PSA Class 9.0
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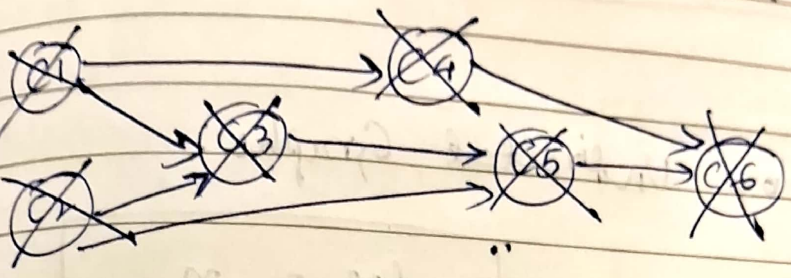
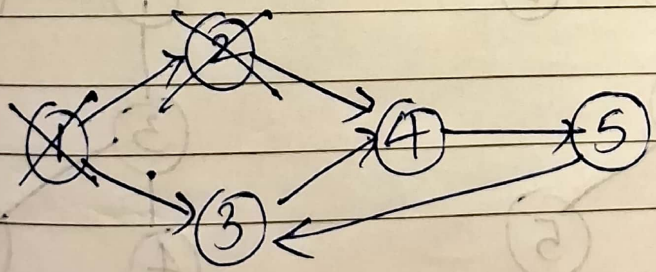


No Fan ins → choose these nodes in every iteration

C1, C2
C4, C3
C5
C6

∴ Topological Sort : | C1 | C2 | C4 | C3 | C5 | C6 |
0 1 2 3 4 5

There should be no loop



No Fan ins
1
2

But All 3, 4, 5 have fan ins that cannot be removed since there is a loop

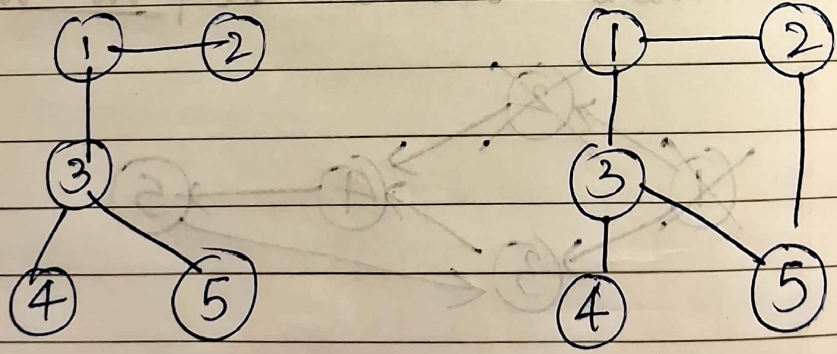
A directed graph that has no loop is called as a DAG - Directed Acyclic Graph

For Undirected Graphs:

If No. of nodes = n
No. of edges = n - 1
Then, the undirected graph has no loop → Also a tree

If No. of nodes = n
No. of edges > n - 1
Then, the undirected graph has a loop

Ex:



Tree: Every node has a father, expt root node → it has no father

DFS - Depth First Search
BFS - Breadth First Search

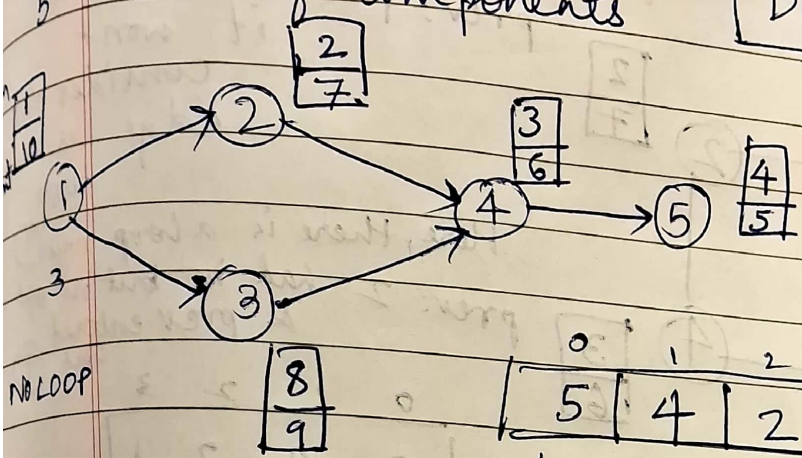
Traversal :

1. Visit all nodes exactly once
2. Get DFS Order
3. Find largest path
4. Detects loop
5. No. of components

DFS Using Timestamp

Space $\approx \Theta(n)$

Complexity $= O(V+E)$

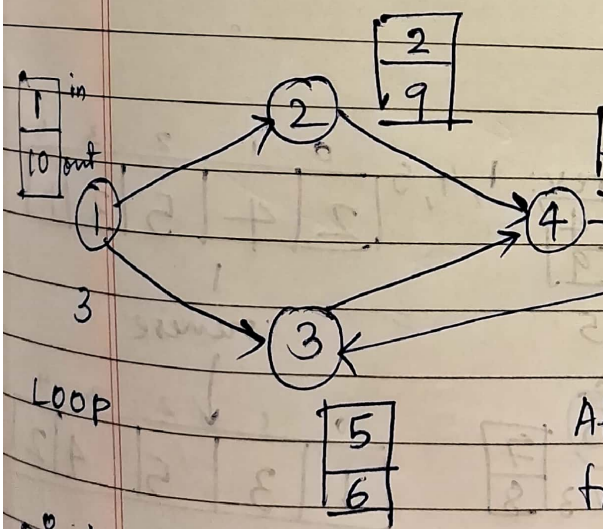


0	1	2	3	4
5	4	2	3	1

reverse

Topologically Sorted →

0	1	2	3	4
1	3	2	4	5



At time 6, we are trying to enter into (4) that means there is a loop in the graph since we have not exited from (4)

