

or

$$\underline{I_D \approx 7.96x10^{-11}\ A}$$

For $V_a = 0.5\ V$

$$I_D = 1.39x10^{-13} + 3.36x10^{-9}$$

or

$$\underline{I_D \approx 3.36x10^{-9}\ A}$$

(b)

Reverse-bias; ratio of generation to ideal diffusion current:

$$\frac{I_{gen}}{I_S} = \frac{6.15x10^{-13}}{5.75x10^{-22}}$$

Ratio $\underline{= 1.07x10^{9}}$

Forward bias: Ratio of recombination to ideal diffusion current:

For $V_a = 0.3\,V$

$$\frac{I_{rec}}{I_D} = \frac{7.96x10^{-11}}{6.17x10^{-17}}$$

Ratio $\underline{= 1.29x10^{6}}$

For $V_a = 0.5\,V$

$$\frac{I_{rec}}{I_D} = \frac{3.36x10^{-9}}{1.39x10^{-13}}$$

Ratio $\underline{= 2.42x10^{4}}$

8.34
Computer Plot

8.35
Computer Plot

8.36
Computer Plot

8.37
We have that

$$R = \frac{np - n_i^2}{\tau_{pO}(n+n') + \tau_{nO}(p+p')}$$

Let $\tau_{pO} = \tau_{nO} = \tau_O$ and $n' = p' = n_i$

We can write

$$n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

We also have

$$(E_{Fn} - E_{Fi}) + (E_{Fi} - E_{Fp}) = eV_a$$

so that

$$(E_{Fi} - E_{Fp}) = eV_a - (E_{Fn} - E_{Fi})$$

Then

$$p = n_i \exp\left[\frac{eV_a - (E_{Fn} - E_{Fi})}{kT}\right]$$

$$= n_i \exp\left(\frac{eV_a}{kT}\right) \cdot \exp\left[\frac{-(E_{Fn} - E_{Fi})}{kT}\right]$$

Define

$$\eta_a = \frac{eV_a}{kT} \quad \text{and} \quad \eta = \left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

Then the recombination rate can be written as

$$R = \frac{(n_i e^{\eta})(n_i e^{\eta_a} \cdot e^{-\eta}) - n_i^2}{\tau_O\left[n_i e^{\eta} + n_i + n_i e^{\eta_a} \cdot e^{-\eta} + n_i\right]}$$

or

$$R = \frac{n_i(e^{\eta_a} - 1)}{\tau_O(2 + e^{\eta} + e^{\eta_a} \cdot e^{-\eta})}$$

To find the maximum recombination rate, set

$$\frac{dR}{d\eta} = 0$$

$$= \frac{n_i(e^{\eta_a} - 1)}{\tau_O} \cdot \frac{d}{dx}\left[2 + e^{\eta} + e^{\eta_a} \cdot e^{-\eta}\right]^{-1}$$

or

$$0 = \frac{n_i(e^{\eta_a} - 1)}{\tau_O} \cdot (-1)\left[2 + e^{\eta} + e^{\eta_a} \cdot e^{-\eta}\right]^{-2} \times\left[e^{\eta} - e^{\eta_a} \cdot e^{-\eta}\right]$$

which simplifies to

$$0 = \frac{-n_i(e^{\eta_a} - 1)}{\tau_O} \cdot \frac{\left[e^{\eta} - e^{\eta_a} \cdot e^{-\eta}\right]}{\left[2 + e^{\eta} + e^{\eta_a} \cdot e^{-\eta}\right]^2}$$

The denominator is not zero, so we have

$$e^{\eta} - e^{\eta_a} \cdot e^{-\eta} = 0 \Rightarrow$$

$$e^{2\eta} = e^{\eta_a} \Rightarrow \eta = \frac{1}{2}\eta_a$$

Then the maximum recombination rate becomes

$$R_{\max} = \frac{n_i(e^{\eta_a} - 1)}{\tau_O\left[2 + e^{\eta_a/2} + e^{\eta_a} \cdot e^{-\eta_a/2}\right]}$$

$$= \frac{n_i(e^{\eta_a} - 1)}{\tau_O\left[2 + e^{\eta_a/2} + e^{\eta_a/2}\right]}$$

or

$$R_{\max} = \frac{n_i(e^{\eta_a} - 1)}{2\tau_O(e^{\eta_a/2} + 1)}$$

which can be written as

$$R_{\max} = \frac{n_i\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]}{2\tau_O\left[\exp\left(\frac{eV_a}{2kT}\right) + 1\right]}$$

If $V_a >> \left(\frac{kT}{e}\right)$, then we can neglect the (-1) term in the numerator and the (+1) term in the denominator so we finally have

$$R_{max} = \frac{n_i}{2\tau_O}\exp\left(\frac{eV_a}{2kT}\right)$$

Q.E.D.

8.38

We have

$$J_{gen} = \int_0^W eG\,dx$$

In this case, $G = g' = 4x10^{19}\ cm^{-3}s^{-1}$, that is a constant through the space charge region. Then

$$J_{gen} = eg'W$$

We find

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(5x10^{15})(5x10^{15})}{(1.5x10^{10})^2}\right] = 0.659\ V$$

and

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.659+10)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(\frac{5x10^{15}+5x10^{15}}{(5x10^{15})(5x10^{15})}\right)\right]^{1/2}$$

or

$$W = 2.35x10^{-4}\ cm$$

Then

$$J_{gen} = (1.6x10^{-19})(4x10^{19})(2.35x10^{-4})$$

or

$$J_{gen} = 1.5x10^{-3}\ A/cm^2$$

8.39

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= (1.6x10^{-19})(1.5x10^{10})^2\left[\frac{1}{3x10^{16}}\sqrt{\frac{18}{10^{-7}}}\right.$$

$$\left.+\frac{1}{10^{18}}\sqrt{\frac{6}{10^{-7}}}\right]$$

or

$$J_S = 1.64x10^{-11}\ A/cm^2$$

Now

$$J_D = J_S\exp\left(\frac{V_D}{V_t}\right)$$

Also

$$J = 0 = J_G - J_D$$

or

$$0 = 25x10^{-3} - 1.64x10^{-11}\exp\left(\frac{V_D}{V_t}\right)$$

which yields

$$\exp\left(\frac{V_D}{V_t}\right) = 1.52x10^9$$

or

$$V_D = V_t\ln(1.52x10^9)$$

so

$$V_D = 0.548\ V$$

8.40

$$V_B = \frac{\in E_{crit}^2}{2eN_B}$$

or

$$30 = \frac{(11.7)(8.85x10^{-14})(4x10^5)^2}{2(1.6x10^{-19})N_B}$$

which yields

$$N_B = N_d = 1.73x10^{16}\ cm^{-3}$$

8.41

For the breakdown voltage, we need $N_d = 3x10^{15}\ cm^{-3}$ and for this doping, we find $\mu_p = 430\ cm^2/V-s$. Then

$$D_p = (430)(0.0259) = 11.14\ cm^2/s$$

For the p^+n junction,

$$J_S = en_i^2\cdot\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}$$

$$= \frac{(1.6x10^{-19})(1.5x10^{10})^2}{3x10^{15}}\sqrt{\frac{11.14}{10^{-7}}}$$

or

$$J_S = 1.27x10^{-10}\ A/cm^2$$

Then

$$I = J_S A \exp\left(\frac{V_a}{V_t}\right)$$

$$2x10^{-3} = \left(1.27x10^{-10}\right) A \exp\left(\frac{0.65}{0.0259}\right)$$

Finally

$$\underline{A = 1.99x10^{-4}\ cm^2}$$

8.42

GaAs, n^+p, and $N_a = 10^{16}\ cm^{-3}$

From Figure 8.25

$$\underline{V_B \approx 75\ V}$$

8.43

$$E_{max} = \frac{eN_d x_n}{\in}$$

We can write

$$x_n = \frac{E_{max} \in}{eN_d}$$

$$= \frac{\left(4x10^5\right)(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(5x10^{16}\right)}$$

or

$$x_n = 5.18x10^{-5}\ cm$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5x10^{16}\right)\left(5x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.778\ V$$

Now

$$x_n = \left[\frac{2\in\left(V_{bi}+V_R\right)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

or

$$\left(5.18x10^{-5}\right)^2 = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{1.6x10^{-19}} \times \left(V_{bi}+V_R\right)\left(\frac{5x10^{16}}{5x10^{16}}\right)\left(\frac{1}{5x10^{16}+5x10^{16}}\right)\right]$$

which yields

$$2.68x10^{-9} = 1.29x10^{-10}\left(V_{bi}+V_R\right)$$

so

$$V_{bi}+V_R = 20.7 \Rightarrow \underline{V_R = 19.9\ V}$$

8.44

For a silicon p^+n junction with

$N_d = 5x10^{15}\ cm^{-3}$ and $V_B \approx 100\ V$

Neglecting V_{bi} compared to V_B

$$x_n \approx \left[\frac{2\in V_B}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(100)}{\left(1.6x10^{-19}\right)\left(5x10^{15}\right)}\right]^{1/2}$$

or

$$\underline{x_n(\min) = 5.09\ \mu m}$$

8.45

We find

$$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(10^{18}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.933\ V$$

Now

$$E_{max} = \frac{eN_d x_n}{\in}$$

so

$$10^6 = \frac{\left(1.6x10^{-19}\right)\left(10^{18}\right)x_n}{(11.7)\left(8.85x10^{-14}\right)}$$

which yields

$$x_n = 6.47x10^{-6}\ cm$$

Now

$$x_n = \left[\frac{2\in\left(V_{bi}+V_R\right)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

Then

$$\left(6.47x10^{-6}\right)^2 = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{1.6x10^{-19}} \times \left(V_{bi}+V_R\right)\left(\frac{10^{18}}{10^{18}}\right)\left(\frac{1}{10^{18}+10^{18}}\right)\right]$$

which yields

$$V_{bi}+V_R = 6.468\ V$$

or

$$\underline{V_R = 5.54\ V}$$

8.46

Assume silicon: For an n^+p junction

$$x_p = \left[\frac{2\in\left(V_{bi}+V_R\right)}{eN_a}\right]^{1/2}$$

Assume $V_{bi} << V_R$

(a)

For $x_p = 75\ \mu m$

Then

$$\left(75x10^{-4}\right)^2 = \frac{2(11.7)\left(8.85x10^{-14}\right)V_R}{\left(1.6x10^{-19}\right)\left(10^{15}\right)}$$

which yields $V_R = 4.35x10^3\ V$

(b)

For $x_p = 150\ \mu m$, we find

$V_R = 1.74x10^4\ V$

From Figure 8.25, the breakdown voltage is approximately 300 V. So, in each case, breakdown is reached first.

8.47

Impurity gradient

$$a = \frac{2x10^{18}}{2x10^{-4}} = 10^{22}\ cm^{-4}$$

From the figure

$V_B = 15\ V$

8.48

(a) If $\dfrac{I_R}{I_F} = 0.2$

Then we have

$$erf\sqrt{\frac{t_s}{\tau_{pO}}} = \frac{I_F}{I_F + I_R} = \frac{1}{1+\dfrac{I_R}{I_F}} = \frac{1}{1+0.2}$$

or

$$erf\sqrt{\frac{t_S}{\tau_{pO}}} = 0.833$$

We find

$$\sqrt{\frac{t_S}{\tau_{pO}}} = 0.978 \Rightarrow \underline{\frac{t_S}{\tau_{pO}} = 0.956}$$

(b)

If $\dfrac{I_R}{I_F} = 1.0$, then

$$erf\sqrt{\frac{t_S}{\tau_{pO}}} = \frac{1}{1+1} = 0.5$$

which yields

$$\underline{\frac{t_S}{\tau_{pO}} = 0.228}$$

8.49

We want

$$\frac{t_S}{\tau_{pO}} = 0.2$$

Then

$$erf\sqrt{\frac{t_S}{\tau_{pO}}} = \frac{1}{1+\dfrac{I_R}{I_F}} = erf\sqrt{0.2}$$

where

$$erf\sqrt{0.2} = erf(0.447) = 0.473$$

We obtain

$$\frac{I_R}{I_F} = \frac{1}{0.473} - 1 \Rightarrow \underline{\frac{I_R}{I_F} = 1.11}$$

We have

$$erf\sqrt{\frac{t_2}{\tau_{pO}}} + \frac{\exp\left(\dfrac{-t_2}{\tau_{pO}}\right)}{\sqrt{\pi\left(\dfrac{t_2}{\tau_{pO}}\right)}} = 1 + (0.1)\left(\frac{I_R}{I_F}\right) = 1.11$$

By trial and error,

$$\underline{\frac{t_2}{\tau_{pO}} = 0.65}$$

8.50

$C_j = 18\ pF$ at $V_R = 0$

$C_j = 4.2\ pF$ at $V_R = 10\ V$

We have $\tau_{nO} = \tau_{pO} = 10^{-7}\ s$, $I_F = 2\ mA$

And $I_R \approx \dfrac{V_R}{R} = \dfrac{10}{10} = 1\ mA$

So

$$t_S \approx \tau_{pO} \ln\left(1 + \frac{I_F}{I_R}\right) = \left(10^{-7}\right)\ln\left(1 + \frac{2}{1}\right)$$

or

$$t_S = 1.1x10^{-7}\ s$$

Also

$$C_{avg} = \frac{18 + 4.2}{2} = 11.1\ pF$$

The time constant is

$$\tau_S = RC_{avg} = (10^4)(11.1x10^{-12}) = 1.11x10^{-7}\ s$$

Now

Turn-off time $= t_S + \tau_S = (1.1 + 1.11) \times 10^{-7}\ s$

Or

$$\underline{2.21x10^{-7}\ s}$$

8.51

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{19})^2}{(1.5x10^{10})^2}\right] = 1.14\ V$$

We find

$$W = \left[\frac{2 \in (V_{bi} - V_a)}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(1.14 - 0.40)}{1.6x10^{-19}} \times \left(\frac{5x10^{19} + 5x10^{19}}{(5x10^{19})^2}\right)\right]^{1/2}$$

which yields

$$\underline{W = 6.19x10^{-7}\ cm = 61.9\ \overset{\circ}{A}}$$

8.52

Sketch

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Chapter 9

Problem Solutions

9.1

(a) We have

$$e\phi_n = eV_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ eV$$

(c)

$$\phi_{BO} = \phi_m - \chi = 4.28 - 4.01$$

or

$$\underline{\phi_{BO} = 0.27\ V}$$

and

$$V_{bi} = \phi_{BO} - \phi_n = 0.27 - 0.206$$

or

$$V_{bi} = 0.064\ V$$

Also

$$x_d = \left[\frac{2 \in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.064)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 9.1x10^{-6}\ cm}$$

Then

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(9.1x10^{-6})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 1.41x10^4\ V/cm}$$

(d)

Using the figure, $\underline{\phi_{Bn} = 0.55\ V}$

So

$$V_{bi} = \phi_{Bn} - \phi_n = 0.55 - 0.206$$

or

$$V_{bi} = 0.344\ V$$

We then find

$\underline{x_d = 2.11x10^{-5}\ cm}$ and $\underline{E_{max} = 3.26x10^4\ V/cm}$

9.2

(a) $\phi_{BO} = \phi_m - \chi = 5.1 - 4.01$

or

$$\underline{\phi_{BO} = 1.09\ V}$$

(b)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{15}}\right) = 0.265\ V$$

Then

$$V_{bi} = \phi_{BO} - \phi_n = 1.09 - 0.265$$

or

$$\underline{V_{bi} = 0.825\ V}$$

(c)

$$W = x_d = \left[\frac{2 \in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.825)}{(1.6x10^{-19})(10^{15})}\right]^{1/2}$$

or

$$\underline{W = 1.03x10^{-4}\ cm}$$

(d)

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{15})(1.03x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 1.59x10^4\ V/cm}$$

9.3

(a) Gold on n-type GaAs

$\chi = 4.07\ V$ and $\phi_m = 5.1\ V$

$$\phi_{BO} = \phi_m - \chi = 5.1 - 4.07$$

and

$$\underline{\phi_{BO} = 1.03\ V}$$

(b)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{5x10^{16}}\right)$$

or

$$\underline{\phi_n = 0.0580\ V}$$

(c)

$$V_{bi} = \phi_{BO} - \phi_n = 1.03 - 0.058$$

or

$$\underline{V_{bi} = 0.972\ V}$$

(d)

$$x_d = \left[\frac{2\in(V_{bi}+V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.972+5)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.416\ \mu m}$$

(e)

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(5x10^{16})(0.416x10^{-4})}{(13.1)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 2.87x10^5\ V/cm}$$

9.4

$\phi_{Bn} = 0.86\ V$ and $\phi_n = 0.058\ V$ (Problem 9.3)

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.86 - 0.058$$

or

$$\underline{V_{bi} = 0.802\ V}$$

and

$$x_d = \left[\frac{2\in(V_{bi}+V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.802+5)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or]

$$\underline{x_d = 0.410\ \mu m}$$

Also

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(5x10^{16})(0.410x10^{-4})}{(13.1)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 2.83x10^5\ V/cm}$$

9.5

Gold, n-type silicon junction. From the figure,

$$\phi_{Bn} = 0.81\ V$$

For $N_d = 5x10^{15}\ cm^{-3}$, we have

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{5x10^{15}}\right) = \phi_n = 0.224\ V$$

Then

$$V_{bi} = 0.81 - 0.224 = 0.586\ V$$

(a)

Now

$$C' = \left[\frac{e\in N_d}{2(V_{bi}+V_R)}\right]^{1/2}$$

$$= \left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})(5x10^{15})}{2(0.586+4)}\right]^{1/2}$$

or

$$C' = 9.50x10^{-9}\ F/cm^2$$

For $A = 5x10^{-4}\ cm^2$, $C = C'A$

So

$$\underline{C = 4.75\ pF}$$

(b)

For $N_d = 5x10^{16}\ cm^{-3}$, we find

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{5x10^{16}}\right) = 0.164\ V$$

Then

$$V_{bi} = 0.81 - 0.164 = 0.646\ V$$

Now

$$C' = \left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})(5x10^{16})}{2(0.646+4)}\right]^{1/2}$$

or

$$C' = 2.99x10^{-8}\ F/cm^2$$

and

$$C = C'A$$

so

$$\underline{C = 15\ pF}$$

9.6

(a) From the figure, $\underline{V_{bi} = 0.90\ V}$

(b) We find

$$\frac{\Delta\left(\frac{1}{C'}\right)^2}{\Delta V_R} = \frac{3x10^{15} - 0}{2 - (-0.9)} = 1.03x10^{15}$$

and

$$1.03x10^{15} = \frac{2}{e \in N_d}$$

Then we can write

$$N_d = \frac{2}{(1.6x10^{-19})(13.1)(8.85x10^{-14})(1.03x10^{15})}$$

or

$$\underline{N_d = 1.05x10^{16}\ cm^{-3}}$$

(c)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{1.05x10^{16}}\right)$$

or

$$\underline{\phi_n = 0.0985\ V}$$

(d)

$$\phi_{Bn} = V_{bi} + \phi_n = 0.90 + 0.0985$$

or

$$\underline{\phi_{Bn} = 0.9985\ V}$$

9.7

From the figure, $\phi_{Bn} = 0.55\ V$

(a)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ V$$

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.55 - 0.206$$

or

$$\underline{V_{bi} = 0.344\ V}$$

We find

$$x_d = \left[\frac{2 \in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.344)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.211\ \mu m}$$

Also

$$|\mathrm{E}_{\max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(0.211x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|\mathrm{E}_{\max}| = 3.26x10^4\ V/cm}$$

(b)

$$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in}} = \left[\frac{(1.6x10^{-19})(3.26x10^4)}{4\pi(11.7)(8.85x10^{-14})}\right]^{1/2}$$

or

$$\underline{\Delta\phi = 20.0\ mV}$$

Also

$$x_m = \sqrt{\frac{e}{16\pi \in \mathrm{E}}}$$

$$= \left[\frac{(1.6x10^{-19})}{16\pi(11.7)(8.85x10^{-14})(3.26x10^4)}\right]^{1/2}$$

or

$$\underline{x_m = 0.307x10^{-6}\ cm}$$

(c)

For $V_R = 4\ V$

$$x_d = \left[\frac{2(11.7)(8.85x10^{-14})(0.344 + 4)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.75\ \mu m}$$

and

$$|\mathrm{E}_{\max}| = \frac{(1.6x10^{-19})(10^{16})(0.75x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|\mathrm{E}_{\max}| = 1.16x10^5\ V/cm}$$

We find

$$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in}} \Rightarrow \underline{\Delta\phi = 37.8\ mV}$$

and

$$x_m = \sqrt{\frac{e}{16\pi \in \mathrm{E}}} \Rightarrow \underline{x_m = 0.163x10^{-6}\ cm}$$

9.8

We have

$$-\phi(x) = \frac{-e}{16\pi \in x} - \mathrm{E}x$$

or

$$e\phi(x) = \frac{e^2}{16\pi \in x} + \mathrm{E}ex$$

Now

$$\frac{d(e\phi(x))}{dx} = 0 = \frac{-e^2}{16\pi \in x^2} + \mathrm{E}e$$

Solving for x^2, we find

$$x^2 = \frac{e}{16\pi \in \mathrm{E}}$$

or

$$\underline{x = x_m = \sqrt{\frac{e}{16\pi \in \mathrm{E}}}}$$

Substituting this value of $x_m = x$ into the equation for the potential, we find

$$\Delta\phi = \frac{e}{16\pi \in \sqrt{\dfrac{e}{16\pi \in \mathrm{E}}}} + \mathrm{E}\sqrt{\frac{e}{16\pi \in \mathrm{E}}}$$

which yields

$$\underline{\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in}}}$$

9.9

Gold, n-type GaAs, from the figure $\phi_{Bn} = 0.87\ V$

(a)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{5x10^{16}}\right) = 0.058\ V$$

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.87 - 0.058$$

or

$$\underline{V_{bi} = 0.812\ V}$$

Also

$$x_d = \left[\frac{2 \in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.812)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.153\ \mu m}$$

Then

$$|\mathrm{E}_{max}| = \frac{eN_d x_d}{\in}$$

$$= \left[\frac{(1.6x10^{-19})(5x10^{16})(0.153x10^{-4})}{(13.1)(8.85x10^{-14})}\right]$$

or

$$\underline{|\mathrm{E}_{max}| = 1.06x10^5\ V/cm}$$

(b)

We want $\Delta\phi$ to be 7% or ϕ_{Bn},

So

$$\Delta\phi = (0.07)(0.87) = 0.0609\ V$$

Now

$$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in}} \Rightarrow \mathrm{E} = \frac{(\Delta\phi^2)(4\pi \in)}{e}$$

so

$$\mathrm{E} = \frac{(0.0609)^2(4\pi)(13.1)(8.85x10^{-14})}{1.6x10^{-19}}$$

or

$$\mathrm{E}_{max} = 3.38x10^5\ V/cm$$

Now

$$\mathrm{E}_{max} = \frac{eN_d x_d}{\in} \Rightarrow x_d = \frac{\in \mathrm{E}}{eN_d}$$

so

$$x_d = \frac{(13.1)(8.85x10^{-14})(3.38x10^5)}{(1.6x10^{-19})(5x10^{16})}$$

or

$$x_d = 0.49\ \mu m$$

Then

$$x_d = 0.49x10^{-4} = \left[\frac{2 \in (V_{bi} + V_R)}{eN_d}\right]^{1/2}$$

or we can write

$$(V_{bi} + V_R) = \frac{eN_d x_d^2}{2 \in}$$

$$= \frac{(1.6x10^{-19})(5x10^{16})(0.49x10^{-4})^2}{2(13.1)(8.85x10^{-14})}$$

or

$$V_{bi} + V_R = 8.28\ V = 0.812 + V_R$$

or

$$\underline{V_R = 7.47\ V}$$

9.10

Computer Plot

9.11

(a) $\phi_{BO} = \phi_m - \chi = 5.2 - 4.07$

or

$$\underline{\phi_{BO} = 1.13\ V}$$

(b)

We have

$$\left(E_g - e\phi_O - e\phi_{Bn}\right) = \frac{1}{eD_{it}}\sqrt{2e \in N_d\left(\phi_{Bn} - \phi_n\right)}$$

$$-\frac{\in_i}{eD_{it}\delta}\left[\phi_m - \left(\chi + \phi_{Bn}\right)\right]$$

which becomes

$$e\left(1.43 - 0.60 - \phi_{Bn}\right)$$

$$= \frac{1}{e\left(\dfrac{10^{13}}{e}\right)}\left[2\left(1.6x10^{-19}\right)(13.1)\left(8.85x10^{-14}\right)\right.$$

$$\left.\times\left(10^{16}\right)\left(\phi_{Bn} - 0.10\right)\right]^{1/2}$$

$$-\frac{\left(8.85x10^{-14}\right)}{e\left(\dfrac{10^{13}}{e}\right)\left(25x10^{-8}\right)}\left[5.2 - \left(4.07 + \phi_{Bn}\right)\right]$$

or

$$0.83 - \phi_{Bn}$$

$$= 0.038\sqrt{\phi_{Bn} = 0.10} - 0.221\left(1.13 - \phi_{Bn}\right)$$

We then find

$$\underline{\phi_{Bn} = 0.858\ V}$$

(c)

If $\phi_m = 4.5\ V$, then

$$\phi_{BO} = \phi_m - \chi = 4.5 - 4.07$$

or

$$\underline{\phi_{BO} = 0.43\ V}$$

From part (b), we have

$$0.83 - \phi_{Bn}$$

$$= 0.038\sqrt{\phi_{Bn} - 0.10} - 0.221\left[4.5 - \left(4.07 + \phi_{Bn}\right)\right]$$

We then find

$$\underline{\phi_{Bn} = 0.733\ V}$$

With interface states, the barrier height is less sensitive to the metal work function.

9.12

We have that

$$\left(E_g - e\phi_O - e\phi_{Bn}\right)$$

$$= \frac{1}{eD_{it}}\sqrt{2e \in N_d\left(\phi_{Bn} - \phi_n\right)}$$

$$-\frac{\in_i}{eD_{it}\delta}\left[\phi_m - \left(\chi + \phi_{Bn}\right)\right]$$

Let $eD_{it} = D'_{it}\left(cm^{-2}eV^{-1}\right)$. Then we can write

$$e(1.12 - 0.230 - 0.60)$$

$$= \frac{1}{D'_{it}}\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\right.$$

$$\left.\times\left(5x10^{16}\right)(0.60 - 0.164)\right]^{1/2}$$

$$-\frac{\left(8.85x10^{-14}\right)}{D'_{it}\left(20x10^{-8}\right)}\left[4.75 - (4.01 + 0.60)\right]$$

We find that

$$\underline{D'_{it} = 4.97x10^{11}\ cm^{-2}eV^{-1}}$$

9.13

(a) $\phi_n = V_t \ln\left(\dfrac{N_C}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right)$$

or

$$\underline{\phi_n = 0.206\ V}$$

(b)

$$V_{bi} = \phi_{Bn} - \phi_n = 0.89 - 0.206$$

or

$$\underline{V_{bi} = 0.684\ V}$$

(c)

$$J_{ST} = A^*T^2 \exp\left(\frac{-e\phi_{Bn}}{kT}\right)$$

For silicon, $A^* = 120\ A/cm^2/°K^2$

Then

$$J_{ST} = (120)(300)^2 \exp\left(\frac{-0.89}{0.0259}\right)$$

or

$$\underline{J_{ST} = 1.3x10^{-8}\ A/cm^2}$$

(d)

$$J_n = J_{ST}\exp\left(\frac{eV_a}{kT}\right)$$

or

$$V_a = V_t\ln\left(\frac{J_n}{J_{ST}}\right) = (0.0259)\ln\left(\frac{2}{1.3x10^{-8}}\right)$$

or

$$\underline{V_a = 0.488\ V}$$

9.14

(a) From the figure, $\phi_{Bn} = 0.68\ V$

Then

$$J_{ST} = A^*T^2\exp\left(\frac{-\phi_{Bn}}{V_t}\right)$$

$$= (120)(300)^2\exp\left(\frac{-0.68}{0.0259}\right)$$

or

$$J_{ST} = 4.28x10^{-5}\ A/cm^2$$

For $I = 10^{-3}\ A \Rightarrow J_n = \dfrac{10^{-3}}{5x10^{-4}} = 2\ A/cm^2$

We have

$$V_a = V_t\ln\left(\frac{J_n}{J_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{2}{4.28x10^{-5}}\right)$$

or

$$\underline{V_a = 0.278\ V}$$

For $I = 10\ mA \Rightarrow J_n = 20\ A/cm^2$

And

$$V_a = (0.0259)\ln\left(\frac{20}{4.28x10^{-5}}\right)$$

or

$$\underline{V_a = 0.338\ V}$$

For $I = 100\ mA \Rightarrow J_n = 200\ A/cm^2$

And

$$V_a = (0.0259)\ln\left(\frac{200}{4.28x10^{-5}}\right)$$

or

$$\underline{V_a = 0.398\ V}$$

(b)

For $T = 400K$, $\phi_{Bn} = 0.68\ V$

Now

$$J_{ST} = (120)(400)^2\exp\left[\frac{-0.68}{(0.0259)(400/300)}\right]$$

or

$$J_{ST} = 5.39x10^{-2}\ A/cm^2$$

For $I = 1\ mA$,

$$V_a = (0.0259)\left(\frac{400}{300}\right)\ln\left(\frac{2}{5.39x10^{-2}}\right)$$

or

$$\underline{V_a = 0.125\ V}$$

For $I = 10\ mA$,

$$V_a = (0.0259)\left(\frac{400}{300}\right)\ln\left[\frac{20}{5.39x10^{-2}}\right]$$

or

$$\underline{V_a = 0.204\ V}$$

For $I = 100\ mA$,

$$V_a = (0.0259)\left(\frac{400}{300}\right)\ln\left(\frac{200}{5.39x10^{-2}}\right)$$

or

$$\underline{V_a = 0.284\ V}$$

9.15

(a) From the figure, $\phi_{Bn} = 0.86\ V$

$$J_{ST} = A^*T^2\exp\left(\frac{-\phi_{Bn}}{V_t}\right)$$

$$= (1.12)(300)^2\exp\left(\frac{-0.86}{0.0259}\right)$$

or

$$J_{ST} = 3.83x10^{-10}\ A/cm^2$$

Now

$$J_n = J_{ST}\exp\left(\frac{V_a}{V_t}\right)$$

and we can write, for $J_n = 5\ A/cm^2$

$$V_a = V_t\ln\left(\frac{J_n}{J_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{5}{3.83x10^{-10}}\right)$$

or

$$\underline{V_a = 0.603\ V}$$

(b)
For $J_n = 10\ A/cm^2$

$$V_a = (0.0259)\ln\left(\frac{10}{3.83x10^{-10}}\right) = 0.621\ V$$

so

$$\Delta V_a = 0.621 - 0.603 \Rightarrow$$

$$\underline{\Delta V_a = 18\ mV}$$

9.16
Computer Plot

9.17
From the figure, $\phi_{Bn} = 0.86\ V$

$$J_{ST} = A^* T^2 \exp\left(\frac{-\phi_{Bn}}{V_t}\right)\exp\left(\frac{\Delta\phi}{V_t}\right)$$

$$= (120)(300)^2 \exp\left(\frac{-0.68}{0.0259}\right)\exp\left(\frac{\Delta\phi}{V_t}\right)$$

or

$$J_{ST} = 4.28x10^{-5}\exp\left(\frac{\Delta\phi}{V_t}\right)$$

We have

$$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in}}$$

Now

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ V$$

and

$$V_{bi} = \phi_{Bn} - \phi_n = 0.68 - 0.206 = 0.474\ V$$

(a)
We find for $V_R = 2\ V$,

$$x_d = \left[\frac{2\in(V_{bi} + V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(2.474)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 0.566\ \mu m$$

Then

$$|\mathrm{E}_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(0.566x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$|\mathrm{E}_{max}| = 8.75x10^4\ V/cm$$

Now

$$\Delta\phi = \left[\frac{(1.6x10^{-19})(8.75x10^4)}{4\pi(11.7)(8.85x10^{-14})}\right]^{1/2}$$

or

$$\Delta\phi = 0.0328\ V$$

Then

$$J_{R1} = 4.28x10^{-5}\exp\left(\frac{0.0328}{0.0259}\right)$$

or

$$J_{R1} = 1.52x10^{-4}\ A/cm^2$$

For $A = 10^{-4}\ cm^2$, then

$$\underline{I_{R1} = 1.52x10^{-8}\ A}$$

(b)
For $V_R = 4\ V$,

$$x_d = \left[\frac{2(11.7)(8.85x10^{-14})(4.474)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 0.761\ \mu m$$

$$|\mathrm{E}_{max}| = \frac{(1.6x10^{-19})(10^{16})(0.761x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$|\mathrm{E}_{max}| = 1.18x10^5\ V/cm$$

and

$$\Delta\phi = \left[\frac{(1.6x10^{-19})(1.18x10^5)}{4\pi(11.7)(8.85x10^{-14})}\right]^{1/2}$$

or

$$\Delta\phi = 0.0381\ V$$

Now

$$J_{R2} = 4.28x10^{-5}\exp\left(\frac{0.0381}{0.0259}\right)$$

or

$$J_{R2} = 1.86x10^{-4}\ A/cm^2$$

Finally,

$$\underline{I_{R2} = 1.86x10^{-8}\ A}$$

9.18
We have that

$$J_{s\to m}^{-} = \int_{E_C}^{\infty} v_x dn$$

The incremental electron concentration is given by

$$dn = g_C(E) f_F(E) dE$$

We have

$$g_C(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}$$

and, assuming the Boltzmann approximation

$$f_F(E) = \exp\left[\frac{-(E - E_F)}{kT}\right]$$

Then

$$dn = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}\cdot\exp\left[\frac{-(E - E_F)}{kT}\right]dE$$

If the energy above E_C is kinetic energy, then

$$\frac{1}{2}m_n^* v^2 = E - E_C$$

We can then write

$$\sqrt{E - E_C} = v\sqrt{\frac{m_n^*}{2}}$$

and

$$dE = \frac{1}{2}m_n^*\cdot 2v dv = m_n^* v dv$$

We can also write

$$E - E_F = (E - E_C) + (E_C - E_F) = \frac{1}{2}m_n^* v^2 + e\phi_n$$

so that

$$dn = 2\left(\frac{m_n^*}{h}\right)^3 \exp\left(\frac{-e\phi_n}{kT}\right)\cdot\exp\left(\frac{-m_n^* v^2}{2kT}\right)\cdot 4\pi v^2 dv$$

