Capacity Perspective of Space-Time Block Codes

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Abstract

Next generation wireless system requires high capacity, reliability and data rate compared to the current communication system. To achieve such expectation space-time coding and MIMO systems are widely regarded as the most likely candidates for futuristic high data rate systems. Although, the Shannon capacity for MIMO channels has been known since few years, attainment of this capacity is challenging issue in many cases. On the other hand, Space-time (ST) coding has been proved effective in combating fading, and enhancing data rates [1,2] thus paving the way towards the attainment of the promised capacities.

In this project, firstly, we will study capacity of MIMO channel and it’s result as in [8] Then, we will examine Space-Time Block Codes from its capacity perspective [6] that is, at what conditions can we use space-time block codes in order to achieve optimal capacity. We also simulate the obtained capacity result to have better insight. Finally, we’ll try to extend our study to tensor space–time coding [17], which allows spreading and multiplexing the transmitted symbols into data streams.

Abbreviations, Mathematical notations and Operations

A^H	conjugate transpose of A
trA	trace of A
detA	determinant of A
rankA	rank of A
$\|A\|_F$	Frobenius norm of A
a_{ij}	$(i,j)^{th}$ entry of A
a^+	max{0,a}
I_a	Identity matrix of order a x a
E	Expected value
Var	Variance
$\mathbb{R}$	Set of real numbers
$\mathbb{C}$	Set of complex numbers
$\mathbb{R}^n$	Set of n-dimensional real vectors
$\mathbb{C}^n$	Set of n-dimensional complex vectors
$\mathbb{C}^{n\times n}$	Set of nxn complex matrices
AWGN	Additive white Gaussian noise
BER	Bit error rate
i.i.d	Identical independent distribution

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Chapter 1:

1.1 Introduction

In Modern days, there is a rapid growth in the wireless communication technology. The optimal design and successful deployment of high-performance communication network has number of technical challenges. The bottleneck to build high-performance wireless network is its channel because the fundamental characteristics and capacity limits of the wireless channel affects all aspects of the wireless communication design. Communication systems in use today are predominantly single-antenna systems. Because of the multiple-path propagation in wireless channels, the capacity of a single wireless channel can be very low. In order to meet the growing demand for higher data rates, use of multiple antennas at the receiver and transmitter are widely regarded as the most likely candidate. This technology ranges from Single-Input Multiple-Output (SIMO) to Multiple-Input, Multiple-Output (MIMO), which open multiple data pipes over a link and collect more energy to improve the signal to noise ratio (SNR) at the receiver. The key feature of multiple antenna system is its ability to turn multiple-path propagation, which is traditionally regarded as a disadvantage to wireless communications, into a benefit to the users. The main advantages of employing multiple antennas are

(a) the improvement in reliability performance through diversity

(b) increase in data rate through spatial multiplexing.

In [1] it is derived that, efficiencies of MIMO channels can grow approximately linearly with the number of antennas available on each side of the wireless link. These results indicate that multiple-antenna systems have much higher Shannon capacity than single-antenna ones.

However, since Shannon capacity can only be achieved by codes with unbounded complexity and delay, the above results do not reflect the performance of real transmission systems.

In the Single-Input Single-Output (SISO) case, channel capacities predicted by Shannon in 1948 was closely achieved with the advent of Turbo codes (which took about fifty years). In MIMO case theoretically channel capacity was proposed in [2]

The key idea for improving the error performance is to exploit the propagation of several paths jointly, which helps in drastically reducing the probability of all signals fading in at the same time. Thus, mitigating fading and improving reliability, which leads to reduction of error rate.

There are many forms to achieve diversity. They can be classified into four classes, as follows: space, time, frequency, and polarization. A simple Space-Time(ST) code was proposed by Alamouti in [3] a transmit diversity scheme using two transmitters which approach closely the channel capacities. By using simple ST code, both the data rate and the performance are improved by many orders of magnitude with no extra cost of spectrum. This is also the main reason that space-time coding attracts much attention from academic researchers and industrial engineers. Since then many works have proposed a variety of ST transmission schemes in order to attain a good compromise between error performance and transmission rate in different system contexts [4,5]. Although, there is quite rapid advancements in encoding/decoding techniques, there still exist many cases of interest, where more research is needed in order to approach the capacities of MIMO systems in practice.

In this project, firstly, we will study capacity of MIMO channel and it's result as in [8] Then, we will examine Space-Time Block Codes from its capacity perspective [6] that is, at what conditions can we use space-time block codes in order to achieve optimal capacity. We also simulate the obtained result to have better insight. Finally, we'll try to extend our study to tensor space–time coding [17], which allows spreading and multiplexing the transmitted symbols into data streams.