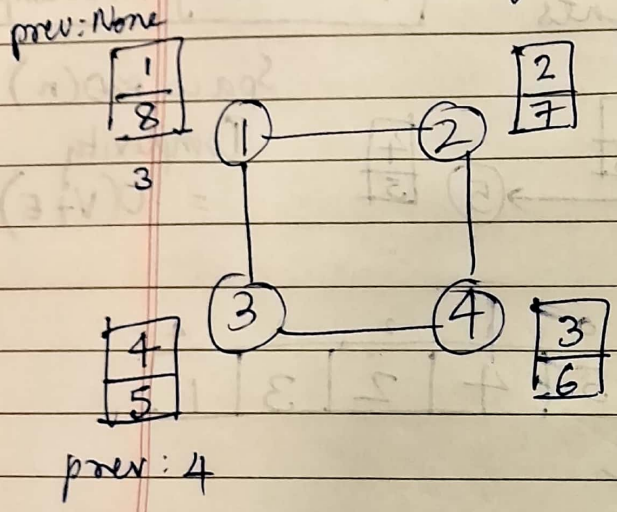
0	1	2	3	4
3	5	4	2	1

reverse

0	1	2	3	4
1	2	4	5	3

no meaning since there is a loop

Previous DFS Using Timestamp will not work exactly with Directed Graph for an Undirected Graph, here we need to make some changes: keep note of prev node so that it won't consider that edge as a loop



Here, there is a loop since (1) had in but no out & prev entered was (4)