We can write

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

The differential volume element is

$$4\pi v^2 dv = dv_x dv_y dv_z$$

The current is due to all x-directed velocities that are greater than v_{Ox} and for all y- and z-directed velocities. Then

$$J_{s\to m}^{-} = 2\left(\frac{m_n^*}{h}\right)^3 \exp\left(\frac{-e\phi_n}{kT}\right) \times \int_{v_{Ox}}^{\infty} v_x \exp\left(\frac{-m_n^* v_x^2}{2kT}\right)dv_x$$
$$\times \int_{-\infty}^{+\infty}\exp\left(\frac{-m_n^* v_y^2}{2kT}\right)dv_y \times \int_{-\infty}^{+\infty}\exp\left(\frac{-m_n^* v_z^2}{2kT}\right)dv_z$$

We can write that

$$\frac{1}{2}m_n^* v_{Ox}^2 = e(V_{bi} - V_a)$$

Make a change of variables:

$$\frac{m_n^* v_x^2}{2kT} = \alpha^2 + \frac{e(V_{bi} - V_a)}{kT}$$

or

$$v_x^2 = \frac{2kT}{m_n^*}\left[\alpha^2 + \frac{e(V_{bi} - V_a)}{kT}\right]$$

Taking the differential, we find

$$v_x dv_x = \left(\frac{2kT}{m_n^*}\right)\alpha d\alpha$$

We may note that when $v_x = v_{Ox}$, $\alpha = 0$.
Other change of variables:

$$\frac{m_n^* v_y^2}{2kT} = \beta^2 \Rightarrow v_y = \left(\frac{2kT}{m_n^*}\right)^{1/2}\cdot\beta$$

$$\frac{m_n^* v_z^2}{2kT} = \gamma^2 \Rightarrow v_z = \left(\frac{2kT}{m_n^*}\right)^{1/2}\cdot\gamma$$

Substituting the new variables, we have

$$J_{s\to m}^{-} = 2\left(\frac{m_n^*}{h}\right)^3\cdot\left(\frac{2kT}{m_n^*}\right)^2 \exp\left(\frac{-e\phi_n}{kT}\right)$$
$$\times\exp\left[\frac{-e(V_{bi} - V_a)}{kT}\right]\cdot\int_0^{\infty}\alpha\exp\left(-\alpha^2\right)d\alpha$$
$$\times\int_{-\infty}^{+\infty}\exp\left(-\beta^2\right)d\beta\cdot\int_{-\infty}^{+\infty}\exp\left(-\gamma^2\right)d\gamma$$

9.19
For the Schottky diode,

$$J_{ST} = 3x10^{-8}\ A/cm^2\ ,\ A = 5x10^{-4}\ cm^2$$

For $I = 1\ mA$,

$$J = \frac{10^{-3}}{5x10^{-4}} = 2\ A/cm^2$$

We have

$$V_a = V_t \ln\left(\frac{J}{J_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{2}{3x10^{-8}}\right)$$

or

$$\underline{V_a = 0.467\ V\ \text{(Schottky diode)}}$$

For the pn junction, $J_S = 3x10^{-12}\ A/cm^2$

Then

$$V_a = (0.0259)\ln\left(\frac{2}{3x10^{-12}}\right)$$

or

$$\underline{V_a = 0.705\ V\ \text{(pn junction diode)}}$$

9.20

For the pn junction diode,

$$J_S = 5x10^{-12}\ A/cm^2\ ,\ A = 8x10^{-4}\ cm^2$$

For $I = 1.2\ mA$,

$$J = \frac{1.2x10^{-3}}{8x10^{-4}} = 1.5\ A/cm^2$$

Then

$$V_a = V_t \ln\left(\frac{J}{J_S}\right)$$

$$= (0.0259)\ln\left(\frac{1.5}{5x10^{-12}}\right) = 0.684\ V$$

For the Schottky diode, the applied voltage will be less, so

$$V_a = 0.684 - 0.265 = 0.419\ V$$

We have

$$I = AJ_{ST}\exp\left(\frac{V_a}{V_t}\right)$$

so

$$1.2x10^{-3} = A\left(7x10^{-8}\right)\exp\left(\frac{0.419}{0.0259}\right)$$

which yields

$$\underline{A = 1.62x10^{-3}\ cm^2}$$

9.21

(a) Diodes in parallel:

We can write

$$I_S = I_{ST}\exp\left(\frac{V_{as}}{V_t}\right)\quad \text{(Schottky diode)}$$

and

$$I_{PN} = I_S\exp\left(\frac{V_{apn}}{V_t}\right)\quad \text{(pn junction diode)}$$

We have $I_S + I_{PN} = 0.5x10^{-3}\ A$, $V_{as} = V_{apn}$

Then

$$0.5x10^{-3} = \left(I_{ST} + I_S\right)\exp\left(\frac{V_a}{V_t}\right)$$

or

$$V_a = V_t \ln\left(\frac{0.5x10^{-3}}{I_S + I_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{0.5x10^{-3}}{5x10^{-8} + 10^{-12}}\right) = 0.239\ V$$

Now

$$I_S = 5x10^{-8}\exp\left(\frac{0.239}{0.0259}\right)$$

or

$$\underline{I_S \approx 0.5x10^{-3}\ A}\quad \text{(Schottky diode)}$$

and

$$I_{PN} = 10^{-12}\exp\left(\frac{0.239}{0.0259}\right)$$

or

$$\underline{I_{PN} = 1.02x10^{-8}\ A}\quad \text{(pn junction diode)}$$

(b) Diodes in Series:

We obtain,

$$V_{as} = (0.0259)\ln\left(\frac{0.5x10^{-3}}{5x10^{-8}}\right)$$

or

$$\underline{V_{as} = 0.239\ V}\quad \text{(Schottky diode)}$$

and

$$V_{apn} = (0.0259)\ln\left(\frac{0.5x10^{-3}}{10^{-12}}\right)$$

or

$$\underline{V_{apn} = 0.519\ V}\quad \text{(pn junction diode)}$$

9.22

(a) For $I = 0.8\ mA$, we find

$$J = \frac{0.8x10^{-3}}{7x10^{-4}} = 1.14\ A/cm^2$$

We have

$$V_a = V_t \ln\left(\frac{J}{J_S}\right)$$

For the pn junction diode,

$$V_a = (0.0259)\ln\left(\frac{1.14}{3x10^{-12}}\right)$$

or

$$\underline{V_a = 0.691\ V}$$

For the Schottky diode,

$$V_a = (0.0259)\ln\left(\frac{1.14}{4x10^{-8}}\right)$$

or

$$\underline{V_a = 0.445\ V}$$

(b)
For the pn junction diode,

$$J_S \propto n_i^2 \propto \left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

Then

$$\frac{J_S(400)}{J_S(300)}$$

$$= \left(\frac{400}{300}\right)^3 \exp\left[\frac{-E_g}{(0.0259)(400/300)} + \frac{E_g}{0.0259}\right]$$

or

$$= 2.37\exp\left[\frac{1.12}{0.0259} - \frac{1.12}{0.03453}\right]$$

We find

$$\frac{J_S(400)}{J_S(300)} = 1.16x10^5$$

Now

$$I = (7x10^{-4})(1.16x10^5)(3x10^{-12})\exp\left(\frac{0.691}{0.03453}\right)$$

or

$$\underline{I = 120\ mA}$$

For the Schottky diode

$$J_{ST} \propto T^2 \exp\left(\frac{-e\phi_{BO}}{kT}\right)$$

Now

$$\frac{J_{ST}(400)}{J_{ST}(300)}$$

$$= \left(\frac{400}{300}\right)^2 \exp\left[\frac{-\phi_{BO}}{(0.0259)(400/300)} + \frac{\phi_{BO}}{0.0259}\right]$$

or

$$= 1.78\exp\left[\frac{0.82}{0.0259} - \frac{0.82}{0.03453}\right]$$

We obtain

$$\frac{J_{ST}(400)}{J_{ST}(300)} = 4.85x10^3$$

and so

$$I = (7x10^{-4})(4.85x10^3)(4x10^{-8})\exp\left(\frac{0.445}{0.03453}\right)$$

or

$$\underline{I = 53.7\ mA}$$

9.23
Computer Plot

9.24
We have

$$R_C = \frac{\left(\frac{kT}{e}\right)\cdot\exp\left(\frac{e\phi_{Bn}}{kT}\right)}{A^*T^2}$$

which can be rewritten as

$$\ln\left[\frac{R_C A^* T^2}{(kT/e)}\right] = \frac{e\phi_{Bn}}{kT}$$

so

$$\phi_{Bn} = \left(\frac{kT}{e}\right)\cdot\ln\left[\frac{R_C A^* T^2}{(kT/e)}\right]$$

$$= (0.0259)\ln\left[\frac{(10^{-5})(120)(300)^2}{0.0259}\right]$$

or

$$\underline{\phi_{Bn} = 0.216\ V}$$

9.25
(b) We need $\phi_n = \phi_m - \chi_s = 4.2 - 4.0 = 0.20\ V$
And

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

or

$$0.20 = (0.0259)\ln\left(\frac{2.8x10^{19}}{N_d}\right)$$

which yields

$$\underline{N_d = 1.24x10^{16}\ cm^{-3}}$$

(c)
Barrier height $\underline{= 0.20\ V}$

9.26
We have that

$$\mathrm{E} = \frac{-eN_d}{\in}(x_n - x)$$

Then

$$\phi = -\int \mathrm{E}dx = \frac{eN_d}{\in}\left(x_n \cdot x - \frac{x^2}{2}\right) + C_2$$

Let $\phi = 0$ at $x = 0 \Rightarrow C_2 = 0$
So

$$\phi = \frac{eN_d}{\in}\left(x_n \cdot x - \frac{x^2}{2}\right)$$

At $x = x_n$, $\phi = V_{bi}$, so

$$\phi = V_{bi} = \frac{eN_d}{\in} \cdot \frac{x_n^2}{2}$$

or

$$x_n = \sqrt{\frac{2 \in V_{bi}}{eN_d}}$$

Also

$$V_{bi} = \phi_{BO} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

For

$$\phi = \frac{\phi_{BO}}{2} = \frac{0.70}{2} = 0.35\ V$$

we have

$$0.35 = \frac{(1.6x10^{-19})N_d}{(11.7)(8.85x10^{-14})}\left[x_n(50x10^{-8}) - \frac{(50x10^{-8})^2}{2}\right]$$

or

$$0.35 = 7.73x10^{-14}\ N_d(x_n - 25x10^{-8})$$

We have

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})V_{bi}}{(1.6x10^{-19})N_d}\right]^{1/2}$$

and

$$V_{bi} = 0.70 - \phi_n$$

By trial and error,

$$\underline{N_d = 3.5x10^{18}\ cm^{-3}}$$

9.27

(b) $\phi_{BO} = \phi_p = V_t \ln\left(\dfrac{N_V}{N_a}\right)$

$$= (0.0259)\ln\left(\frac{1.04x10^{19}}{5x10^{16}}\right) \Rightarrow$$

$$\underline{\phi_{BO} = 0.138\ V}$$

9.28
Sketches

9.29
Sketches

9.30
Electron affinity rule

$$\Delta E_C = e(\chi_n - \chi_p)$$

For GaAs, $\chi = 4.07$; and for AlAs, $\chi = 3.5$,
If we assume a linear extrapolation between GaAs and AlAs, then for

$$Al_{0.3}Ga_{0.7}As \Rightarrow \chi = 3.90$$

Then

$$|E_C| = 4.07 - 3.90 \Rightarrow$$

$$\underline{|E_C| = 0.17\ eV}$$

9.31
Consider an n-P heterojunction in thermal equilibrium. Poisson's equation is

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\in} = -\frac{d\mathrm{E}}{dx}$$

In the n-region,

$$\frac{d\mathrm{E}_n}{dx} = \frac{\rho(x)}{\in_n} = \frac{eN_{dn}}{\in_n}$$

For uniform doping, we have

$$\mathrm{E}_n = \frac{eN_{dn}x}{\in_n} + C_1$$

The boundary condition is
$\mathrm{E}_n = 0$ at $x = -x_n$, so we obtain

$$C_1 = \frac{eN_{dn}x_n}{\in_n}$$

Then

$$\mathrm{E}_n = \frac{eN_{dn}}{\in_n}(x + x_n)$$

In the P-region,

$$\frac{d\mathrm{E}_P}{dx} = -\frac{eN_{aP}}{\in_P}$$

which gives

$$\mathrm{E}_P = -\frac{eN_{aP}x}{\in_P} + C_2$$

We have the boundary condition that
$\mathrm{E}_P = 0$ at $x = x_P$ so that

$$C_2 = \frac{eN_{aP}x_P}{\in_P}$$

Then

$$\mathrm{E}_P = \frac{eN_{aP}}{\in_P}\left(x_P - x\right)$$

Assuming zero surface charge density at $x = 0$, the electric flux density D is continuous, so
$\in_n \mathrm{E}_n(0) = \in_P \mathrm{E}_P(0)$
which yields

$$N_{dn}x_n = N_{aP}x_P$$

We can determine the electric potential as

$$\phi_n(x) = -\int \mathrm{E}_n dx$$

$$= -\left[\frac{eN_{dn}x^2}{2\in_n} + \frac{eN_{dn}x_n x}{\in_n}\right] + C_3$$

Now

$$V_{bin} = \left|\phi_n(0) - \phi_n(-x_n)\right|$$

$$= C_3 - \left[C_3 - \frac{eN_{dn}x_n^2}{2\in_n} + \frac{eN_{dn}x_n^2}{\in_n}\right]$$

or

$$V_{bin} = \frac{eN_n x_n^2}{2\in_n}$$

Similarly on the P-side, we find

$$V_{biP} = \frac{eN_{aP}x_p^2}{2\in_P}$$

We have that

$$V_{bi} = V_{bin} + V_{biP} = \frac{eN_{dn}x_n^2}{2\in_n} + \frac{eN_{aP}x_P^2}{2\in_P}$$

We can write

$$x_P = x_n\left(\frac{N_{dn}}{N_{aP}}\right)$$

Substituting and collecting terms, we find

$$V_{bi} = \left[\frac{e\in_P N_{dn}N_{aP} + e\in_n N_{dn}^2}{2\in_n\in_P N_{aP}}\right]\cdot x_n^2$$

Solving for x_n, we have

$$x_n = \left[\frac{2\in_n\in_P N_{aP}V_{bi}}{eN_{dn}\left(\in_P N_{aP} + \in_n N_{dn}\right)}\right]^{1/2}$$

Similarly on the P-side, we have

$$x_P = \left[\frac{2\in_n\in_P N_{dn}V_{bi}}{eN_{aP}\left(\in_P N_{aP} + \in_n N_{dn}\right)}\right]^{1/2}$$

The total space charge width is then

$$W = x_n + x_P$$

Substituting and collecting terms, we obtain

$$W = \left[\frac{2\in_n\in_P V_{bi}\left(N_{aP} + N_{dn}\right)}{eN_{dn}N_{aP}\left(\in_n N_{dn} + \in_P N_{aP}\right)}\right]^{1/2}$$

Chapter 10

Problem Solutions

10.1
Sketch

10.2
Sketch

10.3

(a) $$|I_S| = \frac{eD_n A_{BE} n_{BO}}{x_B}$$

$$= \frac{(1.6x10^{-19})(20)(10^{-4})(10^4)}{10^{-4}}$$

or $\underline{I_S = 3.2x10^{-14}\ A}$

(b)

(i) $i_C = 3.2x10^{-14} \exp\left(\frac{0.5}{0.0259}\right) \Rightarrow$

$\underline{i_C = 7.75\ \mu A}$

(ii) $i_C = 3.2x10^{-14} \exp\left(\frac{0.6}{0.0259}\right) \Rightarrow$

$\underline{i_C = 0.368\ mA}$

(iii) $i_C = 3.2x10^{-14} \exp\left(\frac{0.7}{0.0259}\right) \Rightarrow$

$\underline{i_C = 17.5\ mA}$

10.4

(a) $\beta = \frac{\alpha}{1-\alpha} = \frac{0.9920}{1-0.9920} \Rightarrow \underline{\beta = 124}$

(b) From 10.3b

(i) For $i_C = 7.75\ \mu A$; $i_B = \frac{i_C}{\beta} = \frac{7.75}{124} \Rightarrow$

$\underline{i_B = 0.0625\ \mu A}$,

$$i_E = \left(\frac{1+\beta}{\beta}\right)\cdot i_C = \left(\frac{125}{124}\right)(7.75) \Rightarrow$$

$\underline{i_E = 7.81\ \mu A}$

(ii) For $i_C = 0.368\ mA$, $\underline{i_B = 2.97\ \mu A}$,

$\underline{i_E = 0.371\ mA}$

(iii) For $i_C = 17.5\ mA$, $\underline{i_B = 0.141\ mA}$,

$\underline{i_E = 17.64\ mA}$

10.5

(a) $\beta = \frac{i_C}{i_B} = \frac{510}{6} \Rightarrow \underline{\beta = 85}$

$\alpha = \frac{\beta}{1+\beta} = \frac{85}{86} \Rightarrow \underline{\alpha = 0.9884}$

$i_E = i_C + i_B = 510 + 6 \Rightarrow \underline{i_E = 516\ \mu A}$

(b)

$\beta = \frac{2.65}{0.05} \Rightarrow \underline{\beta = 53}$

$\alpha = \frac{53}{54} \Rightarrow \underline{\alpha = 0.9815}$

$i_E = 2.65 + 0.05 \Rightarrow \underline{i_E = 2.70\ mA}$

10.6

(c) For $i_B = 0.05\ mA$,

$i_C = \beta i_B = (100)(0.05) \Rightarrow \underline{i_C = 5\ mA}$

We have

$v_{CE} = V_{CC} - i_C R = 10 - (5)(1)$

or

$\underline{v_{CE} = 5\ V}$

10.7

(b) $V_{CC} = I_C R + V_{CB} + V_{BE}$

so

$10 = I_C(2) + 0 + 0.6$

or

$\underline{I_C = 4.7\ mA}$

10.8

(a)

$$n_{pO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

At $x = 0$,

$$n_p(0) = n_{pO} \exp\left(\frac{V_{BE}}{V_t}\right)$$

or we can write

$$V_{BE} = V_t \ln\left(\frac{n_p(0)}{n_{pO}}\right)$$

We want $n_p(0) = 10\% \times 10^{16} = 10^{15}\ cm^{-3}$,
So

$$V_{BE} = (0.0259)\ln\left(\frac{10^{15}}{2.25x10^4}\right)$$

or

$$\underline{V_{BE} = 0.635\ V}$$

(b)
At $x' = 0$,

$$p_n(0) = p_{nO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

where

$$p_{nO} = \frac{n_i^2}{N_E} = \frac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

Then

$$p_n(0) = 2.25x10^3\exp\left(\frac{0.635}{0.0259}\right) \Rightarrow$$

$$\underline{p_n(0) = 10^{14}\ cm^{-3}}$$

(c)
From the B-C space charge region,

$$x_{p1} = \left[\frac{2\in\left(V_{bi}+V_{R1}\right)}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right]^{1/2}$$

We find

$$V_{bi1} = (0.0259)\ln\left[\frac{\left(10^{16}\right)\left(10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.635\ V$$

Then

$$x_{p1} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(0.635+3)}{1.6x10^{-19}} \times\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{15}+10^{16}}\right)\right]^{1/2}$$

or

$$x_{p1} = 0.207\ \mu m$$

We find

$$V_{bi2} = (0.0259)\ln\left[\frac{\left(10^{17}\right)\left(10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.754\ V$$

Then

$$x_{p2} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(0.754-0.635)}{1.6x10^{-19}} \times\left(\frac{10^{17}}{10^{16}}\right)\left(\frac{1}{10^{17}+10^{16}}\right)\right]^{1/2}$$

or

$$x_{p2} = 0.118\ \mu m$$

Now

$$x_B = x_{BO} - x_{p1} - x_{p2} = 1.10 - 0.207 - 0.118$$

or

$$\underline{x_B = 0.775\ \mu m}$$

10.9

(a) $$p_{EO} = \frac{n_i^2}{N_E} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{17}} \Rightarrow$$

$$\underline{p_{EO} = 4.5x10^2\ cm^{-3}}$$

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} \Rightarrow$$

$$\underline{n_{BO} = 2.25x10^4\ cm^{-3}}$$

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{10^{15}} \Rightarrow$$

$$\underline{p_{CO} = 2.25x10^5\ cm^{-3}}$$

(b)

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= \left(2.25x10^4\right)\exp\left(\frac{0.625}{0.0259}\right)$$

or

$$\underline{n_B(0) = 6.80x10^{14}\ cm^{-3}}$$

Also

$$p_E(0) = p_{EO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= \left(4.5x10^2\right)\exp\left(\frac{0.625}{0.0259}\right)$$

or

$$\underline{p_E(0) = 1.36x10^{13}\ cm^{-3}}$$

10.10

(a) $$n_{EO} = \frac{n_i^2}{N_E} = \frac{\left(1.5x10^{10}\right)^2}{10^{18}} \Rightarrow$$

$$\underline{n_{EO} = 2.25x10^2\ cm^{-3}}$$

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{16}} \Rightarrow$$

$\underline{p_{BO} = 4.5x10^3 \ cm^{-3}}$

$$n_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{10^{15}} \Rightarrow$$

$$\underline{n_{CO} = 2.25x10^5 \ cm^{-3}}$$

(b)

$$p_B(0) = p_{BO}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= \left(4.5x10^3\right)\exp\left(\frac{0.650}{0.0259}\right)$$

or

$$\underline{p_B(0) = 3.57x10^{14} \ cm^{-3}}$$

Also

$$n_E(0) = n_{EO}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= \left(2.25x10^2\right)\exp\left(\frac{0.650}{0.0259}\right)$$

or

$$\underline{n_E(0) = 1.78x10^{13} \ cm^{-3}}$$

10.11

We have

$$\frac{d(\delta n_B)}{dx} = \frac{n_{BO}}{\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right.$$

$$\left.\times\left(\frac{-1}{L_B}\right)\cosh\left(\frac{x_B - x}{L_B}\right) - \frac{1}{L_B}\cosh\left(\frac{x}{L_B}\right)\right\}$$

At $x = 0$,

$$\frac{d(\delta n_B)}{dx}\Big|(0) = \frac{-n_{BO}}{L_B\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right.$$

$$\left.\times\cosh\left(\frac{x_B}{L_B}\right) + 1\right\}$$

At $x = x_B$,

$$\frac{d(\delta n_B)}{dx}\Big|(x_B) = \frac{-n_{BO}}{L_B\sinh\left(\frac{x_B}{L_B}\right)}$$

$$\times\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] + \cosh\left(\frac{x_B}{L_B}\right)\right\}$$

Taking the ratio,

$$\frac{\frac{d(\delta n_B)}{dx}\Big|(x_B)}{\frac{d(\delta n_B)}{dx}\Big|(0)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] + \cosh\left(\frac{x_B}{L_B}\right)}{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cosh\left(\frac{x_B}{L_B}\right) + 1}$$

$$\underline{\approx \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)}}$$

(a) For $\frac{x_B}{L_B} = 0.1 \Rightarrow Ratio = \underline{0.9950}$

(b) For $\frac{x_B}{L_B} = 1.0 \Rightarrow Ratio = \underline{0.648}$

(c) For $\frac{x_B}{L_B} = 10 \Rightarrow Ratio = \underline{9.08x10^{-5}}$

10.12

In the base of the transistor, we have

$$D_B\frac{d^2\left(\delta n_B(x)\right)}{dx^2} - \frac{\delta n_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2\left(\delta n_B(x)\right)}{dx^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B\tau_{BO}}$

The general solution to the differential equation is of the form,

$$\delta n_B(x) = A\exp\left(\frac{x}{L_B}\right) + B\exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we have

$$\delta n_B(0) = A + B = n_B(0) - n_{BO}$$

$$= n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]$$

Also

$$\delta n_B(x_B) = A\exp\left(\frac{x_B}{L_B}\right) + B\exp\left(\frac{-x_B}{L_B}\right) = -n_{BO}$$

From the first boundary condition, we can write

$$A = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] - B$$

Substituting into the second boundary condition equation, we find

$$B\left[\exp\left(\frac{x_B}{L_B}\right)-\exp\left(\frac{-x_B}{L_B}\right)\right]$$

$$=n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\exp\left(\frac{x_B}{L_B}\right)+n_{BO}$$

which can be written as

$$B=\frac{n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\exp\left(\frac{x_B}{L_B}\right)+n_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

We then find

$$A=\frac{-n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x_B}{L_B}\right)-n_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

10.13

In the base of the pnp transistor, we have

$$D_B\frac{d^2\left(\delta p_B(x)\right)}{dx^2}-\frac{\delta p_B(x)}{\tau_{BO}}=0$$

or

$$\frac{d^2\left(\delta p_B(x)\right)}{dx^2}-\frac{\delta p_B(x)}{L_B^2}=0$$

where $L_B=\sqrt{D_B\tau_{BO}}$

The general solution is of the form

$$\delta p_B(x)=A\exp\left(\frac{x}{L_B}\right)+B\exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we can write

$$\delta p_B(0)=A+B=p_B(0)-p_{BO}$$

$$=p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]$$

Also

$$\delta p_B(x_B)=A\exp\left(\frac{x_B}{L_B}\right)+B\exp\left(\frac{-x_B}{L_B}\right)=-p_{BO}$$

From the first boundary condition equation, we find

$$A=p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]-B$$

Substituting into the second boundary equation

$$B=\frac{p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cdot\exp\left(\frac{x_B}{L_B}\right)+p_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

and then we obtain

$$A=\frac{-p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x_B}{L_B}\right)-p_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

Substituting the expressions for A and B into the general solution and collecting terms, we obtain

$$\delta p_B(x)=p_{BO}$$

$$\times\left\{\frac{\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cdot\sinh\left(\frac{x_B-x}{L_B}\right)-\sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

10.14

For the idealized straight line approximation, the total minority carrier concentration is given by

$$n_B(x)=n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot\left(\frac{x_B-x}{x_B}\right)$$

The excess concentration is

$$\delta n_B=n_B(x)-n_{BO}$$

so for the idealized case, we can write

$$\delta n_{BO}(x)=n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot\left(\frac{x_B-x}{x_B}\right)-1\right\}$$

At $x=\frac{1}{2}x_B$, we have

$$\delta n_{BO}\left(\frac{1}{2}x_B\right)=n_{BO}\left\{\frac{1}{2}\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]-1\right\}$$

For the actual case, we have

$$\delta n_B\left(\frac{1}{2}x_B\right)=n_{BO}$$

$$\times\left\{\frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\sinh\left(\frac{x_B}{2L_B}\right)-\sinh\left(\frac{x_B}{2L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

(a) For $\frac{x_B}{L_B} = 0.1$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.0500208$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 0.100167$$

Then

$$\frac{\delta n_{BO}\left(\frac{1}{2}x_B\right) - \delta n_B\left(\frac{1}{2}x_B\right)}{\delta n_{BO}\left(\frac{1}{2}x_B\right)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot(0.50 - 0.49937) - 1.0 + 0.99875}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

which becomes

$$= \frac{(0.00063)\exp\left(\frac{V_{BE}}{V_t}\right) - (0.00125)}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

If we assume that $\exp\left(\frac{V_{BE}}{V_t}\right) >> 1$, then we find that the ratio is

$$\frac{0.00063}{0.50} = 0.00126 \Rightarrow \underline{0.126\%}$$

(b)

For $\frac{x_B}{L_B} = 1.0$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.5211$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 1.1752$$

Then

$$\frac{\delta n_{BO}\left(\frac{1}{2}x_B\right) - \delta n_B\left(\frac{1}{2}x_B\right)}{\delta n_{BO}\left(\frac{1}{2}x_B\right)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right](0.50 - 0.4434) - 1.0 + 0.8868}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

which becomes

$$\frac{(0.0566)\exp\left(\frac{V_{BE}}{V_t}\right) - (0.1132)}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

Assuming that $\exp\left(\frac{V_{BE}}{V_t}\right) >> 1$

Then the ratio is

$$= \frac{0.0566}{0.50} = 0.1132 \Rightarrow \underline{11.32\%}$$

10.15

The excess hole concentration at $x = 0$ is

$$\delta p_B(0) = p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right] = 8x10^{14}\ cm^{-3}$$

and the excess hole concentration at $x = x_B$ is

$$\delta p_B(x_B) = -p_{BO} = -2.25x10^4\ cm^{-3}$$

From the results of problem 10.13, we can write

$$\delta p(x) = p_{BO}$$

$$\times\left\{\frac{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right]\cdot\sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

or

$$\delta p_B(x) =$$

$$\frac{(8x10^{14})\sinh\left(\frac{x_B - x}{L_B}\right) - (2.25x10^4)\sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}$$

Let $x_B = L_B = 10\ \mu m$, so that

$$\sinh\left(\frac{x_B}{L_B}\right) = 1.1752$$

Then, we can find $\delta p_B(x)$ for (a) the ideal linear approximation and for (b) the actual distribution as follow:

x	$(a)\ \delta p_B$	$(b)\ \delta p_B$
0	$8x10^{14}$	$8x10^{14}$
$0.25L_B$	$6x10^{14}$	$5.6x10^{14}$
$0.50L_B$	$4x10^{14}$	$3.55x10^{14}$
$0.75L_B$	$2x10^{14}$	$1.72x10^{14}$
$1.0L_B$	$-2.25x10^{4}$	$-2.25x10^{4}$

(c)
For the ideal case when $x_B << L_B$, then $J(0) = J(x_B)$, so that

$$\underline{\frac{J(x_B)}{J(0)} = 1}$$

For the case when $x_B = L_B = 10\ \mu m$

$$J(0) = \frac{eD_B}{\sinh\left(\frac{x_B}{L_B}\right)} \frac{d}{dx}\left\{(8x10^{14})\sinh\left(\frac{x_B - x}{L_B}\right) - (2.25x10^4)\sinh\left(\frac{x}{L_B}\right)\right\}\Big|_{x=0}$$

or

$$J(0) = \frac{eD_B}{\sinh(1)}\left\{\frac{-1}{L_B}(8x10^{14})\cosh\left(\frac{x_B - x}{L_B}\right) - \frac{1}{L_B}(2.25x10^4)\cosh\left(\frac{x}{L_B}\right)\right\}\Big|_{x=0}$$

which becomes

$$= \frac{-eD_B}{L_B\sinh(1)}\cdot\left\{(8x10^{14})\cosh(1) + (2.25x10^4)\cosh(0)\right\}$$

We find

$$J(0) = \frac{-(1.6x10^{-19})(10)}{(10x10^{-4})(1.175)} \times\left[(8x10^{14})(1.543) + (2.25x10^4)(1)\right]$$

or

$$\underline{J(0) = -1.68\ A/cm^2}$$

Now

$$J(x_B) = \frac{-eD_B}{L_B\sinh(1)}\left\{(8x10^{14})\cosh(0) + (2.25x10^4)\cosh(1)\right\}$$

or

$$= \frac{-(1.6x10^{-19})(10)}{(10x10^{-4})(1.175)} \times\left[(8x10^{14})(1) + (2.25x10^4)(1.543)\right]$$

We obtain

$$\underline{J(x_B) = -1.089\ A/cm^2}$$

Then

$$\frac{J(x_B)}{J(0)} = \frac{-1.089}{-1.68} \Rightarrow \underline{\frac{J(x_B)}{J(0)} = 0.648}$$

10.16
(a) npn transistor biased in saturation

$$D_B\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B\tau_{BO}}$

The general solution is of the form

$$\delta n_B(x) = A\exp\left(\frac{x}{L_B}\right) + B\exp\left(\frac{-x}{L_B}\right)$$

If $x_B << L_B$, then also $x << L_B$ so that

$$\delta n_B(x) \approx A\left(1 + \frac{x}{L_B}\right) + B\left(1 - \frac{x}{L_B}\right)$$

$$= (A + B) + (A - B)\left(\frac{x}{L_B}\right)$$

which can be written as

$$\delta n_B(x) = C + D\left(\frac{x}{L_B}\right)$$

The boundary conditions are

$$\delta n_B(0) = C = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]$$

and

$$\delta n_B(x_B) = C + D\left(\frac{x_B}{L_B}\right) = n_{BO}\left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]$$

Then the coefficient D can be written as

$$D = \left(\frac{L_B}{x_B}\right)\left\{ n_{BO}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right] - n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\right\}$$

The excess electron concentration is then given by

$$\delta n_B(x) = n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\left(1-\frac{x}{L_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]\cdot\left(\frac{x}{x_B}\right)\right\}$$

(b)
The electron diffusion current density is

$$J_n = eD_B\frac{d(\delta n_B(x))}{dx}$$

$$= eD_B n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\left(\frac{-1}{x_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]\cdot\left(\frac{1}{x_B}\right)\right\}$$

or

$$J_n = -\frac{eD_B n_{BO}}{x_B}\left\{\exp\left(\frac{V_{BE}}{V_t}\right)-\exp\left(\frac{V_{BC}}{V_t}\right)\right\}$$

(c)
The total excess charge in the base region is

$$Q_{nB} = -e\int_0^{x_B}\delta n_B(x)dx$$

$$= -en_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\left(x-\frac{x^2}{2x_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]\cdot\left(\frac{x^2}{2x_B}\right)\right\}\Bigg|_0^{x_B}$$

which yields

$$Q_{nB} = \frac{-en_{BO}x_B}{2}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right] + \left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]\right\}$$

10.17
(a) Extending the results of problem 10.16 to a pnp transistor, we can write

$$J_P = \frac{eD_B p_{BO}}{x_B}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-\exp\left(\frac{V_{CB}}{V_t}\right)\right]$$

We have

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

Then

$$165 = \frac{(1.6x10^{-19})(10)(2.25x10^3)}{0.7x10^{-4}}\times\left[\exp\left(\frac{0.75}{0.0259}\right)-\exp\left(\frac{V_{CB}}{V_t}\right)\right]$$

or

$$3.208x10^{12} = 3.768x10^{12} - \exp\left(\frac{V_{CB}}{V_t}\right)$$

which yields

$$V_{CB} = (0.0259)\ln(0.56x10^{12}) \Rightarrow$$

$$V_{CB} = 0.70\ V$$

(b)

$$V_{EC}(sat) = V_{EB} - V_{CB} = 0.75 - 0.70 \Rightarrow$$

$$V_{EC}(sat) = 0.05\ V$$

(c)
Again, extending the results of problem 10.16 to a pnp transistor, we can write

$$Q_{pB} = \frac{ep_{BO}x_B}{2}\left\{\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]+\left[\exp\left(\frac{V_{CB}}{V_t}\right)-1\right]\right\}$$

$$= \frac{(1.6x10^{-19})(2.25x10^3)(0.7x10^{-4})}{2}\times\left[3.768x10^{12}+0.56x10^{12}\right]$$

or

$$Q_{pB} = 5.45x10^{-8}\ C/cm^2$$

or

$$\frac{Q_{pB}}{e} = 3.41x10^{11}\ holes/cm^2$$

(d)
In the collector, we have

$$\delta n_p(x) = n_{PO}\left[\exp\left(\frac{V_{CB}}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x}{L_C}\right)$$

