Clustering Part 2

```
import math
import random
import matplotlib
import numpy as np
import pandas as pd
import seaborn as sns
from tqdm.notebook import tqdm
import matplotlib.pyplot as plt
from sklearn.datasets import make_circles
matplotlib.rcParams['figure.figsize'] = (10, 7)
```

DBSCAN Algorithm

DBSCAN(Density-Based Spatial Clustering of Applications with Noise) is a commonly used unsupervised clustering algorithm. DBSCAN does not need to specify the number of clusters. It can automatically detect the number of clusters based on your input data and parameters. More importantly, DBSCAN can find arbitrary shape clusters that k-means are not able to find.

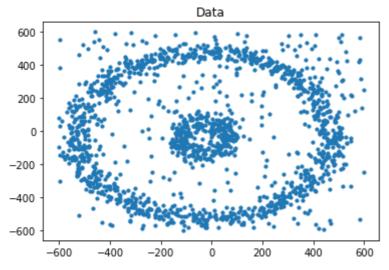
Algorithm:

- a. The algorithm proceeds by arbitrarily picking up a point in the dataset (until all points have been visited).
- b. If there are at least 'minPoint' points within a radius of ' ϵ ' to the point then we consider all these points to be part of the same cluster.
- c. The clusters are then expanded by recursively repeating the neighborhood calculation for each neighboring point

A. Generate "N" spherical training data points.

```
In []: ## write your code here
    ### x = R * sin(theta) ; y = R * cos(theta) ; as x --> (1, n) ==> theta --> (0
    def Circle(r,n=100):
        return [(math.cos(2*math.pi/n*x)*r+np.random.normal(-30,30),math.sin(2*math.pi/n
    df=pd.DataFrame(Circle(500,1000))
    df=df.append(Circle(100,300))
    df=df.append([(np.random.randint(-600,600),np.random.randint(-600,600)) for i in ran
    plt.scatter(df[0],df[1],s=10)
    plt.title('Data')
    plt.show()
```

C:\Users\Shashank\AppData\Local\Temp\ipykernel_26768\3063699069.py:6: FutureWarning:
The frame.append method is deprecated and will be removed from pandas in a future ve
rsion. Use pandas.concat instead.
 df=df.append(Circle(100,300))
C:\Users\Shashank\AppData\Local\Temp\ipykernel_26768\3063699069.py:7: FutureWarning:
The frame.append method is deprecated and will be removed from pandas in a future ve
rsion. Use pandas.concat instead.
 df=df.append([(np.random.randint(-600,600),np.random.randint(-600,600)) for i in r
ange(300)])



B. Perform DBSCAN Algorithm on the above generated data to obtain clusters

```
In [ ]: |
        ## Write your code here
         def plot_label(label, data, title):
             for c in np.unique(label):
                 x_temp = data[label==c, 0]
                 y_temp = data[label==c, 1]
                 plt.scatter(x_temp, y_temp, c=np.random.rand(3,))
             plt.xlabel('x')
             plt.ylabel('y')
             plt.title(title)
             plt.show()
         def neighbours(p, data, eps):
             neighs = []
             for i in range(len(data)):
                 if np.linalg.norm(np.array(data[i])-np.array(data[p]), ord=2)<=eps:</pre>
                     neighs.append(i)
             return neighs
         def expand_cluster(data, labels, p, neighs, cluster_id, eps, min_points):
             labels[p] = cluster_id
             i = 0
             while i<len(neighs):</pre>
                 p next = neighs[i]
                 if labels[p_next]==-1: labels[p_next]=cluster_id
                 elif labels[p next]==0:
                     labels[p_next] = cluster_id
                     p_next_neighs = neighbours(p_next, data, eps)
                     if len(p_next_neighs)>=min_points:
                         neighs+=p_next_neighs
                 i+=1
         def DB_SCAN(eps, min_points, d):
             data = d.tolist()
             labels = [0]*len(data)
```

```
cluster_id = 0
for p in tqdm(range(len(data))):
    if labels[p]: continue
    neighs = neighbours(p, data, eps)
    if len(neighs)<min_points:
        labels[p]=-1
    else:
        cluster_id += 1
        expand_cluster(data, labels, p, neighs, cluster_id, eps, min_points)

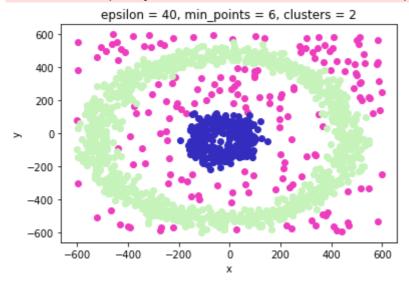
data = np.array(data)
    labels = np.array(labels)
    title = f'epsilon = {eps}, min_points = {min_points}, clusters = {cluster_id}'
    plot_label(labels, data, title)
    return labels

labels = DB_SCAN(40, 6, data)</pre>
```

```
0% | 0/1600 [00:00<?, ?it/s]
```

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C. Experiment by varying the number of min points and epsilon radius and plot your observations

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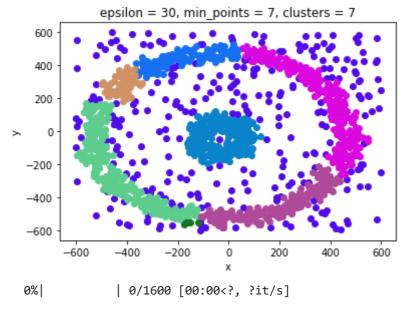
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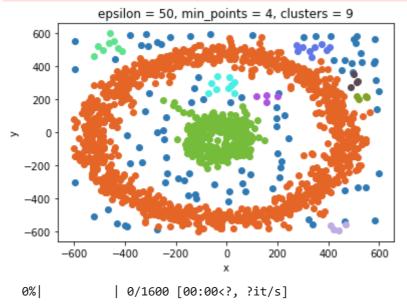
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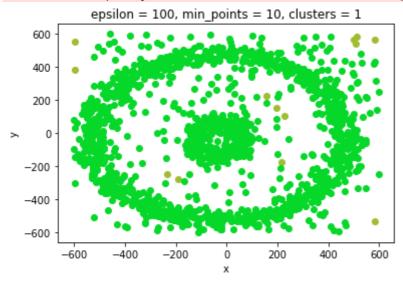
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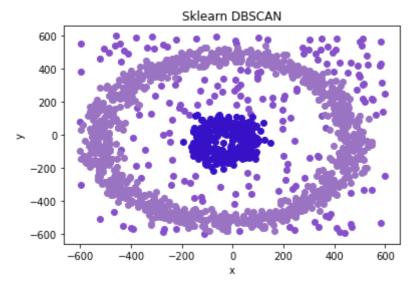


D. Compare your model with the built in DBSCAN in Sci-kit Learn. Also compare you results with GMM and the K-means Algorithm

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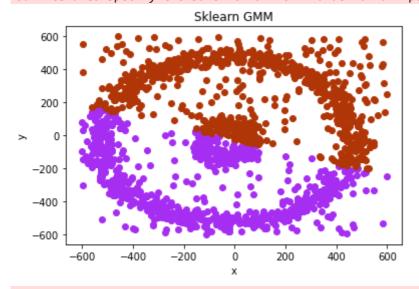
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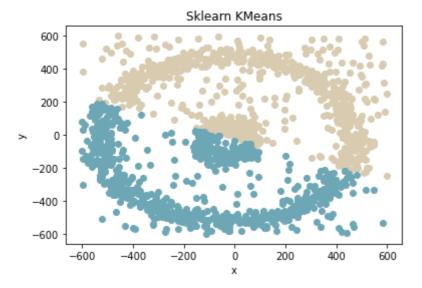
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Fuzzy C-Means Based clustering

