Deep Learning Principles & Applications

Chapter 2 – Linear Classifiers

Sudarsan N.S. Acharya (sudarsan.acharya@manipal.edu)

Classification in Practice

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(Institution of Eminence Deemed to be University)

Classifying a sample into one of the known categories (or classes) is a common challenge across different domains:

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is this sample a $\underbrace{\text{lion/tiger/leopard}}_{\text{known 3 classes}}$?

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Recall that this color image is internally represented as a $337 \times 600 \times 3$-tensor of integer values ranging from 0 to 255.

Classification in Practice – continued

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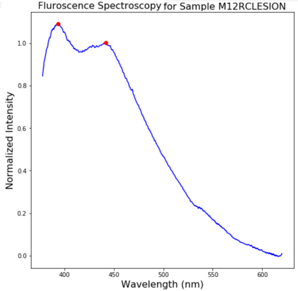
Medical Signal Processing

Classification in Practice – continued

Medical Signal Processing

	0	1	...	1017	1018
M12RCLESION	0.845890	0.861290	...	0.000037	0.001354



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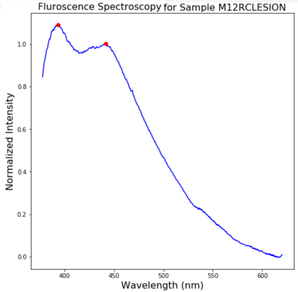

What kind of an oral tumor does this patient have: benign / premalignant / malignant?

known 3 classes

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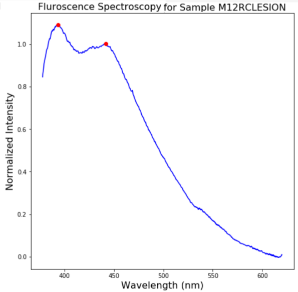

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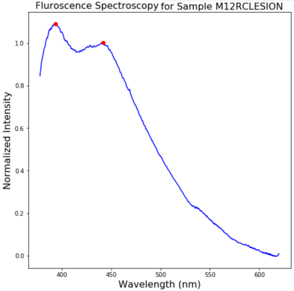

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Language Application

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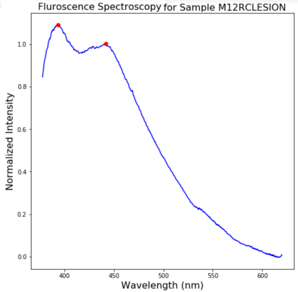

What kind of an oral tumor does this patient have: benign/premalignant/malignant?

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Language Application

The movie was goat

Is this movie review positive/negative?

known 2 classes

Linear Classifier: Basic Idea

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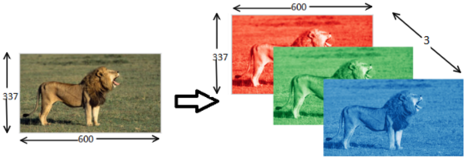

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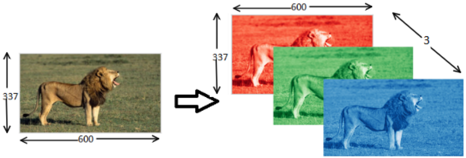

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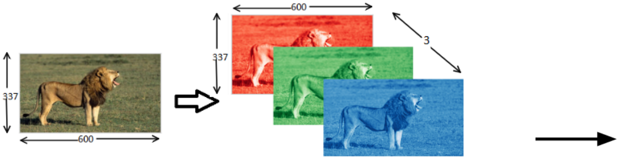

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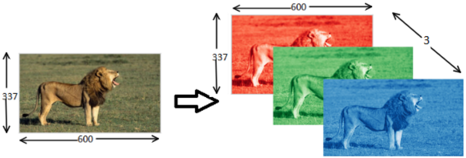

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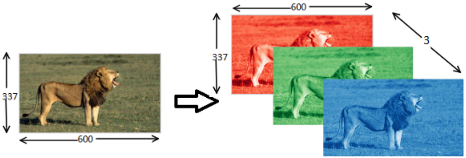

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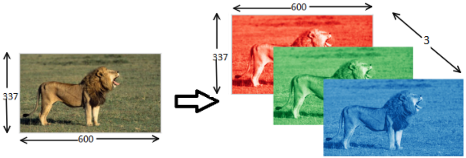

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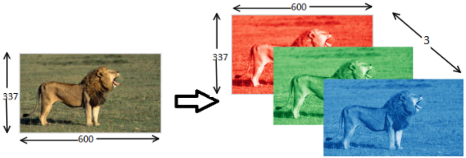

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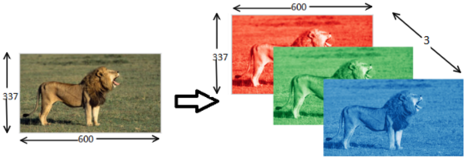

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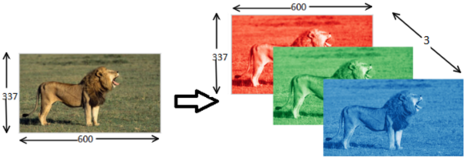

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What are $\mathbf{W}$ and $\mathbf{b}$ (the parameters), and how do we know what they are?

Weights, Bias, & Raw Scores

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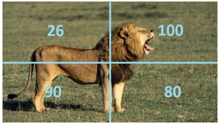

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Loss Function – Intuition

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Loss for Sample-1

$$L_1 = \begin{cases} \max(0, 6.4 - 5.6) \\ + \\ \max(0, -4.6 - 5.6) \end{cases}$$

$$= 0.8$$

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Loss for Sample-2

$$L_2 = \begin{cases} \max(0, -1.8 - 10.2) \\ + \\ \max(0, 3.5 - 10.2) \end{cases}$$

$$= 0$$

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Loss for Sample-3

$$L_3 = \begin{cases} \max(0, 2.0 - (-8.6) \\ \qquad + \\ \max(0, 5.4 - (-8.6)) \end{cases}$$

$$= 24.6$$

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Average training loss

$$\frac{0.8 + 0 + 24.6}{3} = 8.5$$

Hinge Loss Function

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sample vector

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$$L_i = \sum_{j \neq y^{(i)}} \max\left(0, z_j^{(i)} - z_{y^{(i)}}^{(i)} + 1\right)$$

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- The multiclass SVM hinge loss function for the ith sample is

$$L_i = \sum_{j \neq y^{(i)}} \max\left(0, \underset{\substack{\uparrow \\ \text{incorrect class raw score}}}{z_j^{(i)}} - z_{y^{(i)}}^{(i)} + 1\right)$$

Hinge Loss Function

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$$= \sum_{j \neq y^{(i)}} \max\left(0, \left[\mathbf{W}\mathbf{x}^{(i)} + \mathbf{b}\right]_j - \left[\mathbf{W}\mathbf{x}^{(i)} + \mathbf{b}\right]_{y^{(i)}} + 1\right)$$

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- The average training data loss is $\frac{1}{n}\sum_{i=1}^{n} L_i$, which is a function of the weights and bias values.

