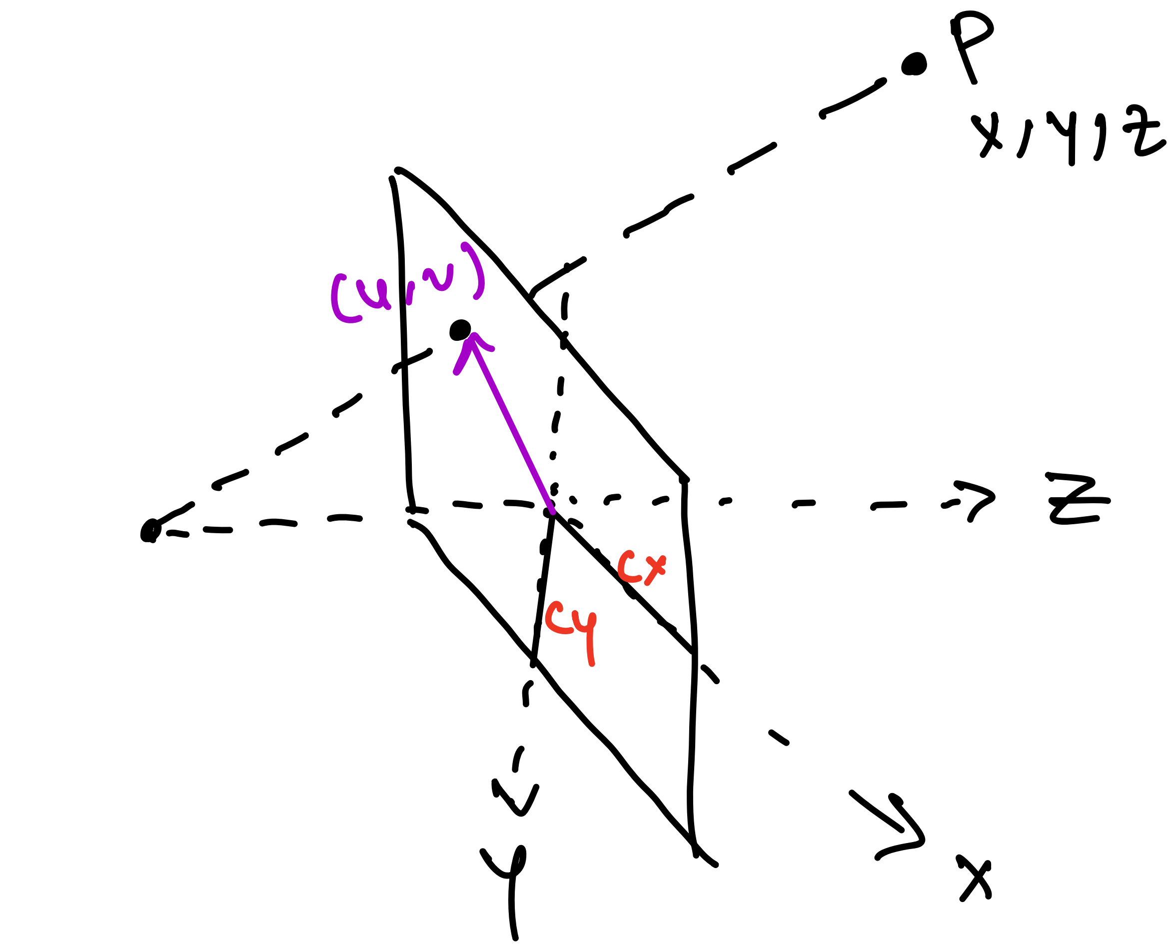
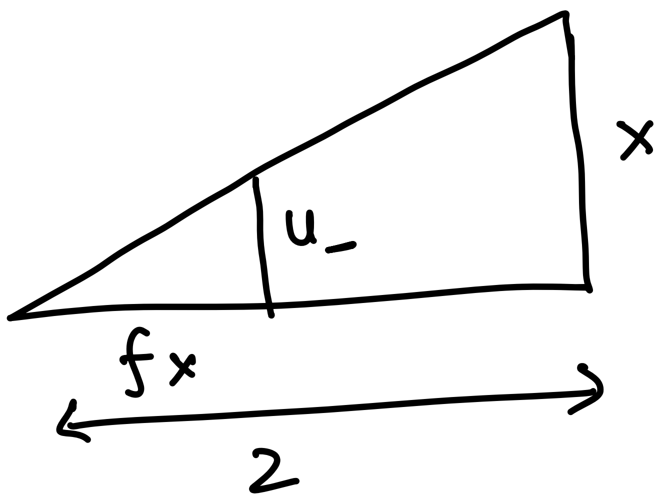
Pinhole





