#**LAB 6 : Regression**

Regression is generally used for curve fitting task. Here we will demonstrate regression task for the following :

1. Fitting of a Line (One Variable and Two Variables)
2. Fitting of a Plane
3. Fitting of M-dimensional hyperplane
4. Practical Example of Regression task

```
import numpy as np
import matplotlib.pyplot as plt
```

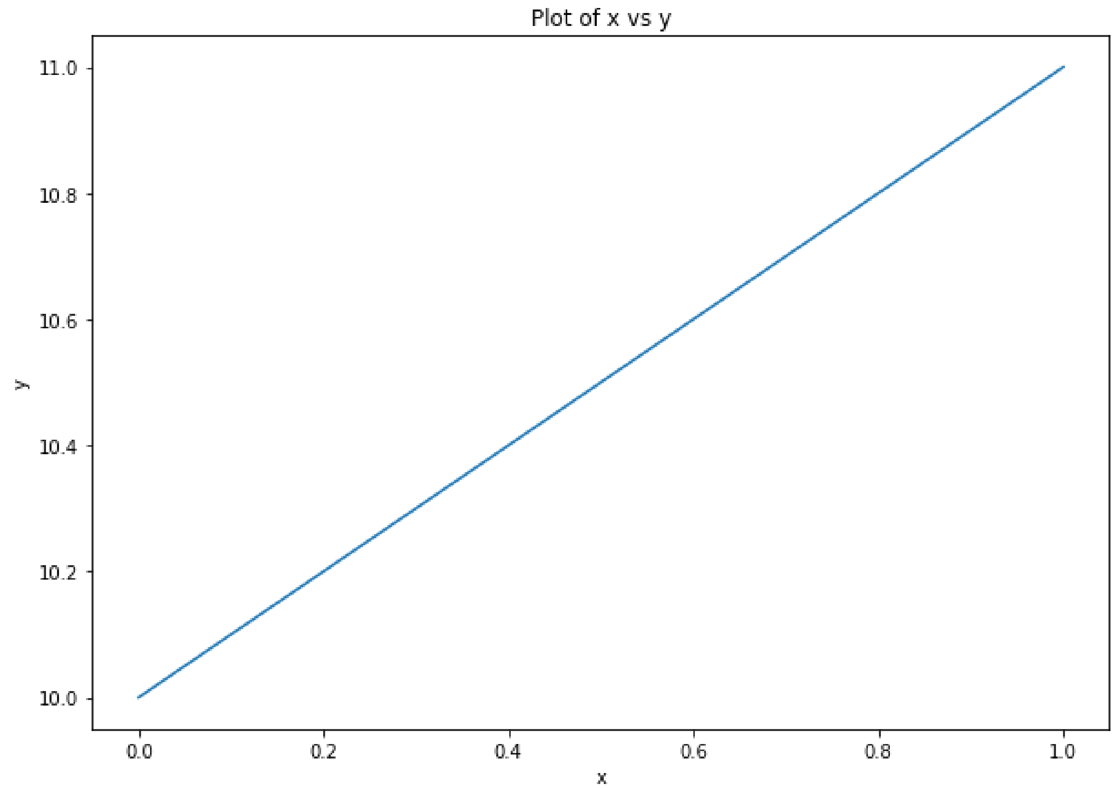
#Fitting of a Line (One Variable)

Generation of line data ($y = w_1 x + w_0$)

1. Generate x, 1000 points from 0-1
2. Take $w_0 = 10$ and $w_1 = 1$ and generate y
3. Plot (x, y)

```
## Write your code here
x = np.linspace(start=0, stop=1, num=1000)
w = np.array([10, 1])
y = np.transpose(np.append(np.ones((1000)), x).reshape(2, 1000)) @ w

plt.figure(figsize=(10, 7))
plt.plot(x, y)
plt.xlabel("x")
plt.ylabel("y")
plt.title("Plot of x vs y")
```

```
Text(0.5, 1.0, 'Plot of x vs y')
```



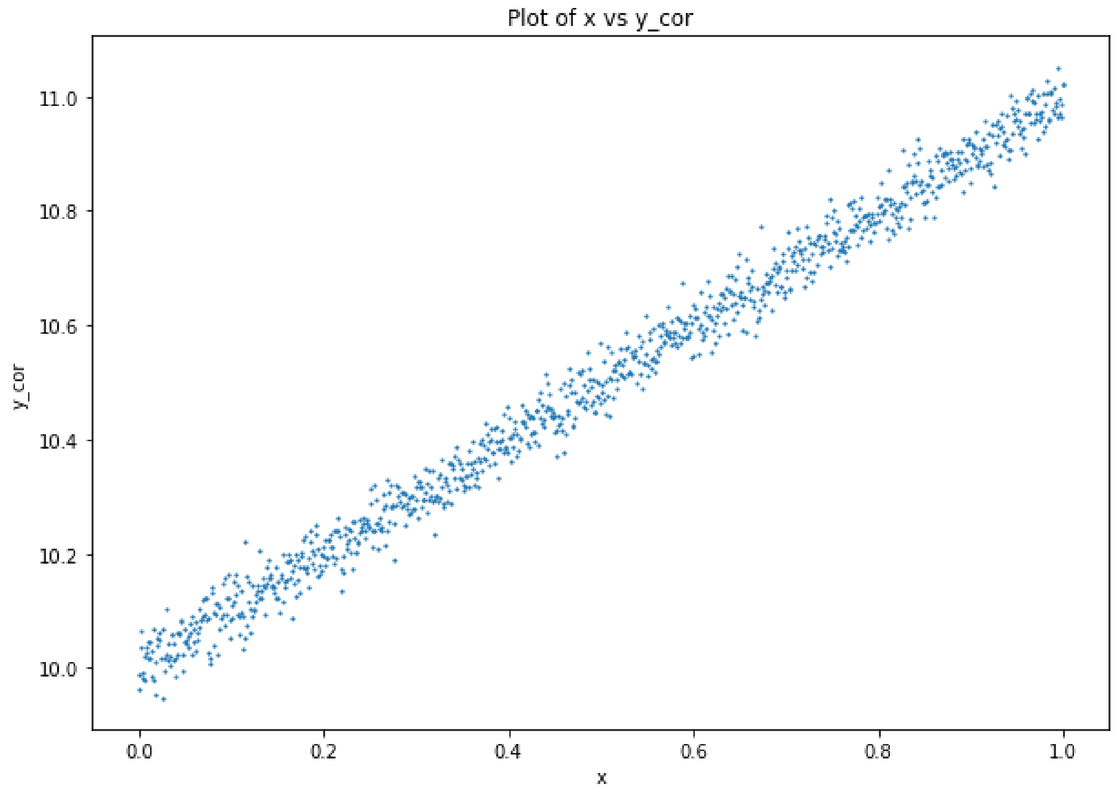
Corruption of data using uniformly sampled random noise

1. Generate random numbers uniformly from (0-1) with same size as y
2. Corrupt y and generate y_{cor} by adding the generated random samples with a weight of 0.1.
3. Plot (x, y_{cor}) (use scatter plot)

```
## Write your code here
y_cor = np.add(y, np.random.normal(0, 0.03, size=(1000,)))
print(y_cor.shape)

plt.figure(figsize=(10, 7))
plt.scatter(x, y_cor, s=1.5)
plt.xlabel("x")
plt.ylabel("y_cor")
plt.title("Plot of x vs y_cor")
```

```
(1000,)
```

```
Text(0.5, 1.0, 'Plot of x vs y_cor')
```



Heuristically predicting the curve (Generating the Error Curve)

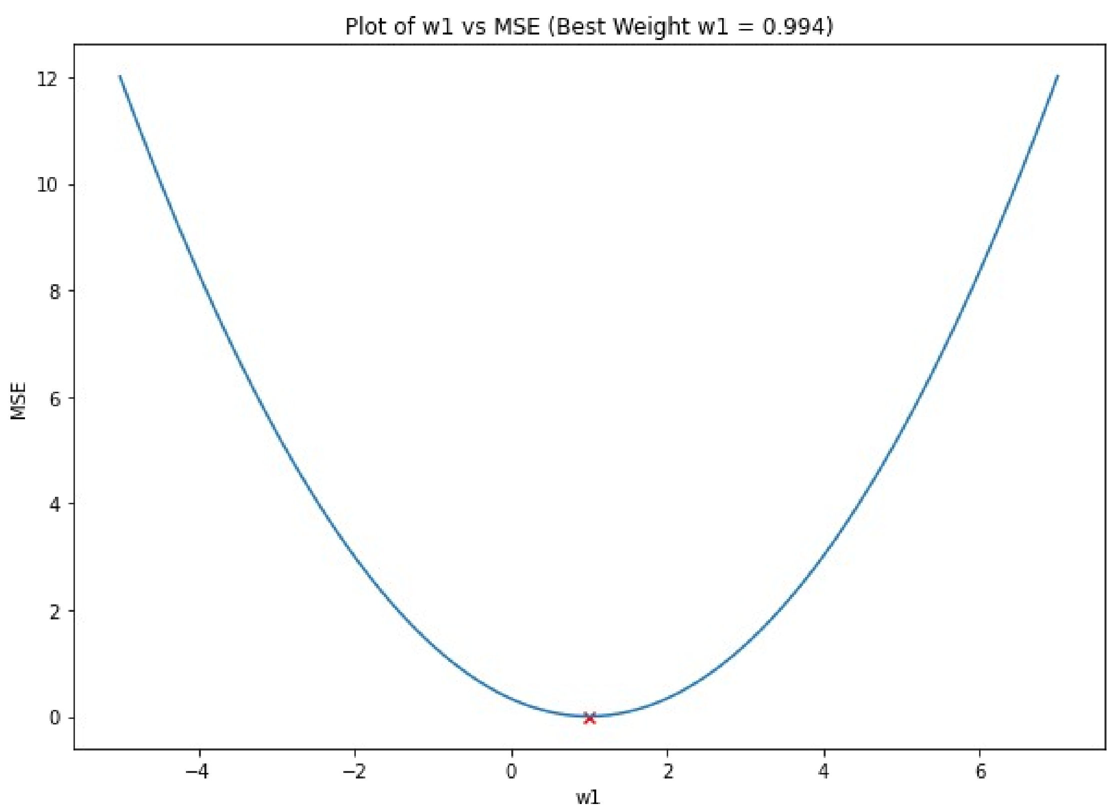
1. Keep $w_0 = 10$ as constant and find w_1
2. Create a search space from -5 to 7 for w_1, by generating 1000 numbers between that
3. Find y_{pred} using each value of w_1
4. The y_{pred} that provide least norm error with y, will be decided as best y_{pred}