Visualizing Loss Functions

Visualizing Loss Functions

Visualizing different loss functions considering contribution from one incorrect class:

Visualizing Loss Functions

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Perceptron Loss

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Visualizing Loss Functions

Visualizing different loss functions considering contribution from one incorrect class:

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Visualizing Loss Functions

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Perceptron Loss | **Hinge Loss** | **Squared Hinge Loss**

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Perceptron Loss

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$\max\left(0, z_j^{(i)} - z_{y^{(i)}}^{(i)}\right)$	$\max\left(0, z_j^{(i)} - z_{y^{(i)}}^{(i)} + 1\right)$	$\max\left(0, z_j^{(i)} - z_{y^{(i)}}^{(i)} + 1\right)^2$

Visualizing Loss Functions

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Perceptron Loss

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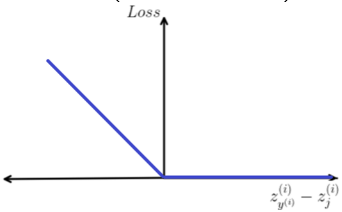

Hinge Loss

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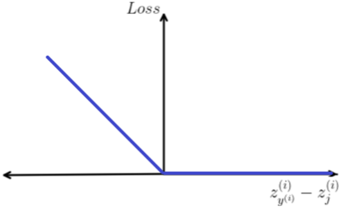

difference between correct and incorrect class raw scores

Hinge Loss

$$\max\left(0, z_j^{(i)} - z_{y^{(i)}}^{(i)} + 1\right)$$

Squared Hinge Loss

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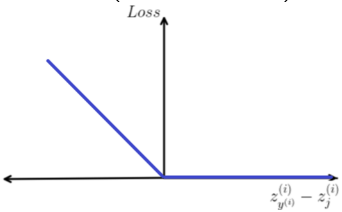

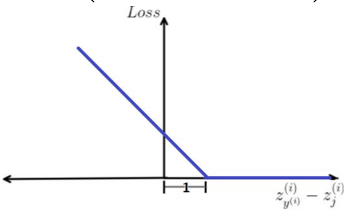

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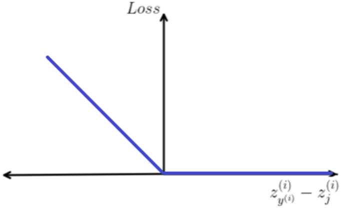

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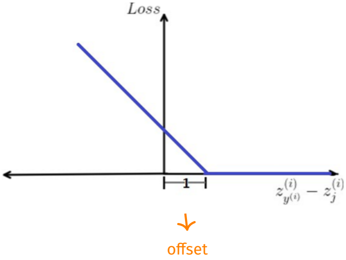

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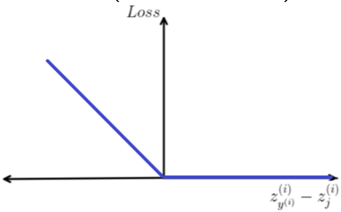

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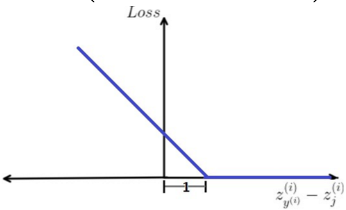

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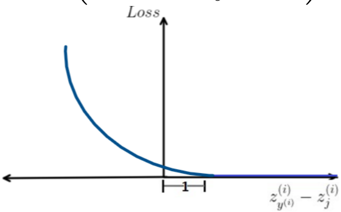

Softmax Function

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Raw score	
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Lion score	5.6		$e^{5.6}$
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Softmax Function

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It is possible to turn the raw scores vector into a a vector of probabilities:

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Softmax Function

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Raise to power of e $\longrightarrow$ normalize $\longrightarrow$

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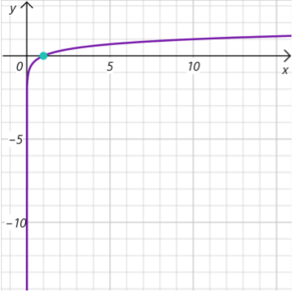

Softmax Loss Function

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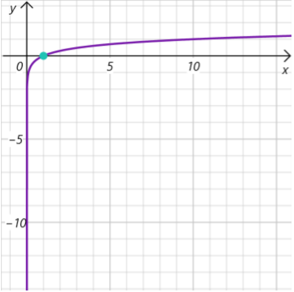

- Note that $\begin{cases} \log(1) = 0, \\ \log(x) \to -\infty \text{ as } x \to 0. \end{cases}$

Softmax Loss Function – continued

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- In plain English, it is the negative of the logarithm of the correct class's probability.
- Note that $\begin{cases} [\text{softmax}(\mathbf{z})]_y = 1 \quad \Rightarrow ☺ \Rightarrow \text{loss} = -\log(1) = 0, \\ [\text{softmax}(\mathbf{z})]_y = 0 \quad \Rightarrow ☹ \Rightarrow \text{loss} = -\log(0) \to \infty. \end{cases}$

Optimization Setup

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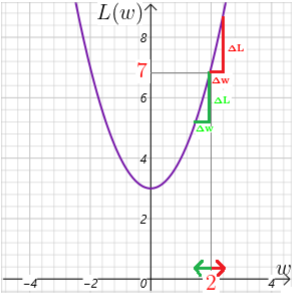

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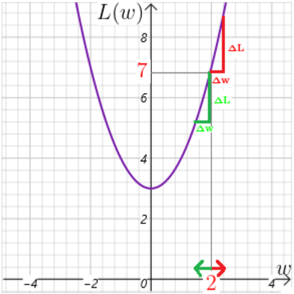

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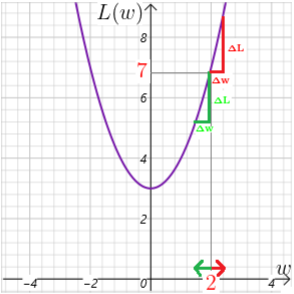

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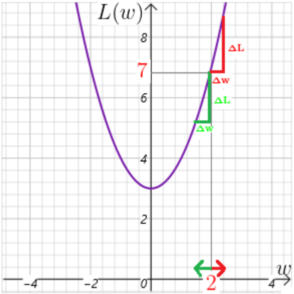
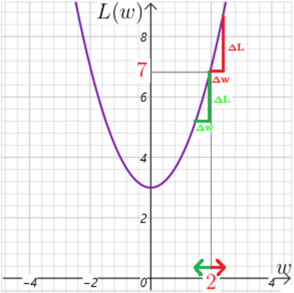


- How can we tweak the input w from it's current value of 2 so that the output L <u>decreases</u> from its current value of 7?
- w can be increased (move right) or decreased (move left) from the current value 2.
- Can we quantify the sensitivity of the output L w.r.t. small changes in the input w?