The project is structures as follows: Chapter 2 provides an overview of MIMO system model and description of MIMO Capacity based on [8]. Chapter 3 is a literature survey that focuses on the optimal capacity of Orthogonal Space-time block codes. A detailed analysis of numerical and simulation of [6] is provided. Chapter 4 is literature survey of tensor space-time coding and description of how we can obtain its capacity.

Chapter 2

2.1 Introduction to MIMO

Multiple-Input Multiple-Output (MIMO) systems is one of the milestone technical breakthroughs in modem communication. MIMO systems are simply defined as the systems containing multiple transmitter antennas and multiple receiver antennas. Communication theory shows that, MIMO technology is considered as the promising technique to support high data rate and high performance by achieving very high capacity in many cases. Since third generation-3G technology different structures of MIMO systems have also been proposed.

2.2 Multiple-Antenna Communication System Model

Figure 1 shows generic architecture of MIMO system comprising *M* transmitter antennas *N* receiver antennas. In-between transmit and receive antennas there exists a channel. Random propagation coefficient h_{mn} represents the channel between mth transmit antenna and the nth receive antenna.

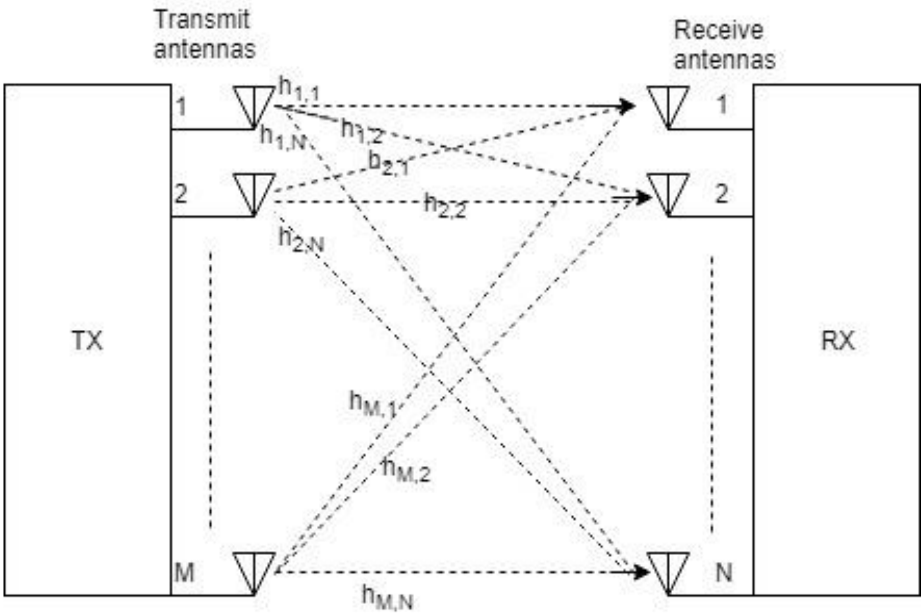

Figure 1:MIMO block diagram

To transmit information from transmitter to the receiver, let $s_{1,} s_2, \dots, s_m$ be the feeds signals at every transmission time, to its m[th] transmission antennas respectively. This transmission antennas then send the signals simultaneously to the receiver. Every signal received at each receiving antenna is superposition of the signals from every transmit antenna through the fading coefficient h_{mn} which is also corrupted by noise w_{n}.

$$x_n = \sum_{m=1}^{M} h_{mn} s_m + w_n \tag{2. 1}$$

This is true for n = 1,2,…..N. Let $s = [s_{1,} s_2, \dots, s_M]$, $x = [x_{1,} x_2, \dots, x_M]$, $w = [w_{1,} w_2, \dots, w_M]$, be transmitted signal, received signal and noise vector respectively and the channel matrix is represented by

$$H = \begin{bmatrix} h_{11} & \cdots & h_{1N} \\ \vdots & \ddots & \vdots \\ h_{M1} & \cdots & h_{MN} \end{bmatrix}$$

Therefore, the system equation can be written as

$$x = Hs + w \tag{2. 2}$$

2.3 MIMO Capacity

As discussed in chapter 1, communication systems with multiple antennas can greatly increase capacity. This is one of the main reasons that multiple-antenna systems are of great interest. This section is about the capacity of multiple-antenna communication systems with Rayleigh fading channels. Two cases are discussed:1) Both the transmitter and the receiver know the channel 2) Only the receiver knows the channel. The following results are based on Telatar's results in [8]

In any communication system, we know that capacity depends on the transmit power P. Therefore, we have power constraint on the transmitted signal. This is given by,

$$E[tr(ss^H)] \le P$$

Case 1: Both the transmitter and receiver know the channel.

