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# README: 4-Star Matching Algorithm

Guidance, Navigation and Controls Subsystem

## Contents



## 1 st_main_4SM_v2.m

**Code Type:** MATLAB - Script
**Code author:** KT Prajwal Prathiksh
**Created on:** 08/06/2020
**Last modified:** {/{/ | -
**Reviewed by:** NOT YET REVIEWED!
**Description:**
This is the main script, which runs the 4-Star Matching Algorithm, with the veri cation step.

**Note:** This script can work with $n$-stars identified by *Feature Extraction block*
**Formula & References:**
Reference:

1. 4-Star Matching Algorithm [1]
2. Geometric Voting Algorithm [2]

**Input parameters:**

1. **st_constants_4SM.mat**: The contents of which are -
   (a) **Focal_Length**
   (b) **st_M_EPS**
   (c) **st_n_GC**
   (d) **st_n_RC**
   (e) **st_DELTA**
   (f) **st_M**
   (g) **st_Q**
   (h) **es_N_EST**
   (i) **st_ITER_MAX_4SM**
   (j) **st_verify_tol**
2. **st_input.mat**: The contents of which are -
   (a) **fe_output**
   (b) **fe_n_str**
3. **st_test_input.mat**: The contents of the test cases which are -
   (a) **fe_output** - Test input
   (b) **fe_n_str** - Test input

**Output:**

1. Writes the following output variables of Star-Matching in ./Star_Matching/4_Star_Matching/Output/st_output.mat directory
   (a) **st_iter_total**
   (b) **st_N_Match**
   (c) **st_Match**
   (d) **st_N_UnMatch**
   (e) **st_UnMatch**
   (f) **st_op_bi**
   (g) **st_op_ri**
   (h) **st_N_Fail**
   (i) **st_Fail**
   (j) **st_N_Verify**

(k) **st_Verify**

2. Writes the following output variables of Star-Matching as input for Estimation in ./Estimation/Input/es_input.mat directory

   (a) **st_N_Verify**

   (b) **st_op_bi**

   (c) **st_op_ri**

## 2 st_gnrt_bi.m

**Code Type:** MATLAB - Function
**Code author:** KT Prajwal Prathiksh
**Created on:** 26/04/2020
**Last modified:** 29/04/2020
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This function generates the body-frame vectors of the stars from the centroids identi ed through means of Feature Extraction
**Formula & References:**
References:

1. Refer Appendix [3]

**Input parameters:**

1. **fe_output**
2. **fe_n_str**
3. **Focal_Length**

**Output:**

1. **st_bi**

## 3 st_4SM.m

**Code Type:** MATLAB - Function
**Code author:** KT Prajwal Prathiksh
**Created on:** 29/04/2020
**Last modified:** {/{/ | -
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This function runs the 4-Star Matching Algorithm. It executes the algorithm to match stars, and returns the result.
**Formula & References:**
Reference:

1. 4-Star Matching Algorithm [1]

**Input parameters:**

1. **st_4SM_input**
2. **st_RF_SC**
3. **st_4SM_constants**

**Output:**

1. **st_n_match**
2. **st_result**

## 4 st_gnrt_ip_4SM.m

**Code Type:** MATLAB - Function
**Code author:** KT Prajwal Prathiksh
**Created on:** 26/04/2020
**Last modified:** 29/04/2020
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This function generates the input for 4-Star Matching Algorithm, using the body-frame vectors of 4 identi ed stars from the image
**Formula & References:**
|
**Input parameters:**

1. **st_4SM_input**

**Output:**

1. **st_c_img_AngDst**
2. **st_c_fe_ID**

## 5 st_gnrt_SIM.m

**Code Type:** MATLAB - Function
**Code author:** KT Prajwal Prathiksh
**Created on:** 26/04/2020
**Last modified:** 29/04/2020
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This function generates the the Star Identi cation Matrix (SIM) from a given array of angular distances between those of four stars, using the Reference Star Catalogue and the Guide Star Catalogue
**Formula & References:**
|
**Input parameters:**

