

PSA Class 10  
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### DFS Implementation:

- Iteration → Using Stack custom, doesn't exhaust stack frame of machine
- Recursion → Easy to implement, exhausts stack frame of machine when deep graphs are tested / used for DFS

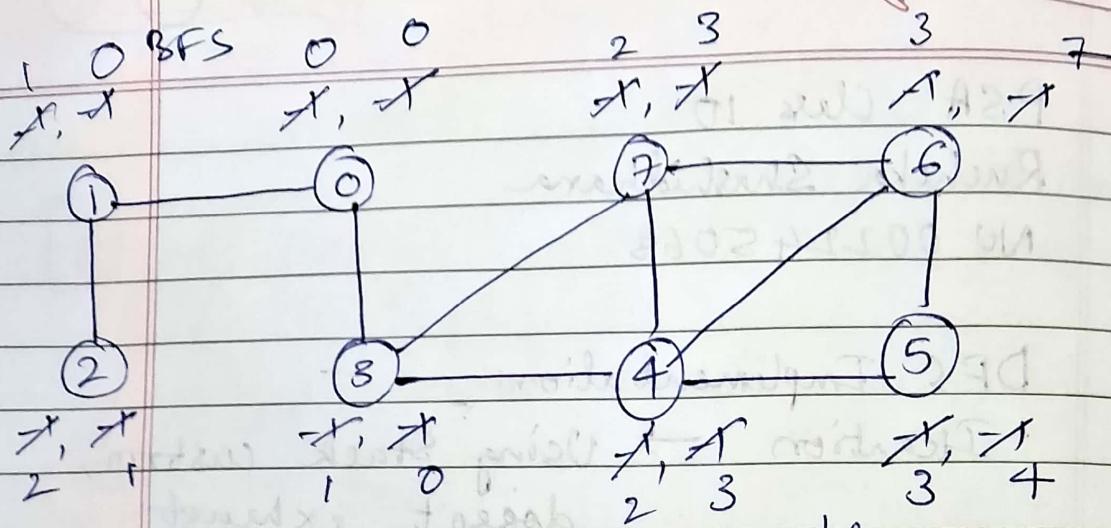
uses Stack

- DFS gives the longest path, if the weights of the graph are uniform / unweighted graph
- DFS can check whether a graph has a loop or not → DAG

- uses Queue
- BFS provides a shortest path, if the weights of the graph are uniform / unweighted graph

- Dijkstra's Algo can give the shortest path to any weighted (not uniform) graph.

- In both DFS & BFS each node is visited once



Starting node will always be given for BFS  
unlike DFS, Here ①

0	1	2	3	4	5	6	7
cost	0	1	2	1	2	3	3
from	0	0	1	0	3	4	7

flow did we reach 7?  
~~1~~ → \* → 7 → 3 → 0 cost = 2

~~2~~ → \* → 0 → 3 → 7 cost = 2  
new

~~3~~ → \* → 0 → 3 → 7 cost = 2  
∴ This is also the shortest path

~~4~~ → \* → 5 → 4 → 3 → 0  
How did we reach 5?  
5 → 4 → 3 → 0  
new

BFS gives shortest path to every node

give us uniformly weighted graph

0 → 3 → 4 → 5

cost = 4

Shortest path from 0 → 5

Space used in this algo, N: Nodes

DS:  $2N$

Queue:  $N$

E: Edges

$\approx \Theta(N)$

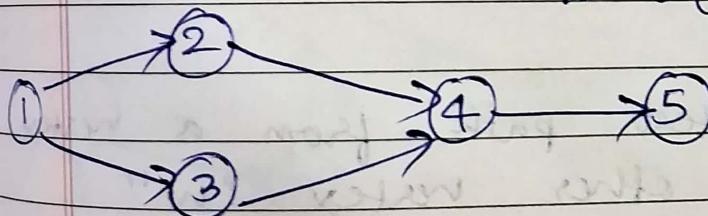
Work Done in this BFS algo:

$N+E$ : Directed Graph

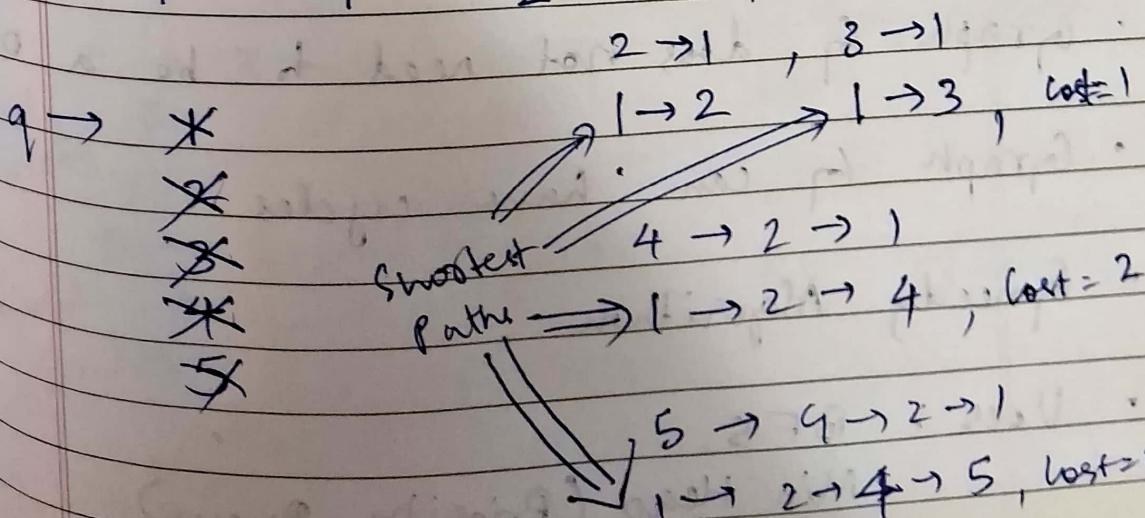
$N+2E$ : Undirected graphs  
 $\approx \Theta(N+E)$

