1.

a

Joint distribution

	$\neg A$	A
$\neg B$	0.25	0
B	0.25	0.5

$$Likelihood = P(D|M) = \Pi_{i=1}^{N} P(D, H = 0|M) + \Pi_{i=1}^{N} P(D, H = 1|M)$$
$$P(D|M) = 0.25 \times 0.25 \times 0.5 \times 0.5 \times (0.5 + 0)$$
$$P(D|M) = 2\log(0.25) + 3\log(0.5)$$
$$= -4.8520$$

b

In order to make the log likelihood of the actual data biggest, we fill in the missing value with 1 in this case.

Hence the joint distribution is

	$\neg A$	A
$\neg B$	0.2	0
B	0.2	0.6

$$Likelihood = P(D|M) = \Pi_{i=1}^{N} P(D, H = 0|M) + \Pi_{i=1}^{N} P(D, H = 1|M)$$
$$P(D|M) = 0.25 \times 0.25 \times 0.6 \times 0.6 \times (0.6 + 0)$$
$$P(D|M) = 2\log(0.2) + 3\log(0.6)$$
$$= -4.7514$$

c

Joint distribution

	$\neg A$	A
$\neg B$	0.2	0.1
B	0.2	0.5

$$Likelihood = P(D|M) = \Pi_{i=1}^{N} P(D, H = 0|M) + \Pi_{i=1}^{N} P(D, H = 1|M)$$
$$P(D|M) = 0.2 \times 0.2 \times 0.5 \times 0.5 \times (0.1 + 0.5)$$
$$\log(P(D|M)) = 2\log(0.2) + 2\log(0.5) + \log(0.6)$$
$$= -5.1160$$

d

new distribution over H

$$P(H|D, M) = P(A, B|M)/P(A|M)$$
$$= 0.5/0.6$$
$$P(H|D, M) = 0.8333$$

Joint distribution

	$\neg A$	A
$\neg B$	0.2	0.0333
B	0.2	0.5666

$$Likelihood = P(D|M) = \Pi_{i=1}^{N} P(D, H = 0|M) + \Pi_{i=1}^{N} P(D, H = 1|M)$$
$$P(D|M) = 0.2\times 0.2\times 0.5666\times 0.5666\times(0.0333 + 0.5666)$$
$$\log(P(D|M)) = 2\log(0.2) + 2\log(0.566) + \log(0.6)$$
$$= -4.8659$$

2.

iteration 1

Given that initial values of $P(H) = 0.4, P(A|H) = 0.55, P(A|\neg H) = 0.6, P(B|H) = 0.42 \; and \; P(B|\neg H) = 0.52$

Distribution over Hidden variable

$$P(H|A,B) = P(A,B|H)\times\frac{P(H)}{P(A \wedge B)}$$
$$= \frac{P(A|H)P(B|H)P(H)}{P(A \wedge B \wedge H) + P(A \wedge B \wedge \neg H)}$$
$$= \frac{0.55\times 0.42\times 0.4}{0.55\times 0.42\times 0.4 + 0.6\times 0.52\times 0.6} = 0.33$$

$$P(H|\neg A, \neg B) = \frac{0.45\times 0.58\times 0.4}{0.45\times 0.58\times 0.4 + 0.6\times 0.48\times 0.4} = 0.4754$$

$$P(H|\neg A, B) = \frac{0.45\times 0.42\times 0.4}{0.45\times 0.42\times 0.4 + 0.6\times 0.4\times 0.52} = 0.3772$$

$$P(H|A, \neg B) = \frac{0.55\times 0.58\times 0.4}{0.55\times 0.58\times 0.4 + 0.6\times 0.6\times 0.48} = 0.4248$$

A	B	#	$P(H\|D,\theta)$
0	0	1	0.4754
0	1	1	0.3772
1	0	1	0.4248
1	1	3	0.3305

$$P(H) = \frac{0.4754 + 0.3772 + 0.4248 + 3\times 0.3305}{6} = 0.3782$$

$$P(A|H) = \frac{P(A \wedge H)}{P(H)} = \frac{\frac{0.3305\times 3 + 0.4248}{6}}{0.3782} = 0.6241$$

$$P(A|\neg H) = \frac{\frac{0.6695\times 3 + 0.5752}{6}}{0.6218} = 0.6925$$

$$P(B|H) = \frac{\frac{0.3305\times 3 + 0.3772}{6}}{0.3782} = 0.6032$$

$$P(B|\neg H) = \frac{\frac{0.6695\times 3 + 0.6228}{6}}{0.6218} = 0.7053$$

likelihood $P(D|\theta) = \Pi_{i=1}^{6} P(D_i|H,\theta)P(D_i|\neg H,\theta)$

$$\log P(D|\theta) = \sum_{i=1}^{6} \sum_{\mathrm{h}\in\{\mathrm{H},\neg\mathrm{H}\}} \log(P(D_i|h,\theta)P(h))$$

