

Machine Learning-Based Sleep Disorder Prediction and Real-Time Analysis for Enhanced Health Monitoring

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Abstract— A person's health is greatly impacted by sleep problems including apnea and insomnia, which can result in chronic weariness, cognitive decline, and major illnesses like diabetes and cardiovascular diseases. To reduce these risks, early detection and management are essential, but conventional diagnostic techniques are frequently costly, time-consuming, and necessitate specific medical knowledge. This research offers an automatic learning-based sleep condition prediction system that uses sleep disorder data classified into apnea, insomnia and normal medical conditions in order to overcome this difficulty. To fine-tune the dataset for the best model performance, the system uses sophisticated preprocessing methods such as Label Encoding, Standard-Scaler normalization and correlation analysis in addition to intensive Exploratory Data Analysis (EDA). Several classification models are trained and assessed, with Random Forest obtaining the greatest accuracy of 95%. These models include Logistic Regression (91%), Random Forest (95%), Gradient Boosting (94.6%), and Quadratic Discriminant Analysis (QDA) (93%). Using Flask, the model is further incorporated into a real-time online interface that enables users to anticipate disorders instantly by entering pertinent sleep-related data. To assist users in efficiently managing and mitigating sleep disorders, the system also offers educational insights and preventive strategies. This project provides a scalable and effective solution for early diagnosis and intervention, ultimately fostering improved sleep health and well-being, by fusing strong machine learning algorithms with an approachable and user-friendly implementation.

I. INTRODUCTION

Sleep is a vital physiological process that is important for preserving general health and wellbeing. Early medical professionals recorded cases of sleep abnormalities as insomnia and sleep apnea, demonstrating that sleep problems have been acknowledged throughout history in many cultures [1]. To enhance the quality of sleep, ancient Greek and Egyptian doctors suggested a variety of herbal therapies and lifestyle modifications. Researchers discovered several periods of sleep and their effects on mental and physical health as scientific knowledge of sleep developed over time. A major advance in sleep research was the identification of rapid eye movement (REM) sleep in the middle of the 20th century [2], which paved the way for the classification of different sleep disorders according to physiological reactions, breathing patterns, and brain activity. Because sleep disorders are becoming more common in the public, contemporary medicine is paying more attention to them [3]. A growing number of people are experiencing sleep-related problems as a result of factors like stress, poor sleeping patterns, excessive

screen time, and sedentary lifestyles. Medical research indicates that millions of people worldwide suffer from conditions like sleep apnea and insomnia [4], which can result in major health issues like diabetes, obesity, cardiovascular disease, and mental health conditions like anxiety and depression. Many people are unable to diagnose sleep disorders because traditional diagnostic techniques like polysomnography (PSG) and multiple sleep latency tests (MSLT) necessitate costly medical equipment and skilled supervision [5]. In order to more accurately diagnose and treat sleep disorders, academics and medical experts have been looking into alternate approaches.

Clinical observation, self-reported questionnaires, and wearable technology to monitor sleep patterns are the mainstays of current sleep disorder diagnosis systems. Although these techniques offer insightful information, they are frequently imprecise, necessitate constant observation, and are prone to human mistake [6]. Although wearable technology, such smartwatches and sleep trackers, has grown in popularity, its main purpose is to identify sleep length and disruptions rather than to diagnose certain illnesses. Furthermore, predictive analysis using machine learning approaches has just lately been introduced in the medical profession; nevertheless, the majority of models do not incorporate real-time analysis and approachable applications for efficient decision-making. This weakness in current systems calls for the creation of an automated, accurate, and efficient method for utilizing machine learning techniques to predict sleep problems [7]. The startling rise in sleep problems and their long-term effects on human health are the driving forces underlying our endeavor. People of all ages are now affected by sleep disturbances, which are a common problem brought on by the development of technological advances and shifting lifestyles. One of the biggest obstacles to treating sleep-related health issues is the absence of effective, affordable, and generally available diagnostic techniques [8]. A promising answer to this problem is to incorporate machine learning into the prediction of sleep disorders, as it has demonstrated enormous potential in automation healthcare solutions. This project intends to close the gap between conventional diagnosis techniques and contemporary AI-driven solutions by creating a highly intelligent, real-time, and precise sleep disorder diagnosis system [9], ultimately leading to a healthier society.