∴ Shortest Path from all cities to other cities Matrix Construction:

work done =  $\Theta(N(N+E))$   
 $\approx \Theta(N^2)$



1	2	3	4	5	
0	1	1	2	3	Cost
0	1	1	2	4	From



Graph Type	Longest Path	Shortest Path
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Unweighted/ Constant weight	DFS	BFS
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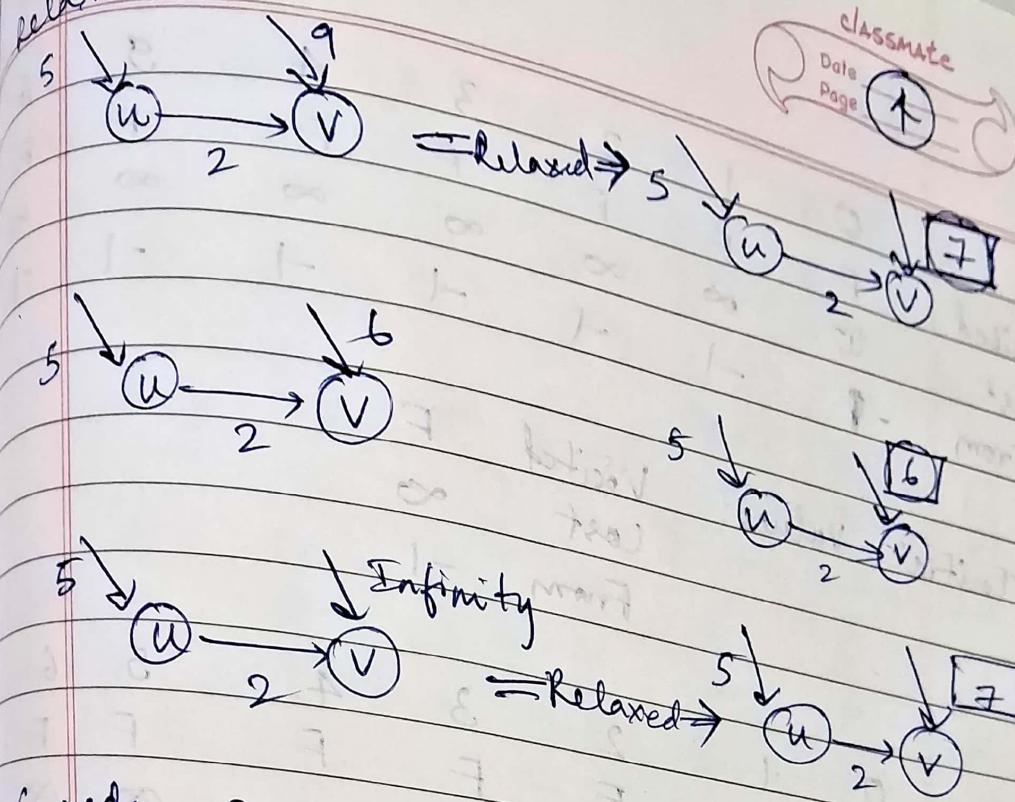
Weight $\geq 0$ (one only)	Currently No Polynomial Time Algo exists NP-complete	Dijkstra's Algo / DP
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Any weight (-ve, 0, +ve)	Currently No polynomial Time Algo exists NP-complete	Bellman-Ford's Algo
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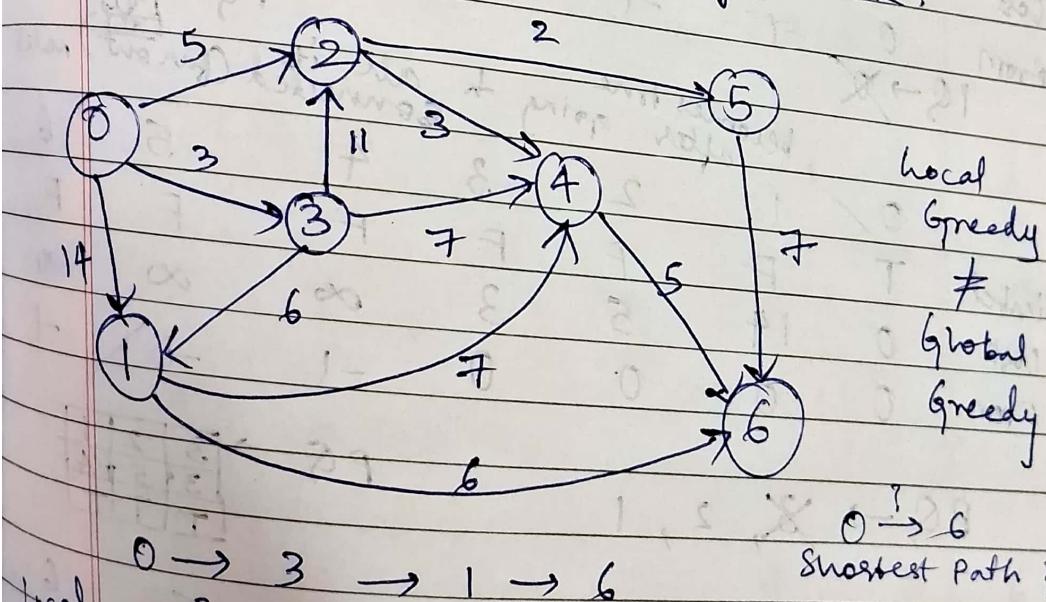
### Dijkstra's Algorithm:

- Finds the shortest path from a vertex "s" to any other vertex "w"
- Graph  $G$  must be directed/ <sup>undirected</sup> & all edge weights must be  $\geq 0$
- Graph  $G$  does not need to be a DAG
- Graph  $G$  can have cycles.
- Greedy Algorithm
- Uses: BFS  
Min heap (Priority Queue)  
Relaxation

relaxation:



greedy may not always work:



local greedy:  $0 \rightarrow 3 \rightarrow 1 \rightarrow 6$  cost = 15

global greedy:  $0 \rightarrow 2 \rightarrow 4 \rightarrow 6$  cost = 13

global greedy:

	0	1	2	3	4	5	6
Visited	F	F	F	F	F	F	F
Cost	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
From	-1	-1	-1	-1	-1	-1	-1

Initial Values:

	Visited	F
Cost	$\infty$	
From	-1	

	0	1	2	3	4	5	6
Visited	F	F	F	F	F	F	F
Cost	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
From	0	-1	-1	-1	-1	-1	-1

~~PQ → X~~PQ → 

0	0
0	0

becomes true after going to all connected (fan out) nodes

	0	1	2	3	4	5	6
Visited	T	F	F	F	F	F	F
Cost	0	14	5	3	$\infty$	$\infty$	$\infty$
From	0	0	0	0	-1	-1	-1

~~PQ → X, 2, 1~~PQ → 

3	2	1
3	5	14
T	T	T

	0	1	2	3	4	5	6
Visited	T	F	F	T	F	F	F
Cost	0	9	5	3	10	$\infty$	$\infty$
From	0	3	0	0	3	-1	-1

~~PQ → X, 1, 4~~PQ → 

2	1	4	1
5	9	10	14
T	F	T	F

Visited	0	1	2	3	4	5	6
Cost	T	F	T	T	F	T	F
From	0	9	5	3	8	7	2
	0	3	0	0	2	2	0

PQ → ~~\*~~, 4, 1

PQ →	5	4	1	4	1
	7	8	9	10	14
	T	T	T	F	P

Visited	0	1	2	3	4	5	6
Cost	T	F	T	T	F	T	F
From	0	9	5	3	8	7	14
	0	3	0	0	2	2	14

PQ → ~~\*~~, 1, 6

PQ →	4	1	4	1	6
	8	9	10	14	14
	T	T	F	F	T

Visited	0	1	2	3	4	5	6
Cost	T	F	T	T	T	T	F
From	0	9	5	3	8	7	13
	0	3	0	0	2	2	4

PQ → ~~\*~~, 6

PQ →	1	4	6	1	6
	9	10	13	14	14
	T	F	T	F	F

Visited	0	1	2	3	4	5	6
Cost	T	T	T	T	T	T	F
From	0	9	5	3	8	7	13
	0	3	0	0	2	2	4

PQ → ~~\*~~

PQ →	6	1	6
	13	14	14
	T	F	F

	0	1	2	3	4	5	6
Visited	T	T	T	T	T	T	T
Cost	0	9	5	3	8	7	13
From	0	3	0	0	2	2	4

$$6 \rightarrow 4 \rightarrow 2 \rightarrow 0 \quad \leftarrow \text{Cost} = 13$$

$$\cdot 13 \cdot 8 \cdot 5$$

$$5 \cdot 3 \cdot 8 \cdot 5$$

$$0 \rightarrow 2 \rightarrow 4 \rightarrow 6 \quad \text{Cost} = 13$$

: Shortest Path with  
least cost

~~$$5 \rightarrow 2 \rightarrow 0 \quad \leftarrow 29$$~~

~~$$0 \rightarrow 2 \rightarrow 5 \quad \text{Cost} = 7$$~~

~~$$4 \rightarrow 2 \rightarrow 0$$~~

~~$$0 \rightarrow 2 \rightarrow 4 \quad \text{Cost} = 8$$~~

~~$$3 \rightarrow 0$$~~

~~$$0 \rightarrow 3 \quad \text{Cost} = 3$$~~

~~$$2 \rightarrow 0$$~~

~~$$0 \rightarrow 2 \quad \text{Cost} = 5$$~~

~~$$1 \rightarrow 3 \rightarrow 0$$~~

~~$$0 \rightarrow 3 \rightarrow 1 \quad \text{Cost} = 9$$~~

Dijkstra's Algo gives the shortest path with least cost for any given node from a single starting node.

Space used

$$\begin{array}{l} \frac{1}{N} \leftarrow 3N: D's \\ \frac{1}{N} \leftarrow IN: PQ \\ \approx \Theta(N) \end{array}$$

N: Vertex / Nodes

E: Edges

Work Done:  $O(\log N * (N + E))$

$\approx O(N \log N)$  since we are using PQ (Min heap)

rather than Python list.

