2D Fourier Transform and Frequency Mixing of Images

Governing Equations:

The **2D Discrete Fourier Transform (DFT)** of an image f(x,y) of size M×Nis defined as:

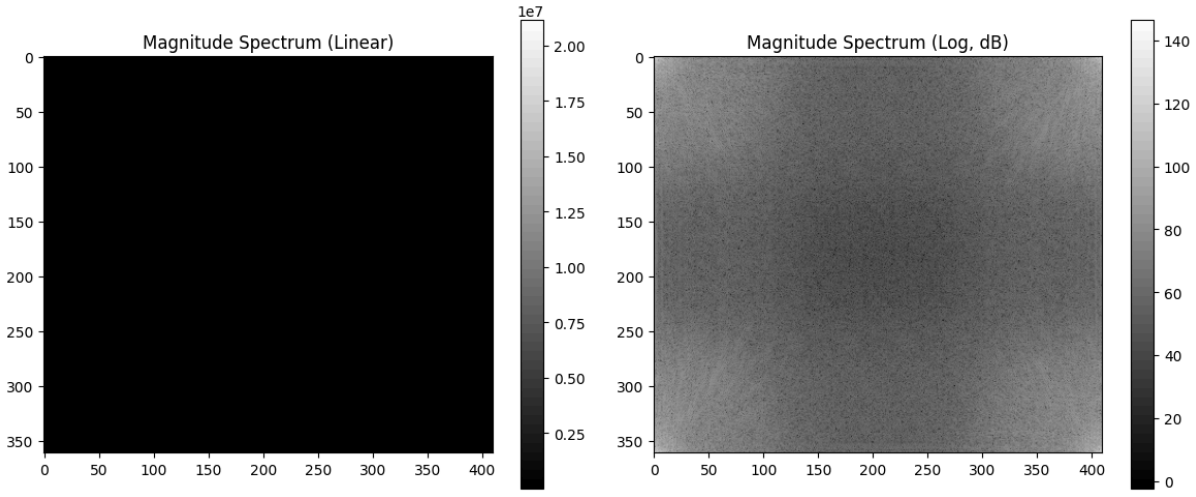
$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{j2\pi(ux/M+vy/N)}$$

where:

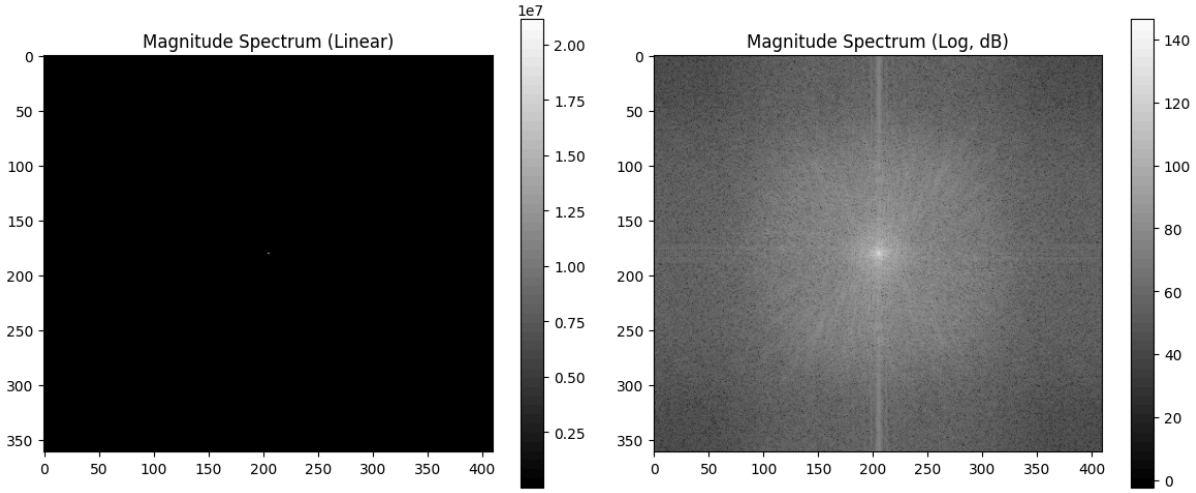
- (x,y) are spatial domain coordinates,
- (u,v) are frequency domain coordinates,
- F(u,v) is the complex Fourier coefficient.