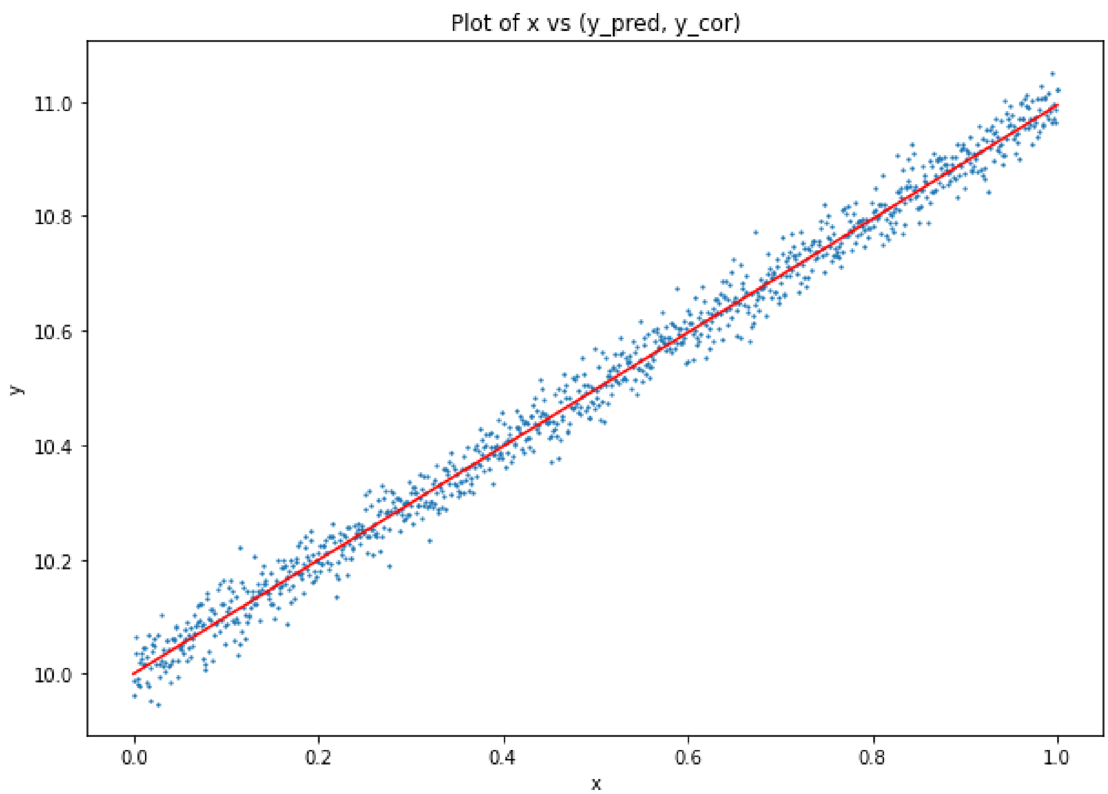
$$error = \frac{1}{m} \sum_{i=1}^{M} \left(y_i - y_{pred_i} \right)^2$$

5. Plot error vs search_$w1$
6. First plot the scatter plot (x, y_{cor}), over that plot $(x, y_{bestpred})$

```
## Write your code here
w0_guess = 10
w1_space = np.linspace(-5, 7, 1000)
w_best = None
best_error = float('inf')

errors = []

for w1 in w1_space:
    w = np.array([w0_guess, w1])
    y_pred = np.append(np.ones((1000)), x).reshape(2, 1000).T @ w
    error = np.sum(np.subtract(y_cor, y_pred)**2)/1000
    errors.append(error)
```

```
    if error<best_error:
        w_best = w
        best_error = error

errors = np.array(errors)

plt.figure(figsize=(10, 7))
plt.plot(w1_space, errors)
plt.scatter(w_best[1], best_error, marker='x', c='red')
plt.xlabel("w1")
plt.ylabel("MSE")
plt.title(f'Plot of w1 vs MSE (Best Weight w1 = {round(w_best[1],
4)})')
plt.show()

y_pred = np.transpose(np.append(np.ones((1000)), x).reshape(2, 1000))
@ w_best

plt.figure(figsize=(10, 7))
plt.plot(x, y_pred, c='red')
plt.scatter(x, y_cor, s=1.5)
plt.xlabel("x")
plt.ylabel("y")
plt.title("Plot of x vs (y_pred, y_cor)")
plt.show()
```





Using Gradient Descent to predict the curve

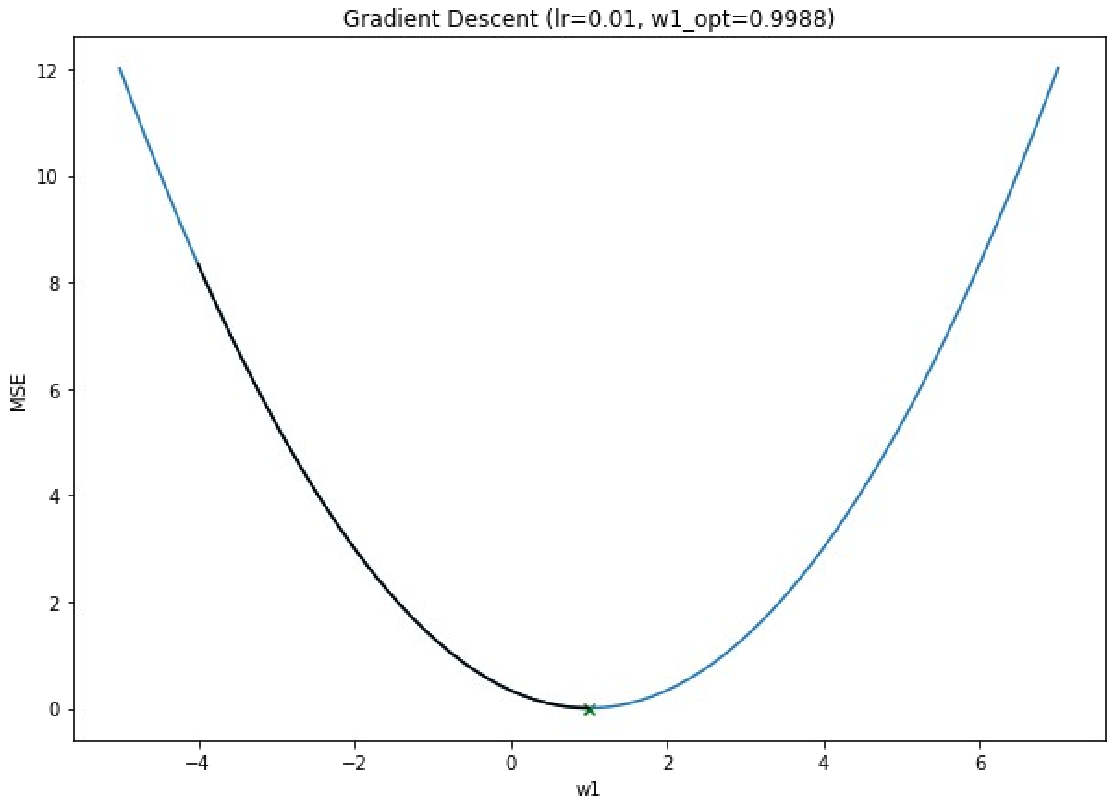
1. $Error = \frac{1}{m}\sum_{i=1}^{M}\left(y_i - y_{pred_i}\right)^2 = \frac{1}{m}\sum_{i=1}^{M}\left(y_i - \left(w_0 + w_1 x_i\right)\right)^2$

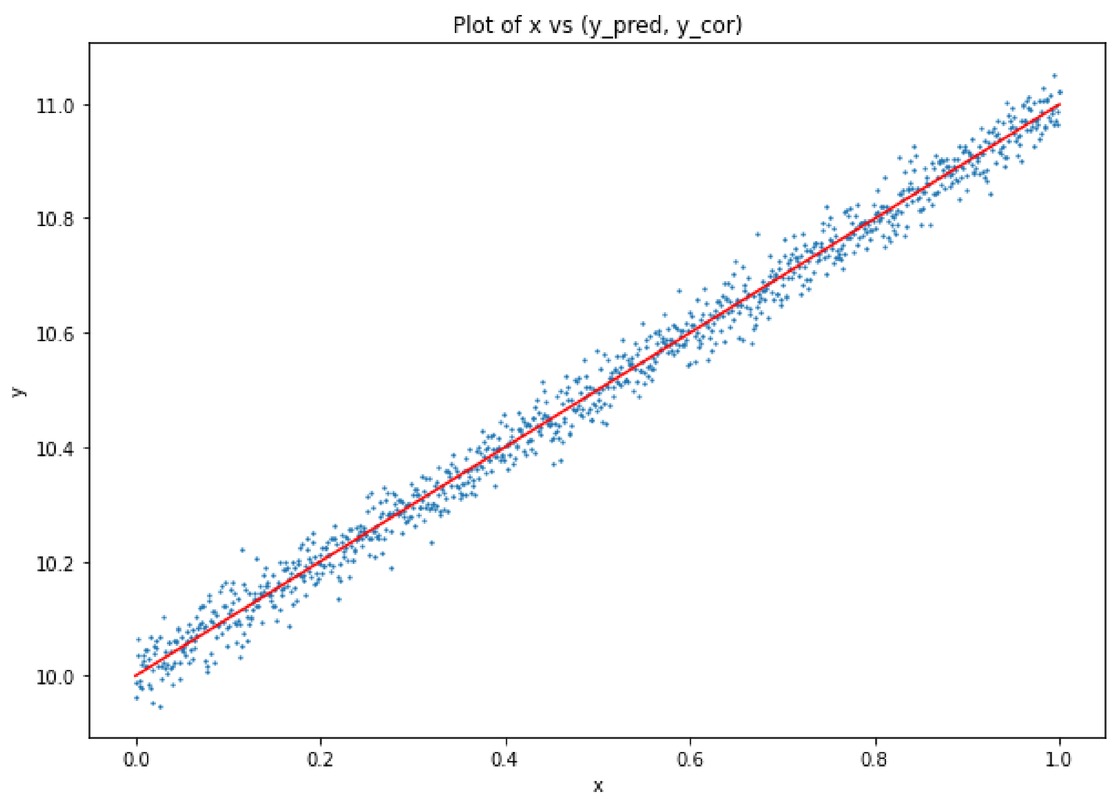
2. $\nabla Error¿_{w1} = \frac{-2}{M}\sum_{i=1}^{M}\left(y_i - y_{pred_i}\right) \times x_i$