1. **st_c_img_AngDst**
2. **st_RF_SC**

3. **st_4SM_constants**

**Output:**

1. **st_SIM**

## 6 st_gnrt_CSPA.m

**Code Type:** MATLAB - Function
**Code author:** KT Prajwal Prathiksh
**Created on:** 26/04/2020
**Last modified:** 29/04/2020
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This function generates the candidate star pair array (CSPA) from the value of angular distance calculated between two image stars
**Formula & References:**
Reference:

1. Refer Section - 2 [1]

**Input parameters:**

1. **st_AngDst**
2. **st_DELTA**
3. **st_M**
4. **st_Q**
5. **st_RF_SC**

**Output:**

1. **st_CSPA**
2. **st_INDEX**

## 7 st_gnrt_SMM.m

**Code Type:** MATLAB - Function
**Code author:** KT Prajwal Prathiksh
**Created on:** 26/04/2020
**Last modified:** 29/04/2020
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This function evaluates the Star Identi cation Matrix to identify the stars that have been matched through the 4-Star Matching Algorithm to generate the Star Matched Matrix (SMM).
Of the four stars that are provided as input to the 4-Star Matching Algorithm, the $S_i$ star is said to have been matched, if only one such row of the Star Identi cation Matrix as given below is present:

1. $S_1 - [1, 1, 1, 0, 0, 0]$
2. $S_2 - [1, 0, 0, 1, 1, 0]$
3. $S_3 - [0, 1, 0, 1, 0, 1]$
4. $S_4 - [0, 0, 1, 0, 1, 1]$

This pattern arises out of the fact that angular distances that are provided as input follow the order: $(S_1, S_2); (S_1, S_3); (S_1, S_4); (S_2, S_3); (S_2, S_4); (S_3, S_4)$
**Formula & References:**
Reference:

1. Refer Section - 2 [1]

**Input parameters:**

1. **st_SIM**
2. **st_n_GC**
3. **st_c_fe_ID**

**Output:**

1. **st_SMM**
2. **st_mtch_rows**

## 8 st_update_match_matrix.m

**Code Type:** MATLAB - Function
**Code author:** KT Prajwal Prathiksh
**Created on:** 29/04/2020
**Last modified:** {/{/ | -
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This function updates the Match Matrices associated with 4-Star Matching Algorithm.
**Formula & References:**
{
**Input parameters:**

1. **st_result**
2. **st_Match**
3. **st_UnMatch**
4. **st_N_Match**
5. **st_N_UnMatch**

**Output:**

1. **st_Match**
2. **st_UnMatch**
3. **st_N_Match**
4. **st_N_UnMatch**

## 9 st_verify_4SM.m

**Code Type:** MATLAB - Function
**Code author:** KT Prajwal Prathiksh
**Created on:** 09/06/2020
**Last modified:** {/{/ | -
**Reviewed by: NOT YET REVIEWED!**
**Description:**
Performs the veri cation step for 4-Star Matching Algorithm. It assigns votes to the matched stars that get veri ed. Returns only those stars which have 100% votes
**Formula & References:**
References:

1. Geometric voting algorithm, veri cation step [2]

**Input parameters:**

1. **st]_Match**
2. **st_N_Match**
3. **st_GD_SC**
4. **st_verify_tol**

**Output:**

1. **st_N_Verify**
2. **st_Verify**
3. **st_N_Fail**
4. **st_Fail**

## 10 st_check_ID_err.m

**Code Type:** MATLAB - Function
**Code author:** KT Prajwal Prathiksh
**Created on:** 29/04/2020
**Last modified:** {/{/ | -
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This function evaluates the result of Star-Matching with one of the input stars having already been matched, and checks for discrepancy error (if any) in the previously matched SSP-ID, and newly generated SSP-ID.

**NOTE: This code is currently under development! DO NOT USE IT!**
**Formula & References:**
|
**Input parameters:**

1. **st_result**

2. **st_ref_ID**
3. **st_N**

**Output:**

1. **st_flag_ID_err**

## 11 st_gnrt_op_4SM_v2.m

**Code Type:** MATLAB - Function
**Code author:** KT Prajwal Prathiksh
**Created on:** 26/04/2020
**Last modified:** 09/06/2020
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This function evaluates the Star Matched Matrix (SMM), to generate the output in the format required by Estimation - Body-frame vectors, and Inertial-frame vectors of the matched stars
**Formula & References:**
|
**Input parameters:**