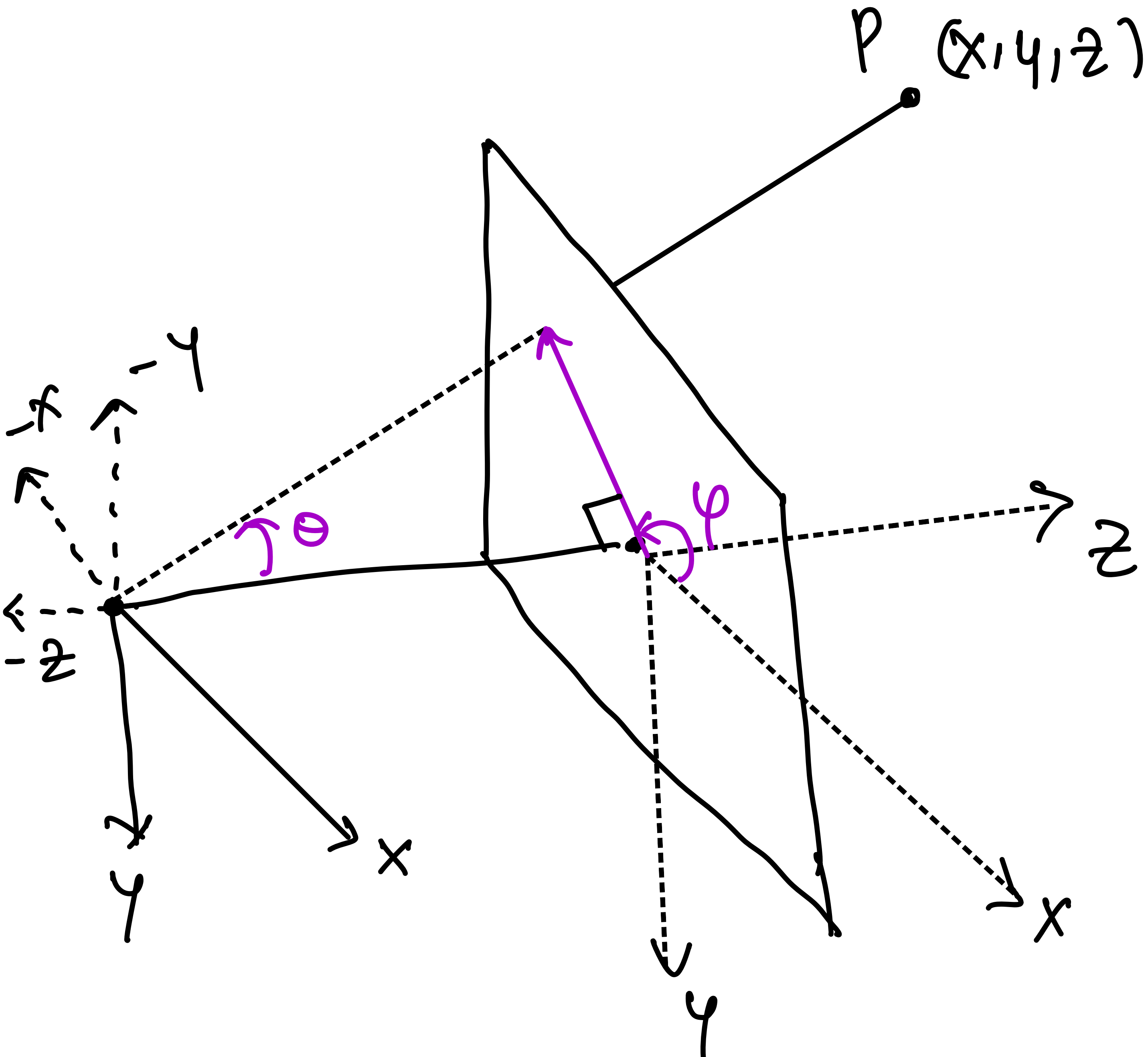
$$\frac{x}{z} = \frac{u\_}{f_x}$$

$$u\_ = \frac{x f_x}{z}$$

$$u = u\_ + c_x$$

$$\Rightarrow \boxed{\begin{aligned} u &= \frac{x f_x}{z} + c_x \\ v &= \frac{y f_y}{z} + c_y \end{aligned}}$$

Pinhole - Polar coordinates



$$\frac{u_-}{f_x} = \frac{x}{z}$$

$$u_- = \frac{x f_x}{z} \Rightarrow u = u_- + c_x$$

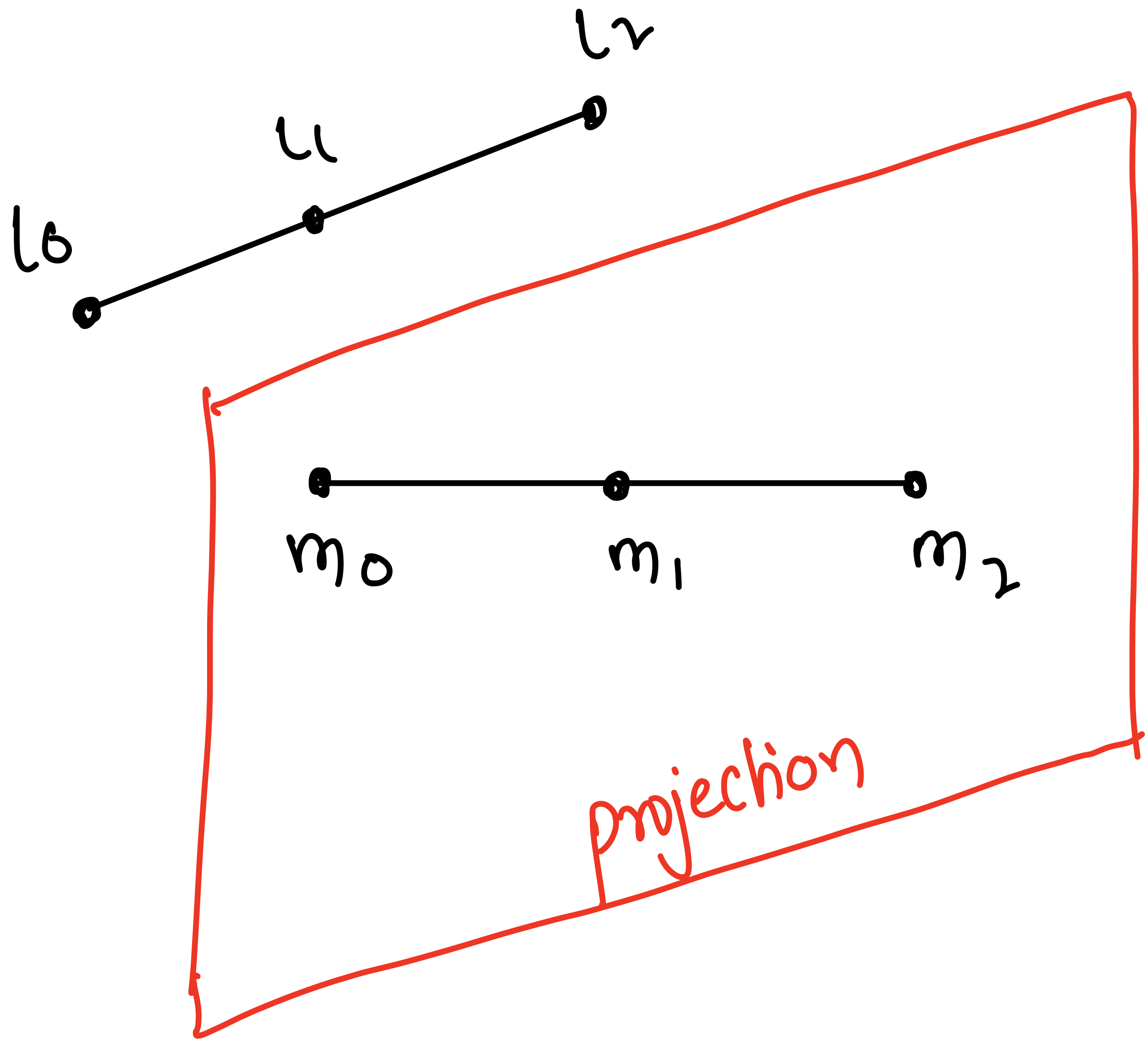
$$\Rightarrow \boxed{\begin{aligned} u &= c_x + f_x \tan\theta \cos\varphi \\ v &= c_y + f_x \tan\theta \sin\varphi \end{aligned}}$$