We have matrix H matrix to be deterministic. Now, taking singular value decomposition of H: $H = UDV^H$, where U is an M x M unitary matrix, V is an N x N unitary matrix, and D is an M x N diagonal matrix with non-negative diagonal entries. we define $\tilde{x} = U^{*}x$, $\tilde{s} = V^{*}s$, and $\widetilde{w} = U^{*}w$, then the system equation (2.2) is equivalent to

$$\tilde{x} = D\tilde{s} + \widetilde{w} \tag{2. 3}$$

Where $\widetilde{w}$ and w is circularly symmetric complex Gaussian with mean zero and variance I_N. Since the rank of H is min {M, N}, at most min{M, N} of its singular values are non-zero. Let $\sqrt{\lambda_i}$ be denoted as the non-zero singular values of H. Then system equation can be written component-wisely to get

$$\tilde{x}_i = \sqrt{\lambda_i}\tilde{s}_i + \widetilde{w}_i \ \ for\ 1 \le i \ \le min\{M,N\}. \tag{2. 4}$$

Therefore, the channel is equivalent to min{M,N} single-antenna systems with uncorrelated channels. In [8] it is proved that the capacity achieving distribution of $\tilde{s}_i$ is circularly symmetric Gaussian and the capacity for the i^th independent channel is $\log(1+\lambda_i P_i)$, where $P_i = E\{\tilde{s}_i \tilde{s}_i^{\,*}\}$is the power consumed in the i^th independent channel. Hence, to maximize the mutual information, $\tilde{s}_i$ should be independent circularly symmetric Gaussian distributed and the transmit power should be allocated to the equivalent independent channels optimally. This optimal power allocation is done by mechanism called “water-filling” [8] and the capacity of such system is given by

$$C = \sum_{i=1}^{min\{M,N\}} log(\mu\lambda_i) \tag{2. 5}$$

where μ is chosen to meet the power constraint such that $\sum_{i=1}^{\min\{M,N\}}\left(\mu - {\lambda_i}^{-1}\right)^{+} = P$. Hence C increases linearly in min {M, N}.

Case 2: Only receiver knows the channel

In this case only the receiver knows the channel matrix H, Hence the transmitter cannot perform the adaptive power allocation (water filling mechanism). It is proved in [8] that the channel capacity of this case is given by

$$C = log\ det(I_N + (P/M)H^H H) \tag{2. 6}$$

which is achieved when s is circularly symmetric complex Gaussian with mean zero and variance $(P/M)I_M$.

When the channel matrix is random and Rayleigh distribution, the expected capacity is

$$C = E\{log\ det(I_N + (P/M)H^H H)\ \} \tag{2. 7}$$

Now, let’s say we have fixed the number of received antenna N, then according to law of large numbers $lim_{M\to\infty} \frac{1}{M} H^H H = I_N$ with probability 1. Thus, the capacity is $Nlog(1 + P)$ with probability 1. From this, we can clearly see that capacity grows linearly with N, the number of receive antenna. Similarly, for fixed transmitter antenna M, it can be shown that with probability 1, capacity increases almost linearly with M.

Therefore, comparing with the single antenna capacity

$$log(1 + P) \tag{2. 8}$$

the capacity of multiple-antenna systems increases almost linearly in min{M,N}. Thus, Multiple-antenna systems gives significant capacity improvement than single antenna systems.

Chapter 3

In chapter 2, we gave an overview MIMO channel capacity. Now we will study detail analysis of Orthogonal Space-Time Codes(OSTC) with information-theoretic insights. The results stated here represent a detailed analysis of work in [6].

3.1 Analysis of Space-Time Coded Systems

In this section, we present a selection of well-known Space-Time Coded(STC) techniques. In particular, we investigate orthogonal space-time block codes (OSTBC), which is sub-class of linear space-time block codes. Within which. We will perform the analysis corresponding system capacities and numerical evaluate the obtain results.

Before getting into detailed analysis we shall first introduce few terminologies used in the analysis of the STC techniques.

Space-Time Block Code(STBC) maps the set of K complex data symbols $\{s_1, \ldots, s_K\}$ onto a matrix S Thus multiplexing the symbol stream onto the M transmit antennas.

Definition 3.1: STBC which transmits K complex data symbols during T symbol time instants. Then its transmission rate is given as:

$$R = K/T \text{ symbols/channel use} \qquad \textit{(3.1)}$$

A code is called full-rate if and only if R = 1.

