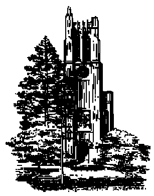
Tuesday May 17, 2016
Lecture 02

Propositional Logic



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Notables

- Homework#1
  - Page 13, problem 8
  - Page 14, problems 16, and 18
  - Page 15, Problem 32
  - Due tomorrow Wed May 18, 2016

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Disjunction

- Definition: Let p and q be propositions. The disjunction of p and q, denoted $p \vee q$, is the proposition that is:
  - false when both p and q are false, and true otherwise.
- Example:
  - Let p: "the butler did it" and let q: "the cook did it." What does $p \vee q$ say?
  - Solution: "Either the butler or the cook did it, or both."

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Truth Table for $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Note the difference between *inclusive* and *exclusive* OR in English

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Exclusive OR

- Definition: Let p and q be propositions. The *Exclusive OR* of p and q, denoted $p \oplus q$, is the proposition that is:
  - true when *exactly one* of p and q is *true*, and false otherwise.

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Truth table for $p \oplus q$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

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Conditional Proposition (Implication)

- Definition: Let p and q be propositions. The conditional proposition, also called *implication*,
  $p \rightarrow q$
  is the proposition that is:
  - false when p is true and q is false, and true otherwise.

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Truth table for $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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More on $p \rightarrow q$

- The conditional operator is interpreted as a guarantee; If the condition is true then the conclusion is expected (guaranteed) to be true, for the implication to be true.
  - Ex: Let p = "It is raining", q = "There are clouds in the sky"
  - $p \rightarrow q$
  - If it is raining then there are clouds in the sky.
  - Note that if it is not raining and there are clouds in the sky, the guarantor has not lied.
- There may not be a *cause* and *effect* relationship here.
  - Ex: The proposition *"If snow is black then I can fly"* is true (as we know it!).

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More on $p \rightarrow q$ …

- Terminologies used to express $p \rightarrow q$:
  - If p then q
  - p implies q
  - p only if q
  - p is a sufficient condition for q
  - q follows from p
  - q is a necessary condition for p
  - q if p
  - q when p
  - q whenever p
  - q unless $\neg p$

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More on $p \rightarrow q$ …

- Other propositions *related* to $p \rightarrow q$:
  - $q \rightarrow p$ is called the converse;
    - If there are clouds in the sky then it is raining .
  - $\neg p \rightarrow \neg q$ is called the inverse;
    - If it is not raining then there are no clouds in the sky
  - $\neg q \rightarrow \neg p$ is the contrapositive.
    - If there no clouds in the sky then it is not raining.

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Conditional Propositions

p	q	$p \rightarrow q$	$q \rightarrow p$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T				
T	F				
F	T				
F	F				

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Conditional Propositions…

p	q	$p \rightarrow q$	$q \rightarrow p$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

p = "It is raining", q = "There are clouds in the sky"

Biconditional

- Definition: Let p and q be propositions. The biconditional $p \leftrightarrow q$ is the proposition that is true when p and q have the same truth values, and false otherwise.
- The biconditional $p \leftrightarrow q$ is true when both the implications $p \rightarrow q$ and $q \rightarrow p$ are true.
  - 'p if and only if q'
  - You are a U.S. citizen if and only if you are eligible to hold a U.S. passport.

Truth Table for $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

NOR

- Definition: Let p and q be propositions. The proposition p NOR q, *denoted as* $p \downarrow q$, is the proposition that is true when both p and q false, and false otherwise.

Truth Table for $p \downarrow q$

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

Summary of logical operations

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$	$p \downarrow q$
T	T	F	T	T	F	T	T	F
T	F	F	F	T	T	F	F	F
F	T	T	F	T	T	T	F	F
F	F	T	F	F	F	T	T	T

How many distinct logical operators can we define?