PQ Implementation:

Create a new DS that stores Node

Cost

Bool → represents if val is correct/reduced or redundant (T)

Whenever we change cost in DS, mark the node in the PQ's  $Bool = F$  & insert a new ele in PQ with new cost &  $Bool = T$

If we encounter a ele with  $Bool = F$ , when we pop from heap, ignore it

## Dynamic Programming:

	0	1	2	3	4	5	6
Visited	0	-1	-1	-1	-1	-1	-1
Cost	0	-1	-1	-1	-1	-1	-1
From	0	-1	-1	-1	-1	-1	-1

DPS Order: (0) 3 2 5 1 4 6

Using Timesamp (Topologically Sorted Order)

In this algo, we traverse DFS, then in DPS order we check which were fan ins → all fan ins must be visited  
 fan outs →

	0	1	2	3	4	5	6
Visited	0	-1	-1	-1	-1	-1	-1
Cost	0	-1	-1	-1	-1	-1	-1
From	0	-1	-1	0	-1	-1	-1

DPS → ✗ ✗ (2) 5 1 4 6

	0	1	2	3	4	5	6
Visited	0	-1	-1	-1	-1	-1	-1
Cost	0	-1	-1	-1	-1	-1	-1
From	0	-1	-1	0	-1	-1	-1

DFS → ✗ ✗ ✗ (5) 1 4 6

	0	1	2	3	4	5	6
Visited	0	-1	-1	-1	-1	-1	-1
Cost	0	-1	-1	-1	-1	-1	-1
From	0	-1	-1	0	-1	2	-1

DFS → ✗ ✗ ✗ ✗ ① + 6

	0	1	2	3	4	5	6
Visited	0	1	2	3	4	5	6
Cost	0	9	5	3	-1	7	-1
From	0	3	0	0	-1	2	-1

DFS → ✗ ✗ ✗ ✗ ✗ ✗ ④ 6

	0	1	2	3	4	5	6
Visited	0	1	2	3	4	5	6
Cost	0	9	5	3	8	7	-1
From	0	3	0	0	2	2	-1

DPS → ✗ ✗ ✗ ✗ ✗ ✗ ⑥

	0	1	2	3	4	5	6
Visited	0	1	2	3	4	5	13
Cost	0	9	5	3	8	7	4
From	0	3	0	0	2	2	4

DFS → ✗ ✗ ✗ ✗ ✗ ✗ ✗

DP only works if graph has  
no loop ∵ DPS order is correct  
for DAG / No Loop Graph

# Finding Shortest Paths

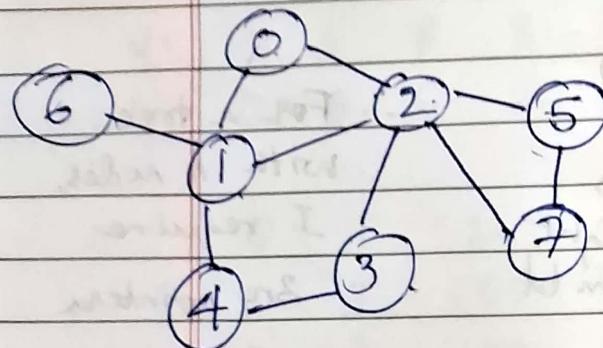
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BFS Undirected Graph

Constant Weight



⑥ → ⑦ ?

	0	1	2	3	4	5	6	7
Cost	0	-1	-1	-1	-1	-1	-1	-1
From	0	-1	-1	-1	-1	-1	-1	-1

9 → ✗

	0	1	2	3	4	5	6	7
Cost	0	1	1	1	-1	-1	-1	-1
From	0	0	0	-1	-1	-1	-1	-1

9 → ✗ 2

	0	1	2	3	4	5	6	7
Cost	0	1	1	-1	2	-1	2	-1
From	0	0	0	-1	1	-1	1	-1

9 → ✗ 6 4

	0	1	2	3	4	5	6	7
Cost	0	1	1	2	2	2	2	2
From	0	0	0	2	1	1	1	1

9 → ✗ ✗ ✗ ✗ ✗

Cost = 2

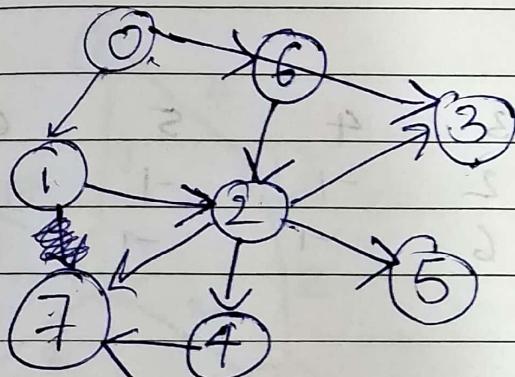
Elements = {7, 2, 0}

Order rev  $\Rightarrow$  0  $\xrightarrow{①}$  2  $\xrightarrow{①}$  7

BFS

Directed Graph

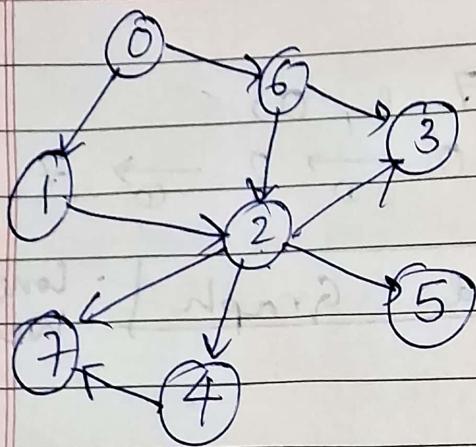
Constant Weight

 $0 \rightarrow 7$ 

0	1	2	3	4	5	6	7
Cost	0	-1	1	-1	-1	-1	-1
From	0	-1	-1	1	-1	-1	-1
$q \rightarrow \times$							