3. $w_1|\square_{new} = w_1|_{old} - \lambda \nabla Error|\square_{w1} = w_1|_{old} + \frac{2\lambda}{M}\sum_{i=1}^{M}\left(y_i - y_{pred_i}\right) \times x_i$

```
## Write your code here
tol = 1e-6
max_iter = int(1e5)
_lambda = 0.01

w0_guess = 10
w1_init = -4

errors2 = []
w1_vals = []

X = np.transpose(np.append(np.ones((1000)), x).reshape(2, 1000))

for i in range(max_iter):
```

```
    w = np.array([w0_guess, w1_init])
    y_pred = X @ w

    error = np.sum(np.subtract(y_cor, y_pred)**2)/1000
    errors2.append(error)
    w1_vals.append(w1_init)

    grad = 2* np.dot(x, (y_pred - y_cor))/1000
    w1_init = w1_init - _lambda * grad

    if abs(grad)<tol: break

plt.figure(figsize=(10, 7))
plt.plot(w1_space, errors)
plt.plot(w1_vals, errors2, c='black')
plt.scatter(w1_init, error, c='green', marker='x')
plt.xlabel("w1")
plt.ylabel("MSE")
plt.title(f"Gradient Descent (lr={_lambda}, w1_opt={round(w1_init,
4)})")
plt.show()

y_pred = X @ w
plt.figure(figsize=(10, 7))
plt.plot(x, y_pred, c='red')
plt.scatter(x, y_cor, s=1.5)
plt.xlabel("x")
plt.ylabel("y")
plt.title("Plot of x vs (y_pred, y_cor)")
plt.show()
```





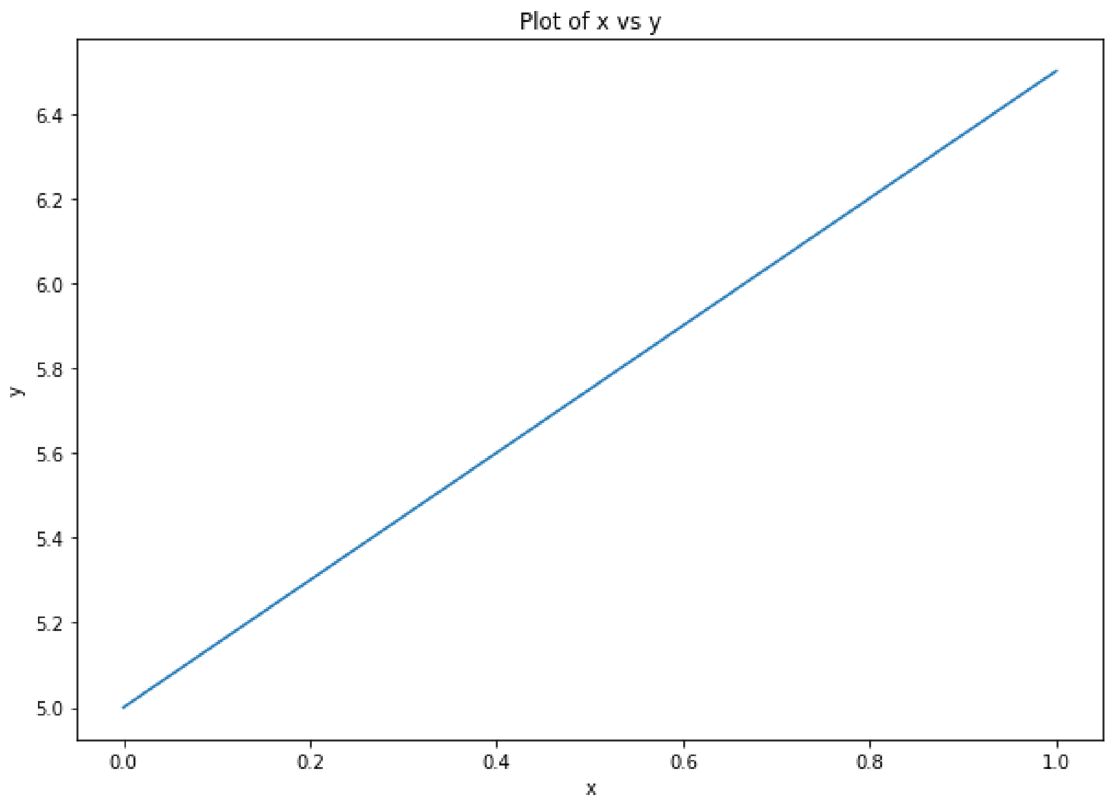
#Fitting of a Line (Two Variables)

Generation of Line Data ($y = w_1 x + w_0$)

1. Generate x, 1000 points from 0-1
2. Take $w_0 = 5$ and $w_1 = 1.5$ and generate y
3. Plot (x, y)

```
## Write your code here
x = np.linspace(start=0, stop=1, num=1000)
w = np.array([5, 1.5])
y = np.transpose(np.append(np.ones((1000)), x).reshape(2, 1000)) @ w

plt.figure(figsize=(10, 7))
plt.plot(x, y)
plt.xlabel("x")
plt.ylabel("y")
plt.title("Plot of x vs y")
```

```
Text(0.5, 1.0, 'Plot of x vs y')
```



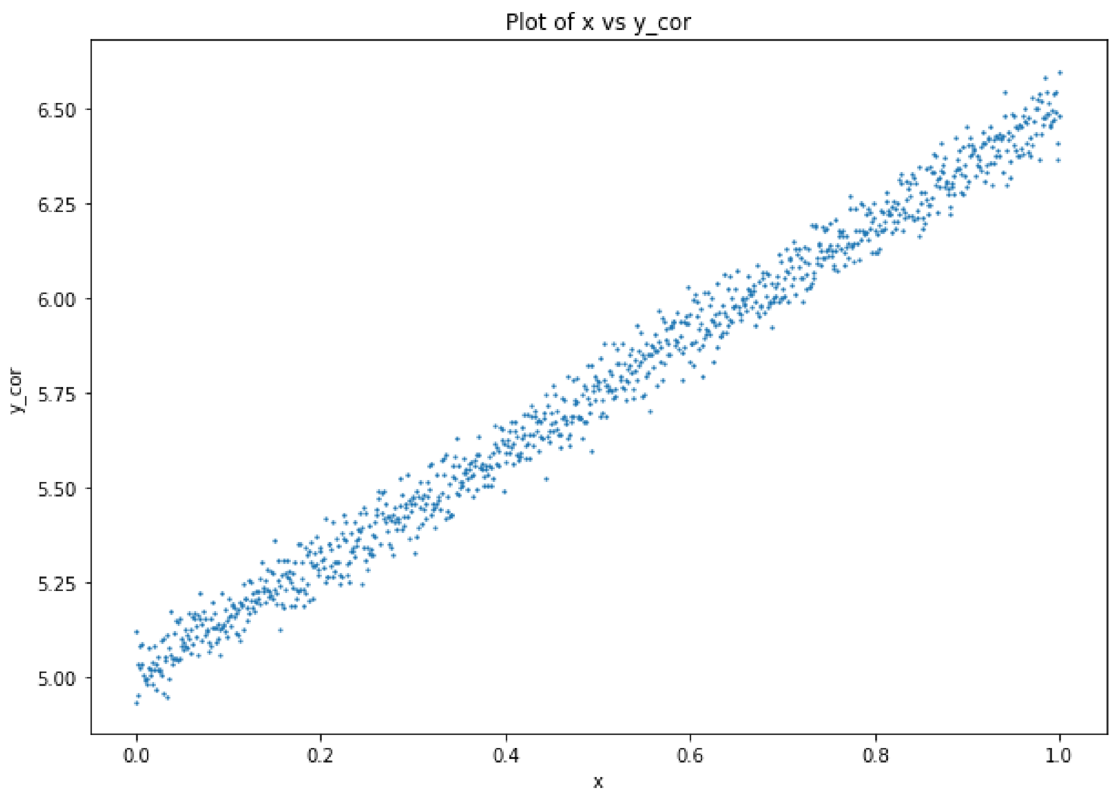
Corrupt the data using uniformly sampled random noise

1. Generate random numbers uniformly from (0-1) with same size as y
2. Corrupt y and generate y_{cor} by adding the generated random samples with a weight of 0.1
3. Plot (x, y_{cor}) (use scatter plot)

```
## Write your code here
y_cor = np.add(y, np.random.normal(0, 0.05, size=(1000,)))
print(y_cor.shape)

plt.figure(figsize=(10, 7))
plt.scatter(x, y_cor, s=1.5)
plt.xlabel("x")
plt.ylabel("y_cor")
plt.title("Plot of x vs y_cor")
```

```
(1000,)
```

```
Text(0.5, 1.0, 'Plot of x vs y_cor')
```



