Suppose we have 3 frames

we want to optimize $x = [\underbrace{\xi_1 \quad \xi_2 \quad \xi_3}_{\text{6DOF pose}} \quad \underbrace{s}_{\text{scale}} \quad \underbrace{z}_{\text{shape code}}] \in R^{(19+C)\times 1}$

$3\times 6+1$ ↑ $(19+C)\times 1$ ↓ C: code length.

we have the x_0 available

now we estimate Δx by Gauss-Newton

$$\underbrace{\Delta x}_{(19+C)\times 1} = \underbrace{(J^T J)}_{(19+C)\times(19+C)}{}^{-1} \underbrace{(J^T b)}_{(19+C)\times 1}$$

So we need to calculate J and b

for each point, either ● ● or ●, we predict its SDF with DeepSDF D_θ

the result is the residual $b_i = f = D_\theta(p, z)$

the gradient w.r.t p and z are g_p and g_z, can be calculated by pytorch.

Suppose we have N points in total, then $b \in R^{N\times 1}$

J has the shape $N\times(19+C)$, each point contributes to a row $\underset{1\times(19+C)}{J_i}$

$$● \; J_i = [\underbrace{\frac{\partial f}{\partial \xi_1}}_{1\times 6} \quad \underbrace{\frac{\partial f}{\partial \xi_2}}_{1\times 6} \quad \underbrace{\frac{\partial f}{\partial \xi_3}}_{1\times 6} \quad \underbrace{\frac{\partial f}{\partial s}}_{1\times 1} \quad \underbrace{\frac{\partial f}{\partial z}}_{1\times C}]$$

if P_i is ● from frame 1 $\quad \frac{\partial f}{\partial \xi_2}, \frac{\partial f}{\partial \xi_3}$ should be 0

if P_i is ● from frame 2 $\quad \frac{\partial f}{\partial \xi_1} \quad \frac{\partial f}{\partial \xi_3}$ should be 0

. . . .

$$\frac{\partial f}{\partial \xi_1} = [\ \overset{1\times3}{g_p} \quad \overset{1\times3}{g_p}\overset{3\times3}{P_\times}\ ] \in R^{1\times6}$$

$\times$ means skew $\quad [a, b, c]_\times = \begin{bmatrix} 0 & a & c \\ -a & 0 & b \\ -c & -b & 0 \end{bmatrix}$

$$\frac{\partial f}{\partial s} = \overset{1\times3}{g_p}\overset{3\times1}{P} \quad \in R^{1\times1}$$

$$\frac{\partial f}{\partial z} = g_z \quad \in R^{1\times C}$$

note, these P are all under canonical frame

$P = \underbrace{T_{cj}}_{Exp(\xi_j)} P^j$

now we know J_i

we can get $J = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_N \end{bmatrix}$ similarly

we also already know b.

So we can calculate $\Delta\chi$

$\chi' = \chi_0 + \Delta\chi$, with the new χ', we transform P accordingly.

$\hookrightarrow$ and we do the iteration till convergence