1. **st_Match**
2. **st_GD_SC**

**Output:**

1. **st_op_bi**
2. **st_op_ri**

## 12 st_pseudo_input.m

**Code Type:** MATLAB - Script
**Code author:** KT Prajwal Prathiksh
**Created on:** 26/04/2020
**Last modified:** 29/04/2020
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This script generates  ctitious input, that resembles the output expected out of Feature Extraction.
**NOTE:** The values are random, and therefore should not be treated as test-cases
**Formula & References:**
|
**Input parameters:**

1. **fe_n_str**

**Output:**

1. Writes the following output variable in ./Star_Matching/4_Star_Matching/Input/st_input.mat directory:

(a) **st_input.mat**: The contents of which are -

i. **fe_output**

ii. **fe_n_str** :

## 13 st_main_4SM_deprecated.m

**Code Type:** MATLAB - Script
**Code author:** KT Prajwal Prathiksh
**Created on:** 26/04/2020
**Last modified:** 29/04/2020
**Reviewed by: NOT YET REVIEWED!**
**Description:**
This is the main script, which runs the 4-Star Matching Algorithm.
**Note: This script can only work with 4-stars identified by *Feature Extraction block*, and therefore has been DEPRECATED!**
**Formula & References:**
Reference:

1. 4-Star Matching Algorithm [1]

**Input parameters:**

1. **st_constants_4SM.mat**: The contents of which are -
   (a) **Focal_Length**
   (b) **st_M_EPS**
   (c) **st_n_GC**
   (d) **st_n_RC**
   (e) **st_DELTA**
   (f) **st_M**
   (g) **st_Q**
   (h) **es_N_EST**
   (i) **st_ITER_MAX_4SM**
2. **st_input.mat**: The contents of which are -
   (a) **fe_output**
   (b) **fe_n_str**

**Output:**

1. Writes the following output variables of Star-Matching in ./Star_Matching/4_Star_Matching/Output/st_output_deprecated.mat directory
   (a) **n_st_strs**
   (b) **st_mtch_rows**
   (c) **st_op_bi**
   (d) **st_op_ri**

(e) **st_SIM**

(f) **st_SMM**

2. Writes the following output variables of Star-Matching as input for Estimation in ./Estimation/Input/es_input_deprecated.mat directory

   (a) **n_st_strs**

   (b) **st_op_bi**

   (c) **st_op_ri**

# 14 Variable Description

1. **Focal_Length** : (Float) The focal length of the optics system. Units in mm
2. **st_M_EPS** : (Float) The machine epsilon of the platform where the algorithm will be executed
3. **st_n_GC** : (Integer) The number of stars (= number of rows) in the Guide Star Catalogue
4. **st_n_RC** : (Integer) The number of star-pairs (= number of rows) in the Reference Star Catalogue
5. **st_DELTA** : (Float) The $\delta$ constant that determines the tolerance of the size window when searching for an angular distance value in the Reference Star Catalogue
6. **st_M** : (Float) The slope of the Z-vector line
7. **st_Q** : (Float) The y-intercept of the Z-vector line
8. **es_N_EST** : (Integer) The minimum number of pairs of body-frame ($b_i$ and inertial-frame $r_i$ vectors required by $Estimation\ block$ to provide the attitude within the accuracy requirements
9. **st_ITER_MAX_4SM** : (Integer) The max number of iterations of 4-Star Matching that will be allowed to execute irregardless of whether all the stars match or not - owing to the time constraint on the $Star\text{-}Matching\ block$
10. **st_verify_tol** : (Double) The tolerance within which the angular distance (in $cos(\theta)$) of a pair of image stars should have match with the corresponding pair of stars from the Star Catalogue. Units in $percentage$
11. **fe_output** : ( (N, 3) - Matrix) - The output of Feature Extraction block - which contains the centroids of the identi ed stars as well ID of the identi ed star. The centroid is represented in $(X, Y)$ format, with the origin at the center of the sensor. Unit of centroid: mm
12. **fe_n_str** : (Integer) - The number of stars identi ed by feature extraction
13. **st_iter_total** : (Integer) - Counter variable which keeps track of the number of iterations of 4-Star Matching that has occurred
14. **st_N_Match** : (Integer) - Number of matched stars in st_Match matrix

