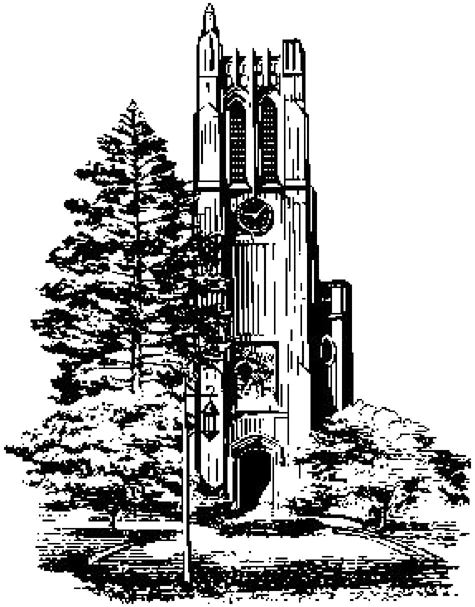
Lecture 02



Notables

- Homework #1 is posted
  - Due Thursday September 15th
  - Read: Top 10 Simple Things Every Computer User Should Know How to Do
  - http://lifehacker.com/5941496/top-10-simple-things-every-computer-user-should-know-how-to-do
- Udacity lecture by Steve Huffman (heads up for HW#2)
  - https://www.udacity.com/course/viewer#!/c-cs253/l-48737165/m-48723400
- Activate your Piazza account once notified
- Forthcoming topics:
  - Math review
  - System review

What is a *Set* ?

- Definition: A *set* is an *unordered* collection of objects, called the *elements* or *members* of the set. A set is called to *contain* its elements.
  - Example: {car, 201, pen, ipod, 6.9, river, {10101, key}}
- Note that the definition of a set does not require any relationship among the members of a set.
- **<u>In a set, repeated elements are ignored.</u>**
- The order of the elements in a set is irrelevant; it does not make sense to ask for the k-th element of a set.
- To indicate the fact that:
  - x is an element of the set S, we write: $x \in S$
  - x is not an element of the set S, we write: $x \notin S$

Some Important Sets

- The set of Natural Numbers
  $\boldsymbol{N}$ ={0, 1, 2, …}
- The set of Integers
  $\boldsymbol{Z}$ ={ …, -2, -1, 0, 1, 2, …}
- The set of Positive Integers
  $\boldsymbol{Z}^+$ ={1, 2, …}
- The set of Rational Numbers
  $\boldsymbol{Q}$ ={*p*/*q* | *p* and *q* are integers, and *q* is not zero}
- $\boldsymbol{R}$ = The set of real number
- The empty set

  {}, Ø

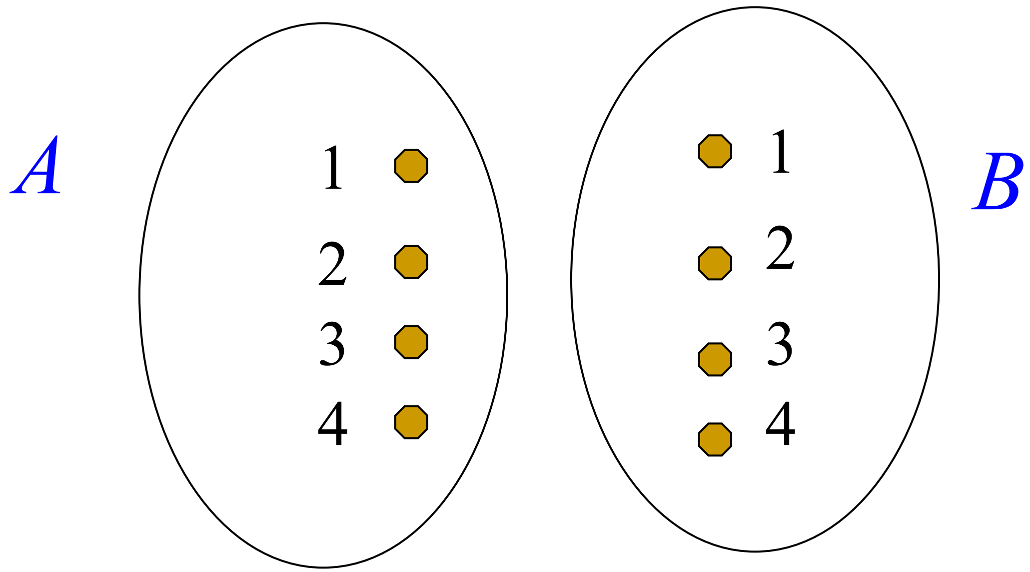
Subsets of a set

- What is a subset of set?
  - X is a subset of a set Y if whatever is in X is also in Y.
  - Example: The subsets of {1,2,5} are {},{1},{2},{5},{1,2},{1,5},{2,5},{1,2,5}
- How many subsets does a set of n elements have?
  - Note that the answer does not depend on the *nature* of the elements in the set, rather on the size of the set
  - 2^n

