Semi-cosmological simulations in a growing dark matter potential

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1 Theory

According to Diemer 2014, the dark matter potential cal be expressed in the form

$$\rho = \rho_{inner}\, f_{trans} + \rho_{outer} \tag{1}$$

The profile of ρ_{inner} can be two choices:
NFW profile:
This profile has density given by

$$\rho(r) = \frac{\rho_0}{r/R_s\,(1 + r/R_s)^2} \tag{2}$$

The corresponding potential is

$$\Phi(r) = -\frac{4\pi G\rho_0 R_s^3}{r}\ln(1 + r/R_s) \tag{3}$$

The gravitational acceleration is given by

$$\overrightarrow{g} = G\frac{M_{vir}}{\ln(1+c) - \frac{c}{1+c}} \times \frac{\frac{r}{r+r_s} - \ln(1 + r/R_s)}{r^3}\overrightarrow{r} \tag{4}$$

where the concentration parameter c is given by $R_{vir} = cR_s$, and the virial mass is $M_{vir} = \frac{4\pi}{3}R_{vir}^3 200\rho_{crit}$.
Einasto profile:
The density profile is given by (Retana-Montenegro 2012)

$$\rho(r) = \rho_s \exp\left(-d_n\left[\left(\frac{r}{r_s}\right)^{1/n} - 1\right]\right) \tag{5}$$

where $\rho_s = \rho(r_s)$. It is more convenient to express this as

$$\rho(r) = \rho_0 \exp\left(-\left(\frac{r}{h}\right)^{1/n}\right) \tag{6}$$

where $\rho_0 = \rho_s e^{d_n}$ and $h = \frac{r_s}{d_n^n}$. Here, the total mass is given by

$$M = 4\pi\rho_0 h^3 n\Gamma(3n) \tag{7}$$

The coefficient d_n can be expressed as a series expansion:

$$d_n \approx 3n - \frac{1}{3} + \frac{8}{1215n} + \frac{184}{229635n^2} + \frac{1048}{31000725n^3} + O(\frac{1}{n^4}) \tag{8}$$

The general formula for a potential due to a spherical mass is

$$\Phi(r) = -4\pi G\left[\frac{1}{r}\int_0^r \rho(r')r'^2 dr' + \int_r^\infty \rho(r')r'dr'\right] \tag{9}$$

For the Einasto profile, we have

$$\Phi(r) = \frac{GM}{hs}\left[1 - \frac{\Gamma(3n, s^{1/n})}{\Gamma(3n)} + \frac{s\Gamma(2n, s^{1/n}}{\Gamma(3n)}\right] \tag{10}$$

where $s = \frac{(d_n)^n r}{r_s}$.
At small radii, the potential is given by

$$\Phi \approx \Phi_0 - \frac{GM}{h} \frac{s^2}{6n\Gamma(3n)} \tag{11}$$

which is a parabolic potential.
For large radii, the potential resembles a Keplerian potential

$$\Phi(r) \approx \frac{GM}{r} \tag{12}$$

Outer profile
Diemer 2014 takes the outer density profile to be

$$\rho_{outer} = \rho_m \left[b_e \left(\frac{r}{5R_{200m}} \right)^{-s_e} + 1 \right] \tag{13}$$

$$= a \left(b \left(\frac{r}{r_0} \right)^{-c} + 1 \right) \tag{14}$$

The corresponding potential is

$$\Phi(r) = 4\pi G a \left(b r_0^c \frac{r^{2-c}}{(3-c)(2-c)} + \frac{r^2}{6} \right) \tag{15}$$

The corresponding acceleration is

$$\overrightarrow{g} = -a \left(b r_0^c \frac{r^{1-c}}{3-c} + \frac{r}{3} \right) \hat{r} \tag{16}$$

Transfer function
The transfer function f_{trans} from Diemer's paper has the property of going to 1 at small radii and going to 0 at large radii. They take the function

$$f_{trans} = \left[1 + \left(\frac{r}{r_t} \right)^{\beta} \right]^{-\frac{\gamma}{\beta}} \tag{17}$$

They take $\beta = 4$ and $\gamma = 8$. This function is plotted here This looks like a sigmoid function. While choosing a

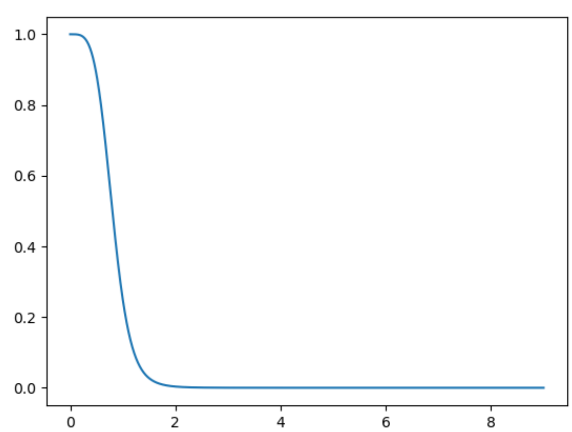

Figure 1: f_{trans} as a function of r/r_t.

transfer function $f(x)$ for the inner to outer transition, it must have the simple property of a sharp transition from 0 to 1 at $x = r/r_t = 1$.
Constructing the potential
From equation 1 for the density profile given by Diemer, I construct the corresponding potential by two methods.