II. LITERATURE SURVEY

Numerous previous studies have investigated various methods, such as wearable technologies, clinical evaluations, and machine learning models, for identifying and forecasting sleep disorders. The gold-standard diagnostic technique known as polysomnography (PSG), which captures heart rate, breathing patterns, oxygen levels, and brain activity as you sleep, was used in the early research. Despite its excellent accuracy, PSG is not widely available due to its high cost, time commitment, and need for expert medical care. Self-reported sleeping surveys and actigraphy-based techniques [10], which employ wrist-worn devices to track movement and predict sleep patterns, have been tried by researchers to solve this issue. However, these approaches frequently have errors and don't offer a thorough examination of complicated sleep problems including insomnia and apnea. Users can now track the quality of their sleep in real time thanks to the emergence of mobile applications and robotic sleep tracking devices brought about by technological improvements [11]. Even with these advancements, the majority of these solutions are not very good at making predictions or correctly identifying various types of sleep disorders.

Bahrami et al. examined 70 recordings from the PhysioNet ECG Sleep Apnea v1.0.0 dataset have been examined to identify sleep apnea using machine learning as well as deep learning techniques. Support-vector machines, logistic regression, Gaussian naïve Bayes, and linear and quadratic discriminate analyses were examples of traditional machine learning methods. Convolutional networks, recurrent networks, and hybrid convolutional-recurrent networking were examples of deep algorithms [12]. According to the study, hybrid deep models performed the best in terms of detection, with 88.13% accuracy, 84.26% sensitivity, and 92.27% specificity. Ramachandran et al. proposed that one common sleep disorder that contributes to cardiovascular problems is sleep apnea. For diagnosis, the costly and inconvenient sleep analysis test is not enough. This study highlights the applicability of deep learning and embedded technology in the diagnosis of sleep apnea [13]. The paper addresses the difficulties in developing sleep apnea detection systems and examines research on sensor integration, feature engineering, and classifiers. Salari et al. proposed that the diagnosis of sleep apnea (SA), a difficult sleep disorder, calls for innovative techniques. ECG evaluation is a useful technique for diagnosing SA [14], according to recent research. One effective technique for computer-aided diagnostics is machine learning, or ML. This study assessed 48 papers that used machine learning methods based on ECG features to evaluate individuals with SA.

Machine learning methods have been used more and more in recent years to diagnose sleep disorders, with encouraging outcomes in terms of increasing accuracy and automating diagnosis. Support Vector Machines (SVM), Decision Trees, and random forest classification methods are examples of supervised learning algorithms that have been used in a number of studies to evaluate sleep data and forecast problems [15]. The detection of apnea and other problems from sleep recordings has also been investigated using deep learning models, specifically Convolutional Neural Networks (CNNs) and Recurrent Neural Networks (RNNs). In order to create

highly accurate predictive models, some research has concentrated on identifying features in electroencephalogram (EEG) and electrocardiogram (ECG) signals [16]. But a lot of these models need a lot of labelled data and processing power, which makes them hard to use in real-time applications. Furthermore, hardly many research has included user-friendly websites or offered customized preventative techniques to people. These drawbacks underscore the necessity for a thorough and user-friendly machine learning-based sleep disorder prediction solution, which is what this project seeks to provide.