- 1. Randomly initialize the centroids and clusters K, and compute the probability that each data point xi is a member of a given cluster k, P(point xi has label k|xi, k).
- 2. Iteration: Recalculate the centroids of the clusters as the weighted centroid given the probabilities of membership of all data points xi:

$$\mu_k(n+1) = rac{\sum_{x_i \in k} x_i * P(\mu_k \mid x_i)^b}{\sum_{x_i \in k} P(\mu_k \mid x_i)^b}$$

1. Implement it on the data for which Kmeans was implemented.

```
In [ ]:
        class FuzzyC:
            # works for only 2D
             """ TODO :
                1 ) first find c centers randomly and calc dist matr and membership matr
                2 ) find new centers
                3 ) find dist matrix
                4 ) find new membership matrix
                 5 ) Do it till convergence
             def init (self, clus, d):
                self.no_clusters = clus
                self.data = d
                self.M = data.shape[0]
                self.means = np.array(random.sample(data.tolist(), self.no clusters))
                self.cols = np.random.rand(self.no clusters, 3)
                self.labels = np.zeros(self.M,)
             def get_dist_matrix(self):
                dist_matrix = np.zeros((self.M, self.no_clusters))
                for i in range(self.M):
                     for j in range(self.no_clusters):
                         tot den = 0
                         for k in range(self.no_clusters):
                             num = np.linalg.norm(data[i]-self.means[j], ord=2)
                             den = np.linalg.norm(data[i]-self.means[k], ord=2)+(1e-15)
                             tot_den += (num/den)**2
                         dist_matrix[i][j] = 1/(tot_den+1e-15)
```

```
return dist_matrix
    def get_new_mean(self, dist_matrix):
        label = self.get_labels(dist_matrix)
        new means = np.zeros((self.no clusters, 2))
        for j in range(self.no_clusters):
            sum = 0
            for i in range(self.M):
                new_means[j] += self.data[i]*(dist_matrix[i][j]**2)
                sum += dist_matrix[i][j]**2
            new_means[j] /= sum+1e-15
        return new_means
    def get_cost(self, dist_matrix):
        cost = 0
        for k in range(self.no_clusters):
            for i in range(self.M):
                cost += dist_matrix[i][k] * np.linalg.norm(data[i]-self.means[k], or
        return cost / (self.M * self.no_clusters)
    def get_labels(self, dist_matrix):
        return np.argmax(dist_matrix, axis=1)
    def doFCM(self):
        present_cost = float('inf')
        delta = present_cost
        it = 1
        while(delta>=1e-15):
            print(self.means)
            dist_mat = self.get_dist_matrix()
            self.means = self.get new mean(dist mat)
            temp_cost = self.get_cost(dist_mat)
            delta = abs(present_cost-temp_cost)
            present_cost = temp_cost
            self.labels = self.get_labels(dist_mat)
            title = f'Iteration: {it}, delta: {delta}'
            self.showGraph(title)
            it+=1
    def showGraph(self, title):
        plt.scatter(self.means[:, 0], self.means[:, 1], c='black', marker='x')
        for k in range(self.no_clusters):
            x temp = data[self.labels==k, 0]
            y_temp = data[self.labels==k, 1]
            plt.scatter(x_temp, y_temp, c=self.cols[k], marker='+')
        plt.xlabel('x')
        plt.ylabel('y')
        plt.title(title)
        plt.show()
K = 4
data = []
means = [(0.5, 0), (5, 5), (5, 1), (10, 1.5)]
for i in range(K):
    d = np.random.multivariate normal(mean=means[i], cov=np.identity(2), size=100)
    data.append(d)
data = np.vstack(data)
# write your code here
fcm = FuzzyC(4, data)
```

```
fcm.doFCM()
fcm.showGraph("Final Clustering")
```

