Machnine Learning (PDEEC0049)

Homework 1

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1 Problem 1

1.1 Question 1

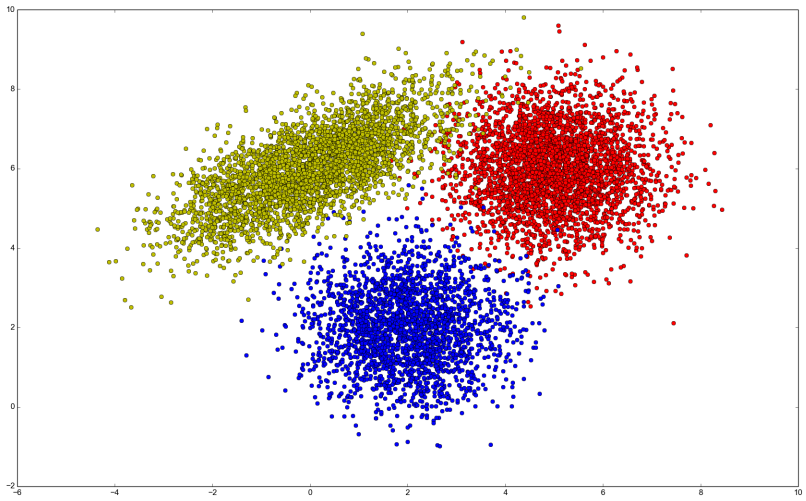

Figure 1: training set plot

1.2 Question 2

Mean of X1 dataset [[5.00287044 5.97561985]]
Mean of X2 dataset [[1.98304658 2.00294107]]
Mean of X3 dataset [[0.01535631 6.01730798]]
Cov of X1 dataset [[1.01607307 0.0134706] [0.0134706 1.01565189]]
Cov of X2 dataset [[1.01827835 0.00437229] [0.00437229 1.00719966]]
Cov of X3 dataset [[1.9229381 0.98014156] [0.98014156 0.98815543]]

1.3 Question 3

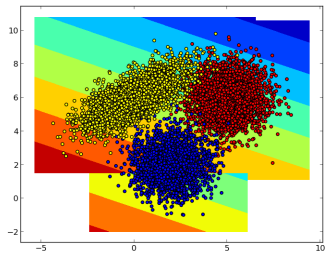

Figure 2: boundry for classification with contours

~~Eng~~ Machine Learning HW 1 (~~2~~0 Oct) SHASHANK

Q.1 Given X_1, X_2, X_3 } XX data sets

for $X_1 \rightarrow \mu_1 = \begin{bmatrix} 5.00287044 \\ 5.9755 1985 \end{bmatrix}$ $\Sigma_1 = \begin{bmatrix} 1.016 & 0.0135 \\ 0.0135 & 1.0157 \end{bmatrix}$

$\Sigma_1^{-1} \begin{bmatrix} 0.9844 & -0.0131 \\ -0.0131 & 0.9848 \end{bmatrix}$

$X_2 \rightarrow \mu_2 = \begin{bmatrix} 1.9830 4658 \\ 2.0029 4107 \end{bmatrix}$ $\Sigma_2 = \begin{bmatrix} 1.0183 & 0.0044 \\ 0.0044 & 1.0072 \end{bmatrix}$

$\Sigma_2^{-1} \begin{bmatrix} 0.9821 & -0.0043 \\ -0.0043 & 0.9929 \end{bmatrix}$

$X_3 \rightarrow \mu_3 = \begin{bmatrix} 0.0153 5631 \\ 6.0173 0758 \end{bmatrix}$ $\Sigma_3 = \begin{bmatrix} 1.9279 & 0.5801 \\ 0.5801 & 0.9882 \end{bmatrix}$

$\Sigma_3^{-1} \begin{bmatrix} 1.0518 & -1.0433 \\ -1.0433 & 2.0463 \end{bmatrix}$

for μ_1 $p_1(x_1) = +\frac{1}{2}(x-\mu_1)^T \Sigma_1^{-1} (x-\mu_1)$

$= +\frac{1}{2}[x_1 - 5.0029 \;\; x_2 - 5.9756] \begin{bmatrix} 0.9844 & -0.0131 \\ -0.0131 & 0.9848 \end{bmatrix}$