Sensitivity

Sensitivity



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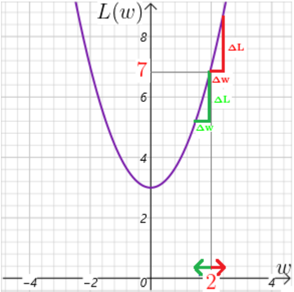

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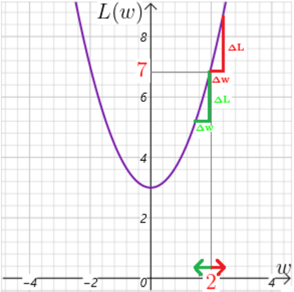
Sensitivity



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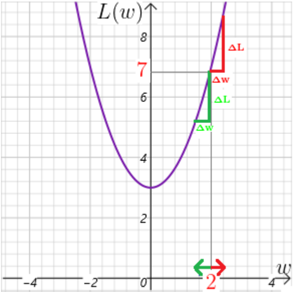

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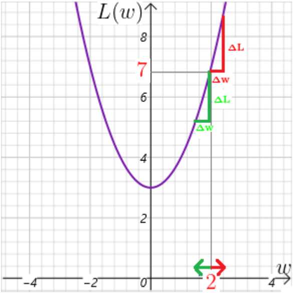

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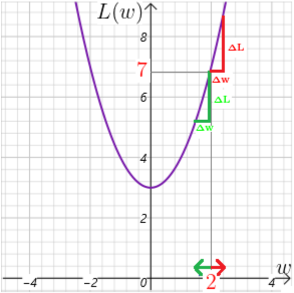
$$\frac{\text{change in output}}{\text{change in input}} : \begin{cases} \text{moving right} & = \\ & \end{cases}$$

Sensitivity



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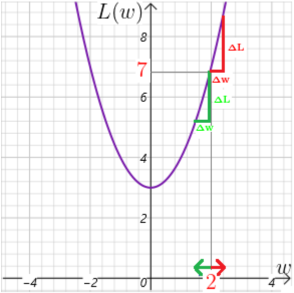

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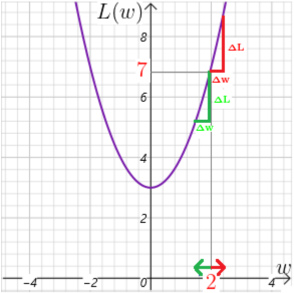

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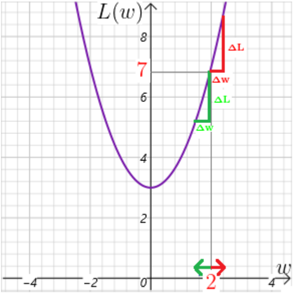

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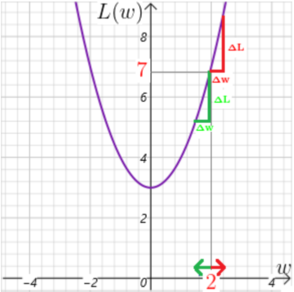

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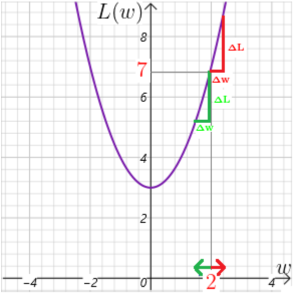

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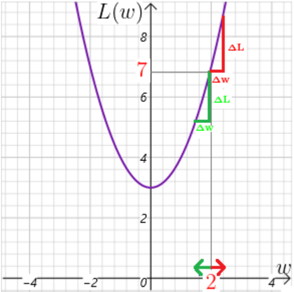

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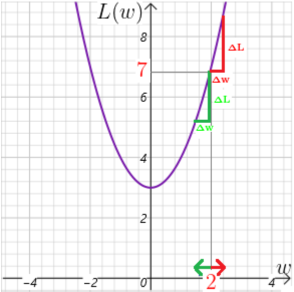

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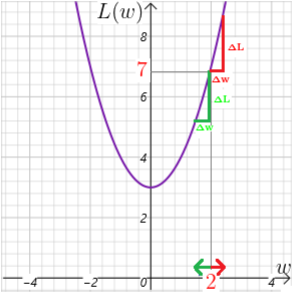

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$+ve$ sensitivity	w increases $\Rightarrow$ L increases & w decreases $\Rightarrow$ L decreases
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In this case, we move left (decrease) w to decrease L.

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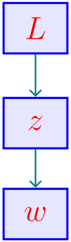

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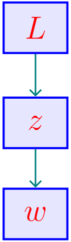

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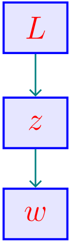

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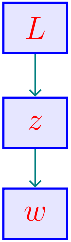

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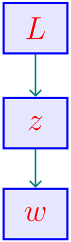

Gradient calculation using chain rule:

$$\begin{aligned} \nabla_w(L) &= \nabla_w(z) \times \nabla_z(L) \\ &= \nabla_w\left(1 + e^{-w}\right) \times \nabla_z\left(\frac{1}{z}\right) \\ &= \qquad\qquad -\frac{1}{z^2} \end{aligned}$$

Chain Rule

The chain rule is used to calculate gradients in a hierarchical way by using intermediate variable(s).

