~~Eng~~ Machine Learning — SHASHANK

HW 1 (~~2~~ 0 oct)

Q.1 Given x_1, x_2, x_3 } XX data sets

for $X_1 \rightarrow \mu_1 = \begin{bmatrix} 5.0029 \\ 5.9756 \end{bmatrix}$ $\Sigma_1 = \begin{bmatrix} 1.016 & 0.0135 \\ 0.0135 & 1.0157 \end{bmatrix}$

$\Sigma_1^{-1} \begin{bmatrix} 0.9844 & -0.0131 \\ -0.0131 & 0.9848 \end{bmatrix}$

$X_2 \rightarrow \mu_2 = \begin{bmatrix} 1.9830 \\ 2.0029 \end{bmatrix}$ $\Sigma_2 = \begin{bmatrix} 1.0183 & 0.0044 \\ 0.0044 & 1.0072 \end{bmatrix}$

$\Sigma_2^{-1} \begin{bmatrix} 0.9821 & -0.0043 \\ -0.0043 & 0.9929 \end{bmatrix}$

$X_3 \rightarrow \mu_3 = \begin{bmatrix} 0.0153 \\ 6.0173 \end{bmatrix}$ $\Sigma_3 = \begin{bmatrix} 1.9229 & 0.9801 \\ 0.9801 & 0.9882 \end{bmatrix}$

$\Sigma_3^{-1} \begin{bmatrix} 1.0519 & -1.0433 \\ -1.0433 & 2.0463 \end{bmatrix}$

for μ_1 $\quad p_1(x_1) = \pm \frac{1}{2} (x-\mu_1)^T \Sigma_1^{-1} (x-\mu_1)$

$= \pm \frac{1}{2} [x_1 - 5.0029 \;\; x_2 - 5.9756] \begin{bmatrix} 0.9844 & -0.0131 \\ -0.0131 & 0.9848 \end{bmatrix} \begin{bmatrix} x_1 - 5.0029 \\ x_2 - 5.9756 \end{bmatrix}$

$= -\frac{1}{2} \{ [x_1 - 5.0029 \;\; x_2 - 5.9756] \begin{bmatrix} 0.9844x_1 - 4.9249 - 0.0131x_2 + 0.0783 \\ -0.0131x_1 + 0.0655 + 0.9848x_2 - 5.9348 \end{bmatrix} \}$

$= \pm \frac{1}{2} \{ [x_1 - 5.0029 \;\; x_2 - 5.9756] \begin{bmatrix} 0.9844x_1 - 0.0131x_2 - 4.8460 \\ -0.0131x_1 + 0.9848x_2 - 5.9193 \end{bmatrix} \}$

$= \pm \frac{1}{2} \{ 0.9844x_1^2 - 0.0131x_1x_2 - 4.8460x_1 - 4.9249x_1 + 0.0655x_2 + 24.2441 - 0.0131x_1x_2 + 0.9848x_2^2 - 5.9193x_2 + 0.0783x_1 - 5.8848x_2 + 34.7738 \}$

$= \pm \frac{1}{2} \{ 0.9844x_1^2 + 0.9848x_2^2 - 0.0262x_1x_2 - 9.6926x_1 - 11.8041x_2 + 59.0173 \}$

$= \boxed{0.4922x_1^2 + 0.4924x_2^2 - 0.0131x_1x_2 - 4.8463x_1 - 5.8193x_2 + 29.5086}$

$$P_2(X_2) = \frac{1}{2}(x-M_2)^T \Sigma_2^{-1}(x-M_2)$$

$$= \frac{1}{2}\left\{ \begin{bmatrix} x_1 - 1.9830 & x_2 - 2.0029 \end{bmatrix} \begin{bmatrix} 0.9921 & -0.0043 \\ -0.0043 & 0.9929 \end{bmatrix} \begin{bmatrix} x_1 - 1.9830 \\ x_2 - 2.0029 \end{bmatrix} \right\}$$