Method 1: Direct integration. Given the density profile, we can calculate $M_{enc}(r) = \int_0^r 4\pi r'^2 \rho(r')dr'$. From this, the radial derivative of the potential, i.e. the radial acceleration, can be calculated as

$$\frac{d\Phi(r)}{dr} = \frac{GM_{enc}(r)}{r^2} \tag{18}$$

The potential is then calculated by integration of the above equation.
Method 2: Using equation 10. I use the scipy module quadpack to perform the integration, as the potential is continuous and smooth.

Fitting Potential

The potential is fitted according to

$$\Phi(r) = w_1\Phi_1 + w_2\Phi_2 \tag{19}$$

where $\Phi_1 = \Phi_{NFW}(r)$ (equation 4), $\Phi_2 = \Phi_{outer}(r)$ (equation 16), and w_1 and w_2 are weight functions.
I take the weight functions as:

$$w_1 = 1 - \left(\frac{1 + erf(x)}{c_e}\right) \qquad w_2 = \left(1 - w_1^4\right)^{1/4} \tag{20}$$

where $erf(x)$ is the error function, $x = \log(r/r_t)$, and c_e ranges from 5 to 10, dependent on ν, R_{200m}, s_e, and other input parameters.
The results for the fitting potential and the potential constructed by the two methods are shown below. The input parameters are $\nu = 4, b_e = 1.5, s_e = 1.5, M_{200} = 10^{12}M_\odot, z = 0$.

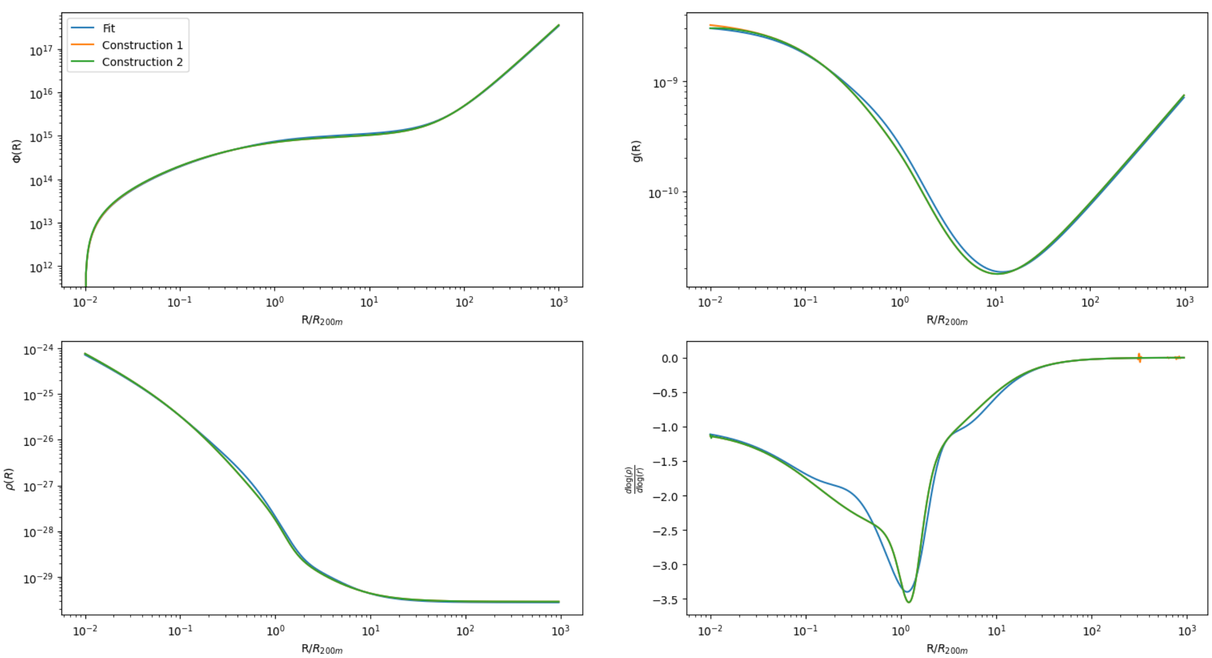

Figure 2: The potential, acceleration, density, and local power law index for the fit and the constructions.