15. **st_Match** : ( (st_N_Match, 5) - Matrix ) - This matrix contains the entries of the stars that have been matched so far. The columns are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}, 3^{rd}, 4^{th}$ columns - $(X, Y, Z)$ unit body-frame vector

    (c) $5^{th}$ column - The matched SSP-ID

16. **st_N_UnMatch** : (Integer) - Number of stars yet to be matched in st_UnMatch matrix

17. **st_UnMatch** : ( (st_N_UnMatch, 4) - Matrix ) - This matrix contains the entries of the stars that are yet to be matched so far. The columns are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}, 3^{rd}, 4^{th}$ columns - $(X, Y, Z)$ unit body-frame vector

18. **st_op_bi** : ( (st_N_Verify, 4) - Matrix ) - The body-frame vectors $b_i$ of the image stars that have been matched through Star-Matching. The columns include:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}, 3^{rd}, 4^{th}$ columns - $(X, Y, Z)$ unit body-frame vector

19. **st_op_ri** : ( (st_N_Verify, 4) - Matrix ) - The corresponding inertial-frame vectors $r_i$ of the matched stars obtained from the Guide Star Catalogue. The columns include:

    (a) $1^{st}$ column - Corresponding SSP-ID of the matched star

    (b) $2^{nd}, 3^{rd}, 4^{th}$ columns - $(X, Y, Z)$ unit inertial-frame vector

    **NOTE:**

    (a) $(N)$ in both cases should be equal!

    (b) The $i^{th}$ body-frame and the $i^{th}$ inertial-frame vector should correspond to the same star!

20. **st_N_Fail** : (Integer) - Number of stars that failed the veri cation step

21. **st_Fail** : ( (st_N_Fail_, 6) - Matrix ) - This matrix contains the entries of the stars that failed the veri cation step. The columns are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}, 3^{rd}, 4^{th}$ columns - $(X, Y, Z)$ unit body-frame vector

    (c) $5^{th}$ column - The matched SSP-ID

    (d) $6^{th}$ column - Number of votes

22. **st_N_Verify** : (Integer) - Number of stars that passed the veri cation step

23. **st_Verify** : ( (st_N_verify, 6) - Matrix ) - This matrix contains the entries of the stars that have been veri ed. The columns are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}, 3^{rd}, 4^{th}$ columns - $(X, Y, Z)$ unit body-frame vector

    (c) $5^{th}$ column - The matched SSP-ID

(d) $6^{th}$ column - Number of votes

24. **st_bi** : ( (N, 4) - Matrix ) - Generates the body-frame vectors for all the stars identi ed by Feature Extraction using the centroids. The body-frame vectors are unit-vectors represented in $(X, Y, Z)$ format, with the origin at the center of the sensor and positive z-axis pointing out of the lens. The columns of matrix are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}, 3^{rd}, 4^{th}$ columns - $(X, Y, Z)$ unit body-frame vector

25. **st_4SM_input** : ( (4, 4) - Matrix ) - The input to 4-Star Matching Algorithm - four stars identi ed by Feature Extraction, along with their body-frame vectors (a unit-vector represented in $(X, Y, Z)$ format, with the origin at the center of the sensor and positive z-axis pointing out of the lens). The columns of the matrix are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}, 3^{rd}, 4^{th}$ columns - $(X, Y, Z)$ unit body-frame vector

26. **st_RF_SC** : ( (st_n_RC, 3) - Matrix ) - The Reference Star Catalogue, which has the following columns:

    (a) **SSP_ID_1** - The SSP-ID of $i^{th}$ star

    (b) **SSP_ID_2** - The SSP-ID of $j^{th}$ star

    (c) **K_Vec** - The K-Vector value determined uniquely using the dot product of the Cartesian unit vector corresponding to the $i^{th}$ and $j^{th}$ star ($i \neq j, \forall\ i, j$)

27. **st_4SM_constants** : ( (4, 1) - Matrix ) - An array that contains the following constants:

    (a) **st_n_GC**

    (b) **st_M**

    (c) **st_Q**

    (d) **st_DELTA**

28. **st_n_match** : (Integer) - The number of stars that have been matched

29. **st_result** : ( (4,2) - Matrix ) - Output of 4-Star Matching. The columns of the matrix are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}$ column - The matched SSP-ID ($If\ no\ match\ was\ made$ - 0 $is\ assigned$)