The total number of excess electrons in the collector is

$$N_{Coll} = \int_0^\infty \delta n_P(x)dx$$

$$= -n_{PO}L_C\left[\exp\left(\frac{V_{CB}}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x}{L_C}\right)\Big|_0^\infty$$

$$= n_{PO}L_C\left[\exp\left(\frac{V_{CB}}{V_t}\right)-1\right]$$

We have

$$n_{PO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{15}} = 4.5x10^4\ cm^{-3}$$

Then the total number of electrons is

$$N_{Coll} = \left(4.5x10^4\right)\left(35x10^{-4}\right)\left(0.56x10^{12}\right)$$

or

$$\underline{N_{Coll} = 8.82x10^{13}\ electrons/cm^2}$$

10.18

(b) $$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

and

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{7x10^{15}} = 3.21x10^4\ cm^{-3}$$

At $x = x_B$,

$$n_B(x_B) = n_{BO}\exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= \left(2.25x10^3\right)\exp\left(\frac{0.565}{0.0259}\right)$$

or

$$n_B(x_B) = 6.7x10^{12}\ cm^{-3}$$

At $x'' = 0$,

$$p_C(0) = p_{CO}\exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= \left(3.21x10^4\right)\exp\left(\frac{0.565}{0.0259}\right)$$

or

$$p_C(0) = 9.56x10^{13}\ cm^{-3}$$

(c)
From the B-C space-charge region,

$$V_{bi1} = (0.0259)\ln\left[\frac{\left(10^{17}\right)\left(7x10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.745\ V$$

Then

$$x_{p1} = \left\{\frac{2(11.7)\left(8.85x10^{-14}\right)(0.745-0.565)}{1.6x10^{-19}}\times\left(\frac{7x10^{15}}{10^{17}}\right)\left(\frac{1}{7x10^{15}+10^{17}}\right)\right\}^{1/2}$$

or

$$x_{p1} = 1.23x10^{-6}\ cm$$

From the B-E space-charge region,

$$V_{bi2} = (0.0259)\ln\left[\frac{\left(10^{19}\right)\left(10^{17}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.933\ V$$

Then

$$x_{p2} = \left\{\frac{2(11.7)\left(8.85x10^{-14}\right)(0.933+2)}{1.6x10^{-19}}\times\left(\frac{10^{19}}{10^{17}}\right)\left(\frac{1}{10^{19}+10^{17}}\right)\right\}^{1/2}$$

or

$$x_{p2} = 1.94x10^{-5}\ cm$$

Now

$$x_B = x_{BO} - x_{p1} - x_{p2} = 1.20 - 0.0123 - 0.194$$

or

$$\underline{x_B = 0.994\ \mu m}$$

10.19

Low injection limit is reached when $p_C(0) = (0.10)N_C$, so that

$$p_C(0) = (0.10)\left(5x10^{14}\right) = 5x10^{13}\ cm^{-3}$$

We have

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{14}} = 4.5x10^5\ cm^{-3}$$

Also

$$p_C(0) = p_{CO}\exp\left(\frac{V_{CB}}{V_t}\right)$$

or

$$V_{CB} = V_t\ln\left(\frac{p_C(0)}{p_{CO}}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{13}}{4.5x10^5}\right)$$

or

$$\underline{V_{CB} = 0.48\ V}$$

10.20
(a)
$$\alpha = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}}$$
$$= \frac{1.18}{1.20 + 0.20 + 0.10} \Rightarrow \underline{\alpha = 0.787}$$
(b)
$$\gamma = \frac{J_{nE}}{J_{nE} + J_{pE}}$$
$$= \frac{1.20}{1.20 + 0.10} \Rightarrow \underline{\gamma = 0.923}$$
(c)
$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \frac{1.18}{1.20} \Rightarrow \underline{\alpha_T = 0.983}$$
(d)
$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}}$$
$$= \frac{1.20 + 0.10}{1.20 + 0.20 + 0.10} \Rightarrow \underline{\delta = 0.867}$$
(e)
$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.787}{1 - 0.787}$$
or
$$\underline{\beta = 3.69}$$

10.21
$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3 \ cm^{-3}$$
Then
$$n_B(0) = n_{BO} \exp\left(\frac{V_{BE}}{V_t}\right)$$
$$= \left(2.25x10^3\right)\exp\left(\frac{0.50}{0.0259}\right)$$
or
$$\underline{n_B(0) = 5.45x10^{11} \ cm^{-3}}$$
As a good approximation,
$$I_C = \frac{eD_B A n_B(0)}{x_B}$$
$$= \frac{\left(1.6x10^{-19}\right)(20)\left(10^{-3}\right)\left(5.45x10^{11}\right)}{10^{-4}}$$
or
$$\underline{I_C = 17.4 \ \mu A}$$
(b)
Base transport factor
$$\alpha_T = \frac{1}{\cosh\left(x_B/L_B\right)}$$
We find
$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(20)\left(10^{-7}\right)} = 1.41x10^{-3} \ cm$$
so that
$$\alpha_T = \frac{1}{\cosh(1/14.1)} \Rightarrow \underline{\alpha_T = 0.9975}$$
Emitter injection efficiency
Assuming $D_E = D_B$, $x_B = x_E$, and $L_E = L_B$; then
$$\gamma = \frac{1}{1 + \dfrac{N_B}{N_E}} = \frac{1}{1 + \dfrac{10^{17}}{10^{18}}} \Rightarrow \underline{\gamma = 0.909}$$
Then
$$\alpha = \gamma\alpha_T\delta = (0.909)(0.9975)(1) \Rightarrow$$
$$\underline{\alpha = 0.9067}$$
and
$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9067}{1 - 0.9067} \Rightarrow \underline{\beta = 9.72}$$
For $I_E = 1.5 \ mA$,
$$I_C = \alpha I_E = (0.9067)(1.5) \Rightarrow \underline{I_C = 1.36 \ mA}$$
(c)
For $I_B = 2 \ \mu A$,
$$I_C = \beta I_B = (9.72)(2) \Rightarrow \underline{I_C = 19.4 \ \mu A}$$

10.22
(a) We have
$$J_{nE} = \frac{eD_B n_{BO}}{L_B}\left\{\frac{1}{\sinh\left(\dfrac{x_B}{L_B}\right)} + \frac{\left[\exp\left(\dfrac{V_{BE}}{V_t}\right) - 1\right]}{\tanh\left(\dfrac{x_B}{L_B}\right)}\right\}$$
We find that
$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{16}} = 4.5x10^3 \ cm^{-3}$$
and
$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(15)\left(5x10^{-8}\right)} = 8.66x10^{-4} \ cm$$
Then

$$J_{nE} = \frac{(1.6x10^{-19})(15)(4.5x10^3)}{8.66x10^{-4}} \times \left\{ \frac{1}{\sinh\left(\frac{0.70}{8.66}\right)} + \frac{\exp\left(\frac{0.60}{0.0259}\right)}{\tanh\left(\frac{0.70}{8.66}\right)} \right\}$$

or

$$\underline{J_{nE} = 1.79 \ A/cm^2}$$

We also have

$$J_{pE} = \frac{eD_E p_{EO}}{L_E}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] \cdot \frac{1}{\tanh\left(\frac{x_E}{L_E}\right)}$$

Also

$$p_{EO} = \frac{n_i^2}{N_E} = \frac{(1.5x10^{10})^2}{10^{18}} = 2.25x10^2 \ cm^{-3}$$

and

$$L_E = \sqrt{D_E \tau_{EO}} = \sqrt{(8)(10^{-8})} = 2.83x10^{-4} \ cm$$

Then

$$J_{pE} = \frac{(1.6x10^{-19})(8)(2.25x10^2)}{2.83x10^{-4}} \times \left[\exp\left(\frac{0.60}{0.0259}\right) - 1\right] \cdot \frac{1}{\tanh\left(\frac{0.8}{2.83}\right)}$$

or

$$\underline{J_{pE} = 0.0425 \ A/cm^2}$$

We can find

$$J_{nC} = \frac{eD_B n_{BO}}{L_B}\left\{ \frac{\left[\exp\left(\frac{0.60}{0.0259}\right) - 1\right]}{\sinh\left(\frac{x_B}{L_B}\right)} + \frac{1}{\tanh\left(\frac{x_B}{L_B}\right)} \right\}$$

$$= \frac{(1.6x10^{-19})(15)(4.5x10^3)}{8.66x10^{-4}} \times \left\{ \frac{\exp\left(\frac{0.60}{0.0259}\right)}{\sinh\left(\frac{0.7}{8.66}\right)} + \frac{1}{\tanh\left(\frac{0.7}{8.66}\right)} \right\}$$

or

$$\underline{J_{nC} = 1.78 \ A/cm^2}$$

The recombination current is

$$J_R = J_{rO} \exp\left(\frac{eV_{BE}}{2kT}\right)$$

$$= (3x10^{-8}) \exp\left(\frac{0.60}{2(0.0259)}\right)$$

or

$$\underline{J_R = 3.22x10^{-3} \ A/cm^2}$$

(b)
Using the calculated currents, we find

$$\gamma = \frac{J_{nE}}{J_{nE} + J_{pE}} = \frac{1.79}{1.79 + 0.0425} \Rightarrow$$

$$\underline{\gamma = 0.977}$$

We find

$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \frac{1.78}{1.79} \Rightarrow \quad \underline{\alpha_T = 0.994}$$

and

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} = \frac{1.79 + 0.0425}{1.79 + 0.00322 + 0.0425}$$

or

$$\underline{\delta = 0.998}$$

Then

$$\alpha = \gamma\alpha_T\delta = (0.977)(0.994)(0.998) \Rightarrow$$

$$\underline{\alpha = 0.969}$$

Now

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.969}{1 - 0.969} \Rightarrow \quad \underline{\beta = 31.3}$$

10.23

(a) $\gamma = \dfrac{1}{1 + \dfrac{N_B}{N_E} \cdot \dfrac{D_E}{D_B} \cdot \dfrac{x_B}{x_E}} \approx 1 - \dfrac{N_B}{N_E} \cdot \dfrac{D_E}{D_B} \cdot \dfrac{x_B}{x_E}$

or

$$\gamma \approx 1 - K \cdot \frac{N_B}{N_E}$$

(i)

$$\frac{\gamma(B)}{\gamma(A)} = \frac{1 - \frac{2N_{BO}}{N_E} \cdot K}{1 - \frac{N_{BO}}{N_E} \cdot K}$$

$$\approx \left(1 - \frac{2N_{BO}}{N_E} \cdot K\right)\left(1 + \frac{N_{BO}}{N_E} \cdot K\right)$$

$$\approx 1 - \frac{2N_{BO}}{N_E} \cdot K + \frac{N_{BO}}{N_E} \cdot K$$

or
$$\frac{\gamma(B)}{\gamma(A)} \approx 1 - \frac{N_{BO}}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

(ii)
$$\frac{\gamma(C)}{\gamma(A)} = 1$$

(b) (i)
$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

(ii)
$$\frac{\alpha_T(C)}{\alpha_T(A)} = \frac{\left(1 - \frac{1}{2} \cdot \frac{(x_{BO}/2)}{L_B}\right)^2}{\left(1 - \frac{1}{2} \cdot \frac{x_{BO}}{L_B}\right)^2}$$
$$\approx \frac{\left(1 - \frac{x_{BO}}{2L_B}\right)}{\left(1 - \frac{x_{BO}}{L_B}\right)} \approx \left(1 - \frac{x_{BO}}{2L_B}\right)\left(1 + \frac{x_{BO}}{L_B}\right)$$
$$\approx 1 - \frac{x_{BO}}{2L_B} + \frac{x_{BO}}{L_B}$$

or
$$\frac{\alpha_T(C)}{\alpha_T(A)} \approx 1 + \frac{x_{BO}}{2L_B}$$

(c) Neglect any change in space charge width. Then
$$\delta = \frac{1}{1 + \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)}$$
$$\approx 1 - \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)$$

(i)
$$\frac{\delta(B)}{\delta(A)} = \frac{1 - \frac{K}{J_{sOB}}}{1 - \frac{K}{J_{sOA}}} \approx \left(1 - \frac{K}{J_{sOB}}\right)\left(1 + \frac{K}{J_{sOA}}\right)$$
$$\approx 1 - \frac{K}{J_{sOB}} + \frac{K}{J_{sOA}}$$

Now
$$J_{sO} \propto n_{BO} = \frac{n_i^2}{N_B}$$

so
$$\frac{\delta(B)}{\delta(A)} \approx 1 - \frac{2N_{BO}K}{C} + \frac{N_{BO}K}{C} = 1 - \frac{N_{BO}K}{C}$$

Then
$$\frac{\delta(B)}{\delta(A)} \approx 1 - \frac{J_{rO}\exp\left(\frac{-V_{BE}}{2V_t}\right)}{\left(\frac{eD_B n_{BO}}{x_B}\right)}$$

(ii)
We find
$$\frac{\delta(C)}{\delta(A)} \approx 1 + \frac{J_{rO}\exp\left(\frac{-V_{BE}}{2V_t}\right)}{\left(\frac{eD_B n_{BO}}{x_B}\right)}$$

(d)
Device C has the largest β. Base transport factor as well as the recombination factor increases.

10.24
(a)
$$\gamma = \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}} = \frac{1}{1 + K \cdot \frac{N_B}{N_E}}$$

or
$$\gamma \approx 1 - K \cdot \frac{N_B}{N_E}$$

(i) Then
$$\frac{\gamma(B)}{\gamma(A)} = \frac{1 - K \cdot \frac{N_B}{2N_{EO}}}{1 - K \cdot \frac{N_B}{N_{EO}}}$$
$$\approx \left(1 - K \cdot \frac{N_B}{2N_{EO}}\right) \cdot \left(1 + K \cdot \frac{N_B}{N_{EO}}\right)$$
$$\approx 1 - K \cdot \frac{N_B}{2N_{EO}} + K \cdot \frac{N_B}{N_{EO}}$$

or
$$= 1 + K \cdot \frac{N_B}{2N_{EO}}$$

or

$$\frac{\gamma(B)}{\gamma(A)} = 1 + \frac{N_B}{2N_{EO}} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

(ii)
Now

$$\gamma = \frac{1}{1 + K' \cdot \frac{x_B}{x_E}} \approx 1 - K' \cdot \frac{x_B}{x_E}$$

Then

$$\frac{\gamma(C)}{\gamma(A)} = \frac{1 - K' \cdot \frac{x_B}{(x_{EO}/2)}}{1 - K' \cdot \frac{x_B}{x_{EO}}}$$

$$\approx \left(1 - K' \cdot \frac{2x_B}{x_{EO}}\right) \cdot \left(1 + K' \cdot \frac{x_B}{x_{EO}}\right)$$

$$\approx 1 - 2K' \cdot \frac{x_B}{x_{EO}} + K' \cdot \frac{x_B}{x_{EO}}$$

$$= 1 - K' \cdot \frac{x_B}{x_{EO}}$$

or

$$\frac{\gamma(C)}{\gamma(A)} = 1 - \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_{EO}}$$

(b)

$$\alpha_T = 1 - \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2$$

so
(i)

$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

and
(ii)

$$\frac{\alpha_T(C)}{\alpha_T(A)} = 1$$

(c)
Neglect any change in space charge width

$$\delta = \frac{1}{1 + \frac{J_{rO}}{J_{SO}} \exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

$$= \frac{1}{1 + \frac{k}{J_{SO}}} \approx 1 - \frac{k}{J_{SO}}$$

(i)

$$\frac{\delta(B)}{\delta(A)} = \frac{1 - \frac{k}{J_{SOB}}}{1 - \frac{k}{J_{SOA}}} \approx \left(1 - \frac{k}{J_{SOB}}\right)\left(1 + \frac{k}{J_{SOA}}\right)$$

$$\approx 1 - \frac{k}{J_{SOB}} + \frac{k}{J_{SOA}}$$

Now

$$J_{SO} \propto \frac{1}{N_E x_E}$$

so
(i)

$$\frac{\delta(B)}{\delta(A)} = 1 - k'(2N_{EO}) + k'(N_{EO})$$

or

$$\frac{\delta(B)}{\delta(A)} = 1 - k' \cdot (N_{EO})$$

(recombination factor decreases)
(ii)
We have

$$\frac{\delta(C)}{\delta(A)} = 1 - k'' \cdot \left(\frac{x_{EO}}{2}\right) + k'' \cdot (x_{EO})$$

or

$$\frac{\delta(C)}{\delta(A)} = 1 + \frac{1}{2} k'' \cdot x_{EO}$$

(recombination factor increases)

10.25
(b)

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{17}} = 2.25x10^3 \ cm^{-3}$$

Then

$$n_B(0) = n_{BO} \exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= (2.25x10^3)\exp\left(\frac{0.6}{0.0259}\right) = 2.59x10^{13} \ cm^{-3}$$

Now

$$J_{nC} = \frac{eD_B n_B(0)}{x_B}$$

$$= \frac{(1.6x10^{-19})(20)(2.59x10^{13})}{10^{-4}}$$

or

$$J_{nC} = 0.829\ A/cm^2$$

Assuming a long collector,

$$J_{pC} = \frac{eD_C p_{nO}}{L_C} \exp\left(\frac{V_{BC}}{V_t}\right)$$

where

$$p_{nO} = \frac{n_i^2}{N_C} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

and

$$L_C = \sqrt{D_C \tau_{CO}} = \sqrt{(15)(2x10^{-7})} = 1.73x10^{-3}\ cm$$

Then

$$J_{pC} = \frac{(1.6x10^{-19})(15)(2.25x10^4)}{1.73x10^{-3}} \exp\left(\frac{0.6}{0.0259}\right)$$

or

$$J_{pC} = 0.359\ A/cm^2$$

The collector current is

$$I_C = (J_{nC} + J_{pC}) \cdot A = (0.829 + 0.359)(10^{-3})$$

or

$$\underline{I_C = 1.19\ mA}$$

The emitter current is

$$I_E = J_{nC} \cdot A = (0.829)(10^{-3})$$

or

$$\underline{I_E = 0.829 mA}$$

10.26

(a)

$$\alpha_T = \frac{1}{\cosh(x_B/L_B)} \qquad \beta = \frac{\alpha_T}{1-\alpha_T}$$

x_B/L_B	α_T	β
0.01	0.99995	19,999
0.10	0.995	199
1.0	0.648	1.84
10.0	0.0000908	≈ 0

(b)

For $D_E = D_B$, $L_E = L_B$, $x_E = x_B$, we have

$$\gamma = \frac{1}{1+(p_{EO}/n_{BO})} = \frac{1}{1+(N_B/N_C)}$$

and

$$\beta = \frac{\gamma}{1-\gamma}$$

N_B/N_E	γ	β
0.01	0.990	99
0.10	0.909	9.99
1.0	0.50	1.0
10.0	0.0909	0.10

(c)

For $x_B/L_B < 0.10$, the value of β is unreasonably large, which means that the base transport factor is not the limiting factor. For $x_B/L_B > 1.0$, the value of β is very small, which means that the base transport factor will probably be the limiting factor.

If $N_B/N_E < 0.01$, the emitter injection efficiency is probably not the limiting factor. If, however, $N_B/N_E > 0.01$, then the current gain is small and the emitter injection efficiency is probably the limiting factor.

10.27

We have

$$J_{sO} = \frac{eD_B n_{BO}}{L_B \tanh(x_B/L_B)}$$

Now

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(25)(10^{-7})} = 15.8x10^{-4}\ cm$$

Then

$$J_{sO} = \frac{(1.6x10^{-19})(25)(2.25x10^3)}{(15.8x10^{-4})\tanh(0.7/15.8)}$$

or

$$J_{sO} = 1.3x10^{-10}\ A/cm^2$$

Now

$$\delta = \frac{1}{1+\dfrac{J_{rO}}{J_{sO}} \cdot \exp\left(\dfrac{-V_{BE}}{2V_t}\right)}$$

$$= \frac{1}{1+\dfrac{2x10^{-9}}{1.3x10^{-10}} \cdot \exp\left(\dfrac{-V_{BE}}{2(0.0259)}\right)}$$

or

(a)

$$\delta = \frac{1}{1+(15.38)\exp\left(\frac{-V_{BE}}{0.0518}\right)}$$

and
(b)

$$\beta = \frac{\delta}{1-\delta}$$

Now

V_{BE}	δ	β
0.20	0.755	3.08
0.40	0.993	142
0.60	0.99986	7,142

(c)
If $V_{BE} < 0.4\ V$, the recombination factor is likely the limiting factor in the current gain.

10.28

For $\beta = 120 = \frac{\alpha}{1-\alpha} \Rightarrow \alpha = \frac{\beta}{1+\beta}$

So

$$\alpha = \frac{120}{121} = 0.9917$$

Now

$$\alpha = \gamma\alpha_T\delta = 0.9917 = (0.998)x^2$$

where

$$x = \alpha_T = \gamma = 0.9968$$

We have

$$\alpha_T = \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)} = 0.9968$$

which means

$$\frac{x_B}{L_B} = 0.0801$$

We find

$$L_B = \sqrt{D_B\tau_{BO}} = \sqrt{(25)(10^{-7})} = 15.8\ \mu m$$

Then

$$x_B(\max) = (0.0801)(15.8) \Rightarrow$$

$$\underline{x_B(\max) = 1.26\ \mu m}$$

We also have

$$\gamma = \frac{1}{1+\frac{p_{EO}}{n_{BO}}\cdot\frac{D_E}{D_B}\cdot\frac{L_B}{L_E}\cdot\frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

where

$$L_E = \sqrt{D_E\tau_{EO}} = \sqrt{(10)(5x10^{-8})} = 7.07\ \mu m$$

Then

$$0.9968 = \frac{1}{1+\frac{p_{EO}}{n_{BO}}\cdot\left(\frac{10}{25}\right)\left(\frac{15.8}{7.07}\right)\frac{\tanh(1.26/15.8)}{\tanh(0.5/7.07)}}$$

which yields

$$\frac{p_{EO}}{n_{BO}} = 0.003186 = \frac{N_B}{N_E}$$

Finally

$$N_E = \frac{N_B}{0.003186} = \frac{10^{16}}{0.003186} \Rightarrow$$

$$\underline{N_E = 3.14x10^{18}\ cm^{-3}}$$

10.29

(a) We have $J_{rO} = 5x10^{-8}\ A/cm^2$

We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{5x10^{16}} = 4.5x10^3\ cm^{-3}$$

and

$$L_B = \sqrt{D_B\tau_{BO}} = \sqrt{(25)(10^{-7})} = 15.8\ \mu m$$

Then

$$J_{sO} = \frac{eD_Bn_{BO}}{L_B\tanh(x_B/L_B)}$$

$$= \frac{(1.6x10^{-19})(25)(4.5x10^3)}{(15.8x10^{-4})\tanh(x_B/L_B)}$$

or

$$J_{sO} = \frac{1.14x10^{-11}}{\tanh(x_B/L_B)}$$

We have

$$\delta = \frac{1}{1+\frac{J_{rO}}{J_{sO}}\cdot\exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

For $T = 300K$ and $V_{BE} = 0.55\ V$,

$\delta = 0.995 =$

$$\frac{1}{1+\left(\frac{5x10^{-8}}{1.14x10^{-11}}\right)\cdot\tanh\left(\frac{x_B}{L_B}\right)\cdot\exp\left(\frac{-0.55}{2(0.0259)}\right)}$$

which yields

$$\frac{x_B}{L_B} = 0.047$$

or

$$x_B = (0.047)(15.8x10^{-4}) \Rightarrow$$

$$\underline{x_B = 0.742\ \mu m}$$

(b)

For $T = 400K$ and $J_{rO} = 5x10^{-8}\ A/cm^2$,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = \left(\frac{400}{300}\right)^3 \cdot \frac{\exp\left[\frac{-E_g}{(0.0259)(400/300)}\right]}{\exp\left[\frac{-E_g}{(0.0259)}\right]}$$

For $E_g = 1.12\ eV$,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = 1.17x10^5$$

or

$$n_{BO}(400) = (1.17x10^5)(4.5x10^3)$$

$$= 5.27x10^8\ cm^{-3}$$

Then

$$J_{sO} = \frac{(1.6x10^{-19})(25)(5.27x10^8)}{(15.8x10^{-4})\tanh(0.742/15.8)}$$

or

$$J_{sO} = 2.84x10^{-5}\ A/cm^2$$

Finally,

$$\delta = \frac{1}{1 + \frac{5x10^{-8}}{2.84x10^{-5}} \cdot \exp\left[\frac{-0.55}{2(0.0259)(400/300)}\right]}$$

or

$$\underline{\delta = 0.9999994}$$

10.30
Computer plot

10.31
Computer plot

10.32
Computer plot

10.33
Computer plot

10.34
Metallurgical base width $= 1.2\ \mu m = x_B + x_n$

We have

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

and

$$p_B(0) = p_{BO}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= (2.25x10^4)\exp\left(\frac{0.625}{0.0259}\right)$$

$$= 6.8x10^{14}\ cm^{-3}$$

Now

$$J_p = eD_B\frac{dp_B}{dx} = eD_B\left(\frac{p_B(0)}{x_B}\right)$$

$$= \frac{(1.6x10^{-19})(10)(6.8x10^{14})}{x_B}$$

or

$$J_p = \frac{1.09x10^{-3}}{x_B}$$

We have

$$x_n = \left\{\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right\}^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5x10^{10})^2}\right] = 0.635\ V$$

We can write

$$x_n = \left\{\frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_R)}{1.6x10^{-19}} \times\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{15}+10^{16}}\right)\right\}^{1/2}$$

or

$$x_n = \{(1.177x10^{-10})(V_{bi}+V_R)\}^{1/2}$$

We know

$$x_B = 1.2x10^{-4} - x_n$$

For $V_R = V_{BC} = 5\ V$

$$x_n = 0.258x10^{-4}\ cm \Rightarrow x_B = 0.942x10^{-4}\ cm$$

Then

$$\underline{J_p = 11.6\ A/cm^2}$$

For $V_R = V_{BC} = 10\ V$,

$x_n = 0.354x10^{-4}\ cm \Rightarrow x_B = 0.846x10^{-4}\ cm$

Then

$$\underline{J_p = 12.9\ A/cm^2}$$

For $V_R = V_{BC} = 15\,V$,

$x_n = 0.429x10^{-4}\ cm \Rightarrow x_B = 0.771x10^{-4}\ cm$

Then

$$\underline{J_p = 14.1\ A/cm^2}$$

(b)

We can write

$$J_p = g'(V_{EC} + V_A)$$

where

$$g' = \frac{\Delta J_p}{\Delta V_{EC}} = \frac{\Delta J_p}{\Delta V_{BC}} = \frac{14.1 - 11.6}{10}$$

or

$$g' = 0.25\ mA/cm^2/V$$

Now

$J_p = 11.6\ A/cm^2$ at

$$V_{EC} = V_{BC} + V_{EB} = 5 + 0.626 = 5.626\,V$$

Then

$$11.6 = (0.25)(5.625 + V_A)$$

which yields

$$\underline{V_A = 40.8\,V}$$

10.35 We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{3x10^{16}} = 7.5x10^3\ cm^{-3}$$

and

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right)$$
$$= (7.5x10^3)\exp\left(\frac{0.7}{0.0259}\right)$$

or

$$n_B(0) = 4.10x10^{15}\ cm^{-3}$$

We have

$$J = eD_B\frac{dn_B}{dx} = \frac{eD_Bn_B(0)}{x_B}$$
$$= \frac{(1.6x10^{-19})(20)(4.10x10^{15})}{x_B}$$

or

$$J = \frac{1.312x10^{-2}}{x_B}$$

Neglecting the space charge width at the B-E junction, we have

$$x_B = x_{BO} - x_p$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{16})(5x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.705\,V$$

and

$$x_p = \left\{\frac{2\in(V_{bi} + V_{CB})}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C + N_B}\right)\right\}^{1/2}$$
$$= \left\{\frac{2(11.7)(8.85x10^{-14})(V_{bi} + V_{CB})}{1.6x10^{-19}}\right.$$
$$\left.\times\left(\frac{5x10^{15}}{3x10^{16}}\right)\left(\frac{1}{5x10^{15} + 3x10^{16}}\right)\right\}^{1/2}$$

or

$$x_p = \left\{(6.163x10^{-11})(V_{bi} + V_{CB})\right\}^{1/2}$$

Now, for $V_{CB} = 5\,V$, $x_p = 0.1875\ \mu m$, and

For $V_{CB} = 10\,V$, $x_p = 0.2569\ \mu m$

(a)

$x_{BO} = 1.0\ \mu m$

For $V_{CB} = 5\,V$, $x_B = 1.0 - 0.1875 = 0.8125\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.8125x10^{-4}} = 161.5\ A/cm^2$$

For $V_{CB} = 10\,V$, $x_B = 1.0 - 0.2569 = 0.7431\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.7431x10^{-4}} = 176.6\ A/cm^2$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}(V_{CE} + V_A)$$

where

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{176.6 - 161.5}{5}$$
$$= 3.02\ A/cm^2/V$$

Then

$$161.5 = 3.02(5.7 + V_A) \Rightarrow$$
$$\underline{V_A = 47.8\,V}$$

(b)

$x_{BO} = 0.80\ \mu m$

For $V_{CB} = 5\ V$, $x_B = 0.80 - 0.1875 = 0.6125\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.6125x10^{-4}} = 214.2\ A/cm^2$$

For $V_{CB} = 10\ V$, $x_B = 0.80 - 0.2569 = 0.5431\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.5431x10^{-4}} = 241.6\ A/cm^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{241.6 - 214.2}{5}$$

$$= 5.48\ A/cm^2/V$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}(V_{CE} + V_A)$$

or

$$214.2 = 5.48(5.7 + V_A) \Rightarrow$$

$$\underline{V_A = 33.4\ V}$$

(c)

$x_{BO} = 0.60\ \mu m$

For $V_{CB} = 5\ V$, $x_B = 0.60 - 0.1875 = 0.4124\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.4125x10^{-4}} = 318.1\ A/cm^2$$

For $V_{CB} = 10\ V$, $x_B = 0.60 - 0.2569 = 0.3431\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.3431x10^{-4}} = 382.4\ A/cm^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{382.4 - 318.1}{5}$$

$$= 12.86\ A/cm^2/V$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}(V_{CE} + V_A)$$

so

$$318.1 = 12.86(5.7 + V_A) \Rightarrow$$

$$\underline{V_A = 19.0\ V}$$

10.36

Neglect the B-E space charge region

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

Then

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= 2.25x10^3 \exp\left(\frac{0.60}{0.0259}\right) = 2.59x10^{13}\ cm^{-3}$$

$$J = eD_B\frac{dn_B}{dx} = \frac{eD_B n_B(0)}{x_B}$$

$$= \frac{(1.6x10^{-19})(20)(2.59x10^{13})}{x_B}$$

or

$$J = \frac{8.29x10^{-5}}{x_B}$$

(a)

Now $x_B = x_{BO} - x_p$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{17})}{(1.5x10^{10})^2}\right] = 0.754\ V$$

Also

$$x_p = \left[\frac{2\in(V_{bi} + V_{CB})}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C + N_B}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi} + V_{CB})}{1.6x10^{-19}} \times\left(\frac{10^{16}}{10^{17}}\right)\left(\frac{1}{10^{16} + 10^{17}}\right)\right]^{1/2}$$

or

$$x_p = \left[(1.177x10^{-11})(V_{bi} + V_{CB})\right]^{1/2}$$

For $V_{CB} = 1\ V$, $x_p(1) = 4.544x10^{-6}\ cm$

For $V_{CB} = 5\ V$, $x_p(5) = 8.229x10^{-6}\ cm$

Now

$$x_B = x_{BO} - x_p = 1.1x10^{-4} - x_p$$

Then

For $V_{CB} = 1\ V$, $x_B(1) = 1.055\ \mu m$

For $V_{CB} = 5\ V$, $x_B(5) = 1.018\ \mu m$

So

$$\Delta x_B = 1.055 - 1.018 \Rightarrow$$

or

$$\underline{\Delta x_B = 0.037\ \mu m}$$

(b)
Now

$$J(1) = \frac{8.29x10^{-5}}{1.055x10^{-4}} = 0.7858\ A/cm^2$$

and

$$J(5) = \frac{8.29x10^{-5}}{1.018x10^{-4}} = 0.8143\ A/cm^2$$

and

$$\Delta J = 0.8143 - 0.7858$$

or

$$\underline{\Delta J = 0.0285\ A/cm^2}$$

10.37

Let $x_E = x_B$, $L_E = L_B$, $D_E = D_B$

Then the emitter injection efficiency is

$$\gamma = \frac{1}{1 + \frac{p_{EO}}{n_{BO}}} = \frac{1}{1 + \frac{n_{iE}^2}{N_E} \cdot \frac{N_B}{n_{iB}^2}}$$

where $n_{iB}^2 = n_i^2$

For no bandgap narrowing, $n_{iE}^2 = n_i^2$.

With bandgap narrowing, $n_{iE}^2 = n_i^2 \exp\left(\frac{\Delta E_g}{kT}\right)$,

Then

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E} \exp\left(\frac{\Delta E_g}{kT}\right)}$$

(a)
No bandgap narrowing, so $\Delta E_g = 0$.

$\alpha = \gamma \alpha_T \delta = \gamma (0.995)^2$. We find

N_E	γ	α	β
E17	0.5	0.495	0.980
E18	0.909	0.8999	8.99
E19	0.990	0.980	49
E20	0.9990	0.989	89.9

(b)
Taking into account bandgap narrowing, we find

N_E	$\Delta E_g (meV)$	γ	α	β
E17	0	0.5	0.495	0.98
E18	25	0.792	0.784	3.63
E19	80	0.820	0.812	4.32
E20	230	0.122	0.121	0.14

10.38

(a) We have

$$\gamma = \frac{1}{1 + \frac{p_{EO} D_E L_B}{n_{BO} D_B L_E} \cdot \frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

For $x_E = x_B$, $L_E = L_B$, $D_E = D_B$, we obtain

$$\gamma = \frac{1}{1 + \frac{p_{EO}}{n_{BO}}} = \frac{1}{1 + \frac{(n_i^2/N_E)\exp(\Delta E_g/kT)}{(n_i^2/N_B)}}$$

or

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}\exp\left(\frac{\Delta E_g}{kT}\right)}$$

For $N_E = 10^{19}\ cm^{-3}$, we have $\Delta E_g = 80\ meV$.