Evolution of the Halo

The dynamical evolution and growth of the DM halo is given by van den Bosch:

$$\log(M(z)/M_0) = -0.301\left[\frac{\log(1+z)}{\log(1+z_f)}\right]^\nu \tag{21}$$

where z_f is the formation redshift ($M(z_f) = M_0/2$) and ν is given by

$$\nu = 1.211 + 1.858\log(1+z_f) + 0.308\Omega_\Lambda^2 - 0.032\log(M_0/10^{11}h^{-1}M_\odot) \tag{22}$$

The concentration parameter of the NFW profile halo evolves as (Zhao et al 2009)

$$c = 4\left(1 + \left(\frac{t}{3.75 t_{0.04}}\right)^{8.4}\right)^{1/8} \tag{23}$$

Therefore, the evolution of the DM halo following the NFW profile is completely specified by its present day mass M_0 and its formation redshift z_f.
Finally, to implement the evolution in PLUTO, the redshift must be converted into lookback time. If the cosmological parameters Ω_m, Ω_Λ, H_0 are specified, with the assumption of a flat universe ($\Omega_m + \Omega_\Lambda = 1$), we have

$$t(z) = \frac{1}{H_0}\frac{2}{3\sqrt{\Omega_\Lambda}} \ln\left[\frac{\sqrt{\Omega_\Lambda (1+z)^{-3}} + \sqrt{\Omega_\Lambda (1+z)^{-3} + \Omega_m}}{\sqrt{\Omega_m}}\right] \tag{24}$$

2 Results

The initial conditions are set as

$$\rho_g = 0.2 \times \rho_{DM} \quad v_r = H_0 E_z(z_0) \times r \quad P = K\rho_g^\gamma \tag{25}$$

where ρ_{DM} is the Diemer density profile and K is a numerical factor (defined in PPC paper).
I also assume a formation redshift $z_f = 2$ for all the simulations (this has to be improved). The peak height is set to $\nu = 4$.
I run the simulation from $z = 6$ to $z = 0$.
In figure ??, I have plotted the density, pressure, and velocity profiles for $M_0 = 10^{12} M_\odot$.

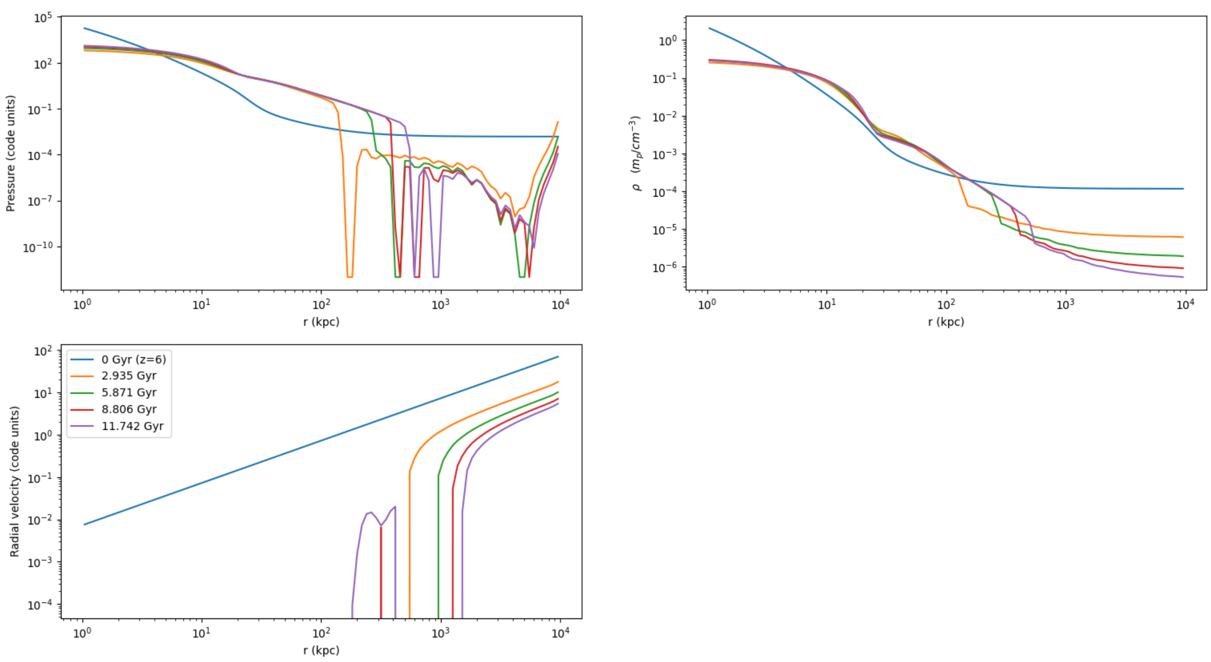

Figure 3: The density, pressure, and velocity profiles for $M_0 = 10^{12} M_\odot$.

