

CSE 331 Homework 1

Problem 1

$$\begin{aligned}n^{\log_b(m)} &= m^{\log_b(n)} \\ \log_b[n^{\log_b(m)}] &= \log_b[m^{\log_b(n)}] \\ \log_b(n) \times \log_b(m) &= \log_b(m) \times \log_b(n) \\ \Rightarrow n^{\log_b(m)} &= m^{\log_b(n)}\end{aligned}$$

Problem 2

a)

n	1	2	3	4
Binary	1	11	111	111
Decimal sum	2^0	2^0+2^1	$2^0+2^1+2^2$	$2^0+2^1+2^2$

$$a_1 = 2^0 = 1 \quad r = 2$$

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{1-2^n}{1-2} = \frac{1-2^n}{-1} = 2^n - 1$$

b)

n	1	2	3	4
Value	9	99	999	9999
Sum	$9[10^0]$	$9[10^0+10^1]$	$9[10^0+10^1+10^2]$	$9[10^0+10^1+10^2+10^3]$

$$a_1 = 10^0 = 1 \quad r = 10$$

$$S_n = 9 \left[\frac{a_1(1-r^n)}{1-r} \right] = 9 \left[\frac{1-10^n}{1-10} \right] = 9 \left[\frac{1-10^n}{-9} \right] = \frac{1-10^n}{-1} = 10^n - 1$$

Problem 3

a) $A = [31, 41, 59, 26, 41, 58]$

31	41	59	26	41	58
-----------	-----------	----	----	----	----

$41 > 31 \rightarrow$ don't swap

31	41	59	26	41	58
----	-----------	-----------	----	----	----

$59 > 41 \rightarrow$ don't swap

31	41	59	26	41	58
----	----	-----------	-----------	----	----

$26 < 59 \rightarrow$ swap. $26 < 41 \rightarrow$ swap. $26 < 31 \rightarrow$ swap

26	31	41	59	41	58
----	----	----	-----------	-----------	----

$41 < 59 \rightarrow$ swap. $41 = 41 \rightarrow$ don't swap

26	31	41	41	59	58
----	----	----	----	-----------	-----------

$58 < 59 \rightarrow$ swap

26	31	41	41	58	59
----	----	----	----	----	----

b) Insertion-sort to sort data in decreasing order:

for $j \leftarrow 2$ to $\text{length}[A]$

do $\text{key} \leftarrow A[j]$

$i \leftarrow j-1$

while $i > 0$ and $A[i] < \text{key}$

do $A[i+1] \leftarrow A[i]$

$i \leftarrow i-1$

$A[i+1] \leftarrow \text{key}$

Problem 4

a)

	LINEAR SEARCH PSEUDOCODE
1	for $i \leftarrow 1$ to n
2	do $key \leftarrow A[i]$
3	if $A[i] = \text{value}$
4	return true
5	return false

b)

	LINEAR SEARCH PSEUDOCODE	Cost	Times
1	for $i \leftarrow 1$ to n	C_1	n
2	do $key \leftarrow A[i]$	C_2	$n-1$
3	if $A[i] = \text{value}$	C_3	$n-1$
4	return true	C_4	$n-1$
5	return false	C_5	1
	$T(n) = C_1n + (n-1)(C_2 + C_3 + C_4) + C_5$		

1. For an array of size n , each element has a $1/n$ probability of being the element searched for, the average number of elements to be checked are $(1+2+\dots+n)/n = (n+1)/2$
2. The best case is when the item you are looking for is the first element in the list. This way, the program doesn't have to iterate through the rest of the list.
3. The worst case is if the item is not in the list because not only do you have to iterate through the entire list, you also have to execute the "return false" statement at the end.
4. $T(n) = C_1n + (n-1)(C_2 + C_3 + C_4) + C_5$
 - a. Worst case: $\Theta(n)$ since you check n elements
 - b. Average case: $\Theta(n)$ since you check $(n+1)/2$ elements