Then

$$0.996 = \frac{1}{1 + \frac{N_B}{10^{19}}\exp\left(\frac{0.080}{0.0259}\right)}$$

which yields

$$\underline{N_B = 1.83x10^{15}\ cm^{-3}}$$

(b)
Neglecting bandgap narrowing, we would have

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}} \Rightarrow 0.996 = \frac{1}{1 + \frac{N_B}{10^{19}}}$$

which yields

$$\underline{N_B = 4.02x10^{16}\ cm^{-3}}$$

10.39

(a)

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{(S/2)}{e\mu_p N_B (L x_B)}$$

Then

$$R = \frac{4x10^{-4}}{\left(1.6x10^{-19}\right)(400)\left(10^{16}\right)\left(100x10^{-4}\right)\left(0.7x10^{-4}\right)}$$

or

$$\underline{R = 893\ \Omega}$$

(b)

$$V = IR = \left(10x10^{-6}\right)(893) \Rightarrow$$

$$\underline{V = 8.93\ mV}$$

(c)

At $x = 0$,

$$n_p(0) = n_{pO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

and at $x = \frac{S}{2}$,

$$n_p'(0) = n_{pO}\exp\left(\frac{V_{BE} - 0.00893}{V_t}\right)$$

Then

$$\frac{n_p'(0)}{n_p(0)} = \frac{n_{pO}\exp\left(\frac{V_{BE} - 0.00893}{V_t}\right)}{n_{pO}\exp\left(\frac{V_{BE}}{V_t}\right)}$$

$$= \exp\left(\frac{-0.00893}{0.0259}\right) = 0.7084$$

or

$$\underline{\frac{n_p'(0)}{n_p(0)} = 70.8\%}$$

10.40

From problem 10.39(c), we have

$$\frac{n_p'(0)}{n_p(0)} = \exp\left(\frac{-V}{V_t}\right)$$

where V is the voltage drop across the $S/2$ length. Now

$$0.90 = \exp\left(\frac{-V}{0.0259}\right)$$

which yields $V = 2.73\ mV$

We have

$$R = \frac{V}{I} = \frac{2.73x10^{-3}}{10x10^{-6}} = 273\ \Omega$$

We can also write

$$R = \frac{S/2}{e\mu_p N_B\left(Lx_B\right)}$$

Solving for S, we find

$$S = 2R\mu_p eN_B Lx_B$$

$$= 2(273)(400)\left(1.6x10^{-19}\right)\left(10^{16}\right) \times\left(100x10^{-4}\right)\left(0.7x10^{-4}\right)$$

or

$$\underline{S = 2.45\ \mu m}$$

10.41

(a)

$$N_B = N_B(0)\exp\left(\frac{-ax}{x_B}\right)$$

where

$$a = \ln\left(\frac{N_B(0)}{N_B(x_B)}\right) > 0$$

and is a constant. In thermal equilibrium

$$J_p = e\mu_p N_B \mathrm{E} - eD_p\frac{dN_B}{dx} = 0$$

so that

$$\mathrm{E} = \frac{D_p}{\mu_p}\cdot\frac{1}{N_B}\cdot\frac{dN_B}{dx} = \left(\frac{kT}{e}\right)\cdot\frac{1}{N_B}\cdot\frac{dN_B}{dx}$$

which becomes

$$\mathrm{E} = \left(\frac{kT}{e}\right)\cdot\frac{1}{N_B}\cdot N_B(0)\cdot\left(\frac{-a}{x_B}\right)\cdot\exp\left(\frac{-ax}{x_B}\right)$$

$$= \left(\frac{kT}{e}\right)\cdot\left(\frac{-a}{x_B}\right)\cdot\frac{1}{N_B}\cdot N_B$$

or

$$\underline{\mathrm{E} = -\left(\frac{a}{x_B}\right)\left(\frac{kT}{e}\right)}$$

which is a constant.

(b)

The electric field is in the negative x-direction which will aid the flow of minority carrier electrons across the base.

(c)

$$J_n = e\mu_n n\mathrm{E} + eD_n\frac{dn}{dx}$$

Assuming no recombination in the base, J_n will be a constant across the base. Then

$$\frac{dn}{dx} + \left(\frac{\mu_n}{D_n}\right)n\mathrm{E} = \frac{J_n}{eD_n} = \frac{dn}{dx} + n\left(\frac{\mathrm{E}}{V_t}\right)$$

where $V_t = \left(\frac{kT}{e}\right)$

The homogeneous solution to the differential equation is found from

$$\frac{dn_H}{dx} + An_H = 0$$

where $A = \dfrac{\mathrm{E}}{V_t}$

The solution is of the form

$$n_H = n_H(0)\exp(-Ax)$$

The particular solution is found from

$$n_P \cdot A = B$$

where $B = \dfrac{J_n}{eD_n}$

The particular solution is then

$$n_P = \frac{B}{A} = \frac{\left(\dfrac{J_n}{eD_n}\right)}{\left(\dfrac{\mathrm{E}}{V_t}\right)} = \frac{J_n V_t}{eD_n \mathrm{E}} = \frac{J_n}{e\mu_n \mathrm{E}}$$

The total solution is then

$$n = \frac{J_n}{e\mu_n \mathrm{E}} + n_H(0)\exp(-Ax)$$

and

$$n(0) = n_{pO}\exp\left(\frac{V_{BE}}{V_t}\right) = \frac{n_i^2}{N_B(0)}\exp\left(\frac{V_{BE}}{V_t}\right)$$

Then

$$n_H(0) = \frac{n_i^2}{N_B(0)}\exp\left(\frac{V_{BE}}{V_t}\right) - \frac{J_n}{e\mu_n \mathrm{E}}$$

10.42

(a) The basic pn junction breakdown voltage from the figure for $N_C = 5x10^{15}\ cm^{-3}$ is approximately $BV_{CBO} = 90\ V$.

(b)
We have

$$BV_{CEO} = BV_{CBO}\sqrt[n]{1-\alpha}$$

For $n = 3$ and $\alpha = 0.992$, we obtain

$$BV_{CEO} = 90 \cdot \sqrt[3]{1-0.992} = (90)(0.20)$$

or

$$BV_{CEO} = 18\ V$$

(c)
The B-E breakdown voltage, for $N_B = 10^{17}\ cm^{-3}$, is approximately,

$$BV_{BE} = 12\ V$$

10.43

We want $BV_{CEO} = 60\ V$

So then

$$BV_{CEO} = \frac{BV_{CBO}}{\sqrt[n]{\beta}} \Rightarrow 60 = \frac{BV_{CBO}}{\sqrt[3]{50}}$$

which yields

$$BV_{CBO} = 221\ V$$

For this breakdown voltage, we need

$$N_C \approx 1.5x10^{15}\ cm^{-3}$$

The depletion width into the collector at this voltage is

$$x_C = x_n = \left\{\frac{2\in(V_{bi}+V_{BC})}{e}\left(\frac{N_B}{N_C}\right)\left(\frac{1}{N_B+N_C}\right)\right\}^{1/2}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(1.5x10^{15})(10^{16})}{(1.5x10^{10})^2}\right] = 0.646\ V$$

and $V_{BC} = BV_{CEO} = 60\ V$

so that

$$x_C = \left\{\frac{2(11.7)(8.85x10^{-14})(0.646+60)}{1.6x10^{-19}} \times\left(\frac{10^{16}}{1.5x10^{15}}\right)\left(\frac{1}{10^{16}+1.5x10^{15}}\right)\right\}^{1/2}$$

or

$$x_C = 6.75\ \mu m$$

10.44

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{16})(5x10^{17})}{(1.5x10^{10})^2}\right] = 0.824\ V$$

At punch-through, we have

$$x_B = 0.70x10^{-4} = x_p(V_{BC} = V_{th}) - x_p(V_{BC} = 0)$$

$$= \left\{\frac{2\in(V_{bi}+V_{pt})}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right\}^{1/2} - \left\{\frac{2\in V_{bi}}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right\}^{1/2}$$

which can be written as

$0.70x10^{-4}$

$$= \left\{ \frac{2(11.7)(8.85x10^{-14})(V_{bi} + V_{pt})}{1.6x10^{-19}} \times \left(\frac{5x10^{17}}{3x10^{16}} \right) \left(\frac{1}{5x10^{17} + 3x10^{16}} \right) \right\}^{1/2}$$

$$- \left\{ \frac{2(11.7)(8.85x10^{-14})(0.824)}{1.6x10^{-19}} \times \left(\frac{5x10^{17}}{3x10^{16}} \right) \left(\frac{1}{5x10^{17} + 3x10^{16}} \right) \right\}^{1/2}$$

which becomes

$$0.70x10^{-4} = (0.202x10^{-4})\sqrt{V_{bi} + V_{pt}} - (0.183x10^{-4})$$

We obtain

$$V_{bi} + V_{pt} = 19.1\ V$$

or

$$\underline{V_{pt} = 18.3\ V}$$

Considering the junction alone, avalanche breakdown would occur at approximately $BV \approx 25\ V$.

10.45

(a) Neglecting the B-E junction depletion width,

$$V_{pt} = \frac{eW_B^2}{2\in} \cdot \frac{N_B(N_C + N_B)}{N_C}$$

$$= \left\{ \frac{(1.6x10^{-19})(0.5x10^{-4})^2}{2(11.7)(8.85x10^{-14})} \cdot \frac{(10^{17})(10^{17} + 7x10^{15})}{(7x10^{15})} \right\}$$

or

$$\underline{V_{pt} = 295\ V}$$

However, actual junction breakdown for these doping concentrations is $\approx 70\ V$. So punch-through will not be reached.

10.46

At punch-through,

$$x_O = \left\{ \frac{2\in(V_{bi} + V_{pt})}{e} \cdot \left(\frac{N_C}{N_B} \right) \left(\frac{1}{N_B + N_C} \right) \right\}^{1/2}$$

Since $V_{pt} = 25\ V$, we can neglect V_{bi}.

Then we have

$$(0.75x10^{-4}) = \left\{ \frac{2(11.7)(8.85x10^{-14})(25)}{1.6x10^{-19}} \times \left(\frac{10^{16}}{N_B} \right) \left(\frac{1}{10^{16} + N_B} \right) \right\}^{1/2}$$

We obtain

$$\underline{N_B = 1.95x10^{16}\ cm^{-3}}$$

10.47

$$V_{CE}(sat) = \left(\frac{kT}{e} \right) \cdot \ln \left[\frac{I_C(1-\alpha_R) + I_B}{\alpha_F I_B - (1-\alpha_F) I_C} \cdot \frac{\alpha_F}{\alpha_R} \right]$$

We can write

$$\exp\left(\frac{V_{CE}(sat)}{0.0259} \right) = \frac{(1)(1-0.2) + I_B}{(0.99)I_B - (1-0.99)(1)} \left(\frac{0.99}{0.20} \right)$$

or

$$\exp\left(\frac{V_{CE}(sat)}{0.0259} \right) = \left(\frac{0.8 + I_B}{0.99 I_B - 0.01} \right)(4.95)$$

(a)

For $V_{CE}(sat) = 0.30\ V$, we find

$$\exp\left(\frac{0.30}{0.0259} \right) = 1.0726x10^5 = \left(\frac{0.8 + I_B}{0.99 I_B - 0.01} \right)(4.95)$$

We find

$$\underline{I_B = 0.01014\ mA}$$

(b)

For $V_{CE}(sat) = 0.20\ V$, we find

$$\underline{I_B = 0.0119\ mA}$$

(c)

For $V_{CE}(sat) = 0.10\ V$, we find

$$\underline{I_B = 0.105\ mA}$$

10.48

For an npn in the active mode, we have $V_{BC} < 0$, so that $\exp\left(\frac{V_{BC}}{V_t} \right) \approx 0$.

Now

$$I_E + I_B + I_C = 0 \Rightarrow I_B = -(I_C + I_E)$$

Then we have

$$I_B = -\left\{\alpha_F I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]+I_{CS}\right\}$$

$$-\left\{-\alpha_R I_{CS} - I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\right\}$$

or

$$\underline{I_B = (1-\alpha_F)I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]-(1-\alpha_R)I_{CS}}$$

10.49

We can write

$$I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]$$

$$= \alpha_R I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]-I_E$$

Substituting, we find

$$I_C = \alpha_F\left\{\alpha_R I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]-I_E\right\}$$

$$-I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

From the definition of currents, we have $I_E = -I_C$ for the case when $I_B = 0$. Then

$$I_C = \alpha_F\alpha_R I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

$$+\alpha_F I_C - I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

When a C-E voltage is applied, then the B-C becomes reverse biased, so $\exp\left(\frac{V_{BC}}{V_t}\right)\approx 0$. Then

$$I_C = -\alpha_F\alpha_R I_{CS} + \alpha_F I_C + I_{CS}$$

We find

$$\underline{I_C = I_{CEO} = \frac{I_{CS}(1-\alpha_F\alpha_R)}{(1-\alpha_F)}}$$

10.50

We have

$$I_C = \alpha_F I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]$$

$$-I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

For $V_{BC} <\approx 0.1\,V$, $\exp\left(\frac{V_{BC}}{V_t}\right)\approx 0$ and

$I_C \approx$ constant. This equation does not include the base width modulation effect.

For $V_{BE} = 0.2\,V$,

$$I_C = (0.98)\left(10^{-13}\right)\exp\left(\frac{0.2}{0.0250}\right)+5x10^{-13}$$

or

$$\underline{I_C = 2.22x10^{-10}\ A}$$

For $V_{BE} = 0.4\,V$,

$$\underline{I_C = 5x10^{-7}\ A}$$

For $V_{BE} = 0.6\,V$,

$$\underline{I_C = 1.13x10^{-3}\ A}$$

10.51

Computer Plot

10.52

(a)

$$r_\pi' = \left(\frac{kT}{e}\right)\cdot\frac{1}{I_E} = \frac{0.0259}{0.5x10^{-3}} = 51.8\ \Omega$$

So

$$\tau_e = r_\pi' C_{je} = (51.8)\left(0.8x10^{-12}\right)\Rightarrow$$

or

$$\underline{\tau_e = 41.4\ ps}$$

Also

$$\tau_b = \frac{x_B^2}{2D_n} = \frac{\left(0.7x10^{-4}\right)^2}{2(25)}\Rightarrow$$

or

$$\underline{\tau_b = 98\ ps}$$

We have

$$\tau_c = r_c\left(C_\mu + C_s\right) = (30)(2)\left(0.08x10^{-12}\right)\Rightarrow$$

or

$$\underline{\tau_c = 4.8\ ps}$$

Also

$$\tau_d = \frac{x_{dc}}{v_s} = \frac{2x10^{-4}}{10^{+7}}\Rightarrow$$

or

$$\underline{\tau_d = 20\ ps}$$

(b)

$$\tau_{ec} = \tau_e + \tau_b + \tau_c + \tau_d$$

$$= 41.4 + 98 + 4.8 + 20 \Rightarrow$$

or

$$\tau_{ec} = 164.2\ ps$$

Then

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(164.2x10^{-12}\right)} \Rightarrow$$

or

$$f_T = 970\ MHz$$

Also

$$f_\beta = \frac{f_T}{\beta} = \frac{970}{50} \Rightarrow$$

or

$$f_\beta = 19.4\ MHz$$

10.53

$$\tau_b = \frac{x_B^2}{2D_B} = \frac{\left(0.5x10^{-4}\right)^2}{2(20)} = 6.25x10^{-11}\ s$$

We have $\tau_b = 0.2\tau_{ec}$,
So that

$$\tau_{ec} = 3.125x10^{-10}\ s$$

Then

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(3.125x10^{-10}\right)} \Rightarrow$$

or

$$f_T = 509\ MHz$$

10.54
We have

$$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$$

We are given

$\tau_b = 100\ ps$ and $\tau_e = 25\ ps$

We find

$$\tau_d = \frac{x_d}{v_s} = \frac{1.2x10^{-4}}{10^7} = 1.2x10^{-11}\ s$$

or

$$\tau_d = 12\ ps$$

Also

$$\tau_c = r_c C_c = (10)\left(0.1x10^{-12}\right) = 10^{-12}\ s$$

or

$$\tau_c = 1\ ps$$

Then

$$\tau_{ec} = 25 + 100 + 12 + 1 = 138\ ps$$

We obtain

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(138x10^{-12}\right)} = 1.15x10^9\ Hz$$

or

$$f_T = 1.15\ GHz$$

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Chapter 12

Problem Solutions

12.1
(a)
$$I_D = 10^{-15}\exp\left[\frac{V_{GS}}{(2.1)V_t}\right]$$
For $V_{GS} = 0.5\,V$,
$$I_D = 10^{-15}\exp\left[\frac{0.5}{(2.1)(0.0259)}\right] \Rightarrow$$
$$\underline{I_D = 9.83x10^{-12}\ A}$$
For $V_{GS} = 0.7\,V$,
$$\underline{I_D = 3.88x10^{-10}\ A}$$
For $V_{GS} = 0.9\,V$,
$$\underline{I_D = 1.54x10^{-8}\ A}$$
Then the total current is:
$$I_{Total} = I_D\left(10^6\right)$$
For $V_{GS} = 0.5\,V$, $\underline{I_{Total} = 9.83\ \mu A}$
For $V_{GS} = 0.7\,V$, $\underline{I_{Total} = 0.388\ mA}$
For $V_{GS} = 0.9\,V$, $\underline{I_{Total} = 15.4\ mA}$
(b)
Power: $P = I_{Total}\cdot V_{DD}$
Then
For $V_{GS} = 0.5\,V$, $\underline{P = 49.2\ \mu W}$
For $V_{GS} = 0.7\,V$, $\underline{P = 1.94\ mW}$
For $V_{GS} = 0.9\,V$, $\underline{P = 77\ mW}$

12.2
We have
$$\Delta L = \sqrt{\frac{2\in}{eN_a}}\cdot\left[\sqrt{\phi_{fp} + V_{DS}(sat) + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$
where
$$\phi_{fp} = V_t\ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right)$$
or
$$\phi_{fp} = 0.347\ V$$
We find
$$\sqrt{\frac{2\in}{eN_a}} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$
$$= 0.360\ \mu m / V^{1/2}$$
We have
$$V_{DS}(sat) = V_{GS} - V_T$$
(a)
For $V_{GS} = 5\ V \Rightarrow V_{DS}(sat) = 4.25\ V$
Then
$$\Delta L = 0.360\left[\sqrt{0.347+5} - \sqrt{0.347+4.25}\right]$$
or
$$\Delta L = 0.0606\ \mu m$$
If ΔL is 10%of L, then $\underline{L = 0.606\ \mu m}$
(b)
For $V_{DS} = 5\ V$, $V_{GS} = 2\ V \Rightarrow V_{DS}(sat) = 1.25\ V$
Then
$$\Delta L = 0.360\left[\sqrt{0.347+5} - \sqrt{0.347+1.25}\right]$$
or
$$\Delta L = 0.377\ \mu m$$
Now if ΔL is 10% of L, then $\underline{L = 3.77\ \mu m}$

12.3
$$\Delta L = \sqrt{\frac{2\in}{eN_a}}\cdot\left[\sqrt{\phi_{fp} + V_{DS}(sat) + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$
where
$$\phi_{fp} = V_t\ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{4x10^{16}}{1.5x10^{10}}\right)$$
or
$$\phi_{fp} = 0.383\ V$$
and
$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$
$$= \left[\frac{4(11.7)\left(8.85x10^{-14}\right)(0.383)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$
or
$$x_{dT} = 0.157\ \mu m$$
Then
$$\left|Q'_{SD}(\max)\right| = eN_a x_{dT}$$

$$= \left(1.6x10^{-19}\right)\left(4x10^{16}\right)\left(0.157x10^{-4}\right)$$

or

$$\left|Q'_{SD}(\max)\right| = 10^{-7}\ C/cm^2$$

Now

$$V_T = \left(\left|Q'_{SD}(\max)\right| - Q'_{SS}\right)\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$

so that

$$V_T = \frac{\left[10^{-7} - \left(1.6x10^{-19}\right)\left(3x10^{10}\right)\right]\left(400x10^{-8}\right)}{(3.9)\left(8.85x10^{-14}\right)} + 0 + 2(0.383)$$

or

$$V_T = 1.87\ V$$

Now

$$V_{DS}(sat) = V_{GS} - V_T = 5 - 1.87 = 3.13\ V$$

We find

$$\sqrt{\frac{2\in}{eN_a}} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2} = 1.80x10^{-5}$$

Now

$$\Delta L = 1.80x10^{-5}\cdot\left[\sqrt{0.383 + 3.13 + \Delta V_{DS}} - \sqrt{0.383 + 3.13}\right]$$

or

$$\Delta L = 1.80x10^{-5}\left[\sqrt{3.513 + \Delta V_{DS}} - \sqrt{3.513}\right]$$

We obtain

ΔV_{DS}	$\Delta L(\mu m)$
0	0
1	0.0451
2	0.0853
3	0.122
4	0.156
5	0.188

12.4
Computer plot

12.5
Plot

12.6
Plot

12.7

(a) Assume $V_{DS}(sat) = 1\ V$, We have

$$E_{sat} = \frac{V_{DS}(sat)}{L}$$

We find

$L(\mu m)$	$E_{sat}(V/cm)$
3	$3.33x10^3$
1	10^4
0.5	$2x10^4$
0.25	$4x10^4$
0.13	$7.69x10^4$

(b)

Assume $\mu_n = 500\ cm^2/V-s$, we have

$v = \mu_n E_{sat}$

Then

For $L = 3\ \mu m$, $\underline{v = 1.67x10^6\ cm/s}$

For $L = 1\ \mu m$, $\underline{v = 5x10^6\ cm/s}$

For $L \le 0.5\ \mu m$, $\underline{v \approx 10^7\ cm/s}$

12.8

We have $I'_D = L(L-\Delta L)^{-1} I_D$

We may write

$$g_O = \frac{\partial I'_D}{\partial V_{DS}} = (-1)L(L-\Delta L)^{-2} I_D\left(\frac{-\partial(\Delta L)}{\partial V_{DS}}\right) = \frac{L}{(L-\Delta L)^2}\cdot I_D\cdot\frac{\partial(\Delta L)}{\partial V_{DS}}$$

We have

$$\Delta L = \sqrt{\frac{2\in}{eN_a}}\cdot\left[\sqrt{\phi_{fp} + V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$

We find

$$\frac{\partial(\Delta L)}{\partial V_{DS}} = \sqrt{\frac{2\in}{eN_a}}\cdot\frac{1}{2\sqrt{\phi_{fp} + V_{DS}}}$$

(a)

For $V_{GS} = 2\ V$, $\Delta V_{DS} = 1\ V$, and

$$V_{DS}(sat) = V_{GS} - V_T = 2 - 0.8 = 1.2\ V$$

Also

$$V_{DS} = V_{DS}(sat) + \Delta V_{DS} = 1.2 + 1 = 2.2\ V$$

and

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$$

Now

$$\sqrt{\frac{2\in}{eN_a}} = \left[\frac{2(11.7)(8.85x10^{-14})}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$
$$= 0.2077\ \mu m / V^{1/2}$$

We find

$$\Delta L = 0.2077\left[\sqrt{0.376+2.2} - \sqrt{0.376+1.2}\right]$$
$$= 0.0726\ \mu m$$

Then

$$\frac{\partial(\Delta L)}{\partial V_{DS}} = \frac{0.2077}{2}\cdot\frac{1}{\sqrt{0.376+2.2}}$$
$$= 0.0647\ \mu m / V$$

From the previous problem,

$I_D = 0.48\ mA$, $L = 2\ \mu m$

Then

$$g_O = \frac{2}{(2-0.0726)^2}(0.48x10^{-3})(0.0647)$$

or

$$g_O = 1.67x10^{-5}\ S$$

so that

$$r_O = \frac{1}{g_O} = 59.8\ k\Omega$$

(b)

If $L = 1\ \mu m$, then from the previous problem, we would have $I_D = 0.96\ mA$, so that

$$g_O = \frac{1}{(1-0.0726)^2}(0.96x10^{-3})(0.0647)$$

or

$$g_O = 7.22x10^{-5}\ S$$

so that

$$r_O = \frac{1}{g_O} = 13.8\ k\Omega$$

12.9

(a)

$$I_D(sat) = \frac{W\mu_n C_{ox}}{2L}(V_{GS}-V_T)^2$$
$$= \left(\frac{10}{2}\right)(500)(6.9x10^{-8})(V_{GS}-1)^2$$

or

$$I_D(sat) = 0.173(V_{GS}-1)^2\ (mA)$$

and

$$\sqrt{I_D(sat)} = \sqrt{0.173}(V_{GS}-1)\ \ (mA)^{1/2}$$

(b)

Let $\mu_{eff} = \mu_O\left(\frac{E_{eff}}{E_C}\right)^{-1/3}$

Where $\mu_O = 1000\ cm^2/V-s$ and $E_C = 2.5x10^4\ V/cm$.

Let $E_{eff} = \frac{V_{GS}}{t_{ox}}$

We find

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} \Rightarrow t_{ox} = \frac{\in_{ox}}{C_{ox}} = \frac{(3.9)(8.85x10^{-14})}{6.9x10^{-8}}$$

or

$$t_{ox} = 500\ \overset{\circ}{A}$$

Then

V_{GS}	E_{eff}	μ_{eff}	$\sqrt{I_D(sat)}$
1	--	--	0
2	4E5	397	0.370
3	6E5	347	0.692
4	8E5	315	0.989
5	10E5	292	1.27

(c)

The slope of the variable mobility curve is not constant, but is continually decreasing.

12.10

Plot

12.11

$$V_T = V_{FB} + \frac{|Q'_{SD}(\max)|}{C_{ox}} + 2\phi_{fp}$$

We find

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{5x10^{16}}{1.5x10^{10}}\right)$$

or

$$\phi_{fp} = 0.389\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$
$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.389)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.142\ \mu m$$

Now

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$
$$= (1.6x10^{-19})(5x10^{16})(0.142x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.14x10^{-7}\ C/cm^2$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{400x10^{-8}} = 8.63x10^{-8}\ F/cm^2$$

Then

$$V_T = -1.12 + \frac{1.14x10^{-7}}{8.63x10^{-8}} + 2(0.389)$$

or

$$\underline{V_T = +0.90\ V}$$

(a)

$$I_D = \frac{W\mu_n C_{ox}}{2L}\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right]$$

and

$$V_{DS}(sat) = V_{GS} - V_T$$

We have

$$I_D = \left(\frac{20}{2}\right)\left(\frac{1}{2}\right)(400)(8.63x10^{-8})$$
$$\times\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right]$$

or

$$I_D = 0.173\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right](mA)$$

For $V_{DS} = V_{DS}(sat) = V_{GS} - V_T = 1\,V$,

$$\underline{I_D(sat) = 0.173\ mA}$$

For $V_{DS} = V_{DS}(sat) = V_{GS} - V_T = 2\,V$,

$$\underline{I_D(sat) = 0.692\ mA}$$

(b)

For $V_{DS} \le 1.25\,V$, $\mu = \mu_n = 400\ cm^2/V-s$. The curve for $V_{GS} - V_T = 1\,V$ is unchanged. For $V_{GS} - V_T = 2\,V$ and $0 \le V_{DS} \le 1.25\,V$, the curve in unchanged. For $V_{DS} \ge 1.25\,V$, the current is constant at

$$I_D = 0.173\left[2(2)(1.25) - (1.25)^2\right] = 0.595\ mA$$

When velocity saturation occurs, $V_{DS}(sat) = 1.25\,V$ for the case of $V_{GS} - V_T = 2\,V$.

12.12

Plot

12.13

(a) Non-saturation region

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right]$$

We have

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} \Rightarrow \frac{C_{ox}}{k}$$

and

$$W \Rightarrow kW,\ L \Rightarrow kL$$

also

$$V_{GS} \Rightarrow kV_{GS},\ V_{DS} \Rightarrow kV_{DS}$$

So

$$I_D = \frac{1}{2}\mu_n\left(\frac{C_{ox}}{k}\right)\left(\frac{kW}{kL}\right)\left[2(kV_{GS}-V_T)kV_{DS} - (kV_{DS})^2\right]$$

Then

$$\underline{I_D \Rightarrow \approx kI_D}$$

In the saturation region

$$I_D = \frac{1}{2}\mu_n\left(\frac{C_{ox}}{k}\right)\left(\frac{kW}{kL}\right)\left[kV_{GS} - V_T\right]^2$$

Then

$$\underline{I_D \Rightarrow \approx kI_D}$$

(b)

$$\underline{P = I_D V_{DD} \Rightarrow (kI_D)(kV_{DD}) \Rightarrow k^2 P}$$

12.14

$$I_D(sat) = WC_{ox}(V_{GS} - V_T)v_{sat}$$
$$\Rightarrow (kW)\left(\frac{C_{ox}}{k}\right)(kV_{GS} - V_T)v_{sat}$$

or

$$\underline{I_D(sat) \approx kI_D(sat)}$$

12.15

(a)

(i) $I_D = K_n(V_{GS} - V_T)^2 = (0.1)(5-0.8)^2$

or

$$\underline{I_D = 1.764\ mA}$$

(ii)

$$I_D = \left(\frac{0.1}{0.6}\right)\left[(0.6)(5) - 0.8\right]^2$$

or

$$\underline{I_D = 0.807\ mA}$$

(b)

(i) $P = (1.764)(5) \Rightarrow \underline{P = 8.82\ mW}$

(ii) $P = (0.807)(0.6)(5) \Rightarrow \underline{P = 2.42\ mW}$

(c)

Current: Ratio $= \dfrac{0.807}{1.764} = 0.457$

Power: Ratio $= \dfrac{2.42}{8.82} = 0.274$

12.16

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

Now

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right)$$

or

$$\phi_{fp} = 0.347\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{450x10^{-8}}$$

$$= 7.67x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = -\frac{(1.6x10^{-19})(10^{16})(0.3x10^{-4})}{7.67x10^{-8}} \times\left\{\frac{0.3}{1}\left[\sqrt{1+\frac{2(0.3)}{0.3}}-1\right]\right\}$$

or

$$\underline{\Delta V_T = -0.137\ V}$$

12.17

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.376)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.180\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{800x10^{-8}}$$

or

$$C_{ox} = 4.31x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = -0.20 = -\frac{(1.6x10^{19})(3x10^{16})(0.18x10^{-4})}{4.31x10^{-8}} \times\left\{\frac{0.6}{L}\left[\sqrt{1+\frac{2(0.18)}{0.6}}-1\right]\right\}$$

or

$$= -0.20 = -\frac{0.319}{L}$$

which yields

$$\underline{L = 1.59\ \mu m}$$

12.18

We have

$$L' = L-(a+b)$$

and from the geometry

(1) $\left(a+r_j\right)^2 + x_{dT}^2 = \left(r_j + x_{dS}\right)^2$

and

(2) $\left(b+r_j\right)^2 + x_{dT}^2 = \left(r_j + x_{dD}\right)^2$

From (1),

$$\left(a+r_j\right)^2 = \left(r_j + x_{dS}\right)^2 - x_{dT}^2$$

so that

$$a = \sqrt{\left(r_j + x_{dS}\right)^2 - x_{dT}^2} - r_j$$

which can be written as

$$a = r_j\left[\sqrt{\left(1+\frac{x_{dS}}{r_j}\right)^2 - \left(\frac{x_{dT}}{r_j}\right)^2} - 1\right]$$

or

$$a = r_j\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\left(\frac{x_{dS}}{r_j}\right)^2-\left(\frac{x_{dT}}{r_j}\right)^2}-1\right]$$

Define

$$\alpha^2 = \frac{x_{dS}^2 - x_{dT}^2}{r_j^2}$$

We can then write

$$a = r_j\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\alpha^2}-1\right]$$

Similarly from (2), we will have

$$b = r_j\left[\sqrt{1+\frac{2x_{dD}}{r_j}+\beta^2}-1\right]$$

where

$$\beta^2 = \frac{x_{dD}^2 - x_{dT}^2}{r_j^2}$$

The average bulk charge in the trapezoid (per unit area) is

$$|Q_B'| \cdot L = eN_a x_{dT}\left(\frac{L+L'}{2}\right)$$

or

$$|Q_B'| = eN_a x_{dT}\left(\frac{L+L'}{2L}\right)$$

We can write

$$\frac{L+L'}{2L} = \frac{1}{2}+\frac{L'}{2L} = \frac{1}{2}+\frac{1}{2L}\left[L-(a+b)\right]$$

which is

$$= 1-\frac{(a+b)}{2L}$$

Then

$$|Q_B'| = eN_a x_{dT}\left[1-\frac{(a+b)}{2L}\right]$$

Now $|Q_B'|$ replaces $|Q_{SD}'(\max)|$ in the threshold equation. Then

$$\Delta V_T = \frac{|Q_B'|}{C_{ox}} - \frac{|Q_{SD}'(\max)|}{C_{ox}}$$

$$= \frac{eN_a x_{dT}}{C_{ox}}\left[1-\frac{(a+b)}{2L}\right] - \frac{eN_a x_{dT}}{C_{ox}}$$

or

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}} \cdot \frac{(a+b)}{2L}$$

Then substituting, we obtain

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}} \cdot \frac{r_j}{2L}\left\{\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\alpha^2}-1\right] + \left[\sqrt{1+\frac{2x_{dD}}{r_j}+\beta^2}-1\right]\right\}$$

Note that if $x_{dS} = x_{dD} = x_{dT}$, then $\alpha = \beta = 0$ and the expression for ΔV_T reduces to that given in the text.

12.19

We have $L' = 0$, so Equation (12.25) becomes

$$\frac{L+L'}{2L} \Rightarrow \frac{L}{2L} = \frac{1}{2} = \left\{1-\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

or

$$\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right] = \frac{1}{2}$$

Then Equation (12.26) is

$$|Q_B'| = eN_a x_{dT}\left(\frac{1}{2}\right)$$

The change in the threshold voltage is

$$\Delta V_T = \frac{|Q_B'|}{C_{ox}} - \frac{|Q_{SD}'(\max)|}{C_{ox}}$$

or

$$\Delta V_T = \frac{(1/2)(eN_a x_{dT})}{C_{ox}} - \frac{(eN_a x_{dT})}{C_{ox}}$$

or

$$\Delta V_T = -\left(\frac{1}{2}\right)\frac{(eN_a x_{dT})}{C_{ox}}$$

12.20

Computer plot

12.21

Computer plot

12.22

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

$$\Rightarrow -\frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)}\left\{\frac{kr_j}{kL}\left[\sqrt{1+\frac{2kx_{dT}}{kr_j}}-1\right]\right\}$$

or

$$\underline{\Delta V_T = k\Delta V_T}$$

12.23

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

We find

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) = 0.347\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{450x10^{-8}}$$

or

$$C_{ox} = 7.67x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = \frac{(1.6x10^{-19})(10^{16})(0.3x10^{-4})}{7.67x10^{-8}} \times\left[\frac{(\pi/2)(0.3x10^{-4})}{2.5x10^{-4}}\right]$$

or

$$\underline{\Delta V_T = +0.118\ V}$$

12.24

Additional bulk charge due to the ends:

$$\Delta Q_B = eN_a L\left(\frac{1}{2}x_{dT}^2\right)\cdot 2 = eN_a Lx_{dT}(\xi x_{dT})$$

where $\xi = 1$.