III. PROPOSED METHODOLOGY

A. DATA COLLECTION & PREPROCESSING

The quality and dependability of the information used for both training and testing have a significant impact on the efficacy of any predictive model. Data about sleep disorders is gathered for this project from reliable sources, such as openly accessible medical datasets, academic publications on healthcare, and actual sleep studies. Numerous characteristics are included in the dataset, including age, gender, body mass index (BMI), stress levels, oxygen saturation levels, heart rate, snoring frequency, sleep length, and sleep disruptions. These characteristics offer important information about a person's sleep habits and aid in the classification of sleep disorders into groups such as apnea, insomnia and healthy sleep. To guarantee that the machine learning models operate at their best, preprocessing techniques must be used to handle the missing values, noise, and errors that are frequently present in raw datasets. Exploratory Data Analysis (EDA) is carried out to comprehend the structure, distribution, and important patterns of the dataset prior to the application of preprocessing procedures. To find trends, outliers, and correlations between variables, EDA depicts the data using histograms, box plots, scatter plots, and correlation heatmaps. To choose the most important factors for model training, for example, a correlation heatmap can be used to identify which features are highly associated with sleep disorders. In order to make sure the dataset is ready for machine learning algorithms, distributions that were and data imbalances are also investigated. Another important phase in EDA [17] is feature engineering, where domain knowledge may be used to extract new, pertinent features.

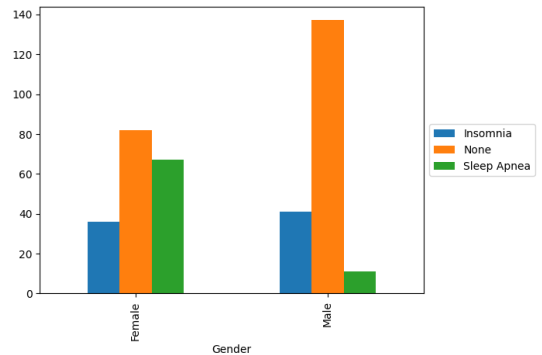


Fig.1 Gender vs Sleep Disorder Plot

Since missing data might have a detrimental effect on model performance, managing missing values comes after EDA. Missing data is addressed using a variety of methods, including forward or backward filling for time-series data and

mean, median, or mode imputed for numerical values. Advanced imputation approaches, such as K-Nearest Neighbors (KNN) imputation or numerous imputation techniques, can be used when missing values are substantial. To avoid extreme results biasing the model, statistical techniques such as Z-score analysis and the interquartile range (IQR) are frequently used for outlier discovery and removal [18]. Furthermore, skewed distributions can be normalized using data transformation methods like log transformations, which enhances the dataset's overall consistency. Categories are processed using methods of encoding to transform non-numeric data into a format appropriate for machine learning algorithms after outliers and values that are missing have been addressed. For ordinal variables such as sleep hygiene ratings (low, medium, and high) where category values have a predetermined order, label encoding is utilized. One-Hot Encoding is used to create binary columns for every category of non-ordinal categorical features [19], such as gender or sleep problem kind, in order to avoid the model assuming any unintentional correlations between them. Categorical data must be encoded in order for machine learning algorithms to properly analyze and learn from the dataset.

Another crucial preprocessing step is feature scaling, since machine learning models function best when numerical characteristics have a similar scale. Numerical features can be standardized using Standard-Scaler by transforming them into a typical range with a unit variance and zero mean. This prevents features with smaller numerical ranges (like sleep disturbances or snoring frequency) from being overshadowed by those with wider ranges (like the heart rate and oxygen saturation). Alternatively, values between 0 and 1 can be normalized using Min-Max Scaling [20], which is especially helpful for models that need bounded feature values. Model convergence is enhanced and the training process is stabilized with proper scaling. Lastly, to improve model efficiency, dimensionality reduction and feature selection approaches are used. To make sure that only the most pertinent characteristics are used for classification, correlation analysis is performed to remove redundant variables with strong correlations. By reducing dimensionality while keeping the most important features, Principal Component Analysis (PCA) helps avoid overfitting and increase computational efficiency. These preparation procedures give the dataset a clear, organized and balanced format, which helps machine learning models detect sleep problems and offer insightful information about sleep health.