3.2 Linear Space-Time Block Codes

Definition 3.2: In [9] Mapping of a linear STBC is defined as

$$S = \sum_{n=1}^{K} (Re\{s_n\}A_n + j\, Im\{s_n\}B_n) \qquad \textit{(3.2)}$$

Where A_n and B_n can be interpreted as modulation matrices, because they modulate the real and imaginary part of the complex data symbols onto the transmitted matrix S.

3.3 Orthogonal STBC

Orthogonal STBC are subclass of linear block codes that as unitary properties. This theory was started by Alamouti [3], which was then extended in [10].

3.3.1 Properties of OSTBC code

1) For OSTBC code design set of matrices $\{A_n, B_n\}$ should satisfy the following conditions

a) $A_n A_n^H = I_M$,

b) $B_n B_n^H = I_M$,

c) $A_n A_p^H = -A_n A_p^H$, (n≠p)

d) $B_n B_p^H = -B_n B_p^H$, (n≠p)

e) $A_n B_p^H = B_n A_p^H$, (n≠p) , where n,p = 1,2,…,K.

2) Unitary property: If matrices $\{A_n, B_n\}$ satisfy above property. Then we can define an OSTBC to be a STC that satisfies the below property:

$$SS^H = \sum_{n=1}^{K} |s_n|^2 \, \mathrm{I}_M , \qquad (3.3)$$

Where I_M is identity matrix of dimension M x M.

3.3.2 Study on OSTBCs capacity

Referring to [6], we will analyse OSTBC in an information theoretic sense. That is, we try to present detailed analysis of the topic mentioned in this paper.

Let's consider ST-MIMO system as show in Figure 1. Let M, N be the number of transmitting and receiving antenna respectively (and M>1). We assume the channel to be flat-fading and quasi-static which is unknown at the transmitter but is known at the receiver. The block fading model assumes that the fading coefficient doesn't change over for T consecutive transmission time. This is approximately equal to the coherence interval of a continues fading process.

From [11] The system equation of T received block can be written as:

$$Y = \sqrt{\frac{\varrho}{M}} HS + W \tag{3.4}$$

Where ϱ is the average SNR for each of the M diversity branches

$$S = \begin{bmatrix} s_{11} & \cdots & s_{1N} \\ \vdots & \ddots & \vdots \\ s_{T1} & \cdots & s_{TM} \end{bmatrix} \; H = \begin{bmatrix} h_{11} & \cdots & h_{1N} \\ \vdots & \ddots & \vdots \\ h_{M1} & \cdots & h_{MN} \end{bmatrix}, W = \begin{bmatrix} w_{11} & \cdots & w_{1N} \\ \vdots & \ddots & \vdots \\ w_{T1} & \cdots & w_{TM} \end{bmatrix}$$

The vectorized MIMO transmission model of equation 3.4

$$y = \mathrm{vec}(Y)$$

$$= \mathrm{vec}\left(\sqrt{\frac{\varrho}{M}} HS + W\right)$$

$$y = \sqrt{\frac{\varrho}{M}} H_e s' + w' \tag{3.5}$$

Where, $s' = \begin{bmatrix} Re\{s\} \\ Im\{s\} \end{bmatrix}, H_e = \left[H_{e,a} \; H_{e,b}\right], H_{e,a} = \{vec(HA_n)\}_{n=1}^{K}, H_{e,b} = \{vec(HB_n)\}_{n=1}^{K}$

$$w' = \mathrm{vec}(W)$$

For scalar AWGN channel, by using the vectorized channel model, the receiver effectively uses 2K virtual antennas. Multiplying $H_e$with y in equation 3.5 can be viewed as Maximum-Ratio Combining (MRC). The effective transmit power MRC can be calculated using the covariance matrix of the MRC signal output. Thus, if receiver combines two virtual antennas we can calculate effective SNR at the receiver.

In [6] effective SNR at the receiver is given by:

$$\frac{2*\frac{\varrho}{M}||H||^4\left(\frac{1}{2}\right)}{||H||^2} = \frac{\varrho}{M}||H||^2 \tag{3.6}$$

The key observation of the above equation is that, set of K parallel independent fading channel has SNR of $\frac{\varrho}{M}\ ||H||^2$ in each channel respectively.

Theorem 3.3.1: MIMO system capacity of OSTBC design is given by:

$$C_{OSTBC} = \frac{K}{T}\ \log(1 + \frac{\varrho}{M}\ ||H||^2)$$

Proof: From [12] capacity of AWGN channel is given by $C = log(1 + SNR)$. As we consider capacity of K parallel independent channels. We multiply capacity of single channel by factor of K channel. We divide this quantity by T because we measure capacity in bits per channel use.