Pinhole linearity

$$u = \frac{f_x x}{z} + c_x$$

$$v = \frac{f_y y}{} + c_y$$

z



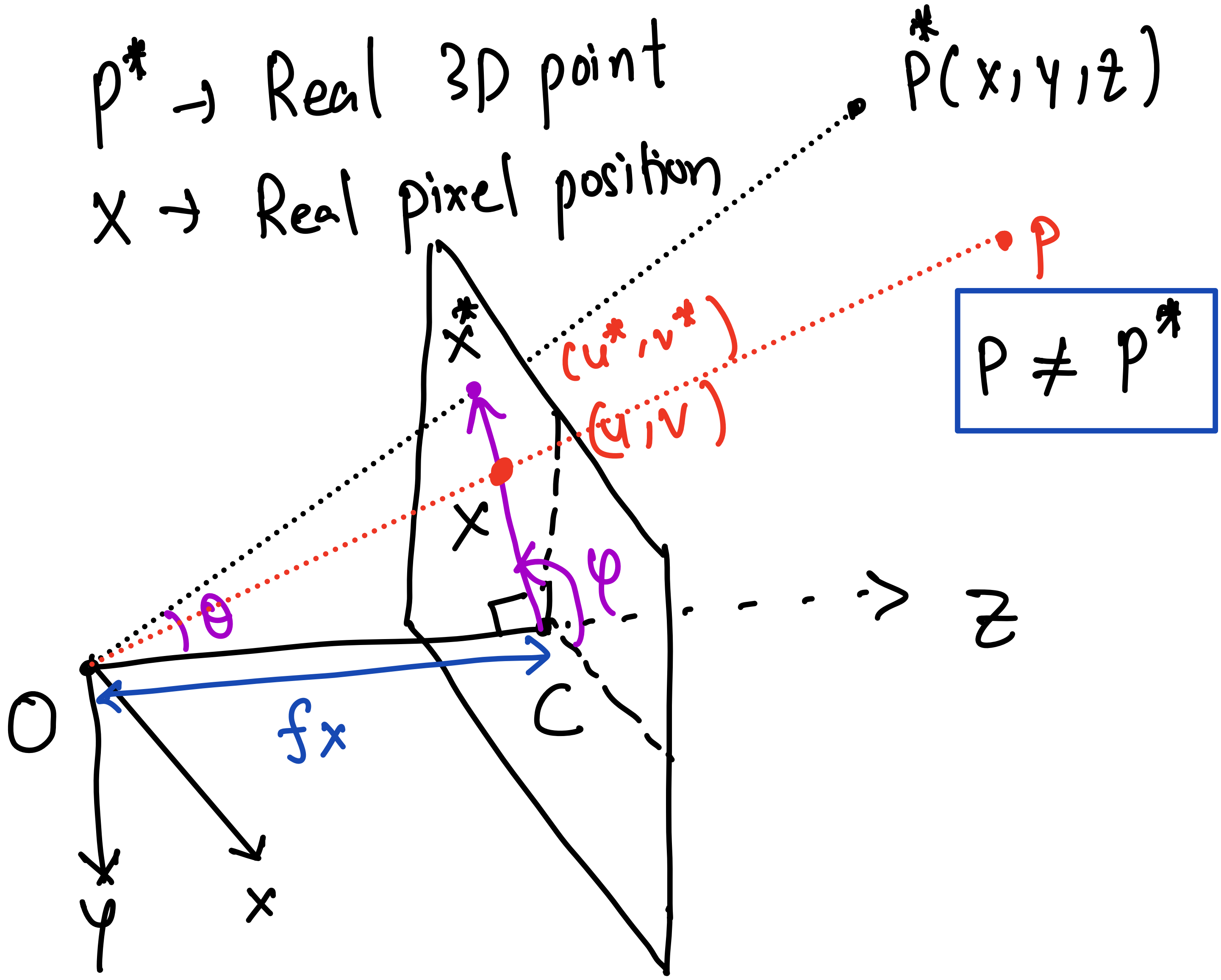
Linear mapping if,

$$l_0, l_1, l_2 \text{ colinear}$$
$$\Downarrow$$
$$m_0, m_1, m_2 \text{ colinear}$$

Spherical Camera Model

$p^* \to$ Real 3D point

$x \to$ Real pixel position



$$CX = u - = u - c_x$$

$$CX^* = f_x \tan\theta$$ linear model

In spherical model,

$$\boxed{\begin{aligned} u &= c_x + f_x\,\theta\cos\varphi \\ v &= c_y + f_y\,\theta\sin\varphi \end{aligned}}$$