Implementation:

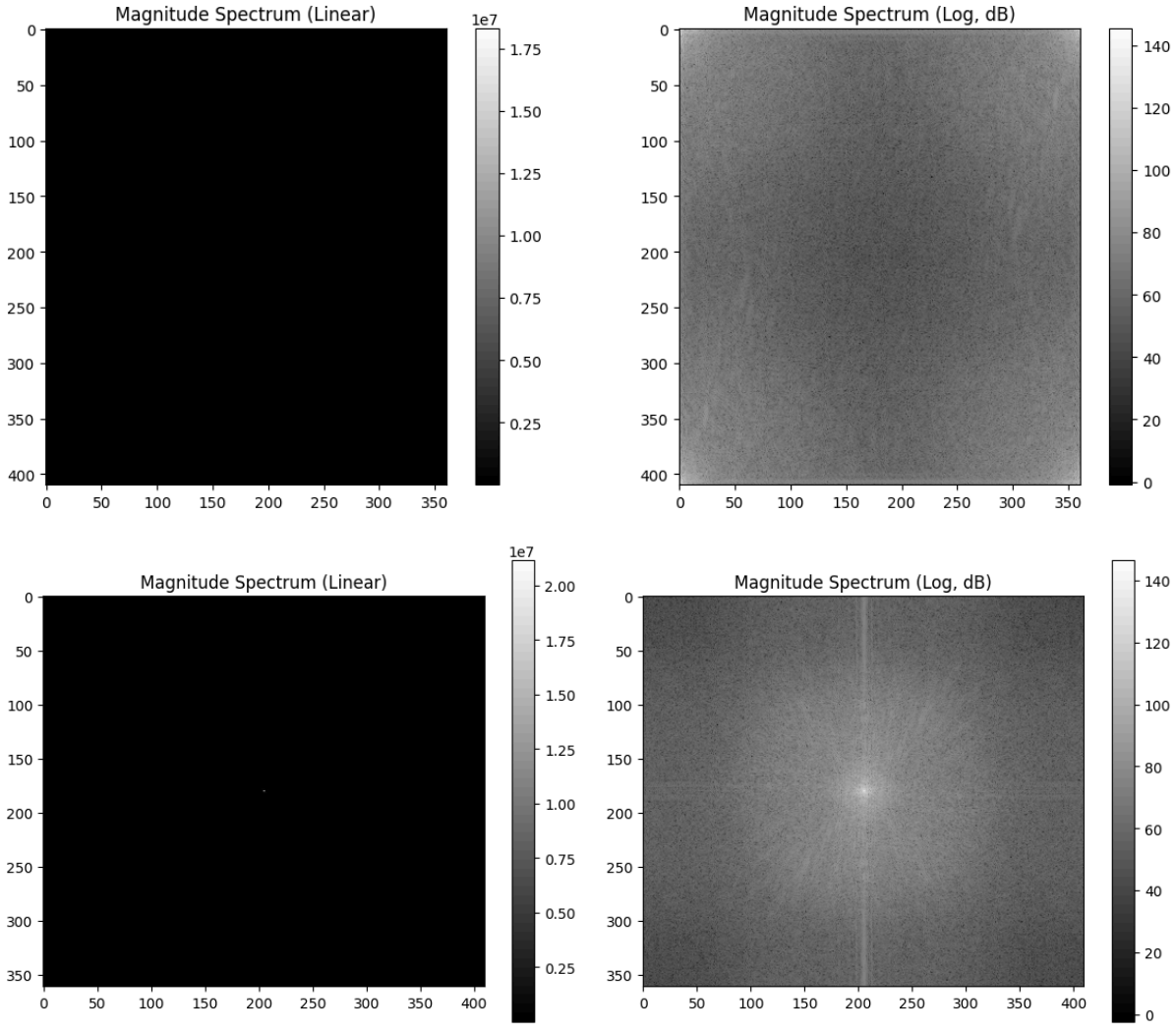
We used the `numpy.fft.fft2` function to calculate the 2D DFT of the images. First we will convert image in a array and then find its 2D Discrete Fourier Transform and then find the magnitude spectrum and decibel magnitude spectrum



Then we will shift the lower frequency to be at center of the graph



Then we will rotate the figure by 90 degrees counter clockwise and find the perform the same procedure



Frequency Mixing (Hybrid Image)

Finally, we designed a simple **frequency mixer** system:

- The cat image provided low-frequency information (overall structure)
- The dog image provided high-frequency details (fine textures)

This was implemented by:

- Designing a **Gaussian low-pass filter** on the cat image in the frequency domain.
- Designing a complementary **Gaussian high-pass filter** on the dog image.
- Combining these frequency components to create a hybrid image.

Mathematically:

Hybrid(u,v) = $F_{cat}(u,v).G_{low}(u,v) + \alpha F_{dog}(u,v).G_{high}(u,v)$

Where: $G_{low}(u,v)$ is the Gaussian low-pass transfer function

$G_{high}(u,v)=1-G_{low}(u,v)$

α is a scaling factor for detail emphasis

This forms a **frequency domain mixing system**, transferring structural and textural cues from separate sources.

