Tutorial 2 ABanerji

Q.1 Let X be a discrete r.v. with probability distribution ~~fxxq~~ fn. given by

$$f_X(x) = \begin{cases} 0, & x \in (-\infty, -2) \\ 1/10, & x \in [-2, -1) \\ 3/10, & x \in [-1, 1) \\ 6/10, & x \in [1, 2) \\ 1, & x \in [2, \infty). \end{cases}$$

Find pmf for X.

Soln. The range for the r.v X is $-2, -1, 1, 2$

$\therefore$ The required prob. mass fn. (pmf) is given by

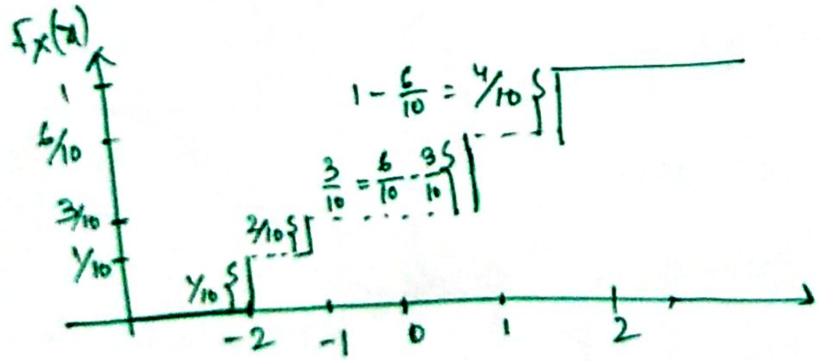
$$p_X(x) = \begin{cases} 1/10, & x = -2 \\ 2/10, & x = -1 \\ 3/10, & x = 1 \\ 4/10, & x = 2 \\ 0, & \text{otherwise} \end{cases}$$

$= \frac{1}{10} - 0 = 1/10.$

$= \frac{3}{10} - \frac{1}{10} = 2/10$

$= \frac{6}{10} - \frac{3}{10} = 3/10$

$= 1 - 6/10 = 4/10$



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Q.2 The cdf of a discrete r.v. X is given by

$$F_X(x) = \begin{cases} 0 & , x < 0 \\ 1/4 & , 0 \leq x < 1 \\ 1/2 & , 1 \leq x < 2 \\ 1 & , x \geq 2. \end{cases}$$

Find the pmf for X and the cdf of $Y = X^2$.

<u>Soln.</u> The range of the r.v. X is 0, 1, 2

$\therefore$ The pmf of X is given by

$$P_X(x) = \begin{cases} \frac{1}{4} & , x = 0 \\ 1/4 & , x = 1 \\ 1/2 & , x = 2 \\ 0 & , \text{otherwise.} \end{cases}$$

Now Y is the derived r.v. $(Y = X^2 = g(X))$

The range for Y is 0, 1, 4.

$\Rightarrow$ Hence, for $x_1 \neq x_2$ we have $g(x_1) \neq g(x_2)$.

$\Rightarrow$ The pmf for Y will remain same as that of X.

$$\therefore f_Y(y) = \begin{cases} \frac{1}{4} & , y = 0 \\ \frac{1}{4} & , y = 1 \\ 1/2 & , y = 4 \\ 0 & , \text{other wise} \end{cases}$$

$\therefore$ required cdf of Y is

$$F_Y(y) = \begin{cases} 0 & , y < 0 \\ 1/4 & , 0 \leq y < 1 \\ 1/2 & , 1 \leq y < 4 \\ 1 & , 4 \leq y. \end{cases}$$

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Q.3. A discrete r.v. X assumes each of the values of the set $\{-10, -9, \ldots, 9, 10\}$ with equal prob. Compute the following probabilities.

(i) Prob $\{4X \le 2\}$, (ii) Prob $\{4x + 4 \le 2\}$

(iii) Prob $\{X^2 - X \le 3\}$ (iv) Prob $\{|X-2| \le 2\}$

Soln. The pmf of X is given by

$P_X(x_i) = \frac{1}{21}$, $\forall x_i \in \{-10, -9, \ldots, 9, 10\}$.

(i) Let $Y = 4X$. $\therefore$ range for Y is $\{-40, -36, \ldots, 36, 40\}$

The pmf of Y is

$P_Y(y_i) = \frac{1}{21}$, $\forall y_i \in \{-40, -36, \ldots, 36, 40\}$.

Im X = $\{-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Im Y = $\{-40, -36, -32, -28, -24, -20, -16, -12, 8, -4, 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40\}$

$\therefore$ Prob $\{Y \le 2\} = \frac{11}{21}$.

soln. of the rest part (do by yourself).

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Q.4. Find the median, made and expected value of the discrete r.v. whose cdf is given by

$$F_X(x) = \begin{cases} 0 & , x < -1 \\ 1/8 & , -1 \le x < 0 \\ 1/4 & , 0 \le x < 1 \\ 1/2 & , 1 \le x < 2 \\ 1 & , x \geq 2 \end{cases}$$

Q.5. The pmf. of a discrete r.v. X is given by

$$p_X(x) = \begin{cases} 0.1 & , x = -2 \\ 0.2 & , x = 0 \\ 0.3 & , x = 2 \\ 0.4 & , x = 5 \\ 0 & , \text{otherwise} \end{cases}$$

find the expectation, second moment (central), variance and standard deviation of X.

Q.6. The pmf. of a discrete r.v. X is given by

$$p_X(x) = \begin{cases} 0.2 & , x = -2 \\ 0.3 & , x = -1 \\ 0.4 & , x = 1 \\ 0.1 & , x = 2 \\ 0 & , \text{otherwise} \end{cases}$$

find the expectation of the following fn. of X.

(a) $Y = 3X - 1$, (b) $Z = -X$, (c) $W = |X|$.

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