RHEINISCHE FRIEDRICH-WILHELMS-UNIVERSITÄT BONN
COMPUTER SCIENCE VI AUTONOMOUS INTELLIGENT SYSTEMS
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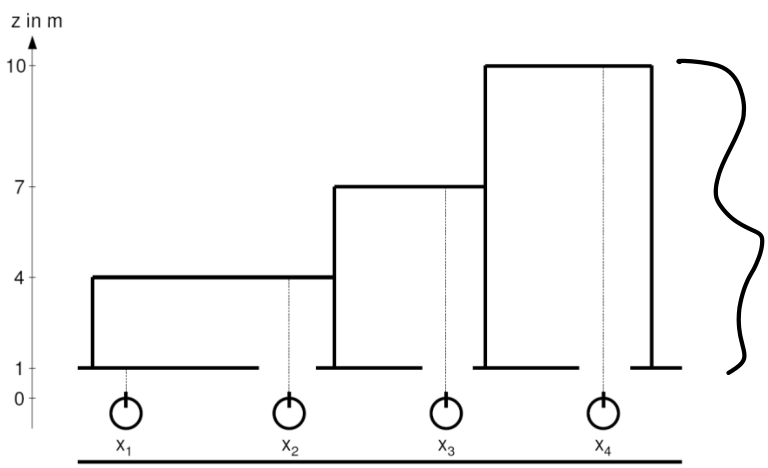
Friedrich-Hirzebruch-Allee 8

Cognitive Robotics

Assignment 2

Due Tuesday, October 31st, before class.

2.1) A robot moves along the middle of a corridor with a given accurate map, as depicted in the figure. At some of the given locations x_i it takes measurements z_k of the forward distance, using one laser beam. Every measurement is corrupted only with additive Gaussian noise $N(\mu, \sigma^2)$ with μ = -0.1 m and σ= (0.25 + 0.05 z_k) m. The scanner range is assumed to be unlimited.



The measured distances are: z_1 = 7.3 m, z_2 = 10.1 m, z_3 = 3.8 m, z_4 = 1.2 m.
The mapping between z_k and x_i is unknown.

(a) For each measurement, determine the most likely robot pose by calculating the probabilities for each position given the measurement using the Bayes rule! Assume an uniformly distributed prior!
The evidence term (denominator) can be neglected, but the probabilities should be scaled such that $\sum_{i=1}^{4} P(x_i|z) = 1$. 4 points

(b) The robot believes that taking measurements at the positions x_2 and x_3 is in general two times as likely as doing so at x_1 and x_4. Use this prior information to recalculate the probabilities of (a)! 4 points

(c) Suppose the laser scanner is not as ideal as above, and reports a faulty measurement of z = 10 m in 10% of all cases, no matter the actual distance.
How does this change the results of (a) and (b)? 4 points

2.2) The robot has a belief about its location x_i:
$p(x_{-1}) = 0$, $p(x_0)=0$, $p(x_1) = 0.1$, $p(x_2) = 0.2$, $p(x_3) = 0.5$, $p(x_4) = 0.2$, $p(x_5) = 0$, $p(x_6) = 0$.
Now it wants to move to the previous location, but this results with probability .1 in no motion, with probability .6 in the intended motion and with probability .2 in moving one location more than intended. Location x_0 is the predecessor of location x_1; and x_{-1} is predecessor of x_0.
What is the probability of being in the locations x_i , i=-1, …, 6 after the move? 4 points

2.3) Compare the Extended Kalman Filter (EKF) to the Unscented Kalman Filter (UKF)! What are commonalities and differences?