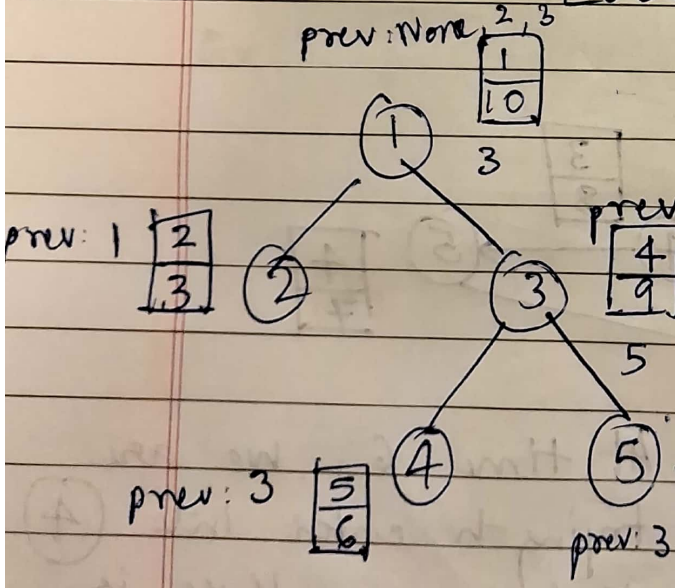
0	1	2	3
3	4	2	1

↓ reverse

0	1	2	3
1	2	4	3

LOOP

Space complexity : $\approx O(n)$



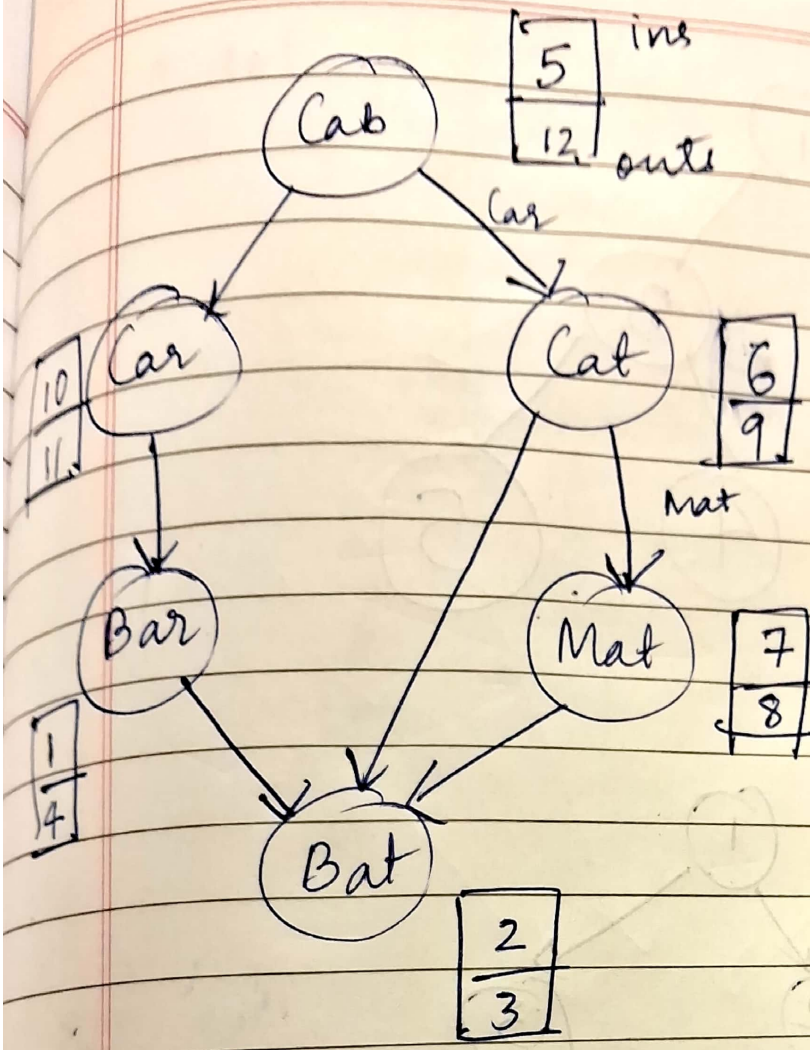
0	1	2	3	4
2	4	5	3	1

↓ reverse

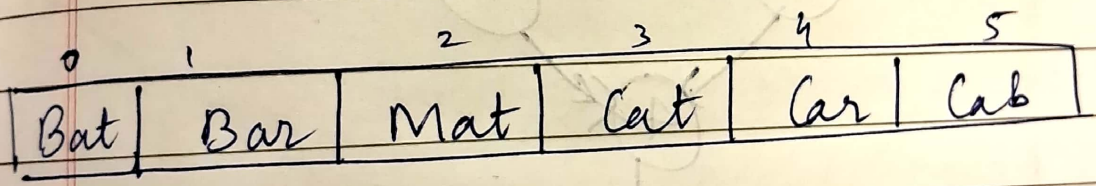
0	1	2	3	4
1	3	5	4	2

NO LOOP

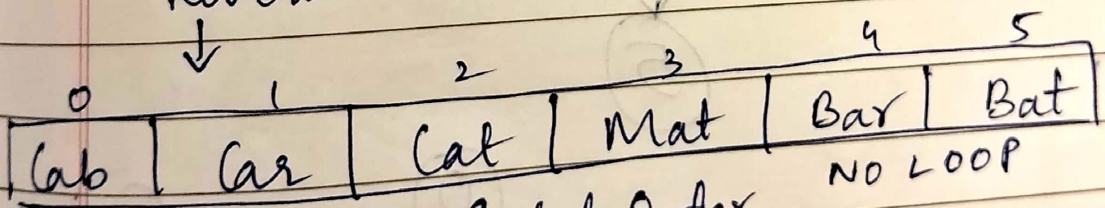
∴ It is in a Topological Sorted Order



If all paths are exhausted, we need to keep traversing until no node has empty ins & outs