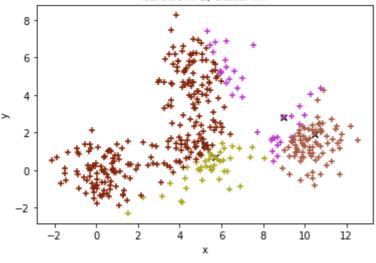
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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```
[[ 5.72035875    0.65426631]
  [ 8.98228383    2.8174128 ]
  [10.4702039    1.93149102]
  [ 5.18426374    1.38046469]]
```



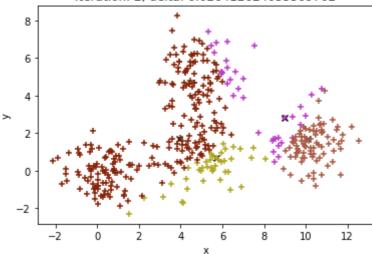


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Iteration: 2, delta: 0.026412624035509702

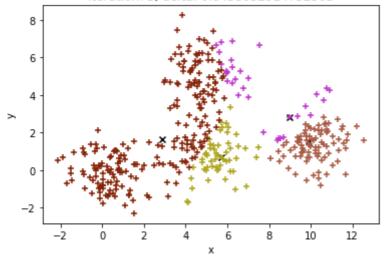


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Iteration: 3, delta: 0.0458032914752563



[[5.72035875 0.65426631] [8.98228383 2.8174128]

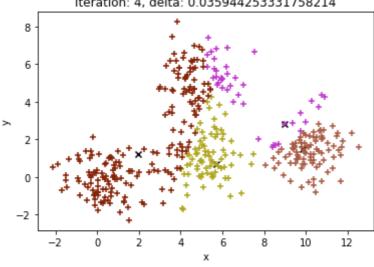
[9.92089512 1.53344824]

[2.85512617 1.65290238]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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Iteration: 4, delta: 0.035944253331758214

[[5.72035875 0.65426631] [8.98228383 2.8174128]

[9.8707813 1.49171328]

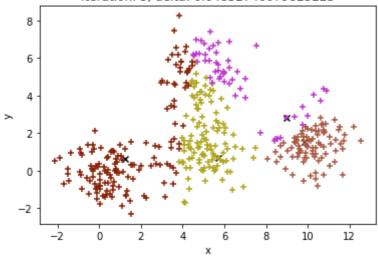
[1.96555914 1.20637571]]

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Iteration: 5, delta: 0.04832746079825223



[[5.72035875 0.65426631] [8.98228383 2.8174128] [9.84455927 1.51217609] [1.24602356 0.6504245]]

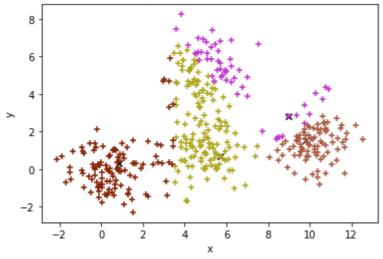
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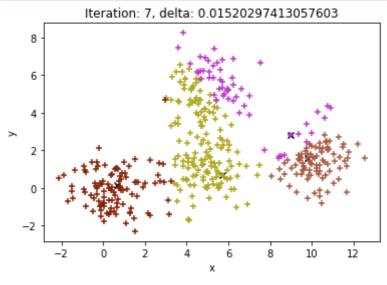
[[5.72035875 0.65426631] [8.98228383 2.8174128] [9.81407354 1.55094301]

[0.82856803 0.3071851]]

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[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.78143171 1.5822272]

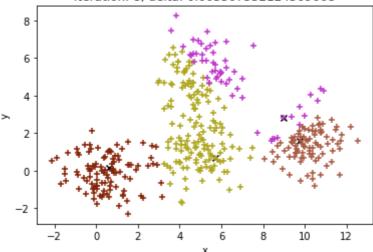
[0.65573534 0.16738889]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 8, delta: 0.003507532124569668



c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

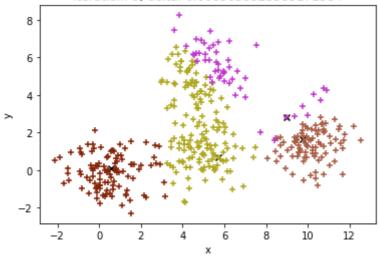
[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.75478109 1.60207176]

[0.59517124 0.11951615]]

Iteration: 9, delta: 0.0005638323383171384



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

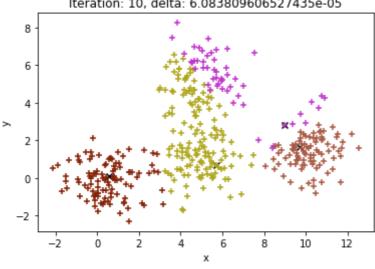
[9.73585737 1.61367048]

[0.57501871 0.10380209]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.



Iteration: 10, delta: 6.083809606527435e-05

[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.72323754 1.62029861]

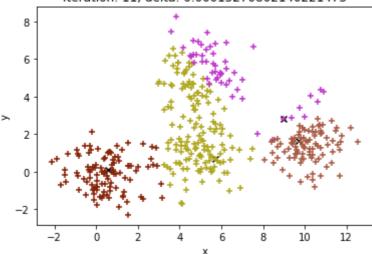
[0.56828378 0.09853613]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 11, delta: 0.00015270802140221473



c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

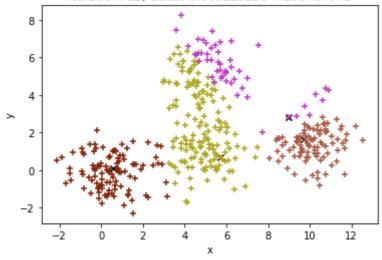
[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.71506037 1.62409613]

[0.56593708 0.09666044]]

Iteration: 12, delta: 0.00012925044198464075



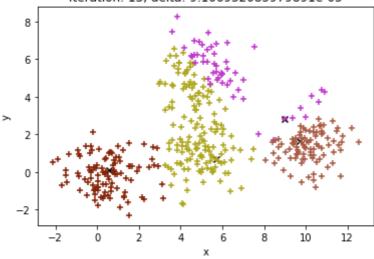
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

[[5.72035875 0.65426631] [8.98228383 2.8174128] [9.70983444 1.62630038] [0.56505357 0.09592549]]





c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

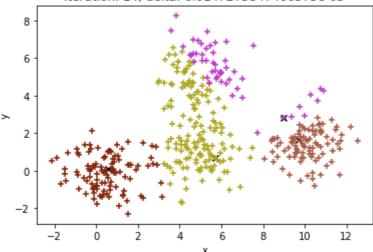
[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70651763 1.62760054]

[0.56468284 0.09560143]]

Iteration: 14, delta: 6.0147275847466375e-05



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70442012 1.62837926]

[0.56450738 0.09544079]]

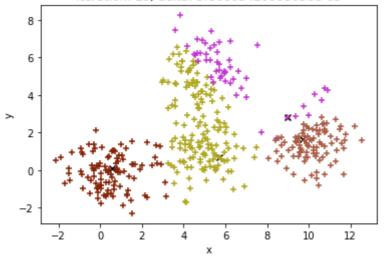
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid

ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 15, delta: 3.86605420895636e-05



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

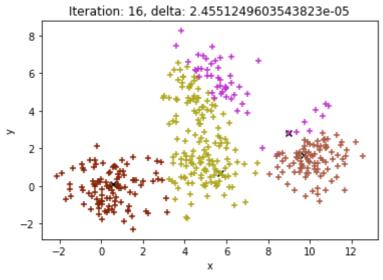
[9.70309632 1.62885176]

[0.56441507 0.09535336]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70226178 1.6291414]

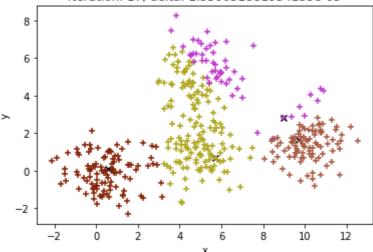
[0.56436267 0.09530269]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 17, delta: 1.550651881954135e-05



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70173604 1.62932032]

[0.5643315 0.09527221]]

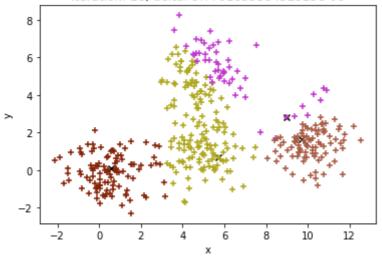
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid

ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 18, delta: 9.770185338431325e-06



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

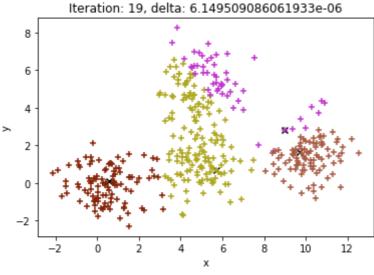
[9.70140498 1.62943145]

[0.56431246 0.09525349]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70119657 1.62950075]

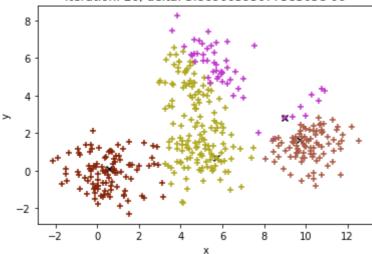
[0.56430067 0.09524186]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 20, delta: 3.8690039507738305e-06



[[5.72035875 0.65426631] [8.98228383 2.8174128] [9.7010654 1.62954409]

[0.56429331 0.09523459]]

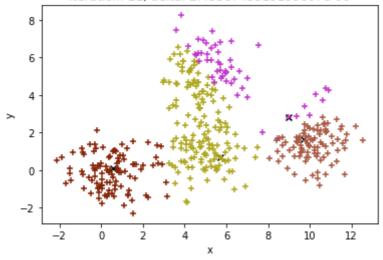
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 21, delta: 2.4338743315199807e-06



[[5.72035875 0.65426631] [8.98228383 2.8174128]

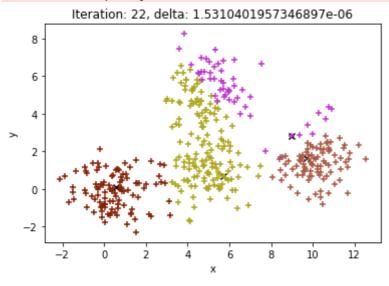
[9.70098284 1.62957125]

[9.70098284 1.62957125] [0.5642887 0.09523004]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.



[[5.72035875 0.65426631] [8.98228383 2.8174128] [9.7009309 1.62958828]

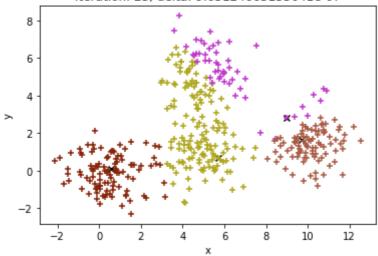
[0.5642858 0.09522718]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 23, delta: 9.63124663133641e-07



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70089821 1.62959898]

[0.56428398 0.09522539]]

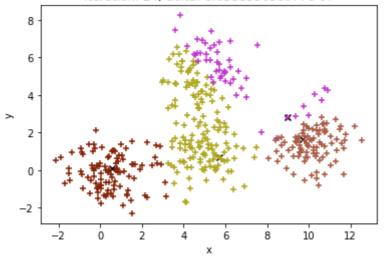
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *v*.

ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 24, delta: 6.058855656077e-07



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

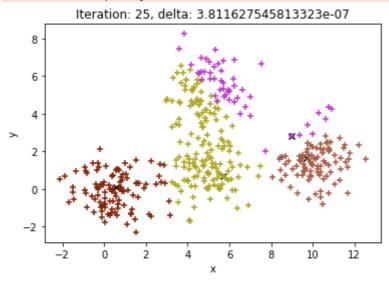
[9.70087764 1.6296057]

[0.56428284 0.09522426]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.7008647 1.62960993]

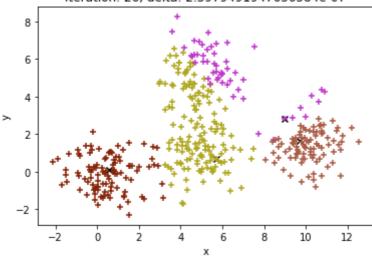
[0.56428212 0.09522355]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 26, delta: 2.3979491947656584e-07



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70085656 1.62961258]

[0.56428167 0.0952231]]

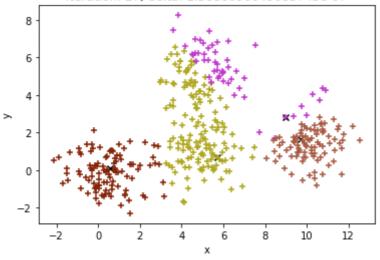
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 27, delta: 1.5086096649685743e-07

ou intend to specify the same RGB or RGBA value for all points.



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

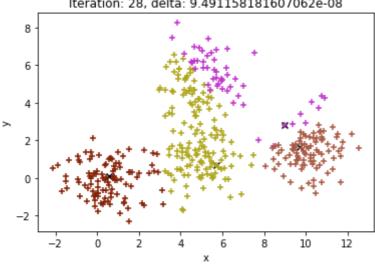
[9.70085143 1.62961425]

[0.56428138 0.09522282]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.