0	1	2	3	4	5	6	7
Cost	0	1	-1	1	-1	-1	-1
From	0	0	-1	-1	-1	-1	0
$q \rightarrow \times 1$							

0	1	2	3	4	5	6	7
Cost	0	1	2	2	-1	-1	-1
From	0	0	6	6	-1	-1	0
$q \rightarrow \times 3 2$							



	0	1	2	3	4	5	6	7
Cost	0	1	2	2	-1	-1	1	-1
From	0	0	6	6	-1	-1	0	-1
9 →	X	2						

	0	1	2	3	4	5	6	7
Cost	0	1	2	2	-1	-1	1	-1
From	0	0	2	6	-1	-1	0	-1
9 →	X	2						

	0	1	2	3	4	5	6	7
Cost	0	1	2	2	3	3	1	3
From	0	0	0	-1	2	2	0	2
9 →	X	5	7					

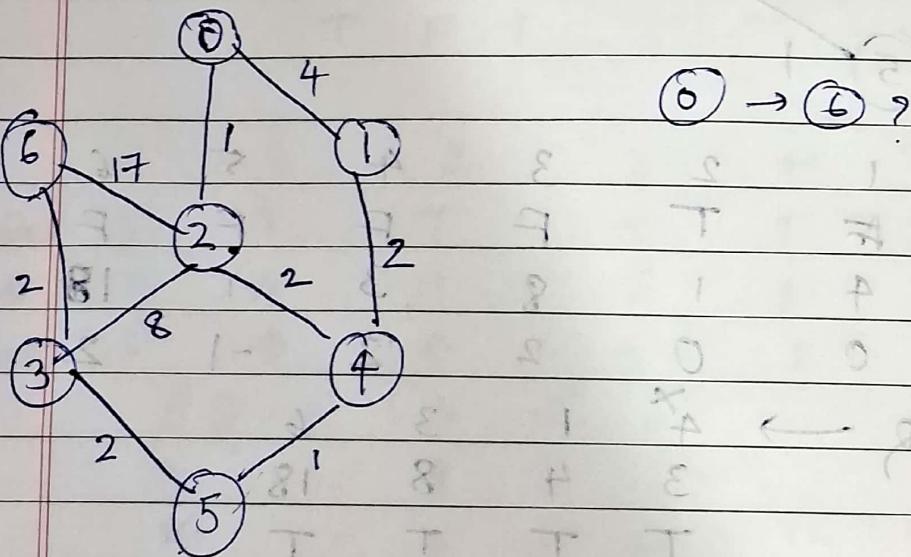
	0	1	2	3	4	5	6	7
Cost	0	1	2	2	3	3	1	3
From	0	0	6	6	2	2	0	2
9 →	X	X						

$$\text{Cost} = 3$$

$$\text{Elements} = \{7, 2, 6, 0\}$$

order new  $\Rightarrow 0 \xrightarrow{①} 6 \xrightarrow{②} 2 \xrightarrow{③} 7$

Dijkstra's Undirected Graph. Any weight



0	1	2	3	4	5	6
---	---	---	---	---	---	---

Visited	F	F	F	F	F	F
---------	---	---	---	---	---	---

Cost	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
------	---	----------	----------	----------	----------	----------

From	-1	-1	-1	-1	-1	-1
------	----	----	----	----	----	----

PQ  $\rightarrow$  0

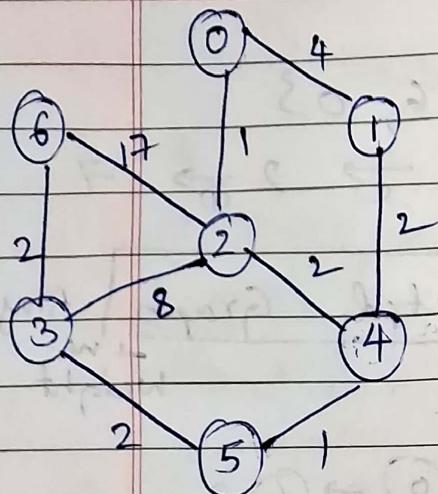
0	1	2	3	4	5	6
---	---	---	---	---	---	---

Visited	T	F	F	F	F	F
---------	---	---	---	---	---	---

Cost	0	4	$\infty$	$\infty$	$\infty$	$\infty$
------	---	---	----------	----------	----------	----------

From	0	0	0	-1	-1	-1
------	---	---	---	----	----	----

PQ  $\rightarrow$  2  
1  
4  
T T



$E = \{0, 1, 2, 3, 4, 5, 6\}$

$PQ \rightarrow 2 \cdot 1$

1 4

T T

	0	1	2	3	4	5	6	7
Visited	T	F	T	F	F	F	F	F
Cost	0	4	1	8	3	-1	18	-
From	0	0	0	2	2	-1	2	-
	$PQ \rightarrow$	4	1	3	6			
		3	4	8	18			
		T	T	T	T			

	0	1	2	3	4	5	6	7
Visited	T	F	T	F	T	F	F	F
Cost	0	4	1	8	3	4	18	-
From	0	0	0	2	2	4	2	-
	$PQ \rightarrow$	1	5	3	6			
		4	4	8	18			
		T	T	T	T			