Thus, $\mathrm{C} = \frac{K}{T}\log(1 + SNR)$

From equation 3.6, SNR of OSTBC is $\frac{\varrho}{M}\ \ ||H||^2$

Therefore $$C_{OSTBC} = \frac{K}{T}\ \log(1 + \frac{\varrho}{M}\ ||H||^2) \tag{3.7}$$

Theorem 3.3.2: Capacity of nonergodic block capacity of a MIMO is greater than capacity of OSTBC, whose code rate <1 and for i.i.d channel matrix H.

$$\mathrm{C(H)} \geq C(H)_{oSTBC}$$

Proof: From chapter 2, The nonergodic block capacity of a MIMO is

$$C(H) = \log\det(I + \frac{\varrho}{M}HH^H)\ \text{ bits/channel}$$

Appling SVD to the channel matrix H: H = U$\sum V^*$, $\sum$ is singular value $\{\lambda\}_{i=1}^{R}$ on diagonal with rank R.

$$C(H) = logdet\left(I + \frac{\varrho}{M} HH^H\right)$$

$$= logdet\left(I + \tfrac{\varrho}{M} U\sum\sum^H U^H\right)$$

Since, U is unitary matrix, thus $U^HU = I$, thus $\det(U^HU) = \det(U)*\det(U^H) = 1$

$$= logdet(U^H)det\left(I + \frac{\varrho}{M} U\sum\sum^H U^H\right) det(U)$$

$$= logdet\left(I + \frac{\varrho}{M} \sum\sum^H\right)$$

$\sum$ is diagonal matrix and has singular value of H ($\sum\sum^H = \lambda_i$, i = 1, 2,…, R),

$$= log \prod_{i=1}^{R} \left(I + \frac{\varrho}{M} \lambda_i\right)$$

Expanding the product and taking $||H||^2 = \sum_{i=1}^{R} \lambda_i^2$ and $\det(HH^*)_R = \prod_{i=1}^{R} \lambda_i^2$

$$C(H) = \log(1 + \tfrac{\varrho}{M} \textstyle\sum_{i=1}^{R} \lambda_i^2 + (\tfrac{\varrho}{M})^2 \sum_{i_1 \neq i_2}^{i_1 < i_2} \lambda_{i1}^2 \lambda_{i2}^2 + \cdots + (\tfrac{\varrho}{M})^R \prod_{i=1}^{R} \lambda_i^2)$$

$$C(H) = \log(1 + \tfrac{\varrho}{M} ||H||^2 + \cdots + (\tfrac{\varrho}{M})^R \det(HH^*)_R) \tag{3.8}$$

By using Equation (3.8) it follows,

$C(H) = \log(1 + \tfrac{\varrho}{M} ||H||^2 + \cdots + (\tfrac{\varrho}{M})^R \det(HH^*)_R) \geq \log(1 + \tfrac{\varrho}{M} ||H||^2)$

$$\geq \tfrac{K}{T} \log(1 + \tfrac{\varrho}{M} ||H||^2) \quad (\because \tfrac{K}{T} < 1)$$

$$= C(H)_{OSTBC}$$

Therefore, $C(H) \geq C(H)_{oSTBC}$

So far, we have studied system capacity of OSTBC capacity is always less then MIMO channel capacity. Now we will investigate under which conditions we can reach the equality in Theorem 3.3.2

To calculate the equality let's compute the difference in the two capacities $\triangle C(H) = C(H) - C(H)_{OSTBC}$

Using Equation (3.7, 3.8)

$$\triangle C(H) = log(1 + \frac{\varrho}{M}||H||^2 + \cdots + (\frac{\varrho}{M})^R det(HH^*)_R) \; - \frac{K}{T}\; log(1 + \frac{\varrho}{M}||H||^2)$$

$$\triangle C(H) = log(1 + \frac{\varrho}{M}||H||^2 + Q\;) - \frac{K}{T}\; log(1 + \frac{\varrho}{M}||H||^2)$$

Where Q = $(\frac{\varrho}{M})^2 \sum_{i_1 \neq i_2}^{i_1 < i_2} \lambda_1^2 \lambda_2^2 + \cdots + \; (\frac{\varrho}{M})^R \prod_{i=1}^{R} \lambda_i^2$.

