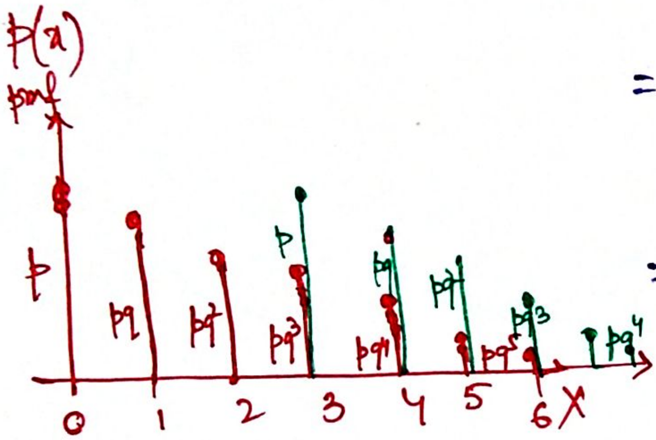
Memoryless property of Geometric distribution

Theorem: Let $X \sim$ geometric (p) and let x and y be any two non-negative integers then

$$P(X \geq x+y \mid X \geq x) = P(X \geq y)$$

Proof:-
$$P(X \geq x+y \mid X \geq x) = \frac{P(X \geq x+y,\ X \geq x)}{P(X \geq x)}$$



$$= \frac{P(X \geq x+y)}{P(X \geq x)} \quad \|\ **$$

$$= \frac{(1-p)^{x+y}}{(1-p)^{x}}$$

$$= (1-p)^{y}$$

$$= P(X \geq y).$$

geometric dist. is the only discrete dist. with the memoryless prop.

Proved.

** $X \sim$ geometric (p)

$$P(X \geq x) = \sum_{i=x}^{\infty} p(i) = \sum_{i=x}^{\infty} q^i p$$

$$= [q^x p + q^{x+1} p + + q^{x+2} p + \cdots]$$

$$= q^x p\,(1 + q + q^2 + \cdots)$$

$$= q^x p \left(\frac{1}{1-q}\right) = q^x p\, \frac{1}{p} = q^x$$

$$\boxed{P(X \geq x) = (1-p)^x} \qquad x = 0, 1, 2, \ldots$$

Lecture 9 p ©️ A Banerji

The Negative Binomial Distribution

Let us consider a sequence of Bernoulli trials with probability of 'success' p and that of failure $1-p=q$.

Let X be a random variable which defines the number of failures before a specified number of successes (say, r) occurs.

Then X is said to follow a negative Binomial distribution with parameter r & p.

The PMF (pmf) of X is given by

$$\boxed{P(X=x)=\binom{x+r-1}{x}(1-q)^r q^x} \; ; \; x=0,1,2,\ldots \quad 0<p<1.$$

$${}^{x+r-1}C_x\, p^r q^x,$$

Example :- In rolling a fair die, let us suppose that getting 6 is a success and getting any other face is a failure. Let X represents the no. of failure before getting 5th success. Then

X is said to follow a negative Binomial distribution with parameters $r=5$, $p=\frac{1}{6}$.

$$MGF \equiv M_X(t)=\left(\frac{1-p}{\cancel{p}1-pe^t}\right)^r, \quad t<-\log p$$

$$CF \equiv \phi_X(t)=\left(\frac{1-p}{1-pe^{it}}\right)^r \quad \forall\, t\in\mathbb{R}$$

$$PGF \equiv G_X(z)=\left(\frac{1-p}{1-pz}\right)^r \quad \forall\, |z|<\frac{1}{p}.$$

$$E(X)=\frac{pr}{1-p}$$

$$Var(X)=\frac{pr}{(1-p)^2}.$$

ABanerji