$$= \frac{1}{2}\left\{ \begin{bmatrix} x_1 - 1.9830 & x_2 - 2.0029 \end{bmatrix} \begin{bmatrix} 0.9921x_1 - 1.9475 - 0.0043x_2 + 0.0086 \\ -0.0043x_1 + 0.0085 + 0.9929x_2 - 1.9887 \end{bmatrix} \right.$$
$$\left. -1.9802 \right.$$

$$= \frac{1}{2}\left\{ \begin{array}{l} 0.9821x_1^2 - 1.9389x_1 - 0.0043x_1x_2 - 1.9475x_1 + 0.0085x_2 + 3.8449 \\ -0.0043x_1x_2 + 0.9929x_2^2 - 1.9802x_2 + 0.0085x_1 \\ -1.9887x_2 + 3.9601 \end{array} \right\}$$

$$= \frac{1}{2}\left\{ \begin{array}{l} 0.9821x_1^2 + 0.9929x_2^2 - 0.0086x_1x_2 - 3.8769x_1 \\ -3.9604x_2 + 7.8103 \end{array} \right\}$$

$$= \boxed{0.4910x_1^2 + 0.4965x_2^2 - 0.0043x_1x_2 - 1.9385x_1 + 1.9802x_2 + 3.9055}$$

$$p(X_3) = \frac{1}{2}\left\{ \begin{bmatrix} x_1 - 0.0154 & x_2 - 6.0173 \end{bmatrix} \begin{bmatrix} 1.0518 & -1.0433 \\ -1.0433 & 2.0469 \end{bmatrix} \begin{bmatrix} x_1 - 0.0154 \\ x_2 - 6.0173 \end{bmatrix} \right\}$$

$$= \frac{1}{2}\left\{ \begin{bmatrix} x_1 - 0.015 & x_2 - 6.0173 \end{bmatrix} \begin{bmatrix} 1.0518x_1 + 6.2616 - 0.0162 - 1.0433x_2 + 6.2778 \\ -1.0433x_1 + 0.0161 + 2.0469x_2 - 12.3162 \end{bmatrix} \right.$$
$$\left. -12.3001 \right.$$

$$= \frac{1}{2}\left\{ \begin{array}{l} 1.0518x_1^2 + 6.2616x_1 - 1.0433x_1x_2 - 0.0162x_1 - 0.0064 \\ +0.0161x_2 - 1.0433x_1x_2 - 12.3001x_2 + 2.0469x_2^2 \\ +6.2778x_1 + 74.0134 - 12.3162x_2 \end{array} \right\}$$

$$= \frac{1}{2}\left\{ \begin{array}{l} 1.0518x_1^2 + 2.0469x_2^2 - 2.0866x_1x_2 + 12.5232x_1 \\ -24.6002x_2 + 73.9170 \end{array} \right\}$$

$$= 0.5259x_1^2 + 1.0234x_2^2 - 1.0433x_1x_2 + 6.2616x_1 - 12.3001x_2 + 36.9585$$

for M_{12}

~~$0.4922x_1^2 + 0.4924x_2^2 - 0.0131x_1x_2 - 4.8463x_1 - 5$~~

$0.4922x_1^2 + 0.4924x_2^2 - 0.0131x_1x_2 - 4.8463x_1 - 5.8193x_2 + 29.5086$

$= 0.4910x_1^2 + 0.4965x_2^2 - 0.0043x_1x_2 - 1.9385x_1 + 1.9802x_2 + 3.9055$

$0.0012x_1^2 - 0.0041x_2^2 - 0.0088x_1x_2 - 2.9078x_1 - 7.7995x_2 + 25.6031$

$= 0$

for M_{13}

~~$0.4922x_1^2 + 0.49$~~

$0.0837x_1^2 + 0.5310x_2^2 - 1.0302x_1x_2 + 11.1079x_1 - 6.4808$

$+ 7.4499$

for M_{23}

$0.0345x_1^2 + 0.5269x_2^2 - 1.0390x_1x_2 + 8.2001x_1$

$- 14.7803x_2 + 33.0530$

M_1 $\quad ax_1^2 + bx_2^2 + cx_1x_2 + dx_1 + ex_2 + f$