E.g. $L(w) = 1/\left(1 + e^{-w}\right) \xrightarrow{\text{use } z=1+e^{-w}} \begin{cases} L(z) & = 1/z, \\ z(w) & = 1 + e^{-w}. \end{cases}$

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&= \left[0 + \left(-e^{-w}\right)\right] \times -\frac{1}{z^2}
\end{aligned}$$

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&= \left[0 + \left(-e^{-w}\right)\right] \times -\frac{1}{z^2} = \frac{e^{-w}}{\left(1 + e^{-w}\right)^2}.
\end{aligned}$$

Gradient in Higher Dimensions

Gradient in Higher Dimensions

Gradient $\nabla_{input}(output)$ has shape $input\ shape \times output\ shape$:

Gradient in Higher Dimensions

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Function	I/O Shapes	Grad. Shape	Gradient

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Function	I/O Shapes	Grad. Shape	Gradient
$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$			

Gradient in Higher Dimensions

Gradient $\nabla_{input}(output)$ has shape $input\ shape \times output\ shape$:

Function	I/O Shapes	Grad. Shape	Gradient
$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$	$input : n$ $output : 1$		

Gradient in Higher Dimensions

Gradient $\nabla_{input}(output)$ has shape $input\ shape \times output\ shape$:

Function	I/O Shapes	Grad. Shape	Gradient
$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$	$input : n$ $output : 1$	$n \times 1$	

Gradient in Higher Dimensions

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Function	I/O Shapes	Grad. Shape	Gradient
$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$	$input : n$ $output : 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = 2\mathbf{w}$

Gradient in Higher Dimensions

Gradient $\nabla_{input}(output)$ has shape $input\ shape \times output\ shape$:

Function	I/O Shapes	Grad. Shape	Gradient
$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$	$input : n$ $output : 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = 2\mathbf{w}$
$L(\mathbf{w}) = \mathbf{x}^{\mathrm{T}}\mathbf{w}$ $\mathbf{x}$ known n-vector			

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$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$	$input:\ n$ $output:\ 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = 2\mathbf{w}$
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$L(\mathbf{w}) = \mathbf{x}^{\mathrm{T}}\mathbf{w}$ $\mathbf{x}$ known n-vector	$input : n$ $output : 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = \mathbf{x}$
$L(\mathbf{w}) = (w_1 - 2)^2 + (w_2 + 3)^2$			

Gradient in Higher Dimensions

Gradient $\nabla_{input}(output)$ has shape $input\ shape \times output\ shape$:

Function	I/O Shapes	Grad. Shape	Gradient
$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$	$input : n$ $output : 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = 2\mathbf{w}$
$L(\mathbf{w}) = \mathbf{x}^{\mathrm{T}}\mathbf{w}$ $\mathbf{x}$ known n-vector	$input : n$ $output : 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = \mathbf{x}$
$L(\mathbf{w}) = (w_1 - 2)^2 + (w_2 + 3)^2$	$input : 2$ $output : 1$		

Gradient in Higher Dimensions

Gradient $\nabla_{input}(output)$ has shape $input\ shape \times output\ shape$:

Function	I/O Shapes	Grad. Shape	Gradient
$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$	$input : n$ $output : 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = 2\mathbf{w}$
$L(\mathbf{w}) = \mathbf{x}^{\mathrm{T}}\mathbf{w}$ $\mathbf{x}$ known n-vector	$input : n$ $output : 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = \mathbf{x}$
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$\mathbf{l}(\mathbf{z}) = \mathbf{W}\mathbf{z}$ W known $p \times k$-matrix			

Gradient in Higher Dimensions

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$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$	$input : n$ $output : 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = 2\mathbf{w}$
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$L(\mathbf{w}) = (w_1 - 2)^2 + (w_2 + 3)^2$	$input : 2$ $output : 1$	2×1	$\nabla_{\mathbf{w}}(L) = \begin{bmatrix} \nabla_{w_1}(L) \\ \nabla_{w_2}(L) \end{bmatrix} = \begin{bmatrix} 2(w_1 - 2) \\ 2(w_2 + 3) \end{bmatrix}$
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Gradient in Higher Dimensions

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$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$	$input : n$ $output : 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = 2\mathbf{w}$
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Gradient in Higher Dimensions

MANIPAL ACADEMY of HIGHER EDUCATION (Institution of Eminence Deemed to be University)

Gradient $\nabla_{input}(output)$ has shape $input\ shape \times output\ shape$:

Function	I/O Shapes	Grad. Shape	Gradient
$L(\mathbf{w}) = \mathbf{w}^{\mathrm{T}}\mathbf{w}$	$input : n$ $output : 1$	$n \times 1$	$\nabla_{\mathbf{w}}(L) = 2\mathbf{w}$
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$L(\mathbf{w}) = (w_1 - 2)^2 + (w_2 + 3)^2$	$input : 2$ $output : 1$	2×1	$\nabla_{\mathbf{w}}(L) = \begin{bmatrix} \nabla_{w_1}(L) \\ \nabla_{w_2}(L) \end{bmatrix} = \begin{bmatrix} 2(w_1 - 2) \\ 2(w_2 + 3) \end{bmatrix}$
$\mathbf{l}(\mathbf{z}) = \mathbf{W}\mathbf{z}$ W known $p \times k$-matrix	$input : k \times 1$ $output : p \times 1$	$k \times p$	$\nabla_{\mathbf{z}}(\mathbf{l}) = \mathbf{W}^{\mathrm{T}}$

Gradient in Higher Dimensions

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$\mathbf{l}(\mathbf{z}) = \mathbf{W}\mathbf{z}$ W known $p \times k$-matrix	$input : k \times 1$ $output : p \times 1$	$k \times p$	$\nabla_{\mathbf{z}}(\mathbf{l}) = \mathbf{W}^{\mathrm{T}}$ note the transpose

Chain Rule Example in Higher Dimensions

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Chain Rule Example in Higher Dimensions

Calculate the gradient of $L(\mathbf{w}) = 1/\left(1 + e^{-\mathbf{w}^{\mathrm{T}}\mathbf{x}}\right)$:

Chain Rule Example in Higher Dimensions

Calculate the gradient of $L(\mathbf{w}) = 1/\left(1 + e^{-\mathbf{w}^{\mathrm{T}}\mathbf{x}}\right)$: $\begin{cases} & \\ z_2(\mathbf{w}) & = -\mathbf{w}^{\mathrm{T}}\mathbf{x}. \end{cases}$

Chain Rule Example in Higher Dimensions

Calculate the gradient of $L(\mathbf{w}) = 1/\left(1 + e^{-\mathbf{w}^{\mathrm{T}}\mathbf{x}}\right)$: $\begin{cases} z_1(z_2) &= 1 + e^{z_2}, \\ z_2(\mathbf{w}) &= -\mathbf{w}^{\mathrm{T}}\mathbf{x}. \end{cases}$

Chain Rule Example in Higher Dimensions

Calculate the gradient of $L(\mathbf{w}) = 1/\left(1 + e^{-\mathbf{w}^\mathrm{T}\mathbf{x}}\right)$: $\begin{cases} L(z_1) & = 1/z_1, \\ z_1(z_2) & = 1 + e^{z_2}, \\ z_2(\mathbf{w}) & = -\mathbf{w}^\mathrm{T}\mathbf{x}. \end{cases}$

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Computation graph:

Chain Rule Example in Higher Dimensions

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Computation graph:

$$\begin{tikzcd} \boxed{L} \arrow[d] \\ \boxed{z_1} \arrow[d] \\ \boxed{z_2} \arrow[d] \\ \boxed{\mathbf{w}} \end{tikzcd}$$