Iteration: 28, delta: 9.491158181607062e-08

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

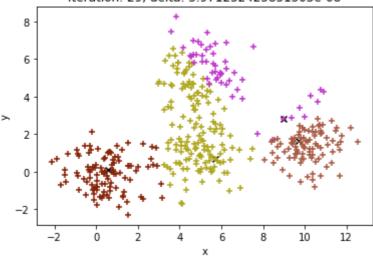
[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70084821 1.6296153]

[0.56428121 0.09522264]]

Iteration: 29, delta: 5.971252425851503e-08



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70084618 1.62961597]

[0.56428109 0.09522253]]

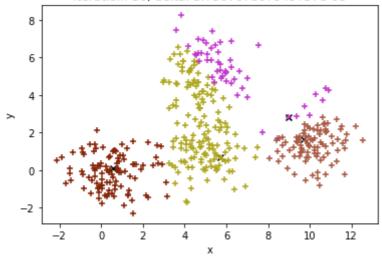
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y

ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid
ed as value-mapping will have precedence in case its length matches with *x* & *y*.

Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 30, delta: 3.756767807949757e-08



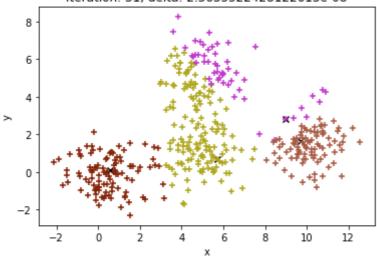
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

[[5.72035875 0.65426631] [8.98228383 2.8174128] [9.70084491 1.62961638] [0.56428102 0.09522246]]

Iteration: 31, delta: 2.3635522428122613e-08



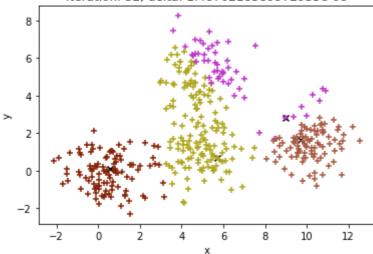
[[5.72035875 0.65426631] [8.98228383 2.8174128] [9.7008441 1.62961664] [0.56428098 0.09522242]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*.

ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid

Iteration: 32, delta: 1.4870218389972933e-08



c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

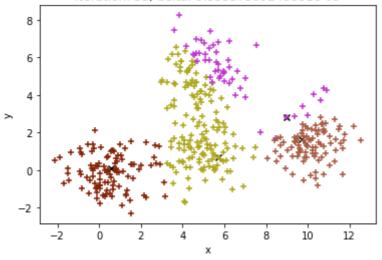
[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.7008436 1.62961681]

[0.56428095 0.09522239]]

Iteration: 33, delta: 9.35557509240681e-09



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

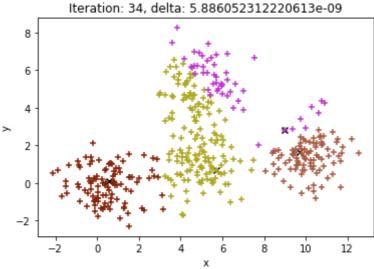
[9.70084328 1.62961691]

[0.56428093 0.09522237]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

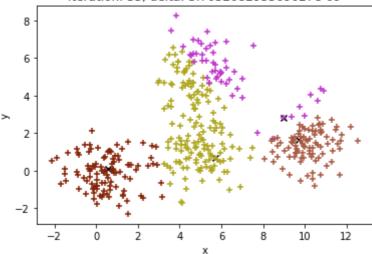
[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70084308 1.62961698]

[0.56428092 0.09522236]]

Iteration: 35, delta: 3.703208295569027e-09



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70084296 1.62961702]

[0.56428091 0.09522235]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

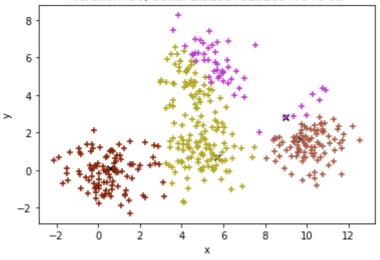
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 36, delta: 2.329875292694794e-09



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

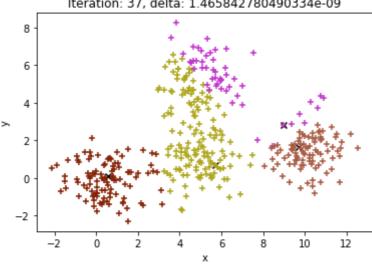
[9.70084288 1.62961704]

[0.56428091 0.09522235]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.



Iteration: 37, delta: 1.465842780490334e-09

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

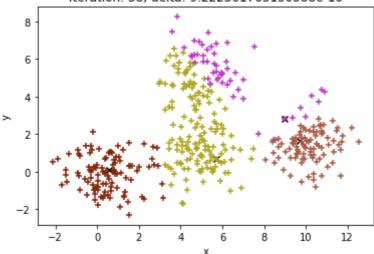
[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70084283 1.62961706]

[0.56428091 0.09522235]]

Iteration: 38, delta: 9.222361763150388e-10



c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

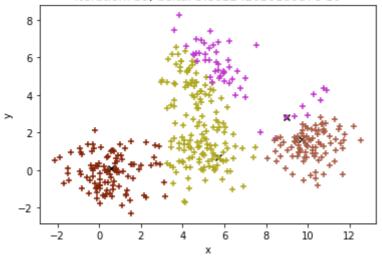
[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70084279 1.62961707]

[0.5642809 0.09522234]]

Iteration: 39, delta: 5.80224202018087e-10



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

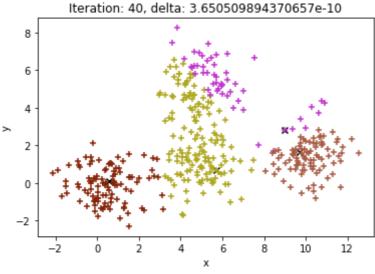
[9.70084278 1.62961708]

[0.5642809 0.09522234]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.



[[5.72035875 0.65426631] [8.98228383 2.8174128]

[9.70084276 1.62961708]

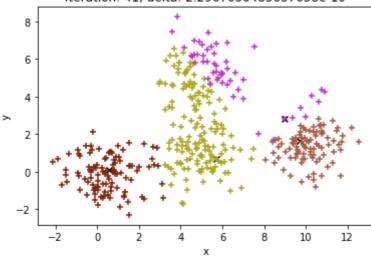
[0.5642809 0.09522234]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points. *c* argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

ou intend to specify the same RGB or RGBA value for all points.

Iteration: 41, delta: 2.2967050483657658e-10



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70084275 1.62961708]

[0.5642809 0.09522234]]

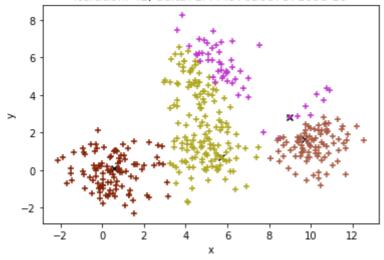
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid

ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 42, delta: 1.444976360787109e-10



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

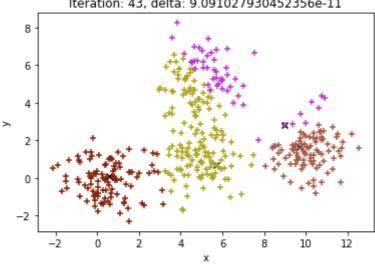
[9.70084275 1.62961708]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.



Iteration: 43, delta: 9.091027930452356e-11

[[5.72035875 0.65426631] [8.98228383 2.8174128]

[9.70084275 1.62961708]

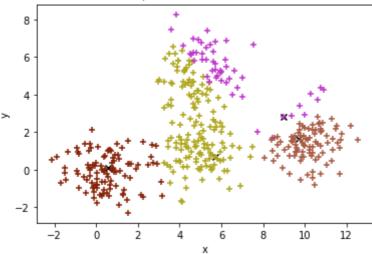
[0.5642809 0.09522234]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 44, delta: 5.719924534020038e-11



[[5.72035875 0.65426631] [8.98228383 2.8174128]

[9.70084274 1.62961709]

[0.5642809 0.09522234]]

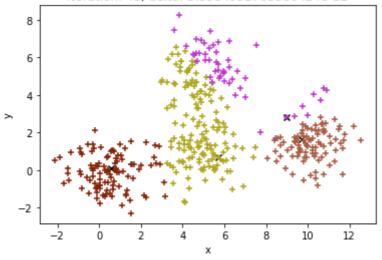
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 45, delta: 3.5984992763360424e-11



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

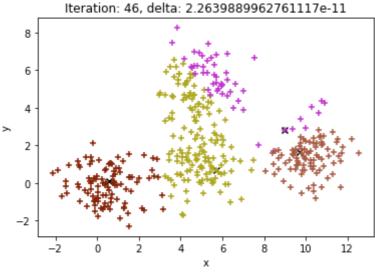
[9.70084274 1.62961709]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70084274 1.62961709]

[0.5642809 0.09522234]]

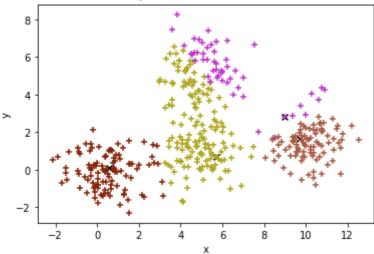
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid as value mapping will have precedence in case its length matches with *x* * * *v*

ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 47, delta: 1.4244827539755534e-11



[8.98228383 2.8174128]

[9.70084274 1.62961709]

[0.5642809 0.09522234]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

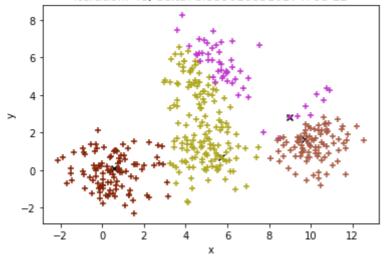
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid

ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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Iteration: 48, delta: 8.959610831027476e-12



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

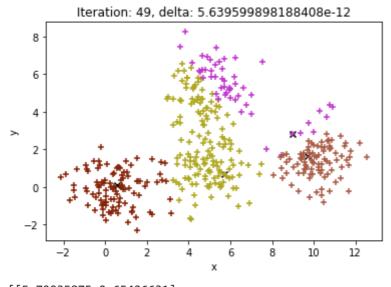
[9.70084274 1.62961709]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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[[5.72035875 0.65426631] [8.98228383 2.8174128]

[9.70084274 1.62961709]

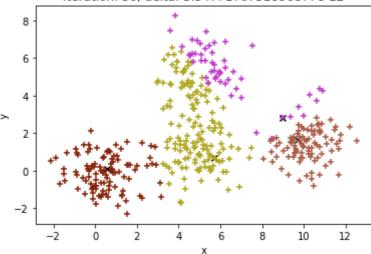
[0.5642809 0.09522234]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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Iteration: 50, delta: 3.5477176751896877e-12

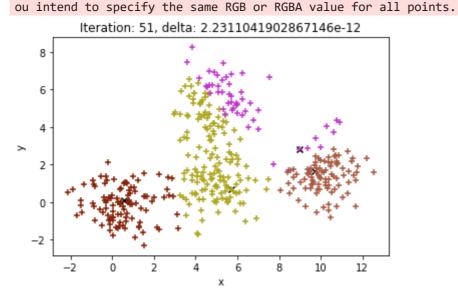


[8.98228383 2.8174128]

[9.70084274 1.62961709]

[0.5642809 0.09522234]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
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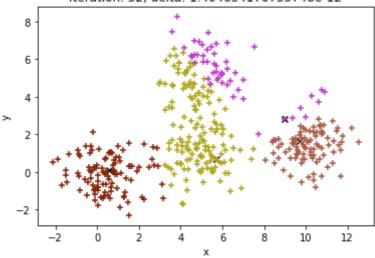
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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[[5.72035875 0.65426631] [8.98228383 2.8174128] [9.70084274 1.62961709] [0.5642809 0.09522234]]