Let's call it

$$\theta\cos\varphi = \frac{u - c_x}{f_x} = m$$

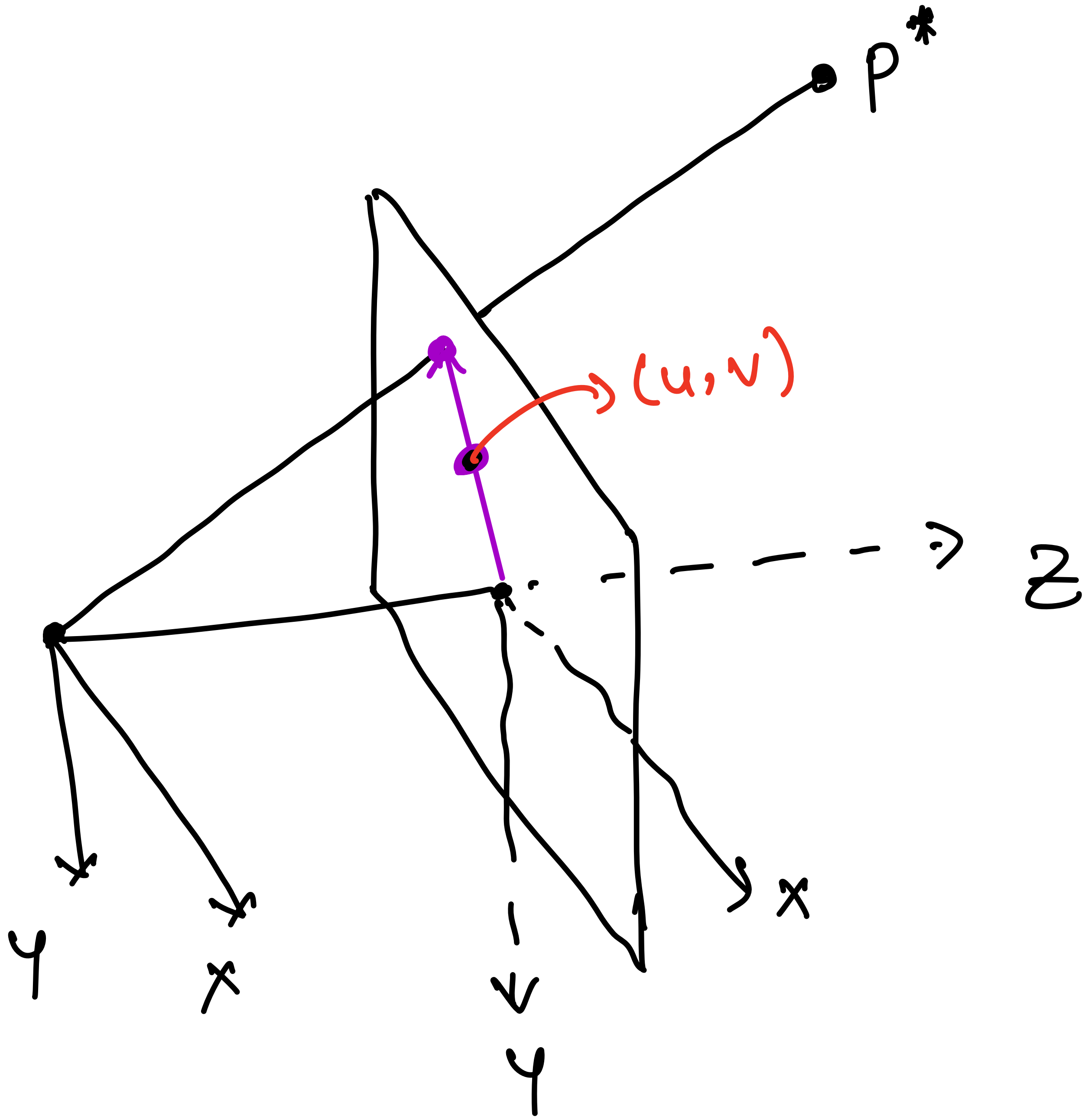
$$\theta\sin\varphi = \frac{v - c_y}{f_y} = n$$

$$\tan\varphi = n/m \Rightarrow \varphi = \tan^{-1}\left(\frac{n}{m}\right)$$

$$\theta = \sqrt{m^2 + n^2}$$

$$\boxed{\varphi = \tan^{-1}\left(\frac{v - c_y \cdot f_x}{f_y \cdot u - c_x}\right) \;\Bigg|\; \theta = \sqrt{\left(\frac{u - c_x}{f_x}\right)^2 + \left(\frac{v - c_y}{f_y}\right)^2}}$$

KB3 - The Kannala Brandt K3 model



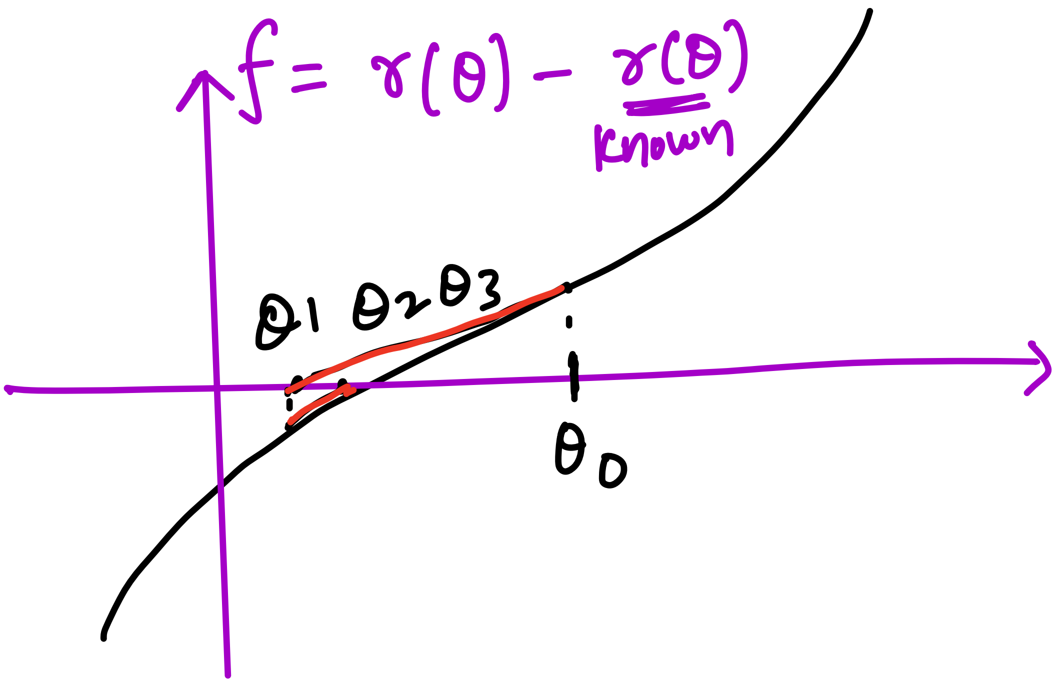
$$u = c_x + f_x\, r(\theta) \cos\varphi$$

$$v = c_y = f_y\, r(\theta) \sin\varphi$$

$$\gamma(\theta) = \theta + k_0\theta^3 + k_1\theta^5 + k_2\theta^7 + \ldots$$

Aria uses 5 params.

* If we have $\gamma(\theta)$, we can get "θ" using Newtons Method



From KB3 → Pinhole
Polar Coordinates

$$\varphi = \tan^{-1}\left(\frac{v - c_y}{f_y} \cdot \frac{f_x}{u - c_x}\right)$$

$$r(\theta) = \sqrt{\left(\frac{u - c_x}{f_x}\right)^2 + \left(\frac{v - c_y}{f_y}\right)^2}$$

θ from $r(\theta)$ using Newtons method.