	0	1	2	3	4	5	6	7
Visited	T	T	T	F	T	F	F	F
Cost	0	4	1	8	3	4	18	-
From	0	0	0	2	2	4	2	-
	$PQ \rightarrow$	5	3	6	1			
		4	8	18	-			
		T	T	T	T			

0	1	2	3	4	5	6
Visited	T	T	F	T	T	F
Cost	0	4	1	6	3	4
From	0	0	0	5	2	4
PQ →	3	3, 6				
	6	8	18			
	T	F	T			

0	1	2	3	4	5	6
Visited	T	T	T	T	T	F
Cost	0	4	1	6	3	4
From	0	0	0	5	2	4
PQ →	3	6	6			
	8	8	18			
	F	T	F	0	→ PQ	

0	1	2	3	4	5	6
Visited	T	T	T	T	T	T
Cost	0	4	1	6	3	4
From	0	0	0	5	2	4

① → ⑥ Cost: 8

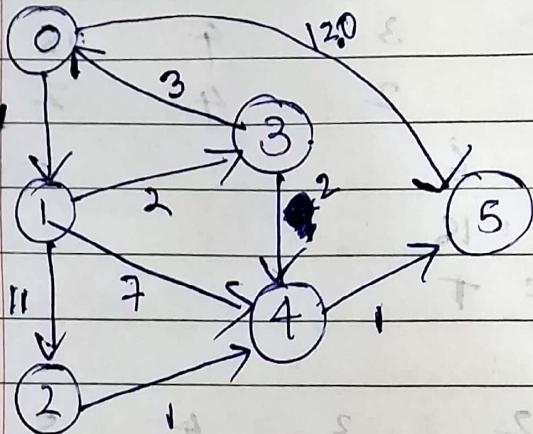
Elements: {6, 5, 4, 12, 03}

rev = {0, 21, 4, 5, 63}

- ① → ② Cost: 4 Elements = {0, 13}
- ② → ③ Cost: 1 Elements = {0, 23}
- ③ → ④ Cost: 6 Elements = {0, 2, 4, 5, 33}
- ④ → ⑤ Cost: 3 Elements = {0, 2, 43}
- ⑤ → ⑥ Cost: 4 Elements = {0, 2, 4, 53}

Dijkstra's Directed Graph

Any true weight

 $0 \rightarrow 5 ?$ 

Visited

T O T I T 2 T 3 4 5

Cost

F F F F F F F F

From

-1 -1 -1 -1 -1 -1

 $PQ \rightarrow 0 \quad T \quad T \quad T$ 

0

Visited

T F F F F F F

Cost

0 1  $\infty$   $\infty$   $\infty$  12

From

0 0 -1 -1 -1 0

 $PQ \rightarrow 1 \quad X \quad 5$ 

1 12

T T

Visited

0 1 2 3 4 5

Cost

T T P F F F

From

0 0 1 3 7 12

$PQ \rightarrow 3^X 4 2 5$

$3 7 12 12$

$T T T T$

	0	1	2	3	4	5
Visited	T	T	F	T	F	F
Cost	0	1	12	3	5	12
From	0	0	1	1	3	0

$PQ \rightarrow 4^X 4 2 5$

$5 7 12 12$

$T F T T$

	0	1	2	3	4	5
Visited	T	T	F	T	T	F
Cost	0	1	12	3	5	6
From	0	0	1	1	3	4

$PQ \rightarrow 5^X 4 2 5$

$6 7 12 12$

$T F T F$

	0	1	2	3	4	5
Visited	T	T	F	T	T	T
Cost	0	1	12	3	5	6
From	0	0	1	1	3	4

$PQ \rightarrow 4^X 2^X 5^X$

$7 12 12$

$F T F$

	0	1	2	3	4	5
Visited	T	T	T	T	T	T
Cost	0	1	12	3	5	6
From	0	0	1	1	3	4

$0 \rightarrow 5$  Cost: 6

Elements: {5, 4, 3, 1, 0}

rev: {0, 1, 3, 4, 5}

$0 \rightarrow 1$  Cost: 11 Elements: {0, 1}

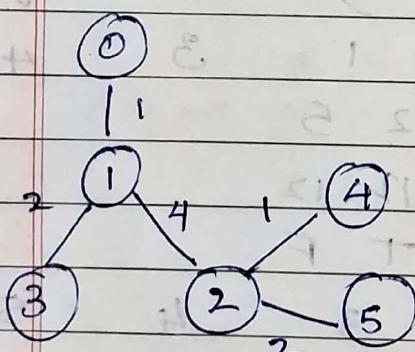
$0 \rightarrow 2$  Cost: 12 Elements: {0, 1, 2}

$0 \rightarrow 3$  Cost: 3 Elements: {0, 1, 3}

$0 \rightarrow 4$  Cost: 5 Elements: {0, 1, 3, 4}

### Dynamic Programming Undirected Graph

- Any tree weight
- DAG



$0 \rightarrow 5$  ?

