EE 201: Data Analysis Project (Autumn 2021)

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Question 1

Consider three random variables, X, Y, N

Let the bias of the coin be p

X : Takes values 0 and 5 with probability $1 - p$ and p respectively.

$N(\mu, \sigma^2)$: Gaussian noise with Mean μ, Variance σ^2

Let $Y = X + N$, according to the problem statement, Y is the random variable of recieved data. Also $X = 0$ for $n \leq 1000$, where n is the trial number. Taking expectation on both sides:

$$\mathbb{E}[Y] = \mathbb{E}[X] + \mathbb{E}[N] \tag{1}$$

When $n \leq 1000$:

$$\mathbb{E}[Y] = \mathbb{E}[N] = \mu \tag{2}$$

$$Var[Y] = Var[0 + N] = Var[N] = \sigma^2 \tag{3}$$

$\mathbb{E}[Y] \approx mean(Y_i,\ i \text{ from } a \text{ to } b) = \mu$ and

$Var[Y] \approx var(Y_i,\ i \text{ from } a \text{ to } b) = \sigma^2$ as n is very large

When $1000 < n \leq 10000$, Using Equations (1) & (2):

$$\begin{aligned}
&\mathbb{E}[Y] = \mathbb{E}[X] + \mu \\
\implies &mean(Y_i, 1001, 10000) = (5p + 0(1-p)) + mean(Y_i, 1, 1000) \\
\implies &mean(Y_i, 1001, 10000) - mean(Y_i, 1, 1000) = 5p
\end{aligned}$$

$$\implies \boxed{p = \frac{mean(Y_i, 1001, 10000) - mean(Y_i, 1, 1000)}{5}}$$

On Running the data through Q1.py program:

$$\boxed{\mu = 2.0} \qquad \boxed{\sigma^2 = 1.5} \qquad \boxed{p = 0.6}$$

PDF of Gaussian Noise:

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi(1.5)}} \exp\left(-\frac{1}{2}\frac{(x-2.0)^{\,2}}{1.5}\right)}$$