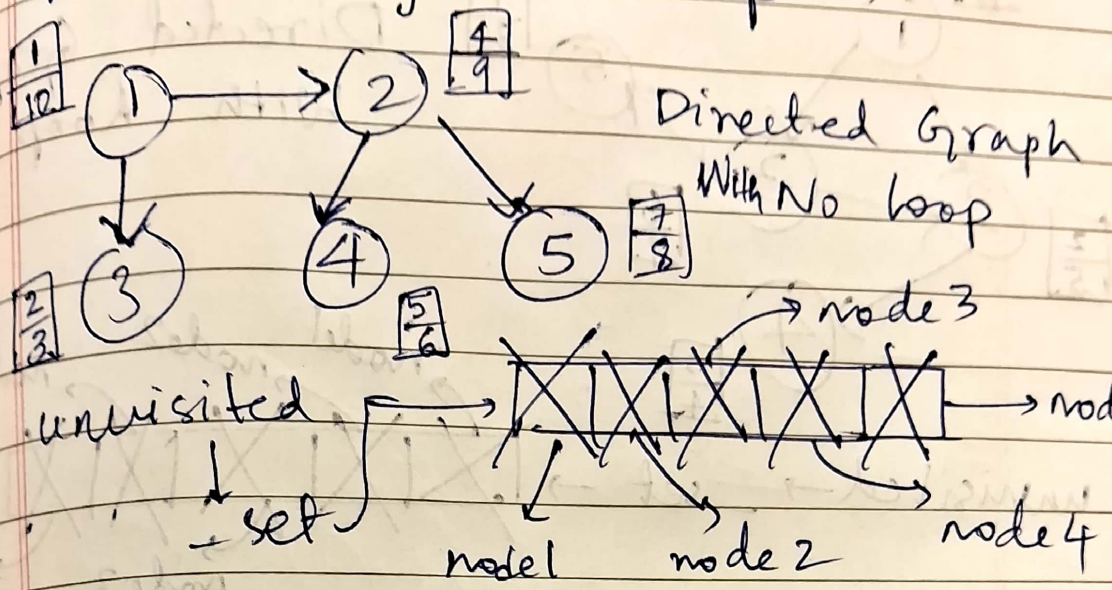
Reverse

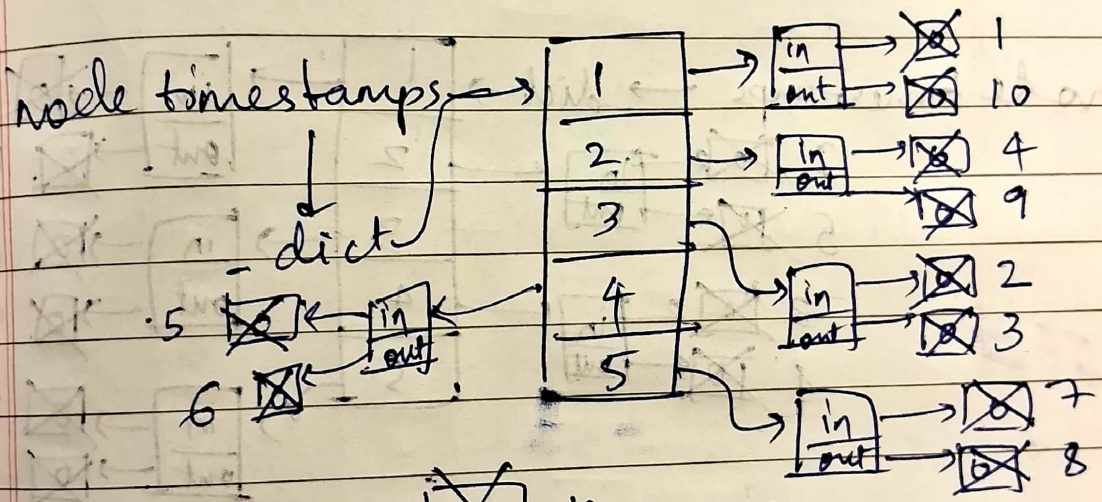


NO LOOP

One Topologically Sorted Order

DFS Using Timestamp





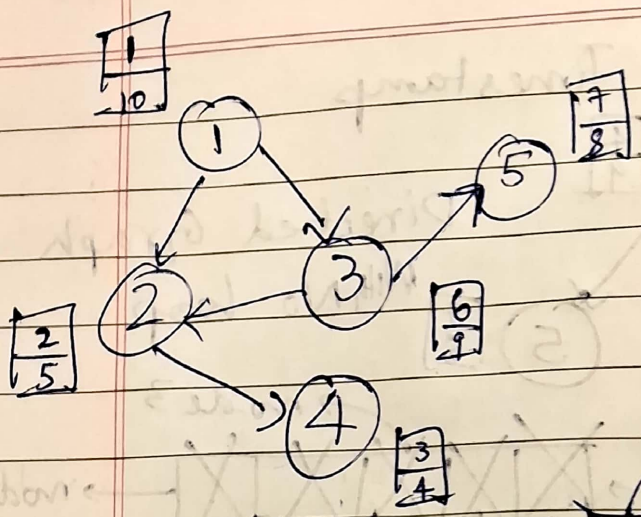
time → 0 X X X X X X X X X X

work → 0 X X X X X 6

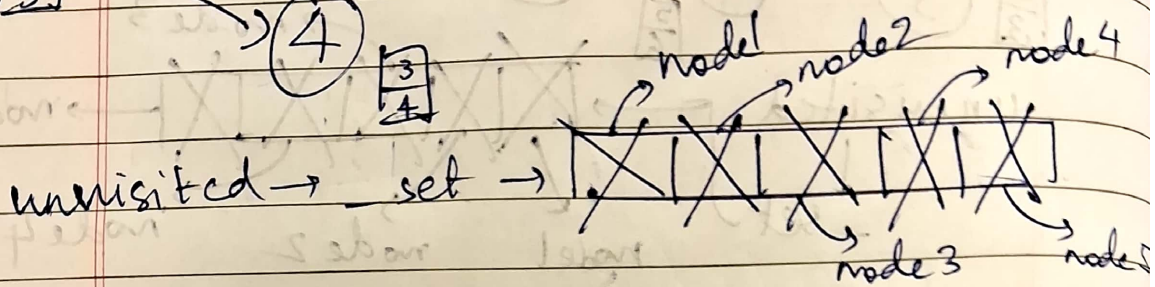
dfs_order → ~~3~~ | ~~4~~ | ~~5~~ | ~~2~~ | ~~1~~