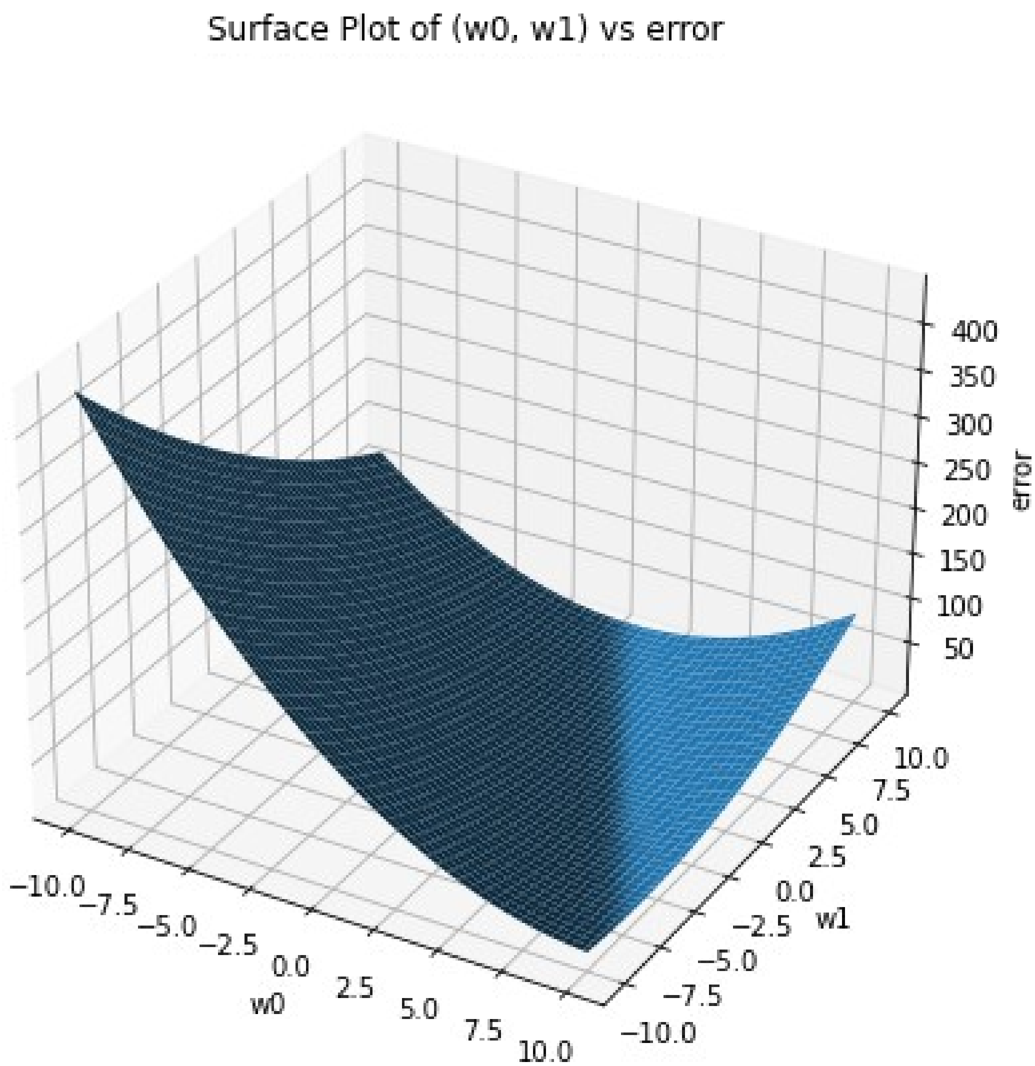
Plot the Error Surface

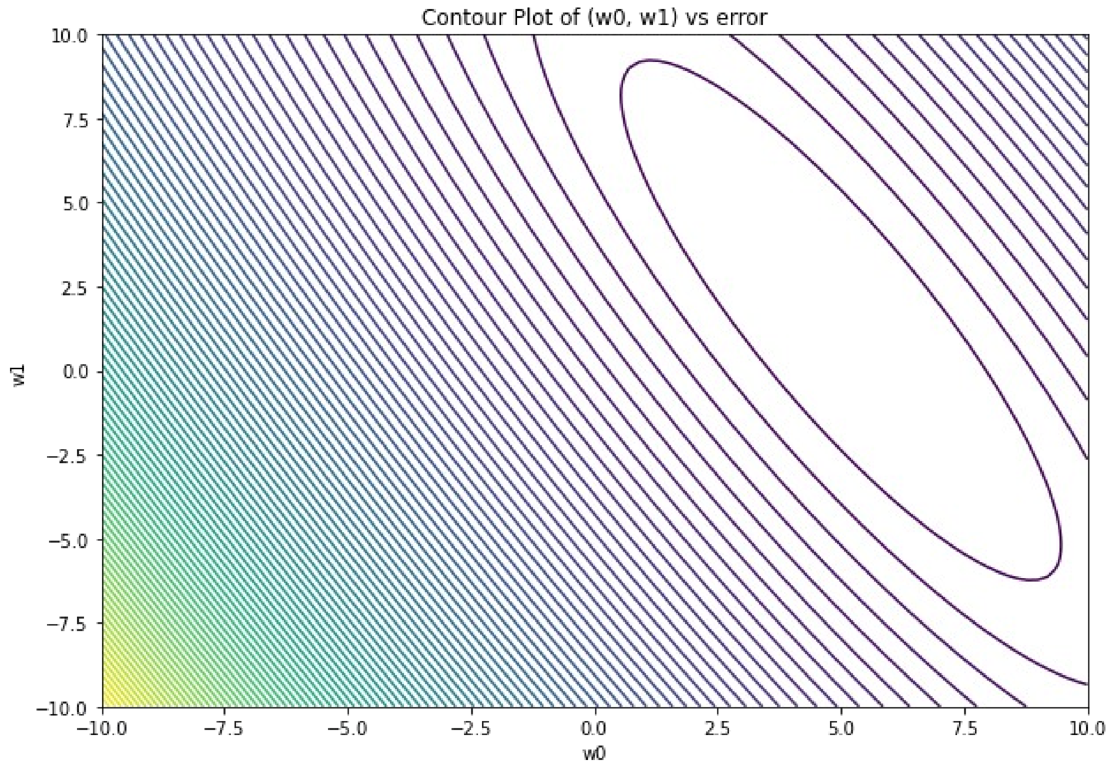
1. we have all the data points available in y_{cor}, now we have to fit a line with it. (i.e from y_{cor} we have to predict the true value of w_1 and w_0)
2. Take w_1 and w_0 from -10 to 10, to get the error surface

```
## Write your code here

w0 = np.linspace(-10, 10, 100)
w1 = np.linspace(-10, 10, 100)
X = np.append(np.ones((1000)), x).reshape(2, 1000)

def cost(w0, w1):
```

```
    w = np.array([w0, w1])
    y_pred = X.T @ w
    return np.sum(np.subtract(y_cor, y_pred)**2)/1000

W0, W1 = np.meshgrid(w0, w1)

W1.shape, W0.shape

errors = np.array([cost(w0_i, w1_i) for w0_i, w1_i in
zip(np.ravel(W0), np.ravel(W1))]).reshape(100, 100)

fig, ax = plt.subplots(subplot_kw={"projection": "3d"}, figsize=(10,
7))

ax.plot_surface(W0, W1, errors)

ax.set_title("Surface Plot of (w0, w1) vs error")
ax.set_xlabel('w0')
ax.set_ylabel('w1')
ax.set_zlabel('error')

fig, ax = plt.subplots(figsize=(10, 7))

ax.contour(W0, W1, errors, 100)

ax.set_title("Contour Plot of (w0, w1) vs error")
ax.set_xlabel('w0')
ax.set_ylabel('w1')
```

```
Text(0, 0.5, 'w1')
```


Surface Plot of (w0, w1) vs error
error
400
350
300
250
200
150
100
50
w0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
w1
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0