	0	1	2	3	4	5
in	0	0	0	0	0	0
out	0	0	0	0	0	0

unvisited set  $\rightarrow$  0 1 2 3 4 5

dfs  $\rightarrow$

order

	0	1	2	3	4	5
in	1	2	0	0	0	0
out	0	0	0	0	0	0

unvisited set  $\rightarrow$  0 1 2 3 4 5

dfs order  $\rightarrow$

	0	1	2	3	4	5
in	1	2	0	3	0	0
out	0	0	0	0	0	0

unvisited set → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5

dfs order → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5

	0	1	2	3	4	5
in	1	2	0	3	0	0
out	0	0	0	4	0	0

unvisited set → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5

dfs order → 3

	0	1	2	3	4	5
in	1	2	4	3	0	0
out	0	0	0	4	0	0

unvisited set → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5

dfs order → 3

	0	1	2	3	4	5
in	1	2	5	3	6	0
out	0	0	0	4	0	0

unvisited set → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5

dfs order → 3

	0	1	2	3	4	5
in	1	2	5	3	6	0
out	0	0	0	4	7	0

unvisited set → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5

dfs order → 3 4

	0	1	2	3	4	5
in	1	2	5	3	6	8
out	0	0	0	4	7	0

unvisited set  $\rightarrow \times \times \times \times \times$   
 dfs order  $\rightarrow 3 4$

	0	1	2	3	4	5
in	1	2	5	3	6	8
out	0	0	0	4	7	9

unvisited set  $\rightarrow$   
 dfs order  $\rightarrow 3 4 5$

	0	1	2	3	4	5
in	1	2	5	3	6	8
out	0	0	10	4	7	9

unvisited set  $\rightarrow$   
 dfs order  $\rightarrow 3 4 5 2$

	0	1	2	3	4	5
in	1	2	5	3	6	8
out	0	11	10	4	7	9

unvisited set  $\rightarrow$   
 dfs order  $\rightarrow 3 4 5 2 1$

	0	1	2	3	4	5
in	1	2	5	3	6	8
out	12	11	10	4	7	9

unvisited set  $\rightarrow$   
 dfs order  $\rightarrow 3 4 5 2 1 0$

(iv)

dfs order  $\rightarrow 0 1 2 5 4 3$

dfs → ~~0~~ 1 2 3 4 5

0	1	2	3	4	5
---	---	---	---	---	---

Visited

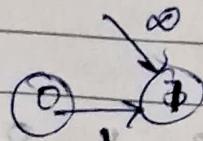
T	F	F	F	FT	F
---	---	---	---	----	---

Cost

0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
---	----------	----------	----------	----------	----------

From

0	-1	-1	-1	-1	-1
---	----	----	----	----	----



dfs → ~~0~~ ~~1~~ 2 3 4 5

0	1	2	3	4	5
---	---	---	---	---	---

Visited

T	T	F	F	F	F
---	---	---	---	---	---

Cost

0	1	$\infty$	$\infty$	$\infty$	$\infty$
---	---	----------	----------	----------	----------

From

0	0	-1	-1	-1	-1
---	---	----	----	----	----

dfs → ~~0~~ ~~1~~ ~~2~~ 3 4 5

0	1	2	3	4	5
---	---	---	---	---	---

Visited

T	T	T	<del>F</del>	F	F
---	---	---	--------------	---	---

Cost

0	1	5	$\infty$	$\infty$	$\infty$
---	---	---	----------	----------	----------

From

0	0	1	-1	-1	-1
---	---	---	----	----	----

dfs → ~~0~~ ~~1~~ ~~2~~ ~~3~~ 4 5

0	1	2	3	4	5
---	---	---	---	---	---

Visited

T	T	T	F	F	T
---	---	---	---	---	---

Cost

0	1	5	<del><math>\infty</math></del>	$\infty$	<del><math>\infty</math></del>
---	---	---	--------------------------------	----------	--------------------------------

From

0	0	1	-1	-1	2
---	---	---	----	----	---

dfs → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5

0	1	2	3	4	5
---	---	---	---	---	---

Visited

T	T	T	F	T	T
---	---	---	---	---	---

Cost

0	1	5	$\infty$	8	7
---	---	---	----------	---	---

From

0	0	1	-1	2	2
---	---	---	----	---	---

		dfs →	X	X	X	X	X	X
0	1	2	3	4	5			
Visited	T	T	T	T	T	T	T	
Cost	0	1	5	3	6	9	7	
From	0	0	1	1	2	0	2	

① → ⑤ : cost : 7

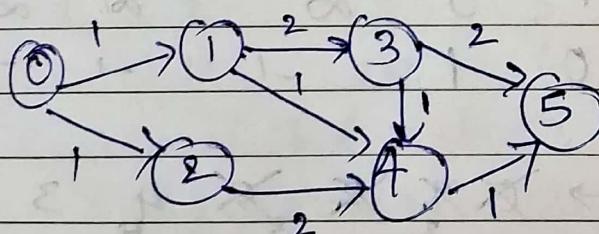
Elements Order: {5, 2, 1, 0}

new: {3, 0, 1, 2, 5}

- ① → ① cost: 0 Elements: {0, 13}
- ① → ② cost: 5 Elements: {0, 1, 23}
- ① → ③ cost: 3 Elements: {0, 1, 33}
- ① → ④ cost: 6 Elements: {0, 1, 2, 43}

### Dynamic Programming Directed Graph

• Any the weight  
• DAG



① → ⑤ ?