Using Pytorch

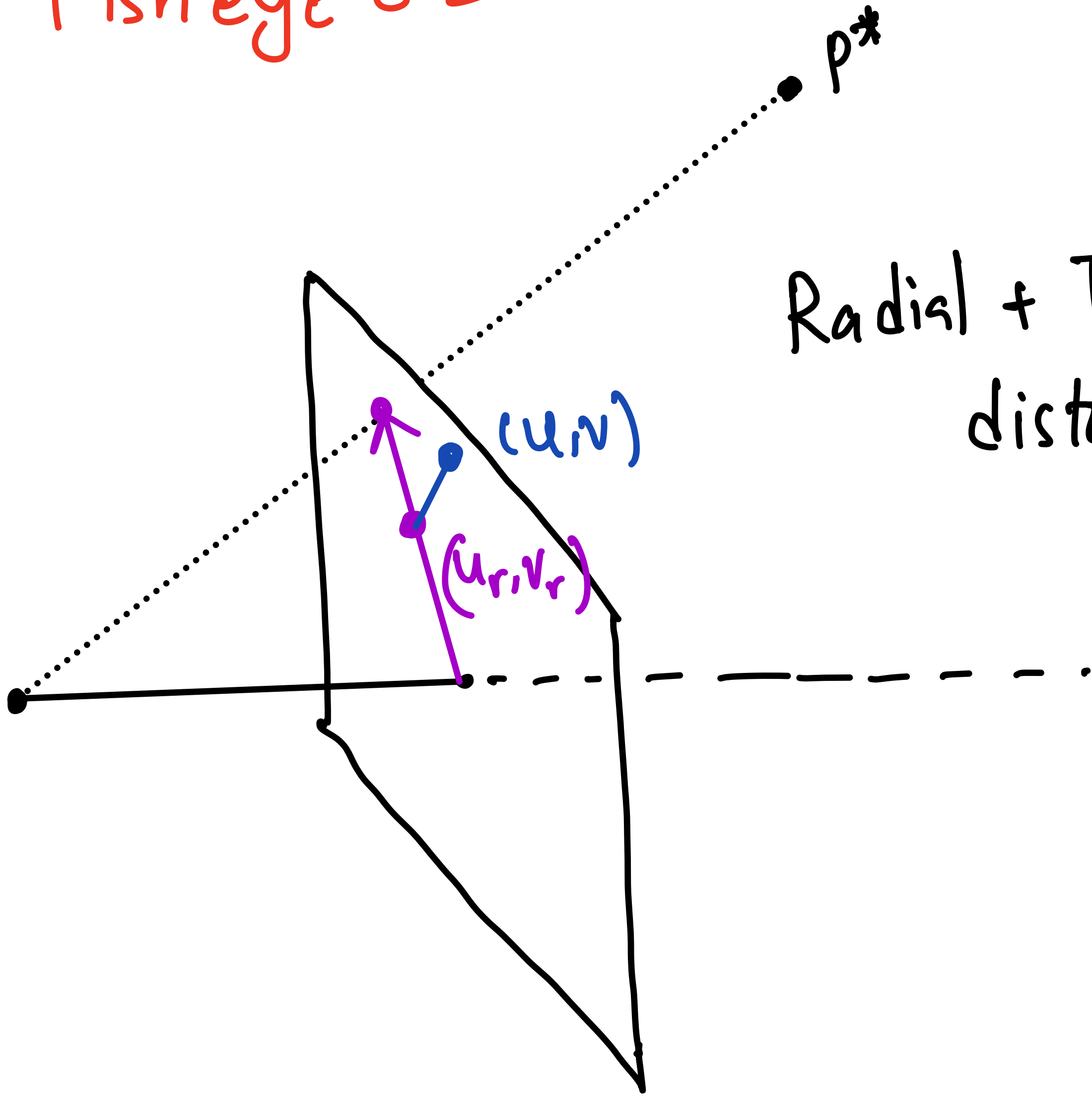
$$\underset{\theta}{\arg\min} \left(\underbrace{r(\theta)}_{\text{functional form}} - \underbrace{r(\theta)}_{\text{known value.}}\right)^2 = \theta$$

$\Downarrow$

polar coordinate

Fisheye 62

p^* → real 3D point



Radial + Tangential distortion

$$u = c_x + f_x u_r + \underline{f_x t_x(u_r, v_r)}$$

tangential part ⇒ function of (f_x, u_r, v_r)

$$u = c_x + f_x\, r(\theta)\cos\varphi + f_x\left[p_0\left(2u_r^2 + r(\theta)^2\right) + 2p_1 u_r v_r\right]$$

Goal is to be able to distort and undistort $\Rightarrow$ changing everything to θ, φ [polar coordinates]

$$\frac{u - c_x}{f_x} = r(\theta)\cos\varphi + 2p_0\, r^2(\theta)\cos^2\varphi + p_0\, r^2(\theta) + 2p_1\, r^2(\theta)\cos\varphi\sin\varphi$$

$\hookrightarrow g_1(r(\theta), \varphi) = 0$

Similarly

$$\frac{v - c_y}{f_y} = r(\theta)\sin\varphi + 2p_1\,\tilde{r}(\theta)\sin^2\varphi + p_1\,\tilde{r}(\theta) + 2p_0\,\tilde{r}(\theta)\sin\varphi\cos\varphi$$