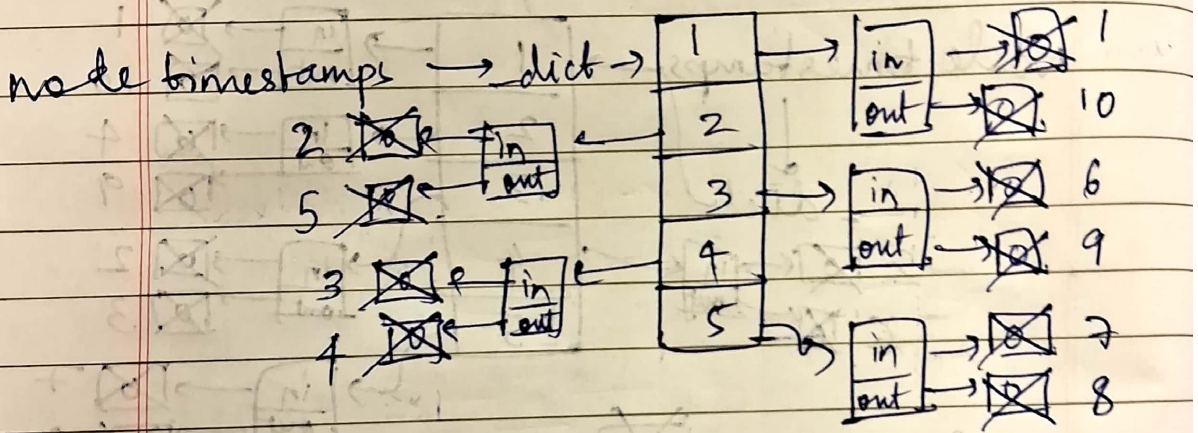
1	2	5	3	4

has loop → False



Directed Graph With loop





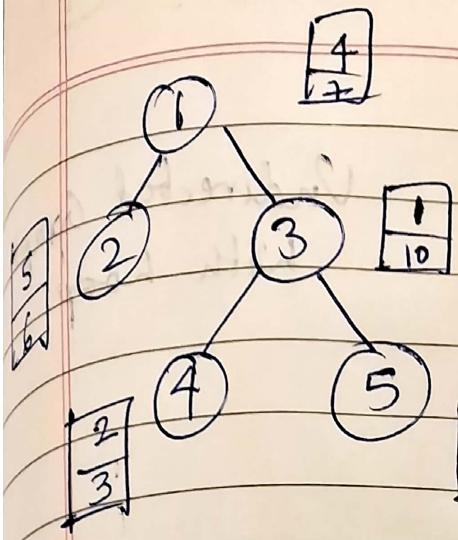
time → 0 ✗ ✗ ✗ ✗ ✗ ✗ ✗ ✗ ✗ ✗

work → 0 ✗ ✗ ✗ ✗ ✗ 6

has_loop → ~~False~~ True

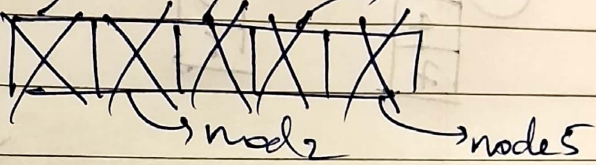
dfs_order → 4 | 2 | ~~5~~ | 3 | 1

1 | 3 | 5 | 2 | 4

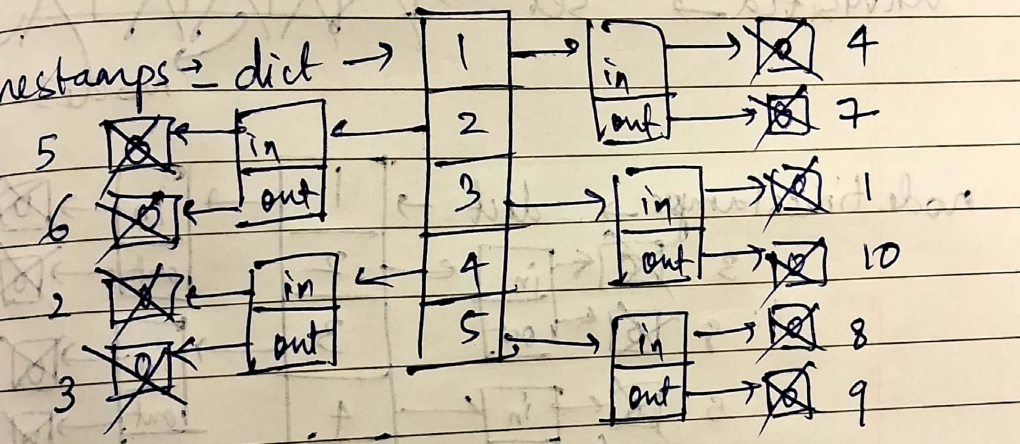


