CE677B: Home assignment 2

GNSS integration with EKF

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Abstract: In this exercise, we are given with navigation data of a moving car i.e. measured pseudoranges from satellite in observation file and coordinates of satellite in .n file. We are required to make use of this data to estimate the trajectory of a car that is installed with a GPS using EKF. Inferences were then draws from obtained trajectory and questions in laboratory manual were answered.

1. Introduction

The Kalman filter algorithm is a set of equations which when applied recursively on navigation data/surveying measurements yields more accurate position, velocity and acceleration of a moving object from traditional surveying measurements. Unlike traditional least squares approach for solving navigation problem, Kalman filter facilitates more reliable prediction of state of system based on 'Measurement model' and 'Dynamic model' of system.In this exercise, we are given with navigation data of a moving car i.e. measured pseudoranges from satellite in observation file and coordinates of satellite in .n file. We are required to make use of this data to estimate the trajectory of a car that is installed with a GPS using EKF. Inferences were then draws from obtained trajectory and questions in laboratory manual were answered. Code is written in python in attached file to solve this problem and yield trajectory. Step by step procedure is explained and questions are answered in subsequent sections of this report.

2. Methodology

Step by step procedure for solving this problem by implementing EKF is as follows:

Following data is given in this exercise:

1. .o file that contains observed pseudoranges from satellite at 0.5 seconds interval.

2. .n file that contains coordinates of these satellite not at every epoch but at some intermediate epochs.

We have to generate trajectory of a car i.e. we have to find X coordinate, Y coordinate, Z coordinate Vx and Vy and Vz at every epoch from this data using EKF algorithm. We are not given with any IMU data hence no need to consider IMU observations while estimating states.

2.1. Assumptions:

Let X_i,Y_i and Z_i denote X and Y and Z coordinate at i^{th} epoch.
Let V_{xi},V_{yi} V_z denote velocity in X and Y and Z direction at i^{th} epoch.

Following standard assumptions are made in EKF:

1. Measurement noise and process noise are uncorrelated. $E[V_k\omega_k] = 0$

2. Measurement noise is white.

3. Process noise is white.

4. Measurements taken from four different control points are independent.

5. Velocity of moving object is constant thus acceleration becomes zero in all three dimensions.

Rest of assumptions concerned with initialization of EKF various covariance matrices and noises are discussed in section 2.5 Initialization of EKF.

2.2. State space model

Trajectory equation of moving object can be written as:

$$
\begin{aligned}
x_{i+1} &= x_i + V_{xi}\Delta t + \omega_x \\
y_{i+1} &= y_i + V_{yi}\Delta t + \omega_y \\
z_{i+1} &= z_i + V_{zi}\Delta t + \omega_z \\
Vx_{i+1} &= V_{xi} + \omega_{vxi} \\
Vy_{i+1} &= V_{yi} + \omega_{vyi} \\
Vz_{i+1} &= V_{zi} + \omega_{vzi}
\end{aligned}
$$

These equations can be written in matrix form as:

$$X_{k+1} = F_k X_k + \omega_k \tag{1}$$

where,

$X_{k+1} = \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ z_{i+1} \\ Vx_{i+1} \\ Vy_{i+1} \\ Vz_{i+1} \end{bmatrix}$ and $X_k = \begin{bmatrix} x_i \\ y_i \\ z_i \\ Vx_i \\ Vy_i \\ Vz_i \\ \Delta r_k \end{bmatrix}$ are state vectors.

Δr_k is receiver clock error in pseudoranges measurement.

F_k is state transition matrix given by:

$$F_k = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

ω_k is matrix of random errors given by:

$$\omega_k = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \omega_{vxi} \\ \omega_{vyi} \\ \omega_{vyi} \\ \omega_{\Delta r_k} \end{bmatrix}$$

Dynamic model is given by:

$$\hat{X_k^-} = F_{k-1}\hat{X_{k-1}^+} + \omega_{k-1} \tag{2}$$

$$P_k^- = F_{k-1}P_{k-1}F_{k-1}^T + Q_{k-1} \tag{3}$$

where Q_{k-1}= Variance covariance matrix of process noise.

These vectors $\hat{X_k^-}$ and P_k^- are calculated at each epoch in dynamic model using corrected vectors at previous epoch from corrector in EKF.

2.3. Measurement model:

Looking at observation model, at each epoch pseudoranges from n satellites are given and where n is not fixed means at different epochs number of satellites that were available to measure pseudo-ranges was different. Thus measurement model will consist of n pseudorange equation at each epoch and corresponding measurement matrix will be thus having n rows at every epoch.

Measurement vector=$Z_k = \begin{bmatrix} \rho_1 \\ \rho_1 \\ \rho_1 \\ . \\ . \\ . \\ . \end{bmatrix}_{n\times 1}$

Where n is number of pseudoranges available at kth epoch.

$$Z_k = h(X_k) + V_k$$

Equation of measurement model can be written as:

$$\rho_i = \sqrt{(X - X_i)^2 + (Y - Y_i)^2 + (Z - Zi)^2} + \Delta r_b$$

H_k is measurement matrix given by:

$H_k = \frac{\partial h}{\partial X_k}|_{X=\hat{X_k^-}}$

$H_k = \begin{bmatrix} \frac{X-X_m}{r_m} & \frac{Y-Y_m}{r_m} & \frac{Z-Z_m}{r_m} & 0 & 0 & 0 & 1 \end{bmatrix}_{n\times 7}$

we have,

$\hat{Z_k} = h(X_k) + v_k$

here $h(X_k)$ in non linear function, linearizing by Taylors series expansion about vector X_N we have,

$$\hat{Z_k} = \frac{\partial h}{\partial X}\bigg|_{X=XN} (X_k - X_N) + (higherorderterms) + v_k$$

$$Z_k = h(X_N) + H_k \Delta X_k + v_k$$

where, $H_k = \frac{\partial h}{\partial X}\Big|_{X=XN}$ is obtained by differentiating each element in $h(X_k)$ w.r.t elements in state matrix X_k and substituting $Xk = X_N$
Vector X_N should be very close to X_k because if this is satisfied then only we can ignore higher order terms in Taylor series expansion of non linear function.
The best estimate for this point can be the state vector $\hat{X_k^-}$ found in prediction step at that particular iteration of EKF. This is thus taken as point about which linearization is done.
Matrix $H_k = \frac{\partial h}{\partial X}\Big|_{X=XN}$ is generated at every iteration by $XN = X_k^-$ at every pass in EKF as

Thus measurement model,

$$Z_k = h(X_N) + H_k \Delta X_k + v_k \tag{4}$$
$$Z_k = Z_{pred} + H_k \Delta X_k + v_k \tag{5}$$

2.4. Corrector steps:

From dynamic model we get $\hat{X_k^-}$ and P_k^-, there are updated in corrector model.

$$\hat{X_k^+} = \hat{X_k^-} + K_k(Z_k - Z_{pred}) \tag{6}$$

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \tag{7}$$

$$P_k^+ = (I - K_k H_k) P_k^- (I - K_k H_k)^T + K_k R_k K_k^T \tag{8}$$

$\hat{X_k^+}$ and P_k^+ obtained in this iteration are now treated as initial values for next iteration and cycle of prediction and correction repeats.
Trajectory vector i.e. values of X coordinate, Y coordinate, V_x and V_y at every epoch are appended with elements in $\hat{X_k^+}$ obtained at every iteration.**[Refer code]**

2.5. Initialization of EKF

1) Initialization of state vector at t=0
X and Y and Z coordinates of point at t=0 from where trajectory starts are found out using least squares technique by observation equation method. Step by step procedure of finding unknown parameters i.e. X and Y coordinate of initial point is followed and explained in python notebook attached named **"Firstpointestimate_Shashank_20103107"**.
Pseudoranges of moving car from six satellite is given at first epoch, these are treated as observations. Parameters are X and Y and Z coordinate at t=0. Iterative solution is found out by linearizing mathematical model and updating parameter at every iteration.
Finally obtained parameters i.e. X and Y ad Z coordinate at initial point are:

$$\begin{bmatrix} X_1 \\ y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} 3520414.8414152 \\ 3534467.07226563 \\ 1807807.4314572 \\ 924552.695111824 \end{bmatrix}$$

Now initial value of V_x and V_y are assumed to be zero assuming that object was stationary at start.
Hence initialization of state vector at first epoch is:

$$X_1^- = \begin{bmatrix} x_1 \\ y_1 \\ V_{x1} \\ V_{y1} \end{bmatrix} = \begin{bmatrix} 3520414.8414152 \\ 3534467.07226563 \\ 1807807.4314572 \\ 0 \\ 0 \\ 0 \\ 924552.695111824 \end{bmatrix}$$

Initialization of variance covariance matrix of state vector:

It is assumed looking at values of distances measured from four points and coordinates of initial point in trajectory that standard deviation in initial estimates of X, Y Z coordinates is 5 meters and in velocity estimation is 1 m/s. Also std deviation in clock bias is 5 m assumed
Also it is assumed that these parameters are uncorrelated. Thus variance covariance matrix at first epoch becomes:

$$P_{X1} = \begin{bmatrix} 25 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 25 \end{bmatrix}$$

2) Initialization of variance covariance matrix of measurement vector and its computation at every epoch:

It is assumed that there is standard deviation of 5m in each pseudo range measurement.
Measurements are also assumed to be independent thus giving rise to off-diagonal elements of variance covariance matrix to be zero.
Thus variance covariance matrix of measurements is n$\times nmatrixwithdiagonalelementsbeing25ateveryepoch.$

3) Generating process error matrix ω_k and V_k at every run pf predictor corrector model:
Process noise and measurement noise are assumed to be normally distributed with mean zero, and standard deviation of 1.
This thus adds a random noise in process model and measurement model at every run of predictor and corrector.
Code for this can be reffered for implementation of above.

2.6. Important discussion

Given .o and .n files were read in matlab and pseudoranges and satellite coordinates data was exported in exel sheet attached in this submission.

This data contains pseudo ranges at 98646 epochs with time difference of 0.5 seconds between successive epochs.

At every epoch, number of satellite that has measured pseudo ranges are different thus a dynamic code is written that fetches all the available pseudo ranges at that particular epoch from .o file.

This code then also fetches coordinates of only those satellites that has given pseudo-ranges at that epoch and uses them in measurement model.

Dimensions of H_k and R_k matrix are adjusted at every epoch H_k being n$\times 7 and$R$_k$ being n X n according to number of pseudo ranges available at that particular epoch and a code is written to calculate these matrices at every epoch.

3. Results

Trajectory information obtained is in attached exel sheet with this document. Plot of trajectory obtained is in attached code.

4. What I understood in this lab:

1) Kalman filter can be successfully implemented by following step by step procedure of correctly modelling process model and measurement model. 2) Choosing initialization matrices for state vector and measurements is important and correct modelling of factors affecting it should be done. 3) Trace of variance covariance matrix initially is more and approches to zero as EKF cycle repeats.