$$\triangle C(H) = \log(1 + \frac{\varrho}{M}\left||H|\right|^2 + Q) - \; \frac{K}{T}\log\left(1 + \frac{\varrho}{M}\left||H|\right|^2\right) + log\left(1 + \frac{\varrho}{M}\left||H|\right|^2\right) - log\left(1 + \frac{\varrho}{M}\left||H|\right|^2\right)$$

$$\triangle C(H) = log(1 + \frac{\varrho}{M}\left||H|\right|^2 + Q) - \; \frac{K}{T}log\left(1 + \frac{\varrho}{M}\left||H|\right|^2\right) + \frac{T}{T}log\left(1 + \frac{\varrho}{M}\left||H|\right|^2\right) - log\left(1 + \frac{\varrho}{M}\left||H|\right|^2\right)$$

$$\triangle C(H) = \; \frac{T-K}{T}\; log\left(1 + \frac{\varrho}{M}\left||H|\right|^2\right) + lo\,g\left(1 + \frac{\varrho}{M}\left||H|\right|^2 + Q\right) - log\left(1 + \frac{\varrho}{M}\left||H|\right|^2\right)$$

$$\triangle C(H) = \; \frac{T-K}{T}\; \log\left(1 + \frac{\varrho}{M}||H||^2\right) + \log\left(1 + \frac{Q}{1+\frac{\varrho}{M}||H||^2}\right) \qquad (3.9)$$

The above result provides loss incurred in outage capacities, we observe that difference is a function of the channel realization. Thus, by imposing constrain on H we can show when OSTBC system capacity coincides with the channel capacity.

Theorem 3.3.3: OSTBC is optimal with respect to channel capacity only when it's rate one and used over a channel of rank one.

Proof: Consider a nontrivial channel $0 < ||H||^2 < \infty$. OSTBC code has optimal capacity if loss incurred is zero ($\triangle C = 0$)

$$\triangle C = \frac{T-K}{T} \log\left(1 + \frac{\varrho}{M}||H||^2\right) + \log\left(1 + \frac{Q}{1+\frac{\varrho}{M}||H||^2}\right) = 0$$

First term in the expression is zero only if K = T (from Definition 3.1 rate is one). The second logarithm term is zero if and only if Q is zero.

That is,

$$\left(\frac{\varrho}{M}\right)^2 \sum_{i_1 \neq i_2}^{i_1 < i_2} \lambda_{i1}^2 \lambda_{i2}^2 + \cdots + \left(\frac{\varrho}{M}\right)^R \prod_{i=1}^{R} \lambda_i^2 = 0 \qquad (3.11)$$

The above equation implies that sum-of-product term in expression is zero. This means that, all singular values except λ_1 is zero describing a channel of rank one. Let's assume that H is rank 2, which means that $\lambda_1^2 \neq 0$ and $\lambda_2^2 \neq 0$. Therefore $\sum_{i_1 \neq i_2}^{i_1 < i_2} \lambda_{i1}^2 \lambda_{i2}^2$ is positive, which means that Q is positive and non-zero. This contradicts the statement. Therefore, H should be rank one.

The consequence of above theorem is that OSTBC can only achieve channel capacity in case of code rate one and a rank 1 channel matrix. To visualize the obtained results, we shall perform simulations of the OSTBC system capacity.

3.4 Simulations and result:

Simulation is carried out in order to visualize difference between OSTBC system capacity and ergodic capacity. We considered two case,each case representing the curve of OSTBC system capacity and ergodic capaciy with two transmit antenna and one or two receive antennas. For this simulation we have considered AWGN channel with 0 mean and 0.5 variance and capacity is calculated by varing SNR from 0 to 25 dB.

