EE 201: Data Analysis Project (Autumn 2021)

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Question 2

Given $Z = X + 10Y \implies Z - X = 10Y$, Taking Expection and Variance on both sides:

$\mathbb{E}\left[Z - X\right] = 10\,\mathbb{E}[Y]$; $Var\left[Z - X\right] = 100\,Var\left[Y\right]$

$X(a,b)$: Uniform random variable with $a = -3$, $b = 3$ and $\mathbb{E}\left[X\right] = \frac{a+b}{2} = 0$

Y: $\sum_{i=1}^{k} W_i$ and W_i's are i.i.d $\implies \mathbb{E}\left[Y\right] = k\,\mathbb{E}\left[W\right]$ and $Var\left[Y\right] = kVar[W]$

The eqautions we get are:

$$\mathbb{E}\left[Z\right] = 10\,k\,\mathbb{E}[W] \tag{1}$$

$$Var\left[Z - X\right]^{*} = 100\,k\,Var[W] \tag{2}$$

** [$Z - X$ can be obtained by subtracting each Z_i data with a uniform R.V. $X(-3,3)$]*

On doing the following operation $\frac{Eq(1)^2}{Eq(2)}$:

$\frac{\mathbb{E}\left[Z\right]^2}{Var\left[Z-X\right]} = k\,\frac{\mathbb{E}[W]^2}{Var[W]} \implies k = \left(\frac{Var[W]}{\mathbb{E}[W]^2}\right)\,\frac{\mathbb{E}\left[Z\right]^2}{Var\left[Z-X\right]}$

- **Exponential distribution**

  $\mathbb{E}\left[W\right] = \frac{1}{\lambda}$ and $Var[W] = \frac{1}{\lambda^2}$

  $\implies \boxed{k = \dfrac{\mathbb{E}\left[Z\right]^2}{Var\left[Z - X\right]}}$ Using (1): $\boxed{\dfrac{1}{\lambda} = \dfrac{\mathbb{E}\left[Z\right]}{10k}}$

- **Rayleigh distribution**

  $\mathbb{E}\left[W\right] = \sigma\sqrt{\frac{\pi}{2}}$ and $Var[W] = \sigma^2(\frac{4-\pi}{2})$

  $\implies \boxed{k = \left(\dfrac{4-\pi}{\pi}\right)\,\dfrac{\mathbb{E}\left[Z\right]^2}{Var\left[Z - X\right]}}$ Using (1): $\boxed{\sigma = \dfrac{\mathbb{E}\left[Z\right]}{10k}\sqrt{\dfrac{2}{\pi}}}$

- **Half-normal distribution**
  $\mathbb{E}[W] = \sigma\sqrt{\frac{2}{\pi}}$ and $Var[W] = \sigma^2(\frac{\pi-2}{\pi})$
  $\implies \boxed{k = \left(\frac{\pi-2}{2}\right)\frac{\mathbb{E}[Z]^2}{Var[Z-X]}}$ Using (1): $\boxed{\sigma = \frac{\mathbb{E}[Z]}{10k}\sqrt{\frac{\pi}{2}}}$

On comparing, we select the distribution whose 'k' value is closest to $\{2, 3, 4\}$. Then using the 'k' value we can compute the parameter of the corresponding distribution as shown above.

Computation for the given data:

Distribution: Exponential

k = 2

Parameter $\frac{1}{\lambda}$ **=** 4 *nearest integer