Expt.No.8: # Stefan's Law Verification

Aim:

1. To verify the Stefan's T^4law by using the filament of the bulb as a back body with ohm's law experimental setup.
2. Assume the power output from the filament is proportional to the power given to the circuit and temp of the filament is proportional to the resistance of the filament.
3. Use LSF method to calculate the slope which verifies the variation of power output against power of temp.
4. Comment on the results.

Apparatus:

Experimental set up; voltmeter, ammeter, rheostat (0-1KΩ), 10-20 watt bulb.

Introduction:

Emission and absorption of Radiation are inverse processes. A perfect emitter of the radiation is perfect absorber of radiation. Therefore at room temperature such an object appears to be perfectly black, hence they are called black body, however the name refers to idealized case. One can't have the perfect black body. In fact, by virtue of temperature all objects in the universe emit thermal radiation. Objects which are at 0 K don't emit thermal radiation, because it doesn't have thermal energy. *A human body emit thermal radiation (T=300K) and absorbs thermal radiation, hence can be approximated to black body. There is a balance between absorption and emission hence human body temperature remains constant, any imbalance results in fever or cold.*

For the laboratory experiment metallic ball or electric bulb can be treated as a nearly black body radiator. During the 20th century Scientists Lord Raleigh, Wein, Max Planck. Stefan and many other used such a black body radiator and studied the radiation process. In 1879 Austrian physicist Stefan using black body arrived at an empirical formula to account for the heat radiation. According to him the energy (E) radiated per unit area by the black body is directly proportional to the fourth power of the absolute temperature (T). Writing in the form of equation,

$$P = A\sigma T^4$$

$$\text{since} \quad R \alpha T$$

$$R = CT$$

$$P = AC^4 \sigma R^4$$

$$P = (constant) x R^4$$

$$\log P = \log(constant) + 4 \log R$$

$$y = mx + c$$

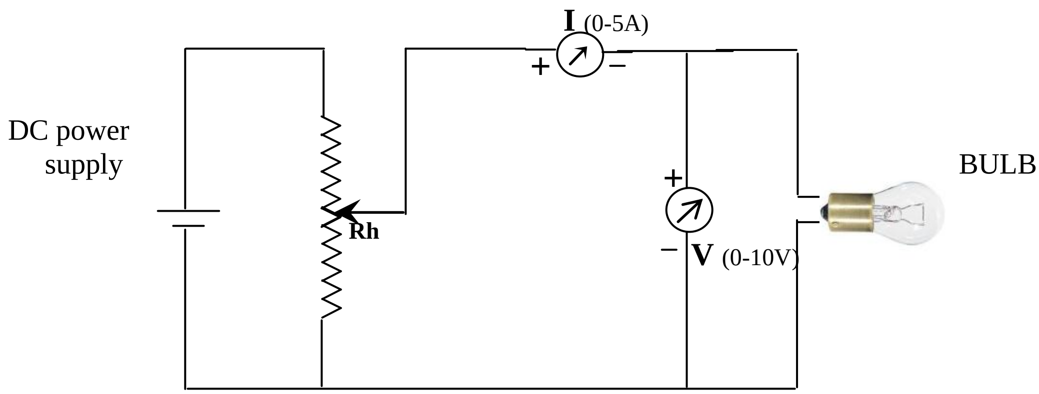
where σ is the proportionality constant known as Stefan's constant.

σ =5.5 $\times 10^{-5}$eargs / $K^4 cm^2$sec. or =5.5 $\times 10^{-8}$joules / $K^4 m^2$sec. (Literature value)

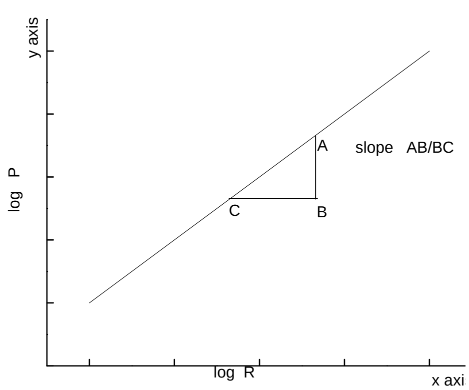
Procedure:

1) The circuit is connected as shown in the circuit diagram.
2) The voltage is set using the rheostat for the bulb just glows and the current I in the ammeter is noted. Power P is calculated using $P = VI$.
3) The resistance is found using: $R = \frac{V}{I}$.
4) The experiment is repeated for different voltages, from the initial setup voltage (increment around 1 volt upto 10 volts)
5) Plot a graph of log(P) .vs. log(R), the slope gives 4, which is the verification of Stefan's T^4 law