c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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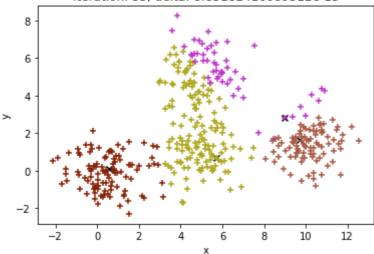
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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[[5.72035875 0.65426631] [8.98228383 2.8174128]

[9.70084274 1.62961709]

Iteration: 53, delta: 8.83182416089312e-13



[8.98228383 2.8174128]

[9.70084274 1.62961709]

[0.5642809 0.09522234]]

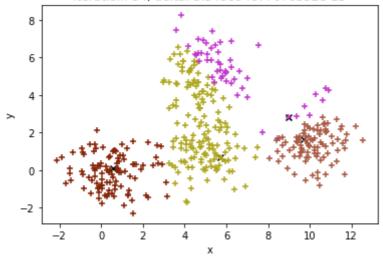
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
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c argument looks like a single numeric RGB on RGBA sequence, which should be avoid

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Iteration: 54, delta: 5.548894677076532e-13



c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

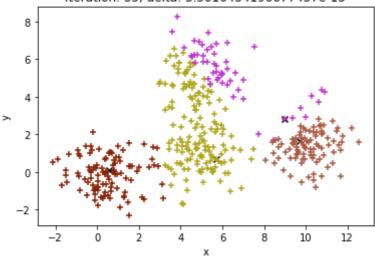
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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[[5.72035875 0.65426631] [8.98228383 2.8174128] [9.70084274 1.62961709] [0.5642809 0.09522234]]

Iteration: 55, delta: 3.5016434196677437e-13



c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

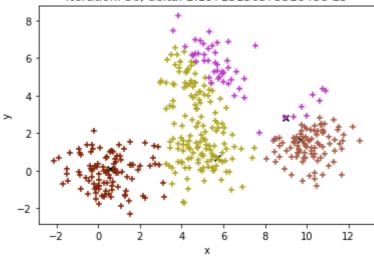
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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[[5.72035875 0.65426631] [8.98228383 2.8174128] [9.70084274 1.62961709]

Iteration: 56, delta: 2.1971313657331848e-13



[[5.72035875 0.65426631] [8.98228383 2.8174128]

[9.70084274 1.62961709]

[0.5642809 0.09522234]]

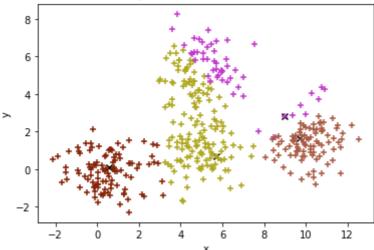
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
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[[5.72035875 0.65426631]

[8.98228383 2.8174128]

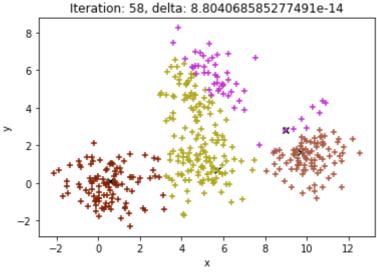
[9.70084274 1.62961709]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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[[5.72035875 0.65426631] [8.98228383 2.8174128]

[9.70084274 1.62961709]

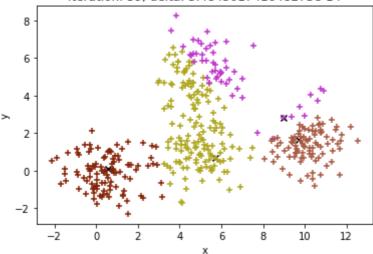
[0.5642809 0.09522234]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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Iteration: 59, delta: 5.484501741648273e-14



[8.98228383 2.8174128]

[9.70084274 1.62961709]

[0.5642809 0.09522234]]

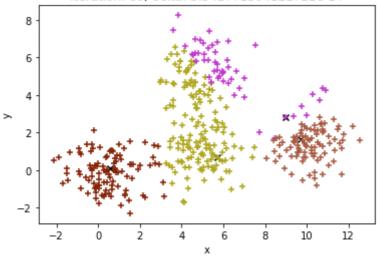
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.
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c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 60, delta: 3.341771304121721e-14



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

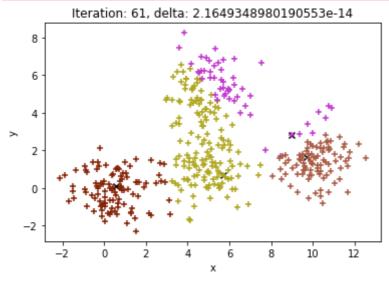
[9.70084274 1.62961709]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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[[5.72035875 0.65426631] [8.98228383 2.8174128]

[9.70084274 1.62961709]

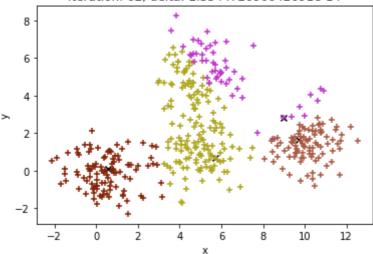
[0.5642809 0.09522234]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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Iteration: 62, delta: 1.354472090042691e-14



[8.98228383 2.8174128]

[9.70084274 1.62961709]

[0.5642809 0.09522234]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

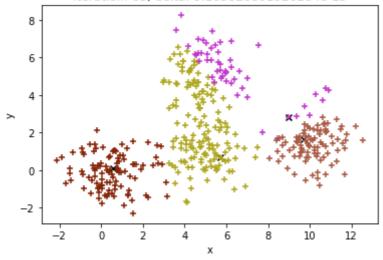
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid

ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

Iteration: 63, delta: 9.103828801926284e-15



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

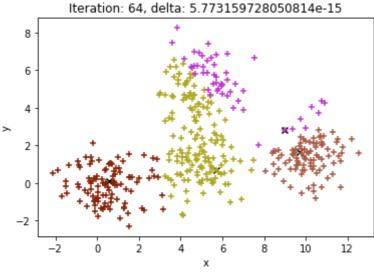
[9.70084274 1.62961709]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70084274 1.62961709]

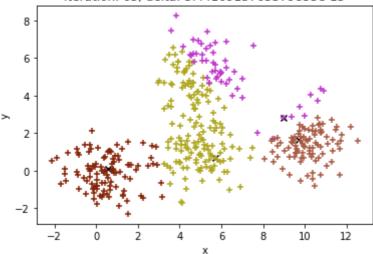
[0.5642809 0.09522234]]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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Iteration: 65, delta: 3.4416913763379853e-15



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

[9.70084274 1.62961709]

[0.5642809 0.09522234]]

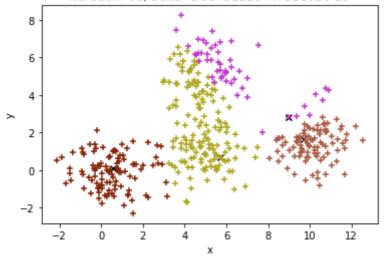
c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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Iteration: 66, delta: 1.5543122344752192e-15



[[5.72035875 0.65426631]

[8.98228383 2.8174128]

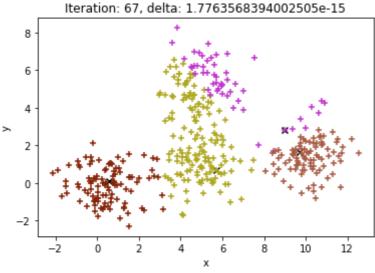
[9.70084274 1.62961709]

c argument looks like a single numeric RGB or RGBA sequence, which should be avoid ed as value-mapping will have precedence in case its length matches with *x* & *y*. Please use the *color* keyword-argument or provide a 2D array with a single row if y ou intend to specify the same RGB or RGBA value for all points.

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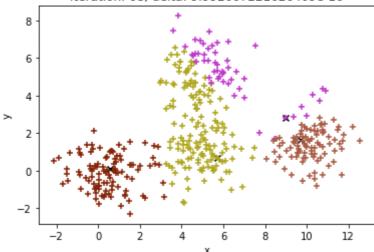
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^{[[5.72035875 0.65426631]} [8.98228383 2.8174128]

^[9.70084274 1.62961709]

^[0.5642809 0.09522234]]

Iteration: 68, delta: 9.992007221626409e-16



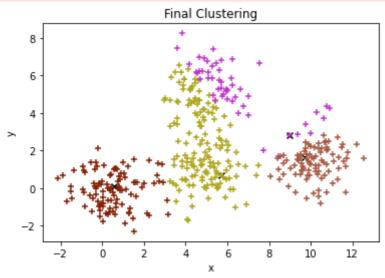
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Hierarchical Clustering

Hierarchical clustering is an unsupervised clustering technique which groups together the unlabelled data of similar characteristics.

There are two types of hierarchical clustering:

Agglomerative Clustering

Divisive Clustering

Agglomerative Clustering:

In this type of hierarchical clustering all data set are considered as indivisual cluster and at every iterations clusters with similar characteristics are merged to give bigger clusters. This is repeated until one single cluster is reached. It is also called bottem-top approach.

Agglomerative Clustering:

Lets start with some dummy example:

X=
$$[x_1,x_2,\ldots,x_5]$$
, with $x_1=egin{bmatrix}1\\1\end{bmatrix}$, $x_2=egin{bmatrix}2\\1\end{bmatrix}$, $x_3=egin{bmatrix}5\\4\end{bmatrix}$, $x_4=egin{bmatrix}6\\5\end{bmatrix}$, $x_5=egin{bmatrix}6.5\\6\end{bmatrix}$

Steps to perform Agglomerative Clustering:

- 1. Compute Distance matrix (N imes N matrix, where N number of vectors present in the dataset): $D(a,b)=||x_a-x_b||_2$
- 2. Replace the diagonal elements with inf and find the index of the minimum element present in the distance matrix (suppose we get the location (l,k)).
- 3. Replace $x_{min(l,k)}=.5 imes[x_l+x_m]$ and delete $x_{max(l,m)}$ vector from X(i.e now (N=N-1)),

repeat from step 1 again untill all the vectors combined to a single cluster.

