UNIVERSITÄT BONN
AIS
RHEINISCHE FRIEDRICH-WILHELMS-UNIVERSITÄT BONN
COMPUTER SCIENCE VI AUTONOMOUS INTELLIGENT SYSTEMS
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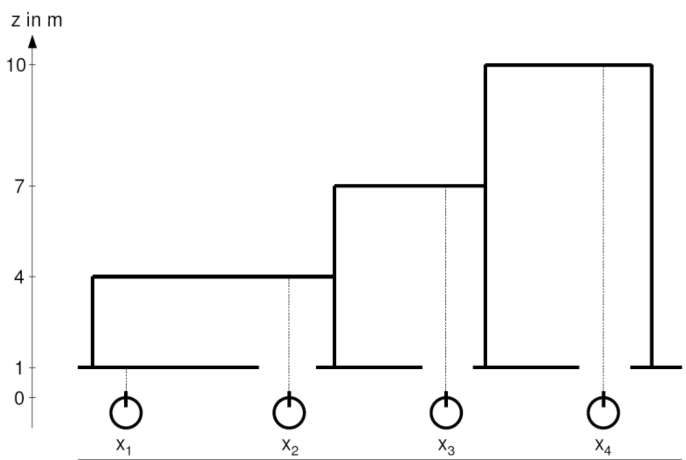
Cognitive Robotics

Assignment 2

Due Tuesday, October 31st, before class.

2.1) A robot moves along the middle of a corridor with a given accurate map, as depicted in the figure. At some of the given locations x_i it takes measurements z_k of the forward distance, using one laser beam. Every measurement is corrupted only with additive Gaussian noise $N(\mu, \sigma^2)$ with μ = -0.1 m and σ= (0.25 + 0.05 z_k) m. The scanner range is assumed to be unlimited.



The measured distances are: z_1 = 7.3 m, z_2 = 10.1 m, z_3 = 3.8 m, z_4 = 1.2 m. Zk

The mapping between z_k and x_i is unknown.

(a) For each measurement, determine the most likely robot pose by calculating the probabilities for each position given the measurement using the Bayes rule! Assume an uniformly distributed prior!
The evidence term (denominator) can be neglected, but the probabilities should be scaled such that $\sum_{i=1}^{4} P(x_i|z) = 1$. 4 points

(b) The robot believes that taking measurements at the positions x_2 and x_3 is in general two times as likely as doing so at x_1 and x_4. Use this prior information to recalculate the probabilities of (a)! 4 points

(c) Suppose the laser scanner is not as ideal as above, and reports a faulty measurement of z = 10 m in 10% of all cases, no matter the actual distance.
How does this change the results of (a) and (b)? 4 points

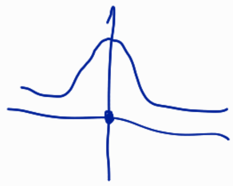
2.2) The robot has a belief about its location x_i:
$p(x_{-1}) = 0$, $p(x_0)=0$, $p(x_1) = 0.1$, $p(x_2) = 0.2$, $p(x_3) = 0.5$, $p(x_4) = 0.2$, $p(x_5) = 0$, $p(x_6) = 0$.
Now it wants to move to the previous location, but this results with probability .1 in no motion, with probability .6 in the intended motion and with probability .2 in moving one location more than intended. Location x_0 is the predecessor of location x_1; and x_{-1} is predecessor of x_0.
What is the probability of being in the locations x_i , i=-1, …, 6 after the move? 4 points

2.3) Compare the Extended Kalman Filter (EKF) to the Unscented Kalman Filter (UKF)!
What are commonalities and differences?

4 points

① $\hat{z}_1 = 1$ $\quad \hat{z}_3 = 7$ $\quad z_A = 7.3$ $\quad z_C = 3.8$

$\hat{z}_2 = 4$ $\quad \hat{z}_4 = 10$ $\quad z_B = 10.1$ $\quad z_D = 1.2$



- For measurement A

$$p(x_1 | z_A) = \eta \, \boxed{p(z_A | x_1)} \cdot p(x_1) \propto 0.24$$

$$\Downarrow$$

$$\left.\begin{aligned} \mu &= \hat{z}_1 - 0.1 \\ \sigma &= 0.25 + 0.05 \cdot z_A \end{aligned}\right\} \Rightarrow f(x) = f(z_A)$$

Gaussian Distribution

$$p(x_2 | z_A) \propto p(z_A | x_2) \cdot p(x_2) = 0.24$$

$$p(x_3 | z_A) \propto p(z_A | x_3) \cdot p(x_3) = 0.275$$

$$p(x_4 | z_A) \propto p(z_A | x_4) \cdot p(x_4) = 0.24$$

Note: This is after normalization $\left(\sum_i p(x_i | z_A) = 1 \right)$

Most likely robot pose for measurement A: X_3

I will just write final results for measurements B, C, D (in notebook you can see calculations)

⊗ Measurement B: X_4

⊗ Measurement C: X_2

⊗ Measurement D: X_1

→ All results are intuitive!