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Gradient calculation using chain rule:

Chain Rule Example in Higher Dimensions

Calculate the gradient of $L(\mathbf{w}) = 1/\left(1 + e^{-\mathbf{w}^\mathrm{T}\mathbf{x}}\right)$: $\begin{cases} L(z_1) &= 1/z_1, \\ z_1(z_2) &= 1 + e^{z_2}, \\ z_2(\mathbf{w}) &= -\mathbf{w}^\mathrm{T}\mathbf{x}. \end{cases}$

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Gradient calculation using chain rule:

$$\nabla_{\mathbf{w}}(L) =$$

Chain Rule Example in Higher Dimensions

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Computation graph:

$$\begin{tikzcd} \boxed{L} \arrow[d] \\ \boxed{z_1} \arrow[d] \\ \boxed{z_2} \arrow[d] \\ \boxed{\mathbf{w}} \end{tikzcd}$$

Gradient calculation using chain rule:

$\nabla_{\mathbf{w}}(L) =$ $\nabla_{z_1}(L)$

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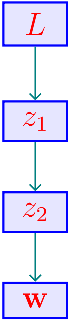
Calculate the gradient of $L(\mathbf{w}) = 1/\left(1 + e^{-\mathbf{w}^{\mathrm{T}}\mathbf{x}}\right)$: $\begin{cases} L(z_1) & = 1/z_1, \\ z_1(z_2) & = 1 + e^{z_2}, \\ z_2(\mathbf{w}) & = -\mathbf{w}^{\mathrm{T}}\mathbf{x}. \end{cases}$

Computation graph:



Gradient calculation using chain rule:

$$\nabla_{\mathbf{w}}(L) = \qquad \nabla_{z_2}(z_1) \times \nabla_{z_1}(L)$$

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Computation graph:

$$
\begin{tikzcd}
\boxed{L} \arrow[d] \\
\boxed{z_1} \arrow[d] \\
\boxed{z_2} \arrow[d] \\
\boxed{\mathbf{w}}
\end{tikzcd}
$$

Gradient calculation using chain rule:

$$\nabla_{\mathbf{w}}(L) = \nabla_{\mathbf{w}}(z_2) \times \nabla_{z_2}(z_1) \times \nabla_{z_1}(L)$$

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Gradient calculation using chain rule:

$$\nabla_{\mathbf{w}}(L) = \nabla_{\mathbf{w}}(z_2) \times \nabla_{z_2}(z_1) \times \nabla_{z_1}(L)$$

$$\nabla_{z_1}\left(\frac{1}{z_1}\right)$$

Chain Rule Example in Higher Dimensions

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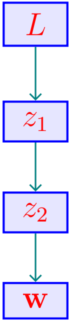

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$$\nabla_{z_1}\left(1 + e^{z_2}\right) \times \nabla_{z_1}\left(\frac{1}{z_1}\right)$$

Chain Rule Example in Higher Dimensions

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Gradient calculation using chain rule:

$$\begin{aligned} \nabla_{\mathbf{w}}(L) &= \nabla_{\mathbf{w}}(z_2) \times \nabla_{z_2}(z_1) \times \nabla_{z_1}(L) \\ &= \nabla_{\mathbf{w}}\left(-\mathbf{w}^{\mathrm{T}}\mathbf{x}\right) \times \nabla_{z_1}\left(1 + e^{z_2}\right) \times \nabla_{z_1}\left(\frac{1}{z_1}\right) \end{aligned}$$

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Calculate the gradient of $L(\mathbf{w}) = 1/\left(1 + e^{-\mathbf{w}^\mathrm{T}\mathbf{x}}\right)$: $\begin{cases} L(z_1) & = 1/z_1, \\ z_1(z_2) & = 1 + e^{z_2}, \\ z_2(\mathbf{w}) & = -\mathbf{w}^\mathrm{T}\mathbf{x}. \end{cases}$

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$$-1/z_1^2$$

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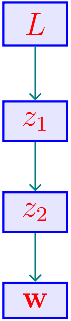

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$$= \nabla_{\mathbf{w}}\left(-\mathbf{w}^{\mathrm{T}}\mathbf{x}\right) \times \nabla_{z_1}\left(1 + e^{z_2}\right) \times \nabla_{z_1}\left(\frac{1}{z_1}\right)$$

$$e^{z_2} \times -1/z_1^2$$

Chain Rule Example in Higher Dimensions

Calculate the gradient of $L(\mathbf{w}) = 1/\left(1 + e^{-\mathbf{w}^{\mathrm{T}}\mathbf{x}}\right)$: $\begin{cases} L(z_1) & = 1/z_1, \\ z_1(z_2) & = 1 + e^{z_2}, \\ z_2(\mathbf{w}) & = -\mathbf{w}^{\mathrm{T}}\mathbf{x}. \end{cases}$

Computation graph:

$$\begin{tikzcd} \boxed{L} \arrow[d] \\ \boxed{z_1} \arrow[d] \\ \boxed{z_2} \arrow[d] \\ \boxed{\mathbf{w}} \end{tikzcd}$$

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Gradient Descent Method

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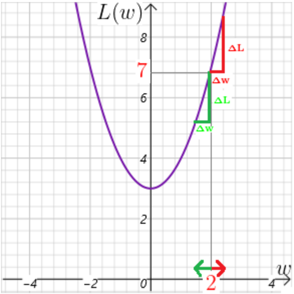

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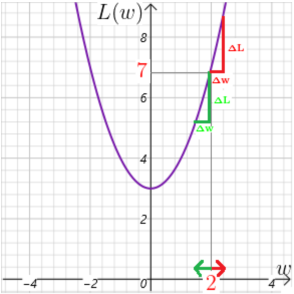

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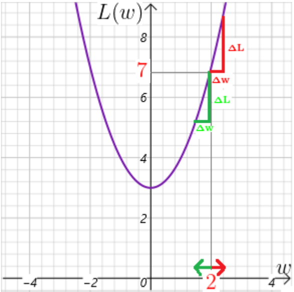

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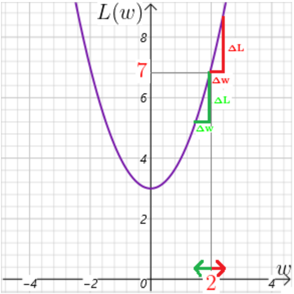

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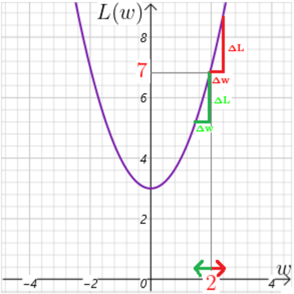

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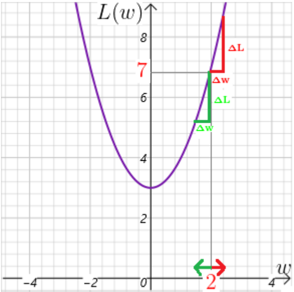

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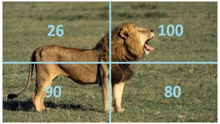

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- This helps in simplifying gradient calculations for computing optimal weights and bias values without having to account for the bias separately.