$\hookrightarrow g_2(r(\theta), \varphi) = 0$

In Blue $\rightarrow$ constants

* Other components are factors of θ, φ

* Let those equations be called g_1, g_2

Using Optimization to get $r(\theta), \varphi$

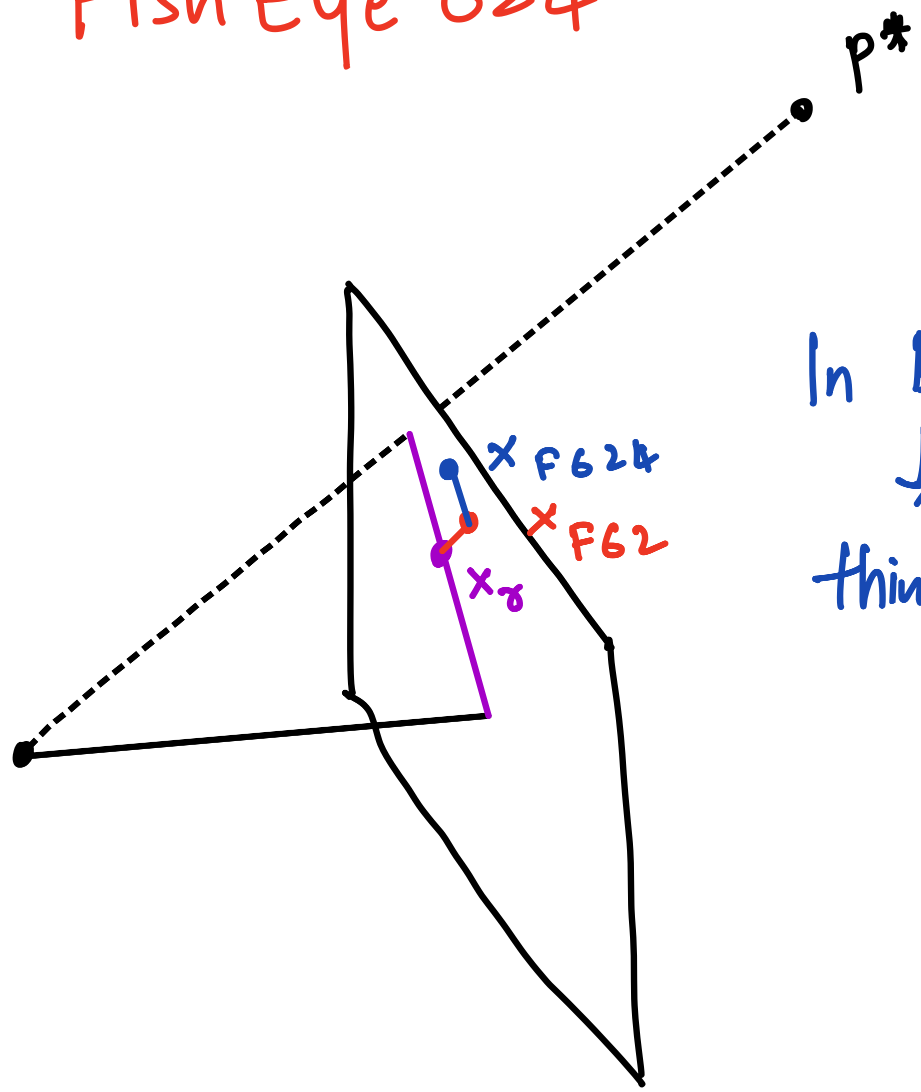
$$\underset{r(\theta),\varphi}{\operatorname{argmin}} \left[\left(g_1(r(\theta), \varphi) - 0\right)^2 + \left(g_2(r(\theta), \varphi) - 0\right)^2 \right]$$

* θ from $r(\theta)$ using Newtons Method

or Optimisation with

$$\theta^* = \underset{\theta}{\arg\min} \left[\left(r(\theta) - \theta - k_0\theta^3 - k_1\theta^5 - \dots \right)^2 \right]$$

FishEye 624



In Blue

$\Downarrow$

thin prism distortion

$$u = c_x + f_x u_r + f_x t_x(u_r, v_r) + f_x t_{px}(u_r, v_r)$$

$$t_{px}(u_r, v_r) = s_0 r(\theta)^2 + s_1 r(\theta)^4$$

$$t_{py}(u_r, v_r) = s_2 r(\theta)^2 + s_3 r(\theta)^4$$

Similar to Fisheye62, convert everything to $r(\theta), \varphi$, then estimate polar coordinates

$$\boxed{\frac{u - c_x}{f_x}} = r(\theta)\cos\varphi + \boxed{2p_0}\, r^2(\theta)\cos^2\varphi + \boxed{p_0}\, r^2(\theta) + \boxed{2p_1}\, r^2(\theta)\cos\varphi\sin\varphi + \boxed{s_0}\, r^2(\theta) + \boxed{s_1}\, r^4(\theta)$$

$\hookrightarrow h_1 = 0$

In Blue constants,

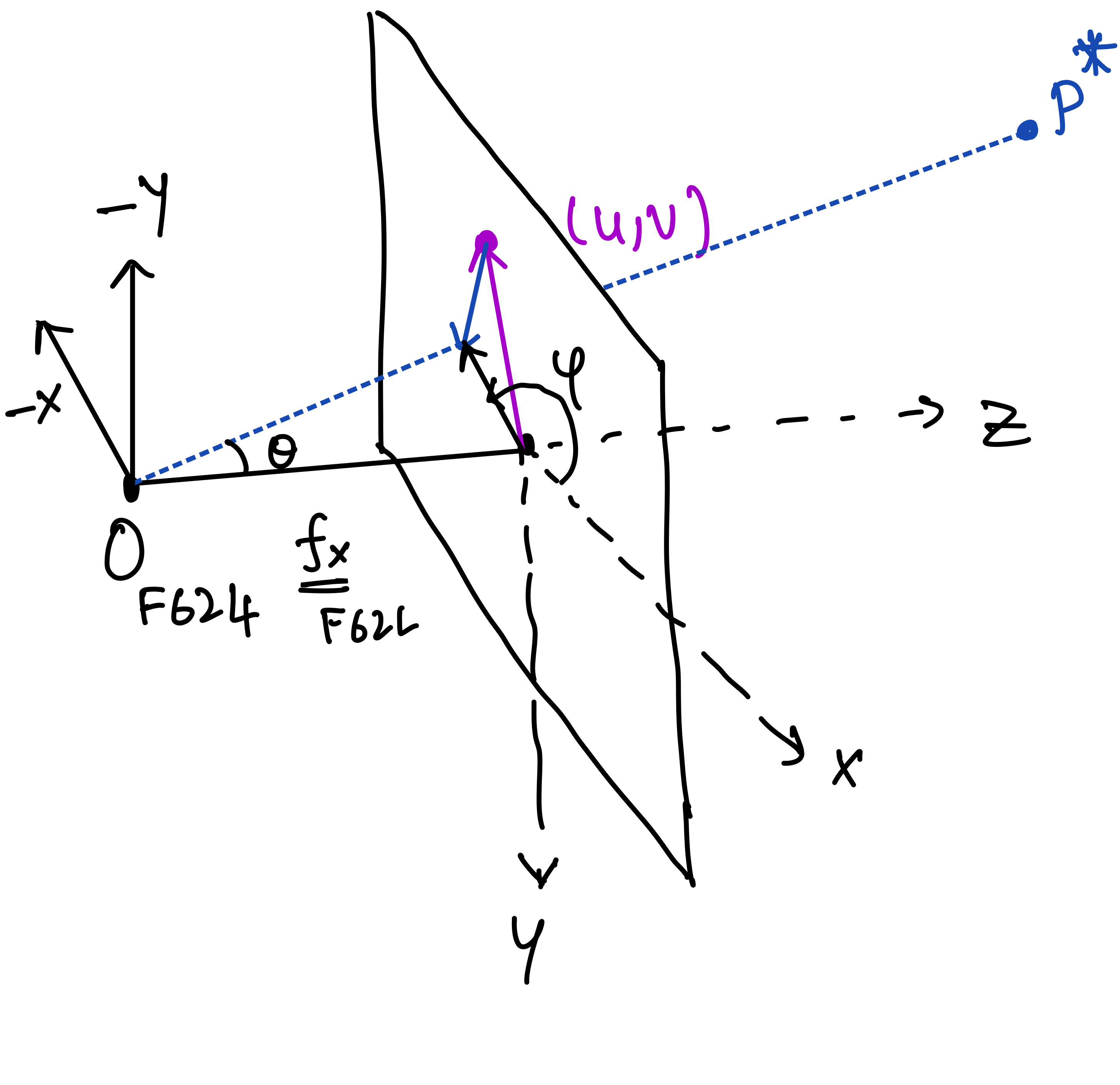
this will be same with y-pixels. $\hookrightarrow h_2 = 0$

So, again.