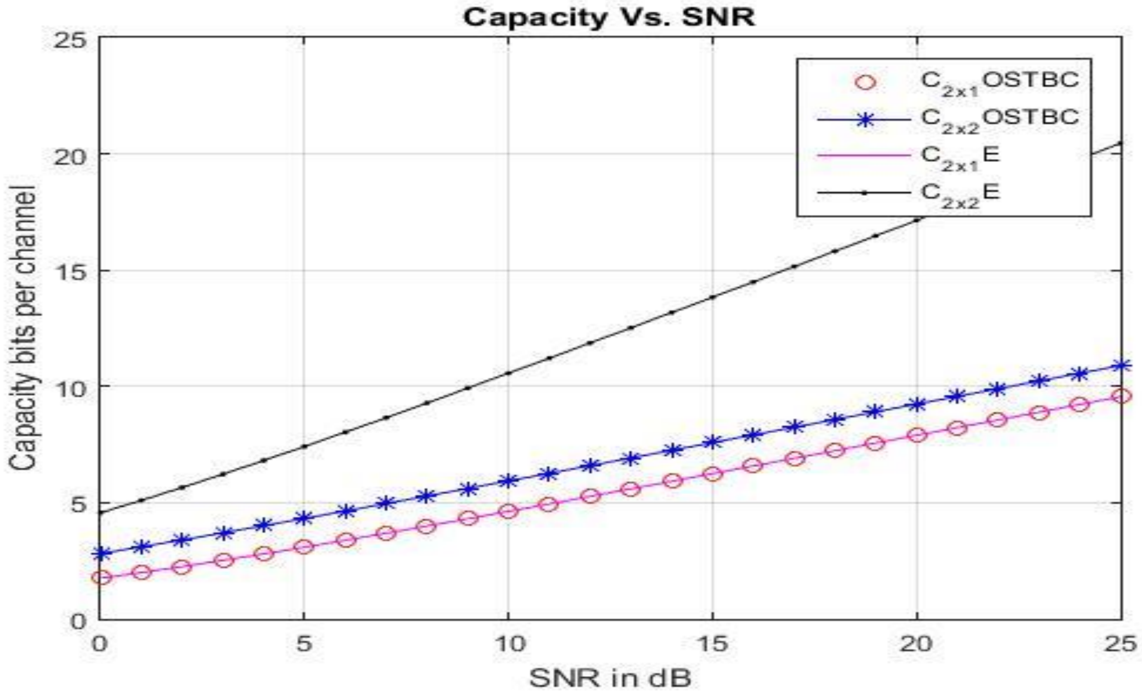

Figure 2: Compression of OSTBC system capacity with ergodic capacity

The results of simulations are shown as in Figure 2. From the result, we observe that OSTBC is optimal in term if capacity in the case where receiving antenna is one. On the other hand, when receiving antenna is two there is drastic change in the curve. That is OSTBC system capacity curve and ergodic channel capacity curve do not coincide anymore. Hence it is clear that, full rate OSTBC over channel with one receive antenna is always optimal with respect to capacity. From the results we also observe that, capacity difference increases significantly between OSTBC and ergodic channel with increase in SNR for receive antenna greater than one.

3.5 Conclusion

From theorem we conclude that, for STBC to be optimal it should be full rate and channel rank one. In other words, STBC doesn’t depend on number of nonzero singular value hence it doesn’t take full advantage of independent subchannel in MIMO. Although, STBC code are simple to implement it comes at the cost of capacity. Even for small number of receive antennas, the loss is significant at desired SNR.

Chapter 4

This chapter deals with the extension of capacity to tensor space time coding for MIMO wireless communication.

4.1 Literature Survey

Alamouti [3] proposed transmit diversity with two transmit antennas to mitigate fading channel. In [4] paper construction criteria for ST codes was developed. After this many works [4,5] have been done in order improve error performance and transmission rate.

During the last decade, tensorial approaches have been employed to exploit diversities of MIMO wireless systems providing a more reliability to recover the transmitted symbols with blind detection [13 14]. A block tensor model for multiple-access MIMO systems with multiple transmit antennas per user was proposed in [14]

Using tensor model spreading and multiplexing operations, and additional diversity on wireless communication systems can be exploited jointly. The tensor decomposition tools are used to model received signal, which take advantage of uniqueness property of the tensor. This uniqueness property helps in recovery of the transmitted signal.

In this section we try to study on the factors that affects tensor space time coding. In particular, tensor based Space time multiplexing [17] which was proposed for achieving variable rate-diversity trade-offs for any transmit-receive antenna.

Linear Dispersion Codes (LDC) introduced in [16] take advantage by combining multiplexing and space-time coding. In [17] paper it is mentioned that tensor based Space time multiplexing can we viewed as a sort of general LDC when formulated using tensor tool.

Extension Idea: Reviewing [16] and [17], We got to know that capacity of Linear Dispersion Codes is closely related to the tensor Space time multiplexing. By deriving the constrain for received signal that includes average SNR, Expectation of allocation matrix and Expectation of signal coding matrix it is possible to analyze the capacity and also, we can determine its optimal capacity limit.

4.2 Linear Dispersion Codes

This method was proposed in [16]

System model

Consider the MIMO system with M transmitting antennas and N receiving antennas. The received system model is given by:

$$y[k] = \sqrt{\frac{\varrho}{M}} HX[k]s + w[k] \tag{4.1}$$

$X[k]s$ is the transmitted matrix. H is channel matrix and w[k] is zero mean Zero-Circular Symmetrix Complex Gaussian noise.