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- The ith sample's softmax loss is $-\log\left(\left[\text{softmax}\left(\mathbf{z}^{(i)}\right)\right]_{y^{(i)}}\right)$.

Softmax Classifier Gradient

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- The average training softmax loss

$$L(\mathbf{W}) = \frac{1}{n}\sum_{i=1}^{n} L_i = \frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{j=1}^{k} e^{\mathbf{w}_j^{\mathrm{T}}\mathbf{x}^{(i)}}}\right).$$

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- The gradient $\nabla_{\mathbf{W}}(L)$ has shape $\underbrace{k \times (p+1)}_{\text{input shape}} \times \underbrace{1}_{\text{output shape}} = k \times (p+1)$.

Softmax Classifier Gradient

- The average training softmax loss

$$L(\mathbf{W}) = \frac{1}{n}\sum_{i=1}^{n} L_i = \frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{j=1}^{k} e^{\mathbf{w}_j^{\mathrm{T}}\mathbf{x}^{(i)}}}\right).$$

- The gradient $\nabla_{\mathbf{W}}(L)$ has shape $\underbrace{k \times (p+1)}_{\text{input shape}} \times \underbrace{1}_{\text{output shape}} = k \times (p+1)$.

- Weights matrix

Softmax Classifier Gradient

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- The average training softmax loss

$$L(\mathbf{W}) = \frac{1}{n}\sum_{i=1}^{n} L_i = \frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{j=1}^{k} e^{\mathbf{w}_j^{\mathrm{T}}\mathbf{x}^{(i)}}}\right).$$

- The gradient $\nabla_{\mathbf{W}}(L)$ has shape $\underbrace{k \times (p+1)}_{\text{input shape}} \times \underbrace{1}_{\text{output shape}} = k \times (p+1)$.

- Weights matrix $\mathbf{W} = \begin{bmatrix} \underbrace{\mathbf{w}_1}_{1\times(p+1)}^{\mathrm{T}} \\ \underbrace{\mathbf{w}_2}_{1\times(p+1)}^{\mathrm{T}} \\ \vdots \\ \underbrace{\mathbf{w}_k}_{1\times(p+1)}^{\mathrm{T}} \end{bmatrix}$,

Softmax Classifier Gradient

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$$L(\mathbf{W}) = \frac{1}{n}\sum_{i=1}^{n} L_i = \frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{j=1}^{k} e^{\mathbf{w}_j^{\mathrm{T}}\mathbf{x}^{(i)}}}\right).$$

- The gradient $\nabla_{\mathbf{W}}(L)$ has shape $\underbrace{k \times (p+1)}_{\text{input shape}} \times \underbrace{1}_{\text{output shape}} = k \times (p+1)$.

- Weights matrix $\mathbf{W} = \begin{bmatrix} \underbrace{\mathbf{w}_1^{\mathrm{T}}}_{1\times(p+1)} \\ \underbrace{\mathbf{w}_2^{\mathrm{T}}}_{1\times(p+1)} \\ \vdots \\ \underbrace{\mathbf{w}_k^{\mathrm{T}}}_{1\times(p+1)} \end{bmatrix}$, gradient $\nabla_{\mathbf{W}}(L) = \begin{bmatrix} \underbrace{\left(\nabla_{\mathbf{w}_1}(L)\right)^{\mathrm{T}}}_{1\times(p+1)} \\ \underbrace{\left(\nabla_{\mathbf{w}_2}(L)\right)^{\mathrm{T}}}_{1\times(p+1)} \\ \vdots \\ \underbrace{\left(\nabla_{\mathbf{w}_k}(L)\right)^{\mathrm{T}}}_{1\times(p+1)} \end{bmatrix}$.

Softmax Classifier Gradient

- The average training softmax loss
$$L(\mathbf{W}) = \frac{1}{n}\sum_{i=1}^{n} L_i = \frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{j=1}^{k} e^{\mathbf{w}_j^{\mathrm{T}}\mathbf{x}^{(i)}}}\right).$$
- The gradient $\nabla_{\mathbf{W}}(L)$ has shape $\underbrace{k \times (p+1)}_{\text{input shape}} \times \underbrace{1}_{\text{output shape}} = k \times (p+1)$.
- Weights matrix $\mathbf{W} = \begin{bmatrix} \underbrace{\mathbf{w}_1{}^{\mathrm{T}}}_{1\times(p+1)} \\ \underbrace{\mathbf{w}_2{}^{\mathrm{T}}}_{1\times(p+1)} \\ \vdots \\ \underbrace{\mathbf{w}_k{}^{\mathrm{T}}}_{1\times(p+1)} \end{bmatrix}$, gradient $\nabla_{\mathbf{W}}(L) = \begin{bmatrix} \underbrace{\left(\nabla_{\mathbf{w}_1}(L)\right)^{\mathrm{T}}}_{1\times(p+1)} \\ \underbrace{\left(\nabla_{\mathbf{w}_2}(L)\right)^{\mathrm{T}}}_{1\times(p+1)} \\ \vdots \\ \underbrace{\left(\nabla_{\mathbf{w}_k}(L)\right)^{\mathrm{T}}}_{1\times(p+1)} \end{bmatrix}$.

focus on term like this

Softmax Classifier Gradient – continued

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L)$$

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right)$$

Softmax Classifier Gradient – continued

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$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

$$\log(a/b) = \log(a) - \log(b)$$

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

$$= -\frac{1}{n}\nabla_{\mathbf{w}_j}\left(\sum_{i=1}^{n}\left[\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right) - \log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right]\right)$$

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

$$= -\frac{1}{n}\nabla_{\mathbf{w}_j}\left(\sum_{i=1}^{n}\left[\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right) - \log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right]\right)$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

Softmax Classifier Gradient – continued

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$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

$$= -\frac{1}{n}\nabla_{\mathbf{w}_j}\left(\sum_{i=1}^{n}\left[\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right) - \log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right]\right)$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

$$\log(e^a) = a$$

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

$$= -\frac{1}{n}\nabla_{\mathbf{w}_j}\left(\sum_{i=1}^{n}\left[\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right) - \log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right]\right)$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

$$= -\frac{1}{n}\nabla_{\mathbf{w}_j}\left(\sum_{i=1}^{n}\left[\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right) - \log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right]\right)$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

$$\begin{cases} = \mathbf{x}^{(i)} & \text{if } j = y^{(i)}, \\ = 0 & \text{if } j \neq y^{(i)}. \end{cases}$$