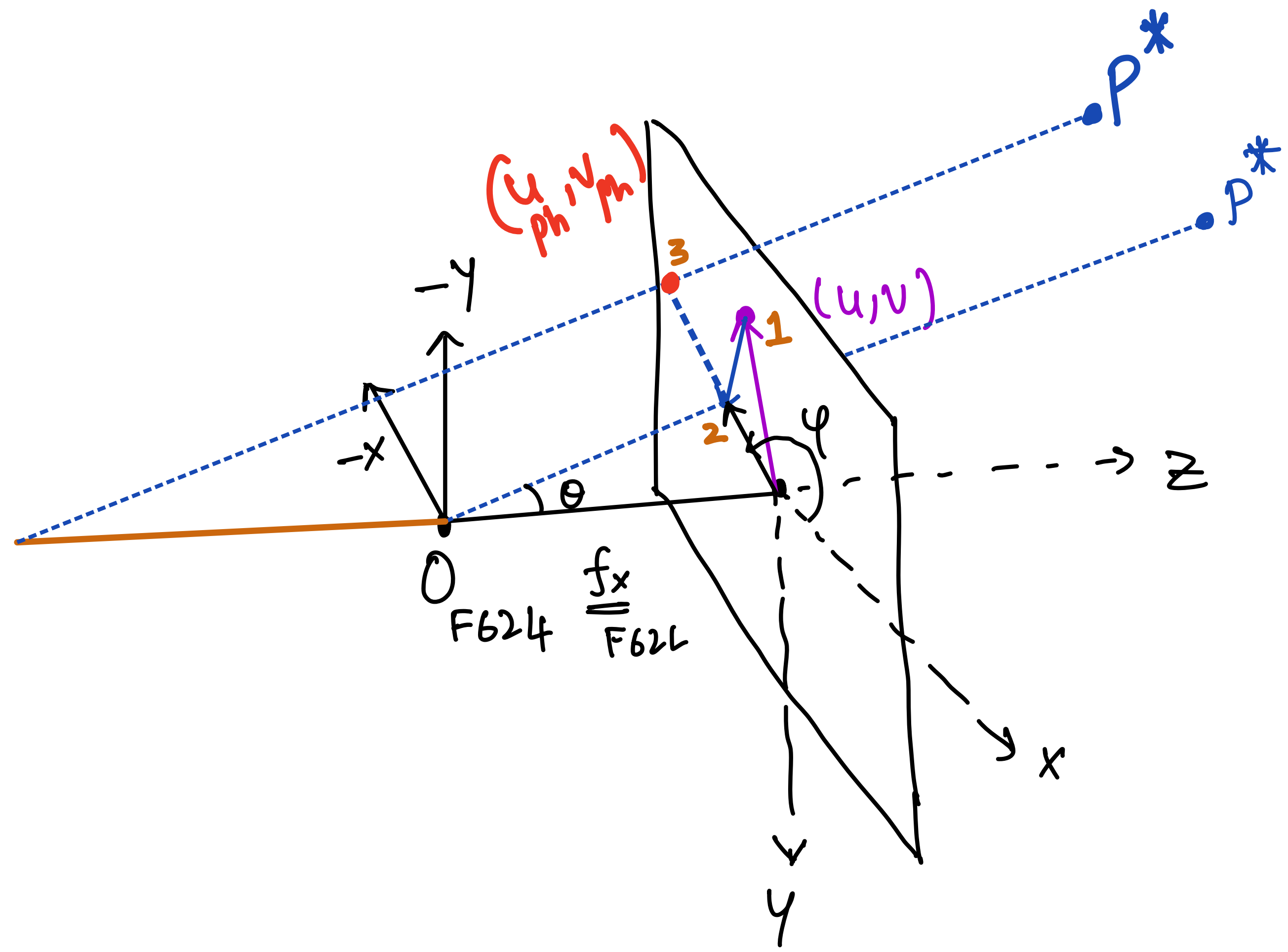
$$\gamma(\theta)^*, \varphi^* = \underset{\gamma(\theta),\varphi}{\operatorname{argmin}} \left[\left(h_1(\gamma(\theta),\varphi) - 0\right)^2 + \left(h_2(\gamma(\theta),\varphi) - 0\right)^2 \right]$$

$$\theta = \underset{\theta}{\operatorname{argmin}} \left[\left(\gamma(\theta) - \theta - K_0\theta^3 - \dots\right)^2 \right]$$

