Homework 1

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1. 2/3
2. 1/6
3. 8/18
4. 25/72

2

Let $P(1to4) = x$, then $P(5) = 3x$ and $P(5) = 3x$
And we know that $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1 \implies 10x = 1$
Probability of rolling 6 is 3/10

3

Given that $P(A) = 0.3$ and $P(B) = 0.6$, A and B are independent events.
For any two events $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$
For independent events A and B, $P(A \wedge B)$ is given as
$P(A \wedge B) = P(A)P(B)$
Therefore,
$P(A \vee B) = P(A) + P(B) - P(A)P(B)$
$P(A \vee B) = 0.3 + 0.6 - 0.3 * 0.6 = 0.72$

4

Given that $P(D) = 0.2, P(+|D) = 0.7, P(+|\neg D) = 0.4$

1. $P(+, D) = P(+|D)(P(D)$
   $= 0.7 \times 0.2 = 0.14$

2. $P(+) = P(+, D) + P(+, \neg D)$
   $= 0.14 + 0.4 \times 0.8 = 0.46$

3. $P(D|+) \times P(+) = P(+|D) \times P(D)$
   $P(D|+) = \frac{P(+,D)}{P(+)}$
   $= \frac{0.14}{0.46} = 0.304$

5

Let D and S are the random variables representing Cancer and Smoker respectively
$P(D) = 0.0002, P(\neg D) = 0.9998, P(S) = 0.04$ and $P(\neg S) = 0.96$
$P(D|S) = 0.002$ and we need to find $P(D|\neg S)$
$P(D|\neg S) = \frac{P(D \wedge \neg S)}{P(\neg S)}$
$P(D \wedge S) = P(D|S)P(S) = 0.002 \times 0.04 = 0.00008$
$P(S|D) = \frac{P(D \wedge S)}{P(D)} = \frac{0.00008}{0.0002} = 0.4$ that gives $P(\neg S|D) = 0.6$
$P(D|\neg S) = \frac{P(\neg S|D) \times P(D)}{P(\neg S)} = \frac{0.6}{0.96} \times 0.0002 = 0.000125$

6

1. $P(R|a, \neg u) = \langle P(r|a, \neg u), P(\neg r|a, \neg u)\rangle$
   $= \alpha\langle 0.06, 0.10\rangle$
   $\alpha = \frac{1}{0.06+0.10} = 6.25$
   $P(R|a, \neg u) = \langle 0.375, 0.625\rangle$

2. $P(R) = \langle 0.48, 0.52\rangle$ and $P(U) = \langle 0.52, 0.48\rangle$
   $P(R = 0, U = 0) = 0.34$
   As $P(R = 0, U = 0) = 0.34 \neq P(R = 0) \times P(U = 0) = 0.48 \times 0.52 = 0.2496$
   R and U are not independent

3. $P(A) = \langle 0.6, 0.4\rangle$
   $P(R = 0|A = 0) = \frac{0.3}{0.6} = 0.5$
   $P(U = 0|A = 0) = \frac{0.36}{0.6} = 0.6$
   $P(R = 0, U = 0|A = 0) = \frac{0.24}{0.6} = 0.4$
   As $P(R = 0, U = 0|A = 0) = 0.4 \neq P(R = 0|A = 0) \times P(U = 0|A = 0) = 0.5 \times 0.6 = 0.3$
   R and U are not independent given A

4. If R and A are independent then $P(R, A) = P(R) \times P(A)$
   $P(R) = \langle 0.48, 0.52\rangle$

$P(A) = \langle 0.6, 0.4 \rangle$
$P(R=0, A=0) = 0.3 \neq P(R=0) \times P(A=0) = 0.48 \times 0.6 = 0.288$
Therefore R and A are not independent

5. $P(U) = \langle 0.52, 0.48 \rangle$
   $P(R=0|U=0) = \frac{0.34}{0.52}$
   $P(A=0|U=0) = \frac{0.36}{0.52}$
   $P(R=0, A=0|U=0) = \frac{0.24}{0.52}$
   $P(R=0|U=0) \times P(A=0|U=0) = \frac{0.34}{0.52} \times \frac{0.36}{0.52} = \frac{0.2335}{0.52}$
   Thus, $P(R=0, A=0|U=0) \neq P(R=0|U=0) \times P(A=0|U=0)$
   Therefore A and R are not independent given U

7

Given that $P(a) = 0.2, P(b) = 0.3, P(c) = 0.4$

1. As A and B are mutually exclusive $P(A \wedge B) = 0$
   Therefore,
   $P(A \vee B) = P(A) + P(B) = 0.2 + 0.3 = 0.5$

2. As B and C are independent $P(B \wedge C) = P(B) \times P(C)$
   $P(B \vee C) = P(B) + P(C) - P(B) \times P(C) = 0.3 + 0.4 - 0.3 \times 0.4 = 0.58$