

Polynomial Regression Model:

X	Y
7.6	157
7.1	174

Step 1: Read dataset, $n=0.1$, epochs = 1, $m_1=1$, $m_2=1$, $c=1$

Step 2: iter = 1

Step 3: Sample $i=1$

$$\text{Step 4: } y'_p = m_2 (x_i)^2 + m_1 x_i + c$$

$$y'_p = (1)(7.6)^2 + (1)(7.6) + 1 = 64.36$$

$$\text{Step 5: } E = \frac{1}{2} (y_i - y'_p)^2$$

$$= \frac{1}{2} (157 - 64.36)^2$$

$$E = 4291.08$$

$$\text{Step 6: } \frac{\partial E}{\partial m_1} = [y_i - m_2 x_i^2 - m_1 x_i - c] x_i$$

$$= -[157 - (1)(7.6)^2 - (1)(7.6) + 1](7.6)$$

$$\frac{\partial E}{\partial m_1} = -704.06$$

$$\frac{\partial E}{\partial m_2} = -[y_i - m_2 x_i^2 - m_1 x_i - c] x_i^2$$

$$= -[157 - (1)(7.6)^2 - (1)(7.6) + 1](7.6)^2$$

$$\frac{\partial E}{\partial m_2} = -5350.88$$

$$\frac{\partial E}{\partial c} = -[y_i - m_2 x_i^2 - m_1 x_i - c]$$

$$= -[157 - (1)(7.6)^2 - (1)(7.6) + 1]$$

$$\frac{\partial E}{\partial c} = -92.64$$

$$\text{Step 7: } \Delta m_1 = -\eta \frac{\partial E}{\partial m_1} = -(0.1)(-704.06) = 70.4$$

$$\Delta m_2 = -\eta \frac{\partial E}{\partial m_2} = -(0.1)(-5350.88) = 535.08$$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = -(0.1)(-92.64) = 9.26$$

$$\text{Step 8: } m_1 = m_1 + \Delta m_1 = 1 + 70.4 = 71.4$$

$$m_2 = m_2 + \Delta m_2 = 1 + 535.08 = 536.08$$

$$c = c + \Delta c = -1 + 92.6 = 8.26$$

$$\text{Step 9: Sample } \Rightarrow i = i + 1 = 1 + 1 = 2 \text{ \& } i \leq n \text{ \& } T \rightarrow \text{Step 4}$$

$$\begin{aligned} \text{Step 4 } y_p^i &= m_2(x_i)^2 + m_1 x_i + c \\ &= (536.08)(7.1)^2 + (71.4)(7.1) + 8.26 \\ &= 27023.79 + 506.94 + 8.26 \\ y_p^i &= 27538.99 \end{aligned}$$

$$\begin{aligned} \text{Step 5: } E &= \frac{1}{2}(y_i - y_p^i)^2 = \frac{1}{2}(174 - 27538.99)^2 \\ E &= 374421338.9 \end{aligned}$$

$$\begin{aligned} \text{Step 6: } \frac{\partial E}{\partial m_1} &= -[y_i - m_2 x_i^2 - m_1 x_i - c] x_i \\ &= -[174 - (536.08)(7.1)^2 - (71.4)(7.1) - 8.26](7.1) \\ &= -(174 - 27023.79 - 506.94 - 8.26)(7.1) \\ &= -(-27364.99)(7.1) \end{aligned}$$

$$\frac{\partial E}{\partial m_1} = 194291.429$$

$$\begin{aligned} \frac{\partial E}{\partial m_2} &= -[y_i - m_2 x_i^2 - m_1 x_i - c] x_i^2 \\ &= -(-27364.99)(7.1)^2 \end{aligned}$$

$$\frac{\partial E}{\partial m_1} = 1379469.14$$

$$\frac{\partial E}{\partial c} = -[y_i - m_2 x_i^2 - m_1 x_i - c]$$

$$= -(-27364.99)$$

$$\frac{\partial E}{\partial c} = 27364.99$$

$$\text{Step 7: } \Delta m_1 = -\eta \frac{\partial E}{\partial m_1} = -(0.1)(194291.429) = -19429.14$$

$$\Delta m_2 = -\eta \frac{\partial E}{\partial m_2} = -(0.1)(1379469.14) = -137946.91$$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = -(0.1)(27364.99) = -2736.49$$

$$\text{Step 8: } m_1 = m_1 + \Delta m_1 = 71.4 - 19429.14 = -19357.74$$

$$m_2 = m_2 + \Delta m_2 = 536.08 - 137946.91 = -137410.83$$

$$c = c + \Delta c = 8.26 - 2736.49 = -2728.23$$

$$\text{Step 9: Sample } i = i + 1 = 2 + 1 = 3 \text{ } \left. \begin{matrix} \text{if } j \leq \frac{n}{2} \\ \} \end{matrix} \right\} \rightarrow \text{next step}$$

$$\text{Step 10: } \text{iter} = \text{iter} + 1 = 1 + 1 = 2, \text{iter} > \frac{\text{epoch}}{1} \rightarrow \text{next step}$$

Step 11: End step