Now by stacking T received vector we obtain block signal model:

$$\begin{bmatrix} y[1] \\ \cdot \\ \cdot \\ y[T] \end{bmatrix} = \sqrt{\frac{\varrho}{M}} \mathcal{H} \begin{bmatrix} X[1] \\ \cdot \\ \cdot \\ X[T] \end{bmatrix} s + \begin{bmatrix} w[1] \\ \cdot \\ \cdot \\ w[T] \end{bmatrix} \tag{4.2}$$

$$\mathcal{Y} = \sqrt{\frac{\varrho}{M}} \mathcal{H}\mathcal{X}s + \mathcal{W} \tag{4.3}$$

$\mathcal{H} = I_T \otimes H$ is a matrix of dimension NT x MT, $\mathcal{Y} = [\, y[1], \ldots, y[T] \,]^T$ is received signal vector of dimension NTx1, $\mathcal{X} = [\, X[1]^T, \ldots, X[T]^T \,]^T$ is a matrix of dimension MT x Q (Q is complex data symbols which is modulated by code matrix).

The spatial rate of these code defined in [16] is R = Q/T. if Q=T then spatial rate is one and if Q = TM then spatial rate is M. Thus, rate is bounded between 1 and M.

The ergodic capacity of this signaling scheme is given by [4],

$$C = \max_{tr(\mathcal{X}^H \mathcal{X}) \leq T} \frac{1}{T} E\left\{log_2 det\left(I_{MT} + \frac{\rho}{M}\mathcal{H}\mathcal{X}\mathcal{X}^H\mathcal{H}^H\right)\right\} \quad (4.5)$$

This expression can be derived from Equation 2.7. in [] it is shown that the capacity of LDC is optimal if Q=MT that is choose $\mathcal{X}$ such that $\mathcal{X}\mathcal{X}^H = \frac{1}{MI_M}$.

Now, we shall analysis capacity for tensor model based Space time multiplexing code

From [17], we can write the received signal model as:

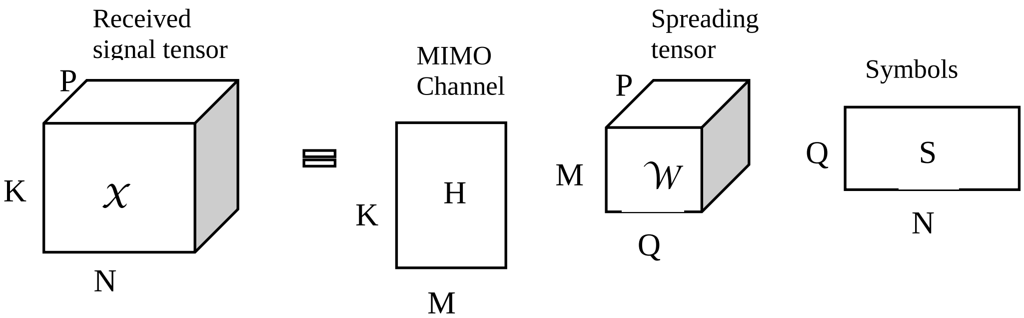

Figure 3: Received signal tensor model

$$X_n = \sqrt{\frac{\rho}{M}} H(s_n^T \otimes I_M)W^T + V_n \quad (4.6)$$

Where H is channel matrix, $C_n = \sum_{q=1}^{Q} s_n^{(q)} W^{(q)} = (s_n^T \otimes I_M)W^T$, $(s_n^T \otimes I_M)W^T$ indicate every symbol spread across the M transmit antennas. W = [$W^{(1)}, \dots, W^{(Q)}$] ∈ $\mathbb{C}^{P \times MQ}$ overall multiplexing matrix.

The Rate (R) is given by R = $\left(\frac{Q}{P}\right)\log(J)$ bits/channel use, where J being the modulation cardinality.

So here, we observe that, for tensor space time multiplexing to achieve its optimal capacity it not only depends of Q spread signals and P symbol periods, but it also depends on modulation cardinality. That is, the entries of S are chosen from an arbitrary J-Phase Shift-Keying (PSK) or J-Quadrature Amplitude Modulation (QAM) constellation. Next, to attain maximum mutual information it should also satisfy power constrain $E[tr(S^H S)]=NQ$, where N time-slots and S being symbol matrix that concatenates Q data sub-streams that are simultaneously transmitted.

Future Scope

If we have knowledge of frequency as in frequency-selective channel or OFDM channel it is more feasible to exploited frequency diversity to minimize bit error rate and to increase capacity conditions.

Another potential area of interest could be to explore GFDM channel as a third order tensor and to find the condition on which optimal capacity is achieved. For this we first need a complete frame work of tensor model of GFDM.

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