$= -\frac{1}{2}\left\{[x_1 - 5.0029 \;\; x_2 - 5.9756] \begin{bmatrix} 0.9844x_1 - 4.9249 - 0.0131x_2 + 0.0783 \\ -0.0131x_1 + 0.0655 + 0.9848x_2 - 5.9148 \end{bmatrix}\right\}$

$= +\frac{1}{2}\left\{[x_1 - 5.0029 \;\; x_2 - 5.9756] \begin{bmatrix} 0.9844x_1 - 0.0131x_2 - 4.8460 \\ -0.0131x_1 + 0.9848x_2 - 5.8193 \end{bmatrix}\right\}$

$= +\frac{1}{2}\{0.9844x_1^2 - 0.0131x_1x_2 - 4.8460x_1 - 4.9249x_1 + 0.0655x_2 + 24.2441 - 0.0131x_1x_2 + 0.9848x_2^2 - 5.8193x_2 + 0.0783x_1 - 5.8848x_2 + 34.7738\}$

$= +\frac{1}{2}\{0.9844x_1^2 + 0.9848x_2^2 - 0.0262x_1x_2 - 9.0926x_1 - 11.6386x_2 + 59.0173\}$

$= \boxed{0.4922x_1^2 + 0.4924x_2^2 - 0.0131x_1x_2 - 4.8463x_1 - 5.8193x_2 + 29.5086}$

$$p_2(X_2) = \frac{1}{2}(x-\mu_2)^T \Sigma_2^{-1}(x-\mu_2)$$

$$= \frac{1}{2}\left\{ \begin{bmatrix} x_1 - 1.9830 & x_2 - 2.0029 \end{bmatrix} \begin{bmatrix} 0.9921 & -0.0043 \\ -0.0043 & 0.9929 \end{bmatrix} \begin{bmatrix} x_1 - 1.9830 \\ x_2 - 2.0029 \end{bmatrix} \right\}$$

$$= \frac{1}{2}\left\{ \begin{bmatrix} x_1 - 1.9830 & x_2 - 2.0029 \end{bmatrix} \begin{bmatrix} 0.9821x_1 - 1.9475 - 0.0043x_2 + 0.0086 \\ -0.0043x_1 + 0.0085 + 0.9929x_2 - 1.9887 \end{bmatrix} \right.$$
$$\left. -1.9389 \qquad -1.9802 \right.$$

$$= \frac{1}{2}\left\{ \begin{array}{l} 0.9821x_1^2 - 1.9389x_1 - 0.0043x_1x_2 - 1.9475x_1 + 0.0085x_2 + 3.8449 \\ -0.0043x_1x_2 + 0.9929x_2^2 - 1.9802x_2 + 0.0085x_1 \\ -1.9887x_2 + 3.9661 \end{array} \right\}$$

$$= \frac{1}{2}\left\{ \begin{array}{l} 0.9821x_1^2 + 0.9929x_2^2 - 0.0086x_1x_2 - 3.8769x_1 \\ -3.9604x_2 + 7.8109 \end{array} \right\}$$

$$= \boxed{0.4910x_1^2 + 0.4965x_2^2 - 0.0043x_1x_2 - 1.9385x_1 + 1.9802x_2 + 3.9055}$$

$$p(X_3) = \frac{1}{2}\left\{ \begin{bmatrix} x_1 - 0.0154 & x_2 - 6.0173 \end{bmatrix} \begin{bmatrix} 1.0518 & -1.0433 \\ -1.0433 & 2.0468 \end{bmatrix} \begin{bmatrix} x_1 - 0.0154 \\ x_2 - 6.0173 \end{bmatrix} \right.$$

$$= \frac{1}{2}\left\{ \begin{bmatrix} x_1 - 0.015 & x_2 - 6.0173 \end{bmatrix} \begin{bmatrix} 1.0518x_1 + 6.2616 - 0.0162 - 1.0433x_2 + 6.2778 \\ -1.0433x_1 + 0.0161 + 2.0468x_2 - 12.3162 \end{bmatrix} \right.$$
$$\left. -0.0162 \qquad -12.3001 \right.$$

