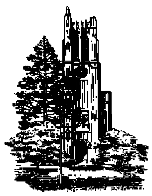
Thursday May 19, 2016
Lecture 04

Predicate Logic



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Notables

- Homework#3
  - Page 64, Problem 2
  - Page 65, Problems 10
  - Due Monday May 23, 2016

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Quantifiers - summary

Proposition	*When True?*	*When False?*
$\forall x\, P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x\, P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

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Binding Variables

- Consider a propositional function $P(x)$
- A variable x is said to be *bound* when:
  - a value is assigned to x, or
  - a quantifier is used on x,

  Otherwise, x is said to be *free*.
- When we have more than one variables in a propositional function, the order of the quantifiers is important (unless the quantifiers are of the same type).

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Free and Bound Variables

- All the variables of a propositional function are bound when it becomes a proposition.
  Let $P(x, y)$ denote "$x < y$".
  - In $\exists x\, P(x, y)$, variable x is bound but y is free, and $\exists x\, P(x, y)$ is not yet a proposition.
  - $\exists x\, P(x, 2)$ is now a proposition; all the variables are bound now.
  - $\forall x\, \exists y\, P(x, y)$ is also a proposition, all the variables are bound now.
  - Note that once a variable is bound using a quantifier, it cannot be given a value. For example, $\exists x\, P(3, y)$ does not make sense.

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Scope of the Quantifiers

- The *scope* of a quantifier ($\exists$ or $\forall$) is the part of the expression that it applies to.
  - In $\forall x\ (S(x) \rightarrow P(x))$, the scope of $\forall$ is $S(x) \rightarrow P(x)$.
  - In $(\exists x\, P(x)) \wedge Q(x)$, the scope of $\exists$ is $P(x)$. That expression is the same as $(\exists x\, P(x)) \wedge Q(y)$. Note that, here x is a bound variable whereas y is not.

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Negations

- $\neg\forall x\ P(x) \equiv \exists x\ \neg P(x)$.
- $\neg\exists x\ P(x) \equiv \forall x\ \neg P(x)$.

- Remember when the universe of discourse is finite, we have:
  $\forall x\ P(x) \equiv P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$

  $\exists x\ P(x) \equiv P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$

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Example

- The negation of "Lansing is a small city" is "Lansing is not a small city".
- The negation of "every city in Michigan is clean" is "not every city in Michigan is clean" which is the same as saying, "some cities in Michigan are not clean." However, the negation is not "every city in Michigan is not clean."

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Negating Quantifiers

Negation	***When True?***	***When False?***
$\neg\exists x\ P(x)$ ($\forall x\ \neg P(x)$)	$P(x)$ is false for every x.	There is an x for which $P(x)$ is true.
$\neg\forall x\ P(x)$ ($\exists x\ \neg P(x)$)	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

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Translating English Sentences into Logical Expressions

- "Some students in CSE260 have visited Mexico"
- *Solution:* Let
  - the universe of discourse be the set of CSE260 students, and
  - $M(x)$ be the predicate "x has visited Mexico."

  Answer:
  - $\exists x\ M(x)$
  - $\exists x$ (x has visited Mexico)

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Example

- "Every CSE260 student has visited either Canada or Mexico"
- *Solution:* Let
  - the universe of discourse be the set of CSE260 students,
  - $M(x)$ be the predicate "x has visited Mexico,"
  - $C(x)$ be the predicate "x has visited Canada."

  Answer:
  - $\forall x\ (C(x) \vee M(x))$

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In-class Exercise

- Consider the following predicates
  - $P(x)$ = "x is a prime number"
  - $Q(x,y)$ = "x is evenly divisible by y"

    Using the above predicates, symbolize the following proposition. Assume the universe of discourse is the set of all positive integers.

  Every integer which is evenly divisible by 10 is also evenly divisible by 5 and evenly divisible by 2.

  Answer: $\forall x\ [Q(x,10) \rightarrow (Q(x,5) \wedge Q(x,2))]$

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In-class Exercise

- Consider the following predicates
  - $P(x)$ = "x is a prime number"
  - $Q(x,y)$ = "x is evenly divisible by y"

  Using the above predicates, symbolize the following proposition. Assume the universe of discourse is the set of all positive integers.

There exists an odd integer which is not prime.

Answer: $\exists x\, (\neg Q(x,2) \wedge \neg P(x))$

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Guidelines in Translating….

- The proposition
  $$\forall y\, (P(y) \rightarrow Q(y))$$
  can be translated as "All individuals having property P also have property Q"
- What is the difference between
  $$\forall y\, (P(y) \rightarrow Q(y))$$
  and
  $$\forall y\, (P(y) \wedge Q(y))$$
- The proposition
  $$\exists\, y\, (P(y) \wedge Q(y))$$
  can be translated as "Some individuals having property P also have property Q"

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