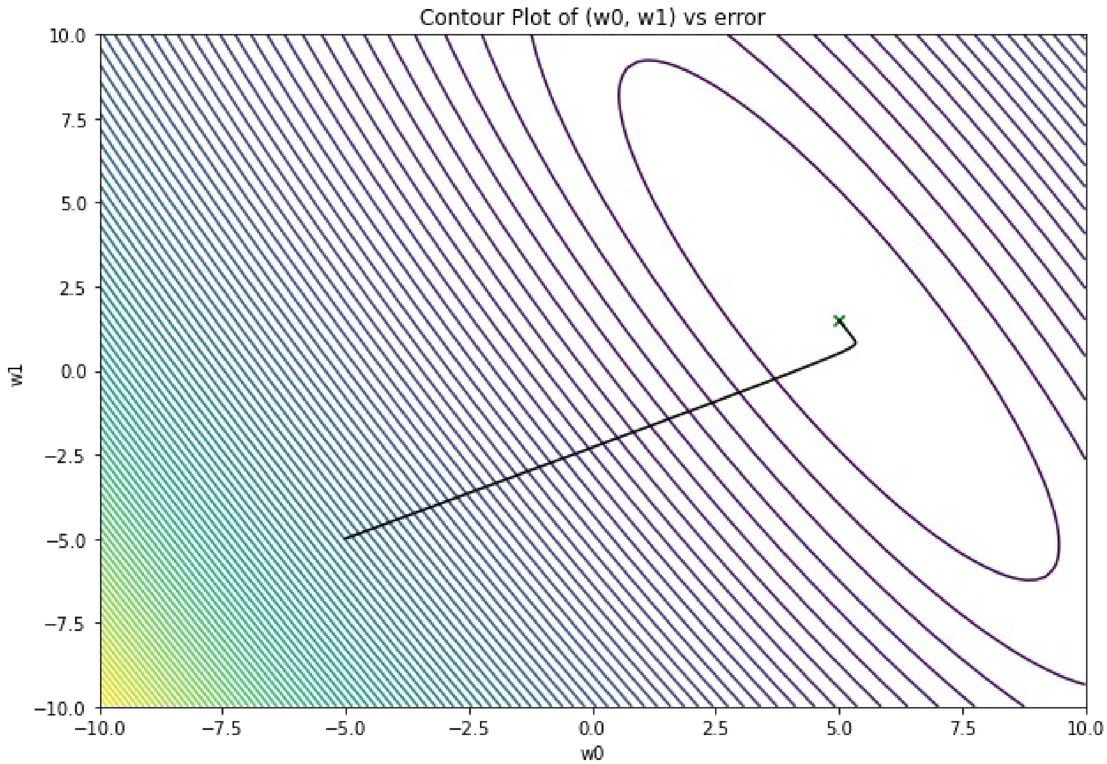


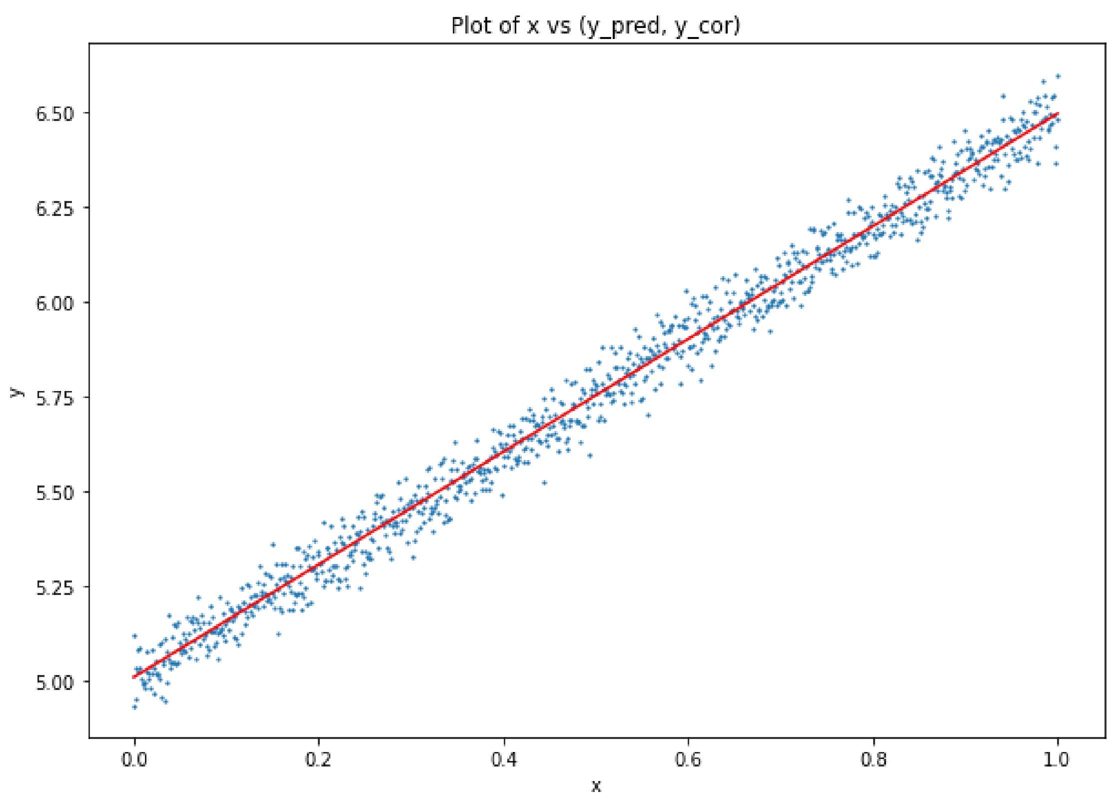
Gradient Descent to find optimal Values

```
## Write your code here
tol = 1e-6
max_iter = int(1e5)
_lambda = 0.1

w_init = np.array([-5, -5])

errors2 = []
w_vals = []

X = np.append(np.ones((1000)), x).reshape(2, 1000)

for i in range(max_iter):
    y_pred = X.T @ w_init

    error_temp = np.sum(np.subtract(y_cor, y_pred)**2)/1000
    errors2.append(error_temp)
    w_vals.append(w_init.copy())

    grad = 2*(X @ (y_pred - y_cor))/1000
    w_init = w_init - _lambda * grad

    if np.sum(grad**2)<tol: break
```

```python
print(f'Optimal value of w0 is : {w_init[0]}')
print(f'Optimal value of w1 is : {w_init[1]}')

fig, ax = plt.subplots(figsize=(10, 7))

w_vals_plot = np.array(w_vals)
ax.contour(W0, W1, errors, 100)
ax.plot(w_vals_plot[:, 0], w_vals_plot[:, 1], c='black')
ax.scatter(w_init[0], w_init[1], c='green', marker='x')

ax.set_title("Contour Plot of (w0, w1) vs error")
ax.set_xlabel('w0')
ax.set_ylabel('w1')

y_pred = X.T @ w_init
print(X.T, w_init)
plt.figure(figsize=(10, 7))
plt.plot(x, y_pred, c='red')
plt.scatter(x, y_cor, s=1.5)
plt.xlabel("x")
plt.ylabel("y")
plt.title("Plot of x vs (y_pred, y_cor)")
plt.show()
```

```
Optimal value of w0 is : 5.007910783677688
Optimal value of w1 is : 1.4865282443736134
[[1.       0.      ]
 [1.       0.001001]
 [1.       0.002002]
 ...
 [1.       0.997998]
 [1.       0.998999]
 [1.       1.      ]] [5.00791078 1.48652824]
```



