In the previous challenge, you wrote code to perform an *Insertion Sort* on an unsorted array. But how would you prove that the code is correct? I.e. how do you show that for any input your code will provide the right output?

### **Loop Invariant**

In computer science, you could prove it formally with a *loop invariant*, where you state that a desired property is maintained in your loop. Such a proof is broken down into the following parts:

- *Initialization*: It is true (in a limited sense) before the loop runs.
- Maintenance: If it's true before an iteration of a loop, it remains true before the next iteration.
- *Termination*: It will terminate in a useful way once it is finished.

#### **Insertion Sort's Invariant**

Say, you have some InsertionSort code, where the outer loop goes through the whole array A:

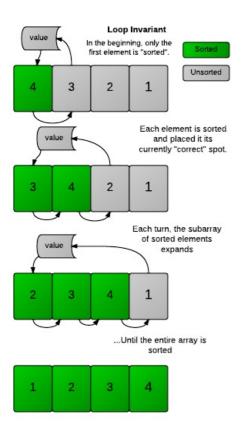
```
for(int i = 1; i < A.length; i++){
//insertion sort code</pre>
```

You could then state the following loop invariant:

At the start of every iteration of the outer loop (indexed with i), the subarray until ar[i] consists of the original elements that were there, but in sorted order.

To prove Insertion Sort is correct, you will then demonstrate it for the three stages:

- Initialization The subarray starts with the first element of the array, and it is (obviously) sorted to begin with.
- Maintenance Each iteration of the loop expands the subarray, but keeps the sorted property. An element V gets inserted into the array only when it is greater than the element to its left. Since the elements to its left have already been sorted, it means V is greater than all the elements to its left, so the array remains sorted. (In *Insertion Sort 2* we saw this by printing the array each time an element was properly inserted.)
- *Termination* The code will terminate after *i* has reached the last element in the array, which means the sorted subarray has expanded to encompass the entire array. The array is now fully sorted.



You can often use a similar process to demonstrate the correctness of many algorithms. You can see <u>these notes</u> for more information.

### Challenge

In the InsertionSort code below, there is an error. Can you fix it? Print the array only once, when it is fully sorted.

# **Input Format**

There will be two lines of input:

- **s** the size of the array
- *arr* the list of numbers that makes up the array

### **Constraints**

$$\begin{array}{l} 1 \leq s \leq 1000 \\ -1500 \leq V \leq 1500, V \in arr \end{array}$$

# **Output Format**

Output the numbers in order, space-separated on one line.

# **Sample Input**

6 7 4 3 5 6 2

# **Sample Output**

2 3 4 5 6 7

# **Explanation**

The corrected code returns the sorted array.