The effect of dark matter halo can be modelled using either the potential or the acceleration vector. In figure ??, I plot the percentage difference between the two. As they differ by only a few percent (max), I use vector as it has a lesser running time.

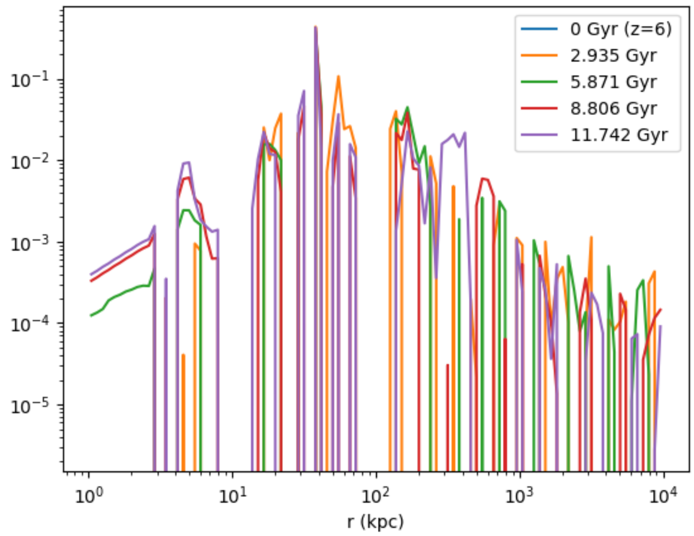

Figure 4: Relative difference between the potential and vector profiles: $(\rho_p - \rho_v)/\rho_p$

I also plot the baryon fraction withing R_{200c} as a function of time. The baryon fraction is defined as

$$f_b = \frac{\int_0^{R_{200c}} \rho_g r^2}{\int_0^{R_{200c}} \rho_{DM} r^2 + \int_0^{R_{200c}} \rho_g r^2} \tag{26}$$

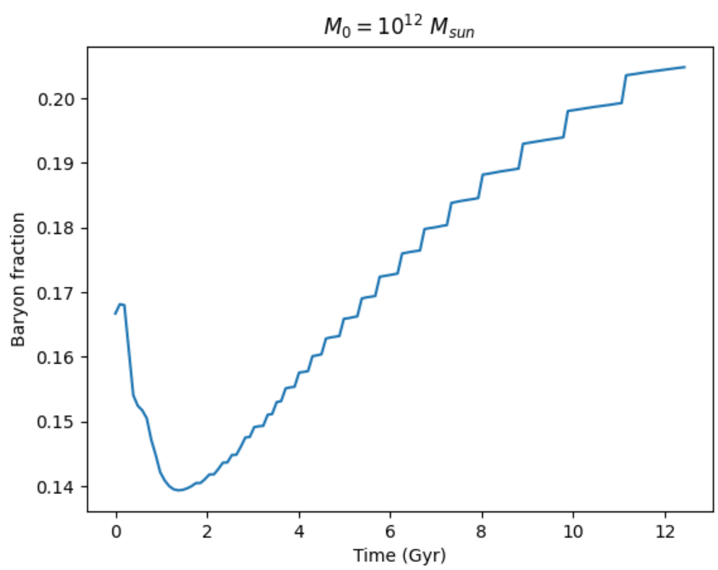

Figure 5: Evolution of baryon fraction over time.

In figure ??, I plot the evolution of the gas for different values of M_0.
In figure ??, I vary the value of γ.

2.1 Self-similarity and comparision

The x axis is normalized with respect to the virial radius R_{200c} at the present time (z=0), while the density is normalized with respect to $\rho_0 = M_{200}/R_{200c}^3$, with M_{200} being the virial mass at $z = 0$.

