Problems set - 2

(1) Show the following implications;
  (a) $(P \wedge Q) \implies (P \to Q)$
  (b) $P \implies (Q \to P)$
  (c) $(P \to (Q \to R)) \implies (P \to Q) \to (Q \to R)$

(2) Show the following implications(without constructing the truth tables);
  (a) $P \to Q \implies P \to (P \wedge Q)$
  (b) $(P \to Q) \to Q \implies P \vee Q$
  (c) $((P \vee \neg P) \to Q) \to ((P \vee \neg P) \to R) \implies (Q \to R)$
  (d) $(Q \to (P \vee \neg P)) \to (R \to (P \vee \neg P)) \implies (R \to Q)$

(3) Show the following equivalences;
  (a) $\neg(P \wedge Q) \iff \neg P \vee \neg Q$
  (b) $\neg(P \vee Q) \iff \neg P \wedge \neg Q$
  (c) $\neg(P \to Q) \iff P \wedge \neg Q$
  (d) $\neg(P \leftrightarrow Q) \iff (P \wedge \neg Q) \vee (\neg P \wedge Q)$

(4) Write the formulas which are equivalent to the formulas given below and which contain connectives $\wedge$ and $\vee$ only.
  (a) $\neg(P \leftrightarrow (Q \to (R \vee P)))$
  (b) $((P \vee Q) \wedge R) \to (P \vee R)$

(5) Prove that the followings sets are not functionally complete:

$$\{\vee, \wedge\}, \ \{\vee\}, \ \{\neg\}$$

(6) If $A(P, Q, R)$ is given by $P \uparrow (Q \wedge \neg(R \downarrow P))$, then find its dual A^*. Also find the formulas which are equivalent to A and A^* involving the connectives $\wedge, \vee$, and $\neg$ only.

(7) Express the formula $P \to (\neg P \to Q)$ in terms of $\uparrow$ only and also in terms of $\downarrow$ only.

(8) Express $P \uparrow Q$ in terms of $\downarrow$ only.

(9) For the formula $(P \wedge Q) \vee (\neg R \wedge \neg P)$, draw a corresponding circuit diagram using (a) NOT, AND, and OR gates (b) $NAND$ gates only.