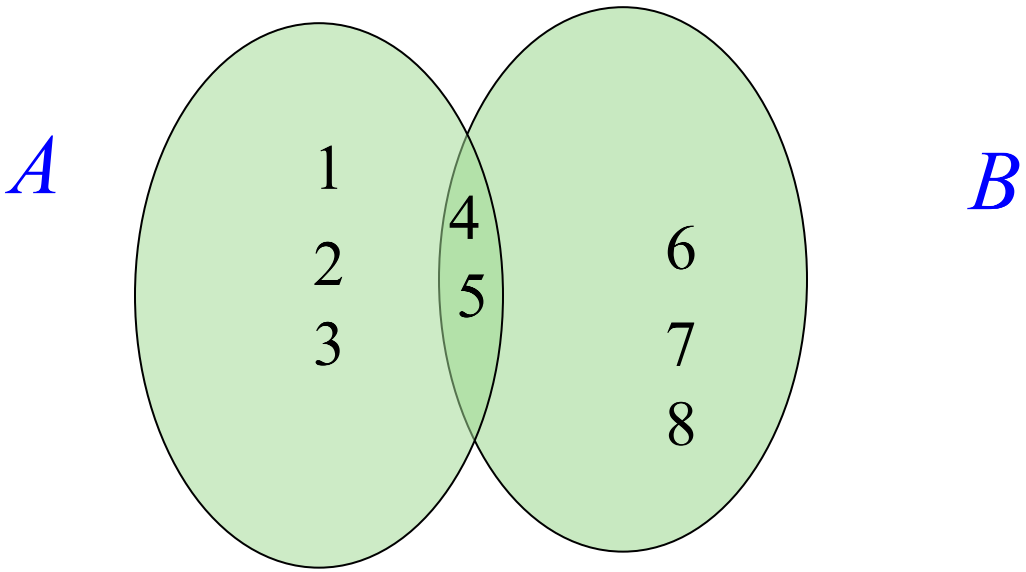
Operations on Set

- Set equality
- Set union
- Set intersection
- Set complement

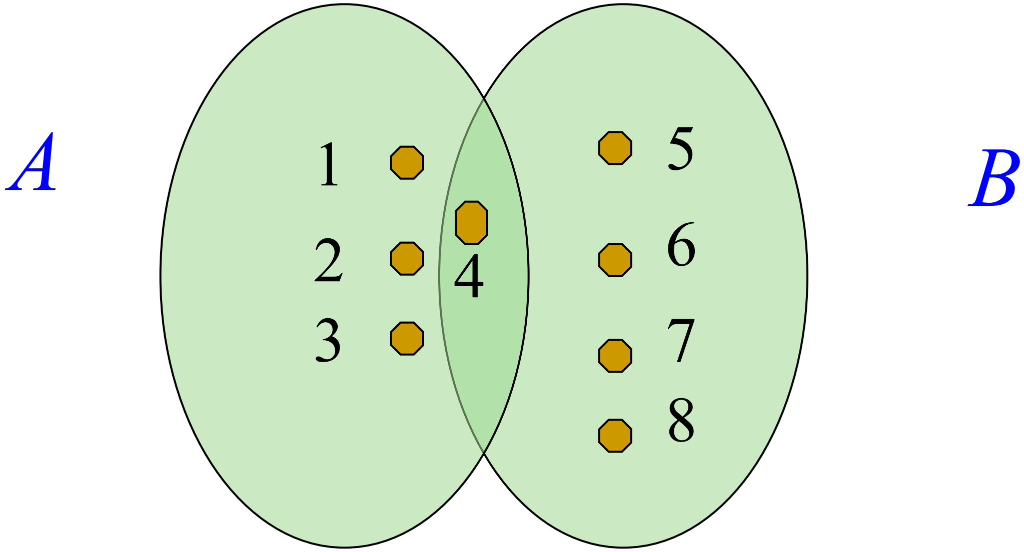
Set Equality



Set Union

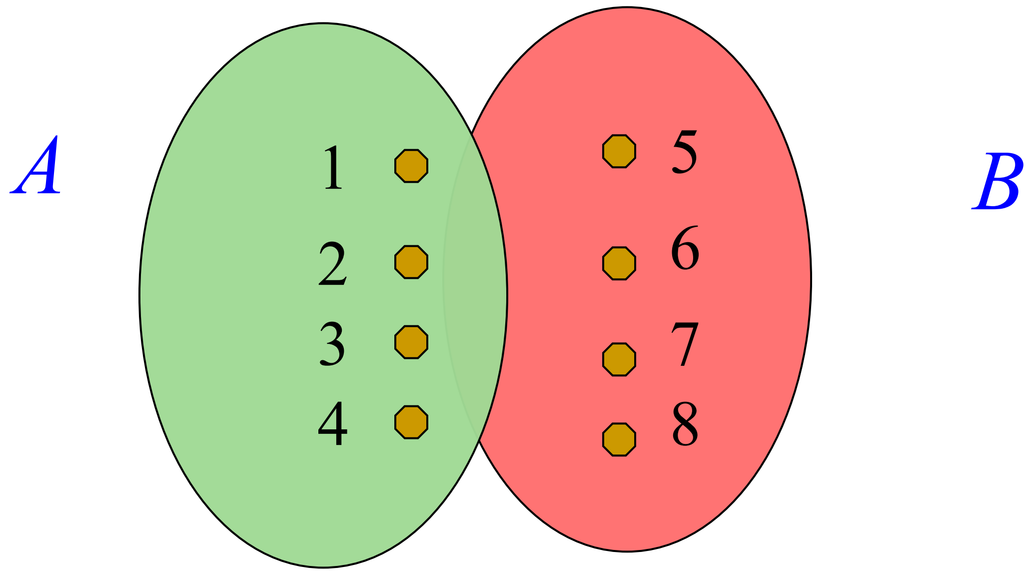


Set Intersection

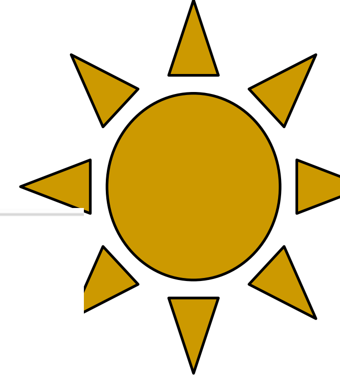


Set Compliment/Difference

If A and B are sets, then the **relative complement** of A in B, also termed the **set-theoretic difference** of B and A, is the set of elements in B but not in A.



Activity: Question 1 ICAT1



Let X and Y be the following sets:

$$X = \{1, 2, 8, 9\}$$
$$Y = \{0, 1, 2, 4, 6, 8, 9\}$$

Which of the following is the set $X \cup Y$?

- ○ $\{1, 2, 8, 9\}$
- ○ $\{0, 4, 6\}$
- ○ $\{0, 1, 2, 4, 6, 8, 9\}$
- ○ $\{\}$

Activity: Question 1 Answer

Let X and Y be the following sets:

$$X = \{1, 2, 8, 9\}$$

$$Y = \{0, 1, 2, 4, 6, 8, 9\}$$

Which of the following is the set $X \cup Y$?

- ○ $\{1, 2, 8, 9\}$
- ○ $\{0, 4, 6\}$
- ● $\{0, 1, 2, 4, 6, 8, 9\}$
- ○ $\{\}$

Activity: Question 2 ICAT1

Let X and Y be the following sets:

$$X = \{1, 3, 5, 7\}$$

$$Y = \{2, 4, 6, 8\}$$

Which of the following is the set $X \cap Y$?

- ○ $\{2, 4, 6, 8\}$
- ○ $\{\}$
- ○ $\{1, 3, 5, 7\}$
- ○ $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Activity: Question 2 Answer

Let X and Y be the following sets:

$$X = \{1, 3, 5, 7\}$$

$$Y = \{2, 4, 6, 8\}$$

Which of the following is the set $X \cap Y$?

- $\{2, 4, 6, 8\}$
- $\{\}$
- $\{1, 3, 5, 7\}$
- $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Activity: Question 3 ICAT1

Let X and Y be the following sets:

$$X = \{15, 12\}$$

$$Y = \{12, 15, 2\}$$

Which of the following is the set $X \setminus Y$?

- $\{2\}$
- $\{\}$
- $\{12, 15\}$
- $\{2, 12, 15\}$

1 / 3 $\setminus$ is the difference between the sets. $X \setminus Y$ is the set of elements that are in X **but not in** Y.

Activity: Question 3 Answer

Let X and Y be the following sets:

$X = \{15, 12\}$

$Y = \{12, 15, 2\}$

Which of the following is the set $X \setminus Y$?

- ○ $\{2\}$
- ◉ $\{\}$
- ○ $\{12, 15\}$
- ○ $\{2, 12, 15\}$

1 / 3 $\setminus$ is the difference between the sets. $X \setminus Y$ is the set of elements that are in X **but not in** Y.

2 / 3 Notice that there aren't any elements that match this description. Every element that is in X is also in Y.

Activity ICAT1

- Find the equality, union, intersection and the A/B of the following sets
- A ={ 1, 2, 3 }
- B= { 2, 3, 4 }

DO LISTS EXIST IN PROGRAMMING LANGUAGES?

Python Lists

- Collection of items
  - Items can be repeated
  - Items are ordered; it makes sense to ask for the k^{th} item of the list
  - Items are "indexed" left to right, starting at 0

We use functions for modularity

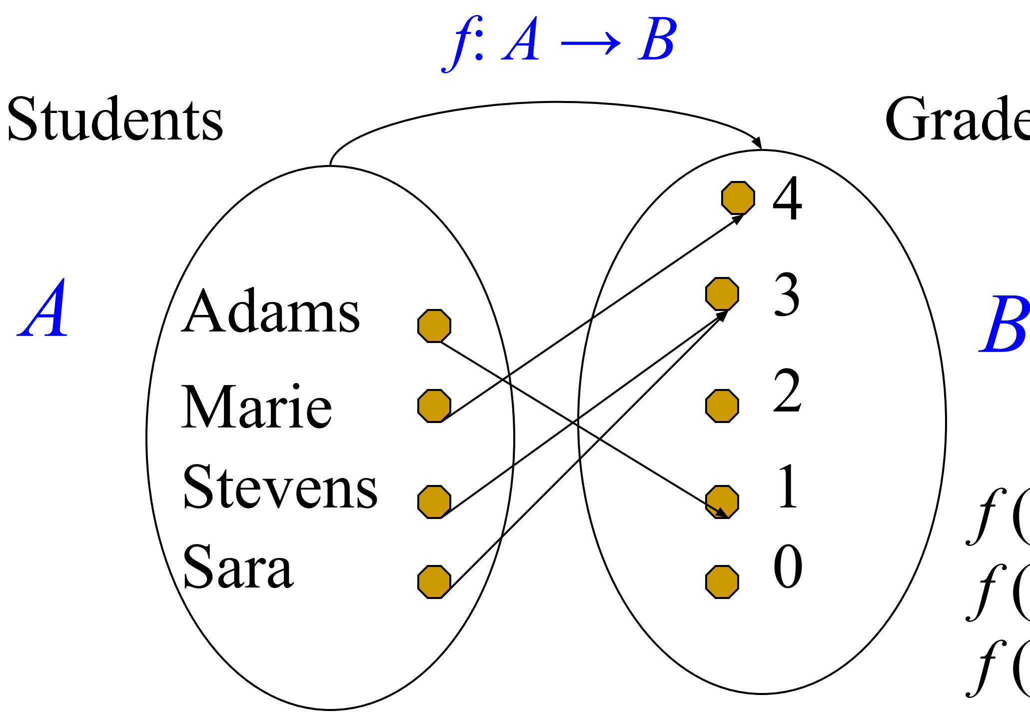
FUNCTIONS

What is a *function?*

- Example
  - Consider final grades in CSE 201. Your grade will be one of the values from the set

    {4, 3.5, 3, 2.5, 2, 1.5, 1, 0, I}
- What kind of properties does this assignment have?
- Definition: Let A, B be sets. A *function* f from set A to set B, denoted $f : A \rightarrow B$, is an assignment of <u>exactly one</u> element of B to <u>each</u> element of A.
- Notation:
  - $f: A \rightarrow B$
  - We write $f(a) = b$, if b is the element of B assigned under f to the element a of A.
  - We also say f *maps* A to B