Then

$$\Delta V_T = \frac{eN_a x_{dT}^2}{C_{ox}W}$$

We find

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.376)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.180\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{800x10^{-8}}$$

or

$$C_{ox} = 4.31x10^{-8}\ F/cm^2$$

Now, we can write

$$W = \frac{eN_a x_{dT}^2}{C_{ox}(\Delta V_T)}$$

$$= \frac{(1.6x10^{-19})(3x10^{16})(0.18x10^{-4})^2}{(4.31x10^{-8})(0.25)}$$

or

$$\underline{W = 1.44\ \mu m}$$

12.25

Computer plot

12.26

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

Assume that ξ is a constant

$$\Rightarrow \frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)}\left(\frac{\xi\cdot kx_{dT}}{kW}\right)$$

or

$$\underline{\Delta V_T = k\Delta V_T}$$

12.27

(a)

$$V_{BD} = (6x10^6)t_{ox} = (6x10^6)(250x10^{-8})$$

or

$$\underline{V_{BD} = 15\ V}$$

(b)
With a safety factor of 3,

$$V_{BD} = \frac{1}{3}\cdot 15 \Rightarrow \underline{V_{BD} = 5\ V}$$

12.28
We want $V_G = 20\ V$. With a safety factor of 3, then $V_{BD} = 60\ V$, so that

$$60 = \left(6x10^6\right)t_{ox} \Rightarrow \underline{t_{ox} = 1000\ \mathring{A}}$$

12.29
Snapback breakdown means $\alpha M = 1$, where

$$\alpha = (0.18)\log_{10}\left(\frac{I_D}{3x10^{-9}}\right)$$

and

$$M = \frac{1}{1-\left(\frac{V_{CE}}{V_{BD}}\right)^m}$$

Let $V_{BD} = 15\ V$, $m = 3$. Now when

$$\alpha M = 1 = \frac{\alpha}{1-\left(\frac{V_{CE}}{15}\right)^3}$$

we can write this as

$$1-\left(\frac{V_{CE}}{15}\right)^3 = \alpha \Rightarrow V_{CE} = 15\sqrt[3]{1-\alpha}$$

Now

I_D	α	V_{CE}
E-8	0.0941	14.5
E-7	0.274	13.5
E-6	0.454	12.3
E-5	0.634	10.7
E-4	0.814	8.6
E-3	0.994	2.7

12.30
One Debye length is

$$L_D = \left[\frac{\in(kT/e)}{eN_a}\right]^{1/2}$$

$$= \left[\frac{(11.7)\left(8.85x10^{-14}\right)(0.0259)}{\left(1.6x10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$

or

$$L_D = 4.09x10^{-6}\ cm$$

Six Debye lengths:

$$6\left(4.09x10^{-6}\right) = 0.246\ \mu m$$

From Example 12.4, we have $x_{dO} = 0.336\ \mu m$, which is the zero-biased source-substrate junction width.
At near punch-through, we will have

$$x_{dO} + 6L_D + x_d = L$$

where x_d is the reverse-biased drain-substrate junction width. Now

$$0.336 + 0.246 + x_d = 1.2 \Rightarrow x_d = 0.618\ \mu m$$ at near punch-through.
We have

$$x_d = \left[\frac{2\in\left(V_{bi}+V_{DS}\right)}{eN_a}\right]^{1/2}$$

or

$$V_{bi}+V_{DS} = \frac{x_d^2 eN_a}{2\in}$$

$$= \frac{\left(0.618x10^{-4}\right)^2\left(1.6x10^{-19}\right)\left(10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

which yields

$$V_{bi}+V_{DS} = 2.95\ V$$

From Example 12.4, we have $V_{bi} = 0.874\ V$, so that

$$\underline{V_{DS} = 2.08\ V}$$

which is the near punch-through voltage. The ideal punch-through voltage was

$$\underline{V_{DS} = 4.9\ V}$$

12.31

$$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{19}\right)\left(3x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.902\ V$$

The zero-biased source-substrate junction width:

$$x_{dO} = \left[\frac{2\in V_{bi}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(0.902)}{\left(1.6x10^{-19}\right)\left(3x10^{16}\right)}\right]^{1/2}$$

or

$$x_{dO} = 0.197\ \mu m$$

The Debye length is

$$L_D = \left[\frac{\in(kT/e)}{eN_a}\right]^{1/2}$$

$$= \left[\frac{(11.7)(8.85x10^{-14})(0.0259)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$L_D = 2.36x10^{-6}\ cm$$

so that

$$6L_D = 6(2.36x10^{-6}) = 0.142\ \mu m$$

Now

$$x_{dO} + 6L_D + x_d = L$$

We have for $V_{DS} = 5\ V$,

$$x_d = \left[\frac{2\in(V_{bi}+V_{DS})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.902+5)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_d = 0.505\ \mu m$$

Then

$$L = 0.197 + 0.142 + 0.505$$

or

$$\underline{L = 0.844\ \mu m}$$

12.32

With a source-to-substrate voltage of 2 volts,

$$x_{dO} = \left[\frac{2\in(V_{bi}+V_{SB})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.902+2)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_{dO} = 0.354\ \mu m$$

We have $6L_D = 0.142\ \mu m$ from the previous problem.

Now

$$x_d = \left[\frac{2\in(V_{bi}+V_{DS}+V_{SB})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.902+5+2)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_d = 0.584\ \mu m$$

Then

$$L = x_{dO} + 6L_D + x_d$$
$$= 0.354 + 0.142 + 0.584$$

or

$$\underline{L = 1.08\ \mu m}$$

12.33

(a) $\phi_{fp} = (0.0259)\ln\left(\frac{2x10^{15}}{1.5x10^{10}}\right) = 0.306\ V$

and

$$\phi_{ms} = -\left(\frac{E_g}{2e} + \phi_{fp}\right) = -\left(\frac{1.12}{2} + 0.306\right)$$

or

$$\phi_{ms} = -0.866\ V$$

Also

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.306)}{(1.6x10^{-19})(2x10^{15})}\right]^{1/2}$$

or

$$x_{dT} = 0.629\ \mu m$$

Now

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$
$$= (1.6x10^{-19})(2x10^{15})(0.629x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 2.01x10^{-8}\ C/cm^2$$

We have

$$Q'_{SS} = (2x10^{11})(1.6x10^{-19}) = 3.2x10^{-8}\ C/cm^2$$

Then

$$V_T = (|Q'_{SD}(\max)| - Q'_{SS})\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$

$$= \frac{(2.01x10^{-8} - 3.2x10^{-8})(650x10^{-8})}{(3.9)(8.85x10^{-14})} - 0.866 + 2(0.306)$$

which yields

$$\underline{V_T = -0.478\ V}$$

(b) We need a shift in threshold voltage in the positive direction, which means we must add acceptor atoms. We need

$$\Delta V_T = +0.80 - (-0.478) = 1.28\ V$$

Then

$$D_I = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(1.28)(3.9)(8.85x10^{-14})}{(1.6x10^{-19})(650x10^{-8})}$$

or

$$\underline{D_I = 4.25x10^{11}\ cm^{-2}}$$

12.34

(a) $\phi_{fn} = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) = 0.347\ V$

and

$$\phi_{ms} = \phi'_{ms} - \left(\chi' + \frac{E_g}{2e} - \phi_{fn}\right)$$

$$= 3.2 - (3.25 + 0.56 - 0.347)$$

or

$$\phi_{ms} = -0.263\ V$$

Also

$$x_{dT} = \left[\frac{4\in\phi_{fn}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

Now

$$|Q'_{SD}(\max)| = eN_d x_{dT}$$

$$= (1.6x10^{-19})(10^{16})(0.30x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 4.8x10^{-8}\ C/cm^2$$

We have

$$Q'_{SS} = (5x10^{11})(1.6x10^{-19}) = 8x10^{-8}\ C/cm^2$$

Now

$$V_T = -(|Q'_{SD}(\max)| + Q'_{SS})\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} - 2\phi_{fn}$$

$$= \frac{-(4.8x10^{-8} + 8x10^{-8})(750x10^{-8})}{(3.9)(8.85x10^{-14})}$$

$$-0.263 - 2(0.347)$$

which becomes

$$\underline{V_T = -3.74\ V}$$

(b)

We want $V_T = -0.50\ V$. Need to shift V_T in the positive direction which means we need to add acceptor atoms.

So

$$\Delta V_T = -0.50 - (-3.74) = 3.24\ V$$

Now

$$D_I = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(3.24)(3.9)(8.85x10^{-14})}{(1.6x10^{-19})(750x10^{-8})}$$

or

$$\underline{D_I = 9.32x10^{11}\ cm^{-2}}$$

12.35

(a) $\phi_{fp} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.288)}{(1.6x10^{-19})(10^{15})}\right]^{1/2}$$

or

$$x_{dT} = 0.863\ \mu m$$

Now

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$

$$= (1.6x10^{-19})(10^{15})(0.863x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.38x10^{-8}\ C/cm^2$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{750x10^{-8}}$$

or

$$C_{ox} = 4.6x10^{-8}\ F/cm^2$$

Then

$$V_T = V_{FB} + 2\phi_{fp} + \frac{|Q'_{SD}(\max)|}{C_{ox}}$$

$$= -1.50 + 2(0.288) + \frac{1.38x10^{-8}}{4.6x10^{-8}}$$

or

$$\underline{V_T = -0.624\ V}$$

(b)

Want $V_T = +0.90\ V$, which is a positive shift and we must add acceptor atoms.

$$\Delta V_T = 0.90 - (-0.624) = 1.52\ V$$

Then

$$D_I = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(1.52)(4.6x10^{-8})}{1.6x10^{-19}}$$

or

$$\underline{D_I = 4.37x10^{11}\ cm^{-2}}$$

(c)
With an applied substrate voltage,

$$\Delta V_T = \frac{\sqrt{2e \in N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

$$= \frac{\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(10^{15}\right)\right]^{1/2}}{4.6x10^{-8}} \times\left[\sqrt{2(0.288)+2} - \sqrt{2(0.288)}\right]$$

or

$$\underline{\Delta V_T = +0.335\ V}$$

Then the threshold voltage is

$$V_T = +0.90 + 0.335$$

or

$$\underline{V_T = 1.24\ V}$$

12.36
The total space charge width is greater than x_I, so from chapter 11,

$$\Delta V_T = \frac{\sqrt{2e \in N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{14}}{1.5x10^{10}}\right) = 0.228\ V$$

and

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{500x10^{-8}}$$

or

$$C_{ox} = 6.90x10^{-8}\ F / cm^2$$

Then

$$\Delta V_T = \frac{\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(10^{14}\right)\right]^{1/2}}{6.90x10^{-8}} \times\left[\sqrt{2(0.228)+V_{SB}} - \sqrt{2(0.228)}\right]$$

or

$$\Delta V_T = 0.0834\left[\sqrt{0.456 + V_{SB}} - \sqrt{0.456}\right]$$

Then

$V_{SB}(V)$	$\Delta V_T(V)$
1	0.0443
3	0.0987
5	0.399

11.37

(a) $\phi_{fn} = (0.0259)\ln\left(\frac{10^{17}}{1.5x10^{10}}\right) = 0.407\ V$

and

$$x_{dT} = \left[\frac{4(11.7)\left(8.85x10^{-14}\right)(0.407)}{\left(1.6x10^{-19}\right)\left(10^{17}\right)}\right]^{1/2}$$

or

$$x_{dT} = 1.026x10^{-5}\ cm$$

n^+ poly on $n \Rightarrow \phi_{ms} = -0.32\ V$

We have

$$\left|Q'_{SD}(\max)\right| = \left(1.6x10^{-19}\right)\left(10^{17}\right)\left(1.026x10^{-5}\right)$$

or

$$\left|Q'_{SD}(\max)\right| = 1.64x10^{-7}\ C / cm^2$$

Now

$$V_{TP} = \left[-1.64x10^{-7} - \left(1.6x10^{-19}\right)\left(5x10^{10}\right)\right] \times\frac{\left(80x10^{-8}\right)}{(3.9)\left(8.85x10^{-14}\right)} - 0.32 - 2(0.407)$$

or

$\underline{V_{TP} = -1.53\ V}$, Enhancement PMOS

(b)
For $V_T = 0$, shift threshold voltage in positive direction, so implant acceptor ions

$$\Delta V_T = \frac{eD_I}{C_{ox}} \Rightarrow D_I = \frac{(\Delta V_T)C_{ox}}{e}$$

so

$$D_I = \frac{(1.53)(3.9)\left(8.85x10^{-14}\right)}{\left(80x10^{-8}\right)\left(1.6x10^{-19}\right)}$$

or

$$\underline{D_I = 4.13x10^{12}\ cm^{-2}}$$

12.38
Shift in negative direction means implanting donor ions. We have

$$\Delta V_T = \frac{eD_I}{C_{ox}}$$

where

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{400x10^{-8}}$$

or

$$C_{ox} = 8.63x10^{-8}\ F / cm^2$$

Now

$$D_I = \frac{C_{ox}\left(\Delta V_T\right)}{e} = \frac{\left(8.63x10^{-8}\right)(1.4)}{1.6x10^{-19}}$$

or

$$\underline{D_I = 7.55x10^{11}\ cm^{-2}}$$

12.39

The areal density of generated holes is

$$= \left(8x10^{12}\right)\left(10^{5}\right)\left(750x10^{-8}\right) = 6x10^{12}\ cm^{-2}$$

The equivalent surface charge trapped is

$$= (0.10)\left(6x10^{12}\right) = 6x10^{11}\ cm^{-2}$$

Then

$$\Delta V_T = -\frac{Q'_{SS}}{C_{ox}} = -\frac{\left(6x10^{11}\right)\left(1.6x10^{-19}\right)}{(3.9)\left(8.85x10^{-14}\right)}\left(750x10^{-8}\right)$$

or

$$\underline{\Delta V_T = -2.09\ V}$$

12.40

The areal density of generated holes is $6x10^{12}\ cm^{-2}$. Now

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{750x10^{-8}}$$

or

$$C_{ox} = 4.6x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = -\frac{Q'_{SS}}{C_{ox}} = -\frac{\left(6x10^{12}\right)(x)\left(1.6x10^{-19}\right)}{4.6x10^{-8}}$$

For $\Delta V_T = -0.50\ V$

Where the parameter x is the maximum fraction of holes that can be trapped. Then we find

$$\underline{x = 0.024 \Rightarrow 2.4\%}$$

12.41

We have the areal density of generated holes as $= (g)(\gamma)\left(t_{ox}\right)$ where g is the generation rate and γ is the dose. The equivalent charge trapped is $= xg\gamma t_{ox}$.

Then

$$\Delta V_T = -\frac{Q'_{SS}}{C_{ox}} = -\frac{exg\gamma t_{ox}}{\left(\epsilon_{ox}/t_{ox}\right)} = -exg\gamma\left(t_{ox}\right)^2$$

so that

$$\underline{\Delta V_T \propto -\left(t_{ox}\right)^2}$$

Chapter 13

Problem Solutions

13.1
Sketch

13.2
Sketch

13.3
p-channel JFET – Silicon
(a)

$$V_{PO} = \frac{ea^2 N_a}{2\in} = \frac{(1.6x10^{-19})(0.5x10^{-4})^2(3x10^{16})}{2(11.7)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 5.79\ V}$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(3x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.884\ V$$

so

$$V_P = V_{PO} - V_{bi} = 5.79 - 0.884$$

or

$$\underline{V_P = 4.91\ V}$$

(b)

$$a - h = a - \left[\frac{2\in(V_{bi} - V_{DS} + V_{GS})}{eN_a}\right]^{1/2}$$

(i)
For $V_{GS} = 1\ V$, $V_{DS} = 0$
Then

$$a - h = 0.5x10^{-4} - \left[\frac{2(11.7)(8.85x10^{-14})(0.884 + 1 - V_{DS})}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$a - h = 0.5x10^{-4} - \left[(4.31x10^{-10})(1.884 - V_{DS})\right]^{1/2}$$

or

$$\underline{a - h = 0.215\ \mu m}$$

(ii) For $V_{GS} = 1\ V$, $V_{DS} = -2.5\ V$

$$\underline{a - h = 0.0653\ \mu m}$$

(iii) For $V_{GS} = 1\ V$, $V_{DS} = -5\ V$

$$\underline{a - h = -0.045\ \mu m}$$

which implies no undepleted region.

13.4
p-channel JFET – GaAs
(a)

$$V_{PO} = \frac{2a^2 N_a}{2\in} = \frac{2(0.5x10^{-4})^2(3x10^{16})}{2(13.1)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 5.18\ V}$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(3x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$$V_{bi} = 1.35\ V$$

so

$$V_P = V_{PO} - V_{bi} = 5.18 - 1.35$$

or

$$\underline{V_P = 3.83\ V}$$

(b)

$$a - h = a - \left[\frac{2\in(V_{bi} - V_{DS} + V_{GS})}{eN_a}\right]^{1/2}$$

(i) For $V_{GS} = 1\ V$, $V_{DS} = 0$
Then

$$a - h = 0.5x10^{-4} - \left[\frac{2(13.1)(8.85x10^{-14})(1.35 + 1 - V_{DS})}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$a - h = 0.5x10^{-4} - \left[(4.83x10^{-10})(2.35 - V_{DS})\right]^{1/2}$$

which yields

$$\underline{a - h = 0.163\ \mu m}$$

(ii) For $V_{GS} = 1\ V$, $V_{DS} = -2.5\ V$

$$\underline{a - h = 0.016\ \mu m}$$

(iii) For $V_{GS} = 1\ V$, $V_{DS} = -5\ V$

$$\underline{a - h = -0.096\ \mu m}$$

which implies no undepleted region.

13.5

(a) $V_{PO} = \dfrac{ea^2 N_d}{2 \in}$

$$= \frac{(1.6x10^{-19})(0.5x10^{-4})^2(8x10^{16})}{2(11.7)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 15.5\ V}$$

(b)

$$a - h = a - \left[\frac{2 \in (V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$$

so

$$0.2x10^{-4} = 0.5x10^{-4} - \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi} - V_{GS})}{(1.6x10^{-19})(8x10^{16})}\right]^{1/2}$$

or

$$9x10^{-10} = 1.618x10^{-10}(V_{bi} - V_{GS})$$

which yields

$$V_{bi} - V_{GS} = 5.56\ V$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{18})(8x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.896\ V$$

Then

$$V_{GS} = 0.897 - 5.56 \Rightarrow \underline{V_{GS} = -4.66\ V}$$

13.6

For GaAs:

(a)

$$V_{PO} = \frac{ea^2 N_d}{2 \in}$$

$$= \frac{(1.6x10^{-19})(0.5x10^{-4})^2(8x10^{16})}{2(13.1)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 13.8\ V}$$

(b)

$$a - h = a - \left[\frac{2 \in (V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$$

$$0.2x10^{-4} = 0.5x10^{-4} - \left[\frac{2(13.1)(8.85x10^{-14})(V_{bi} - V_{GS})}{(1.6x10^{-19})(8x10^{16})}\right]^{1/2}$$

which can be written as

$$9x10^{-10} = 1.811x10^{-10}(V_{bi} - V_{GS})$$

or

$$V_{bi} - V_{GS} = 4.97\ V$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{18})(8x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$$V_{bi} = 1.36\ V$$

Then

$$V_{GS} = V_{bi} - 4.97 = 1.36 - 4.97$$

or

$$\underline{V_{GS} = -3.61\ V}$$

13.7

(a) $V_{PO} = \dfrac{ea^2 N_a}{2 \in}$

$$= \frac{(1.6x10^{-19})(0.3x10^{-4})^2(3x10^{16})}{2(13.1)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 1.863\ V}$$

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(3x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$$V_{bi} = 1.352\ V$$

Then

$$V_P = V_{PO} - V_{bi} = 1.863 - 1.352$$

or

$$\underline{V_P = 0.511\ V}$$

(b) (i)

$$a - h = a - \left[\frac{2 \in (V_{bi} + V_{GS})}{eN_a}\right]^{1/2}$$

or

$$a - h = (0.3x10^{-4}) - \left[\frac{2(13.1)(8.85x10^{-14})(1.352)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

which yields

$$\underline{a-h = 4.45x10^{-6}\ cm}$$

(ii)

$$a-h = (0.3x10^{-4}) - \left[\frac{2(13.1)(8.85x10^{-14})(1.351+1)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

which yields

$$\underline{a-h = -3.7x10^{-6}\ cm}$$

which implies no undepleted region.

13.8

(a) n-channel JFET – Silicon

$$V_{PO} = \frac{ea^2 N_d}{2\in} = \frac{(1.6x10^{-19})(0.35x10^{-4})^2(4x10^{16})}{2(11.7)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 3.79\ V}$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(4x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.892\ V$$

so that

$$V_P = V_{bi} - V_{PO} = 0.892 - 3.79$$

or

$$\underline{V_P = -2.90\ V}$$

(b)

$$a-h = a - \left[\frac{2\in(V_{bi}+V_{DS}-V_{GS})}{eN_d}\right]^{1/2}$$

We have

$$a-h = 0.35x10^{-4} - \left[\frac{2(11.7)(8.85x10^{-14})(0.892+V_{DS}-V_{GS})}{(1.6x10^{-19})(4x10^{16})}\right]^{1/2}$$

or

$$a-h = 0.35x10^{-4} - \left[(3.24x10^{-10})(0.892+V_{DS}-V_{GS})\right]^{1/2}$$

(i) For $V_{GS} = 0$, $V_{DS} = 1\ V$,

$$\underline{a-h = 0.102\ \mu m}$$

(ii) For $V_{GS} = -1\ V$, $V_{DS} = 1\ V$,

$$\underline{a-h = 0.044\ \mu m}$$

(iii) For $V_{GS} = -1\ V$, $V_{DS} = 2\ V$,

$$\underline{a-h = -0.0051\ \mu m}$$

which implies no undepleted region

13.9

$$V_{bi} = (0.0259)\ln\left(\frac{(5x10^{18})(4x10^{16})}{(1.8x10^{6})^2}\right)$$

or

$$V_{bi} = 1.359\ V$$

$$a-h = a - \left[\frac{2\in(V_{bi}+V_{DS}-V_{GS})}{eN_D}\right]^{1/2}$$

or

$$a-h = 0.35x10^{-4} - \left[\frac{2(13.1)(8.85x10^{-14})(1.359+V_{DS}-V_{GS})}{(1.6x10^{-19})(4x10^{16})}\right]^{1/2}$$

We want $a-h = 0.05x10^{-4}\ cm$,

Then

$$0.05x10^{-4} = 0.35x10^{-4} - \left[(3.623x10^{-10})(1.359+V_{DS}-V_{GS})\right]^{1/2}$$

(a)

For $V_{DS} = 0$, we find

$$\underline{V_{GS} = -1.125\ V}$$

(b)

For $V_{DS} = 1\ V$, we find

$$\underline{V_{GS} = -0.125\ V}$$

13.10

(a)

$$I_{P1} = \frac{\mu_n(eN_d)^2 Wa^3}{6\in L}$$

$$= \frac{(1000)[(1.6x10^{-19})(10^{16})]^2}{6(11.7)(8.85x10^{-14})} \times \frac{(400x10^{-4})(0.5x10^{-4})^3}{(20x10^{-4})}$$

or

$$\underline{I_{P1} = 1.03\ mA}$$

(b)

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$= \left[\frac{(1.6x10^{-19})(0.5x10^{-4})^2(10^{16})}{2(11.7)(8.85x10^{-14})} \right]$$

or

$$\underline{V_{PO} = 1.93\ V}$$

Also

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{19})(10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.874\ V$$

Now

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$
$$= 1.93 - 0.874 + V_{GS}$$

or

$$V_{DS}(sat) = 1.06 + V_{GS}$$

We have

$$V_P = V_{bi} - V_{PO} = 0.874 - 1.93$$

or

$$\underline{V_P = -1.06\ V}$$

Then

(i) $V_{GS} = 0 \Rightarrow \underline{V_{DS}(sat) = 1.06\ V}$

(ii) $V_{GS} = \frac{1}{4}V_P = -0.265\ V \Rightarrow$
$\underline{V_{DS}(sat) = 0.795\ V}$

(iii) $V_{GS} = \frac{1}{2}V_P = -0.53\ V \Rightarrow$
$\underline{V_{DS}(sat) = 0.53\ V}$

(iv) $V_{GS} = \frac{3}{4}V_P = -0.795\ V \Rightarrow$
$\underline{V_{DS}(sat) = 0.265\ V}$

(c)

$$I_{D1}(sat)$$
$$= I_{P1}\left[1 - 3\left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)\left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)\right]$$
$$= 1.03\left[1 - 3\left(\frac{0.874 - V_{GS}}{1.93}\right)\right.$$
$$\left.\times\left(1 - \frac{2}{3}\sqrt{\frac{0.874 - V_{GS}}{1.93}}\right)\right]$$

(i) For $V_{GS} = 0 \Rightarrow \underline{I_{D1}(sat) = 0.258\ mA}$

(ii) For $V_{GS} = -0.265\ V \Rightarrow$
$\underline{I_{D1}(sat) = 0.140\ mA}$

(iii) For $V_{GS} = -0.53\ V \Rightarrow$
$\underline{I_{D1}(sat) = 0.061\ mA}$

(iv) For $V_{GS} = -0.795\ V \Rightarrow$
$\underline{I_{D1}(sat) = 0.0145\ mA}$

13.11

$$g_d = G_{O1}\left[1 - \left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)^{1/2}\right]$$

where

$$G_{O1} = \frac{3I_{P1}}{V_{PO}} = \frac{3(1.03x10^{-3})}{1.93} = 1.60x10^{-3}$$

or

$$G_{O1} = 1.60\ mS$$

Then

$\underline{V_{GS}}$	$\underline{[(V_{bi} - V_{GS})/V_{PO}]}$	$\underline{g_d(mS)}$
0	0.453	0.523
-0.265	0.590	0.371
-0.53	0.727	0.236
-0.795	0.945	0.112
-1.06	1.0	0

13.12

n-channel JFET – GaAs

(a)

$$G_{O1} = \frac{e\mu_n N_d Wa}{L}$$
$$= \frac{(1.6x10^{-19})(8000)(2x10^{16})(30x10^{-4})(0.35x10^{-4})}{10x10^{-4}}$$

or

$$\underline{G_{O1} = 2.69x10^{-3}\ S}$$

(b)

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$

We have

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$
$$= \frac{(1.6x10^{-19})(0.35x10^{-4})^2(2x10^{16})}{2(13.1)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 1.69\ V}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(2x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$$V_{bi} = 1.34\ V$$

Then

$$V_P = V_{bi} - V_{PO} = 1.34 - 1.69$$

or

$$\underline{V_P = -0.35\ V}$$

We then obtain

$$V_{DS}(sat) = 1.69 - (1.34 - V_{GS}) = 0.35 + V_{GS}$$

For $V_{GS} = 0 \Rightarrow$ $\underline{V_{DS}(sat) = 0.35\ V}$

For $V_{GS} = \dfrac{1}{2}V_P = -0.175\ V \Rightarrow$

$$\underline{V_{DS}(sat) = 0.175\ V}$$

(c)

$$I_{D1}(sat)$$

$$= I_{P1}\left[1 - 3\left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)\left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)\right]$$

where

$$I_{P1} = \frac{\mu_n (eN_d)^2 Wa^3}{6 \in L}$$

$$= \frac{(8000)\left[(1.6x10^{-19})(2x10^{16})\right]^2}{6(13.1)(8.85x10^{-14})} \times \frac{(30x10^{-4})(0.35x10^{-4})^3}{(10x10^{-4})}$$

or

$$\underline{I_{P1} = 1.51\ mA}$$

Then

$$I_{D1}(sat) = 1.51\left[1 - 3\left(\frac{1.34 - V_{GS}}{1.69}\right) \times \left(1 - \frac{2}{3}\sqrt{\frac{1.34 - V_{GS}}{1.69}}\right)\right](mA)$$

For

$V_{GS} = 0 \Rightarrow$ $\underline{I_{D1}(sat) = 0.0504\ mA}$

and for

$V_{GS} = -0.175\ V \Rightarrow$ $\underline{I_{D1}(sat) = 0.0123\ mA}$

13.13

$$g_{mS} = \frac{3I_{P1}}{V_{PO}}\left(1 - \sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)$$

We have

$$I_{P1} = 1.03\ mA,\ V_{PO} = 1.93\ V,\ V_{bi} = 0.874\ V$$

The maximum transconductance occurs when $V_{GS} = 0$

Then

$$g_{mS}(\max) = \frac{3(1.03)}{1.93}\left(1 - \sqrt{\frac{0.874}{1.93}}\right)$$

or

$$g_{mS} = 0.524\ mS$$

For $W = 400\ \mu m$,

We have

$$g_{mS}(\max) = \frac{0.524\ mS}{400x10^{-4}\ cm}$$

or

$$\underline{g_{mS} = 13.1\ mS/cm = 1.31\ mS/mm}$$

13.14

The maximum transconductance occurs for $V_{GS} = 0$, so we have

(a)

$$g_{mS}(\max) = \frac{3I_{P1}}{V_{PO}}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right) = G_{O1}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

We found

$$G_{O1} = 2.69\ mS,\ V_{bi} = 1.34\ V,\ V_{PO} = 1.69\ V$$

Then

$$g_{mS}(\max) = (2.69)\left(1 - \sqrt{\frac{1.34}{1.69}}\right)$$

or

$$\underline{g_{mS}(\max) = 0.295\ mS}$$

This is for a channel length of $L = 10\ \mu m$.

(b)

If the channel length is reduced to $L = 2\ \mu m$, then

$$g_{mS}(\max) = (0.295)\left(\frac{10}{2}\right) \Rightarrow$$

$$\underline{g_{mS}(\max) = 1.48\ mS}$$

13.15
n-channel MESFET – GaAs
(a)

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(0.5x10^{-4}\right)^2\left(1.5x10^{16}\right)}{2(13.1)\left(8.85x10^{-14}\right)}$$

or

$$V_{PO} = 2.59\ V$$

Now

$$V_{bi} = \phi_{Bn} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right) = (0.0259)\ln\left(\frac{4.7x10^{17}}{1.5x10^{16}}\right)$$

or

$$\phi_n = 0.0892\ V$$

so that

$$V_{bi} = 0.90 - 0.0892 = 0.811\ V$$

Then

$$V_T = V_{bi} - V_{PO} = 0.811 - 2.59$$

or

$$\underline{V_T = -1.78\ V}$$

(b)
If $V_T < 0$ for an n-channel device, the device is a depletion mode MESFET.

13.16
n-channel MESFET – GaAs
(a)
We want $V_T = +0.10\ V$
Then

$$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$$

so

$$V_T = 0.10 = 0.89 - V_t \ln\left(\frac{N_C}{N_d}\right) - \frac{ea^2 N_d}{2\in}$$

which can be written as

$$(0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right) + \frac{\left(1.6x10^{-19}\right)\left(0.35x10^{-4}\right)^2 N_d}{2(13.1)\left(8.85x10^{-14}\right)} = 0.89 - 0.10$$

or

$$(0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right) + \left(8.45x10^{17}\right)N_d = 0.79$$

By trial and error

$$\underline{N_d = 8.1x10^{15}\ cm^{-3}}$$

(b)
At $T = 400K$,

$$N_C(400) = N_C(300)\cdot\left(\frac{400}{300}\right)^{3/2}$$

$$= \left(4.7x10^{17}\right)(1.54)$$

or

$$N_C(400) = 7.24x10^{17}\ cm^{-3}$$

Also

$$V_t = (0.0259)\left(\frac{400}{300}\right) = 0.03453$$

Then

$$V_T = 0.89 - (0.03453)\ln\left(\frac{7.24x10^{17}}{8.1x10^{15}}\right) - \left(8.45x10^{-17}\right)\left(8.1x10^{15}\right)$$

which becomes

$$\underline{V_T = +0.051\ V}$$

13.17
We have

$$a - h = a - \left[\frac{2\in\left(V_{bi} + V_{DS} - V_{GS}\right)}{eN_d}\right]^{1/2}$$

where

$$V_{bi} = \phi_{Bn} - \phi_n$$

Now

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{5x10^{16}}\right) = 0.058\ V$$

Then

$$V_{bi} = 0.80 - 0.058 = 0.742\ V$$

For $V_{GS} = 0.5\ V$,

$$a - h = \left(0.8x10^{-4}\right) - \left[\frac{2(13.1)\left(8.85x10^{-14}\right)\left(0.742 + V_{DS} - 0.5\right)}{\left(1.6x10^{-19}\right)\left(5x10^{16}\right)}\right]^{1/2}$$

or

$$a - h = \left(0.80x10^{-4}\right) - \left[\left(2.898x10^{-10}\right)\left(0.242 + V_{DS}\right)\right]^{1/2}$$

Then

$V_{DS}(V)$	$a-h$ (μm)
0	0.716
1	0.610
2	0.545
5	0.410

13.18

$$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$$

We want

$$V_T = 0 \Rightarrow \phi_n + V_{PO} = \phi_{Bn}$$

Device 1: $N_d = 3x10^{16}\ cm^{-3}$

Then

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{3x10^{16}}\right) = 0.0713\ V$$

so that

$$V_{PO} = 0.89 - 0.0713 = 0.8187\ V$$

Now

$$V_{PO} = \frac{ea^2 N_d}{2\in} \Rightarrow a = \left[\frac{2\in V_{PO}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.8187)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$\underline{a = 0.199\ \mu m}$$

Device 2: $N_d = 3x10^{17}\ cm^{-3}$

Then

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{3x10^{17}}\right) = 0.0116\ V$$

so that

$$V_{PO} = 0.89 - 0.0116 = 0.8784\ V$$

Now

$$a = \left[\frac{2\in V_{PO}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.8784)}{(1.6x10^{-19})(3x10^{17})}\right]^{1/2}$$

or

$$\underline{a = 0.0651\ \mu m}$$

13.19

$$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$$

We want $V_T = 0.5\ V$, so

$$0.5 = 0.85 - \phi_n - V_{PO}$$

Now

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right)$$

and

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$= \frac{(1.6x10^{-19})(0.25x10^{-4})^2 N_d}{2(13.1)(8.85x10^{-14})}$$

or

$$V_{PO} = (4.31x10^{-17})N_d$$

Then

$$0.5 = 0.85 - (0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right) - (4.31x10^{-17})N_d$$

By trial and error, we find

$$\underline{N_d = 5.45x10^{15}\ cm^{-3}}$$

13.20

n-channel MESFET – silicon

(a) For a gold contact, $\phi_{Bn} = 0.82\ V$.