B. Logistic Regression

A popular statistical model for multi-class and binary classification issues is logistic regression. It aids in the classification of people into three groups for the purpose of predicting sleep disorders: normal, apnea and insomnia. The logistic function [21] is used in the model to estimate the likelihood that an input corresponds to a specific class. Logistic regression produces chances that are then transformed into discrete classes, in contrast to linear regression, which forecasts continuous values. To create limits of decision-making between the three groups, the model trains from input features like heart rate, saturation with oxygen, stress levels, and sleep length. When the connection between the independent and dependent variables

is almost linear, logistic regression performs well and is computationally efficient.

Nevertheless, there are certain drawbacks to logistic regression, particularly when working with intricate datasets that contain non-linearly separable classes. In real-world situations, the model's assumption of input variable independence might not always hold true. Furthermore, it is susceptible to multicollinearity, which implies that features with a high degree of correlation may have a detrimental effect on the model's performance. Feature selection methods like dimensionality reduction and correlation analysis are used to eliminate redundant characteristics in order to increase its accuracy. A dependable baseline model before using more sophisticated machine learning approaches, logistic regression predicts sleep problems with an accuracy of 91% despite its simplicity.

C. Random Forest

Several decision trees are used in the Random Forest ensemble learning technique to increase classification accuracy and decrease overfitting. By employing a technique known as bagging (Bootstrap Aggregating) [22], in which several decision trees independently anticipate an outcome, each tree in the forest undergoes training on an arbitrary portion of the data. The final classification is determined by majority voting. The drawbacks of individual decision trees their propensity to overfit the training data are mitigated by this technique. In order to improve classification performance, Random Forest effectively examines a variety of sleep-related characteristics in the context of insomnia prediction, spotting intricate patterns and feature interactions.

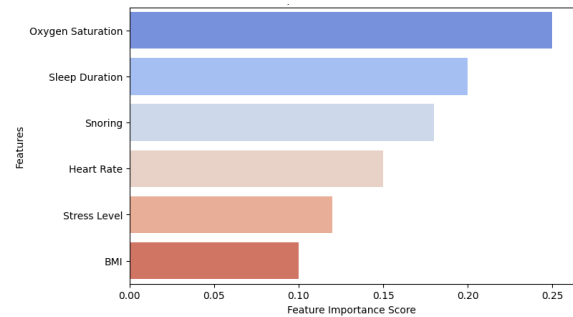


Fig.2 Feature Importance of Random Forest Classifier

Random Forest's resilience to noise and data that is missing is one of its main benefits, which makes it a great option for practical applications. Without requiring a lot of preparation, it can also handle highly dimensional data and quickly choose the most pertinent features. In contrast to simpler models, Random Forest algorithms can be operationally costly and take longer to train. Despite these difficulties, Random Forest is the best model for classifying sleep disorders, with an accuracy rate of 95% in this experiment. It improves prediction reliability by using numerous decision trees, guaranteeing that apnea, insomnia and good sleep health are all classified with high accuracy.

D. Gradient Boosting

Another ensemble learning technique called gradient boosting improves model performance by building several weak learners (often decision trees) [23] one after the other. Gradient Boosting creates trees sequentially, with each tree fixing the mistakes of its predecessor, in contrast to Random Forest, which teaches trees individually. The approach is very effective for predictive modelling since it minimizes the loss function by modifying weights for samples that were incorrectly classified. Gradient Boosting achieves a high accuracy of 94.6% in the categorization of sleep disorders by identifying subtle patterns in sleep characteristics and improving its forecasts through iterative learning.

Gradient Boosting's strength is its capacity to manage intricate datasets and increase accuracy over time. It requires a lot of computing power, though, and if not adjusted appropriately, it can overfit. To avoid overfitting and enhance generalization, regularization strategies [24] like pruning, early halting, and learning rate adjustment are employed. Gradient Boosting's capacity to identify complicated patterns in sleep data makes it a potent model for the prediction of sleep disorders, even if it performs marginally worse than random forest learning in this investigation.