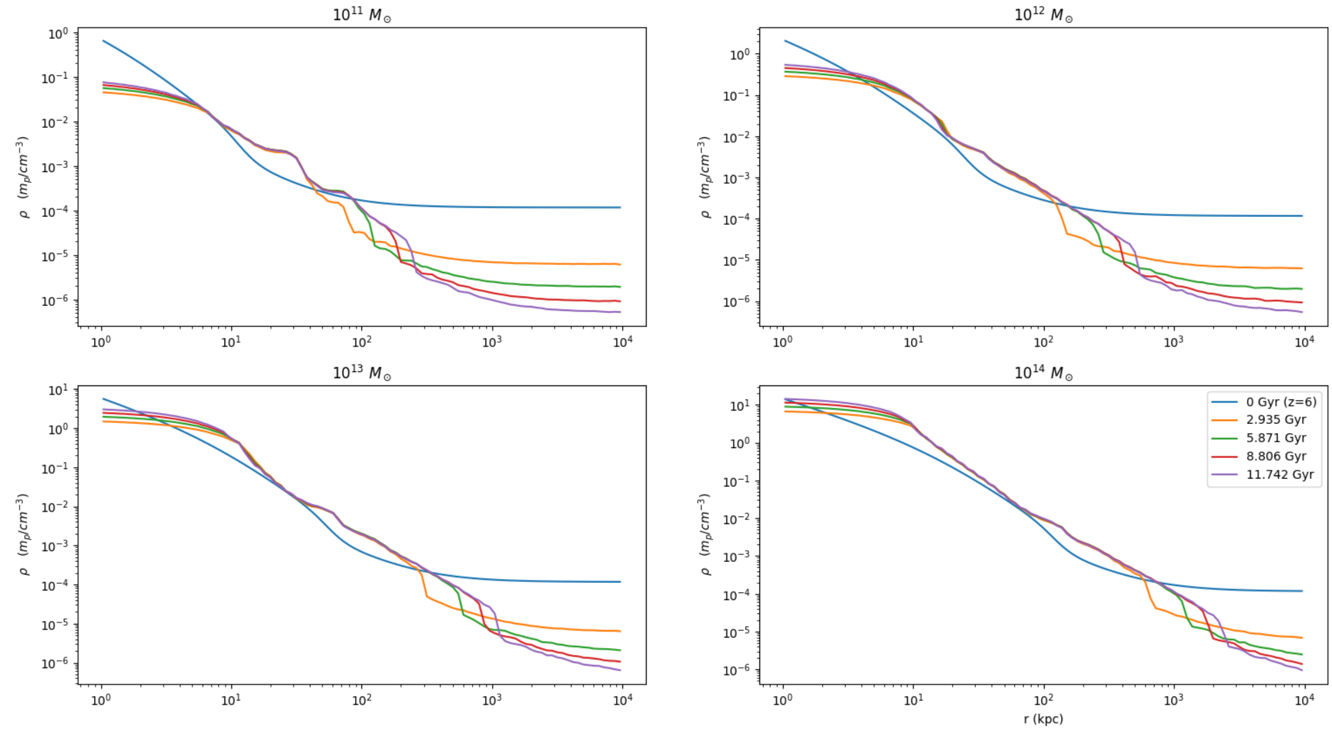

Figure 6: Different values of M_0.

In figure ??, I plot the normalized density vs normalized radius for different values of M_0 at z=0.
In figure ??, I plot the same for different values of γ.
Next, I plot the value of the density power law index, $d\ log(\rho)/d\ log(r)$ as a function of r.
Finally, I plot the value of ρ_g/ρ_{DM} as a function of r.
From the results, the following conclusions can be made:

- The density profiles are self-similar with respect to the current (z=0) virial density and radius.
- The virial shock appears at a radius about 2 times the virial radius (R_{200c}). This is in contradiction to the results from PPC, we have to investigate whether this is due to the difference in choice of potential, or if our simulations are wrong.
- The virial shock moves away from R_{200c} as the value of γ is increased.
- The baryon fraction and the value of ρ_g/ρ_{DM} deviate greatly from their expected values. We have to investigate why this is so.