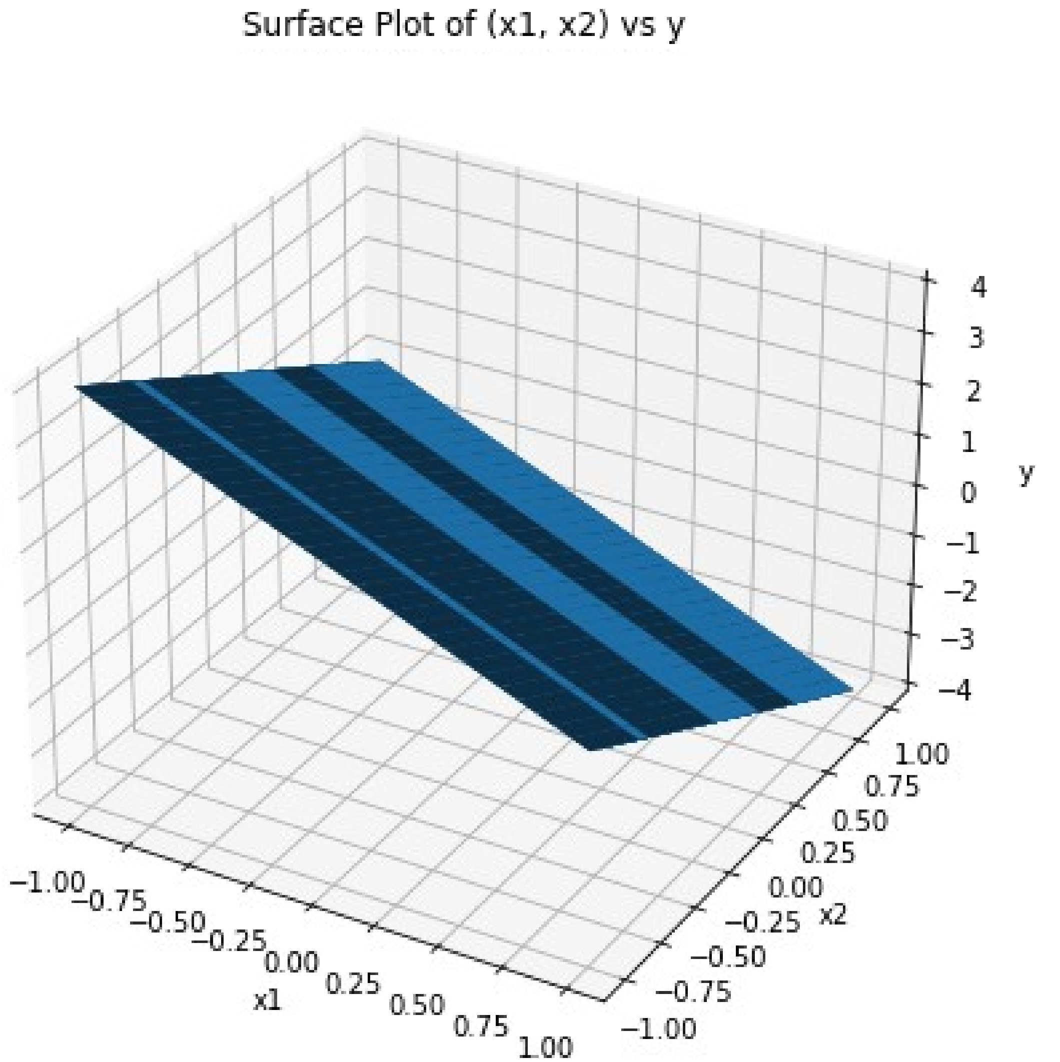


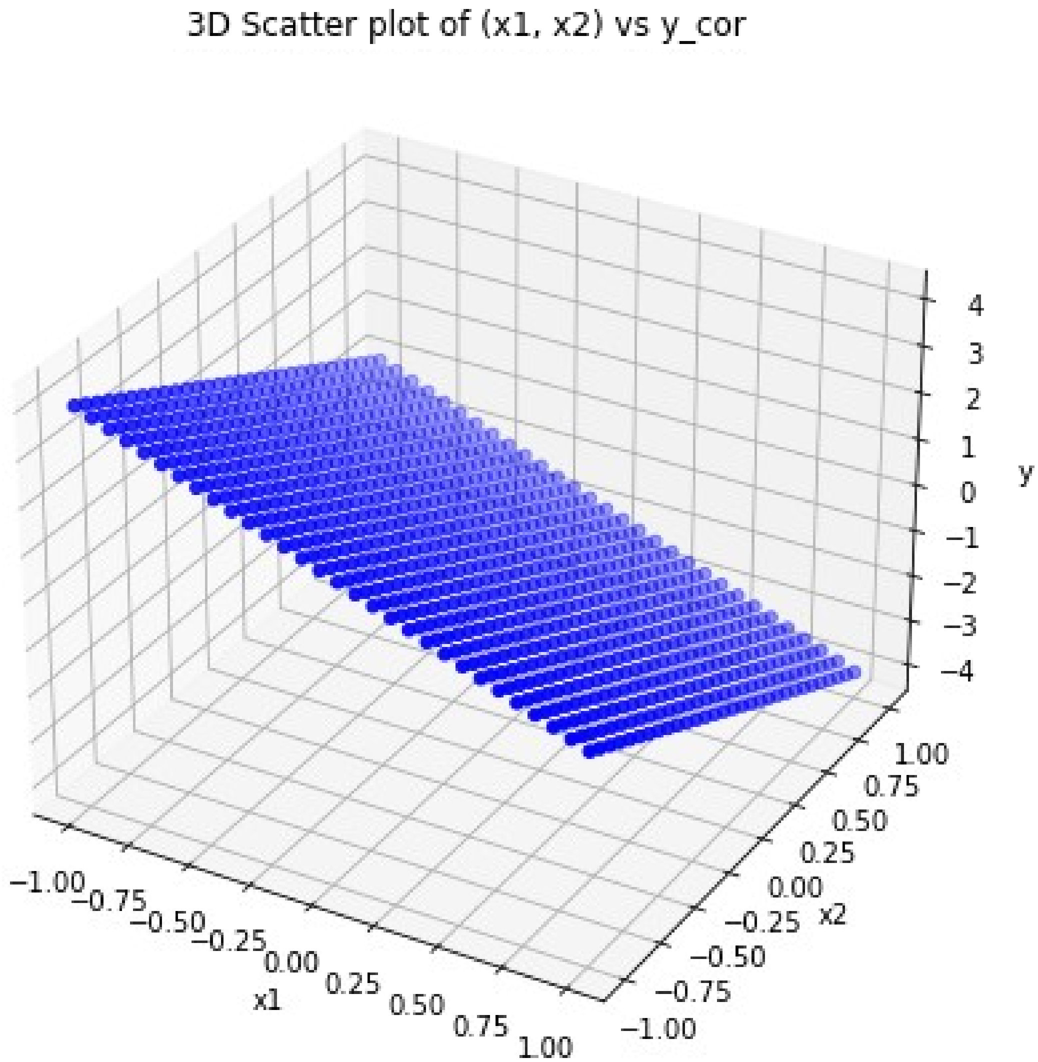
#Fitting of a Plane

Generation of plane data

1. Generate x_1 and x_2 from range -1 to 1, (30 samples)
2. Equation of plane $y = w_0 + w_1 x_1 + w_2 x_2$
3. Here we will fix w_0 and will learn w_1 and w_2

```
## Write your code here
x1 = np.linspace(-1, 1, 30)
x2 = np.linspace(-1, 1, 30)

np.random.shuffle(x2)

w = np.array([0, -2, -2])

X1, X2 = np.meshgrid(x1, x2)

y = -2*X1 -2*X2

fig, ax = plt.subplots(subplot_kw={"projection": "3d"}, figsize=(10,
7))

ax.plot_surface(X1, X2, y)

ax.set_title("Surface Plot of (x1, x2) vs y")
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y')

fig, ax = plt.subplots(subplot_kw={"projection": "3d"}, figsize=(10,
7))

ax.scatter3D(X1, X2, y, color = "blue")
plt.title("3D Scatter plot of (x1, x2) vs y_cor")
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y')

X = np.concatenate((np.ones(30), x1, x2), axis=0,
dtype=object).reshape(3, 30).T
y_cor = X @ w
```


Surface Plot of (x1, x2) vs y
y
4
3
2
1
0
−1
−2
−3
−4
x1
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x2
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00



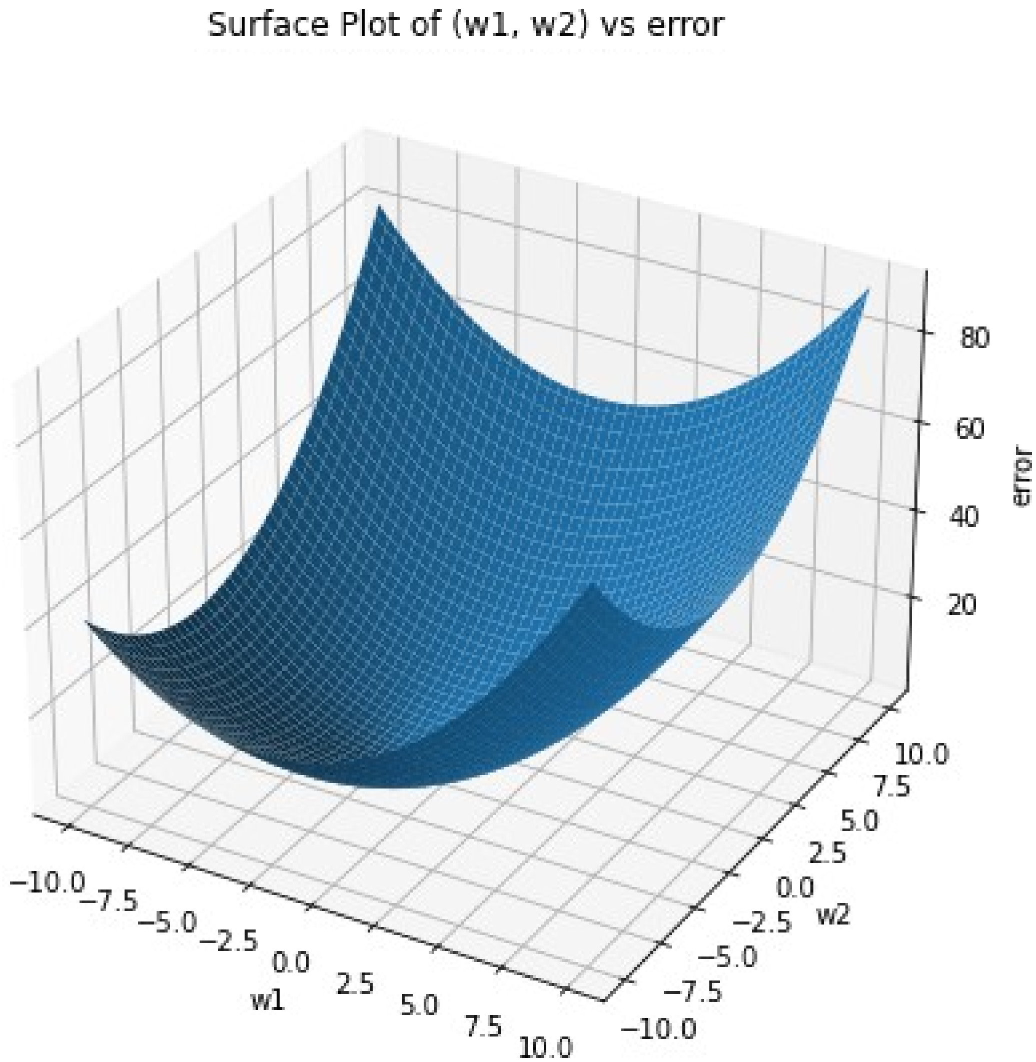
Generate the Error Surface

1. Vary w_1 and w_2 and generate the error surface and find their optimal value
2. Also plot the Contour

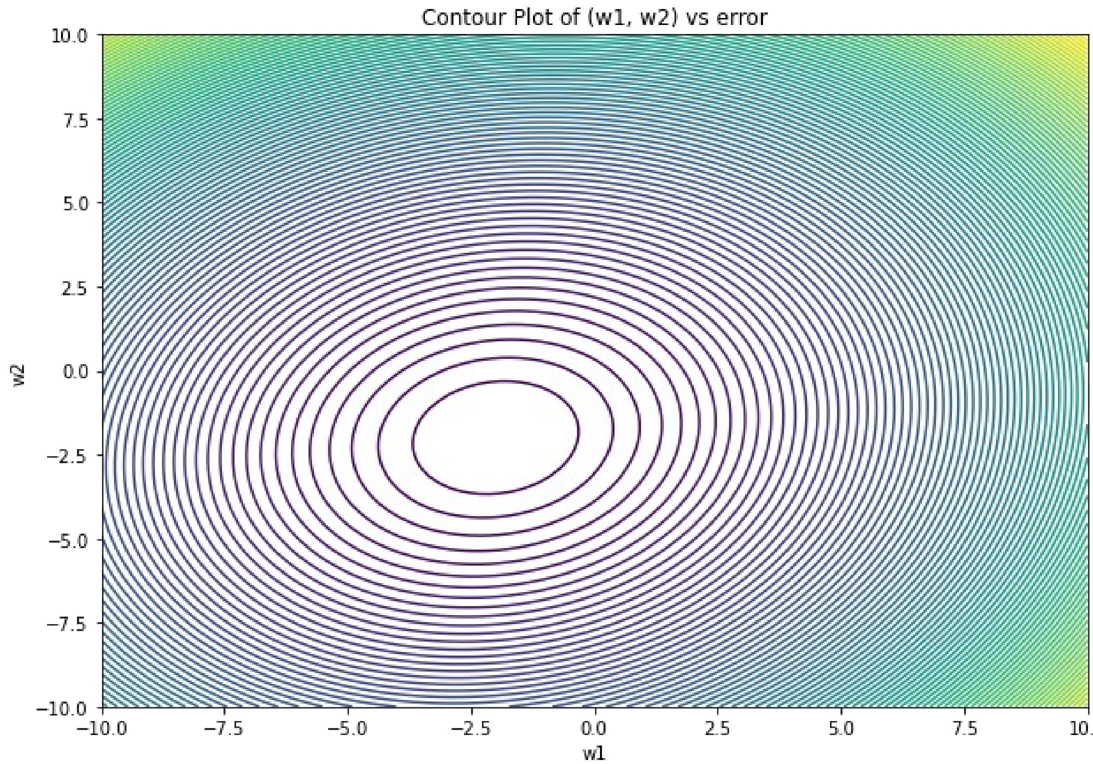
```
## Write your code here
w0 = 0
def cost(w1_i, w2_i):
    w = np.array([w0, w1_i, w2_i])
    y_pred = X @ w
    return np.sum(np.subtract(y_cor, y_pred)**2)/30

w1 = np.linspace(-10, 10, 100)
w2 = np.linspace(-10, 10, 100)

W1, W2 = np.meshgrid(w1, w2)

W1.shape, W2.shape
```

```
errors = np.array([cost(w1_i, w2_i) for w1_i, w2_i in
zip(np.ravel(W1), np.ravel(W2))]).reshape(100, 100)

fig, ax = plt.subplots(subplot_kw={"projection": "3d"}, figsize=(10,
7))

ax.plot_surface(W1, W2, errors)

ax.set_title("Surface Plot of (w1, w2) vs error")
ax.set_xlabel('w1')
ax.set_ylabel('w2')
ax.set_zlabel('error')

fig, ax = plt.subplots(figsize=(10, 7))

ax.contour(W1, W2, errors, 100)

ax.set_title("Contour Plot of (w1, w2) vs error")
ax.set_xlabel('w1')
ax.set_ylabel('w2')
```

```
Text(0, 0.5, 'w2')
```