Circuit diagram:



Nature of Graph:



Tabulation:

Obs No.	Voltage V (volts)	Current I (amps)	$R = \frac{V}{I}$ (Ω)	$P = VI$ J / sec (Watt)	Log R	Log P
Error	±	±				
1	Initial Volt; V=					
2						
3						
4						
5						
6						
7						
8						
9						

Viva Questions:

1) What is black body?
2) What is black body radiation?
3) State Stefan's law of black body radiation
4) State Kirchoff's law of black body radiation

Theory (Estimation of errors)

Measurement is fundamental to the growth and application of science. We know that no physical measurement gives the exact or true value. There will be always some uncertainty or error in measurement. It is therefore necessary to know the ways in which measurement variations arise.

We know that the physical quantities which can be expressed by a single value varies with the conditions under which it is measured and our experiment sets out to find this relation. The simplest of these is linear type of relationship which can be expressed as

$$y = mx + c$$

where x and y are quantities which are measured. The values m and c are constants. These values can be determined by measuring a set of different values $x_1, x_2, x_3 \ldots\ldots x_n$ together with their corresponding $y_1, y_2, y_3 \ldots y_n$ values.

If all the measurements were without error and the linear relationship were true, then all the points (x_1, y_1), (x_2, y_2), $(x_3, y_3) \ldots (x_n, y_n)$ plotted as a Cartesian graph would lie exactly on a straight line. Its slope (dy/dx) would give the unknown m and its intercept on the y-axis would give the second unknown c. however one and perhaps both the measured quantities will be subjected to error and plotted points will therefore lie about the true straight line, close to it if the errors are small, more widely scattered if the errors are large. The problem now is to give the best estimate of this true straight line-to decide what the best line is.

Here we use least square fit method to get the best straight line for experimental points. We assume the precession in the x-value is much greater than y-value. So we plot x-value for each point without error and the value of y is plotted as mx+c by knowing the values of m and c. the values of y = mx + c corresponding to $x_1, x_2, x_3 \ldots\ldots x_n$ would be $y_1', y_2', y_3' \ldots y_n'$ and now all the points (x_1, y_1'), (x_2, y_2'), $(x_3, y_3') \ldots (x_n, y_n')$ lie on the true straight line. The error in the measured values of y at these points would then be

$$\varepsilon_1 = y_1 - y_1' = y_1 - mx_1 - c \ ,$$

$$\varepsilon_2 = y_2 - y_2' = y_2 - mx_2 - c \, .$$

$$.$$

$$.$$

$$.$$

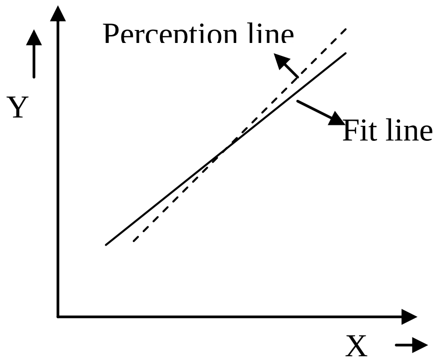
$$. \varepsilon_n = y_n - y_n' = y_n - mx_n - c$$

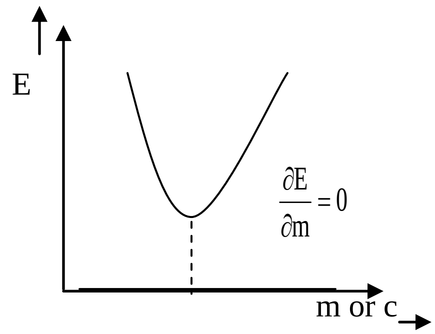
According to leat square principle, the sum of squares of errors should be minimum.

i.e. $E = \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \ldots\ldots\ldots\ldots\ldots + \varepsilon_n^2$

$$E = (y_1 - mx_1 - c)^2 + (y_2 - mx_2 - c)^2 + \ldots\ldots\ldots\ldots\ldots\ldots + (y_n - mx_n - c)^2$$

Here E depends on two assumed true values m and c. The plot E versus m or c values for different straight lines on the basis of perception will be a parabola. Thus the error in m and c will be minimum when the partial derivative of E with respect to m and c are zero.