```
In [ ]: def Euclidian_Dist(x,y):
          return np.linalg.norm(x-y) ## write your code here
        def Dist_mat(X):
         ## write your code here
          mat_dist = np.zeros((X.shape[1],X.shape[1]))
          for i in range(X.shape[1]):
            for j in range(X.shape[1]):
              mat dist[i][j] = round(Euclidian Dist(X[:,i],X[:,j]),1)
          return mat dist
        def combine(X, cluster_comb):
          ## write your code here
          md = Dist_mat(X)
          max_val = np.inf
          md[(md==0)]=max_val
          print(md)
          arr min = np.min(md, axis=0)
          #print(arr min)
          index = np.argwhere(md == np.min(arr_min))
          index = index[0,:]
          #print(index)
          print('Vector of X to be combined: ', index+1)
          cluster comb.append(index+1)
          #print(index[1])
          ret x = X
```

```
ret_x = np.delete(ret_x,max(index),1)
ret_x[:,min(index)] = 0.5*(X[:,index[0]]+X[:,index[1]])
x_new = ret_x
return x_new
```

```
In [ ]: X=np.array([[1,1],[2,1],[5,4],[6,5],[6.5,6]])
        X=X.transpose()
        ## write your code here
        cluster_comb=[]
        x_1 = X
        print(X)
        while x_1.shape!=(2,1):
          x_1 = combine(x_1, cluster\_comb)
          print('\nMean of clusters after every iteration: \n\n',x_1)
        print('\ncluster combination order: \n\n',cluster_comb)
        print("Final X:", X)
        ## validate from inbuilt Dendogram
        import plotly.figure_factory as ff
        lab=np.linspace(1,X.shape[1],X.shape[1])
        fig = ff.create_dendrogram(X.T, labels=lab)
        fig.update_layout(width=800, height=300)
        fig.show()
```