$$= 3\sum_{\mathrm{h}\in\{\mathrm{H},\neg\mathrm{H}\}} \log(P(A,B|h,\theta)P(h)) + \sum_{\mathrm{h}\in\{\mathrm{H},\neg\mathrm{H}\}} \log(P(\neg A,\neg B|h,\theta)P(h))$$
$$+ \sum_{\mathrm{h}\in\{\mathrm{H},\neg\mathrm{H}\}} \log(P(\neg A,B|h,\theta)P(h)) + \sum_{\mathrm{h}\in\{\mathrm{H},\neg\mathrm{H}\}} \log(P(A,\neg B|h,\theta)P(h))$$

$$3\left(\log\big(P(A|H)P(B|H)P(H)\big) + \log\,(P(A|\neg H)P(B|\neg H)P(\neg H)\big)\right.$$
$$= 3 * \big(log(0.3782 * 0.6241 * 0.6032) + log(0.6218 * 0.6925 * 0.7053)\big)$$
$$= -9.4230$$

$$\log\big(P(\neg A|H)P(\neg B|H)P(H)\big) + \log\big(P(\neg A|\neg H)P(\neg B|\neg H)P(\neg H)\big)$$
$$= (log(0.3782 * (1 - 0.6241) * (1 - 0.6032)) + log(0.6218 * (1 - 0.6925)$$
$$* (1 - 0.7053)))$$
$$= -5.7513$$

$$\log\big(P(\neg A|H)P(B|H)P(H)\big) + \log\big(P(\neg A|\neg H)P(B|\neg H)P(\neg H)\big)$$
$$= \Big(log\big(0.3782 * (1 - 0.6241) * (0.6032)\big)$$
$$+ log\big(0.6218 * (1 - 0.6925) * (0.7053)\big)\Big) = -4.4598$$

$$\log\big(P(A|H)P(\neg B|H)P(H)\big) + \log\big(P(A|\neg H)P(\neg B|\neg H)P(\neg H)\big) = -4.4325$$

$$\log P(D|\theta) = -9.4230 - 5.7513 - 4.4598 - 4.4325 = -24.0666$$

iteration 2

$$P(H) = 0.3782, P(A|H) = 0.6241, P(A|\neg H) = 0.6925, P(B|H) = 0.6032 \; and \; P(B|\neg H) = 0.7053$$

$$P(H|A,B) = \frac{0.6241 \times 0.6032 \times 0.3782}{0.6241 \times 0.6032 \times 0.3782 + 0.6218 \times 0.6925 \times 0.7053} = 0.3192$$

$$P(H|\neg A,\neg B) = \frac{0.3759 \times 0.3968 \times 0.3782}{0.3759 \times 0.3968 \times 0.3782 + 0.6218 \times 0.3075 \times 0.2947} = 0.5003$$

$$P(H|\neg A,B) = \frac{0.3759 \times 0.6032 \times 0.3782}{0.3759 \times 0.6032 \times 0.3782 + 0.6218 \times 0.3075 \times 0.7053} = 0.3887$$

$$P(H|A,\neg B) = \frac{0.6241 \times 0.3968 \times 0.3782}{0.6241 \times 0.3968 \times 0.3782 + 0.6218 \times 0.6925 \times 0.2947} = 0.4246$$

A	B	#	$P(H\|D,\theta)$
0	0	1	0.5003
0	1	1	0.3887
1	0	1	0.4246
1	1	3	0.3192

$$P(H) = \frac{0.5003 + 0.3887 + 0.4246 + 3 \times 0.3192}{6} = 0.3785$$

$$P(A|H) = \frac{P(A \wedge H)}{P(H)} = \frac{\frac{0.3192 \times 3 + 0.4246}{6}}{0.3785} = 0.6086$$

$$P(A|\neg H) = \frac{\frac{0.6808 \times 3 + 0.5754}{6}}{0.6215} = 0.702$$

$$P(B|H) = \frac{\frac{0.3192 \times 3 + 0.3887}{6}}{0.3785} = 0.5928$$

$$P(B|\neg H) = \frac{\frac{0.6808 \times 3 + 0.6113}{6}}{0.6215} = 0.7116$$

$$\log P(D|\theta) = \sum_{i=1}^{6} \sum_{\text{h}\in\{\text{H},\neg\text{H}\}} \log(P(D_i|h,\theta)P(h))$$

$$= 3 \sum_{\text{h}\in\{\text{H},\neg\text{H}\}} \log(P(A,B|h,\theta)P(h)) + \sum_{\text{h}\in\{\text{H},\neg\text{H}\}} \log(P(\neg A,\neg B|h,\theta)P(h))$$
$$+ \sum_{\text{h}\in\{\text{H},\neg\text{H}\}} \log(P(\neg A,B|h,\theta)P(h)) + \sum_{\text{h}\in\{\text{H},\neg\text{H}\}} \log(P(A,\neg B|h,\theta)P(h))$$

$$3\left(\log\big(P(A|H)P(B|H)P(H)\big) + \log\left(P(A|\neg H)P(B|\neg H)P(\neg H)\right)\right)$$
$$= 3 * (log(0.3785 * 0.6086 * 0.5928) + log((1 - 0.3785) * 0.702 * 0.7116))$$
$$= -9.4821$$

