Tutorial 5
MA 202

Q.1. Let $X \sim N(30, 5^2)$. Then find the probabilities that (i) $26 \le X \le 40$ (ii) $X \geq 45$ and (iii) $|X-30| > 5$. Given that if $f(t) = \int_0^t \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$ then $f(0.8) = 0.2881$, $f(2) = 0.4772$, $f(3) = 0.49865$, $f(1) = 0.3413$.

Soln. $\mu = 30$, $\sigma = 5$

(i) $P(26 \le X \le 40) = P\left(\frac{26-30}{5} \le \frac{X-30}{5} \le \frac{40-30}{5}\right)$

$= P(-0.8 \le Z \le 2)$, $Z \sim N(0,1)$.

$= P(-0.8 \le Z \le 0) + P(0 \le Z \le 2)$

$= P(0 \le Z \le 0.8) + P(0 \le Z \le 2)$ due to symmetry

$= f(0.8) + f(2)$

$= 0.2881 + 0.4772$.

$= 0.7653$ Ans.

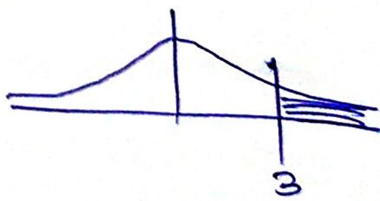
(ii) $P(X \geq 45) = P\left(\frac{X-30}{5} \geq \frac{45-30}{5}\right) = P(Z \geq 3)$

$= 0.5 - P(0 \le Z \le 3)$

$= 0.5 - f(3)$

$= 0.5 - 0.49865$

$= 0.00135$ Ans.



P.T.O

Tutorial 5 P(1) A Banerji.

(iii) $P(|X-30| \leq 5) = P(-5 \leq X-30 \leq 5)$

$= P(25 \leq X \leq 35)$

$= P\left(\frac{25-30}{5} \leq \frac{X-30}{5} \leq \frac{35-30}{5}\right)$

$= P(-1 \leq Z \leq 1)$

$= P(-1 \leq Z \leq 0) + P(0 \leq Z \leq 1)$

$= \underbrace{P(0 \leq Z \leq 1) + P(0 \leq Z \leq 1)}$ due to symmetry

$= 2P(0 \leq Z \leq 1)$

$= 2 \cdot f(1) = 0.6826$

$\therefore P(|X-30| > 5) = 1 - P(|X-30| \leq 5)$

$= 1 - 0.6826$

$= 0.3174$ Ans.

Q.2. The local authorities in a certain city instal 10,000 electric lampls in the streets of the city. If these lamps have an average life of 1,000 burning hrs. with a standard deviation of 200 hrs., assuming that the life of a lamp is normally distributed, find the no. of lamps expected to fail (i) in the first 800 burning hrs. (ii) between 800 and 1,200 burning hrs. After what period of burning hrs. would you

P.T.O.

ABanerjee.

Tutorial 3 P(2)

expect that (a) 10 % of the lamps would fail ?
(b) 10 % of the lamps would be still burning ?

Given : In a normal curve, the area between the ordinates corresponding to $[(x-\mu)/\sigma] = 0$ and $[(x-\mu)/\sigma] = 1$ is 0.3413 and 80% of the area lies between the ordinates corresponding to $[(x-\mu)/\sigma] = \pm 1.28$.

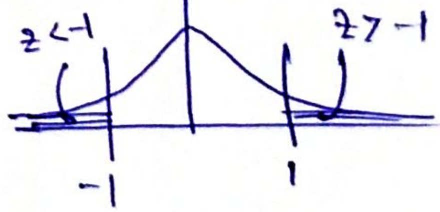
Soln. Let X be the r.v. denoting the life of a lamp in burning hrs.

Given $X \sim N(\mu, \sigma^2)$ where $\mu = 1000$ & $\sigma = 200$.

(i) Let p be the prob. that a lamp fails in the first 800 burning hrs. Then

$$p = P(X < 800) = P\left(\frac{X-\mu}{\sigma} < \frac{800-\mu}{\sigma}\right) = P(Z < -1)$$ where $Z \sim N(0,1)$.

$= P(Z \geq 1)$ [due to symmetry]



$= 0.5 - P(0 \leq Z \leq 1)$

$= 0.5 - 0.3413$

$= 0.1587$.

Hence, the expected no. of lamps which fail in the first 800 burning hrs is

$= 10,000 \times 0.1587 = 1,587$.

Ans.

P.T.O

Lecture 5 P (3). ABanerji.

(iii) Required probability

$P(800 \leq X \leq 1{,}200) = P(-1 \leq Z \leq 1) = 2P(0 \leq Z \leq 1)$

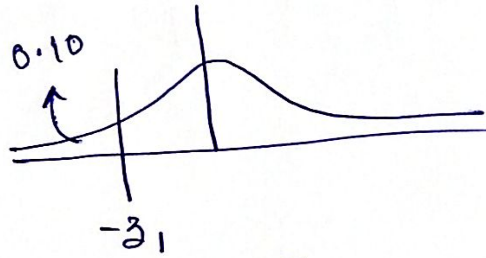
$= 2 \times 0.3413 = 0.6826.$

Hence, the expected no. of lamps with life between 800 and 1,200 burning hrs. is

$10{,}000 \times 0.6826 = 6{,}826.$ Ans.

(a) Let 10% of the lamps fail after x_1 burning hrs.

Then $P(X < x_1) = 0.10$



$\Rightarrow P\left(Z < \dfrac{x_1 - 1000}{200}\right) = 0.10$

$\therefore \boxed{\dfrac{x_1 - 1000}{200} = -z_1}$

$\Rightarrow P(Z < -z_1) = 0.10 \quad \Rightarrow \boxed{P(Z > z_1) = 0.10}$

$\Rightarrow \boxed{P(0 \leq Z \leq z_1) = 0.40}$ —— ①

Also $P(-1.28 < Z < 1.28) = 0.80$ (Given).

$\Rightarrow 2P(0 < Z < 1.28) = 0.80$

$\Rightarrow \boxed{P(0 < Z < 1.28) = 0.40}$ —— ②

Comparing ① & ② we have $z_1 = 1.28$.

$\Rightarrow \dfrac{x_1 - 100}{200} = -1.28 \quad \Rightarrow \boxed{x_1 = 744}$

$\Rightarrow$ After 744 hrs. (burning hrs.), 10% of the lamps will fail.

ABanerji.

Tutorial 5 P(4)

(b) Let 10% of the ~~bulbs~~ lamps be still burning after (say) x_2 burning hrs.

Then $P(X > x_2) = 0.10 \Rightarrow P(Z > z_2) = 0.10$ $\quad z_2 = \dfrac{x_2 - 1000}{200}$

$\Rightarrow P(0 < Z < z_2) = 0.40$

$\Rightarrow z_2 = 1.28$

$\Rightarrow \dfrac{x_2 - 1000}{200} = 1.28$

$\Rightarrow x_2 = 1.256.$

Hence, after 1,256 burning hrs, 10% of the lamps will still be burning.

———— x ————

Tutorial 5P(5) ABanerjee.