Example

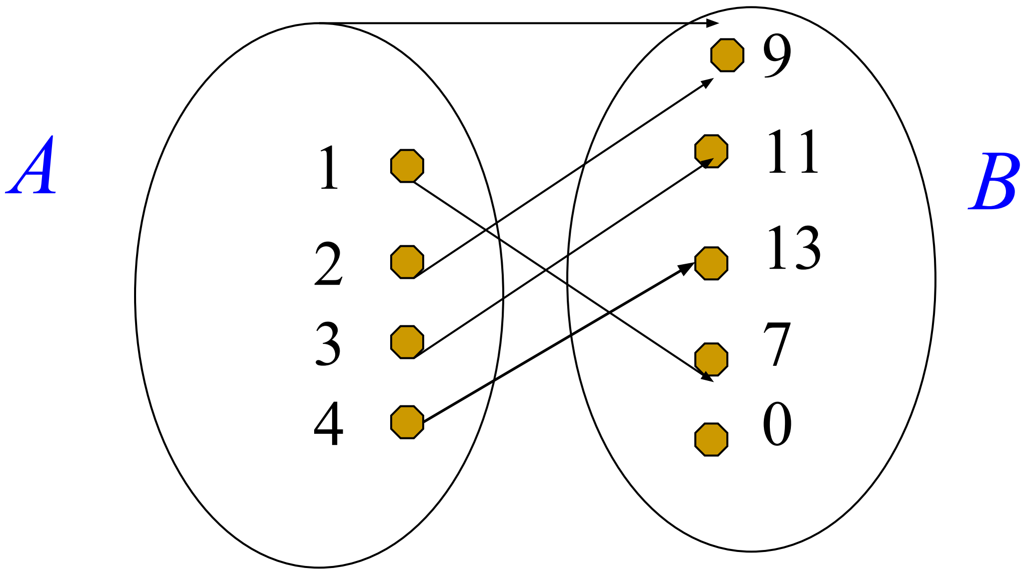


$f\,(\text{Adams}) = 1$
$f\,(\text{Marie}) = 4$
$f\,(\text{Stevens}) = 3$
$f\,(\text{Sara}) = 3$

Functions…

- How to represent functions

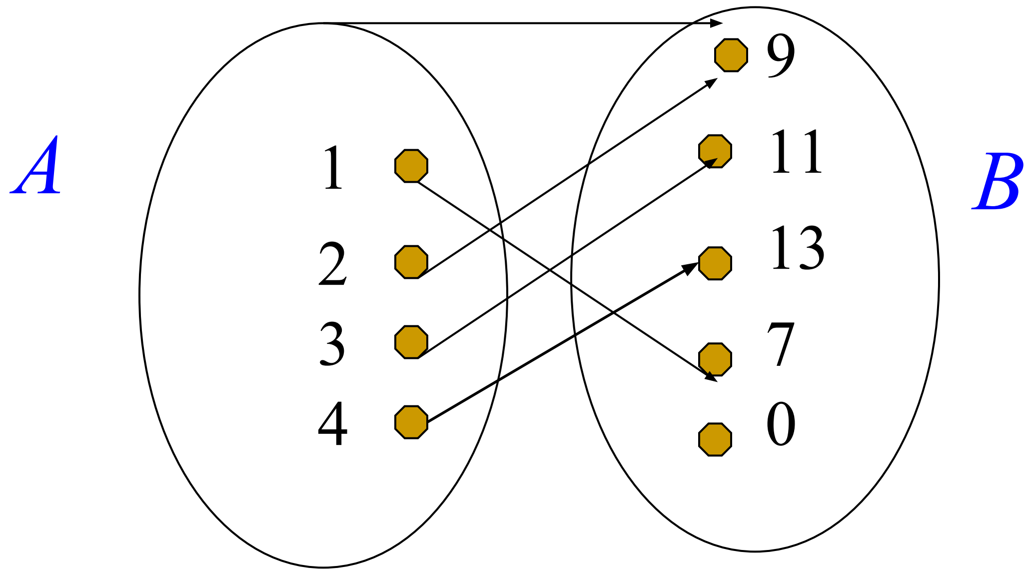
$$f: A \rightarrow B$$



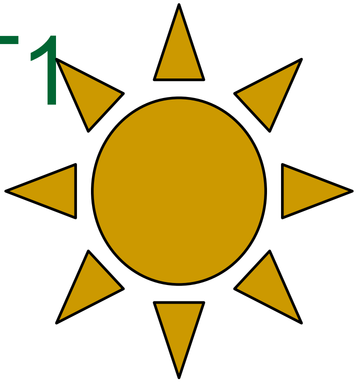
Functions…

- How to represent functions , see function below for integer x between 1 and 4.

f(x): 2x+5



Activity Function Example ICAT1



Evaluate $k(t) = 13t - 2$ at the specified input.

$k(3) =$

Activity Function Example Answer

Evaluate $k(t) = 13t - 2$ at the specified input.

$k(3) =$

1 / 2

To find the value of $k(3)$, we need to substitute $t = 3$ into the function's formula:

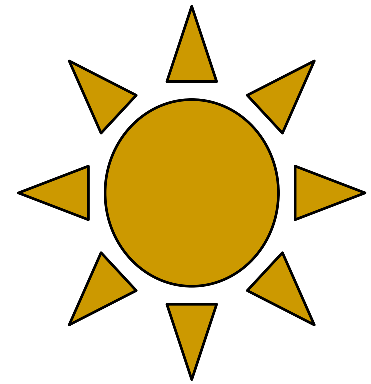
$$k(t) = 13t - 2$$

$$k(3) = 13 \cdot 3 - 2$$

$$= 39 - 2$$

$$= 37$$

Activity Function Example ICAT1



Evaluate $g(r) = 25 - 3r$ at the specified input.

$g(4) = \boxed{}$

Activity Function Example Answer

Evaluate $g(r) = 25 - 3r$ at the specified input.

$g(4) =$ 13

1 / 2

To find the value of $g(4)$, we need to substitute $r = 4$ into the function's formula:

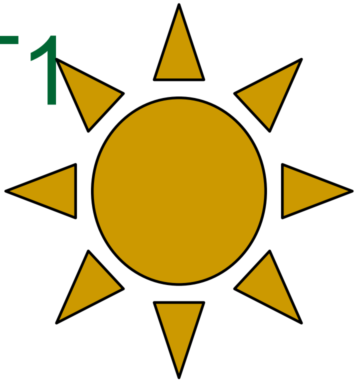
$$g(r) = 25 - 3r$$

$$g(4) = 25 - 3 \cdot 4$$

$$= 25 - 12$$

$$= 13$$

Activity Function Example ICAT1



Evaluate $h(r) = \frac{4}{5}r + 11$ at the specified input.

$h(-5) =$

Activity Function Example Answer

Evaluate $h(r) = \frac{4}{5}r + 11$ at the specified input.

$h(-5) = $ ☐

1 / 2

To find the value of $h(-5)$, we need to substitute $r = -5$ into the function's formula:

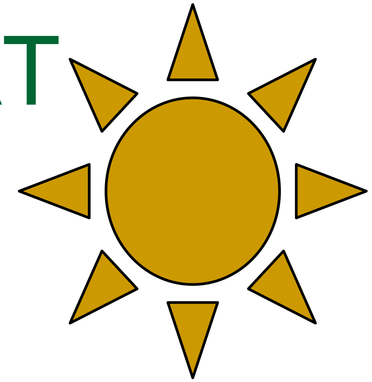
$$h(r) = \frac{4}{5}r + 11$$

$$h(-5) = \frac{4}{5} \cdot (-5) + 11$$

$$= -4 + 11$$

$$= 7$$

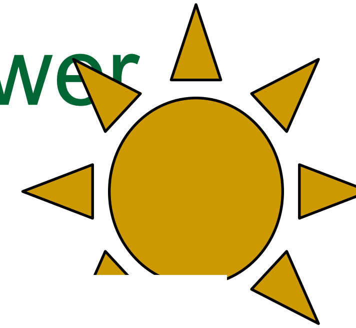
Activity Function Example ICAT



Evaluate $g(r) = -1 - 7r$ at the specified input.

$g(6) =$ $\boxed{}$

Activity Function Example Answer



Evaluate $g(r) = -1 - 7r$ at the specified input.