Undirected Graph with No Loop

visited → set →



node timestamps → dict →

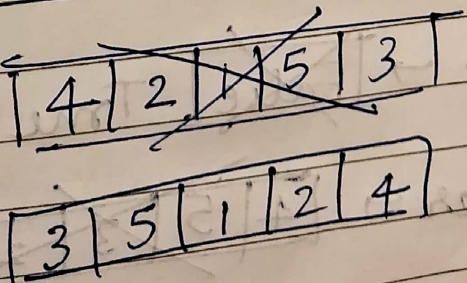


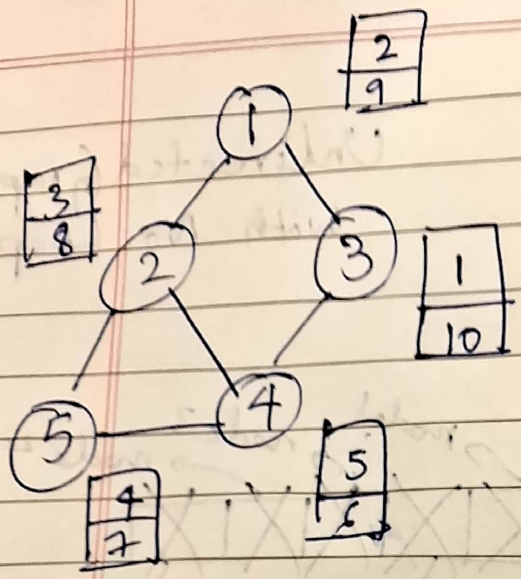
time →

work →

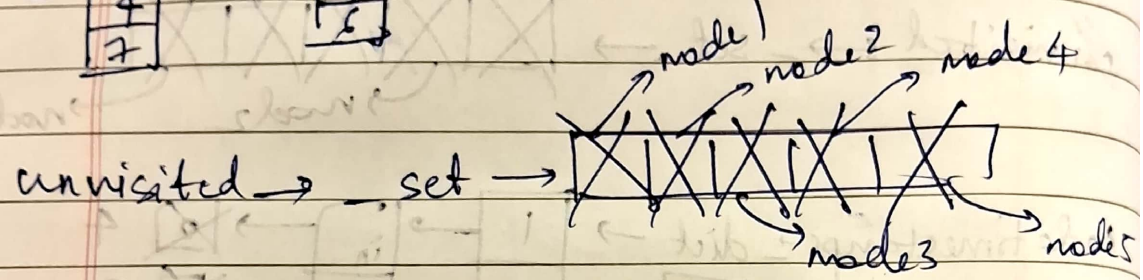
has_loop → False

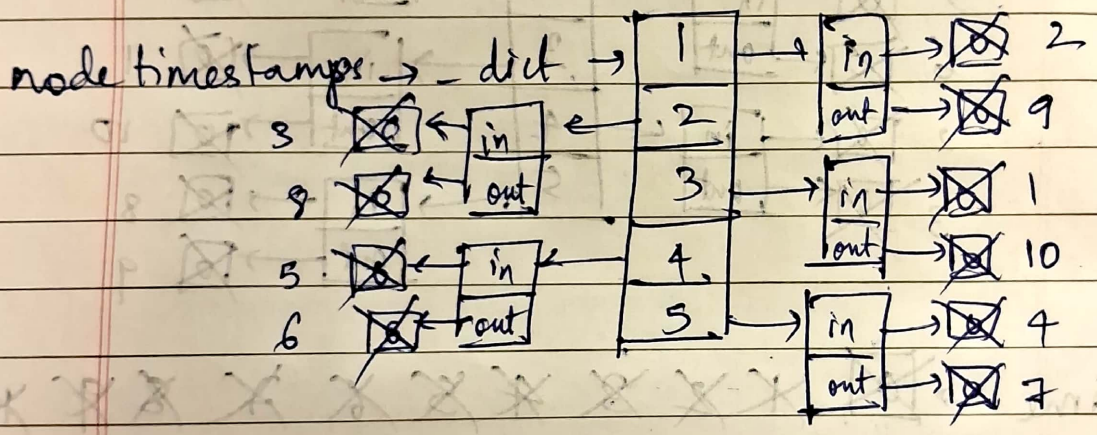
dfs_order →





Undirected Graph with loop