Polar coordinates to Pinhole



Changing one pinhole intrisic to other



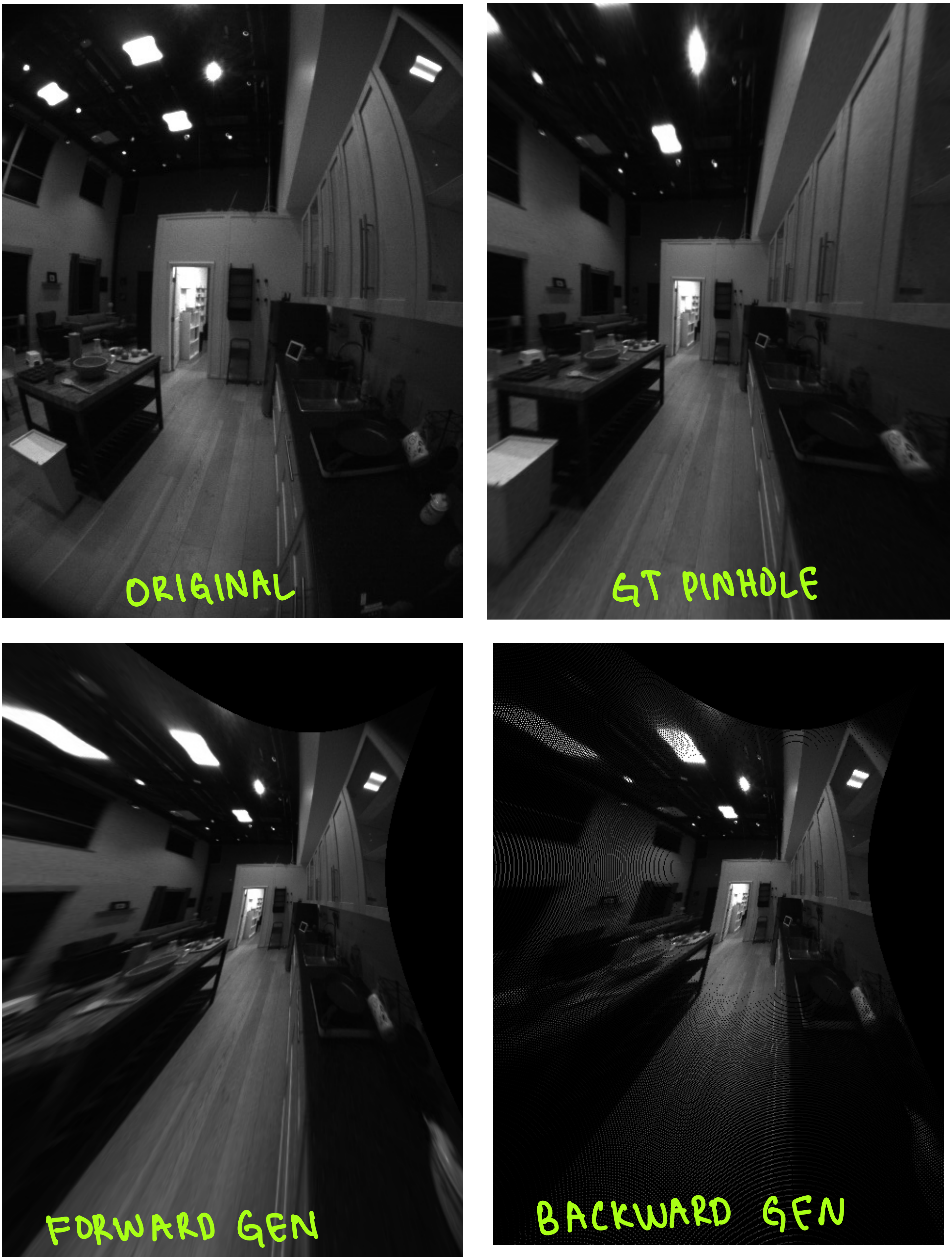
Intrinsincs of destination

f_x, f_y, c_x, c_y

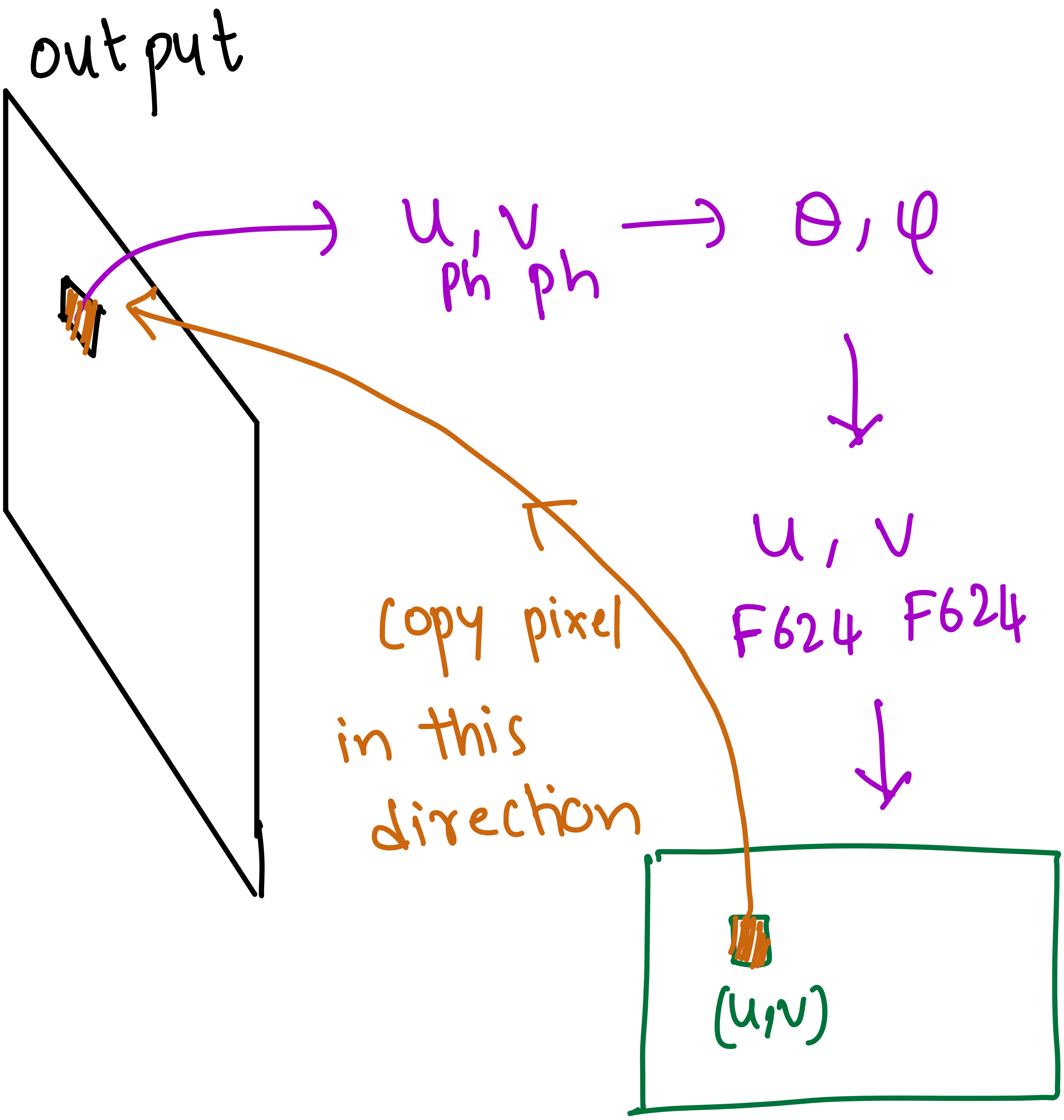
* This will follow-basic pinhole - polar conversion

$$u = c_x + f_x \tan\theta \cos\varphi$$

$$v = c_y + f_x \tan\theta \sin\varphi$$


ORIGINAL
GT PINHOLE
FORWARD GEN
BACKWARD GEN

Forward Gen



output
u, v
ph ph
θ, φ
u, v
F624 F624
copy pixel
in this
direction
(u,v)

Backward Gen