We find

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ V$$

and

$$V_{bi} = \phi_{Bn} - \phi_n = 0.82 - 0.206 = 0.614\ V$$

With $V_{DS} = 0$, $V_{GS} = 0.35\ V$

We find

$$a - h = 0.075x10^{-4}$$

$$= a - \left[\frac{2\in(V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$$

so that

$$a = 0.075x10^{-4} + \left[\frac{2(11.7)(8.85x10^{-14})(0.614 - 0.35)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$a = 0.26\ \mu m$$

Now

$$V_T = V_{bi} - V_{PO} = 0.614 - \frac{ea^2 N_d}{2\in}$$

or

$$V_T = 0.614 - \frac{\left(1.6x10^{-19}\right)\left(0.26x10^{-4}\right)^2\left(10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

We obtain

$$\underline{V_T = 0.092\ V}$$

(b)

$$V_{DS}(sat) = V_{PO} - \left(V_{bi} - V_{GS}\right)$$
$$= \left(V_{bi} - V_T\right) - \left(V_{bi} - V_{GS}\right) = V_{GS} - V_T$$

Now

$$V_{DS}(sat) = 0.35 - 0.092$$

or

$$\underline{V_{DS}(sat) = 0.258\ V}$$

13.21

(a) n-channel MESFET - silicon

$$V_{bi} = \phi_{Bn} - \phi_n$$

and

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{2x10^{16}}\right) = 0.188\ V$$

so

$$V_{bi} = 0.80 - 0.188 \Rightarrow \underline{V_{bi} = 0.612\ V}$$

Now

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$
$$= \frac{\left(1.6x10^{-19}\right)\left(0.4x10^{-4}\right)^2\left(2x10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

or

$$\underline{V_{PO} = 2.47\ V}$$

We find

$$V_T = V_{bi} - V_{PO} = 0.612 - 2.47$$

or

$$\underline{V_T = -1.86\ V}$$

and

$$V_{DS}(sat) = V_{PO} - \left(V_{bi} - V_{GS}\right)$$
$$= 2.47 - (0.612 - (-1))$$

or

$$\underline{V_{DS}(sat) = 0.858\ V}$$

(b)

For $V_{PO} = 4.5\ V$, additional donor atoms must be added.

We have

$$V_{PO} = \frac{ea^2 N_d}{2\in} \Rightarrow N_d = \frac{2\in V_{PO}}{ea^2}$$

so that

$$N_d = \frac{2(11.7)\left(8.85x10^{-14}\right)(4.5)}{\left(1.6x10^{-19}\right)\left(0.4x10^{-4}\right)^2}$$

or

$$\underline{N_d = 3.64x10^{16}\ cm^{-3}}$$

which means that

$$\Delta N_d = 3.64x10^{16} - 2x10^{16}$$

or

$$\underline{\Delta N_d = 1.64x10^{16}\ cm^{-3}}$$

Donors must be added

Then

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{3.64x10^{16}}\right) = 0.172\ V$$

so that

$$V_{bi} = 0.80 - 0.172 = 0.628\ V$$

We find

$$V_T = V_{bi} - V_{PO} = 0.628 - 4.5$$

or

$$\underline{V_T = -3.87\ V}$$

Also

$$V_{DS}(sat) = V_{PO} - \left(V_{bi} - V_{GS}\right)$$
$$= 4.5 - (0.628 - (-1))$$

or

$$\underline{V_{DS}(sat) = 2.87\ V}$$

13.22

(a) $$k_n = \frac{\mu_n \in W}{2aL}$$
$$= \frac{(7800)(13.1)\left(8.85x10^{-14}\right)\left(20x10^{-4}\right)}{2\left(0.30x10^{-4}\right)\left(1.2x10^{-4}\right)}$$

or

$$\underline{k_n = 2.51\ mA/V^2}$$

(b)

$$V_{DS}(sat) = V_{PO} - \left(V_{bi} - V_{GS}\right) = V_{GS} - V_T$$

So for $V_{GS} = 1.5V_T \Rightarrow V_{DS}(sat) = (0.5)(0.12)$

Or

$$\underline{V_{DS}(sat) = 0.06\ V}$$

and for $V_{GS} = 2V_T \Rightarrow V_{DS}(sat) = (1)(0.12)$

or

$$\underline{V_{DS}(sat) = 0.12\ V}$$

(c)

$$I_{D1}(sat) = k_n\left(V_{GS} - V_T\right)^2$$

For $V_{GS} = 1.5V_T \Rightarrow I_{D1}(sat) = (2.51)(0.06)^2$
Or
$$\underline{I_{D1}(sat) = 9.04\ \mu A}$$
and for $V_{GS} = 2V_T \Rightarrow I_{D1}(sat) = (2.51)(0.12)^2$
or
$$\underline{I_{D1}(sat) = 36.1\ \mu A}$$

13.23
(a) We have
$$g_m = 2k_n(V_{GS} - V_T)$$
so that
$$1.75x10^{-3} = 2k_n(0.50 - 0.25)$$
which gives
$$k_n = 3.5x10^{-3}\ A/V^2 = \frac{\mu_n \in W}{2aL}$$
We obtain
$$W = \frac{(3.5x10^{-3})(2)(0.35x10^{-4})(10^{-4})}{(8000)(13.1)(8.85x10^{-14})}$$
or
$$\underline{W = 26.4\ \mu m}$$
(b)
$$I_{D1}(sat) = k_n(V_{GS} - V_T)^2$$
For $V_{GS} = 0.4\ V$,
$$I_{D1}(sat) = (3.5x10^{-3})(0.4 - 0.25)^2$$
or
$$\underline{I_{D1}(sat) = 78.8\ \mu A}$$
For $V_{GS} = 0.65\ V$,
$$I_{D1}(sat) = (3.5x10^{-3})(0.65 - 0.25)^2$$
or
$$\underline{I_{D1}(sat) = 0.56\ mA}$$

13.24
Computer plot

13.25
Computer plot

13.26

We have $L' = L - \frac{1}{2}\Delta L$

Or
$$\frac{L'}{L} = 0.90 = 1 - \frac{1}{2}\cdot\frac{\Delta L}{L}$$
We have
$$\Delta L = \left[\frac{2\in(V_{DS} - V_{DS}(sat))}{eN_d}\right]^{1/2}$$
and
For $V_{GS} = 0$, $V_{DS}(sat) = V_{PO} - V_{bi}$
We find
$$V_{PO} = \frac{ea^2N_d}{2\in}$$
$$= \frac{(1.6x10^{-19})(0.4x10^{-4})^2(3x10^{16})}{2(11.7)(8.85x10^{-14})}$$
or
$$V_{PO} = 3.71\ V$$
and
$$V_{bi} = (0.0259)\ln\left[\frac{(10^{19})(3x10^{16})}{(1.5x10^{10})^2}\right]$$
or
$$V_{bi} = 0.902\ V$$
so that
$$V_{DS}(sat) = 3.71 - 0.902 = 2.81\ V$$
Then
$$\Delta L = \left[\frac{2(11.7)(8.85x10^{-14})(5 - 2.81)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$
or
$$\Delta L = 0.307\ \mu m$$
Now
$$\frac{L'}{L} = 0.90 = 1 - \frac{1}{2}\cdot\frac{\Delta L}{L}$$
or
$$\frac{1}{2}\cdot\frac{\Delta L}{L} = 1 - 0.9 = 0.10$$
so
$$L = \frac{\Delta L}{2(0.10)} = \frac{0.307x10^{-4}}{2(0.10)}$$
or
$$\underline{L = 1.54\ \mu m}$$

13.27

We have that $I'_{D1} = I_{D1}\left(\frac{L}{L-(1/2)\Delta L}\right)$

Assuming that we are in the saturation region, then $I'_{D1} = I'_{D1}(sat)$ and $I_{D1} = I_{D1}(sat)$. We can write

$$I'_{D1}(sat) = I_{D1}(sat)\cdot\frac{1}{1-\frac{1}{2}\cdot\frac{\Delta L}{L}}$$

If $\Delta L << L$, then

$$I'_{D1}(sat) = I_{D1}(sat)\left[1+\frac{1}{2}\cdot\frac{\Delta L}{L}\right]$$

We have that

$$\Delta L = \left[\frac{2\in\left(V_{DS}-V_{DS}(sat)\right)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2\in V_{DS}}{eN_d}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

which can be written as

$$\Delta L = V_{DS}\left[\frac{2\in}{eN_dV_{DS}}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

If we write

$$I'_{D1}(sat) = I_{D1}(sat)(1+\lambda V_{DS})$$

then by comparing equations, we have

$$\lambda = \frac{1}{2L}\left[\frac{2\in}{eN_dV_{DS}}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

The parameter is not independent of V_{DS}. Define

$x = \frac{V_{DS}}{V_{DS}(sat)}$ and consider the function

$f = \frac{1}{x}\left(1-\frac{1}{x}\right)$ which is directly proportional to λ. We find that

x	$f(x)$
1..5	0.222
1.75	0.245
2.0	0.250
2.25	0.247
2.50	0.240
2.75	0.231
3.0	0.222

So that λ is nearly a constant.

13.28

(a) Saturation occurs when $E = 1x10^4\ V/cm$

As a first approximation, let

$$E = \frac{V_{DS}}{L}$$

Then

$$V_{DS} = E\cdot L = \left(1x10^4\right)\left(2x10^{-4}\right)$$

or

$$\underline{V_{DS} = 2\ V}$$

(b)

We have that

$$h_2 = h_{sat} = \left[\frac{2\in\left(V_{bi}+V_{DS}-V_{GS}\right)}{eN_d}\right]^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5x10^{18}\right)\left(4x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.892\ V$$

For $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(0.892+2)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$

or

$$\underline{h_{sat} = 0.306\ \mu m}$$

(c)

We then find

$$I_{D1}(sat) = eN_dv_{sat}\left(a-h_{sat}\right)W$$

$$= \left(1.6x10^{-19}\right)\left(4x10^{16}\right)\left(10^7\right)(0.50-0.306)$$

$$\times\left(10^{-4}\right)\left(30x10^{-4}\right)$$

or

$$\underline{I_{D1}(sat) = 3.72\ mA}$$

(d)

For $V_{GS} = 0$, we have

$$I_{D1}(sat) = I_{P1}\left[1-3\left(\frac{V_{bi}}{V_{PO}}\right)\left(1-\frac{2}{3}\sqrt{\frac{V_{bi}}{V_{PO}}}\right)\right]$$

Now

$$I_{P1} = \frac{\mu_n\left(eN_d\right)^2Wa^3}{6\in L}$$

$$= \frac{(1000)\left[(1.6x10^{-19})(4x10^{16})\right]^2}{6(11.7)(8.85x10^{-14})} \times \frac{(30x10^{-4})(0.5x10^{-4})^3}{(2x10^{-4})}$$

or

$$I_{P1} = 12.4\ mA$$

Also

$$V_{PO} = \frac{ea^2 N_d}{2 \in} = \frac{(1.6x10^{-19})(0.5x10^{-4})^2(4x10^{16})}{2(11.7)(8.85x10^{-14})}$$

or

$$V_{PO} = 7.73\ V$$

Then

$$I_{D1}(sat) = 12.4\left[1 - 3\left(\frac{0.892}{7.73}\right)\left(1 - \frac{2}{3}\sqrt{\frac{0.892}{7.73}}\right)\right]$$

or

$$\underline{I_{D1}(sat) = 9.08\ mA}$$

13.29

(a) If $L = 1\ \mu m$, then saturation will occur when

$$V_{DS} = \mathrm{E} \cdot L = (10^4)(1x10^{-4}) = 1\ V$$

We find

$$h_2 = h_{sat} = \left[\frac{2 \in (V_{bi} + V_{DS} - V_{GS})}{eN_d}\right]^{1/2}$$

We have $V_{bi} = 0.892\ V$ and for $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)(8.85x10^{-14})(0.892 + 1)}{(1.6x10^{-19})(4x10^{16})}\right]^{1/2}$$

or

$$h_{sat} = 0.247\ \mu m$$

Then

$$I_{D1}(sat) = eN_d v_{sat}(a - h_{sat})W$$
$$= (1.6x10^{-19})(4x10^{16})(10^7)(0.50 - 0.247) \times (10^{-4})(30x10^{-4})$$

or

$$\underline{I_{D1}(sat) = 4.86\ mA}$$

If velocity saturation did not occur, then from the previous problem, we would have

$$I_{D1}(sat) = 9.08\left(\frac{2}{1}\right) \Rightarrow \underline{I_{D1}(sat) = 18.2\ mA}$$

(b)
If velocity saturation occurs, then the relation $I_{D1}(sat) \propto (1/L)$ does not apply.

13.30

(a)

$$v = \mu_n \mathrm{E} = (8000)(5x10^3) = 4x10^7\ cm/s$$

Then

$$t_d = \frac{L}{v} = \frac{2x10^{-4}}{4x10^7} \Rightarrow$$

or

$$\underline{t_d = 5\ ps}$$

(b)
Assume $v_{sat} = 10^7\ cm/s$
Then

$$t_d = \frac{L}{v_{sat}} = \frac{2x10^{-4}}{10^7} \Rightarrow$$
$$\underline{t_d = 20\ ps}$$

13.31

(a) $v = \mu_n \mathrm{E} = (1000)(10^4) = 10^7\ cm/s$

$$t_d = \frac{L}{v} = \frac{2x10^{-4}}{10^7} \Rightarrow \underline{t_d = 20\ ps}$$

(b)
For $v_{sat} = 10^7\ cm/s$,

$$t_d = \frac{L}{v_{sat}} = \frac{2x10^{-4}}{10^7} \Rightarrow \underline{t_d = 20\ ps}$$

13.32

The reverse-bias current is dominated by the generation current. We have

$$V_P = V_{bi} - V_{PO}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(3x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.884\ V$$

and

$$V_{PO} = \frac{ea^2 N_d}{2 \in}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(0.3x10^{-4}\right)^2\left(3x10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

or

$$V_{PO} = 2.09\ V$$

Then

$$V_P = 0.884 - 2.09 = -1.21 = V_{GS}$$

Now

$$x_n = \left[\frac{2 \in \left(V_{bi} + V_{DS} - V_{GS}\right)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)\left(0.884 + V_{DS} - (-1.21)\right)}{\left(1.6x10^{-19}\right)\left(3x10^{16}\right)}\right]^{1/2}$$

or

$$x_n = \left[\left(4.31x10^{-10}\right)\left(2.09 + V_{DS}\right)\right]^{1/2}$$

(a)
For $V_{DS} = 0 \Rightarrow,\ x_n = 0.30\ \mu m$
(b)
For $V_{DS} = 1\ V \Rightarrow,\ x_n = 0.365\ \mu m$
(c)
For $V_{DS} = 5\ V \Rightarrow\ x_n = 0.553\ \mu m$

The depletion region volume is

$$Vol = (a)\left(\frac{L}{2}\right)(W) + \left(x_n\right)(2a)(W)$$

$$= \left(0.3x10^{-4}\right)\left(\frac{2.4x10^{-4}}{2}\right)\left(30x10^{-4}\right) + \left(x_n\right)\left(0.6x10^{-4}\right)\left(30x10^{-4}\right)$$

or

$$Vol = 10.8x10^{-12} + x_n\left(18x10^{-8}\right)$$

(a)
For $V_{DS} = 0 \Rightarrow\ Vol = 1.62x10^{-11}\ cm^3$
(b)
For $V_{DS} = 1\ V \Rightarrow\ Vol = 1.74x10^{-11}\ cm^3$
(c)
For $V_{DS} = 5\ V \Rightarrow\ Vol = 2.08x10^{-11}\ cm^3$

The generation current is

$$I_{DG} = e\left(\frac{n_i}{2\tau_O}\right) \cdot Vol = \frac{\left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)}{2\left(5x10^{-8}\right)} \cdot Vol$$

or

$$I_{DG} = \left(2.4x10^{-2}\right) \cdot Vol$$

(a)
For $V_{DS} = 0 \Rightarrow\ \underline{I_{DG} = 0.39\ pA}$
(b)
For $V_{DS} = 1\ V \Rightarrow\ \underline{I_{DG} = 0.42\ pA}$
(c)
For $V_{DS} = 5\ V \Rightarrow\ \underline{I_{DG} = 0.50\ pA}$

13.33

(a) The ideal transconductance for $V_{GS} = 0$ is

$$g_{mS} = G_{O1}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

where

$$G_{O1} = \frac{e\mu_n N_d Wa}{L}$$

$$= \frac{\left(1.6x10^{-19}\right)(4500)\left(7x10^{16}\right)}{1.5x10^{-4}} \times \left(5x10^{-4}\right)\left(0.3x10^{-4}\right)$$

or

$$G_{O1} = 5.04\ mS$$

We find

$$V_{PO} = \frac{ea^2 N_d}{2 \in}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(0.3x10^{-4}\right)^2\left(7x10^{16}\right)}{2(13.1)\left(8.85x10^{-14}\right)}$$

or

$$V_{PO} = 4.35\ V$$

We have

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{7x10^{16}}\right) = 0.049\ V$$

so that

$$V_{bi} = \phi_{Bn} - \phi_n = 0.89 - 0.049 = 0.841\ V$$

Then

$$g_{mS} = 5.04\left(1 - \sqrt{\frac{0.841}{4.35}}\right)$$

or

$$\underline{g_{mS} = 2.82\ mS}$$

(b)
With a source resistance

$$g'_m = \frac{g_m}{1 + g_m r_s} \Rightarrow \frac{g'_m}{g_m} = \frac{1}{1 + g_m r_s}$$

For

$$\frac{g'_m}{g_m} = 0.80 = \frac{1}{1 + (2.82)r_s}$$

which yields

$$\underline{r_s = 88.7\ \Omega}$$

(c)

$$r_s = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{(e\mu_n n)(0.3x10^{-4})(5x10^{-4})}$$

so

$$L = (88.7)(1.6x10^{-19})(4500)(7x10^{16}) \times (0.3x10^{-4})(5x10^{-4})$$

or

$$\underline{L = 0.67\ \mu m}$$

13.34

$$f_T = \frac{g_m}{2\pi C_G}$$

where

$$C_G = \frac{\in WL}{a}$$

$$= \frac{(13.1)(8.85x10^{-14})(5x10^{-4})(1.5x10^{-4})}{0.3x10^{-4}}$$

or

$$C_G = 2.9x10^{-15}\ F$$

We must use g'_m, so we obtain

$$f_T = \frac{(2.82x10^{-3})(0.80)}{2\pi(2.9x10^{-15})} = 124\ GHz$$

We have

$$f_T = \frac{1}{2\pi\tau_C} \Rightarrow \tau_C = \frac{1}{2\pi f_T} = \frac{1}{2\pi(124x10^9)}$$

or

$$\tau_C = 1.28x10^{-12}\ s$$

The channel transit time is

$$t_t = \frac{1.5x10^{-4}}{10^7} = 1.5x10^{-11}\ s$$

The total time constant is

$$\tau = 1.5x10^{-11} + 1.28x10^{-12} = 1.63x10^{-11}\ s$$

so that

$$f_T = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.63x10^{-11})}$$

or

$$\underline{f_T = 9.76\ GHz}$$

13.35

(a) For a constant mobility

$$f_T = \frac{e\mu_n N_d a^2}{2\pi \in L^2}$$

$$= \frac{(1.6x10^{-19})(5500)(10^{17})(0.25x10^{-4})^2}{2\pi(13.1)(8.85x10^{-14})(10^{-4})^2}$$

or

$$\underline{f_T = 755\ GHz}$$

(b)

Saturation velocity model:

$$f_T = \frac{v_{sat}}{2\pi L}$$

Assuming $v_{sat} = 10^7\ cm/s$, we find

$$f_T = \frac{10^7}{2\pi(10^{-4})}$$

or

$$\underline{f_T = 15.9\ GHz}$$

13.36

(a) $$V_{off} = \phi_B - \frac{\Delta E_C}{e} - V_{P2}$$

where

$$V_{P2} = \frac{eN_d d_d^2}{2\in_N}$$

$$= \frac{(1.6x10^{-19})(3x10^{18})(350x10^{-8})^2}{2(12.2)(8.85x10^{-14})}$$

or

$$V_{P2} = 2.72\ V$$

Then

$$V_{off} = 0.89 - 0.24 - 2.72$$

or

$$\underline{V_{off} = -2.07\ V}$$

(b)

$$n_S = \frac{\in_N}{e(d+\Delta d)}(V_g - V_{off})$$

For $V_g = 0$, we have

$$n_S = \frac{(12.2)(8.85x10^{-14})}{(1.6x10^{-19})(350+80)\cdot 10^{-8}}(2.07)$$

or

$$\underline{n_S = 3.25x10^{12}\ cm^{-2}}$$

13.37

(a) We have

$$I_D(sat) = \frac{\in_N W}{(d + \Delta d)}\left(V_g - V_{off} - V_O\right)v_s$$

We find

$$\left(\frac{g_{mS}}{W}\right) = \frac{\partial}{\partial V_g}\left[\frac{I_D(sat)}{W}\right] = \frac{\in_N v_s}{(d + \Delta d)}$$

$$= \frac{(12.2)\left(8.85x10^{-14}\right)\left(2x10^7\right)}{(350 + 80)\cdot 10^{-8}} = 5.02 \frac{S}{cm}$$

or

$$\frac{g_{mS}}{W} = 502 \frac{mS}{mm}$$

(b)

At $V_g = 0$, we obtain

$$\frac{I_D(sat)}{W} = \frac{\in_N}{(d + \Delta d)}\left(-V_{off} - V_O\right)v_S$$

$$= \frac{(12.2)\left(8.85x10^{-14}\right)}{(350 + 80)\cdot 10^{-8}}(2.07 - 1)\left(2x10^7\right)$$

or

$$\frac{I_D(sat)}{W} = 5.37 \ A/cm = 537 \ mA/mm$$

13.38

$$V_{off} = \phi_B - \frac{\Delta E_C}{e} - V_{P2}$$

We want $V_{off} = -0.3\,V$, so

$$-0.30 = 0.85 - 0.22 - V_{P2}$$

or

$$V_{P2} = 0.93\,V = \frac{eN_d d_d^2}{2 \in_N}$$

We can then write

$$d_d^2 = \frac{2 \in_N V_{P2}}{eN_d}$$

$$= \frac{2(12.2)\left(8.85x10^{-14}\right)(0.93)}{\left(1.6x10^{-19}\right)\left(2x10^{18}\right)}$$

We then obtain

$$d_d = 2.51x10^{-6} \ cm = 251 \ \mathring{A}$$

Chapter 14

Problem Solutions

14.1

(a) $\lambda = \frac{1.24}{E}\ \mu m$

Then

Ge: $E_g = 0.66\ eV \Rightarrow \underline{\lambda = 1.88\ \mu m}$

Si: $E_g = 1.12\ eV \Rightarrow \underline{\lambda = 1.11\ \mu m}$

GaAs: $E_g = 1.42\ eV \Rightarrow \underline{\lambda = 0.873\ \mu m}$

(b)

$$E = \frac{1.24}{\lambda}$$

For $\lambda = 570\ nm \Rightarrow \underline{E = 2.18\ eV}$

For $\lambda = 700\ nm \Rightarrow \underline{E = 1.77\ eV}$

14.2

(a) GaAs

$h\nu = 2\ eV \Rightarrow \lambda = 0.62\ \mu m$

so

$$\underline{\alpha \approx 1.5x10^4\ cm^{-1}}$$

Then

$$\frac{I(x)}{I_o} = \exp(-\alpha x) = \exp\left[-\left(1.5x10^4\right)\left(0.35x10^{-4}\right)\right]$$

or

$$\underline{\frac{I(x)}{I_o} = 0.59}$$

so the percent absorbed is (1-0.59), or

<u>41%</u>

(b) Silicon

Again $h\nu = 2\ eV \Rightarrow \lambda = 0.62\ \mu m$

So

$$\underline{\alpha \approx 4x10^3\ cm^{-1}}$$

Then

$$\frac{I(x)}{I_o} = \exp(-\alpha x) = \exp\left[-\left(4x10^3\right)\left(0.35x10^{-4}\right)\right]$$

or

$$\underline{\frac{I(x)}{I_o} = 0.87}$$

so the percent absorbed is (1-0.87), or

<u>13%</u>

14.3

$$g' = \frac{\alpha I(x)}{h\nu}$$

For $h\nu = 1.3\ eV \Rightarrow \lambda = \frac{1.24}{1.3} = 0.95\ \mu m$

For silicon, $\alpha \approx 3x10^2\ cm^{-1}$,

Then for

$I(x) = 10^{-2}\ W/cm^2$

we obtain

$$g' = \frac{\left(3x10^2\right)\left(10^{-2}\right)}{\left(1.6x10^{-19}\right)(1.3)} \Rightarrow$$

$$\underline{g' = 1.44x10^{19}\ cm^{-3}s^{-1}}$$

The excess concentration is

$$\delta n = g'\tau = \left(1.44x10^{19}\right)\left(10^{-6}\right) \Rightarrow$$

$$\underline{\delta n = 1.44x10^{13}\ cm^{-3}}$$

14.4

n-type GaAs, $\tau = 10^{-7}\ s$

(a)

We want

$$\delta n = \delta p = 10^{15}\ cm^{-3} = g'\tau = g'\left(10^{-7}\right)$$

or

$$g' = \frac{10^{15}}{10^{-7}} = 10^{22}\ cm^{-3}s^{-1}$$

We have

$$h\nu = 1.9\ eV \Rightarrow \lambda = \frac{1.24}{1.9} = 0.65\ \mu m$$

so that

$\alpha \approx 1.3x10^4\ cm^{-1}$

Then

$$g' = \frac{\alpha I(x)}{h\nu} \Rightarrow I(x) = \frac{(g')(h\nu)}{\alpha}$$

$$= \frac{\left(10^{22}\right)\left(1.6x10^{-19}\right)(1.9)}{1.3x10^4}$$

or

$$\underline{I(0) = 0.234\ W/cm^2 = I_o}$$

(b)

$$\frac{I(x)}{I_o} = 0.20 = \exp\left[-\left(1.3x10^4\right)x\right]$$

We obtain $\underline{x = 1.24\ \mu m}$

14.5
GaAs
(a)
For $h\nu = 1.65\ eV \Rightarrow \lambda = 0.75\ \mu m$
So

$$\alpha \approx 0.7x10^4\ cm^{-1}$$

For 75% obsorbed,

$$\frac{I(x)}{I_O} = 0.25 = \exp(-\alpha x)$$

Then

$$\alpha x = \ln\left(\frac{1}{0.25}\right) \Rightarrow x = \frac{1}{0.7x10^4}\ln\left(\frac{1}{0.25}\right)$$

or

$$\underline{x = 1.98\ \mu m}$$

(b)
For 75% transmitted,

$$\frac{I(x)}{I_O} = 0.75 = \exp\left[-\left(0.7x10^4\right)x\right]$$

we obtain

$$\underline{x = 0.41\ \mu m}$$

14.6
GaAs
For $x = 1\ \mu m = 10^{-4}\ cm$, we have 50% absorbed or 50% transmitted, then

$$\frac{I(x)}{I_O} = 0.50 = \exp(-\alpha x)$$

We can write

$$\alpha = \left(\frac{1}{x}\right)\cdot\ln\left(\frac{1}{0.5}\right) = \left(\frac{1}{10^{-4}}\right)\cdot\ln(2)$$

or

$$\alpha = 0.69x10^4\ cm^{-1}$$

This value corresponds to

$$\underline{\lambda = 0.75\ \mu m\ ,\ E = 1.65\ eV}$$

14.7
The ambipolar transport equation for minority carrier holes in steady state is

$$D_p\frac{d^2(\delta p_n)}{dx^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$

or

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$

where $L_p^2 = D_p\tau_p$
The photon flux in the semiconductor is

$$\Phi(x) = \Phi_O\exp(-\alpha x)$$

and the generation rate is

$$G_L = \alpha\Phi(x) = \alpha\Phi_O\exp(-\alpha x)$$

so we have

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = -\frac{\alpha\Phi_O}{D_p}\exp(-\alpha x)$$

The general solution is of the form

$$\delta p_n = A\exp\left(\frac{-x}{L_p}\right) + B\exp\left(\frac{+x}{L_p}\right) - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\exp(-\alpha x)$$

At $x \to \infty$, $\delta p_n = 0$
So that $B = 0$, then

$$\delta p_n = A\exp\left(\frac{-x}{L_p}\right) - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\exp(-\alpha x)$$

At $x = 0$, we have

$$D_p\frac{d(\delta p_n)}{dx}\Big|_{x=0} = s\delta p_n\Big|_{x=0}$$

so we can write

$$\delta p_n\Big|_{x=0} = A - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}$$

and

$$\frac{d(\delta p_n)}{dx}\Big|_{x=0} = -\frac{A}{L_p} + \frac{\alpha^2\Phi_O\tau_p}{\alpha^2L_p^2 - 1}$$

Then we have

$$-\frac{AD_p}{L_p} + \frac{\alpha^2\Phi_O\tau_pD_p}{\alpha^2L_p^2 - 1} = sA - \frac{s\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}$$

Solving for A, we find

$$A = \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\cdot\left[\frac{s + \alpha D_p}{s + \left(D_p/L_p\right)}\right]$$

The solution can now be written as

$$\delta p_n = \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\cdot\left\{\frac{s + \alpha D_p}{s + \left(D_p/L_p\right)}\cdot\exp\left(\frac{-x}{L_p}\right) - \exp(-\alpha x)\right\}$$

14.8

We have

$$D_n \frac{d^2(\delta n_p)}{dx^2} + G_L - \frac{\delta n_p}{\tau_n} = 0$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where $L_n^2 = D_n \tau_n$

The general solution can be written in the form

$$\delta n_p = A\cosh\left(\frac{x}{L_n}\right) + B\sinh\left(\frac{x}{L_n}\right) + G_L \tau_n$$

For $s = \infty$ at $x = 0$ means that $\delta n_p(0) = 0$,

Then

$$0 = A + G_L \tau_n \Rightarrow A = -G_L \tau_n$$

At $x = W$,

$$-D_n \frac{d(\delta n_p)}{dx}\Big|_{x=W} = s_o \delta n_p \Big|_{x=W}$$

Now

$$\delta n_p(W) = -G_L \tau_n \cosh\left(\frac{W}{L_n}\right) + B\sinh\left(\frac{W}{L_n}\right) + G_L \tau_n$$

and

$$\frac{d(\delta n_p)}{dx}\Big|_{x=W} = -\frac{G_L \tau_n}{L_n}\sinh\left(\frac{W}{L_n}\right) + \frac{B}{L_n}\cosh\left(\frac{W}{L_n}\right)$$

so we can write

$$\frac{G_L \tau_n D_n}{L_n}\sinh\left(\frac{W}{L_n}\right) - \frac{BD_n}{L_n}\cosh\left(\frac{W}{L_n}\right)$$

$$= s_o\left[-G_L \tau_n \cosh\left(\frac{W}{L_n}\right) + B\sinh\left(\frac{W}{L_n}\right) + G_L \tau_n\right]$$

Solving for *B*, we find

$$B = \frac{G_L\left[L_n \sinh\left(\frac{W}{L_n}\right) + s_o \tau_n \cosh\left(\frac{W}{L_n}\right) - s_o \tau_n\right]}{\frac{D_n}{L_n}\cosh\left(\frac{W}{L_n}\right) + s_o \sinh\left(\frac{W}{L_n}\right)}$$

The solution is then

$$\delta n_p = G_L \tau_n\left[1 - \cosh\left(\frac{x}{L_n}\right)\right] + B\sinh\left(\frac{x}{L_n}\right)$$

where *B* was just given.

14.9

$$V_{OC} = V_t \ln\left(1 + \frac{J_L}{J_S}\right)$$

$$= (0.0259)\ln\left(1 + \frac{30x10^{-3}}{J_S}\right)$$

where

$$J_S = en_i^2\left[\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_n}} + \frac{1}{N_d}\cdot\sqrt{\frac{D_p}{\tau_p}}\right]$$

which becomes

$$J_S = (1.6x10^{-19})(1.8x10^6)^2 \times\left[\frac{1}{N_a}\cdot\sqrt{\frac{225}{5x10^{-8}}} + \frac{1}{10^{19}}\cdot\sqrt{\frac{7}{5x10^{-8}}}\right]$$

or

$$J_S = (5.18x10^{-7})\left[\frac{6.7x10^4}{N_a} + 1.18x10^{-15}\right]$$

Then

N_a	$J_S(A/cm^2)$	$V_{OC}(V)$
1E15	3.47E-17	0.891
1E16	3.47E-18	0.950
1E17	3.48E-19	1.01
1E18	3.53E-20	1.07

14.10

(a)

$$I_L = J_L \cdot A = (25x10^{-3})(2) = 50\ mA$$

We have

$$J_S = en_i^2\left[\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_n}} + \frac{1}{N_d}\cdot\sqrt{\frac{D_p}{\tau_p}}\right]$$

or

$$J_S = (1.6x10^{-19})(1.5x10^{10})^2 \times\left[\frac{1}{3x10^{16}}\cdot\sqrt{\frac{18}{5x10^{-6}}} + \frac{1}{10^{19}}\cdot\sqrt{\frac{6}{5x10^{-7}}}\right]$$

which becomes

$$J_S = 2.29x10^{-12}\ A/cm^2$$

or

$$I_S = 4.58x10^{-12}\ A$$

We have

$$I = I_L - I_S\left[\exp\left(\frac{V}{V}\right) - 1\right]$$

or

$$I = 50x10^{-3} - 4.58x10^{-12}\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$$

We see that when $I = 0$, $\underline{V = V_{OC} = 0.599\ V}$.