$g(6) =$

1 / 2

To find the value of $g(6)$, we need to substitute $r = 6$ into the function's formula:

$$g(r) = -1 - 7r$$

$$g(6) = -1 - 7 \cdot 6$$

$$= -1 - 42$$

$$= -43$$

Activity Function Example ICAT

Evaluate $k(x) = 6x + 100$ at the specified input.

$k(-5) = \boxed{}$

Activity Function Example Answer

Evaluate $k(x) = 6x + 100$ at the specified input.

$k(-5) = $ ☐

1 / 2

To find the value of $k(-5)$, we need to substitute $x = -5$ into the function's formula:

$$k(x) = 6x + 100$$

$$k(-5) = 6 \cdot (-5) + 100$$

$$= -30 + 100$$

$$= 70$$

Important Integer Functions

- Whole numbers constitute the backbone of discrete mathematics and thus computer science. We often need to convert fractions or arbitrary **real** numbers to **integers**. These *integer functions* will help us do that.
- Some important integer functions are:
  - The ***floor*** function,
  - The ***ceiling*** function,
  - The ***mod*** function.

Floor and ceiling functions

The *floor* function maps any real number x onto the **greatest integer less than or equal to x**:

$$\lfloor 3.2 \rfloor = \lfloor 3 \rfloor = 3$$

$$\lfloor -5.2 \rfloor = \lfloor -6 \rfloor = -6$$

- Consider it *rounding towards negative infinity*

The *ceiling* function maps x onto the least **integer greater than or equal to x:**

$$\lceil 3.2 \rceil = \lceil 4 \rceil = 4$$

$$\lceil -5.2 \rceil = \lceil -5 \rceil = -5$$

Floor Function

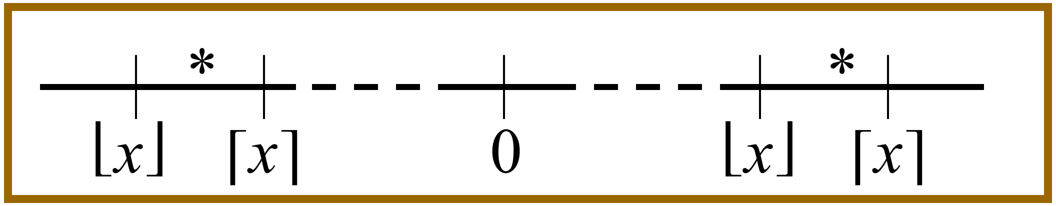
- Definition: The *floor function* from **real numbers** to **integers** assigns to the real number x, the **largest** integer ≤ x.

  The value of the floor function at x is denoted by $\lfloor x \rfloor$.
- Examples:
  - $\lfloor 3.75 \rfloor$
    - $\lfloor 3.75 \rfloor = 3$
  - $\lfloor 18 \rfloor$
    - $\lfloor 18 \rfloor = 18$
  - $\lfloor -4.5 \rfloor$
    - $\lfloor -4.5 \rfloor = -5$

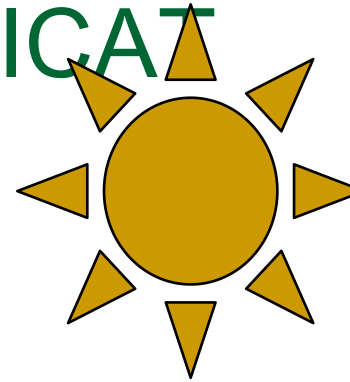
Ceiling Function

- The *ceiling function* from ***R*** to **Z** assigns to the real number x the smallest integer ≥ x. The value of the ceiling function at x is denoted by $\lceil x \rceil$.
- Examples:
  - $\lceil 3.75 \rceil =$
    - $\lceil 3.75 \rceil = 4$
  - $\lceil -18 \rceil =$
    - $\lceil -18 \rceil = -18$
  - $\lceil -4.5 \rceil =$
    - $\lceil -4.5 \rceil = -4$

Floor and Ceiling Functions, recap

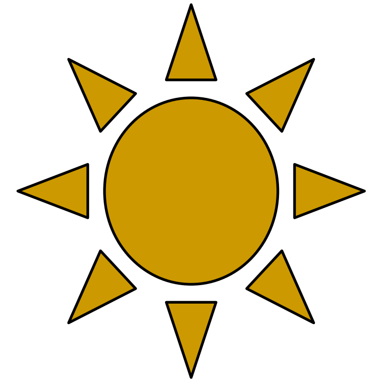


Activity: Floor Function Questions ICAT



- $\lfloor 3.75 \rfloor$ =?
- $\lfloor 0.75 \rfloor$ =?
- $\lfloor 3/4 \rfloor$ = ?
- $\lfloor 1/2 \rfloor$ =?
- $\lfloor 7/8 \rfloor$ =?
- $\lfloor 18 \rfloor$ =?
- $\lfloor -4.5 \rfloor$ =?
- $\lfloor -6.8 \rfloor$ =?

Activity: Floor Function Answers

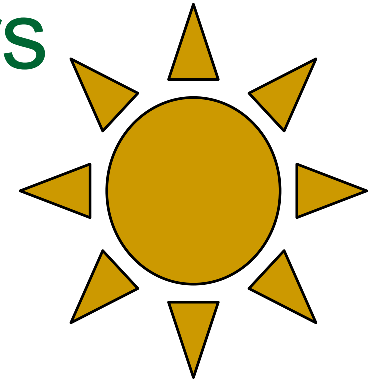


- $\lfloor 3.75 \rfloor = 3$
- $\lfloor 0.75 \rfloor = 0$
- $\lfloor 3/4 \rfloor = \lfloor 0.75 \rfloor = 0$
- $\lfloor 1/2 \rfloor = \lfloor 0.50 \rfloor = 0$
- $\lfloor 7/8 \rfloor = \lfloor 0.875 \rfloor = 0$
- $\lfloor 18 \rfloor = \lfloor 18 \rfloor$
- $\lfloor -4.5 \rfloor = -5$
- $\lfloor -6.8 \rfloor = -7$

Activity: Ceiling Function Questions ICAT

- $\lceil 3.78 \rceil = ?$
- $\lceil 0.75 \rceil = ?$
- $\lceil 3/4 \rceil = ?$
- $\lceil 1/2 \rceil = ?$
- $\lceil 7/8 \rceil = ?$
- $\lceil -18 \rceil = ?$
- $\lceil -4.5 \rceil = ?$
- $\lceil -6.8 \rceil = ?$
- $\lceil -1.2 \rceil = ?$

Activity: Ceiling Function Answers



- $\lceil 3.78 \rceil = 4$
- $\lceil 0.75 \rceil = 1$
- $\lceil 3/4 \rceil = 1$
- $\lceil 1/2 \rceil = 1$
- $\lceil 7/8 \rceil = 1$
- $\lceil -18 \rceil = -18$
- $\lceil -4.5 \rceil = -4$
- $\lceil -6.8 \rceil = -6$
- $\lceil -1.2 \rceil = -1$

Activity: Floor and Ceiling functions ICAT

For the values given, examine if the left side is equal to the right side

$x = 0.25, x = 0.5, x = 1, x = 4, y = 3$

a) $\lceil \lfloor x \rfloor \rceil \stackrel{?}{=} \lfloor x \rfloor$

b) $\lfloor 2x \rfloor \stackrel{?}{=} 2\lfloor x \rfloor$

c) $\lceil x \rceil + \lceil y \rceil - \lceil x + y \rceil \in \{0,1\} \quad ?$

d) $\lceil xy \rceil \stackrel{?}{=} \lceil x \rceil \lceil y \rceil$

e) $\left\lceil \frac{\mathrm{x}}{2} \right\rceil \stackrel{?}{=} \left\lceil \frac{x+1}{2} \right\rceil$

Example: Solution

Answers:

$x = 0.25, x = 0.5, x = 1, x = 4, y = 3$

a) $\lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor$ Answer: True

b) $\lfloor 2x \rfloor \neq 2\lfloor x \rfloor$ Answer: False, try $x = 0.5$

c) $\lceil x \rceil + \lceil y \rceil - \lceil x + y \rceil \in \{0, 1\}$ Answer: True

d) $\lceil xy \rceil \neq \lceil x \rceil \lceil y \rceil$ Answer: False, try $x = 0.25, y = 3$

e) $\left\lceil \frac{\mathrm{x}}{2} \right\rceil \neq \left\lceil \frac{x+1}{2} \right\rceil$ Answer: False, try $x = 4$

Division

- Which relation defines *dividing* 101 by 11?
  - $101 = 11 \times 8 + 13$
  - $101 = 11 \times 11 - 20$
  - $101 = 11 \times 9 + 2$

