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CSA0672 : Analytical Questions - 1

a) Solve the following recurrence relations :-

a) $x(n) = x(n-1) + 5$ for $n > 1$ $x(1) = 0$

Given $x(n) = x(n-1) + 5$

Sub $n = 2$ $x(1) = 0$

$x(2) = x(2-1) + 5$

$\Rightarrow x(1) + 5$

$\Rightarrow 0 + 5$

Sub $n = 3$

$x(3) = x(3-1) + 5$

$\Rightarrow x(2) + 5$

$\Rightarrow 5 + 5 \Rightarrow 10$

$n = 4$

$x(4) = x(4-1) + 5$

$\Rightarrow x(3) + 5$

$\Rightarrow 15.$

the general for the given equation. is $x(n) = x(1) + (n-1)$ In the given equation $d = 5$ and $x(1) = 0.$

$x(n) = 0 + 5(n-1)$

$x(n) = 5(n-1)$

$x(n) = 5(n-1)$ is the recurrence relation

b) $x(n) = 3x(n-1)$ for $n > 1$, $x(1) = 4$.

Given

$x(n) = 3x(n-1)$

Sub $n = 2$ $x(1) = 4$

$3x(2-1)$

$\Rightarrow 3x(1)$

$x(2) \Rightarrow 12$

Sub $n = 3$

$x(3) = 3x(3-1)$

$\Rightarrow 3x(2)$

$\Rightarrow 36$

$n=4$

$x(4) = 3x(4-1)$

$= 3x(3) \Rightarrow 108$

The general form of the given eq^n is $x(n) = 3^{n-1}x(0)$

$$\boxed{x(n) = 3^{n-1}4}$$

c) $x(n) = 3^{n-1}4$ is recurrence relation.

Given $x(n) = x(n/2) + n$

$x(1) = 1$; $n = 2k$

$x(2k) = x(2k/2) + 2k \Rightarrow x(2k) = xk + 2k$

$\underline{k=1}$

$x(2.1) = x(1) + 2$

$\Rightarrow 3$

$\underline{k=2}$

$x(2.2) = x(2) + 2(2)$

$\Rightarrow x(2) + 4$

$\underline{k=3}$

$x(2.3) = x(3) + 2(3)$

$x(6) \Rightarrow x(1.5) + 3$

$\underline{k=4}$

$\Rightarrow 3 + 4$

$\Rightarrow 7$

The general equation for given expressions

$$\boxed{x(2k) = x(k) + 2k}$$

d) $x(n) = x(n/3) + 1$ for $n > 1$ $x(1) = 1$

$x(n) = x(n/3) + 1$

$n = 3k$

$x(3k) = x(3k/3) + 1$

$x(3k) = x(k) + 1$

$\underline{k=1}$

$k=2$

$x(3.1) = x(1)+1$

$\Rightarrow 2$

$x(3.2) = x(2)+1$

$\Rightarrow x(2/3)+1$

The general equation for $x(3k) = 1+\log_3(k)$.

Evaluate the following recurrence completely.

i) $T(n) = T(n/2)+1$, where $n = 2k$ $k \geq 0$.

$n = 2k$; i.e $k = \log n$

$T(2k) = T(2k/k)+1$

$T(2k) = T(k)+1$

$T(2 \cdot k) = T(k/2)+2$ (if k is even)

$T(2 \cdot k) = T(k-1/2)+2$ (if k is odd)

$\Rightarrow$ Recurrence $\Rightarrow \boxed{T(n) = O(\log n)}$

ii) $T(n) = T(n/3) + T(2n/3) + cn$

and n is input size. c is constant

$T(n) = aT(n/b) + f(n)$

$a = 2$; $b = 3$, $f(n) = cn$

using masters theorem

$f(n) = O(n^c)$

where $c > \log_a^b$

$T(n) = \Theta(n \log_a^b)$

$f(n) = \Theta(n \log_b^a)$

$T(n) = \Theta(n \log_a^b \log n)$

$f(n) = \Omega(n^c)$

where $c > \log_b^a$, $af(n/b) \leq k f(n)$ for $k<1$.

$T(n) = \Theta(f(n))$

find $\log_a^b = \log_a^b \Rightarrow \log_3^2$

$f(n) = cn = n \log_a^b$

Recurrence relation $\Rightarrow T(n) = \Theta(n)$

consider the following recursion algorithm

MINI [A [0 ... n-1]]

If n-1 return A [0]

Else temp = min 1 [A [0 ... n_2)]

If temp <= A [n-1] return temp

else

Return A [n-1]

a) what does this algorithm compute?

→ The algorithm computes the minimum element in an array of size n using a recursive approach.

⇒ If the array has only one element ($n==1$) it returns that single element as the minimum

⇒ Recursive case:-

* If the array has more than one element ($n>1$) the function makes a recursive call to find the min element in subarray consisting of the first n-1 elements.

* The result of this receive call (temp) is then compared to the last element of the current array segment ("A[n-1]")

* The function returns the smaller of these two values

b) Setup a recurrence relation for algorithm basic operation count and solve it.

```
Min1 [A[0...n-1])
    If n==1
        return A[0]
    Else
        temp = min1 [A[0..n-2]) .. n-1
If temp <= A[n-1]
        return temp
else
        return A[n-1]
```

$T(n)$: no. of basic operations

If $n=1$ Then $T(1)=0$

" $T(n) = T(n-1)+1$ " is the recurrence relation

$T(1) = 0$

$T(2) = T(2-1)+1$

$T(2) = 1$

$T(3) = T(3-1)+1$

$= 2$

$T(4) = T(4-1)+1$ Time complexity $= O(n)$.

$= 3$

$T(n) = n-1$

4) Analyze the order of growth

$f(n) = 2n^2+5$ & $g(n) = 7n$ $\Omega\, g(n)$

$f(n) = 2n^2+5$ $g(n) = 7n$.

If $n=1$ => $f(n) = 2(1)^2+5$ $g(n) = 7(1) = 7$

$= 7$

$n=2$ => $f(n) = 2(2)^2+5$ $g(n) = 7(2)$

$= 13$ $= 14$

$n=3$ => $f(n) = 2(3)^2+5$ $g(n) = 7(3)$

$= 23$ $= 21$

$n=4$ => $f(n) = 2(4)^2+5$ $g(n) = 7(4)$

$= 37$ $= 28$

$f(n) \geq g(n)$. (condition satisfies at $n=1$ onwards. So, the $\Omega(7n)$ is the recurrence relation $T(n) = \Omega(n)$.