$$\log\big(P(\neg A|H)P(\neg B|H)P(H)\big) + \log\big(P(\neg A|\neg H)P(\neg B|\neg H)P(\neg H)\big)$$
$$= (log(0.3785 * (1 - 0.6086) * (1 - 0.5928)) + log((1 - 0.3785) * (1 - 0.702) * (1 - 0.7116)))$$
$$= -5.7377$$

$$\log\big(P(\neg A|H)P(B|H)P(H)\big) + \log\big(P(\neg A|\neg H)P(B|\neg H)P(\neg H)\big)$$
$$= (log(0.3785 * (1 - 0.6086) * (0.5928)) + log((1 - 0.3785) * (1 - 0.702) * (0.7116))) = -4.4590$$

$$\log\big(P(A|H)P(\neg B|H)P(H)\big) + \log\big(P(A|\neg H)P(\neg B|\neg H)P(\neg H)\big) = -4.4394$$

$$\log P(D|\theta) = -9.4821 - 4.4590 - 5.7377 - 4.4394 = -24.1182$$

3.

Given that $P(C_0) = 1 \; and \; P(H_0) = 0$.

1. $P(H_1)$
$$= P(H_1|C_0)P(C_0) + P(H_1|H_0)P(H_0)$$
$$= 0.4\times 1 + 0.3\times 0 = 0.4$$
$$\therefore P(C_1) = 0.6$$

   $P(H_2)$
$$= P(H_2|C_1)P(C_1) + P(H_2|H_1)P(H_1)$$
$$0.4\times 0.6 + 0.3\times 0.4 = 0.36$$

2. $\lim_{\text{t}\to\infty} P(C_t)$
$$= P(C_t|C_{t-1})P(C_{t-1}) + P(C_t|H_{t-1})P(H_{t-1})$$
$$= 0.6\times P(C_t) + 0.7\times\big(1 - P(C_t)\big)$$
$$let \; P(C_t) = x$$
$$x = 0.6\times x + 0.7\times(1 - x)$$
$$0.9x = 0.7 \Rightarrow x = \frac{7}{9}$$

4.
Given that $P(C_0) = 1 \ and \ P(H_0) = 0$, and we observed that student wear shorts.

1. $P(C_1|S_1)$

$$= \frac{P(S_1|C_1)P(C_1)}{P(S_1)}$$
$$P(S_1) = P(S_1|C_1)P(C_1) + P(S_1|H_1)P(H_1)$$
$$= 0.1 \times 0.6 + 0.8 \times 0.4 = 0.38$$
$$P(C_1|S_1) = \frac{0.1 \times 0.6}{0.38} = \frac{6}{38}$$

2. $P(C_2|S_1)$, according to the given network, C_2 is independent of S_1.

$$P(C_2|S_1) = P(C_2)$$
$$= P(C_2|C_1)P(C_1)|P(C_2|H_1)P(H_1)$$
$$= 0.6 \times 0.6 + 0.7 \times 0.4 = 0.64$$

5.

Stock market prediction

Complex, volatile nature of stock markets has been the biggest challenge for research community. Fluctuations in prices depends on myriad factors such as interest rates, securities, government policies, and company's status. Human traders cannot master such markets. Thus, there is a need for developing an AI system that could deal with the complex nature of stock markets.

Stock markets are the integral part of global economy, any fluctuations in the markets would affect individuals, corporate companies and the economy of a country. Information of a stock value beforehand even by a second yields huge profits.

Previously techniques like auto-regression moving averages (ARMA), artificial neural networks (ANN) support vector machines (SVMs) have been proposed to solve the problem of forecasting [1,2]. However, most of them have their own constraints.

HMMs are good at handling time dependent phenomena such as speech recognition, ECG analysis etc. HMMs have hidden states among which transitions occur, those hidden states are unobservable. Each state is associated with a set of possible observations. Similarly, stock market forecasting has a set of unobservable states that determine the behavior of stock value. Transitions between those states is based on previous day stock prices, company policies, government policies, economic conditions etc.

The aim is to predict the closing stock price based on the historical data, i.e. the open, close, low and high stock prices of previous days. As a part of this project, I would train a HMM on

Apple Inc. stock prices data, which can be collected from [3]. Baum-Welch algorithm would be applied to find the best model parameters such as transition matrix, observation emission matrix, etc.

The experiments would include training the HMM with historical data of different sizes. Testing constitutes of predicting the closing stock price of a day given a sequence of observations till its previous day. Mean absolute percentage error (MAPE) is used as a quantitative metric.

[1] Stock Market Forecasting Using Hidden Markov Model: A New Approach, MR Hassan, B Nath, *International conference on intelligent systems design and applications,* IEEE, 2005.
[2] Stock Market Prediction Using Hidden Markov Models, *Students conference on engineering and systems*, IEEE, 2012.
[3]https://www.quantconnect.com/data - symbol/usa/aapl