Solution for MA202 A-1

September 29, 2020

1. (a) Number of ways to select 13 cards from $52 = \binom{52}{13}$
   Number of ways to select 13 cards in such a way that all the aces come in hand $= \binom{48}{9}$
   Hence, the probabilty of having all the aces in hand of 13 cards $=\frac{\binom{48}{9}}{\binom{52}{13}}$

   (b) Number of ways to select a hand of 13 cards which contains no aces $= \binom{48}{13}$
   probability of having 13 cards in hand which contains no aces $= \frac{\binom{48}{13}}{\binom{52}{13}}$
   Hence, probability of having cards which contains at least one ace $= 1 - \frac{\binom{48}{13}}{\binom{52}{13}}$

2. Let E be the event that no Physics books are placed consecutively.
   So number of ways to arrange physics books so that no Physics books are placed consecutively $n(E) = \binom{8}{3}(3!)(7!)$
   Also, $n(A)$ is the total numbers of ways to arrange 10 books=10! Hence

$$\begin{aligned} P(E) &= \frac{n(E)}{n(A)} \\ P(E) &= \frac{\binom{8}{3}(3!)(7!)}{10!} \\ P(E) &= \frac{7}{15} \end{aligned}$$

3. Let A be the event that student failed in mathematics and B be the event that student failed in chemistry.
   $P(A) = 0.3$, $P(B) = 0.15$ and $P(AB) = 0.1$, so our object here is to find the probability that the selected student failed in chemistry but passed in mathematics i.e. $P(A^c|B)$

   So the probability that selected student fails only in chemistry is

$$P(A^cB) = P(B) - P(AB) = 0.15 - 0.10 = 0.05$$

   Hence,

$$P(A^c|B) = \frac{P(A^cB)}{P(B)} = \frac{1}{3}$$

4. *Given*: A is independent of B, $B \cup C$ and BC.Our object here is to show that A and C are independent events i.e. $P(AC) = P(A)P(C)$
   From the definition of two events are independents, we have

$$\begin{aligned} P(AB) &= P(A)P(B) && (1) \\ P(A(B \cup C)) &= P(A)P(B \cup C) && (2) \\ P(A(BC)) &= P(A)P(BC) && (3) \end{aligned}$$

   By the inclusion-exclusion formula we have,

$$\begin{aligned} P(A(B \cup C)) &= P(AB \cup AC) \\ P(A(B \cup C)) &= P(AB) + P(AC) - P(AB) \\ P(A(B \cup C)) + P(ABC) &= P(AB) + P(AC) \end{aligned}$$

   Adding (2) and (3) and subtracting (1) we get

$$\begin{aligned} P(ABC) + P(A(B \cup C)) - P(AB) &= P(A)(P(B \cup C) + P(BC) - P(B)) \\ p(AB) + P(AC) - P(AB) &= P(A)P(C) \\ P(AC) &= P(A)P(C). \end{aligned}$$

5. Let U_i be the ith urn. Probability to select one urn from three urns is $P(U_i) = \frac{1}{3}, \quad = 1,\ 2,\ 3.$Also let E be the event to draw 1 black and 1 red ball. Our objective here is here is to find the probability that both balls are from urn 1 i.e. $P(U_1|E)$

   Writing bayes formula,

$$\begin{aligned} P(U_1|E) &= \frac{P(E|U_1)P(U_1)}{P(E|U_1)P(U_1) + P(E|U_2)P(U_2) + P(E|U_3)P(U_3)} \\ &= \frac{\frac{1}{5}\frac{1}{3}}{\frac{1}{5}\frac{1}{3} + \frac{2}{9}\frac{1}{3} + \frac{1}{6}\frac{1}{3}} \\ &= \frac{18}{53} \end{aligned}$$

6. **Case 1**: Without replacement
   Given: Urn 1 contains 3 White and 5 Black balls and Urn 2 is empty. Then 4 balls are transferred from urn 1 to urn 2. Also two balls are already drawn from Urn 2 and they turned to be white. Now the possiblities of the balls transferred from Urn 1 to Urn 2 are either 2 white and 2 black balls or 3 white and 1 black ball.

   Object is to find that third ball drawn from the urn 2 is white. So probability of transferring 2W and 2B is

$$P(E = 2W \ and \ 2B) = \frac{\binom{3}{2}\binom{5}{2}}{\binom{8}{4}} = \frac{3}{7}$$

   also probability of transferring 3W and 1B ball to the empty container is:

$$P(F = 3W \ and \ 1B) = \frac{\binom{3}{3}\binom{5}{1}}{\binom{8}{4}} = \frac{1}{14}$$

   $P(2W) = P(2W|E)P(E) + P(2W|F)P(F) = \frac{1}{6} \times \frac{3}{7} + \frac{3}{6} \times \frac{1}{14} = \frac{3}{28}$

   After drawing two white balls from urn 2 we have only 1 white and 1 black ball left. So probability of drawing white ball is,

$$P(W_3|F, 2W) = \frac{1}{2}$$

   Now, $P(W_3|2W) = \frac{P(W_3|F,2W)P(2W|F)P(F)}{P(2W)} = \frac{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{14}}{\frac{3}{28}} = \frac{1}{6}$

   Case 2: With replacement
   Let the two balls be replaced before the third draw then, $P(W_3|E, 2W) = \frac{1}{2}$, $P(W_3|F, 2W) = \frac{3}{4}$, then probability of getting third ball to be white given that two white balls are replaced

$$\begin{aligned} P(W_3|2W) &= \frac{P(W_3|2W, E)P(2W|E)P(E) + P(W_3|2W, F)P(2W|F)P(F)}{P(2W)} \\ &= \frac{\frac{1}{2} \times \frac{1}{6} \times \frac{3}{7} + \frac{3}{4} \times \frac{1}{2} \times \frac{1}{14}}{\frac{3}{28}} \\ &= \frac{7}{12} \end{aligned}$$

7. Let G be the event that chosen cyprion is a Greek and T be the event that chosen cyprion is a turkish. Also, Let E be the event that chosen cyprion speaks english. Therefore by the theorem of Total probability we have, $P(E) = P(E|G)P(G) + P(E|T)P(T) = 0.2 \times 0.75 + 0.1 \times 0.25 = 0.175$

   Now, object is to find the probality that chosen cypriot is a greek and given that he randomly meets to the english speaking cypriot i.e. $P(G|E)$. So by using Baye's formula, we have

$$P(G|E) = \frac{P(E|G)P(G)}{P(E)} = \frac{0.2 \times 0.75}{0.175} = \frac{6}{7}.$$