```
[[1. 2. 5. 6. 6.5]
 [1. 1. 4. 5. 6.]]
[[inf 1. 5. 6.4 7.4]
 [1. inf 4.2 5.7 6.7]
 [5. 4.2 inf 1.4 2.5]
 [6.4 5.7 1.4 inf 1.1]
[7.4 6.7 2.5 1.1 inf]]
Vector of X to be combined: [1 2]
Mean of clusters after every iteration:
[[1.5 5. 6. 6.5]
 [1. 4. 5. 6.]]
[[inf 4.6 6. 7.1]
 [4.6 inf 1.4 2.5]
 [6. 1.4 inf 1.1]
 [7.1 2.5 1.1 inf]]
Vector of X to be combined: [3 4]
Mean of clusters after every iteration:
 [[1.5 5.
           6.25]
 [1.
     4.
           5.5 ]]
[[inf 4.6 6.5]
[4.6 inf 2.]
[6.5 2. inf]]
Vector of X to be combined: [2 3]
Mean of clusters after every iteration:
[[1.5
       5.625]
[1.
       4.75 ]]
[[inf 5.6]
 [5.6 inf]]
Vector of X to be combined: [1 2]
Mean of clusters after every iteration:
 [[3.5625]
 [2.875]]
cluster combination order:
[array([1, 2], dtype=int64), array([3, 4], dtype=int64), array([2, 3], dtype=int6
4), array([1, 2], dtype=int64)]
Final X: [[1. 2. 5. 6. 6.5]
[1. 1. 4. 5. 6.]]
```

Clustering Algorithms on MNIST Digit dataset

Perform Kmeans and gmm clustering on MNIST dataset

1. Load MNIST data from the given images and labels

2. Consider any 2 classes

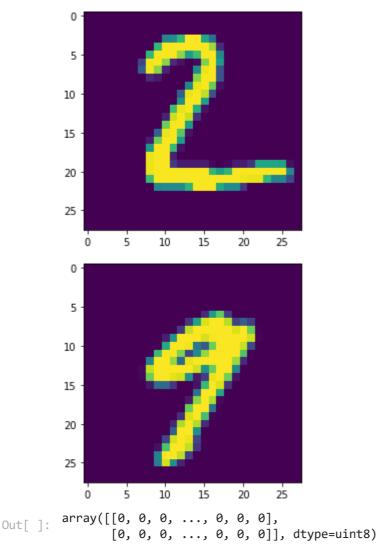
```
In [ ]: !pip install idx2numpy
```

Requirement already satisfied: idx2numpy in c:\users\shashank\.env\.mldev\lib\site-p ackages (1.2.3)

Requirement already satisfied: numpy in c:\users\shashank\.env\.mldev\lib\site-packages (from idx2numpy) (1.22.4)

Requirement already satisfied: six in c:\users\shashank\.env\.mldev\lib\site-package s (from idx2numpy) (1.16.0)