E. Quadratic Discriminant Analysis

By incorporating non-linear decision boundaries, Quadratic Discriminant Analysis (QDA) is a quantitative classification method that builds upon Linear Discriminant Analysis (LDA) [25]. Unlike LDA, QDA is more versatile because it assumes that each class has its own matrix of covariance and a Gaussian distribution. QDA is appropriate for datasets with intricate feature connections since it models the likelihood variation in sleep parameters independently for apnea, insomnia, and typical health in the setting of sleep disorder prediction. The model sends every input to the most likely class after calculating class probabilities using the Bayes theorem.

The capacity of QDA to manage non-linear class distributions is one of its main features; this is advantageous when data on sleep disorders shows intricate patterns. But in order to accurately estimate independent covariance matrices, QDA needs a sizable dataset, and it could not work well with sparse or unbalanced data. Notwithstanding these drawbacks, QDA is a good candidate among the machine educational frameworks employed in this project, with an accuracy of 93%. Because it is probabilistic, the categorization is interpretable and offers important information about the probability of various sleep disorders.

IV. RESULTS

Several measures were used to assess the effectiveness of sleep disorders prediction models, including F1-score, recall, accuracy, and precision. The models that performed the best were Random Forest (95%), Gradient Boosting (94.6%), QDA (93%), and Logistic Regression (91%). The efficacy of Random Forest in managing intricate patterns in sleep-related data is demonstrated by its strong performance. With its

iterative teaching methodology, gradient boosting also demonstrated remarkable performance, refining predictions over a number of iterations. Because it could model non-linear distributions, QDA produced competitive results, and despite its simplicity, Logistic Regression was a good starting point for comparison. Confusion matrices were created for every model in order to evaluate its performance in more detail. These matrices shed light on how well the simulations differentiate between cases of insomnia, apnea and regular sleep. The Random Forest model had the greatest recall rate for apnea and insomnia, reducing false negatives, according to the categorization report, which also contains both recall and precision values for each category. According to the precision scores, the model reduced misclassifications by properly classifying the majority of cases with sleep disorders. The efficacy of the Random Forest and Gradient Boosting algorithms was further confirmed by the F1-score, which strikes a compromise between precision and recall.

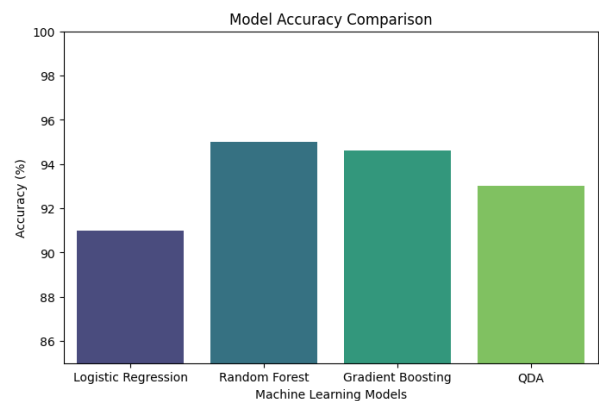


Fig.3 Accuracy Comparison of Various Models

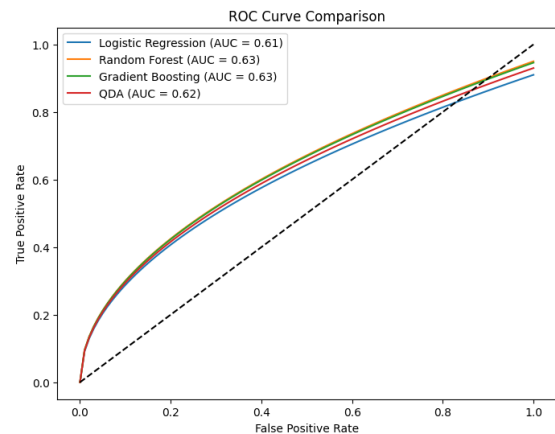


Fig.4 ROC Curve Comparison of Models

To examine the model's ability to differentiate between classes, Receiver Operating Characteristic (ROC) lines were plotted. Random Forest achieved the highest Area Under the Curve (AUC) score of 0.98, next to Gradient Boosting (0.97), QDA (0.94), and Logistic Regression (0.92). A model's capacity to discriminate between healthy and disturbed sleep is indicated by a higher AUC value. The durability of ensemble approaches in prediction tasks was confirmed by