$$= \frac{1}{2}\left\{ \begin{array}{l} 1.0518x_1^2 + 6.2616x_1 - 1.0433x_1x_2 - 0.0162x_1 - 0.0964 \\ +0.0161x_2 - 1.0433x_1x_2 - 12.3001x_2 + 2.0468x_2^2 \\ +6.2778x_1 + 74.0134 - 12.3162x_2 \end{array} \right\}$$

$$= \frac{1}{2}\left\{ \begin{array}{l} 1.0518x_1^2 + 2.0468x_2^2 - 2.0866x_1x_2 + 12.5232x_1 \\ -24.6002x_2 + 73.9170 \end{array} \right\}$$

$$= 0.5259x_1^2 + 1.0234x_2^2 - 1.0433x_1x_2 + 6.2616x_1 - 12.3001x_2 + 36.9585$$

for μ_{12}

~~$0.4922x_1^2 + 0.4924x_2^2 - 0.0131x_1x_2 - 4.8463x_1 - 5$~~

$0.4922x_1^2 + 0.4924x_2^2 - 0.0131x_1x_2 - 4.8463x_1 - 5.8193x_2 + 29.5086$

$= 0.4910x_1^2 + 0.4965x_2^2 - 0.0043x_1x_2 - 1.9385x_1 + 1.9802x_2 + 3.9055$

$0.0012x_1^2 - 0.0041x_2^2 = 0.0088x_1x_2 - 2.9078x_1 - 7.7925x_2 + 25.6031$

$= 0$

for μ_{13}

~~$0.4922x_1^2 + 0.49$~~

$0.0337x_1^2 + 0.5310x_2^2 - 1.0302x_1x_2 + 11.1079x_1 - 6.4809$

$+ 7.4499$

for μ_{23}

$0.0345x_1^2 + 0.5269x_2^2 - 1.0390x_1x_2 + 8.2001x_1$

$- 14.7803x_2 + 33.0530$

μ_1 $\quad ax_1^2 + bx_2^2 + cx_1x_2 + dx_1 + ex_2 + f$

1.4 Question 4

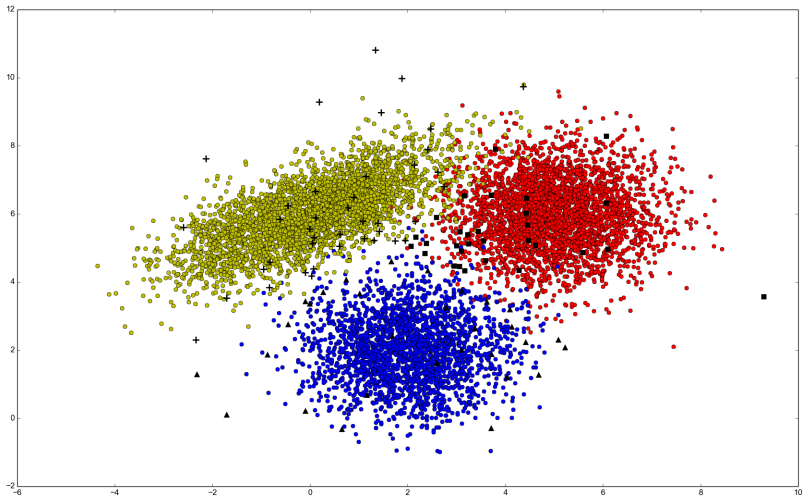