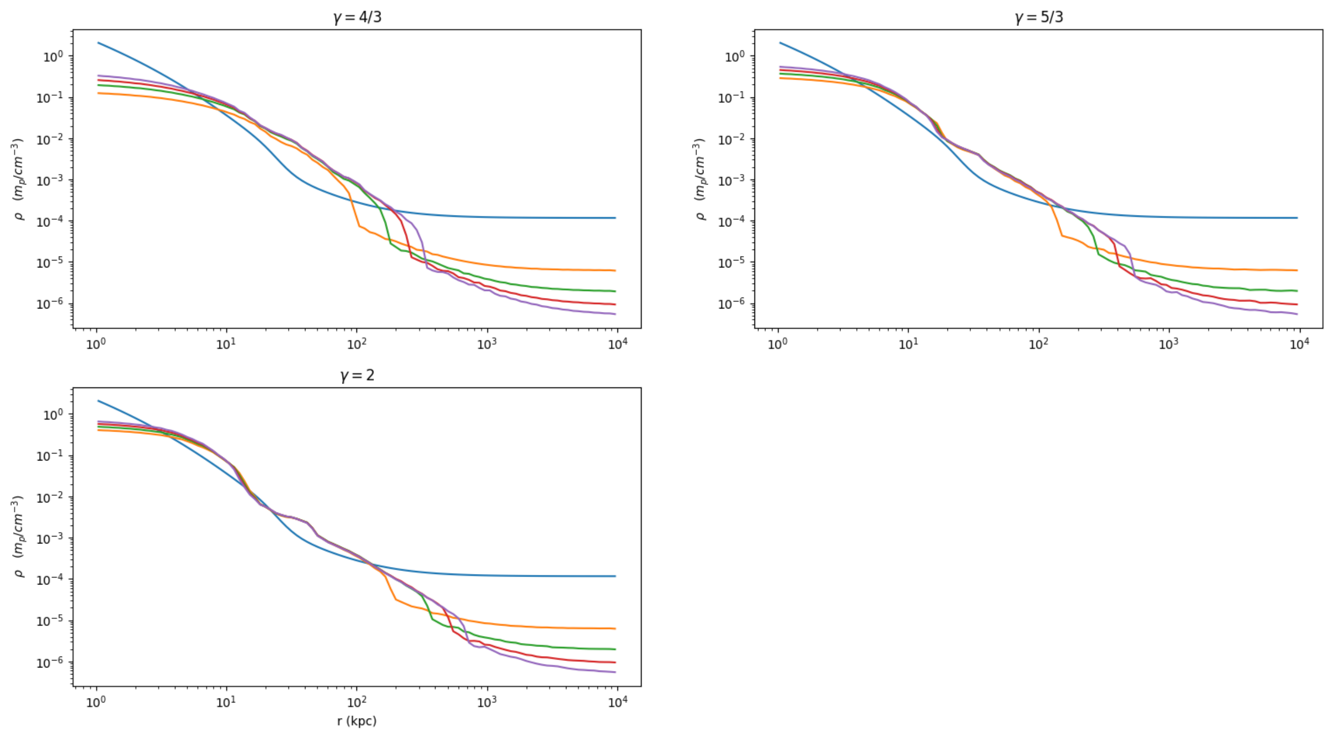

Figure 7: Different values of γ.

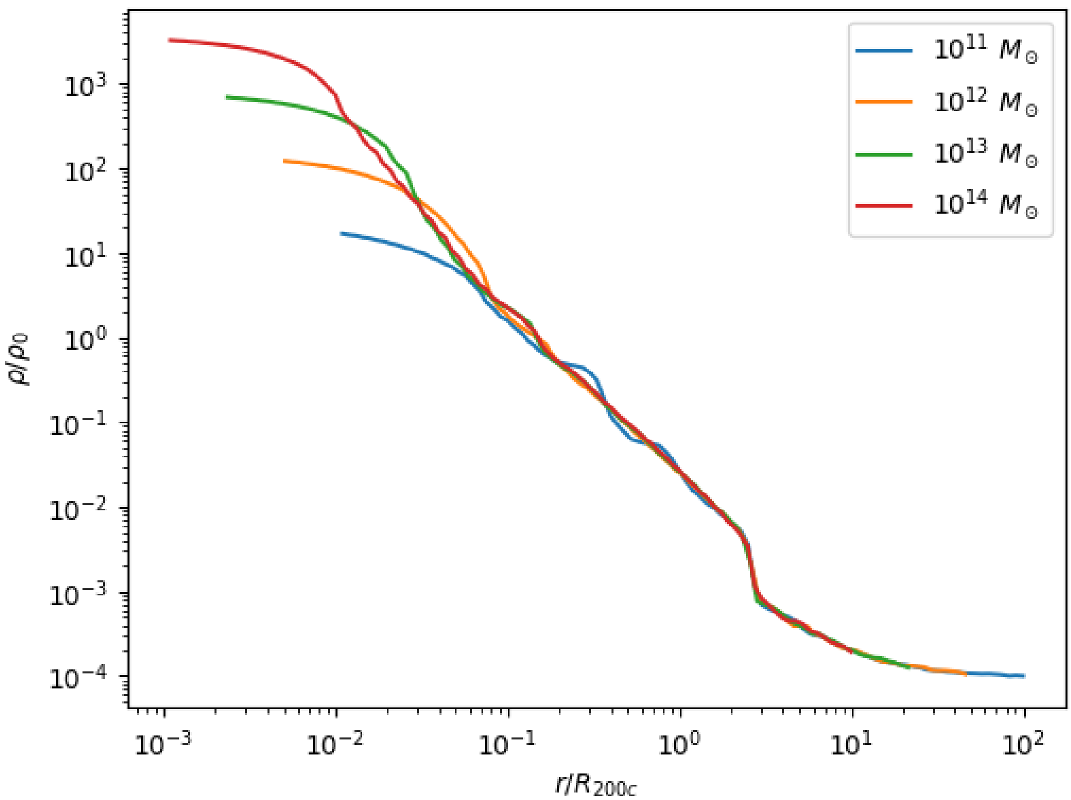

Figure 8: Self similar profiles at $z = 0$ of ρ vs r for different M_0.