Surface Plot of (w1, w2) vs error
80
60
40
20
error
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
w1
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
w2



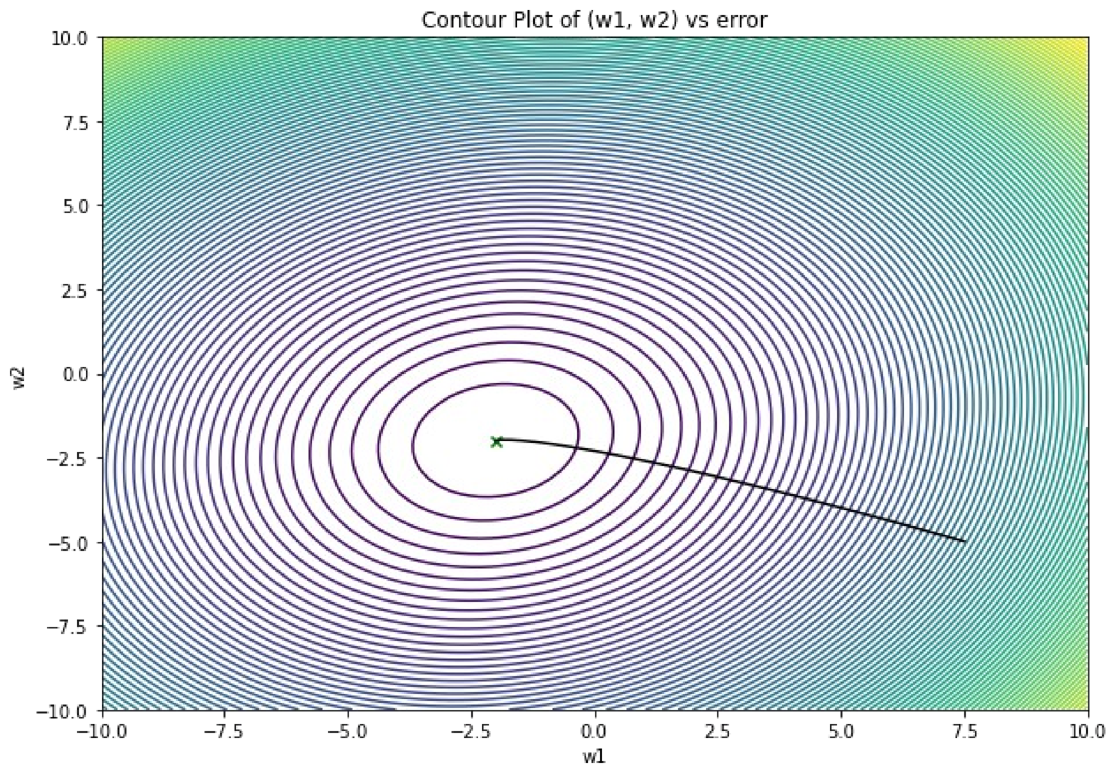
Prediction using Gradient Descent

```
## Write your code here
tol = 1e-6
max_iter = int(1e5)
_lambda = 0.1

w_init = np.array([0, 7.5, -5])

errors2 = []
w_vals = []

for i in range(max_iter):
    y_pred = X @ w_init

    error_temp = np.sum(np.subtract(y_cor, y_pred)**2)/50
    errors2.append(error_temp)
    w_vals.append(w_init.copy())

    grad = 2*(X.T @ (y_pred - y_cor))/50
    w_init = w_init - _lambda * grad

    if np.sum(grad**2)<tol: break

print(f'Optimal value of w1 is : {w_init[1]}')
print(f'Optimal value of w2 is : {w_init[2]}')
```

```
fig, ax = plt.subplots(figsize=(10, 7))

w_vals_plot = np.array(w_vals)
ax.contour(W1, W2, errors, 100)
ax.plot(w_vals_plot[:, 1], w_vals_plot[:, 2], c='black')
ax.scatter(w_init[1], w_init[2], c='green', marker='x')

ax.set_title("Contour Plot of (w1, w2) vs error")
ax.set_xlabel('w1')
ax.set_ylabel('w2')
```

```
Optimal value of w1 is : -1.997843690521284
Optimal value of w2 is : -1.998799080461982
```

```
Text(0, 0.5, 'w2')
```



#Fitting of M-dimentional hyperplane (M-dimention, both in matrix inversion and gradient descent)

Here we will vectorize the input and will use matrix method to solve the regression problem.

let we have M- dimensional hyperplane we have to fit using regression, the inputs are $x1, x2, x3, \ldots, x_M$. in vector form we can write $\left[x1, x2, \ldots, x_M\right]^T$, and similarly the weights are $w1, w2, \ldots w_M$ can be written as a vector $[w1, w2, \ldots w_M]^T$, Then the equation of the plane can be written as:

$$y = w1x1 + w2x2 + \ldots + w_M x_M$$

$w1, w2, \ldots, wM$ are the scalling parameters in M different direction, and we also need a offset parameter w0, to capture the offset variation while fitting.

The final input vector (generally known as augmented feature vector) is represented as $[1, x1, x2, \ldots, x_M]^T$ and the weight matrix is $[w0, w1, w2, \ldots w_M]^T$, now the equation of the plane can be written as:

$$y = w0 + w1x1 + w2x2 + \ldots + w_M x_M$$

In matrix notation: $y = x^T w$ (for a single data point), but in general we are dealing with N-data points, so in matrix notation

$$Y = X^T W$$

where Y is a $N \times 1$ vector, X is a $M \times N$ matrix and W is a $M \times 1$ vector.

$$Error = \frac{1}{N} \| Y - X^T W \|^2$$

it looks like a optimization problem, where we have to find W, which will give minimum error.

1. **By computation:**

$\nabla Error = 0$ will give us W_{opt}, then W_{opt} can be written as:

$$W_{opt} = (X X^T)^{-1} X Y$$

1. **By gradient descent:**

$$W_{new} = W_{old} + \frac{2\lambda}{N} X (Y - X^T W_{old})$$

1. Create a class named Regression
2. Inside the class, include constructor, and the following functions:

   a. grad_update: Takes input as previous weight, learning rate, x, y and returns the updated weight.

   b. error: Takes input as weight, learning rate, x, y and returns the mean squared error.

   c. mat_inv: This returns the pseudo inverse of train data which is multiplied by labels.

d. Regression_grad_des: Here, inside the for loop, write a code to update the weights. Also calulate error after each update of weights and store them in a list. Next, calculate the deviation in error with new_weights and old_weights and break the loop, if it's below a threshold value mentioned the code.