We find

$V(V)$	$I(mA)$
0	50
0.1	50
0.2	50
0.3	50
0.4	49.9
0.45	49.8
0.50	48.9
0.55	42.4
0.57	33.5
0.59	14.2

(b)
The voltage at the maximum power point is found from

$$\left[1 + \frac{V_m}{V_t}\right] \cdot \exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_S}$$

$$= 1 + \frac{50x10^{-3}}{4.58x10^{-12}} = 1.092x10^{10}$$

By trial and error,

$$\underline{V_m = 0.520\ V}$$

At this point, we find

$$\underline{I_m = 47.6\ mA}$$

so the maximum power is

$$P_m = I_m V_m = (47.6)(0.520)$$

or

$$\underline{P_m = 24.8\ mW}$$

(c)
We have

$$V = IR \Rightarrow R = \frac{V}{I} = \frac{V_m}{I_m} = \frac{0.520}{47.6x10^{-3}}$$

or

$$\underline{R = 10.9\ \Omega}$$

14.11
If the solar intensity increases by a factor of 10, then I_L increases by a factor of 10 so that $I_L = 500\ mA$. Then

$$I = 500x10^{-3} - 4.58x10^{-12}\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$$

At the maximum power point

$$\left[1 + \frac{V_m}{V_t}\right] \cdot \exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_S}$$

$$= 1 + \frac{500x10^{-3}}{4.58x10^{-12}} = 1.092x10^{11}$$

By trial and error, we find

$$\underline{V_m = 0.577\ V}$$

and the current at the maximum power point is

$$\underline{I_m = 478.3\ mA}$$

The maximum power is then

$$\underline{P_m = I_m V_m = 276\ mW}$$

The maximum power has increased by a factor of 11.1 compared to the previous problem, which means that the efficiency has increased slightly.

14.12
Let $x = 0$ correspond to the edge of the space charge region in the p-type material. Then

$$D_n \frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{\tau_n} = -G_L$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where

$$G_L = \alpha\Phi(x) = \alpha\Phi_O \exp(-\alpha x)$$

Then we have

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{\alpha\Phi_O}{D_n}\exp(-\alpha x)$$

The general solution is of the form

$$\delta n_p = A\exp\left(\frac{-x}{L_n}\right) + B\exp\left(\frac{+x}{L_p}\right)$$

$$-\frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}\exp(-\alpha x)$$

At $x \to \infty$, $\delta n_p = 0$ so that $B = 0$, then

$$\delta n_p = A\exp\left(\frac{-x}{L_n}\right) - \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1} \cdot \exp(-\alpha x)$$

We also have $\delta n_p(0) = 0 = A - \dfrac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}$,

which yields

$$A = \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}$$

We then obtain

$$\delta n_p = \frac{\alpha \Phi_O \tau_n}{\alpha^2 L_n^2 - 1}\left[\exp\left(\frac{-x}{L_n}\right) - \exp(-\alpha x)\right]$$

where Φ_O is the incident flux at $x = 0$.

14.13

For 90% absorption, we have

$$\frac{\Phi(x)}{\Phi_O} = \exp(-\alpha x) = 0.10$$

Then

$$\exp(+\alpha x) = \frac{1}{0.1} = 10$$

or

$$x = \left(\frac{1}{\alpha}\right) \cdot \ln(10)$$

For $h\nu = 1.7\ eV$, $\alpha \approx 10^4\ cm^{-1}$

Then

$$x = \left(\frac{1}{10^4}\right) \cdot \ln(10) \Rightarrow \underline{x = 2.3\ \mu m}$$

and for $h\nu = 2.0\ eV$, $\alpha \approx 10^5\ cm^{-1}$, so that

$$\underline{x = 0.23\ \mu m}$$

14.14

$G_L = 10^{20}\ cm^{-3}s^{-1}$ and $N_d > N_a$ so holes are the minority carrier.

(a)

$$\delta p = g'\tau = G_L \tau_p$$

so that

$$\delta p = \delta n = \left(10^{20}\right)\left(10^{-7}\right)$$

or

$$\underline{\delta p = \delta n = 10^{13}\ cm^{-3}}$$

(b)

$$\Delta\sigma = e(\delta p)\left(\mu_n + \mu_p\right)$$

$$= \left(1.6x10^{-19}\right)\left(10^{13}\right)(1000 + 430)$$

or

$$\underline{\Delta\sigma = 2.29x10^{-3}\ (\Omega - cm)^{-1}}$$

(c)

$$I_L = J_L \cdot A = \frac{(\Delta\sigma)AV}{L}$$

$$= \frac{\left(2.29x10^{-3}\right)\left(10^{-3}\right)(5)}{100x10^{-4}}$$

or

$$\underline{I_L = 1.15\ mA}$$

(d)

The photoconductor gain is

$$\Gamma_{ph} = \frac{\tau_p}{t_n}\left(1 + \frac{\mu_p}{\mu_n}\right)$$

where

$$t_n = \frac{L}{\mu_n \mathrm{E}} = \frac{L^2}{\mu_n V}$$

Then

$$\Gamma_{ph} = \frac{\tau_p \mu_n V}{L^2}\left(1 + \frac{\mu_p}{\mu_n}\right) = \frac{\tau_p V}{L^2}\left(\mu_n + \mu_p\right)$$

or

$$\Gamma_{ph} = \frac{\left(10^{-7}\right)(5)}{\left(100x10^{-4}\right)^2}(1000 + 430)$$

or

$$\underline{\Gamma_{ph} = 7.15}$$

14.15

n-type, so holes are the minority carrier

(a)

$$\delta p = G_L \tau_p = \left(10^{21}\right)\left(10^{-8}\right)$$

so that

$$\underline{\delta p = \delta n = 10^{13}\ cm^{-3}}$$

(b)

$$\Delta\sigma = e(\delta p)\left(\mu_n + \mu_p\right)$$

$$= \left(1.6x10^{-19}\right)\left(10^{13}\right)(8000 + 250)$$

or

$$\underline{\Delta\sigma = 1.32x10^{-2}\ (\Omega - cm)^{-1}}$$

(c)

$$I_L = J_L \cdot A = (\Delta\sigma)A\mathrm{E} = \frac{(\Delta\sigma)AV}{L}$$

$$= \frac{\left(1.32x10^{-2}\right)\left(10^{-4}\right)(5)}{100x10^{-4}}$$

or $\underline{I_L = 0.66\ mA}$

(d)

$$\Gamma_{ph} = \frac{\tau_p}{t_n}\left(1 + \frac{\mu_p}{\mu_n}\right) = \frac{\tau_p V}{L^2}\left(\mu_n + \mu_p\right)$$

$$= \frac{\left(10^{-8}\right)(5)}{\left(100x10^{-4}\right)^2}(8000 + 250)$$

or $\underline{\Gamma_{ph} = 4.13}$

14.16

$\Phi(x) = \Phi_O \exp(-\alpha x)$

The electron-hole generation rate is

$g' = \alpha\Phi(x) = \alpha\Phi_O \exp(-\alpha x)$

and the excess carrier concentration is

$\delta p = \tau_p \alpha\Phi(x)$

Now

$\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$

and

$J_L = \Delta\sigma \mathrm{E}$

The photocurrent is now found from

$$I_L = \iint \Delta\sigma\mathrm{E}\cdot dA = \int_0^W dy \int_0^{x_O} \Delta\sigma\mathrm{E}\cdot dx$$

$$= We(\mu_n + \mu_n)\mathrm{E}\int_0^{x_O} \delta p \cdot dx$$

Then

$$I_L = We(\mu_n + \mu_p)\mathrm{E}\alpha\Phi_O\tau_p \int_0^{x_O} \exp(-\alpha x)dx$$

$$= We(\mu_n + \mu_p)\mathrm{E}\alpha\Phi_O\tau_p \left[-\frac{1}{\alpha}\exp(-\alpha x)\right]_0^{x_O}$$

which becomes

$$I_L = We(\mu_n + \mu_p)\mathrm{E}\Phi_O\tau_p[1 - \exp(-\alpha x_O)]$$

Now

$$I_L = (50x10^{-4})(1.6x10^{-19})(1200 + 450)(50) \times(10^{16})(2x10^{-7})[1 - \exp(-(5x10^4)(10^{-4}))]$$

or

$$\underline{I_L = 0.131\ \mu A}$$

14.17

(a)

$$V_{bi} = (0.0259)\ln\left[\frac{(2x10^{16})(10^{18})}{(1.5x10^{10})^2}\right] = 0.832\ V$$

The space charge width is

$$W = \left[\frac{2\in(V_{bi} + V_R)}{e}\left(\frac{N_a + N_d}{N_aN_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.832 + 5)}{1.6x10^{-19}} \times\left(\frac{2x10^{16} + 10^{18}}{(2x10^{16})(10^{18})}\right)\right]^{1/2}$$

or

$W = 0.620\ \mu m$

The prompt photocurrent density is

$$J_{L1} = eG_LW = (1.6x10^{-19})(10^{21})(0.620x10^{-4})$$

or

$$\underline{J_{L1} = 9.92\ mA/cm^2}$$

(b)

The total steady-state photocurrent density is

$$J_L = e(W + L_n + L_p)G_L$$

We find

$$L_n = \sqrt{D_n\tau_n} = \sqrt{(25)(2x10^{-7})} = 22.4\ \mu m$$

and

$$L_p = \sqrt{D_p\tau_p} = \sqrt{(10)(10^{-7})} = 10.0\ \mu m$$

Then

$$J_L = (1.6x10^{-19})(0.62 + 22.4 + 10.0)(10^{-4})(10^{21})$$

or

$$\underline{J_L = 0.528\ A/cm^2}$$

14.18

In the n-region under steady state and for $\mathrm{E} = 0$, we have

$$D_p\frac{d^2(\delta p_n)}{dx'^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$

or

$$\frac{d^2(\delta p_n)}{dx'^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$

where $L_p^2 = D_p\tau_p$ and where x' is positive in the negative x direction. The homogeneous solution is found from

$$\frac{d^2(\delta p_{nh})}{dx'^2} - \frac{\delta p_{nh}}{L_p^2} = 0$$

The general solution is found to be

$$\delta p_{nh} = A\exp\left(\frac{-x'}{L_p}\right) + B\exp\left(\frac{+x'}{L_p}\right)$$

The particular solution is found from

$$\frac{-\delta p_{np}}{L_p^2} = \frac{-G_L}{D_p}$$

which yields

$$\delta p_{np} = \frac{G_LL_p^2}{D_p} = G_L\tau_p$$

The total solution is the sum of the homogeneous and particular solutions, so we have

$$\delta p_n = A\exp\left(\frac{-x'}{L_p}\right) + B\exp\left(\frac{+x'}{L_p}\right) + G_L\tau_p$$

One boundary condition is that δp_n remains finite as $x' \to \infty$ which means that $B = 0$. Then
At $x' = 0$, $p_n(0) = 0 = \delta p_n(0) + p_{nO}$, so that

$$\delta p_n(0) = -p_{nO}$$

We find that

$$A = -(p_{nO} + G_L\tau_p)$$

The solution is then written as

$$\delta p_n = G_L\tau_p - (G_L\tau_p + p_{nO})\exp\left(\frac{-x'}{L_p}\right)$$

The diffusion current density is found as

$$J_p = -eD_p \frac{d(\delta p_n)}{dx}\bigg|_{x'=0}$$

But

$$\frac{d(\delta p_n)}{dx} = -\frac{d(\delta p_n)}{dx'}$$

since x and x' are in opposite directions.
So

$$J_p = +eD_p \frac{d(\delta p_n)}{dx'}\bigg|_{x'=0}$$

$$= eD_p\left[-(G_L\tau_p + p_{nO})\right]\left(\frac{-1}{L_p}\right)\exp\left(\frac{-x'}{L_p}\right)\bigg|_{x'=0}$$

Then

$$\underline{J_p = eG_L L_p + \frac{eD_p p_{nO}}{L_p}}$$

14.19
We have

$$J_L = e\Phi_O\left[1 - \exp(-\alpha W)\right]$$

$$= (1.6x10^{-19})(10^{17})\left[1 - \exp(-(3x10^3)W)\right]$$

or

$$J_L = 16\left[1 - \exp(-(3x10^3)W)\right] (mA)$$

Then for $W = 1\ \mu m = 10^{-4}\ cm$, we find

$$\underline{J_L = 4.15\ mA}$$

For $W = 10\ \mu m \Rightarrow$ $\underline{J_L = 15.2\ mA}$

For $W = 100\ \mu m \Rightarrow$ $\underline{J_L = 16\ mA}$

14.20
The minimum α occurs when $\lambda = 1\ \mu m$ which gives $\alpha = 10^2\ cm^{-1}$. We want

$$\frac{\Phi(x)}{\Phi_O} = \exp(-\alpha x) = 0.10$$

which can be written as

$$\exp(+\alpha x) = \frac{1}{0.1} = 10$$

Then

$$x = \frac{1}{\alpha}\ln(10) = \frac{1}{10^2}\ln(10)$$

or

$$\underline{x = 230\ \mu m}$$

14.21
For the $Al_xGa_{1-x}As$ system, a direct bandgap for $0 \le x \le 0.45$, we have

$$E_g = 1.424 + 1.247x$$

At $x = 0.45$, $E_g = 1.985\ eV$, so for the direct bandgap

$$\underline{1.424 \le E_g \le 1.985\ eV}$$

which yields

$$\underline{0.625 \le \lambda \le 0.871\ \mu m}$$

14.22
For $x = 0.35$ in $GaAs_{1-x}P_x$, we find
(a) $\underline{E_g = 1.85\ eV}$ and (b) $\underline{\lambda = 0.670\ \mu m}$

14.23
(a)
For GaAs, $\bar{n}_2 = 3.66$ and for air, $\bar{n}_1 = 1.0$.
The critical angle is

$$\theta_C = \sin^{-1}\left(\frac{\bar{n}_1}{\bar{n}_2}\right) = \sin^{-1}\left(\frac{1}{3.66}\right) = 15.9°$$

The fraction of photons that will not experience total internal reflection is

$$\frac{2\theta_C}{360} = \frac{2(15.9)}{360} \Rightarrow \underline{8.83\%}$$

(b)
Fresnel loss:

$$R = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2 = \left(\frac{3.66 - 1}{3.66 + 1}\right)^2 = 0.326$$

The fraction of photons emitted is then

$$(0.0883)(1 - 0.326) = 0.0595 \Rightarrow \underline{5.95\%}$$

14.24

We can write the external quantum efficiency as

$$\eta_{ext} = T_1 \cdot T_2$$

where $T_1 = 1 - R_1$ with R_1 is the reflection coefficient (Fresnel loss), and the factor T_2 is the fraction of photons that do not experience total internal reflection. We have

$$R_1 = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1} \right)^2$$

so that

$$T_1 = 1 - R_1 = 1 - \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1} \right)^2$$

which reduces to

$$T_1 = \frac{4\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2}$$

Now consider a solid angle from the source point. The surface area described by the solid angle is πp^2. The factor T_1 is given by

$$T_1 = \frac{\pi p^2}{4\pi R^2}$$

From the geometry, we have

$$\sin\left(\frac{\theta_C}{2}\right) = \frac{p/2}{R} \Rightarrow p = 2R\sin\left(\frac{\theta_C}{2}\right)$$

Then the area is

$$A = \pi p^2 = 4R^2\pi\sin^2\left(\frac{\theta_C}{2}\right)$$

Now

$$T_1 = \frac{\pi p^2}{4\pi R^2} = \sin^2\left(\frac{\theta_C}{2}\right)$$

From a trig identity, we have

$$\sin^2\left(\frac{\theta_C}{2}\right) = \frac{1}{2}(1 - \cos\theta_C)$$

Then

$$T_1 = \frac{1}{2}(1 - \cos\theta_C)$$

The external quantum efficiency is now

$$\eta_{ext} = T_1 \cdot T_2 = \frac{4\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2} \cdot \frac{1}{2}(1 - \cos\theta_C)$$

or

$$\underline{\eta_{ext} = \frac{2\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2}(1 - \cos\theta_C)}$$

14.25

For an optical cavity, we have

$$N\left(\frac{\lambda}{2}\right) = L$$

If λ changes slightly, then N changes slightly also. We can write

$$\frac{N_1\lambda_1}{2} = \frac{(N_1 + 1)\lambda_2}{2}$$

Rearranging terms, we find

$$\frac{N_1\lambda_1}{2} - \frac{(N_1 + 1)\lambda_2}{2} = \frac{N_1\lambda_1}{2} - \frac{N_1\lambda_2}{2} - \frac{\lambda_2}{2} = 0$$

If we define $\Delta\lambda = \lambda_1 - \lambda_2$, then we have

$$\frac{N_1}{2}\Delta\lambda = \frac{\lambda_2}{2}$$

We can approximate $\lambda_2 = \lambda$, then

$$\frac{N_1\lambda}{2} = L \Rightarrow N_1 = \frac{2L}{\lambda}$$

Then

$$\frac{1}{2} \cdot \frac{2L}{\lambda}\Delta\lambda = \frac{\lambda}{2}$$

which yields

$$\underline{\Delta\lambda = \frac{\lambda^2}{2L}}$$

14.26

For GaAs,

$$h\nu = 1.42\ eV \Rightarrow \lambda = \frac{1.24}{E} = \frac{1.24}{1.42}$$

or

$$\lambda = 0.873\ \mu m$$

Then

$$\Delta\lambda = \frac{\lambda^2}{2L} = \frac{(0.873x10^{-4})^2}{2(0.75x10^{-4})} = 5.08x10^{-7}\ cm$$

or

$$\underline{\Delta\lambda = 5.08x10^{-3}\ \mu m}$$

Chapter 2

Exercise Solutions

E2.1

(a) $$E = h\nu = \frac{hc}{\lambda} = \frac{(6.625x10^{-34})(3x10^{10})}{10{,}000x10^{-8}}$$
or
$$\underline{E = 1.99x10^{-19}\ J}$$
Also
$$E = \frac{1.99x10^{-19}}{1.6x10^{-19}} \Rightarrow \underline{E = 1.24\ eV}$$
(b)
$$E = \frac{hc}{\lambda} = \frac{(6.625x10^{-34})(3x10^{10})}{10x10^{-8}}$$
or
$$\underline{E = 1.99x10^{-16}\ J}$$
Also
$$E = \frac{1.99x10^{-16}}{1.6x10^{-19}} \Rightarrow \underline{E = 1.24x10^{3}\ eV}$$

E2.2

(a) $$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{6.625x10^{-34}}{180x10^{-10}}$$
or
$$p = 3.68x10^{-26}\ kg - m/s$$
Then
$$E = \frac{p^2}{2m} = \frac{(3.68x10^{-26})^2}{2(5x10^{-31})}$$
or
$$\underline{E = 1.35x10^{-21}\ J = 8.46x10^{-3}\ eV}$$
(b)
$$E = 0.020\ eV = 3.2x10^{-21}\ J$$
$$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE}$$
So
$$p = \sqrt{2(9.11x10^{-31})(3.2x10^{-21})}$$
or
$$p = 7.64x10^{-26}\ kg - m/s$$

Then
$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{7.64x10^{-26}} \Rightarrow \underline{\lambda = 86.7\ \mathring{A}}$$

E2.3
$$\Delta p \Delta x = \hbar$$
Then
$$\Delta p = \frac{\hbar}{\Delta x} = \frac{6.625x10^{-34}}{2\pi(12x10^{-10})} \Rightarrow$$
or
$$\underline{\Delta p = 8.79x10^{-26}\ kg - m/s}$$
Then
$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{(8.79x10^{-26})^2}{2(9.11x10^{-31})} \Rightarrow$$
or
$$\underline{\Delta E = 4.24x10^{-21}\ J = 0.0265\ eV}$$

E2.4
$$\Delta E \Delta t = \hbar$$
Now
$$\Delta E = 1.2\ eV \Rightarrow 1.92x10^{-19}\ J$$
So
$$\Delta t = \frac{\hbar}{\Delta E} = \frac{6.625x10^{-34}}{2\pi(1.92x10^{-19})} \Rightarrow$$
or
$$\underline{\Delta t = 5.49x10^{-16}\ s}$$

E2.5
$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{(1.054x10^{-34})^2 n^2 \pi^2}{2(9.11x10^{-31})(10x10^{-10})^2}$$
or
$$E_n = n^2(6.02x10^{-20})\ J = n^2(0.376)\ eV$$
Then
$$\underline{E_1 = 0.376\ eV}$$
$$\underline{E_2 = 1.50\ eV}$$
$$\underline{E_3 = 3.38\ eV}$$

E2.6

$$m=\frac{\hbar^2\pi^2}{2E_1a^2}$$

Now

$$E_1=(0.025)\left(1.6x10^{-19}\right)=4x10^{-21}\ J$$

Then

$$m=\frac{\left(1.054x10^{-34}\right)^2\pi^2}{2\left(4x10^{-21}\right)\left(100x10^{-10}\right)^2}\Rightarrow$$

or

$$\underline{m=1.37x10^{-31}\ kg}$$

E2.7

$$E=\frac{1}{2}mv^2=\frac{1}{2}\left(9.11x10^{-31}\right)\left(10^5\right)^2$$

$$=4.56x10^{-21}\ J$$

Now

$$K_2=\sqrt{\frac{2m}{\hbar^2}\left(V_O-E\right)}\qquad \text{Set }\ V_O=3E$$

Then

$$K_2=\frac{1}{\hbar}\sqrt{2m(2E)}$$

$$=\frac{\left[2\left(9.11x10^{-31}\right)(2)\left(4.56x10^{-21}\right)\right]^{1/2}}{1.054x10^{-34}}$$

or

$$K_2=1.22x10^9\ m^{-1}$$

(a) $d=10\ \overset{\circ}{A}=10x10^{-10}\ m$

$$P=\exp\left[-2\left(1.22x10^9\right)\left(10x10^{-10}\right)\right]$$

or

$$\underline{P=0.0872\Rightarrow 8.72\%}$$

(b) $d=100\ \overset{\circ}{A}=100x10^{-10}\ m$

$$P=\exp\left[-2\left(1.22x10^9\right)\left(100x10^{-10}\right)\right]$$

or

$$\underline{P=2.53x10^{-11}\Rightarrow 2.53x10^{-9}\%}$$

E2.8

$$K_2=\sqrt{\frac{2m\left(V_O-E\right)}{\hbar^2}}$$

$$=\left\{\frac{2\left(9.11x10^{-31}\right)(1-0.2)\left(1.6x10^{-19}\right)}{\left(1.054x10^{-34}\right)^2}\right\}^{1/2}$$

or

$$K_2=4.58x10^9\ m^{-1}$$

Now

$$T\cong 16\left(\frac{0.2}{1}\right)\left(1-\frac{0.2}{1}\right)\exp\left[-2\left(4.58x10^9\right)\left(15x10^{-10}\right)\right]$$

or

$$\underline{T\cong 2.76x10^{-6}}$$

E2.9 Computer plot

E2.10

$$10^{-5}\cong 16\left(\frac{0.04}{0.4}\right)\left(1-\frac{0.04}{0.4}\right)\exp\left(-2K_2a\right)$$

so

$$\exp\left(+2K_2a\right)=1.44x10^5$$

or

$$2K_2a=11.88$$

Now

$$K_2=\sqrt{\frac{2m\left(V_O-E\right)}{\hbar^2}}$$

$$=\left\{\frac{2\left(9.11x10^{-31}\right)(0.4-0.04)\left(1.6x10^{-19}\right)}{\left(1.054x10^{-34}\right)^2}\right\}^{1/2}$$

or

$$K_2=3.07x10^9\ m^{-1}$$

Then

$$a=\frac{11.88}{2\left(3.07x10^9\right)}=1.93x10^{-9}\ m$$

or\

$$\underline{a=19.3\ \overset{\circ}{A}}$$

E2.11

$$E_1=\frac{-me^4}{2\left(4\pi\in_o\right)^2\hbar^2}$$

$$=\frac{-\left(9.11x10^{-31}\right)\left(1.6x10^{-19}\right)^4}{2\left[4\pi\left(8.85x10^{-12}\right)\right]^2\left(1.054x10^{-34}\right)^2}$$

or

$$\underline{E_1=-2.17x10^{-18}\ J=-13.6\ eV}$$

Chapter 3

Exercise Solutions

E3.1

$$-1 = 10\frac{\sin\alpha a}{\alpha a} + \cos\alpha a$$

By trial and error, $\alpha a = 5.305\ rad$

Now

$$\sqrt{\frac{2mE_2}{\hbar^2}}\cdot a = 5.305$$

so

$$E_2 = \frac{(5.305)^2\hbar^2}{2ma^2} = \frac{(5.305)^2\left(1.054x10^{-34}\right)^2}{2\left(9.11x10^{-31}\right)\left(5x10^{-10}\right)^2}$$

or

$$E_2 = 6.86x10^{-19}\ J = 4.29\ eV$$

Also

$$\sqrt{\frac{2mE_1}{\hbar^2}}\cdot a = \pi$$

so

$$E_1 = \frac{(\pi)^2(\hbar)^2}{2ma^2} = \frac{(\pi)^2\left(1.054x10^{-34}\right)^2}{2\left(9.11x10^{-31}\right)\left(5x10^{-10}\right)^2}$$

or

$$E_1 = 2.41x10^{-19}\ J = 1.50\ eV$$

Then

$$\Delta E = E_2 - E_1 = 4.29 - 1.50$$

or

$$\underline{\Delta E = 2.79\ eV}$$

E3.2

$$g_c(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E-E_c}$$

Then

$$g_T = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\int_{E_c}^{E_c+kT}\left(E-E_C\right)^{1/2}dE$$

$$= \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)\left(E-E_c\right)^{3/2}\Big|_{E_c}^{E_c+kT}$$

or

$$g_T = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)(kT)^{3/2}$$

which yields

$$g_T = \frac{4\pi\left[2(1.08)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(\frac{2}{3}\right)$$

$$\times\left[(0.0259)\left(1.6x10^{-19}\right)\right]^{3/2}$$

which yields

$$\underline{g_T = 2.12x10^{25}\ m^{-3} = 2.12x10^{19}\ cm^{-3}}$$

E3.3

We have

$$g_T = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\int_{E_v-kT}^{E_v}\left(E_v-E\right)^{1/2}dE$$

which yields

$$g_T = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left(E_v-E\right)^{3/2}\Big|_{E_v-kT}^{E_v}$$

or

$$g_T = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left[0-(kT)^{3/2}\right]$$

$$= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)(kT)^{3/2}$$

Then

$$g_T = \frac{4\pi\left[2(0.56)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(\frac{2}{3}\right)$$

$$\times\left[(0.0259)\left(1.6x10^{-19}\right)\right]^{3/2}$$

or

$$\underline{g_T = 7.92x10^{24}\,m^{-3} = 7.92x10^{18}\ cm^{-3}}$$

E3.4

(a)

$$f_F = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{E_c-E_F}{kT}\right)}$$

or

$$f_F = \frac{1}{1+\exp\left(\frac{0.30}{0.0259}\right)} \Rightarrow \underline{f_F = 9.32x10^{-6}}$$

(b)

$$f_F = \frac{1}{1+\exp\left(\dfrac{0.30+0.0259}{0.0259}\right)} \Rightarrow$$

$$\underline{f_F = 3.43x10^{-6}}$$

E3.5

(a)

$$1-f_F = 1-\frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$$

$$= \frac{\exp\left(\dfrac{E-E_F}{kT}\right)}{1+\exp\left(\dfrac{E-E_F}{kT}\right)} = \frac{1}{1+\exp\left(\dfrac{E_F-E}{kT}\right)}$$

Then

$$1-f_F = \frac{1}{1+\exp\left(\dfrac{0.35}{0.0259}\right)}$$

so

$$\underline{1-f_F = 1.35x10^{-6}}$$

(b)

$$1-f_F = \frac{1}{1+\exp\left(\dfrac{0.35+0.0259}{0.0259}\right)}$$

or

$$\underline{1-f_F = 4.98x10^{-7}}$$

E3.6

$$kT = (0.0259)\left(\frac{400}{300}\right) = 0.03453$$

(a)

$$f_F = \frac{1}{1+\exp\left(\dfrac{0.30}{0.03453}\right)} \Rightarrow \underline{f_F = 1.69x10^{-4}}$$

(b)

$$f_F = \frac{1}{1+\exp\left(\dfrac{0.30+0.03453}{0.03453}\right)} \Rightarrow$$

$$\underline{f_F = 6.20x10^{-5}}$$

E3.7

$$kT = 0.03453\ eV$$

(a)

$$1-f_F = \frac{1}{1+\exp\left(\dfrac{0.35}{0.03453}\right)} \Rightarrow$$

$$\underline{1-f_F = 3.96x10^{-5}}$$

(b)

$$1-f_F = \frac{1}{1+\exp\left(\dfrac{0.35+0.03453}{0.03453}\right)} \Rightarrow$$

$$\underline{1-f_F = 1.46x10^{-5}}$$

Chapter 4

Exercise Solutions

E4.1

$$n_o = 2.8x10^{19} \exp\left(\frac{-0.22}{0.0259}\right)$$

or

$$\underline{n_o = 5.73x10^{15}\ cm^{-3}}$$

Now

$$E_F - E_v = 1.12 - 0.22 = 0.90\ eV$$

So

$$p_o = 1.04x10^{19} \exp\left(\frac{-0.90}{0.0259}\right)$$

or

$$\underline{p_o = 8.43x10^{3}\ cm^{-3}}$$

E4.2

$$p_o = 7.0x10^{18} \exp\left(\frac{-0.30}{0.0259}\right)$$

or

$$\underline{p_o = 6.53x10^{13}\ cm^{-3}}$$

Now

$$E_c - E_F = 1.42 - 0.30 = 1.12\ eV$$

So

$$n_o = 4.7x10^{17} \exp\left(\frac{-1.12}{0.0259}\right)$$

or

$$\underline{n_o = 0.0779\ cm^{-3}}$$

E4.3

(a)

For $200K$: $kT = (0.0259)\left(\frac{200}{300}\right) = 0.01727$

Now

$$n_i^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{200}{300}\right)^3 \exp\left(\frac{-1.12}{0.01727}\right)$$

or

$$n_i^2 = 5.90x10^{9}$$

Then

$$\underline{n_i = 7.68x10^{4}\ cm^{-3}}$$

(b)

For $400K$: $kT = (0.0259)\left(\frac{400}{300}\right) = 0.03453$

Now

$$n_i^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{400}{300}\right)^3 \exp\left(\frac{-1.12}{0.03453}\right)$$

or

$$n_i^2 = 5.65x10^{24}$$

Then

$$\underline{n_i = 2.38x10^{12}\ cm^{-3}}$$

E4.4

(a) $200K$

$$n_i^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{200}{300}\right)^3 \exp\left(\frac{-1.42}{0.01727}\right)$$

or

$$n_i^2 = 1.904$$

Then

$$\underline{n_i = 1.38\ cm^{-3}}$$

(b) $400K$

$$n_i^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{400}{300}\right)^3 \exp\left(\frac{-1.42}{0.03453}\right)$$

or

$$n_i^2 = 1.08x10^{19}$$

Then

$$\underline{n_i = 3.28x10^{9}\ cm^{-3}}$$

E4.5

(a) $200K$

$$n_i^2 = \left(1.04x10^{19}\right)\left(6x10^{18}\right)\left(\frac{200}{300}\right)^3 \exp\left(\frac{-0.66}{0.01727}\right)$$

or

$$n_i^2 = 4.67x10^{20}$$

Then

$$\underline{n_i = 2.16x10^{10}\ cm^{-3}}$$

(b) $400K$

$$n_i^2 = \left(1.04x10^{19}\right)\left(6x10^{18}\right)\left(\frac{400}{300}\right)^3 \exp\left(\frac{-0.66}{0.03453}\right)$$

or

$$n_i^2 = 7.39x10^{29}$$

Then

$$\underline{n_i = 8.6x10^{14}\ cm^{-3}}$$

E4.6

$$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$
$$= \frac{3}{4}(0.0259)\ln\left(\frac{0.067}{0.48}\right)$$

or

$$\underline{E_F - E_{midgap} = -38.2\ meV}$$

E4.7

$$\frac{r_n}{a_o} = n^2 \in_r \left(\frac{m_o}{m^*}\right) = (1)(13.1)\left(\frac{1}{0.067}\right)$$

so

$$\underline{\frac{r_1}{a_o} = 195.5}$$

E4.8

For $\eta_F = 0$, $F_{1/2}(\eta_F) = 0.60$

Then

$$n_o = \frac{2}{\sqrt{\pi}}N_C F_{1/2}(\eta_F) = \frac{2}{\sqrt{\pi}}(2.8x10^{19})(0.60)$$

or

$$\underline{n_o = 1.9x10^{19}\ cm^{-3}}$$

E4.9

$$\frac{p_a}{p_o + p_a} = \frac{1}{1 + \dfrac{N_V}{4N_a}\exp\left[\dfrac{-(E_a - E_v)}{kT}\right]}$$
$$= \frac{1}{1 + \dfrac{1.04x10^{19}}{4(10^{17})}\exp\left[\dfrac{-0.045}{0.0259}\right]}$$

or

$$\underline{\frac{p_a}{p_o + p_a} = 0.179}$$

E4.10 Computer plot

E4.11

$$p_o = N_a - N_d = 2x10^{16} - 5x10^{15}$$

or

$$\underline{p_o = 1.5x10^{16}\ cm^{-3}}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8x10^6)^2}{1.5x10^{16}}$$

or

$$\underline{n_o = 2.16x10^{-4}\ cm^{-3}}$$

E4.12 (b)

$$n = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

Then

$$1.1x10^{15} = 5x10^{14} + \sqrt{(5x10^{14})^2 + n_i^2}$$

which yields

$$n_i^2 = 11x10^{28}$$

and

$$n_i^2 = N_C N_V \exp\left[\frac{-E_g}{kT}\right] = 11x10^{28}$$
$$= (2.8x10^{19})(1.04x10^{19})\left(\frac{T}{300}\right)^3$$
$$\times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

By trial and error

$$\underline{T \cong 552K}$$

E4.13

$$E_F - E_v = (0.0259)\ln\left[\frac{7x10^{18}}{5x10^{16} - 4x10^{15}}\right]$$
$$\underline{= 0.130\,eV}$$

E4.14

$$E_F - E_{Fi} = (0.0259)\ln\left[\frac{1.7x10^{17}}{1.5x10^{10}}\right]$$

or

$$\underline{E_F - E_{Fi} = 0.421\ eV}$$

Chapter 5

Exercise Solutions

E5.1

$$n_o = 10^{15} - 10^{14} = 9x10^{14}\ cm^{-3}$$

so

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5x10^{10}\right)^2}{9x10^{14}} = 2.5x10^5\ cm^{-3}$$

Now

$$J_{drf} = e\left(\mu_n n_o + \mu_p p_o\right)\mathrm{E} \approx e\mu_n n_o \mathrm{E}$$

$$= \left(1.6x10^{-19}\right)(1350)\left(9x10^{14}\right)(35)$$

or

$$\underline{J_{drf} = 6.80\ A/cm^2}$$

E5.2

$$J_{drf} \cong e\mu_p p_o \mathrm{E}$$

Then

$$120 = \left(1.6x10^{-19}\right)(480)p_o(20)$$

so

$$\underline{p_o = 7.81x10^{16}\ cm^{-3} = N_a}$$

E5.3

Use Figure 5.2

(a)

(i) $\mu_n \cong 500\ cm^2/V-s$, (ii) $\cong 1500\ cm^2/V-s$

(b)