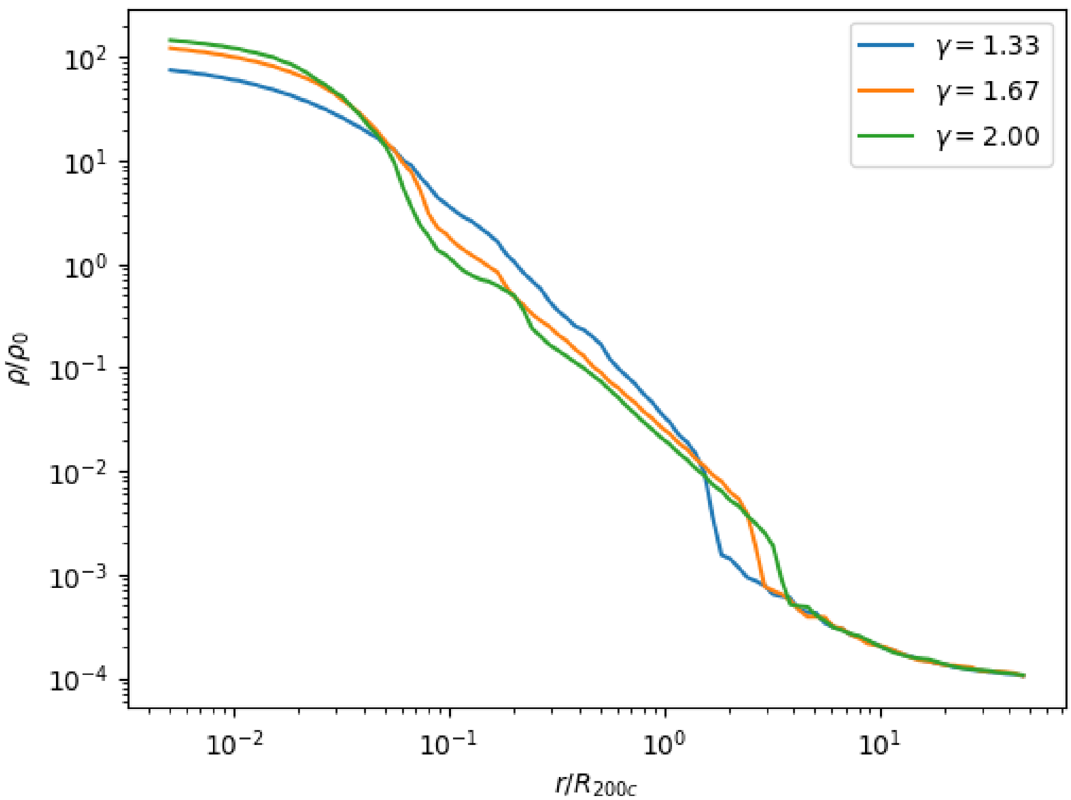

Figure 9: Self similar profiles at $z = 0$ of ρ vs r for different γ.