Division

- Let n be an integer and m a positive integer. Then there are *unique* integers q and r, with $\mathbf{0 \le r < m}$, such that $\mathbf{n = mq + r}$
  - n is called the ***dividend***
  - m is called the ***divisor***
  - q is called the ***quotient***
  - r is called the ***remainder***
  - Examples: $101 = \mathbf{11} \times 9 + \mathbf{2}$
    - How about: $101 = \mathbf{11} \times 8 + \mathbf{13}$
  - Examples: $-11 = 3(-4) + 1$
    - How about: $-11 = 3(-3) - \mathbf{2}$
  - **Remainder cannot be a negative number**

Special Function…

- Consider dividing a positive integer n by an integer m:
  - $\boldsymbol{n = mq + r}$
  - $29 = 8 \times 3 + 5$
  - Can we express q as a function of n and m?
    - $\boldsymbol{q = \lfloor n/m \rfloor}$
  - Can we express r as a function of n and m?
    - **Note that** $\boldsymbol{0 \le r < m}$

The mod Function

- When dividing an integer n by a number m , the quotient of the division is $\lfloor n/m \rfloor$. What about a simple notation for the remainder of this division?
- That's what the mod function is about:
- n mod m
- m is called *modulus*
- $n = \underbrace{\lfloor n/m \rfloor}_{\text{quotient}} \times m + \underbrace{n \bmod m}_{\text{remainder}}$

The mod Function…

- So, the *mod* function returns the remainder of a division
- n mod m = the remainder in dividing n by m.
  - Therefore, n mod m will always return a non-negative number less than m, that is,

$$0 \leq n \bmod m < m$$

- Example:
  - 5 mod 3 = 2
  - -7 mod 4 = 1

Take notes

ADDITIONAL EXAMPLES OF MOD SOLVED DOC CAMERA

LOGARITHMS

Logarithm

http://serc.carleton.edu/quantskills/methods/quantlit/logarithms.html

$$\log_b x = y \quad \Leftrightarrow \quad x = b^y; \text{ assume } b > 1$$

$$\log_2 64 = 6 \quad \text{since } 64 = 2^6$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

For example:

$$\log_2 64 = \frac{\log_{10} 64}{\log_{10} 2} = \frac{1.806}{0.301} = 6$$

Logarithms

We will begin with a review of logarithms:

If $n = e^m$, we define

$$m = \ln(n)$$

It is always true that $e^{\ln(n)} = n$; however, $\ln(e^n) = n$ requires that n is real