Transfer Function 2D Plots

The Gaussian low-pass filter is:

$$G_{low}(u,v) = \exp\left(\frac{-D^2(u,v)}{2\sigma^2}\right)$$

where:

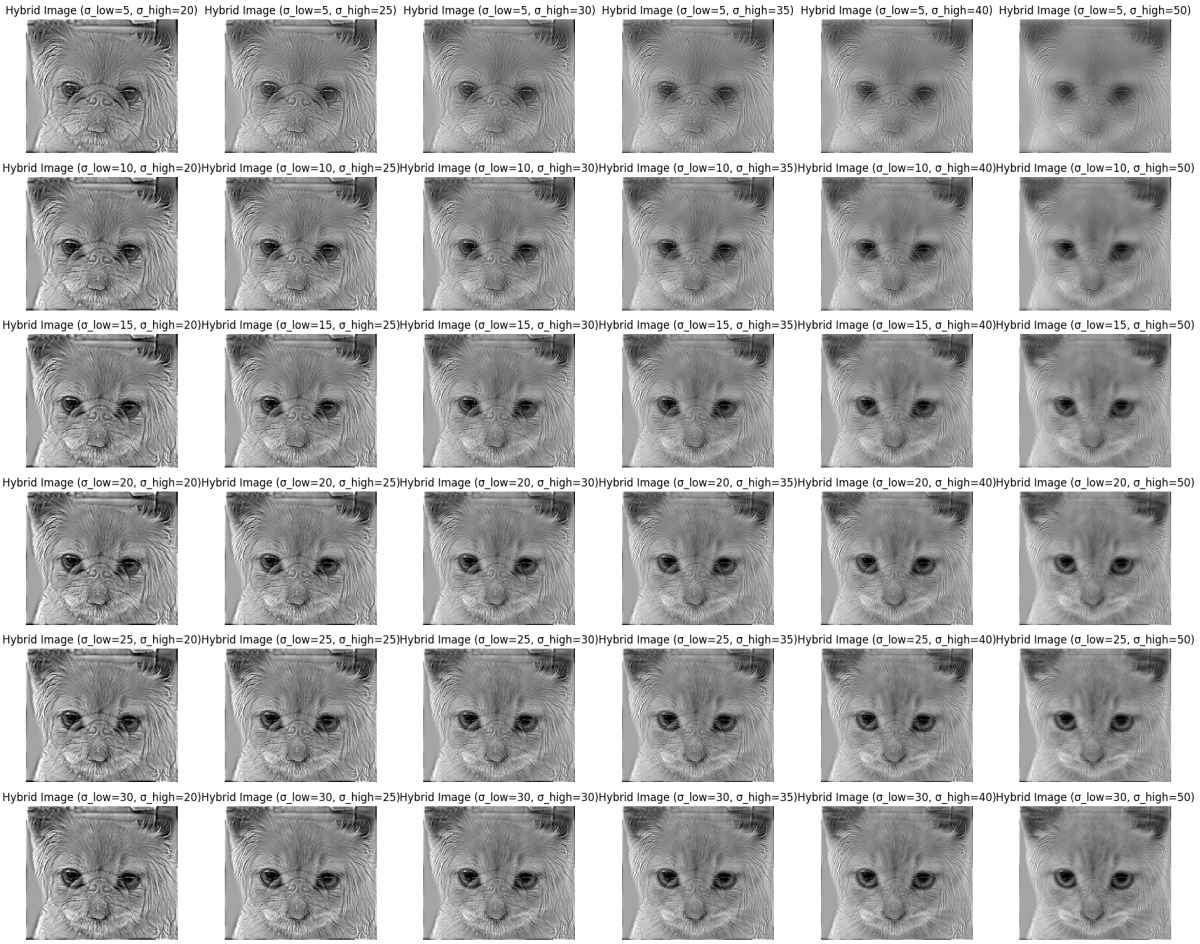
- D(u,v) is the Euclidean distance from the center frequency
- σ controls bandwidth

The complementary high-pass filter:

$$G_{high}(u,v)=1-G_{low}(u,v)$$

These transfer functions can be visualized as 2D Gaussian bell-shaped functions in the frequency domain.

I have tried multiple combinations the best one is



The frequency mixer successfully fused low frequencies from the cat and high frequencies from the dog, resulting in a hybrid image that visually preserves the structural shape of the cat while introducing textural details from the dog.

Best Hybrid Image (σ_low=15, σ_high=20)