4 points

Q1) $\int p(x \mid z_1)\, dx \Rightarrow 1$

$$\Rightarrow \sum_{i=1}^{4} p(x_i \mid z_1) = 1$$

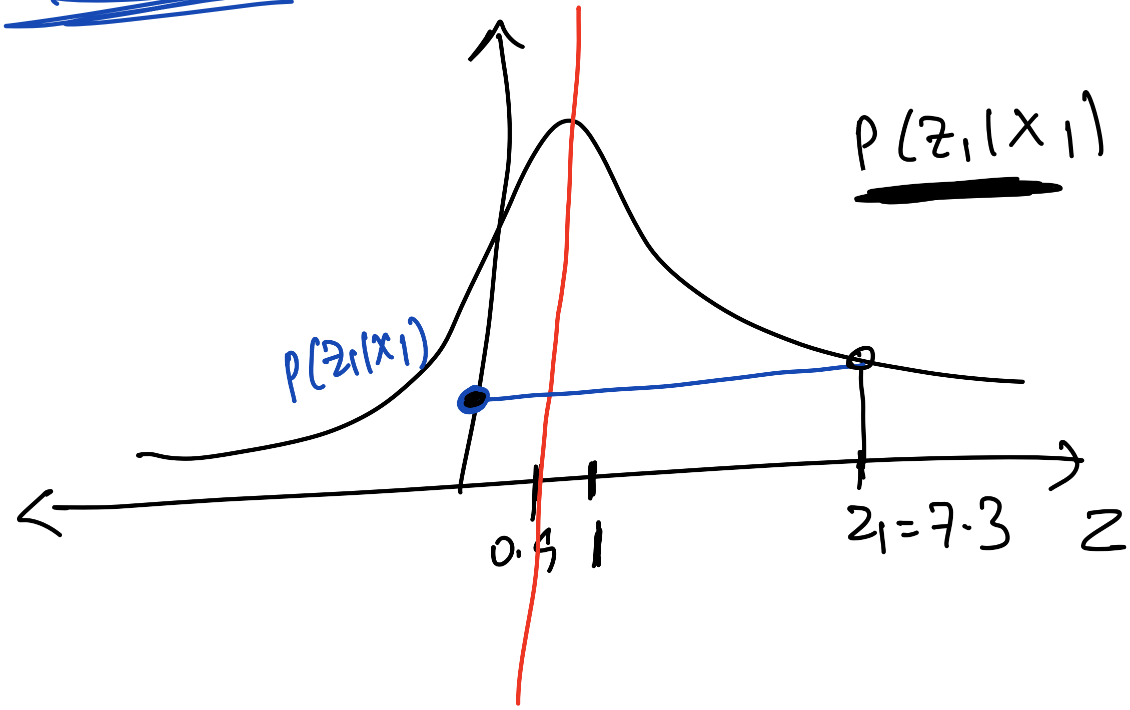
$$p(x_i \mid z_1) = \frac{p(z_1 \mid x_i)\, \underline{p(x_i)}}{p(z_1)}$$