b) In this case, everything remains the same, only prior $p(x)$ change

$$a + 2a + 2a + a = 1$$

$$a = \frac{1}{6}$$

$p(X_1) = \frac{1}{6}$ $p(X_3) = \frac{1}{3}$

$p(X_2) = \frac{1}{3}$ $p(X_4) = \frac{1}{6}$

→ Repeating same, only for different prior, results are

Measurement A: X_3

Measurement B: X_4

Measurement C: X_2

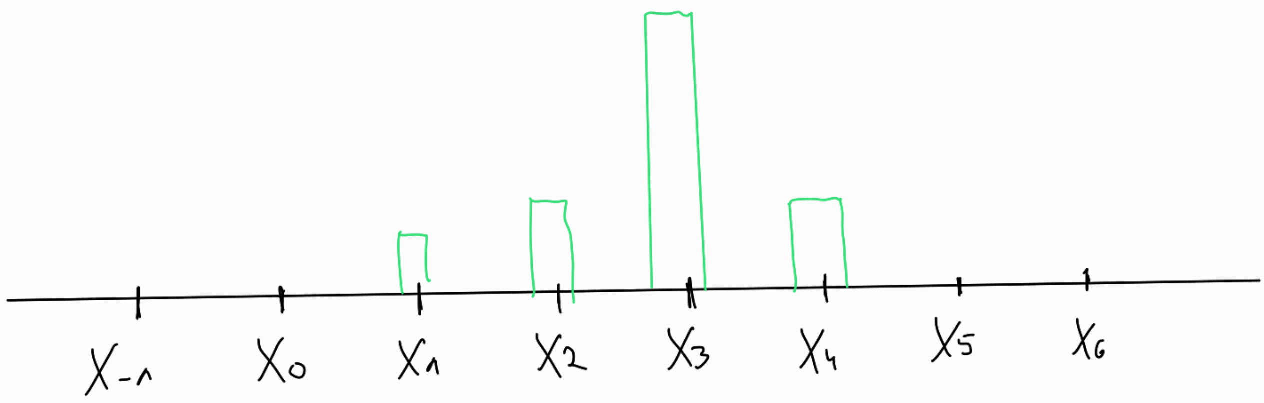
Measurement D: X_1

(2)

$p(X_{-1}) = 0$
$p(X_0) = 0$
$p(X_1) = 0.1$

$p(X_2) = 0.2$
$p(X_3) = 0.5$
$p(X_4) = 0.2$

$p(X_5) = 0$
$p(X_6) = 0$



$$u = \begin{cases} 0.1 & \to \text{no motion} \\ 0.6 & \to +1 \\ 0.3 & \to +2 \end{cases}$$

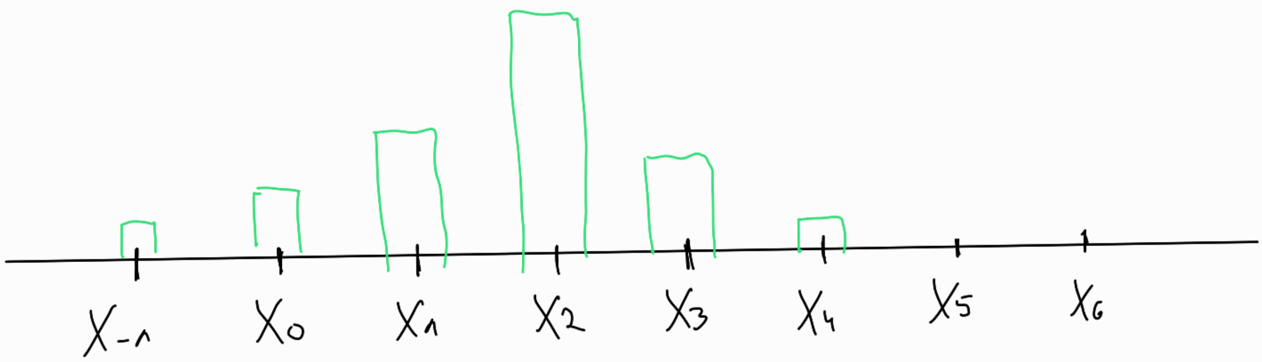
→ We will apply prediction step in Bayes Filter to calculate predicted belief.

$$\overline{bel}(X_t) = \int p(X_t \mid u_t, X_{t-1}) \cdot bel(X_{t-1})\, dX_{t-1}$$

$$= \sum_{X_{t-1}} \underline{p(X_t \mid u_t, X_{t-1})} \cdot \underline{bel(X_{t-1})}$$

$$\overline{bel}(x_t = X_{-1}) = 0.1 \cdot 0 + 0.6 \cdot 0 + 0.3 \cdot 0.1 = 0.03$$
$$\overline{bel}(x_t = X_{0}) = 0.1 \cdot 0 + 0.6 \cdot 0.1 + 0.3 \cdot 0.2 = 0.12$$
$$\overline{bel}(x_t = X_{1}) = 0.1 \cdot 0.1 + 0.6 \cdot 0.2 + 0.3 \cdot 0.5 = 0.28$$
$$\overline{bel}(x_t = X_{2}) = 0.1 \cdot 0.2 + 0.6 \cdot 0.5 + 0.3 \cdot 0.2 = 0.38$$
$$\overline{bel}(x_t = X_{3}) = 0.1 \cdot 0.5 + 0.6 \cdot 0.2 + 0.3 \cdot 0 = 0.17$$
$$\overline{bel}(x_t = X_{4}) = 0.1 \cdot 0.2 + 0.6 \cdot 0 + 0.3 \cdot 0 = 0.02$$
$$\overline{bel}(x_t = X_{5}) = 0.1 \cdot 0 + 0.6 \cdot 0 + 0.3 \cdot 0 = 0$$
$$\overline{bel}(x_t = X_{6}) = 0.1 \cdot 0 + 0.6 \cdot 0 + 0.3 \cdot 0 = 0$$

↳ sums up to 1



③

EKF:

1) Uses Gaussian to represent state
2) Linearize function around the state
3) Computes posterior state by putting 1) through 2)
4) Issues can occur if motion/observation model is very non-linear around linearization point and if standard deviation of 1) is big. Then 3) is not good

UKF:

1) Uses Gaussian to represent state
2) Instead of linearization, it is taking sigma points from 1)
3) Based on sigma points, it calculates mean and std of posterior
4) Handles better non-linearity of motion/observation functions