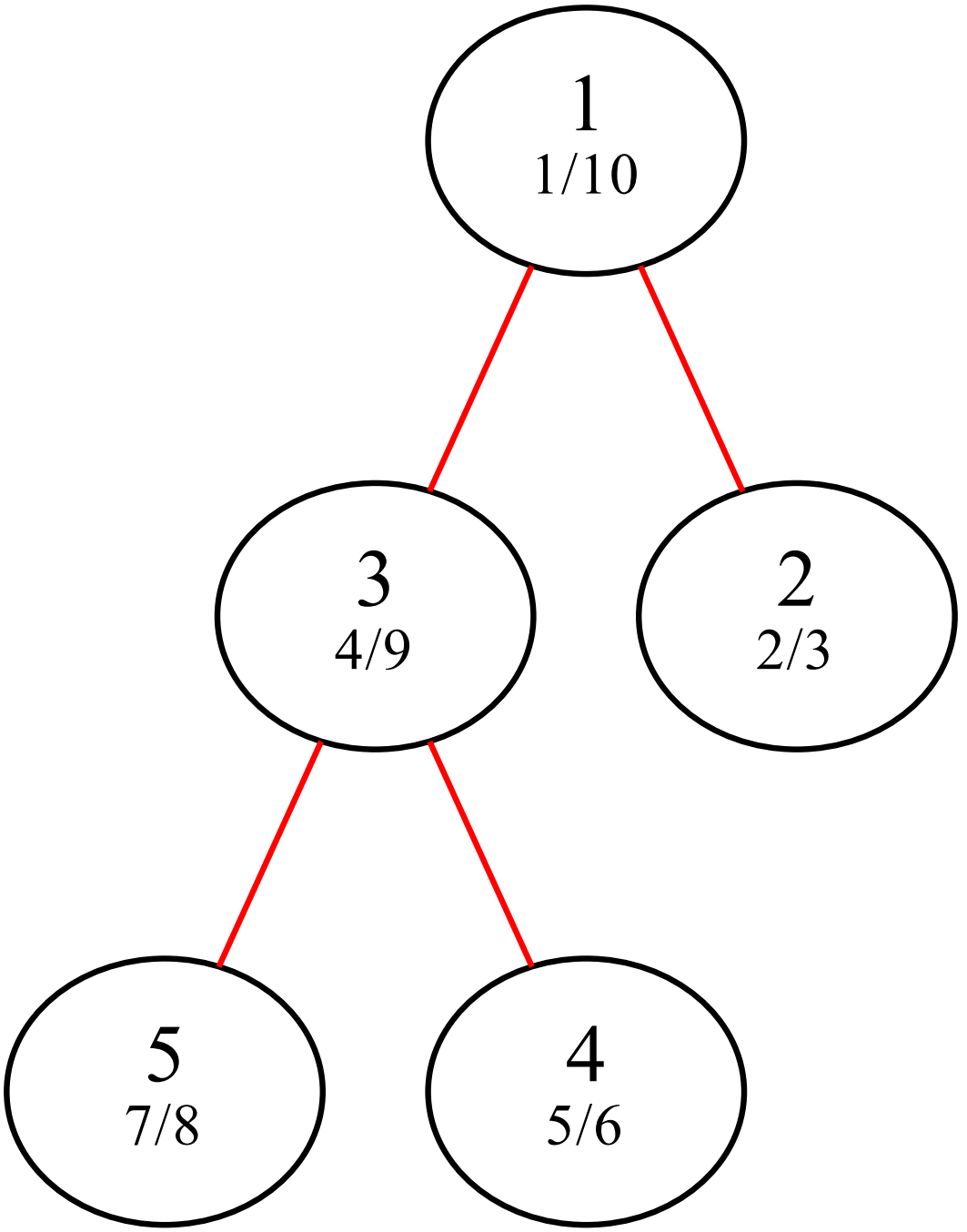
time → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ 10

work → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5

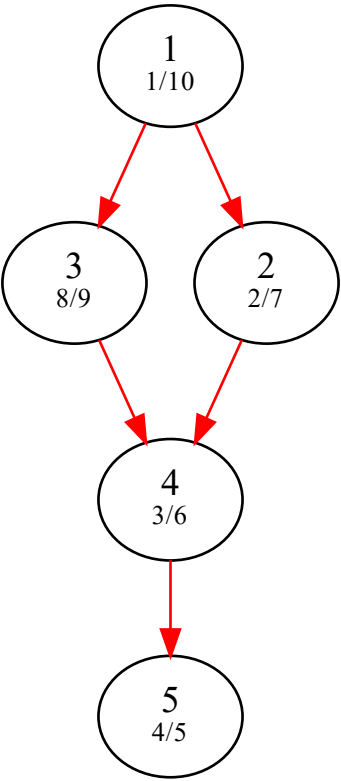
has_loop → ~~False~~ True

dfs_order → [4 | 5 | ~~3~~ | ~~2~~ | 1]

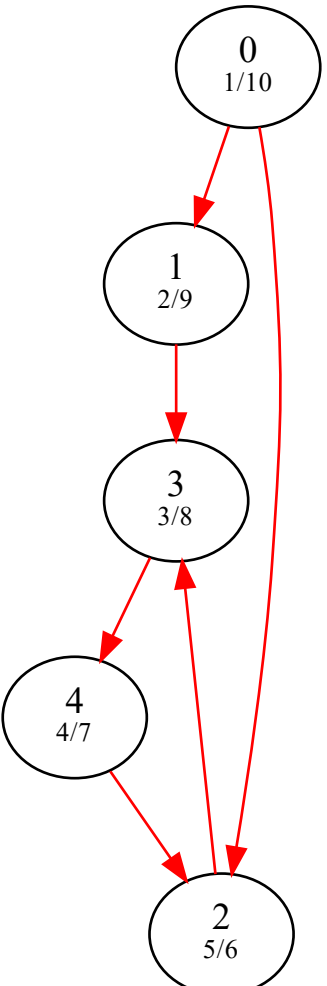
[1 | 2 | 3 | 4 | 5]