Softmax Classifier Gradient – continued

$$\begin{aligned}
\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) &= \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right) \\
&= -\frac{1}{n}\nabla_{\mathbf{w}_j}\left(\sum_{i=1}^{n}\left[\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right) - \log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right]\right) \\
&= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right] \\
&= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]
\end{aligned}$$

$$I\left(y^{(i)} = j\right)\mathbf{x}^{(i)}$$

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

$$= -\frac{1}{n}\nabla_{\mathbf{w}_j}\left(\sum_{i=1}^{n}\left[\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right) - \log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right]\right)$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

use chain rule: $\begin{cases} L &= \log(z), \\ z &= \sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}. \end{cases}$

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

$$= -\frac{1}{n}\nabla_{\mathbf{w}_j}\left(\sum_{i=1}^{n}\left[\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right) - \log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right]\right)$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}\right) - \underbrace{\nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)}_{\boxed{\frac{e^{\mathbf{w}_j^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}}\mathbf{x}^{(i)}}\right]$$

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

$$= -\frac{1}{n}\nabla_{\mathbf{w}_j}\left(\sum_{i=1}^{n}\left[\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right) - \log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right]\right)$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[-I\left(y^{(i)} = j\right)\mathbf{x}^{(i)} + \hat{p}_{ji}\mathbf{x}^{(i)}\right]$$

Softmax Classifier Gradient – continued

$$\nabla_{\mathbf{w}_j}(L) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} L_i\right) = \nabla_{\mathbf{w}_j}\left(\frac{1}{n}\sum_{i=1}^{n} -\log\left(\frac{e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}}{\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}}\right)\right)$$

$$= -\frac{1}{n}\nabla_{\mathbf{w}_j}\left(\sum_{i=1}^{n}\left[\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right) - \log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right]\right)$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\log\left(e^{\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left[\nabla_{\mathbf{w}_j}\left(\mathbf{w}_{y^{(i)}}^{\mathrm{T}}\mathbf{x}^{(i)}\right) - \nabla_{\mathbf{w}_j}\left(\log\left(\sum_{r=1}^{k} e^{\mathbf{w}_r^{\mathrm{T}}\mathbf{x}^{(i)}}\right)\right)\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[-I\left(y^{(i)} = j\right)\mathbf{x}^{(i)} + \underbrace{\hat{p}_{ji}}_{\text{predicted probability that sample-}i\text{ belongs to class }j}\mathbf{x}^{(i)}\right]$$

Softmax Classifier Gradient – continued

Softmax Classifier Gradient – continued

$$\Rightarrow \nabla_{\mathbf{W}}(L) = \begin{bmatrix} \left(\nabla_{\mathbf{w}_1}(L)\right)^{\mathrm{T}} \\ \vdots \\ \left(\nabla_{\mathbf{w}_j}(L)\right)^{\mathrm{T}} \\ \vdots \\ \left(\nabla_{\mathbf{w}_k}(L)\right)^{\mathrm{T}} \end{bmatrix}$$

Softmax Classifier Gradient – continued

$$\Rightarrow \nabla_{\mathbf{W}}(L) = \begin{bmatrix} \left(\nabla_{\mathbf{w}_1}(L)\right)^{\mathrm{T}} \\ \vdots \\ \left(\nabla_{\mathbf{w}_j}(L)\right)^{\mathrm{T}} \\ \vdots \\ \left(\nabla_{\mathbf{w}_k}(L)\right)^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{n}\sum_{i=1}^{n}\left[\hat{p}_{1i} - I\left(y^{(i)} = 1\right)\right]\mathbf{x}^{(i)}\right)^{\mathrm{T}} \\ \vdots \\ \left(\frac{1}{n}\sum_{i=1}^{n}\left[\hat{p}_{ji} - I\left(y^{(i)} = j\right)\right]\mathbf{x}^{(i)}\right)^{\mathrm{T}} \\ \vdots \\ \left(\frac{1}{n}\sum_{i=1}^{n}\left[\hat{p}_{ki} - I\left(y^{(i)} = k\right)\right]\mathbf{x}^{(i)}\right)^{\mathrm{T}} \end{bmatrix}$$

Softmax Classifier Gradient – continued

$$\Rightarrow \nabla_{\mathbf{W}}(L) = \begin{bmatrix} \left(\nabla_{\mathbf{w}_1}(L)\right)^{\mathrm{T}} \\ \vdots \\ \left(\nabla_{\mathbf{w}_j}(L)\right)^{\mathrm{T}} \\ \vdots \\ \left(\nabla_{\mathbf{w}_k}(L)\right)^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{n}\sum_{i=1}^{n}\left[\hat{p}_{1i} - I\left(y^{(i)} = 1\right)\right]\mathbf{x}^{(i)}\right)^{\mathrm{T}} \\ \vdots \\ \left(\frac{1}{n}\sum_{i=1}^{n}\left[\hat{p}_{ji} - I\left(y^{(i)} = j\right)\right]\mathbf{x}^{(i)}\right)^{\mathrm{T}} \\ \vdots \\ \left(\frac{1}{n}\sum_{i=1}^{n}\left[\hat{p}_{ki} - I\left(y^{(i)} = k\right)\right]\mathbf{x}^{(i)}\right)^{\mathrm{T}} \end{bmatrix} = \frac{1}{n}\sum_{i=1}^{n}\left(\begin{bmatrix} \left[\hat{p}_{1i} - I\left(y^{(i)} = 1\right)\right]\mathbf{x}^{(i)\,\mathrm{T}} \\ \vdots \\ \left[\hat{p}_{ji} - I\left(y^{(i)} = j\right)\right]\mathbf{x}^{(i)\,\mathrm{T}} \\ \vdots \\ \left[\hat{p}_{ki} - I\left(y^{(i)} = k\right)\right]\mathbf{x}^{(i)\,\mathrm{T}} \end{bmatrix}\right)$$