Logarithms

Exponentials grow faster than any non-constant polynomial

$$\lim_{n\to\infty} \frac{e^n}{n^d} = \infty$$

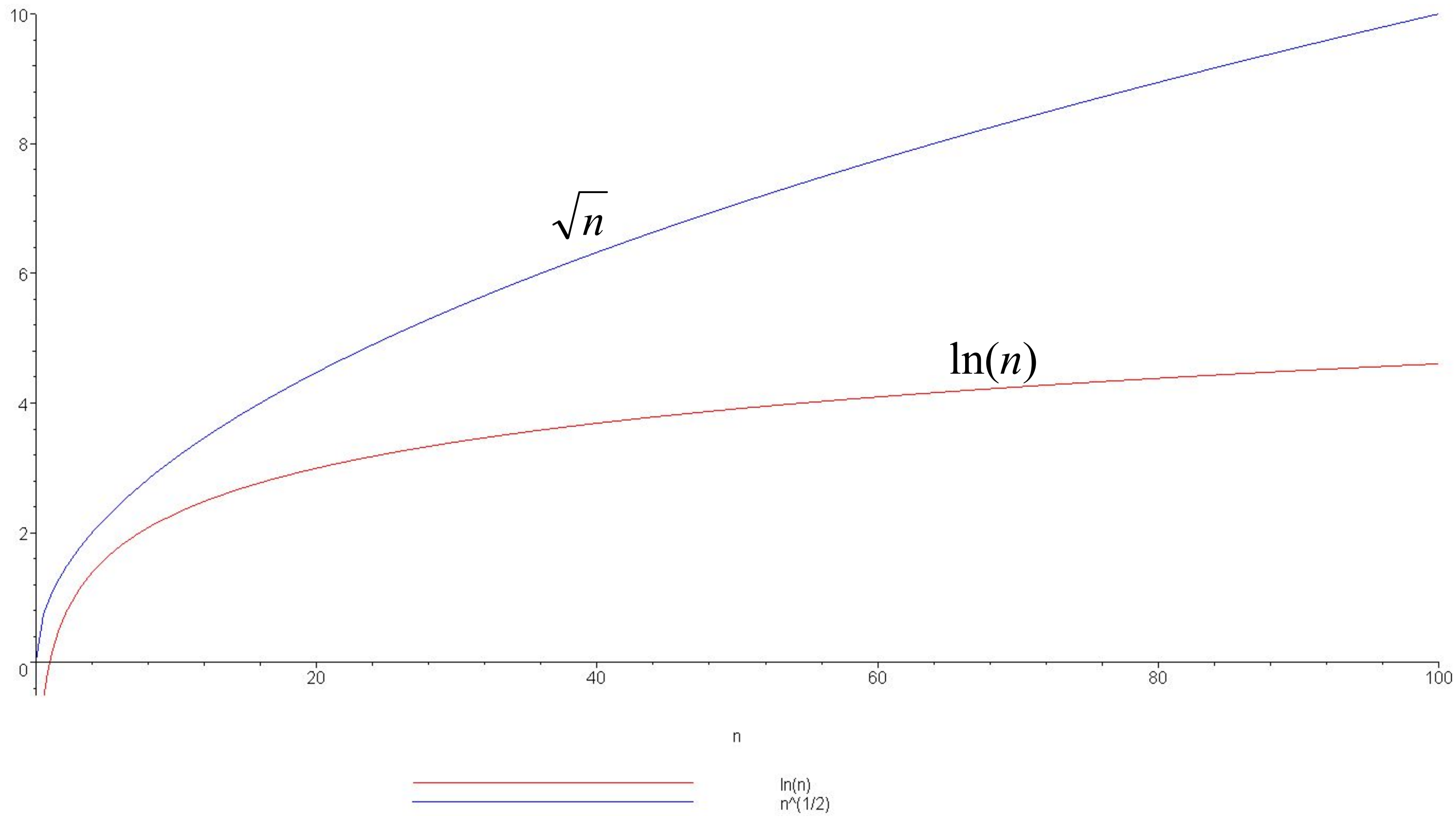
for any $d > 0$

Thus, their inverses—logarithms—grow slower than any polynomial

$$\lim_{n\to\infty} \frac{\ln(n)}{n^d} = 0$$

Logarithms

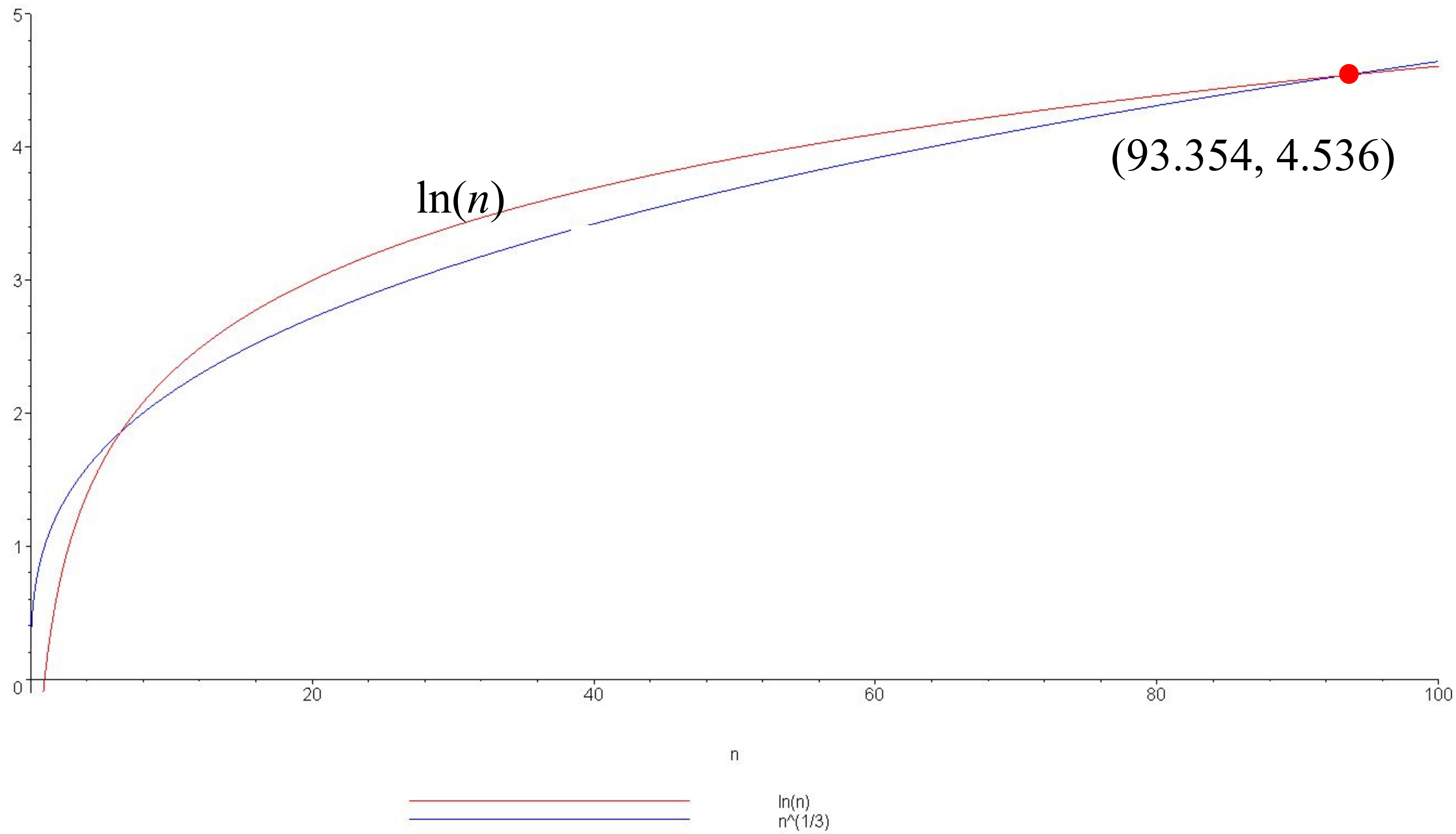
Example: $f(n) = n^{1/2} = \sqrt{n}$ is strictly greater than $\ln(n)$



Logarithms

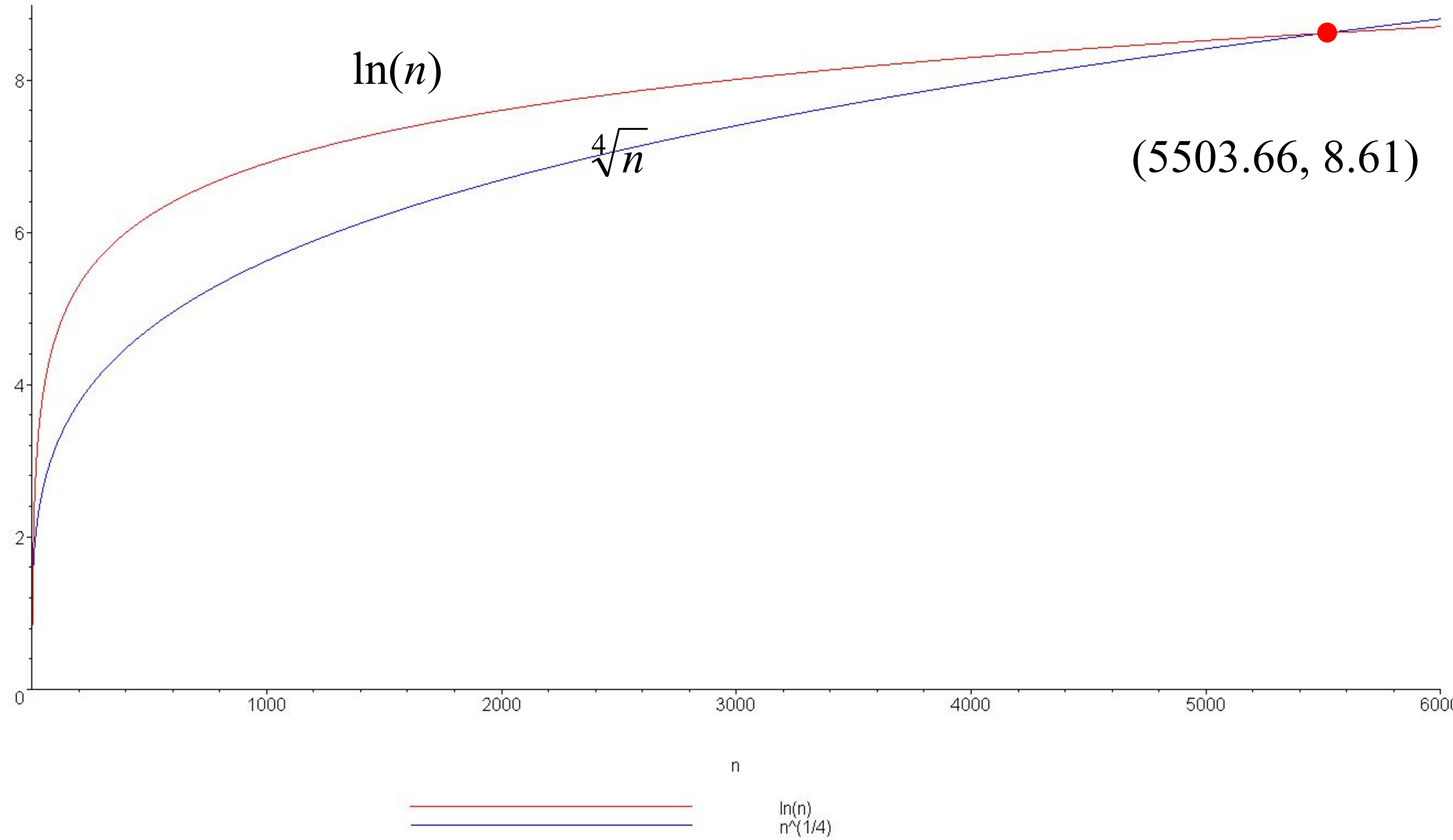
grows slower but only up to

$n = 93$



Logarithms

You can view this with any polynomial



Logarithms

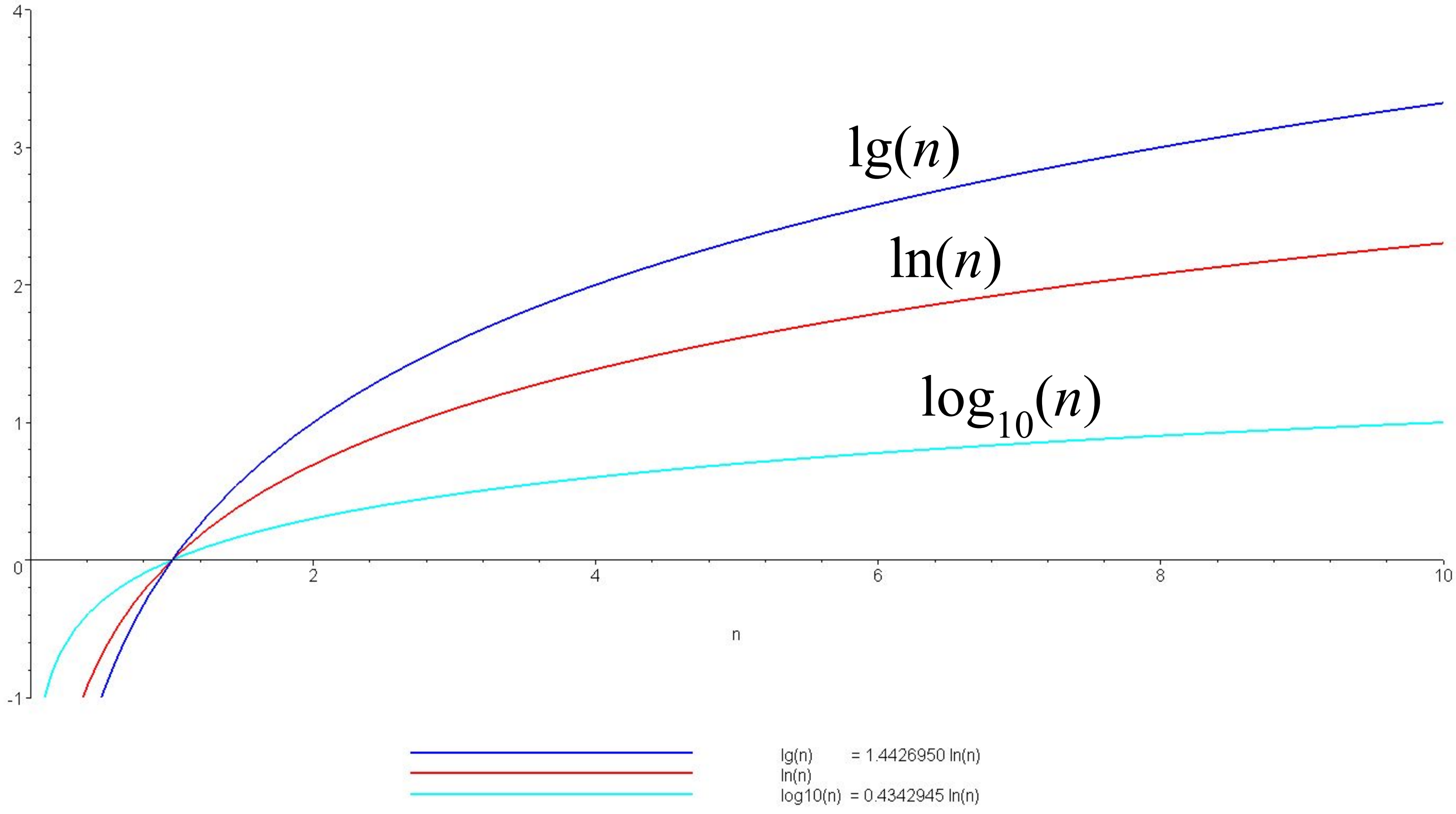
We have compared logarithms and polynomials