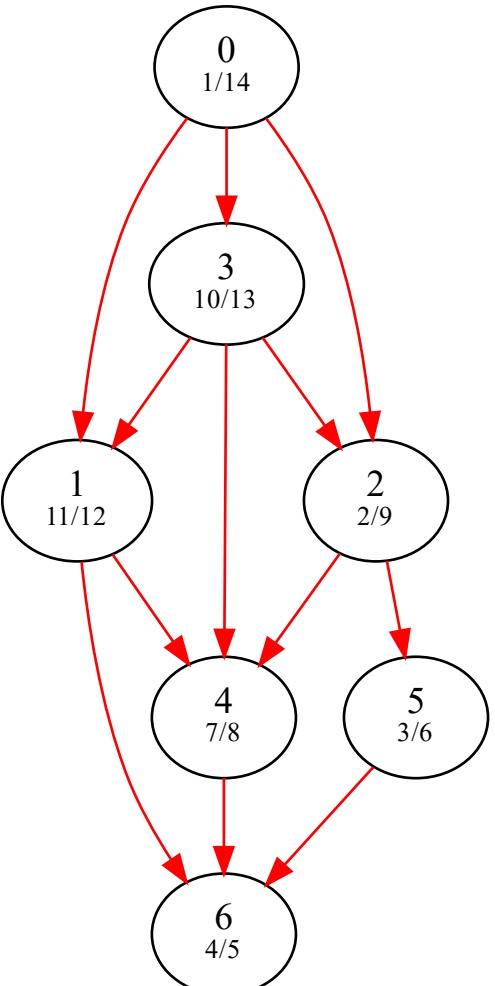
[1 3 5 4 2] NO LOOP



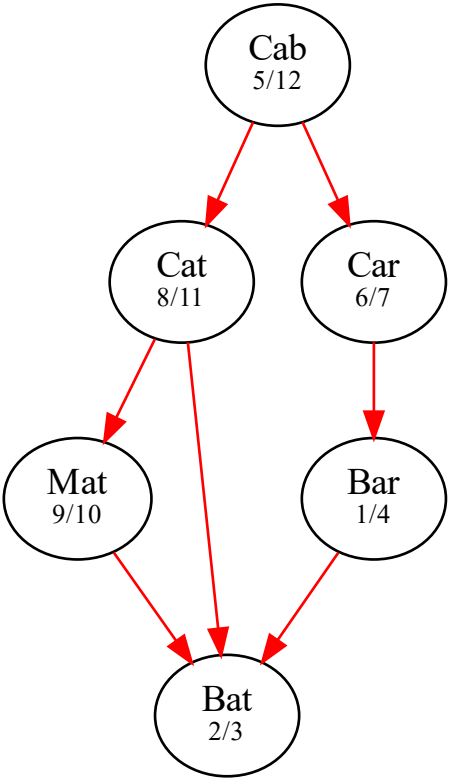
[1 3 2 4 5] NO LOOP



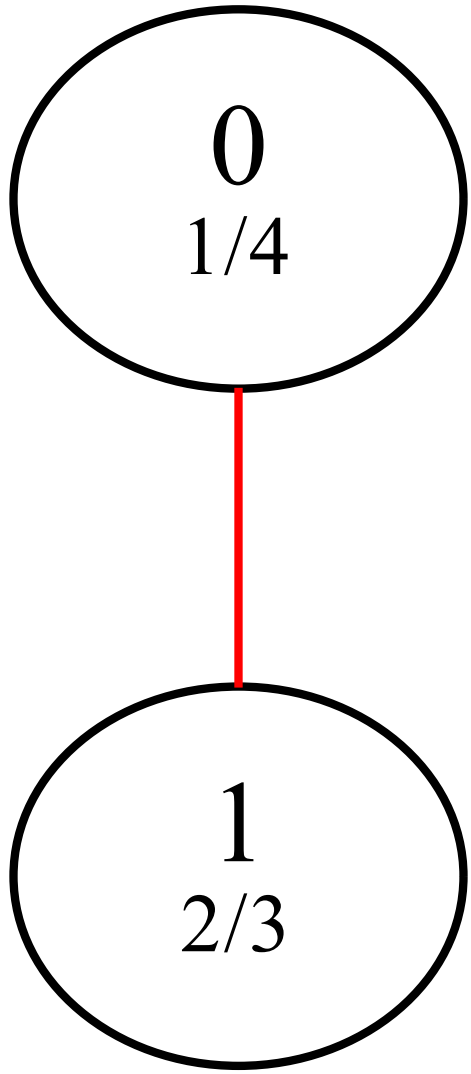
[0 1 3 4 2] LOOP



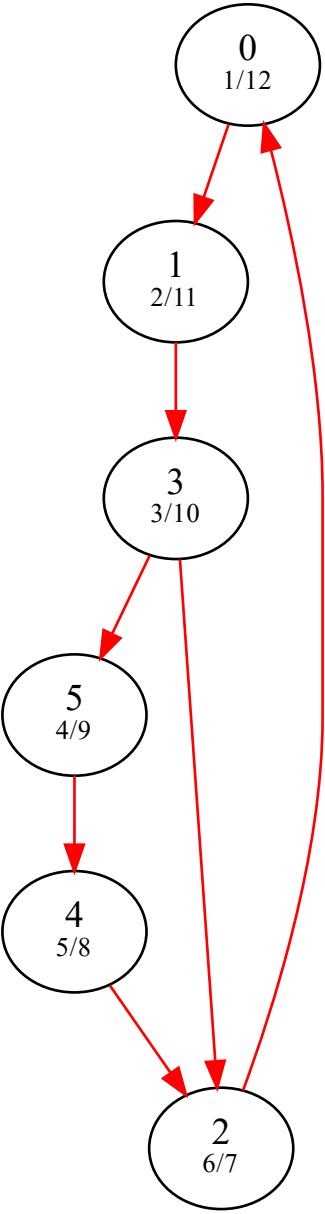
[0 3 1 2 4 5 6] NO LOOP



[Cab Cat Mat Car Bar Bat] NO LOOP



[0 1] NO LOOP



[0 1 3 5 4 2] LOOP