Softmax Classifier Gradient – continued

$$\Rightarrow \nabla_{\mathbf{W}}(L) = \begin{bmatrix} \left(\nabla_{\mathbf{w}_1}(L)\right)^{\mathrm{T}} \\ \vdots \\ \left(\nabla_{\mathbf{w}_j}(L)\right)^{\mathrm{T}} \\ \vdots \\ \left(\nabla_{\mathbf{w}_k}(L)\right)^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{n}\sum_{i=1}^{n}\left[\hat{p}_{1i} - I\left(y^{(i)}=1\right)\right]\mathbf{x}^{(i)}\right)^{\mathrm{T}} \\ \vdots \\ \left(\frac{1}{n}\sum_{i=1}^{n}\left[\hat{p}_{ji} - I\left(y^{(i)}=j\right)\right]\mathbf{x}^{(i)}\right)^{\mathrm{T}} \\ \vdots \\ \left(\frac{1}{n}\sum_{i=1}^{n}\left[\hat{p}_{ki} - I\left(y^{(i)}=k\right)\right]\mathbf{x}^{(i)}\right)^{\mathrm{T}} \end{bmatrix} = \frac{1}{n}\sum_{i=1}^{n}\left(\begin{bmatrix} \left[\hat{p}_{1i} - I\left(y^{(i)}=1\right)\right]\mathbf{x}^{(i)\mathrm{T}} \\ \vdots \\ \left[\hat{p}_{ji} - I\left(y^{(i)}=j\right)\right]\mathbf{x}^{(i)\mathrm{T}} \\ \vdots \\ \left[\hat{p}_{ki} - I\left(y^{(i)}=k\right)\right]\mathbf{x}^{(i)\mathrm{T}} \end{bmatrix}\right)$$

$$= \frac{1}{n}\left(\begin{bmatrix} \hat{p}_{11} - I\left(y^{(1)}=1\right) \\ \vdots \\ \hat{p}_{j1} - I\left(y^{(1)}=j\right) \\ \vdots \\ \hat{p}_{k1} - I\left(y^{(1)}=k\right) \end{bmatrix}\mathbf{x}^{(1)\mathrm{T}} + \begin{bmatrix} \hat{p}_{12} - I\left(y^{(2)}=1\right) \\ \vdots \\ \hat{p}_{j2} - I\left(y^{(2)}=j\right) \\ \vdots \\ \hat{p}_{k2} - I\left(y^{(2)}=k\right) \end{bmatrix}\mathbf{x}^{(2)\mathrm{T}} + \cdots + \begin{bmatrix} \hat{p}_{1n} - I\left(y^{(n)}=1\right) \\ \vdots \\ \hat{p}_{jn} - I\left(y^{(n)}=j\right) \\ \vdots \\ \hat{p}_{kn} - I\left(y^{(n)}=k\right) \end{bmatrix}\mathbf{x}^{(n)\mathrm{T}}\right)$$

Softmax Classifier Gradient – continued

Softmax Classifier Gradient – continued

$$\Rightarrow \nabla_{\mathbf{w}}(L) = \frac{1}{n}\begin{bmatrix} \hat{p}_{11} - I\left(y^{(1)} = 1\right) & \hat{p}_{12} - I\left(y^{(2)} = 1\right) & \dots & \hat{p}_{1n} - I\left(y^{(n)} = 1\right) \\ \vdots & \vdots & \vdots & \\ \hat{p}_{j1} - I\left(y^{(1)} = j\right) & \hat{p}_{j2} - I\left(y^{(2)} = j\right) & \dots & \hat{p}_{jn} - I\left(y^{(n)} = j\right) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{p}_{k1} - I\left(y^{(1)} = k\right) & \hat{p}_{k2} - I\left(y^{(2)} = k\right) & \dots & \hat{p}_{kn} - I\left(y^{(n)} = k\right) \end{bmatrix} \begin{bmatrix} \mathbf{x}^{(1)^{\mathrm{T}}} \\ \mathbf{x}^{(2)^{\mathrm{T}}} \\ \vdots \\ \vdots \\ \mathbf{x}^{(n)^{\mathrm{T}}} \end{bmatrix}$$

Softmax Classifier Gradient – continued

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$$

$$
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for each sample, correct class predicted probability minus one; incorrect class predicted probabilities untouched

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check shape : $(k \times n) \times (n \times (p+1)) = (k \times (p+1))$-matrix

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Logistic Regression Classifier Setup

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- But how do we get the predicted probability that a sample belongs to its correct class?

Logistic Regression Classifier Setup – continued

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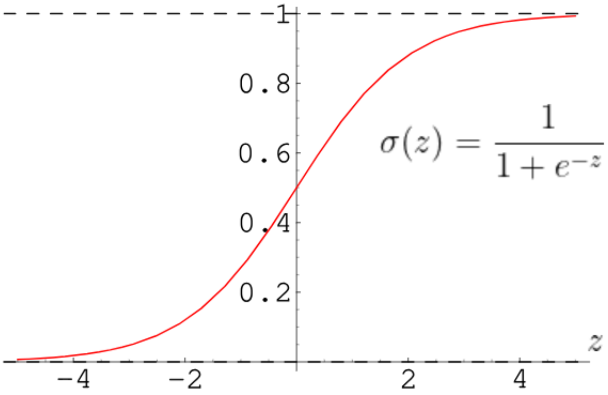

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- Note that we can write this compactly as

$$\hat{y}^{(i)} = \left(\sigma\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}^{(i)}\right)\right)^{y^{(i)}} \left(1 - \sigma\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}^{(i)}\right)\right)^{1-y^{(i)}}.$$

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sigmoid broadcasted to all elements of vector

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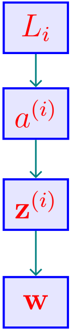

Why Regularizarization?

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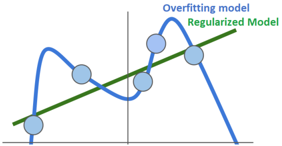
Potentially overfit feature

Oil	Density	Crispy	Fracture	Hardness	Taste
16.5	2955	10	23	97	fair
17.7	2660	14	9	139	excellent
16.2	2870	12	17	143	poor
16.7	2920	10	31	95	good
16.3	2975	11	26	143	fair
19.1	2790	13	16	189	good
18.4	2750	13	17	114	poor



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- λ is a hyperparameter that has to be tuned.

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adding to the average training data loss constraints on the weights (not to the bias values):

$$\underbrace{\frac{1}{n}\sum_{i=1}^{n} L_i(\mathbf{W})}_{\text{average training data loss}} + \begin{cases} \lambda \sum_{r,s} |w_{rs}|^2 \\ (\text{ridge or } L_2) \\ \quad \text{or} \\ \lambda \sum_{r,s} |w_{rs}| \\ (\text{lasso or } L_1) \\ \quad \text{or} \\ \lambda \left[\alpha \sum_{r,s} |w_{rs}| + (1-\alpha) \sum_{r,s} |w_{rs}|^2\right] \\ (\text{elastic-net}), \end{cases}$$

- λ is a hyperparameter that has to be tuned.
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- All regularization approaches tend to drive the weight values close to zero.
- L_1-regularization typically results in a smaller subset of nonzero weights than L_2-regularization.

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- An epoch is when the weights have been updated using all the training samples through batches; that is, the model has seen all the training samples once.