```
In [ ]: import idx2numpy
        from keras.utils import np_utils
        img_path = "t10k-images-idx3-ubyte" ## write your code here
        label_path = "t10k-labels-idx1-ubyte" ## write your code here
        Images = idx2numpy.convert_from_file(img_path)
        labels = idx2numpy.convert_from_file(label_path)
        ## write your code here
        11 = labels==2
        12 = labels == 9
        X = Images[np.logical_or(l1, l2)]
        K = 2
        print(X.shape)
        print(labels.shape)
        print(X[0])
        plt.imshow(X[0])
        plt.show()
        plt.imshow(X[5])
        plt.show()
        X = X.reshape((X.shape[0], 28*28))
        mean_vectors = X[:K, :]
        mean_vectors
```



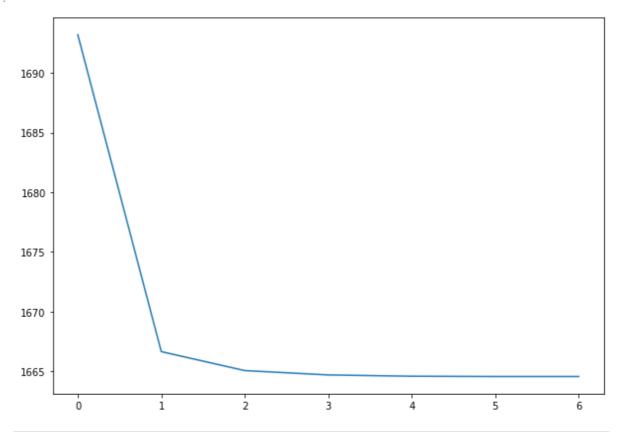
Use the K-means clustering algorithm from the last lab to form the clusters

```
In [ ]:
        ## Write your code here
        tol = 1e-5
        delta = float('inf')
        prev_error = float('inf')
        errors = []
        iter = 0
        while delta>tol:
            assigned = [[] for i in range(K)]
            for i in range(X.shape[0]):
                min_index = np.argmin(np.linalg.norm(X[i] - mean_vectors, axis=1))
                assigned[min_index].append(i)
            assigned clusters = []
            for i in range(K): assigned_clusters.append(X[assigned[i]])
            mean_vectors = np.array([cluster.mean(axis=0) for cluster in assigned_clusters])
            sum error = 0
            for i in range(K): sum_error += np.sum(np.linalg.norm(assigned_clusters[i]-mean_
            sum error
            delta = abs(prev_error - sum_error/X.shape[0])
            prev_error = sum_error/X.shape[0]
```

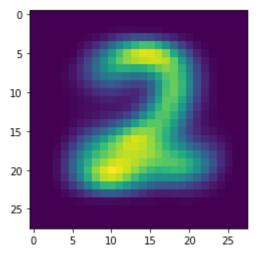
```
prev_error
errors.append(prev_error)
iter+=1

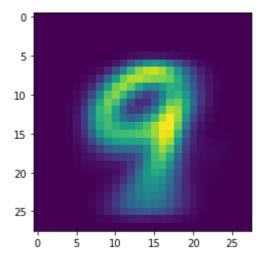
plt.figure(figsize=(10, 7))
plt.plot(range(len(errors)), errors)
```

Out[]: [<matplotlib.lines.Line2D at 0x1b2d5de8400>]



```
In []: gen1 = mean_vectors[0].reshape((28, 28))
    gen2 = mean_vectors[1].reshape((28, 28))
    plt.imshow(gen1)
    plt.show()
    plt.imshow(gen2)
    plt.show()
```

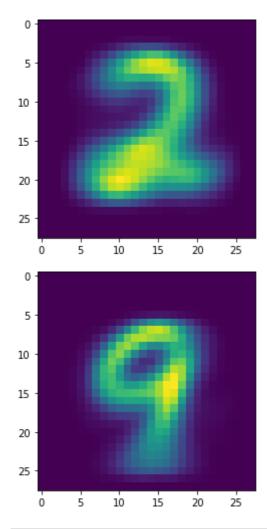




Use the GMM clustering algorithm from the last lab to form the clusters

```
In [ ]: | ## Write your code here
        def initialization(data,K):
          dim = data.shape[1]
          indices = np.random.randint(0, len(data)-1, K)
          mean_vectors = data[indices, :]
          cov_matrix = np.zeros((dim, dim, K))
          for k in range(K):
            cov_matrix[:, :, k] = np.identity(dim)*np.max(data)
          w = np.array([1/K for i in range(K)])
          theta = [mean_vectors, cov_matrix, w]
          return theta
        from scipy.stats import multivariate_normal
        def E_Step_GMM(data,K,theta):
            # write your code here
            mean_vectors = theta[0]
            cov_matrix = theta[1]
            w = theta[2]
             responsibility = np.zeros((len(data), K))
            for i in range(len(data)):
                for k in range(K):
                     p_ik = multivariate_normal.pdf(data[i], mean=mean_vectors[k], cov=cov_ma
                     numerator_ik = w[k] * p_ik
                     denominator ik = 0
                     for k2 in range(K):
                         p_ik_temp = multivariate_normal.pdf(data[i], mean=mean_vectors[k2],
                         numerator_ik_temp = w[k2] * p_ik_temp
                         denominator_ik += numerator_ik_temp
                     responsibility[i][k] = numerator_ik/(denominator_ik+1e-20)
             return responsibility
        def M Step GMM(data,responsibility):
            N = len(data)
            dim = data.shape[1]
            K = responsibility.shape[1]
```

```
N_k = np.sum(responsibility, axis=0)
             N_k = np.reshape(N_k, (K, 1))
             w = N_k/N
             num_k = np.transpose(responsibility) @ data
            mean_vectors = np.divide(num_k, N_k)
             cov_matrix = np.zeros((dim, dim, K))
            for k in range(K):
                for i in range(N):
                     diff = np.reshape((data[i]-mean_vectors[k]), (dim, 1))
                     cov_matrix[:, :, k] += responsibility[i][k] * ( diff @ np.transpose(diff
                 cov_matrix[:, :, k]/=N_k[k]
            theta = [mean_vectors, cov_matrix, w]
             log_likelihood = 0
             for k in range(K):
                inner = 0
                for i in range(N):
                     inner += w[k] * multivariate_normal.pdf(data[i], mean=mean_vectors[k], c
                log_likelihood += np.log(inner)
             return theta, log_likelihood
        log_l=[]
        Itr=50
        eps=10**(-14) # for threshold
        print(X.shape, K)
        theta=initialization(X,K)
        for n in range(Itr):
             responsibility=E_Step_GMM(X,K,theta)
             cluster_label=np.argmax(responsibility,axis=1) #Label Points
             theta,log_likhd=M_Step_GMM(data,responsibility)
             log_l.append(log_likhd)
            Cents=theta[0]
             if n>2:
                if abs(log_l[n]-log_l[n-1])<eps:</pre>
                     break
        (2041, 784) 2
In [ ]: gen1 = theta[0][0].reshape((28, 28))
        gen2 = theta[0][1].reshape((28, 28))
        plt.imshow(gen1)
        plt.show()
        plt.imshow(gen2)
        plt.show()
```



In []: