Tutorial 1: MA : 202 (Prob. & Stat) Odd Sem 2020-2021

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Q.1. If 10 persons are arranged at random (i) in a line (ii) in a ring, find the probability that 2 particular persons will be next to each other.

Soln. (i) * * . . . . . $S \rightarrow$ sample space, $A \rightarrow$ event

$n(S) = 10!$

$n(A) = 9!\,2!$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{9!\,2!}{10!} = \frac{2}{10} = \frac{1}{5}$ Ans.

(ii) $n(S) = 9!$ $\therefore P(A) = \frac{n(A)}{n(S)}$

$n(A) = 8!\,2!$

$= \frac{8!\,2!}{9!}$

$= \frac{2}{9}$ Ans.

Q.2. X and Y stands in a line at random with 10 other people. What is the probability that there are 3 people between x and y.

Soln. $n(S) = 12!$

$n(A) = 8!\ {}^{10}C_3\ 3!\ 2!$

$P(A) = \frac{8!\ {}^{10}C_3\ 3!\ 2!}{12!} = \frac{4}{33}$ Ans.

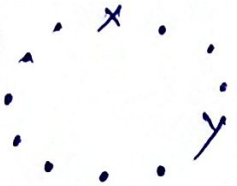
Q3. Twelve persons, amongst whom X and Y are seated at random at a round table. What is the probability that there are 3 persons between X and Y?

Soln. $n(S) = 11!$

$n(A) = 7!\, 2!\, {}^{10}C_3\, 3!$

$\therefore P(A) = \dfrac{7!\, 2!\, {}^{10}C_3\, 3!}{11!}$

$= \dfrac{2}{11}$ Ans.



Q.4. A card is drawn from each of two well-shuffled packs of cards. Find the prob. that at least one of them is an ace.

Soln. $n(S) = 52 \times 52$

$n(A) = 4 \times 48 + 48 \times 4 + 4 \times 4$

$\therefore P(A) = \dfrac{4(48+48+4)}{52 \times 52} = \dfrac{100}{13 \times 52} = \dfrac{25}{169}$.

2nd Method

Let A → event that the card drawn from pack I is an ace

B → " " " " " " II " " "

∴ We required to find $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Now $P(A \cap B) = P(A)P(B)$ as A and B are independent events

Now $P(A) = \dfrac{4}{52} = \dfrac{1}{13}$

$P(B) = \dfrac{4}{52} = \dfrac{1}{13}$

$\therefore P(A \cup B) = \dfrac{25}{169}$. Ans.

Tutorial 1 P(2) Banerji.

Q.5. A box contains twenty tickets of identical appearance, the tickets being numbered 1, 2, 3, ..., 20. If 3 tickets are chosen at random, find the probability that the numbers on the drawn are in arithmatic progression.

Soln. $n(S) = {}^{20}C_3$

The 3 numbers on the drawn tickets will be in A.P. if they have a common difference of either 1, or 2, or 3 or, ..., or at the most 9.

Let d be the common difference

When d = 1, then the no.s are $\underbrace{(1,2,3), (2,3,4), (3,4,5), \ldots, (18,19,20)}_{\text{total } 18}$

when d = 2, then the no.s are $\underbrace{(1,3,5), (2,4,6), (3,5,7), \ldots, (16,18,20)}_{\text{total } 16}$

when d = 3, then the no.s are $\underbrace{(1,4,7), (2,5,8), \ldots, (14,17,20)}_{\text{total } 14}$

$\vdots$

when d = 8, then no.s are $\underbrace{(1,9,17), (2,10,18), (3,11,19), (4,12,20)}_{\text{total } 4}$

d = 9, " " " $\underbrace{(1,10,19), (2,11,20)}_{\text{total } 2}$

Now $n(A) = 2 + 4 + 6 + \ldots + 14 + 16 + 18$

$=$

$$P(A) = \frac{90}{{}^{20}C_3} = \frac{90}{\frac{20!}{3!17!}} = \frac{90.3!17!}{20!} = \frac{90.3!}{18.19.20}$$

$$= \frac{1.2.3}{2.19.2} = \frac{3}{38} \text{ Ans.}$$

Q.6. One urn contains 2 white and 2 black balls; a second urn contains 2 white and 4 black balls.
(i) If one ball is chosen from each urn, what is the probability that they will be of the same colour? (ii) If an urn is selected at random and one ball is drawn from it, what is the probability that it will be white ball?

(i) $\frac{2}{4} \times \frac{2}{6} + \frac{2}{4} \times \frac{4}{6} = \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6}$

prob. that a white ball is chosen from urn I $= \frac{3}{6} = \frac{1}{2}$

prob. that a white ball is drawn from urn II

prob. that a black ball is drawn from urn I

prob. that a black ball is drawn from urn II.

(ii) $\frac{1}{2} \times \frac{2}{4} + \frac{1}{2} \times \frac{2}{6} = \frac{1}{4} + \frac{1}{6} = \frac{3+2}{12} = \frac{5}{12}$

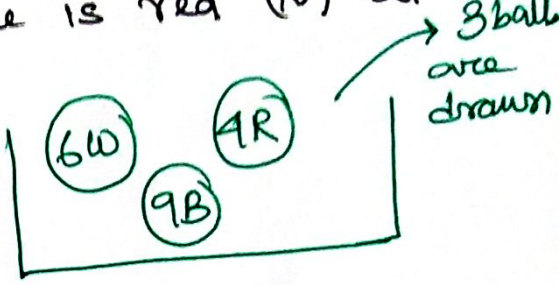
prob. of choosing urn I

prob. of drawing a white ball from urn I

prob. of selecting urn II

prob. of drawing a white ball from urn II.

Q7. An urn contain 6 white, 4 red and 9 black balls. If 3 balls are drawn at random, find the probability that: (i) two of the balls drawn are white (ii) one is of each colour (iii) none is red (iv) at least one is white.



Soln. Total no. of balls = 19

$$n(S) = {}^{19}C_3.$$

(i) $P(A) = \dfrac{{}^{6}C_2 \, {}^{13}C_1}{{}^{19}C_3}$

(ii) $P(B) = \dfrac{{}^{6}C_1 \, {}^{4}C_1 \, {}^{9}C_1}{{}^{19}C_3}$

(iii) $P(C) = \dfrac{{}^{15}C_3}{{}^{19}C_3}$

(iv) $P(\text{None is white}) = \dfrac{{}^{13}C_1}{{}^{19}C_3}$

$\therefore P(\text{at least one is white}) = 1 - \dfrac{{}^{13}C_1}{{}^{19}C_3}.$

Q.8 (i) If the letters of the word 'REGULATIONS' are arranged at random, what is the prob. there will be exactly 4 letters between R and E?

(ii) What is the prob. that 4 S's come consecutively in the word 'MISSISSIPPI'?

Soln. See next page

Soln. Q. 7. (Tutorial 1).

(i) R . . . . E . . . . .

$n(S) = 11!$

$n(A) = 2! \; {}^9C_4 \; 4! \; 6!$

$\therefore P(A) = \dfrac{2! \; {}^9C_4 \; 4! \; 6!}{11!}$

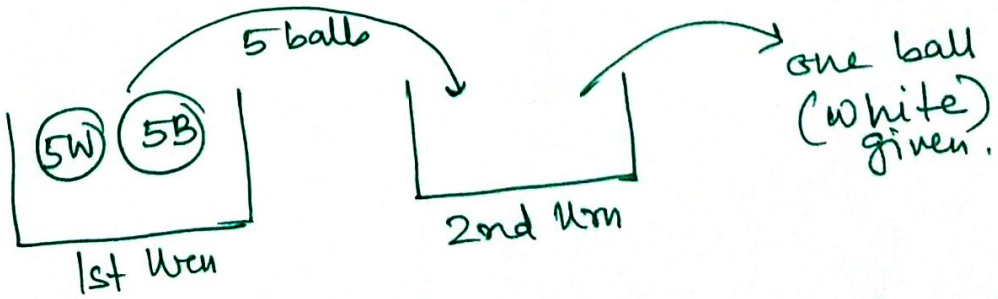
$= \dfrac{12}{110} = \dfrac{6}{55}$ Ans.

(ii) $n(S) = \dfrac{11}{4!4!2!}$ $\because$ I → 1, S → 4, P → 2, M → 1

$n(A) = \dfrac{8!}{4!\,2!}$

$\therefore P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{165}$ Ans.

Q.8. From an urn containing 5 white and 5 black balls, 5 balls are transferred at random into an empty second urn from which one ball is drawn and it is found to be white. What is the prob. that all balls transferred from the first urn are white.



Soln. See the next page.

Soln. (Q.8, Tutorial 1).

Let A_i be the event s.t

$A_i \rightarrow$ i white ball and $(5-i)$ black balls are transferred from 1st urn to the 2nd urn

$\forall i = 0, 1, 2, 3, 4, 5.$

$$\therefore P(A_0) = \frac{{}^5C_5}{{}^{10}C_5} = \frac{1}{252}, \quad P(A_1) = \frac{{}^5C_4\,{}^5C_4}{{}^{10}C_5} = \frac{25}{252}$$

$$P(A_2) = \frac{{}^5C_2\,{}^5C_3}{{}^{10}C_5} = \frac{25}{63}, \quad P(A_3) = \frac{{}^5C_3\,{}^5C_2}{{}^{10}C_5} = \frac{25}{63}$$

$$P(A_4) = \frac{{}^5C_4\,{}^5C_4}{{}^{10}C_5} = \frac{25}{252}, \quad P(A_5) = \frac{{}^5C_5}{{}^{10}C_5} = \frac{1}{252}.$$

Let X be the r.v denoting that one white ball is drawn from 2nd urn.

Now $P(X|A_0) = 0$, $P(X|A_1) = \frac{1}{5}$, $P(X|A_2) = \frac{2}{5}$,

$P(X|A_3) = \frac{3}{5}$, $P(X|A_4) = \frac{4}{5}$, $P(X|A_5) = 1$.

Hence required prob. will be obtained using Bayes' theorem.

$$P(A_5|X) = \frac{P(X|A_5)\,P(A_5)}{\sum_{i=0}^{5} P(X|A_i)\,P(A_i)} = \frac{1}{126}. \text{ Ans.}$$

Q.9. The chance that a doctor will diagonose a certain disease correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor who had the disease dies. what is the prob. that the disease was diagnosed correctly.

Soln. Let A be the event which define that the doctor diagnose a certain disease correctly.

D be another event which defines that the patient dies.

$\therefore P(A) = \frac{60}{100} = \frac{3}{5}$

$P(\bar{A}) = 1 - P(A) = 2/5$

$P(D|A) = \frac{40}{100} = 2/5$

$P(D|\bar{A}) = \frac{70}{100} = 7/10.$

Now we need to find $P(A|D)$

$\therefore$ By Baye's theorem

$$P(A|D) = \frac{P(D|A)P(A)}{P(A \cap D) + P(\bar{A} \cap D)}$$

$$= \frac{P(D|A)P(A)}{P(D|A)P(A) + P(D|\bar{A})P(\bar{A})}$$

$$= \frac{2/5 \times 3/5}{2/5 \times 3/5 + \frac{7}{10} \times \frac{2}{5}} = \frac{6}{13} \text{ Ans.}$$

Tutorial 1 P(8) Bayes'