- How about $\log_2(n)$ versus $\ln(n)$ versus $\log_{10}(n)$

All logarithms are scalar multiples of each others

Logarithms

A plot of $\log_2(n) = \lg(n)$, $\ln(n)$, and $\log_{10}(n)$



Logarithms

Note: the base-2 logarithm $\log_2(n)$ is written as $\lg(n)$

It is an industry standard to implement the natural logarithm $\ln(n)$ as

```
double log( double );
```

The *common* logarithm $\log_{10}(n)$ is implemented as

```
double log10( double );
```

Logarithms

A more interesting observation we will repeatedly use:

$$n^{\log_b(m)} = m^{\log_b(n)},$$

a consequence of :

$$\begin{aligned} n^{\log_b(m)} &= (b^{\log_b(n)})^{\log_b(m)} \\ &= b^{\log_b(n)\log_b(m)} \\ &= (b^{\log_b(m)})^{\log_b(n)} \\ &= m^{\log_b(n)} \end{aligned}$$

Logarithms

You should also, be aware of the relationship:

$$\lg(2^{10}) = \lg(1024) \quad = 10$$

$$\lg(2^{20}) = \lg(1\,048\,576) \quad = 20$$

and consequently:

$$\lg(10^{3}) = \lg(1000) \quad \approx 10 \quad \text{kilo}$$

$$\lg(10^{6}) = \lg(1\,000\,000) \quad \approx 20 \text{mega}$$

$$\lg(10^{9}) \quad \approx 30 \quad \text{giga}$$

$$\lg(10^{12}) \quad \approx 40 \quad \text{tera}$$

Take Notes

SEE MORE EXAMPLES ON DOC CAMERA

Periodic Functions

Definition [edit]

A function *f* is said to be **periodic** with period *P* (*P* being a nonzero constant) if we have

$$f(x+P) = f(x)$$

for all values of *x* in the domain.[citation needed] If there exists a least positive[1] constant *P* with this property, it is called the **fundamental period** (also **primitive period**, **basic period**, or **prime period**.) A function with period *P* will repeat on intervals of length *P*, and these intervals are referred to as **periods**.

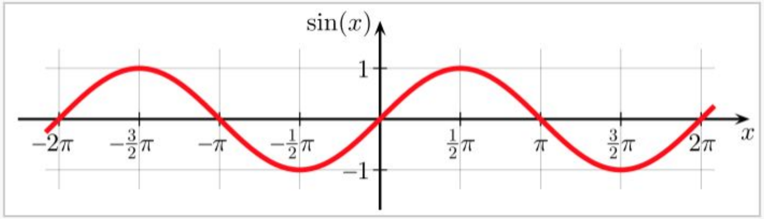
Periodic Functions

Examples [edit]

For example, the sine function is periodic with period 2π, since

$$\sin(x + 2\pi) = \sin x$$

for all values of x. This function repeats on intervals of length 2π (see the graph to the right).



A graph of the sine function, showing two complete periods

Everyday examples are seen when the variable is *time*; for instance the hands of a clock or the phases of the moon show periodic behaviour. **Periodic motion** is motion in which the position(s) of the system are expressible as periodic functions, all with the *same* period.

For a function on the real numbers or on the integers, that means that the entire graph can be formed from copies of one particular portion, repeated at regular intervals.

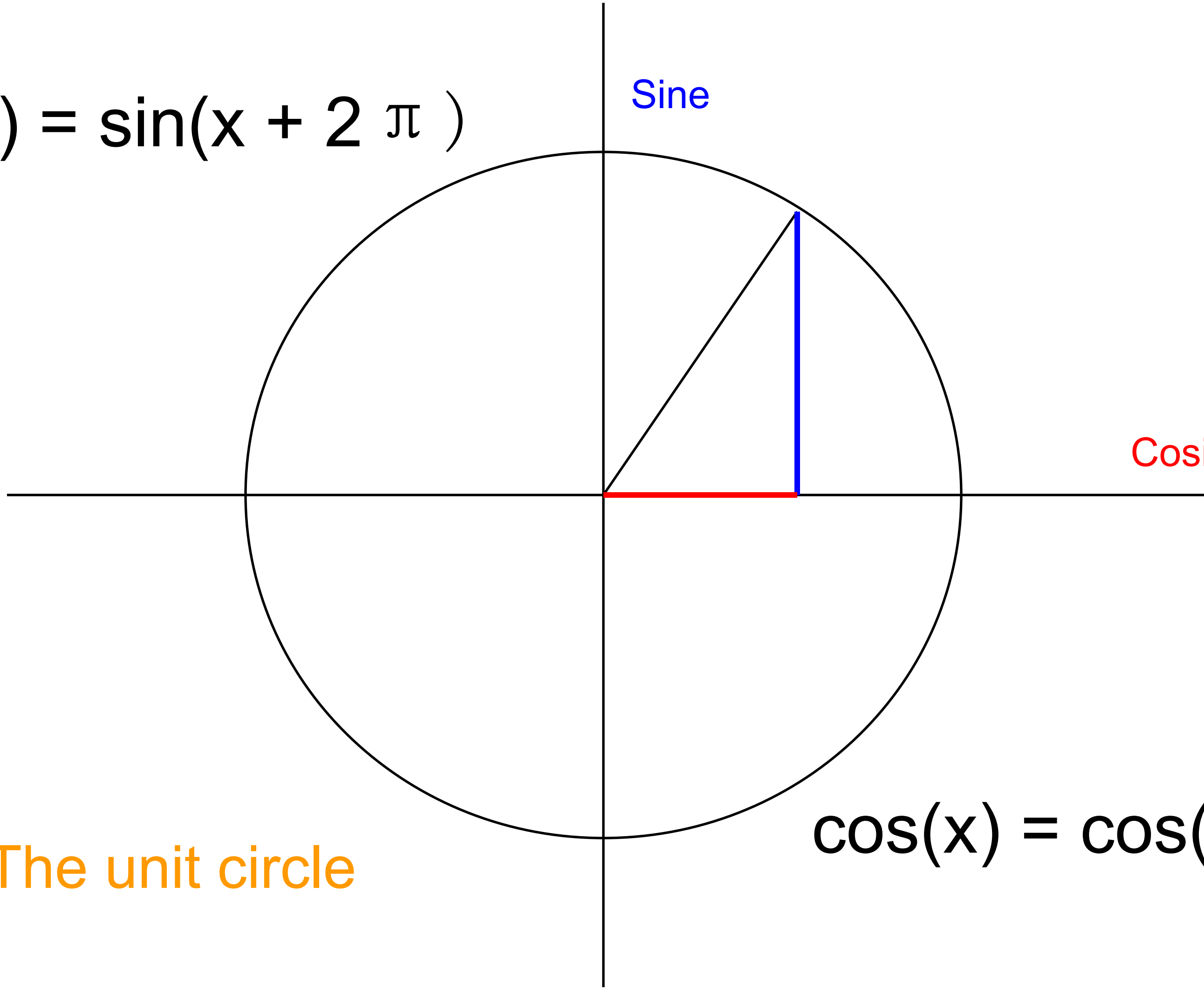
A simple example of a periodic function is the function f that gives the "fractional part" of its argument. Its period is 1. In particular,

$$f(0.5) = f(1.5) = f(2.5) = \ldots = 0.5.$$

The graph of the function f is the sawtooth wave.