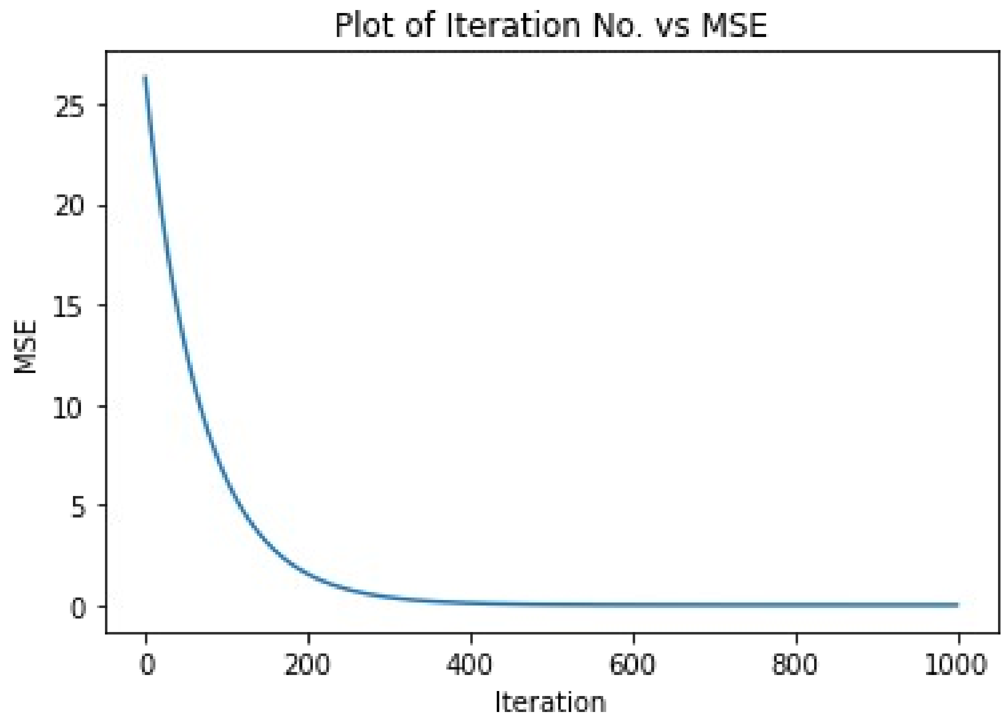
```
class regression:
  # Constructor
  def __init__(self, name='reg'):
    self.name = name  # Create an instance variable

  def grad_update(self, w_old, lr, y, x):
    w = w_old + 2 * lr * (x @ ( y - x.T@w_old ))/x.shape[1]
    return w

  def error(self,w,y,x):
    return np.sum((y - x.T@w)**2)/x.shape[1]

  def mat_inv(self, y, x_aug):
    return np.linalg.pinv(x_aug @ x_aug.T) @ x_aug @ y

  # By Gradien descent

  def Regression_grad_des(self, x, y, lr):

    w_old = np.random.uniform(0, 1, (x.shape[0], 1))
    err = []
    for i in range(1000):

      w_pred = self.grad_update(w_old, lr, y, x)
      err.append(self.error(w_pred, y, x))

      dev = np.linalg.norm(np.subtract(w_pred, w_old), ord=1)
      if dev<=0.000001:
        break

      w_old = w_pred

    w_pred = w_old
    return w_pred, err


#######################################################################
#######
# Generation of data

sim_dim=5
sim_no_data=1000
x=np.random.uniform(-1,1,(sim_dim,sim_no_data))
print("Initial data shape: ", x.shape)
```

```
w = np.array([1, 2, 3, 4, 5, 6]).reshape(6, 1) ## Write your code here
(Initialise the weight matrix) (W=[w0,w1,.....,wM]')
print("Dimension of Weight matrix: ", w.shape)

## Augment the Input
ones =np.ones(sim_no_data)
x_aug = np.vstack([ones, x]) ## Write your code here (Augment the data
so as to include x0 also which is a vector of ones)
print("Data shape after augmenting: ", x_aug.shape)

y=x_aug.T @ w  # vector multiplication
print("Shape of Output: ", y.shape)

## Corrupt the input by adding noise
noise = np.random.uniform(0, 1, y.shape)
y = y+0.1*noise

### The data (x_aug and y) is generated ###

######################################################################
#######

# By Computation (Normal Equation)
reg = regression()
w_opt=reg.mat_inv(y, x_aug)
print("Optimal weights obatained by computation: ", w_opt)

# By Gradient descent
lr=0.01
w_pred, err = reg.Regression_grad_des(x_aug, y, lr)
print("Optimal weights obatained by Gradient descent: ", w_pred)

plt.plot(err)
plt.xlabel("Iteration")
plt.ylabel("MSE")
plt.title("Plot of Iteration No. vs MSE")
```

```
Initial data shape:  (5, 1000)
Dimension of Weight matrix:  (6, 1)
Data shape after augmenting:  (6, 1000)
Shape of Output:  (1000, 1)
Optimal weights obatained by computation:  [[1.0490547 ]
 [2.00007289]
 [3.00094711]
 [4.0027456 ]
 [4.99931453]
 [6.00039595]]
Optimal weights obatained by Gradient descent:  [[1.0494999 ]
```

```
 [2.0010318 ]
 [3.00068978]
 [4.00185538]
 [4.99215704]
 [5.99304317]]
```

```
Text(0.5, 1.0, 'Plot of Iteration No. vs MSE')
```



#Practical Example (Salary Prediction)

1. Read data from csv file
2. Do train test split (90% and 10%)
3. Compute optimal weight values and predict the salary using the regression class created above (Use both the methods)
4. Find the mean square error in test.
5. Also find the optimal weight values using regression class from the Sci-kit learn library

```
# Read data from CSV

import pandas as pd
from sklearn.preprocessing import MinMaxScaler

scaler = MinMaxScaler()

data = pd.read_csv("./salary_pred_data.csv",
index_col=False).astype(float)
data = scaler.fit_transform(data)
```

```python
data[:5]
```

```
array([[0.33333333, 0.55555556, 0.35897436, 1.        , 0.25      ,
        0.28249927],
       [1.        , 0.72222222, 0.20512821, 0.        , 0.375     ,
        0.45696576],
       [0.        , 0.66666667, 0.8974359 , 0.66666667, 0.125     ,
        0.13569616],
       [1.        , 1.        , 0.69230769, 0.        , 0.75      ,
        0.78014892],
       [0.33333333, 0.5       , 0.1025641 , 0.33333333, 0.625     ,
        0.57951662]])
```

```python
from sklearn.model_selection import train_test_split

X = data[:, :-1]   # independant features
y = data[:, -1]

X_train, X_test, y_train, y_test = train_test_split(X, y,
test_size=0.1, random_state=42)

X_train = np.vstack([np.ones(X_train.shape[0]), X_train.T])
X_test = np.vstack([np.ones(X_test.shape[0]), X_test.T])
y_train= y_train.reshape(y_train.shape[0], 1)
y_test = y_test.reshape(y_test.shape[0], 1)

X_train.shape, X_test.shape, y_train.shape, y_test.shape
```

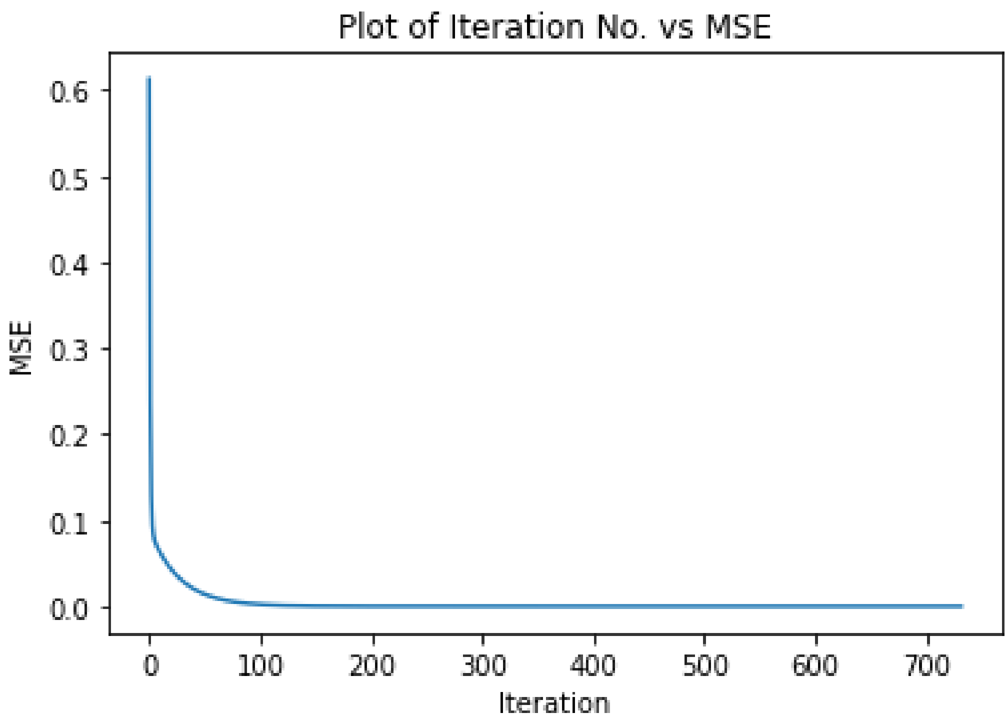
```
((6, 900), (6, 100), (900, 1), (100, 1))
```

```python
# Manually FInding Optimal Weights

# By Computation (Normal Equation)
reg = regression()
w_opt = reg.mat_inv(y_train, X_train)
print("Optimal weights obatained by computation: ", w_opt)

# By Gradient descent
lr=0.1
w_pred, err = reg.Regression_grad_des(X_train, y_train, lr)
print("Optimal weights obatained by Gradient descent: ", w_pred)

plt.plot(err)
plt.xlabel("Iteration")
plt.ylabel("MSE")
plt.title("Plot of Iteration No. vs MSE")
```

```
Optimal weights obatained by computation:  [[-0.00719664]
 [ 0.1247972 ]
```

```
 [ 0.03743916]
 [ 0.00162236]
 [ 0.01871958]
 [ 0.83198136]]
Optimal weights obatained by Gradient descent:  [[-0.00723657]
 [ 0.12480666]
 [ 0.03746076]
 [ 0.00164404]
 [ 0.01873025]
 [ 0.83199419]]
```

```
Text(0.5, 1.0, 'Plot of Iteration No. vs MSE')
```




```
# Mean squared error for testing set

print("Mean square error in test (Normal Equation): ",
reg.error(w_opt, y_test, X_test))
print("Mean square error in test (Gradient Descent): ",
reg.error(w_pred, y_test, X_test))
```

```
Mean square error in test (Normal Equation):  5.519031184464351e-30
Mean square error in test (Gradient Descent):  1.5268912771185229e-10
```

```
# Linear regression using Sci-Kit learn
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error

LR = LinearRegression()
```

```
X_train, X_test, y_train, y_test = train_test_split(X, y,
test_size=0.1, random_state=42)

LR.fit(X_train, y_train)

w_sk = np.append(LR.intercept_, LR.coef_).reshape(X_train.shape[1]+1,
1)

print("Optimal weights from Sci-Kit learn: ", w_sk)

prediction = LR.predict(X_test)
error = mean_squared_error(y_test, prediction)

print("MSE for Sci-Kit Learn: ", error)
```

```
Optimal weights from Sci-Kit learn:  [[-0.00719664]
 [ 0.1247972 ]
 [ 0.03743916]
 [ 0.00162236]
 [ 0.01871958]
 [ 0.83198136]]
MSE for Sci-Kit Learn:  1.7858305779952167e-32
```