30. **st_c_img_AngDst** : ( (6,1) - Matrix ) - Has the angular distances (in cos($\theta$)) between those of four stars in the following order:
$(S_1, S_2)$; $(S_1, S_3)$; $(S_1, S_4)$; $(S_2, S_3)$; $(S_2, S_4)$; $(S_3, S_4)$;
**NOTE: THE ABOVE ORDER IS IMPORTANT, AND SHOULD BE FOLLOWED!**

31. **st_c_fe_ID** : ( (4,1) - Matrix ) - Has the Feature Extraction IDs of stars that are used to generate st_c_img_AngDst, in the following order:
[$S_1; S_2; S_3; S_4$]

32. **st_SIM** : ( (st_n_GC, 6) - Matrix ) - The Star Identi cation Matrix (SIM). The $i^{th}$ row of the matrix corresponds to the $i^{th}$ SSP-ID star. There are thus as many rows in SIM as there are in the Guide Star Catalogue = (st_n_GC). The $j^{th}$ column corresponds to a boolean value. If an element $SIM_{ij}$ is 1 $\implies$ $S_j^{th}$ input star matched to the $i^{th}$ star of the Guide Star Catalogue

33. **st_AngDst** : (Float) - The angular distance calculated between two stars ( in cos($\theta$) )

34. **st_CSPA** : ( (X,1) - Matrix ) - The possible SSP-ID matches for given angular distance

35. **st_INDEX** : ( (1,2) - Matrix ) - The start and stop indices of possible matches generated for given angular distance

36. **st_SMM** : ( (4,3) - Matrix ) - The Star Matched Matrix. The columns of the matrix are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}$ column - The matched SSP-ID ($If\ no\ match\ was\ made$ - 0 $is\ assigned$)

    (c) $3^{rd}$ column - The number of rows in SIM that matched the condition of $S_i$ star

37. **st_mtch_rows** : ( (4, X) - Cell Array ) - The columns of the cell array are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}$ column - $i^{th}$ element consists of a (X,1) - Matrix, that stores the SSP-IDs that matched the $i^{th}$ condition

38. **st_result** : ( (4,2) - Matrix ) - Output of 4-Star Matching. The columns of the matrix are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}$ column - The matched SSP-ID ($If\ no\ match\ was\ made$ - 0 $is\ assigned$)

39. **st_ref_ID** : ( (4,2) - Matrix ) - This matrix is stores the previously matched SSP-IDs of the stars taken from st_Match matrix. This is used to check if the newly matched SSP-ID is the same as the previous SSP-ID. The columns of this matrix are as follows:

    (a) $1^{st}$ column - Feature Extraction ID

    (b) $2^{nd}$ column - The previously matched SSP-ID (The unmatched stars have 0 as their ID)

40. **st_N** : (Integer) - Number of previously matched stars used as input

41. **st_flag_ID_err** : (boolean) - Returns 1 if the previous and the new SSP-ID do not match, or if the previously matched star was not matched in st_result. Returns 0 only if they match!

42. **st_GD_SC** : ( (st_n_GC, 4) - Matrix ) - The Guide Star Catalogue, which has the following columns:

    (a) **SSP_ID** - The SSP-ID of Guide Stars

(b) $\mathbf{X}$ - X component of the unit vector, corresponding to the star

(c) $\mathbf{Y}$- Y component of the unit vector, corresponding to the star

(d) $\mathbf{Z}$- Z component of the unit vector, corresponding to the star

## References


[1] Y. Dong, F. Xing, and Z. You, \Brightness independent 4-star matching algorithm for lost-in-space 3-axis attitude acquisition", *Tsinghua Science & Technology*, vol. 11, no. 5, pp. 543{548, 2006.

[2] M. Kolomenkin, S. Pollak, I. Shimshoni, and M. Lindenbaum, \Geometric voting algorithm for star trackers", *IEEE Transactions on Aerospace and Electronic Systems*, vol. 44, no. 2, pp. 441{456, 2008.

[3] A. O. Erlank, \Development of cubestar: A cubesat-compatible star tracker", PhD thesis, Stellenbosch: Stellenbosch University, 2013.