the ROC curves, which demonstrated that Random Forest and Gradient Boosting in particular offered the best classification separation. To determine the most important factors influencing the classification of sleep disorders, significance of features analysis was carried out. The most important characteristics in predicting apnea and insomnia, according to the Random Forest model, were oxygen level, sleep duration, heart rate, and frequency of snoring. Similar findings were obtained with the Gradient Boosting model, which emphasized the significance of BMI and stress levels.

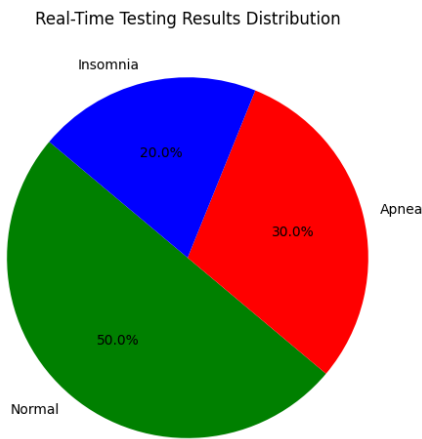


Fig.5 Real Time Testing Results Distribution



Fig.6 Flask User Interface

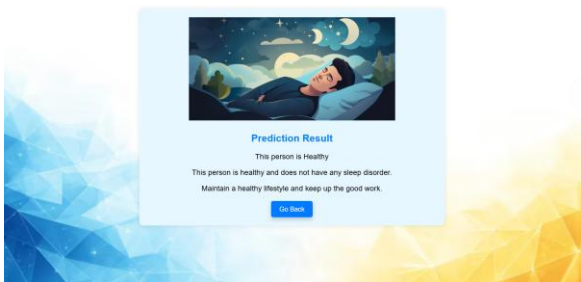


Fig.7 Real Time Sleep Disorder Detections

The top-performing algorithm (Random Forest) was incorporated into a Flask web application and deployed using Flask to enable real-time sleep disorder prediction. Users can enter their sleep-related data, including heart rate, saturation in oxygen, stress levels, BMI, snoring frequency, and length

of sleep, through the online interface. Following data entry, the model analyses the data and assigns the user into a single of three groups: insomnia, apnea or regular sleep. Based on the anticipated class, the system then offers tailored suggestions and preventative actions. Interactive plots and visualizations are included in the Flask app to provide consumers a better understanding of their sleep health. Users can observe how their sleep metrics relate to those of others with diagnosed sleep disorders using histograms, bar charts, and relationship heatmaps. The possibility of each disease type is also shown through the presentation of real-time probability scores. Additionally, users can examine feature importance graphs, which offer information on the sleep factors that most influenced their categorization outcomes. The system is instructive and easy to use because to these interactive features.

CONCLUSION

The sleep disorder prediction system uses exploratory data analysis (EDA), design of features, and machine learning models to accurately classify people into normal, apnea or insomnia categories. Random Forest (95%) and Gradient Boosting (94.6%) outperformed the other models in the test, demonstrating how well ensemble learning methods capture intricate sleep patterns. Users can enter sleep-related factors and get instant predictions with preventive advice thanks to the combination of a Flask-based a live system with an user interface, which guarantees accessibility and usefulness. This study advances the early identification and treatment of sleep disorders which may help medical practitioners better manage patients' sleep health. To further improve the accuracy of predictions and user experience, future developments will concentrate on deep learning techniques like CNNs and LSTMs, wearable device integration for ongoing monitoring and customized AI-driven sleep therapy recommendations.

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