Therefore $\frac{\partial E}{\partial m} = 0$ and $\frac{\partial E}{\partial c} = 0$

These equations simplify to,

$$\frac{\partial E}{\partial m} = 2(y_1 - mx_1 - c)(-x_1) + 2(y_2 - mx_2 - c)(-x_2) + ... + 2(y_n - mx_n - c)(-x_n)$$

$$\Rightarrow 0 = \sum x_n y_n - m\sum x_n^{\ 2} - c\sum x_n \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

and

$$\frac{\partial E}{\partial c} = 2(y_1 - mx_1 - c)(-1) + 2(y_2 - mx_2 - c)(-1) + ... + 2(y_n - mx_n - c)(-1)$$

$$\Rightarrow 0 = \sum y_n - m\sum x_n - nc \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

On solving the simultaneous equations (1) and (2) we get,

$$m = \frac{n\sum x_n y_n - \sum x_n \sum y_n}{n\sum x_n^2 - \left(\sum x_n\right)^2}$$

and

$$c = \frac{\sum x_n^2 \sum y_n - \sum x_n \sum x_n y_n}{n\sum x_n^2 - \left(\sum x_n\right)^2}$$

Thus from set of experimental values and using above equations we can calculate the slope and y-intercept values of the best straight line for the experimental values.

Alternate method:

The equation of straight line is,

$$y = mx + c \ .... \quad (3)$$

generate another equation by multiplying x on both sides, we get,

$$xy = mx^2 + cx \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

on adding n equations (3) and (4) becomes,

$$\sum y_n = m\sum x_n + nc \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (5)$$

and

$$\sum x_n y_n = m\sum x_n^2 + c\sum x_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (6)$$

On solving the simultaneous equations (5) an (6) we get,

$$m = \frac{n\sum x_n y_n - \sum x_n \sum y_n}{n\sum x_n^2 - \left(\sum x_n\right)^2}$$

and

$$c = \frac{\sum x_n^2 \sum y_n - \sum x_n \sum x_n y_n}{n\sum x_n^2 - \left(\sum x_n\right)^2}$$

Standard deviation:

It is the measure of precession of the apparatus. It is also called root mean square deviation. It is calculated by the equation,

$$\sigma_n(y) = \sqrt{\frac{\sum\left(y_n - y_n'\right)^2}{n}}$$

where y is experimental value

y' is least square fit value

n is number of observations

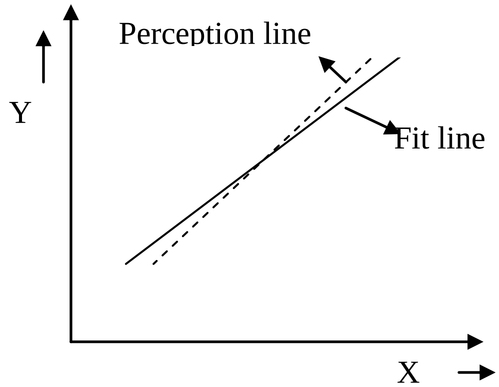
The error in the determined value of m and c by least square fit method are given by

$$\delta(m) = \frac{n\sigma_n(y)}{\left[(n-2)\left[n\sum x_n^2 - \left(\sum x_n\right)^2\right]\right]^{1/2}} \qquad \text{and} \qquad \delta(c) = \frac{n\sigma_n(y)\left(\sum x_n^2\right)^{1/2}}{\left[n(n-2)\left[n\sum x_n^2 - \left(\sum x_n\right)^2\right]\right]^{1/2}}$$

Procedure:

1. Take the verification of Stefan's T^4 law experimental data as a Log (R) is a X component and Log (P) is a Y component.
2. As given in the Table, find out the values.
3. Substitute the values in the given formula and find out slope **m** and intercept **c** and also find out the standard deviation.

Nature of Graph:



Tabulation:

Obs No (n)	X (log(R))	Y (log P)	X^2	XY	Y'	$Y-Y'$	$((Y-Y')^2$
1							
2							
3							
4							
5							
6							
7							
8							
9							

n= $\sum X$ = $\sum Y =$ $\sum X_n^2 =$ $\sum XY =$ $\sum((Y-Y')^2 =$

Calculation:

1. Slope: $m = \dfrac{n\sum x_n y_n - \sum x_n \sum y_n}{n\sum x_n^2 - \left(\sum x_n\right)^2}$

2. intercept $c = \dfrac{\sum x_n^2 \sum y_n - \sum x_n \sum x_n y_n}{n\sum x_n^2 - \left(\sum x_n\right)^2}$

3. Standard deviation: $\sigma_n(y) = \sqrt{\dfrac{\sum\left(y_n - y_n'\right)^2}{n}}$

4. Error in slope m: $\delta(m) = \dfrac{n\sigma_n(y)}{\left[(n-2)\left[n\sum x_n^2 - \left(\sum x_n\right)^2\right]\right]^{1/2}}$

5. Percentage error in slope m = $(\delta(m)/m)$ x 100

Result:

i) Experimental slope =---------

ii) Slope from least square fit method =---------

iii) Difference =---------