Well-formed Formulas

- A *well-formed formula* (wff) can be generated by using one or more of the following rules finitely many times
  - A proposition standing alone is a wff.
  - If p is wff, so is $\neg p$.
  - If p and q are wffs, so are $p \wedge q$, $p \vee q$, $p \to q$, and $p \leftrightarrow q$.

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Precedence of Logical Operators

- 1: Parentheses
- 2: Negation is applied before all others
- 3: Conjunction is prior to Disjunction
- 4: Conditionals at last

Operator	Precedence
$\neg$	1
$\wedge$, $\vee$	2
$\to$, $\leftrightarrow$	3

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Translating English Sentences

- "You can access the Internet from campus only if you are a computer science major or you are not a freshman."
- Let a, c and f represent the propositions:
  - a : "You can access the Internet from campus"
  - c : "You are a computer science major"
  - f : "You are a freshman."
- $a \to (c \vee \neg f)$

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Tautology, Contradiction, Contingency

- Definition: A *tautology* is any proposition (wff) that is always true regardless of the truth values of its "atomic" propositions.
- A tautology is a proposition which is true by virtue of logic.
- Definition: A *contradiction* is any wff that is always false regardless of the truth values of its "atomic" propositions.
- A contradiction is a proposition which is false by virtue of logic.
- A compound proposition that is neither a tautology nor a contradiction is called a *contingency*.

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Example

- Which of the following propositions are tautology?
  - "Sara has red hair"
    - Not a tautology; it is not necessarily either true or false
  - "$1 \neq 2$"
    - Not a tautology; it's truth is based the our number system
  - "Sara has red hair or she does not have red hair"
    - This is a tautology

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Example

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

- $p \vee \neg p$ is a tautology.
- $p \wedge \neg p$ is a contradiction.

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Propositional Equivalences

- Definition: Two wffs p and q are *logically equivalent* (denoted $p \Leftrightarrow q$ *or* $p \equiv q$), if they have the same truth values in all possible cases.
- In other words, the wff p and q are logically equivalent whenever the proposition $p \leftrightarrow q$ is a tautology.

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Example: $\neg (p \vee q) \equiv \neg p \wedge \neg q$

p	q	$p \vee q$	$\neg (p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

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Example: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

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Example: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

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Homework#1

- Page 13, Problem 8
  - Let p and q be the propositions
  - p : I bought a lottery ticket this week.
  - q : I won the million dollar jackpot.
  - Express each of these propositions as an English sentence.
  - **a)** $\neg p$ **b)** $p \vee q$ **c)** $p \rightarrow q$
  - **d)** $p \wedge q$ **e)** $p \leftrightarrow q$ **f)** $\neg p \rightarrow \neg q$
  - **g)** $\neg p \wedge \neg q$ **h)** $\neg p \vee (p \wedge q)$

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Homework#1

- Page 14, Problem 16
  - Determine whether these biconditionals are true or
  - false.
  - **a)** $2 + 2 = 4$ if and only if $1 + 1 = 2$.
  - **b)** $1 + 1 = 2$ if and only if $2 + 3 = 4$.
  - **c)** $1 + 1 = 3$ if and only if monkeys can fly.
  - **d)** $0 > 1$ if and only if $2 > 1$.

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Homework#1

- Page 14, Problem 18
  - Determine whether each of these conditional statements
  - is true or false.
  - **a)** If 1 + 1 = 3, then unicorns exist.
  - **b)** If 1 + 1 = 3, then dogs can fly.
  - **c)** If 1 + 1 = 2, then dogs can fly.
  - **d)** If 2 + 2 = 4, then 1 + 2 = 3.

Homework#1

- Page 15, Problem 32
  - Construct a truth table for each of these compound propositions.
  - **a)** $p \to \neg p$ **b)** $p \leftrightarrow \neg p$
  - **c)** $p \oplus (p \vee q)$ **d)** $(p \wedge q) \to (p \vee q)$
  - **e)** $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$
  - **f)** $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$