Figure 3: training set plot

2 Problem 2

2.1 Question 1

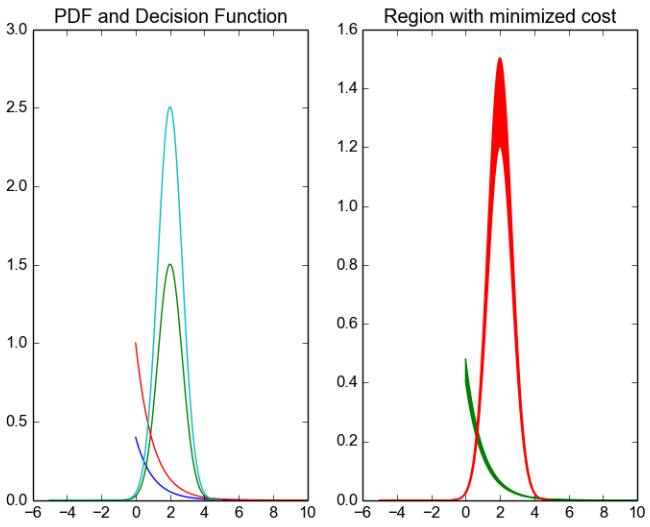

Figure 4: PDF and Decision function in the left plot

Probability Density Function
For A = exp(-x)
For B = (sqrt(2*pi))*exp(-(y-2)**2)
Decision Functions
For A = 0.4*exp(-x)
For B = 0.6*(sqrt(2*pi))*exp(-(y-2)**2)

2.2 Question 2

With 0.30 probability of error, for x = 1 the class would be B with 0.55 probability. Probability fo Class A would be 0.15.

2.3 Question 3

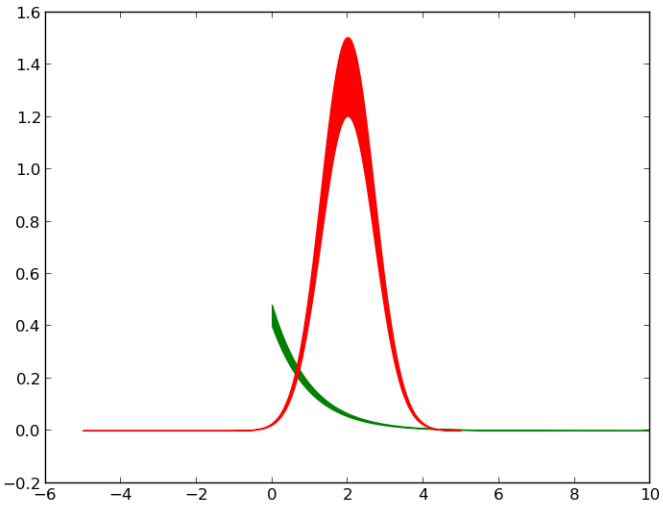

Figure 5: Minimum error regions in shaded

With consideration of Error cost
For A = 1.2*0.4*exp(-x)
For B = 0.8*0.6*(sqrt(2*pi))*exp(-(y-2)**2)

3 Problem 3

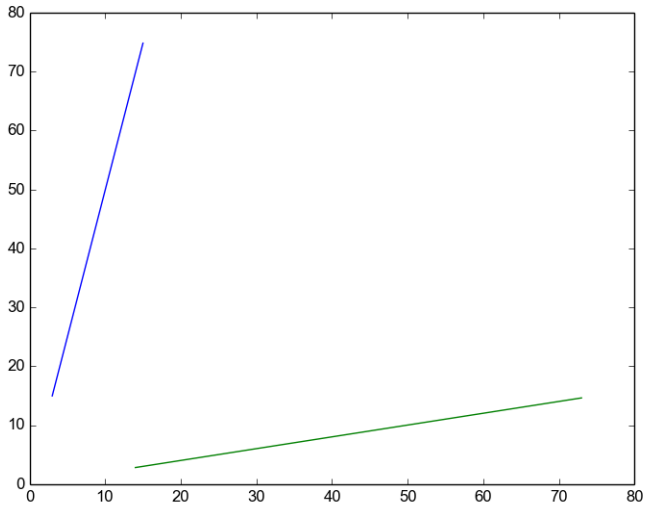

Figure 6: The plots of both F1(x) and F2(x)

3.1 Question 1

for F1(x) the value of a and b are 4.98582995951 0

3.2 Question 2

for F2(y) the value of c and d are 0.200178803641 0

3.3 Question 3

the value of y at x=5 by F1(x) is 24.9291497976
the value of y at x=5 by F2(y) is 24.9776695087

3.4 Question 4

Preferred model is F1(x)