Other special functions; sin(), cos()

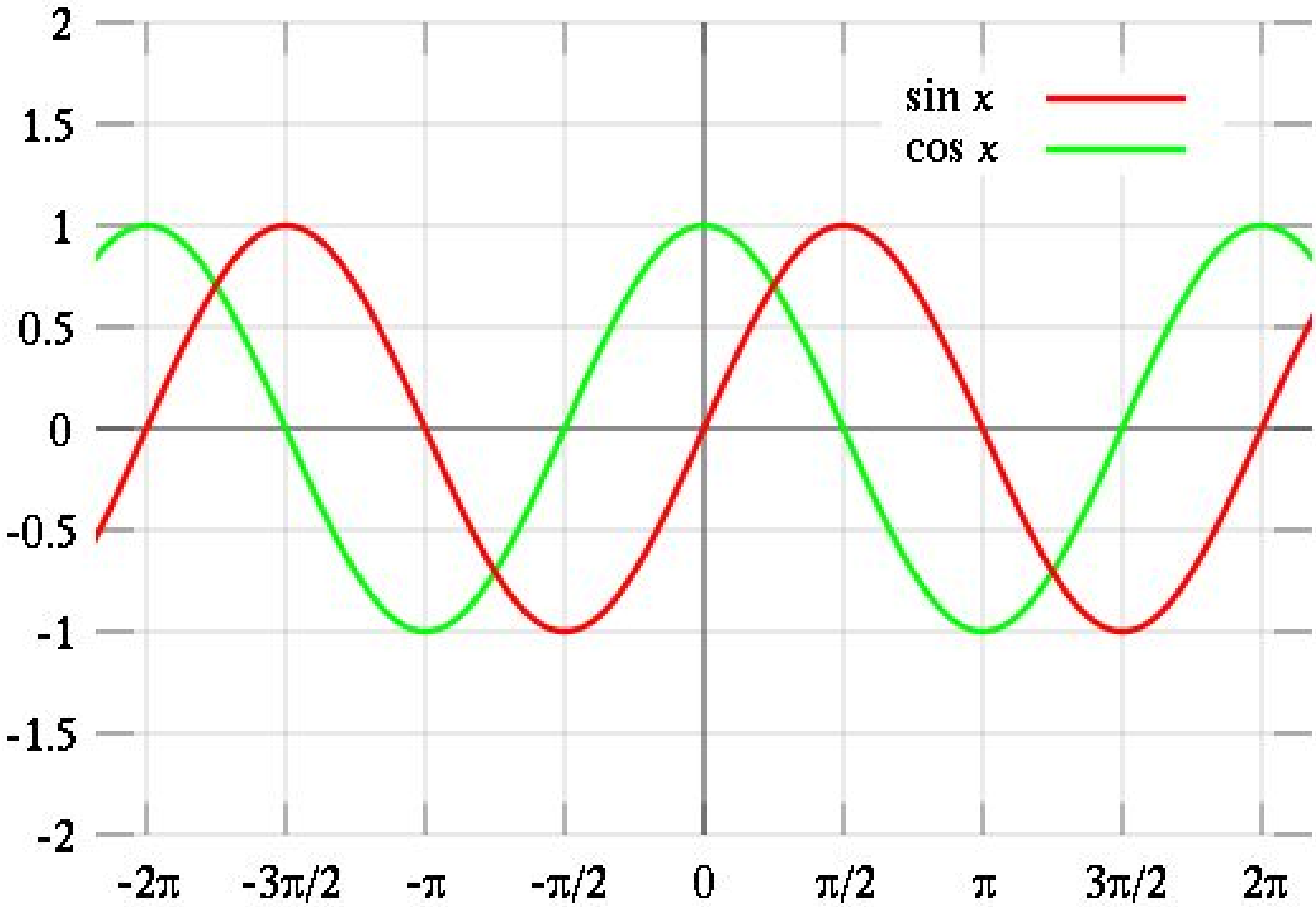
$\sin(x) = \sin(x + 2\pi)$



$\cos(x) = \cos(x + 2\pi)$

The unit circle

Sin and Cos Periodic Function

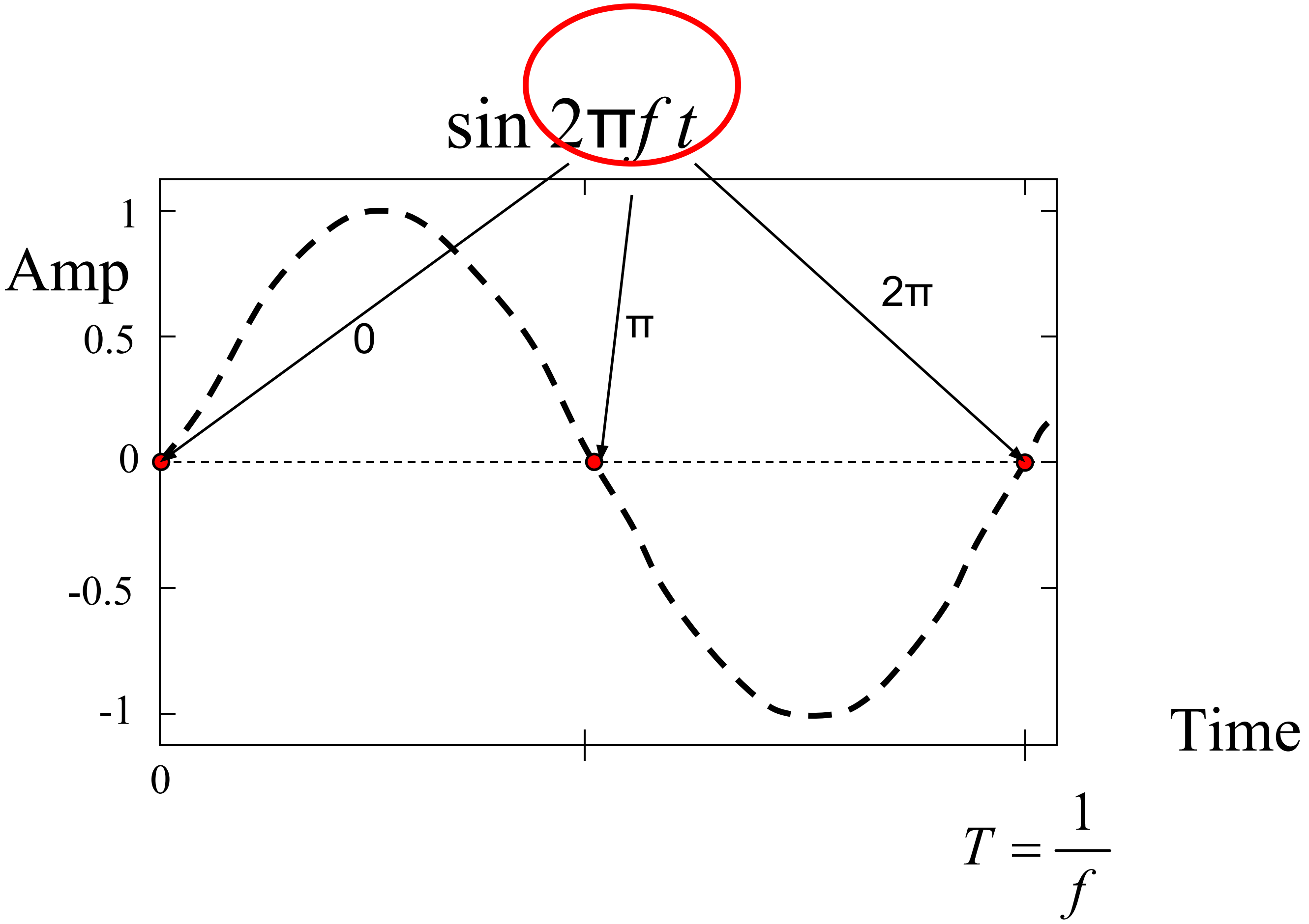


Sin() function

- Consider the following function where *t* is in seconds

- After how short of a time does this function repeat itself?
  - This time interval is called the *period* of the function
  - The *frequency*, the number of times the function repeats itself per second, is $f = 1/T$

Example: Periodic Signals



Example

- Find the frequency and the period of the following periodic function where *t* is in seconds
  - $g(t) = \sin(50t)$
  - $\sin(50t) = \sin 2\pi f\, t$
    - $50 = 2\pi f$. This gives frequency $f = 50/\ 2\pi = 7.95$
    - The period is: $T= 1/f = 1/7.95$

References

- Wikipedia
- Khan Academy