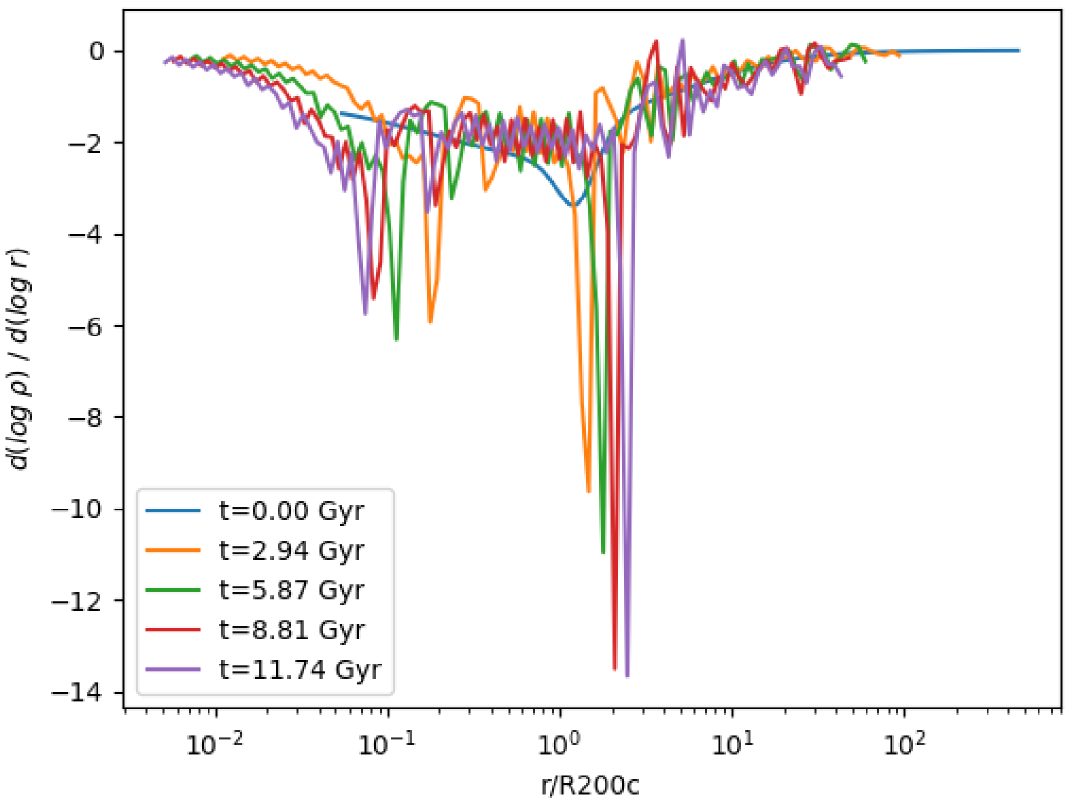

Figure 10: Power law index variation over r. $M_0 = 10^{12} M\odot$.

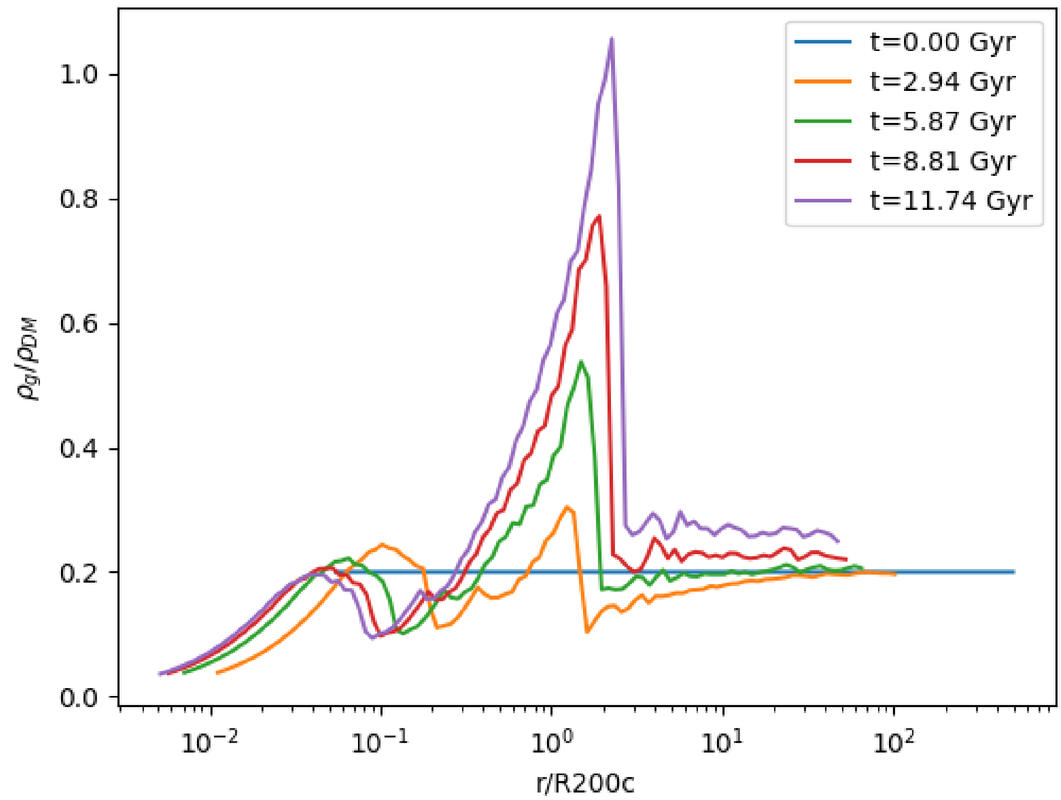

Figure 11: ρ_g/ρ_{DM} for $M_0 = 10^{12} M\odot$.