$$\boxed{p(x_i) \Rightarrow 1/4}$$

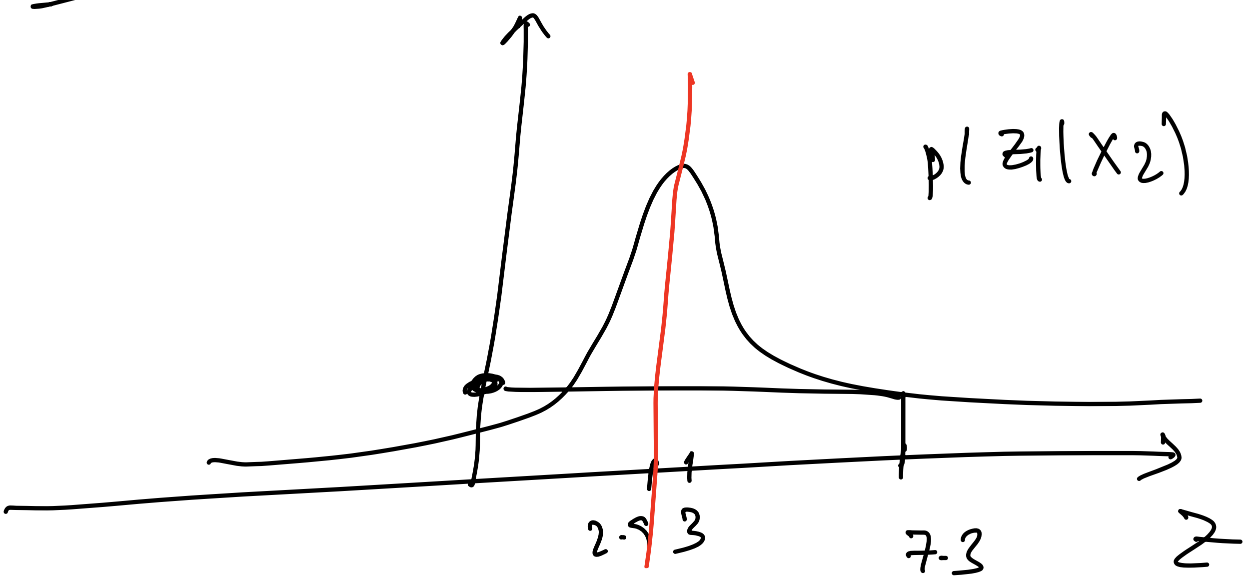
$p(z_1)$ = unknown

$$\Rightarrow p(x_1 \mid z_1) = \frac{\underline{p(z_1 \mid x_1)}\, p(x_1)}{p(z_1)}$$

Given x_1



Given x_2



Given z_1, find the best $\underline{x}$

$$P(x|z_1) = \frac{P(z_1|x)\,P(x)}{P(z_1)}$$

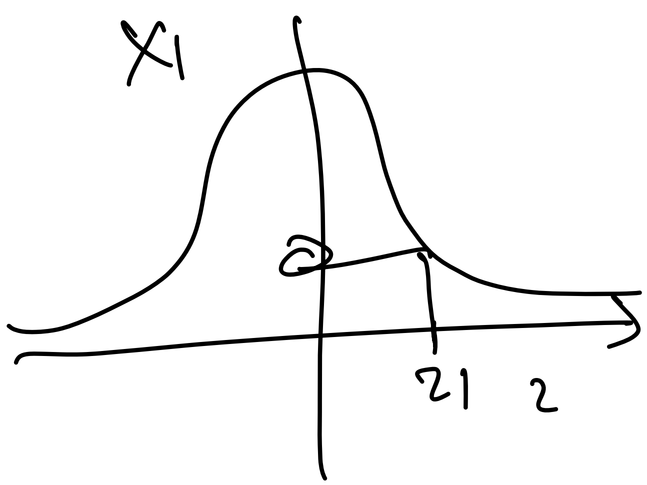
consider $P(x_1|z_1)$, $P(x_2|z_1)$, $P(x_3, z_1)$, $P(x_4|z_1)$

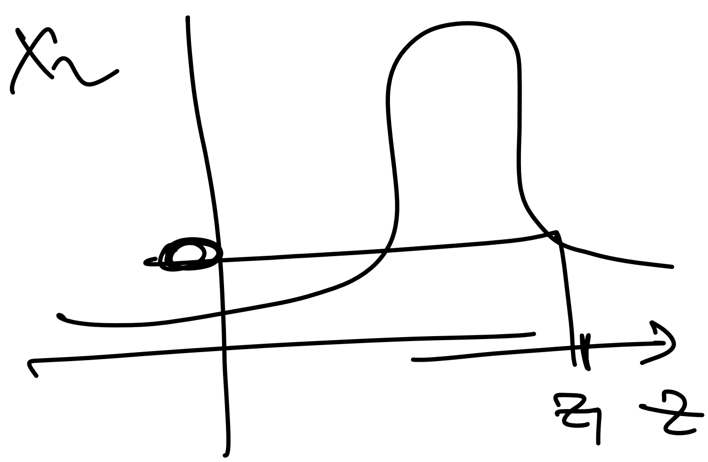
divide by $[P(x)/P(z_1)]$ for all these

order wont change of size

$$\Rightarrow \underbrace{P(\underline{z_1}|x_1),\ P(z_1|x_2),\ P(z_1|x_3),\ P(z_1, x_4)}$$

likelihood also has the same order of probability as the posterior





$$p(z_1|x_1) \Rightarrow p(z = z_1 | x_1)$$

$$\Rightarrow \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_1-\mu}{\sigma}\right)^2}$$

$$\mu = 1-0.1$$

$$\sigma = 0.25 + (0.05)\,7.3$$

$$p(z_1|x_1) \approx 0$$

$$p(z_1|x_2) \approx 0$$

$$p(z_1|x_3) \approx 0.525$$

$$p(z_1|x_4) \approx 0$$

$\Rightarrow$ So, for "z_1" $\Rightarrow$ x_3 is the most probable position

$\Rightarrow$ so, for "z_2" $\Rightarrow$ $\boxed{0.51}$ x_4 is the most probable

