Problems set - 1

(1) Consider the statements;

R : Mark is rich

H : Mark is happy

write the following statements in the symbolic form:

(a) Mark is poor but happy.
(b) Mark is rich or unhappy.
(c) Mark is neither rich nor happy.
(d) Mark is poor or he is both rich and unhappy.

(2) Construct the truth tables for the following formulas:
(a) $\neg(\neg P \vee \neg Q)$
(b) $\neg(\neg P \wedge \neg Q)$
(c) $P \wedge (P \vee Q)$
(d) $P \wedge (Q \wedge P)$
(e) $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$
(f) $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$
(g) $P \vee (Q \wedge R)$
(h) $(P \wedge (Q \wedge R)) \vee \neg((P \vee Q) \wedge (R \vee S))$
(i) $(\neg(P \wedge Q) \vee \neg R) \vee (((\neg P \wedge Q) \vee \neg R) \wedge S)$
(j) $(Q \wedge (P \to Q)) \to P$
(k) $\neg(P \vee (Q \wedge R)) \leftrightarrow ((P \vee Q) \wedge (P \vee R))$
(l) $(\neg(P \wedge Q) \vee \neg R) \vee ((Q \leftrightarrow \neg P) \to (R \vee \neg S))$
(m) $(P \leftrightarrow R) \wedge (\neg Q \to S)$
(n) $(P \vee (Q \to (R \wedge \neg P))) \leftrightarrow (Q \vee \neg S)$

(3) Show that the truth values of the following formulas are independent of their components:
(a) $(P \wedge (P \to Q)) \to Q$
(b) $(P \to Q) \leftrightarrow (\neg P \vee Q)$
(c) $((P \to Q) \wedge (Q \to R)) \to (P \to R)$
(d) $(P \leftrightarrow Q) \leftrightarrow ((P \wedge Q) \vee (\neg P \wedge \neg Q))$

(4) From the formulas given below select those which are well-formed formula and also indicate which ones are tautologies or contradictions.
(a) $(P \to (P \vee Q))$
(b) $((P \to (\neg P)) \to \neg P)$
(c) $((\neg Q \wedge P) \wedge Q)$
(d) $((P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$
(e) $((\neg P \to Q) \to (Q \to P)))$
(f) $((P \wedge Q) \leftrightarrow P)$

(5) Produce the substitution instances of the following formulas for the given substitutions.
(a) $(((P \to Q) \to P) \to P)$; substitute $(P \to Q)$ for P and $((P \wedge Q) \to R)$ for Q.
(b) $((P \to Q) \to (Q \to P))$; substitute Q for P and $(P \wedge \neg P)$ for Q.

(6) Determine the formulas which are substitution instances of other formulas in the list and give the substitutions.
(a) $(P \to (Q \to P))$
(b) $((((P \to Q) \wedge (R \to S)) \wedge (P \vee R)) \to (Q \vee S))$
(c) $(Q \to ((P \to P) \to Q))$
(d) $(P \to ((P \to (Q \to P)) \to P))$
(e) $((((R \to S) \wedge (Q \to P)) \wedge (R \vee Q)) \to (S \vee P))$