(i) $\mu_p \cong 380\ cm^2/V-s$, (ii) $\cong 200\ cm^2/V-s$

E5.4

Use Figure 5.3 [Units of $cm^2/V-s$]

(a) For $N_I = 10^{15}\ cm^{-3}$; $\mu_n \cong 1350$, $\mu_p \cong 480$:

(b) $N_I = 1.5x10^{17}\ cm^{-3}$; $\mu_n \cong 700$, $\mu_p \cong 300$:

(c) $N_I = 1.1x10^{17}\ cm^{-3}$; $\mu_n \cong 800$, $\mu_p \cong 310$:

(d) $N_I = 2x10^{17}\ cm^{-3}$; $\mu_n \cong 4500$, $\mu_p \cong 220$

E5.5

(a) For

$N_I = 7x10^{16}\ cm^{-3}$; $\mu_n \cong 1000\ cm^2/V-s$,

$\mu_p \cong 350\ cm^2/V-s$

(b) $\sigma \cong e\mu_n\left(N_d - N_a\right)$

$$= \left(1.6x10^{-19}\right)(1000)\left(3x10^{16}\right) \Rightarrow$$

$$\underline{\sigma = 4.8\ (\Omega - cm)^{-1}}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{4.8} \Rightarrow \underline{\rho = 0.208\ \Omega - cm}$$

E5.6

$$\sigma = e\mu_n N_d = \frac{1}{\rho}$$

so

$$\left(1.6x10^{-19}\right)\mu_n N_d = \frac{1}{0.1} = 10$$

Then

$$\mu_n N_d = 6.25x10^{19}$$

Using Figure 5.4a, $\underline{N_d \cong 9x10^{16}\ cm^{-3}}$

Then

$$\underline{\mu_n \approx 695\ cm^2/V-s}$$

E5.7

(a) $R = \frac{V}{I} = \frac{5}{2} = 2.5\ k\Omega$

(b) $R = 2.5x10^3 = \frac{\rho\left(1.2x10^{-3}\right)}{10^{-6}} \Rightarrow$

$$\underline{\rho = 2.08\ \Omega - cm}$$

(c) From Figure 5.4a, $\underline{N_a \cong 7x10^{15}\ cm^{-3}}$

E5.8

$$J_{diff} = eD_n\frac{dn}{dx} = -eD_n\left(\frac{10^{15}}{10^{-4}}\right)\exp\left(\frac{-x}{L_n}\right)$$

$$D_n = 25\ cm^2/s,\quad L_n = 10^{-4}\ cm = 1\ \mu m$$

Then

$$J_{diff} = -40\exp\left(\frac{-x}{1}\right)\ A/cm^2$$

(a) $x = 0$; $\underline{J_{diff} = -40\ A/cm^2}$

(b) $x = 1\ \mu m$; $\underline{J_{diff} = -14.7\ A/cm^2}$

(c) $x = \infty$; $\underline{J_{diff} = 0}$

E5.9

$$J_{diff} = -eD_p \frac{dp}{dx}$$

so

$$20 = -\left(1.6x10^{-19}\right)(10)\frac{\Delta p}{(0 - 0.010)}$$

Then

$$\Delta p = 1.25x10^{17} = 4x10^{17} - p$$

or

$$\underline{p(x = 0.01) = 2.75x10^{17}\ cm^{-3}}$$

E5.10

At $x = 0$,

$$J_{diff} = -eD_p \frac{dp}{dx} = -eD_p\left(\frac{2x10^{15}}{-L_p}\right)$$

Then

$$6.4 = \left(1.6x10^{-19}\right)(10)\left(\frac{2x10^{15}}{L_p}\right)$$

Which yields

$$\underline{L_P = 5x10^{-4}\ cm}$$

Chapter 6

Exercise Solutions

E6.1

$$\delta n(t) = \delta n(0)\exp\left(\frac{-t}{\tau_{no}}\right)$$

so

$$\delta n(t) = 10^{15}\exp\left(\frac{-t}{1\ \mu s}\right)$$

(a) $t = 0$; $\underline{\delta n = 10^{15}\ cm^{-3}}$

(b) $t = 1\ \mu s$; $\underline{\delta n = 3.68x10^{14}\ cm^{-3}}$

(c) $t = 4\ \mu s$; $\underline{\delta n = 1.83x10^{13}\ cm^{-3}}$

E6.2

$$R = \frac{\delta n}{\tau_{no}}$$

Then

(a) $R = \dfrac{10^{15}}{10^{-6}} \Rightarrow \underline{R = 10^{21}\ cm^{-3}s^{-1}}$

(b) $R = \dfrac{3.68x10^{14}}{10^{-6}} \Rightarrow \underline{R = 3.68x10^{20}\ cm^{-3}s^{-1}}$

(c) $R = \dfrac{1.83x10^{13}}{10^{-6}} \Rightarrow \underline{R = 1.83x10^{19}\ cm^{-3}s^{-1}}$

E6.3

(a) p-type $\Rightarrow$ Minority carrier = electrons

(b) $\delta n(t) = \delta n(0)\exp\left(\dfrac{-t}{\tau_{no}}\right)$

Then

$$\underline{\delta n(t) = 10^{15}\exp\left(\frac{-t}{5\ \mu s}\right)\quad cm^{-3}}$$

E6.4

(a) p-type $\Rightarrow$ Minority carrier = electrons

(b) $\delta n(t) = g'\tau_{no}\left[1 - \exp\left(\dfrac{-t}{\tau_{no}}\right)\right]$

or

$$\delta n(t) = \left(10^{20}\right)\left(5x10^{-6}\right)\left[1 - \exp\left(\frac{-t}{5\ \mu s}\right)\right]$$

Then

$$\underline{\delta n(t) = 5x10^{14}\left[1 - \exp\left(\frac{t}{5\ \mu s}\right)\right]}$$

(c) As $t \to \infty$, $\underline{\delta n(\infty) = 5x10^{14}\ cm^{-3}}$

E6.5

$$\delta n(x) = \delta p(x) = \delta n(0)\exp\left(\frac{-x}{L_p}\right)$$

$$L_P = \sqrt{D_P\tau_{po}} = \sqrt{(10)\left(10^{-6}\right)} = 31.6\ \mu m$$

Then

$$\underline{\delta n(x) = \delta p(x) = 10^{15}\exp\left(\frac{-x}{31.6\ \mu m}\right)\quad cm^{-3}}$$

E6.6

n-type $\Rightarrow$ Minority carrier = hole

$$J_{diff} = -eD_p\frac{dp}{dx} = -eD_p\frac{d(\delta p(x))}{dx}$$

$$J_{diff} = \frac{-\left(1.6x10^{-19}\right)(10)\left(10^{15}\right)}{-\left(3.16x10^{-3}\right)}\exp\left(\frac{-10}{31.6}\right)$$

or

$\underline{J_{diff} = +0.369\ A/cm^2}$ Hole diffusion current

$$J_{diff}(electrons) = -J_{diff}(holes)$$

so

$\underline{J_{diff} = -0.369\ A/cm^2}$ Electron diffusion current

E6.7

$$\delta p = \frac{\exp\left(-t/\tau_{po}\right)}{\left(4\pi D_p t\right)^{1/2}}$$

(a) $\dfrac{\exp(-1/5)}{\left[(4\pi)(10)\left(10^{-6}\right)\right]^{1/2}} \Rightarrow \underline{\delta p = 73.0}$

(b) $\dfrac{\exp(-5/5)}{\left[(4\pi)(10)\left(5x10^{-6}\right)\right]^{1/2}} \Rightarrow \underline{\delta p = 14.7}$

(c) $\dfrac{\exp(-15/5)}{\left[(4\pi)(10)\left(15x10^{-6}\right)\right]^{1/2}} \Rightarrow \underline{\delta p = 1.15}$

(d) $\dfrac{\exp(-25/5)}{\left[(4\pi)(10)\left(25x10^{-6}\right)\right]^{1/2}} \Rightarrow \underline{\delta p = 0.120}$

$$x = \mu_p \mathrm{E}_o t = (386)(10)t$$

(a) $\underline{x = 38.6\ \mu m}$; (b) $\underline{x = 193\ \mu m}$

(b) $\underline{x = 579\ \mu m}$; (d) $\underline{x = 965 \mu m}$

E6.8

$$\delta p = \frac{\exp(-t/\tau_{po})}{(4\pi D_p t)^{1/2}} \cdot \exp\left[\frac{-(x-\mu_p \mathrm{E}_o t)^2}{4D_p t}\right]$$

(a) (i) $x - \mu_p \mathrm{E}_o t$

$$= 1.093x10^{-2} - (386)(10)(10^{-6}) = 7.07x10^{-3}$$

$$\delta p = \frac{\exp(-1/5)}{\left[(4\pi)(10)(10^{-6})\right]^{1/2}} \cdot \exp\left[\frac{-(7.07x10^{-3})^2}{4(10)(10^{-6})}\right]$$

or

$$\delta p = 73.0 \exp\left[\frac{-(7.07x10^{-3})^2}{4(10)(10^{-6})}\right]$$

Then

$$\underline{\delta p = 20.9}$$

(ii) $x - \mu_p \mathrm{E}_o t$

$$-3.21x10^{-3} - (386)(10)(10^{-6}) = -7.07x10^{-3}$$

$$\delta p = 73.0 \exp\left[\frac{-(-7.07x10^{-3})^2}{4(10)(10^{-6})}\right]$$

or

$$\underline{\delta p = 20.9}$$

(b) (i) $x - \mu_p \mathrm{E}_o t$

$$= 2.64x10^{-2} - (386)(10)(5x10^{-6}) = 7.1x10^{-3}$$

$$\delta p = 14.7 \exp\left[\frac{-(7.1x10^{-3})^2}{4(10)(5x10^{-6})}\right]$$

Then

$$\underline{\delta p = 11.4}$$

(ii) $x - \mu_p \mathrm{E}_o t$

$$= 1.22x10^{-2} - (386)(10)(5x10^{-6}) = -7.1x10^{-3}$$

Then

$$\underline{\delta p = 11.4}$$

(c) (i) $x - \mu_p \mathrm{E}_o t$

$$= 6.50x10^{-2} - (386)(10)(15x10^{-6}) = 7.1x10^{-3}$$

$$\delta p = 1.15 \exp\left[\frac{-(7.1x10^{-3})^2}{4(10)(15x10^{-6})}\right] \quad \underline{\delta p = 1.05}$$

(ii) $x - \mu_p \mathrm{E}_o t$

$$= 5.08x10^{-2} - (386)(10)(15x10^{-6}) = -7.1x10^{-3}$$

Then

$$\underline{\delta p = 1.05}$$

E6.9 Computer Plot

E6.10

(a) $E_F - E_{Fi} = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) \Rightarrow$

$$\underline{E_F - E_{Fi} = 0.3473\ eV}$$

(b) $E_{Fn} - E_{Fi} = (0.0259)\ln\left(\frac{10^{16} + 5x10^{14}}{1.5x10^{10}}\right) \Rightarrow$

$$\underline{E_{Fn} - E_{Fi} = 0.3486\ eV}$$

$$E_{Fi} - E_{Fp} = (0.0259)\ln\left(\frac{5x10^{14}}{1.5x10^{10}}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_{Fp} = 0.2697\ eV}$$

E6.11

(a) p-type

$$E_{Fi} - E_F = (0.0259)\ln\left(\frac{6x10^{15} - 10^{15}}{1.5x10^{10}}\right)$$

$$\underline{E_{Fi} - E_F = 0.3294\ eV}$$

(b) $E_{Fn} - E_{Fi} = (0.0259)\ln\left(\frac{2x10^{14}}{1.5x10^{10}}\right) \Rightarrow$

$$\underline{E_{Fn} - E_{Fi} = 0.2460\ eV}$$

$$E_{Fi} - E_{Fp} = (0.0259)\ln\left(\frac{5x10^{15} + 2x10^{14}}{1.5x10^{10}}\right)$$

$$\underline{E_{Fi} - E_{Fp} = 0.3304\ eV}$$

E6.12

n-type; $n_o = 10^{15}\ cm^{-3}$; $p_o = 2.25x10^5\ cm^{-3}$

$$R = \frac{\left[(n_o + \delta n)(p_o + \delta p) - n_i^2\right]}{\tau_{po}(n_o + \delta n + n_i) + \tau_{no}(p_o + \delta p + n_i)}$$

Then

$$\underline{R = 1.83x10^{20}\ cm^{-3} s^{-1}}$$

Chapter 11

Exercise Solutions

E11.1

(a) $\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$

$$x_{dT} = \left\{\frac{4(11.7)(8.85x10^{-14})(0.376)}{(1.6x10^{-19})(3x10^{16})}\right\}^{1/2}$$

or

$\underline{x_{dT} = 0.180\ \mu m}$

(b) $\phi_{fp} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$

$$x_{dT} = \left\{\frac{4(11.7)(8.85x10^{-14})(0.288)}{(1.6x10^{-19})(10^{15})}\right\}^{1/2}$$

or

$\underline{x_{dT} = 0.863\ \mu m}$

E11.2

$$\phi_{fn} = (0.0259)\ln\left(\frac{8x10^{15}}{1.5x10^{10}}\right) = 0.342\ V$$

$$x_{dT} = \left\{\frac{4(11.7)(8.85x10^{-14})(0.342)}{(1.6x10^{-19})(8x10^{15})}\right\}^{1/2}$$

or

$\underline{x_{dT} = 0.333\ \mu m}$

E11.3

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$$

$$\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp}\right)$$

$$= 3.20 - (3.25 + 0.555 + 0.376)$$

or

$\underline{\phi_{ms} = -0.981\ V}$

E11.4

$\phi_{fp} = 0.376\ V$

$\phi_{ms} = -(0.555 + 0.376) \Rightarrow \underline{\phi_{ms} = -0.931\ V}$

E11.5

$\phi_{fp} = 0.376\ V$

$\phi_{ms} = (0.555 - 0.376) \Rightarrow \phi_{ms} = +0.179\ V$

E11.6

From E11.3, $\phi_{ms} = -0.981\ V$

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{200x10^{-8}} = 1.73x10^{-7}\ F/cm^2$$

Then

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}} = -0.981 - \frac{(1.6x10^{-19})(8x10^{10})}{1.73x10^{-7}}$$

or

$\underline{V_{FB} = -1.06\ V}$

E11.7

From E11.4, $\phi_{ms} = -0.931\ V$

$$V_{FB} = -0.931 - \frac{(1.6x10^{-19})(8x10^{10})}{1.73x10^{-7}}$$

or

$\underline{V_{FB} = -1.01\ V}$

E11.8

From E11.5, $\phi_{ms} = +0.179\ V$

$$V_{FB} = +0.179 - \frac{(1.6x10^{-19})(8x10^{10})}{1.73x10^{-7}}$$

or

$\underline{V_{FB} = +0.105\ V}$

E11.9

From E11.3, $\phi_{ms} = -0.981$ V and $\phi_{fp} = 0.376\ V$

$$x_{dT} = \left\{\frac{4(11.7)(8.85x10^{-14})(0.376)}{(1.6x10^{-19})(3x10^{16})}\right\} = 0.18\ \mu m$$

Now

$$|Q'_{SD}(\max)| = (1.6x10^{-19})(3x10^{16})(0.18x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 8.64x10^{-8}\ C/cm^2$$

From Equation [11.27b]

$$V_{TN} = \left[8.64x10^{-8} - \left(10^{11}\right)\left(1.6x10^{-19}\right)\right]$$
$$\times\left(\frac{250x10^{-8}}{(3.9)\left(8.85x10^{-14}\right)}\right) - 0.981 + 2(0.376)$$

or

$$\underline{V_{TN} = +0.281\ V}$$

E11.10

From Figure 11.15, $\phi_{ms} = +0.97\ V$

$$\phi_{fn} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$$

$$x_{dT} = \left\{\frac{4(11.7)\left(8.85x10^{-14}\right)(0.288)}{\left(1.6x10^{-19}\right)\left(10^{15}\right)}\right\}^{1/2} = 0.863\ \mu m$$

Then

$$\left|Q'_{SD}(\max)\right| = \left(1.6x10^{-19}\right)\left(10^{15}\right)\left(0.863x10^{-4}\right)$$

or

$$\left|Q'_{SD}(\max)\right| = 1.38x10^{-8}\ C/cm^2$$

Also

$$Q'_{ss} = \left(8x10^{10}\right)\left(1.6x10^{-19}\right) = 1.28x10^{-8}\ C/cm^2$$

Now, from Equation [11.28]

$$V_{TP} = \left(-1.38x10^{-8} - 1.28x10^{-8}\right)$$
$$\times\left(\frac{220x10^{-8}}{(3.9)\left(8.85x10^{-14}\right)}\right) + 0.97 - 2(0.288)$$

or

$$\underline{V_{TP} = +0.224\ V}$$

E11.11

By trial and error, let $N_d = 4x10^{16}\ cm^{-3}$, then

$\phi_{fn} = 0.383$, $\phi_{ms} \cong 1.07$,

$\left|Q'_{SD}(\max)\right| = 1x10^{-7}$ and

$\underline{V_{TP} = -0.405\ V}$ which is between the limits specified.

E11.12

$$\frac{C'_{\min}}{C_{ox}} = \frac{\dfrac{\in_{ox}}{t_{ox} + \left(\in_{ox}/\in_s\right)x_{dT}}}{\dfrac{\in_{ox}}{t_{ox}}} = \frac{t_{ox}}{t_{ox} + \left(\dfrac{\in_{ox}}{\in_s}\right)x_{dT}}$$

or

$$\frac{C'_{\min}}{C_{ox}} = \frac{1}{1 + \left(\dfrac{\in_{ox}}{\in_s}\right)\left(\dfrac{x_{dT}}{t_{ox}}\right)}$$

From E11.9, $x_{dT} = 0.18\ \mu m$

Then

$$\frac{C'_{\min}}{C_{ox}} = \frac{1}{1 + \left(\dfrac{3.9}{11.7}\right)\left(\dfrac{0.18x10^{-4}}{250x10^{-8}}\right)} \Rightarrow$$

$$\underline{\frac{C'_{\min}}{C_{ox}} = 0.294}$$

Also

$$\frac{C'_{FB}}{C_{ox}} = \frac{1}{1 + \left(\dfrac{\in_{ox}}{\in_s}\right)\left(\dfrac{1}{t_{ox}}\right)\sqrt{\left(\dfrac{kT}{e}\right)\left(\dfrac{\in_s}{eN_a}\right)}}$$

$$= \frac{1}{1 + \left(\dfrac{3.9}{11.7}\right)\left(\dfrac{1}{220x10^{-8}}\right)\sqrt{\dfrac{(0.0259)(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(3x10^{16}\right)}}}$$

or

$$\underline{\frac{C'_{FB}}{C_{ox}} = 0.736}$$

E11.13

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{200x10^{-8}} \Rightarrow$$

$$C_{ox} = 1.73x10^{-7}\ F/cm^2$$

Now

$$I_D = \frac{1}{2}\left(\frac{W}{L}\right)\mu_n C_{ox}\left(V_{GS} - V_{TN}\right)^2$$
$$= \left(\frac{50}{2}\right)(650)\left(1.73x10^{-7}\right)\left(V_{GS} - 0.4\right)^2$$

or

$$I_D = \left(2.81x10^{-3}\right)\left(V_{GS} - 0.4\right)^2$$

Then

$$\underline{V_{GS} = 1\ V \Rightarrow I_D = 1.01\ mA}$$
$$\underline{V_{GS} = 2\ V \Rightarrow I_D = 7.19\ mA}$$
$$\underline{V_{GS} = 3\ V \Rightarrow I_D = 19\ mA}$$

E11.14

$$I_D = \frac{1}{2}\left(\frac{W}{L}\right)\mu_n C_{ox}\left(V_{GS} - V_{TN}\right)^2$$

Now

$$100x10^{-6} = \left(\frac{W}{L}\right)\frac{(650)\left(1.73x10^{-7}\right)}{2}(1.75 - 0.4)^2$$

which yields

$$\underline{\left(\frac{W}{L}\right) = 0.976}$$

E11.15

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{220x10^{-8}} = 1.57x10^{-7}\ F / cm^2$$

$$I_D = \left(\frac{60}{2}\right)(310)\left(1.57x10^{-7}\right)\left(V_{SG} - 0.4\right)^2$$

or

$$I_D = 1.46x10^{-3}\left(V_{SG} - 0.4\right)^2$$

Then

$$\underline{V_{SG} = 1\,V \Rightarrow I_D = 0.526\ mA}$$

$$\underline{V_{SG} = 1.5\,V \Rightarrow I_D = 1.77\ mA}$$

$$\underline{V_{SG} = 2\,V \Rightarrow I_D = 3.74\ mA}$$

E11.16

$$200x10^{-6} = \left(\frac{W}{L}\right)\left(\frac{310}{2}\right)\left(1.57x10^{-7}\right)(1.25 - 0.4)^2$$

which yields

$$\underline{\left(\frac{W}{L}\right) = 11.4}$$

E11.17

(a)

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{200x10^{-8}} = 1.73x10^{-7}\ F / cm^2$$

Now

$$\gamma = \frac{\sqrt{2e \in_s N_a}}{C_{ox}}$$

$$= \frac{\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(10^{16}\right)\right]^{1/2}}{1.73x10^{-7}}$$

or

$$\underline{\gamma = 0.333\ V^{1/2}}$$

(b) $\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) = 0.347\ V$

(i)

$$\Delta V = (0.333)\left[\sqrt{(2)(0.347)+1} - \sqrt{2(0.347)}\right]$$

or

$$\underline{\Delta V = 0.156\ V}$$

(ii)

$$\Delta V = (0.333)\left[\sqrt{2(0.347)+2} - \sqrt{2(0.347)}\right]$$

or

$$\underline{\Delta V = 0.269\ V}$$

E11.18

$$C_{ox} = 1.73x10^{-7}\ V / cm^2$$

(a)

$$\gamma = \frac{\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(10^{15}\right)\right]^{1/2}}{1.73x10^{-7}}$$

or

$$\underline{\gamma = 0.105\ V^{1/2}}$$

(b) $\phi_{fp} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$

(i)

$$\Delta V = (0.105)\left[\sqrt{2(0.288)+1} - \sqrt{2(0.288)}\right]$$

or

$$\underline{\Delta V = 0.052\ V}$$

(ii)

$$\Delta V = (0.105)\left[\sqrt{2(0.288)+2} - \sqrt{2(0.288)}\right]$$

or

$$\underline{\Delta V = 0.0888\ V}$$

E11.19

$$C_{ox} = 1.73x10^{-7}\ V / cm^2$$

$$g_m = \left(\frac{W}{L}\right)\mu_n C_{ox}\left(V_{GS} - V_T\right)$$

$$= (20)(400)\left(1.73x10^{-7}\right)(2.5 - 0.4)$$

or

$$g_m = 2.91\ mA / V$$

Now

$$\frac{C_M}{C_{gdT}} = 1 + g_m R_L = 1 + (2.91)(100)$$

or

$$\frac{C_M}{C_{gdT}} = 292$$

E11.20

$$f_T = \frac{\mu_n (V_{GS} - V_T)}{2\pi L^2}$$

$$= \frac{(400)(2.5 - 0.4)}{2\pi (0.5x10^{-4})^2}$$

or

$$f_T = 53.5\ GHz$$

Chapter 12

Exercise Solutions

E12.1

$$\frac{I_{D1}}{I_{D2}} = \frac{\exp\left(\frac{V_{GS1}}{V_t}\right)}{\exp\left(\frac{V_{GS2}}{V_t}\right)} = \exp\left(\frac{V_{GS1} - V_{GS2}}{V_t}\right)$$

or

$$V_{GS1} - V_{GS2} = V_t \ln\left(\frac{I_{D1}}{I_{D2}}\right)$$

Then

$$V_{GS1} - V_{GS2} = (0.0259)\ln(10) \Rightarrow$$

$$\underline{V_{GS1} - V_{GS2} = 59.64\ mV}$$

E12.2

$$\phi_{fp} = (0.0259)\ln\left(\frac{2x10^{16}}{1.5x10^{10}}\right) = 0.365\ V$$

$$V_{DS}(sat) = V_{GS} - V_T = 1 - 0.4 = 0.60\ V$$

Now

$$\Delta L = \left[\frac{2(11.7)(8.85x10^{-14})}{(1.6x10^{-19})(2x10^{16})}\right]^{1/2} \times\left[\sqrt{2(0.365)+2.5} - \sqrt{2(0.365)+0.60}\right]$$

or

$$\underline{\Delta L = 0.1188\ \mu m}$$

$$\frac{I'_D}{I_D} = \frac{L}{L - \Delta L} = \frac{1}{1 - 0.1188} \Rightarrow$$

$$\underline{\frac{I'_D}{I_D} = 1.135}$$

E12.3

$$\frac{I'_D}{I_D} = \frac{1}{1 - \left(\frac{\Delta L}{L}\right)} = 1.25 \Rightarrow \frac{\Delta L}{L} = \frac{1.25 - 1}{1.25}$$

or

$$\frac{\Delta L}{L} = 0.20$$

$$V_{DS}(sat) = 0.80 - 0.40 = 0.40\ V$$

Now

$$\Delta L = \left[\frac{2(11.7)(8.85x10^{-14})}{(1.6x10^{-19})(2x10^{16})}\right]^{1/2} \times\left[\sqrt{2(0.365)+2.5} - \sqrt{2(0.365)+0.40}\right]$$

or

$$\Delta L = 0.1867\ \mu m$$

Then

$$0.20 = \frac{0.1867}{L} \Rightarrow$$

$$\underline{L = 0.934\ \mu m}$$

E12.4

(a) $$I_D(sat) = \frac{\mu_n C_{ox} W}{2L}(V_{GS} - V_T)^2$$

$$= \frac{(1000)(10^{-8})(10^{-3})}{2(10^{-4})}(V_{GS} - 0.4)^2$$

or

$$I_D(sat) = 0.50x10^{-4}(V_{GS} - 0.4)^2$$

or

$$\underline{I_D(sat) = 50(V_{GS} - 0.4)^2\ \mu A}$$

(b) $$I_D(sat) = WC_{ox}v_{sat}(V_{GS} - V_T)$$

$$= (10^{-3})(10^{-8})(5x10^{6})(V_{GS} = 0.4)$$

or

$$I_D(sat) = 5x10^{-5}(V_{GS} - 0.4)$$

or

$$\underline{I_D(sat) = 50(V_{GS} - 0.4)\ \mu A}$$

E12.5

$$L \to kL = (0.7)(1) \Rightarrow \underline{L = 0.7\ \mu m}$$

$$W \to kW = (0.7)(10) \Rightarrow \underline{W = 7\ \mu m}$$

$$t_{ox} \to kt_{ox} = (0.7)(250) \Rightarrow \underline{t_{ox} = 175\ \mathring{A}}$$

$$N_a \to \frac{N_a}{k} = \frac{5x10^{15}}{0.7} \Rightarrow \underline{N_a = 7.14x10^{15}\ cm^{-3}}$$

$$V_D \to kV_D = (0.7)(3) \Rightarrow \underline{V_D = 2.1\ V}$$

E12.6

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{250x10^{-8}} = 1.38x10^{-7}$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{15}}{1.5x10^{10}}\right) = 0.316\ V$$

$$x_{dT} = \left\{\frac{4(11.7)(8.85x10^{-14})(0.316)}{(1.6x10^{-19})(3x10^{15})}\right\}^{1/2}$$

or

$$x_{dT} = 0.522x10^{-4}\ cm$$

Now

$$\Delta V_T = \frac{-(1.6x10^{-19})(3x10^{15})(0.522x10^{-4})}{1.38x10^{-7}} \times\left\{\frac{0.3}{0.8}\left[\sqrt{1+\frac{2(0.522)}{0.3}}-1\right]\right\}$$

or

$$\underline{\Delta V_T = -0.076\ V}$$

E12.7

$$\phi_{ms} \cong +0.35$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$$

$$x_{dT} = \left\{\frac{4(11.7)(8.85x10^{-14})(0.288)}{(1.6x10^{-19})(10^{15})}\right\}^{1/2}$$

or

$$x_{dT} = 0.863\ \mu m$$

$$|Q'_{SD}(\max)| = (1.6x10^{-19})(10^{15})(0.863x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.38x10^{-8}$$

$$Q'_{ss} = (1.6x10^{-19})(5x10^{10}) = 8x10^{-9}$$

Then

$$V_{TN} = \frac{(1.38x10^{-8} - 8x10^{-9})(200x10^{-8})}{(3.9)(8.85x10^{-14})} + 0.35 + 2(0.288)$$

or

$$\underline{V_{TN} = +0.959\ V}$$

We find

$$C_{ox} = \frac{(3.9)(8.85x10^{-14})}{200x10^{-8}} = 1.73x10^{-7}\ F/cm^2$$

Now

$$V_T = V_{TO} + \Delta V$$

or

$$+0.4 = +0.959 + \Delta V$$

which yields

$$\underline{\Delta V = -0.559\ V}$$

Implant Donors for negative shift

Now

$$|\Delta V_T| = \frac{eD_I}{C_{ox}} \Rightarrow D_I = \frac{|\Delta V_T| C_{ox}}{e}$$

so

$$D_I = \frac{(0.559)(1.73x10^{-7})}{1.6x10^{-19}} \Rightarrow$$

$$\underline{D_I = 6.03x10^{11}\ cm^{-2}}$$

E12.8

Using the results of E12.7

$$V_{TO} = +0.959\ V$$

$$V_T = V_{TO} + \Delta V$$

or

$$\Delta V = V_T - V_{TO} = -0.4 - 0.959 \Rightarrow$$

$$\underline{\Delta V = -1.359\ V}$$

Implant donors for a negative shift

Now

$$|\Delta V| = \frac{eD_I}{C_{ox}} \Rightarrow D_I = \frac{|\Delta V| C_{ox}}{e}$$

so

$$D_I = \frac{(1.359)(1.73x10^{-7})}{1.6x10^{-19}} \Rightarrow$$

or

$$\underline{D_I = 1.47x10^{12}\ cm^{-2}}$$

Chapter 15

Exercise Solutions

E15.1

(a) Collector Region

$$x_n = \left\{ \frac{2 \in_s (V_{bi} + V_R)}{e} \left(\frac{N_a}{N_d} \right) \left(\frac{1}{N_a + N_d} \right) \right\}^{1/2}$$

Neglecting V_{bi} compared to V_R;

$$x_n = \left\{ \frac{2(11.7)(8.85x10^{-14})(200)}{1.6x10^{-19}} \times \left(\frac{10^{16}}{10^{14}} \right) \left(\frac{1}{10^{16} + 10^{14}} \right) \right\}^{1/2}$$

or

$$\underline{x_n = 50.6\ \mu m}$$

(b) Base Region

$$x_p = \left\{ \frac{2(11.7)(8.85x10^{-14})(200)}{1.6x10^{-19}} \times \left(\frac{10^{14}}{10^{16}} \right) \left(\frac{1}{10^{16} + 10^{14}} \right) \right\}^{1/2}$$

or

$$\underline{x_p = 0.506\ \mu m}$$

E15.2

(a) $V_{CC} = 30\ V$, $V_{CE} = 30 - I_C R_C$

Now, maximum power

$\underline{P_T = 10\ W}$, $P_T = 10 = I_C V_{CE}$

Maximum power at $V_{CE} = \frac{1}{2} V_{CC} = \frac{30}{2} = 15\ V$

Then, maximum power at $I_C = \frac{10}{V_{CE}} = \frac{10}{15} = \frac{2}{3}\ A$

Then,

$$I_C(\max) = 2\left(\frac{2}{3}\right) \Rightarrow \underline{I_C(\max) = 1.33\ A}$$

At the maximum power point,

$$15 = 30 - \left(\frac{2}{3}\right) R_L$$

which yields

$$\underline{R_L = 22.5\ \Omega}$$

(b) $V_{CC} = 15\ V \Rightarrow I_C(\max) = 2\ A$

$V_{CE} = V_{CC} - I_C R_L$

We have

$$0 = 15 - (2) R_L \Rightarrow R_L = 7.5\ \Omega$$

Maximum power at the center of the load line, or at $V_{CE} = 7.5\ V$, $I_C = 1\ A$

Then

$$P(\max) = (1)(7.5) \Rightarrow \underline{P(\max) = 7.5\ W}$$

E15.3

$$V_{CE} = V_{CC} - I_C R_E \Rightarrow V_{CE} = 20 - I_C (0.2)$$

so

$$0 = 20 - I_C(\max)(0.2) \Rightarrow I_C(\max) = 100\ mA$$

Maximum power at the center of the load line, or

$$P(\max) = (0.05)(10) \Rightarrow \underline{P(\max) = 0.5\ W}$$

E15.4

For $V_{DS} = 0$, $I_D(\max) = \frac{24}{20} \Rightarrow$

$$\underline{I_D(\max) = 1.2\ A}$$

For $I_D = 0$, $V_{DS}(\max) = V_{DD} \Rightarrow$

$$\underline{V_{DS}(\max) = 24\ V}$$

Maximum power at the center of the load line, or at $I_D = 0.6\ A$, $V_{DS} = 12\ V$

Then $P(\max) = (0.6)(12) \Rightarrow$

$$\underline{P(\max) = 7.2\ W}$$

E15.5

Power $= I_D V_{DS} = (1)(12) = 12\ W$

(c) Heat sink:

$$T_{snk} = T_{amb} + P \cdot \theta_{snk-amb}$$

or

$$T_{snk} = 25 + (12)(4) \Rightarrow \underline{T_{snk} = 73°\ C}$$

(b) Case:

$$T_{case} = T_{snk} + P \cdot \theta_{case-snk}$$

or

$$T_{case} = 73 + (12)(1) \Rightarrow \underline{T_{case} = 85°\ C}$$

(a) Device:

$$T_{dev} = T_{case} + P \cdot \theta_{dev-case}$$

or

$$T_{dev} = 85 + (12)(3) \Rightarrow \underline{T_{dev} = 121°\ C}$$

E15.6

$$\theta_{dev-case} = \frac{T_{j,\max} - T_{amb}}{P_{D,rated}} = \frac{200 - 25}{50} = 3.5\ °C/W$$

$$P_D(\max) = \frac{T_{j,\max} - T_{amb}}{\theta_{dev-case} + \theta_{case-snk} + \theta_{snk-amb}}$$

$$= \frac{200 - 25}{3.5 + 0.5 + 2} \Rightarrow$$

$$\underline{P_D(\max) = 29.2\ W}$$

Now

$$T_{case} = T_{amb} + P_D(\max)\left[\theta_{case-snk} + \theta_{snk-amb}\right]$$

$$= 25 + (29.2)(0.5 + 2) \Rightarrow$$

$$\underline{T_{case} = 98°\ C}$$