$\Rightarrow$ so, for "z_3" $\Rightarrow$ "x_2" is the most probable

$\Rightarrow$ so, for "z_4" $\Rightarrow$ "x_1" is the most probable

z_1	z_2	z_3	z_4
x_3	x_4	x_2	x_1

(5) New prior $\Rightarrow$ a $2a$ $2a$ a $= 1$

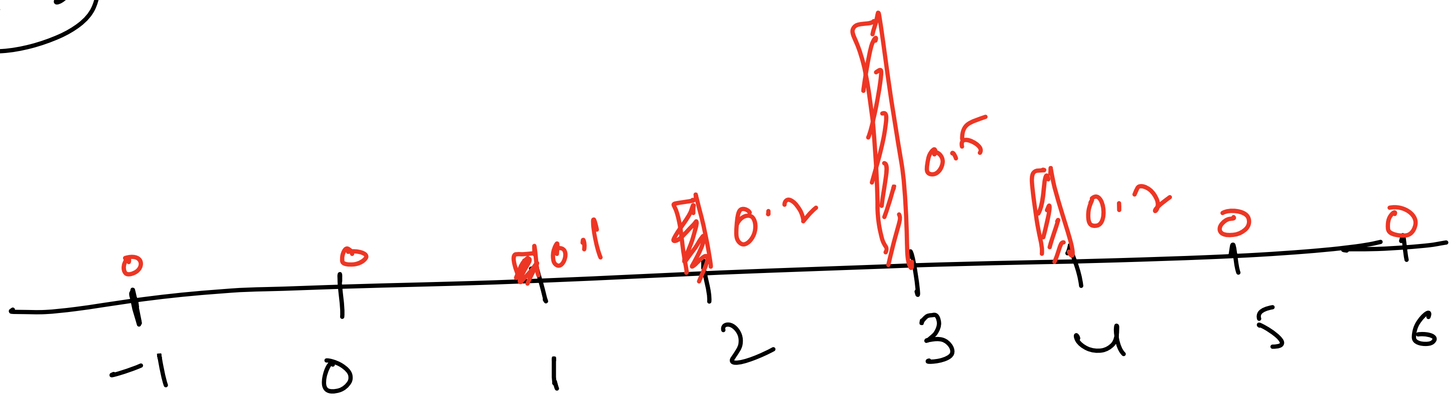
$\Rightarrow a = 1/6$

$$\boxed{\Rightarrow 1/6,\ 1/3,\ 1/3,\ 1/6}$$

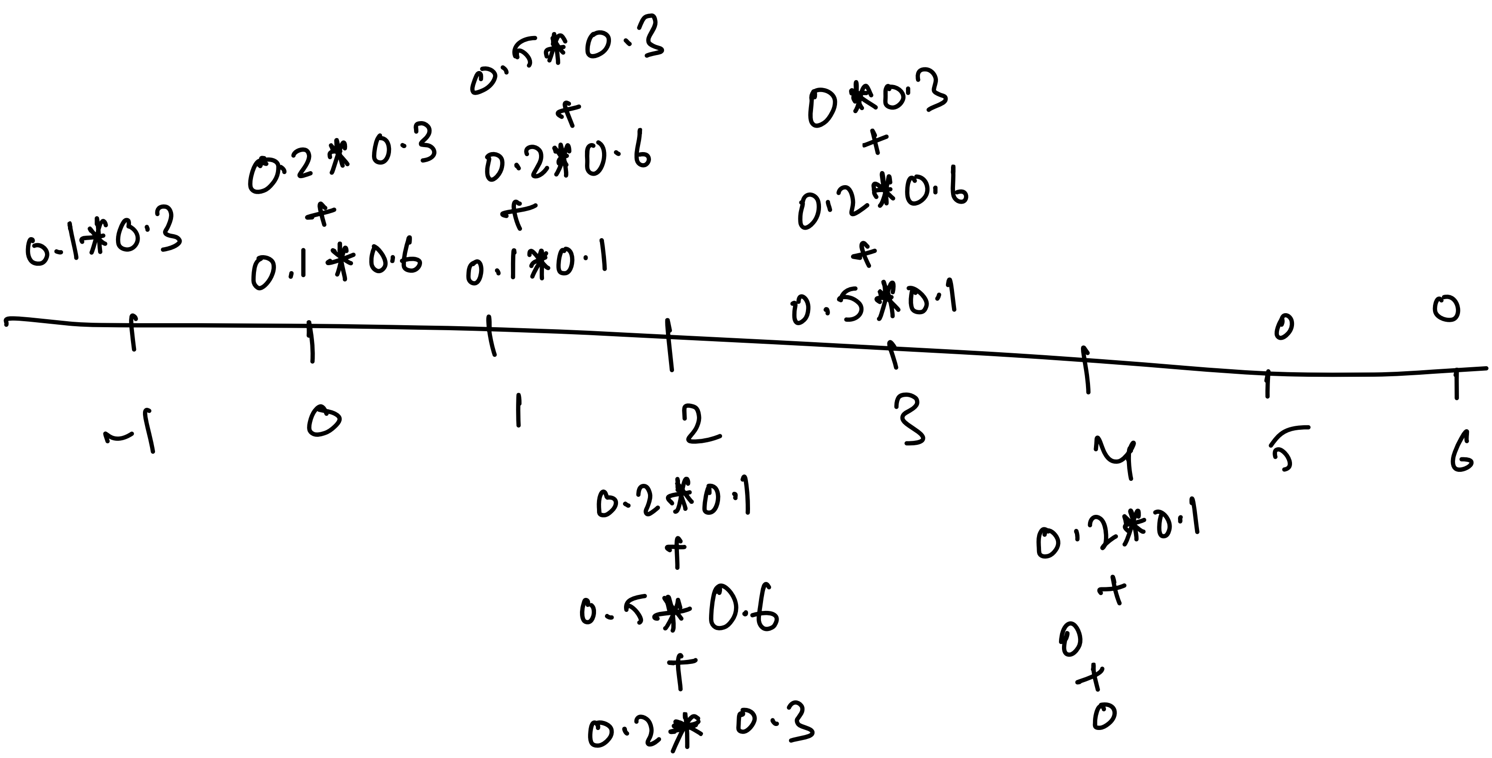