	0	1	2	3	4	5
in	1	0	0	1	0	0
out	0	0	0	0	0	0

unvisited → {1, 2, 3, 4, 5}  
dfs order →

unvisited set  $\rightarrow \cancel{\times} \cancel{\times} 2 3 4 5$

dfs order  $\rightarrow$

0 1 2 3 4 5

in 2 1 2 0 0 0

out 0 0 0 0 0 0

unvisited set  $\rightarrow \cancel{\times} \cancel{\times} 2 \cancel{\times} 4 5$

dfs order  $\rightarrow$

0 1 2 3 4 5

in 2 1 2 0 0 3

out 0 0 0 0 0 0

unvisited set  $\rightarrow \cancel{\times} \cancel{\times} 2 \cancel{\times} \cancel{\times} 5$

dfs order  $\rightarrow$

0 1 2 3 4 5

in 2 1 2 0 3 4

out 0 0 0 0 0 0

unvisited set  $\rightarrow \cancel{\times} \cancel{\times} \cancel{\times} \cancel{\times} \cancel{\times}$

dfs order  $\rightarrow$

0 1 2 3 4 5

in 2 1 2 0 3 4

out 0 0 0 0 0 0

unvisited set  $\rightarrow \cancel{\times} \cancel{\times} 2 \cancel{\times} \cancel{\times} \cancel{\times}$

dfs order  $\rightarrow$

0 1 2 3 4 5

in 1 2 0 3 4 5

out 0 0 0 0 0 6

unvisited set  $\rightarrow \cancel{0} \cancel{1} \cancel{2} \cancel{3} \cancel{4} \cancel{5}$   
 dfs order  $\rightarrow 5 4$

	0	1	2	3	4	5
in	1	2	0	3	4	5
out	0	0	0	0	7	6

dfs order  $\rightarrow 5 4 3$   
 unvisited set  $\rightarrow \cancel{0} \cancel{1} \cancel{2} \cancel{3} \cancel{4} \cancel{5}$

	0	1	2	3	4	5
in	1	2	0	3	4	5
out	0	0	0	8	7	6

unvisited set  $\rightarrow \cancel{0} \cancel{1} \cancel{2} \cancel{3} \cancel{4} \cancel{5}$   
 dfs order  $\rightarrow 5 4 3 1$

	0	1	2	3	4	5
in	1	2	0	3	4	5
out	0	9	0	8	7	6

unvisited set  $\rightarrow \cancel{0} \cancel{1} \cancel{2} \cancel{3} \cancel{4} \cancel{5}$   
 dfs order  $\rightarrow 5 4 3 9$

	0	1	2	3	4	5
in	1	2	10	3	4	5
out	0	9	0	8	7	6

unvisited set  $\rightarrow \cancel{0} \cancel{1} \cancel{2} \cancel{3} \cancel{4} \cancel{5}$   
 dfs order  $\rightarrow 5 4 3 1 2$

	0	1	2	3	4	5
in	1	2	10	3	4	5
out	0	9	11	8	7	6

unvisited set  $\rightarrow 0 \ 1 \ 2 \ 3 \ 4 \ 5$   
 dfs order  $\rightarrow 5 \ 4 \ 3 \ 1 \ 2 \ 0$

0	1	2	3	4	5
in	1	2	10	3	4
out	12	9	11	8	7

(even)  $\rightarrow$  ~~0 1 2 3 4 5~~

dfs order  $\rightarrow \cancel{0} \ 2 \ 1 \ 3 \ 4 \ 5$

0	1	2	3	4	5
Visited	T	F	F	F	F
lost	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
From	0	-1	-1	-1	-1

dfs order  $\rightarrow \cancel{0} \ \cancel{1} \ 2 \ 3 \ 4 \ 5$

0	1	2	3	4	5
Visited	T	F	T	F	F
lost	0	$\infty$	1	$\infty$	$\infty$
From	0	-1	1	-1	-1

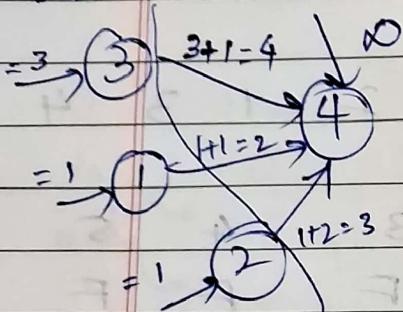
dfs order  $\rightarrow \cancel{0} \ \cancel{1} \ \cancel{2} \ 3 \ 4 \ 5$

0	1	2	3	4	5
Visited	T	T	T	F	F
lost	0	1	1	$\infty$	$\infty$
From	0	0	1	-1	-1

dfs order  $\rightarrow \cancel{0} \ \cancel{1} \ \cancel{2} \ \cancel{3} \ 4 \ 5$

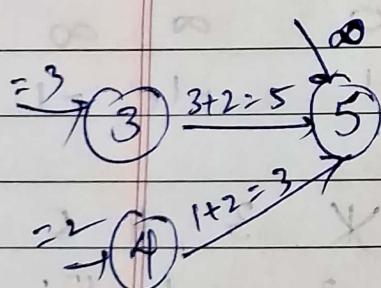
0	1	2	3	4	5
Visited	T	T	T	T	F
lost	0	1	1	3	$\infty$
From	0	1	1	1	-1

	dfs order	1	2	3	4	5
Visited	T	T	T	T	T	F
Cost	0	1	10	3	2	$\infty$
From	0	1	10	10	10	-1



	dfs order	1	2	3	4	5
Visited	T	T	T	T	T	T

Visited	0	1	2	3	4	5
Cost	0	1	1	3	2	3
From	0	1	1	1	1	4



① → ⑤ ? cost = 3

Elements order : {5, 4, 1, 03}

ans : {0, 1, 4, 5}

- ① → ② cost: 1 Elements order: {0, 13}
- ① → ③ cost: 1 Elements order: {0, 23}
- ① → ④ cost: 3 Elements order: {0, 1, 33}
- ① → ⑤ cost: 2 Elements order: {0, 1, 43}