Based on the prev question, our likelihood is so biased that for other poses other than the correct $\Rightarrow$ the probabilities are so low.

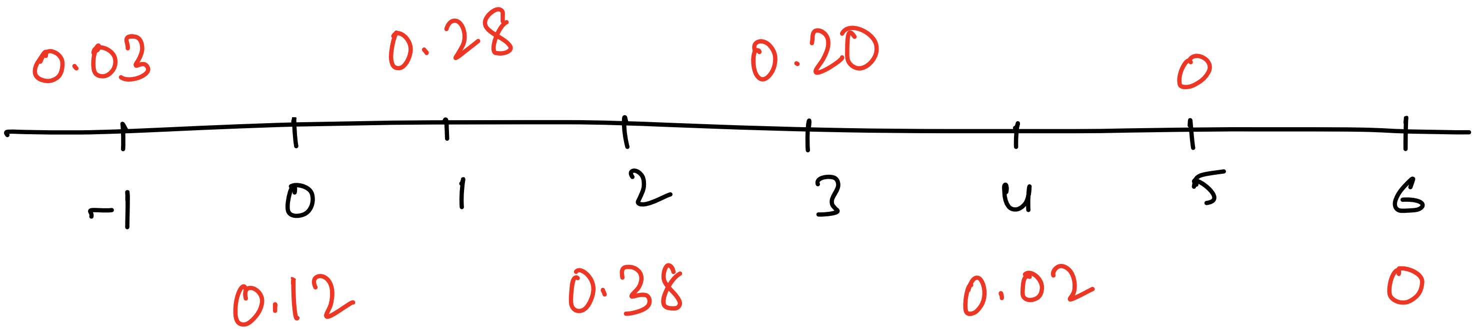
we. think the order will remain the same.

2.2



$$u = -1 \begin{cases} 0.1 \Rightarrow \text{no motion} \\ 0.6 \Rightarrow -1 \\ 0.3 \Rightarrow -2 \end{cases}$$





$$\Rightarrow \frac{[0.03,\ 0.12,\ 0.28,\ 0.38,\ 0.20,\ 0.02,\ 0,\ 0]}{103}$$