Mini Project: 1

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Contributions:

1.

a) $x_i=\{1,2,3,4\}$. $P(x)=\{.5, .125, .125, .25\}$

$E(X)= \sum x*P(x) = 1*.5+ 2*.125+ 3*.125 + 4*.25 = 2.125$

$Var(X)=E(x^2)-(E(x))^2$

$x^2=\{1,4,9,16\}$

$E(x^2)=.5+.5+9/8+4$

$=6.125$

$(E(x))^2= (17/8)^2$

$Var(X)=1.6093.$

b) Here $X \sim f(x)$, consider it be an arbitrary distribution function, where $\sum P_i=1$ and $P(X=x_i)=p_i$
Assume the value of N be not very large, then we may not consider LLN and CLT.
Lets divide the subinterval between [0,1]
$x_1=[0,1/2]$
$x_{2=}[1/2,5/8]$
$x_{3=}[5/8,6/8]$
$x_4=[6/8,1]$
If x_i falls in the respective ranges(sub intervals), then the particular $P[X=x_i]$ is assigned.
$P[X=x_i]=P[U$ falls in subinterval $i]$
$=P[P0+P1+P2+\ldots P_{i-1} < U < P0+P1+P2+P3+\ldots..P_i]$
$=F(P0+P1+P2\ldots.+P_i) – F(P0+P1+\ldots.+P_{I-1})$
$=P0+P1+P2\ldots.+ P_i - P0+P1+\ldots.+P_{I-1}$
$=P_i$

Lets assume that the value of N is large, i.e.; large number of draws then,
Simulate a large number of N independent draws from distribution of $X1,X2,\ldots.,X_N$
(E) Monte Carlo estimator of μ = (X`)sample average = $(1/N)*(\sum X_i)$
(Variance) Mote Carlo estimator of $\sigma^{2=} E((X- \mu)^2) = (1/(N-1))* (\sum X_i- x`)^2$

c) A=replicate(1000,sample(c(1,2,3,4),1,TRUE,c(.5,.125,.125,.25)))

n	Mean	Variance	P(X<=2) (cdf)
1000	2.144	1.668933	.003
1000	2.168	1.665441	.003
1000	2.066	1.581225	.003
1000	2.074	1.544068	.003
1000	2.144	1.604869	.003

d)

N	Mean	Variance	P(X<=2) (cdf)
5000	2.1396	1.601232	0.00625
5000	2.109	1.590237	0.00625
5000	2.092	1.590417	0.00625
5000	2.428	1.619932	0.00625
5000	2.1576	1.613885	0.00625

N	Mean	Variance	P(X<=2) (cdf)
10000	2.1278	1.610628	.0003
10000	2.1272	1.612381	.0003
10000	2.1505	1.627412	.0003
10000	2,1091	1.601957	.0003
10000	2.1299	1.599586	.0003

e) From the results obtained by a,c,d we observe that as the value of N increases the distribution turns out to be normal and the values of μ and σ^2 are almost equal to the true values of μ and σ^2.
Also the variance decreases as value of N increases , thus follows LLN.

2)

a) For large values of n ,ˆ **p-hat** follows a normal distribution.

Explanation: By **CLT**, for large n, **p-hat** ~= N[E(**p-hat**)=p , var(**p-hat**) = p(1-p)/n]

Or Z=(**p-hat**)-(p)/sqrt(p(1-p)/n) ~= N(0,1).

b) For a given (n, p) combination, simulate 500 values of ˆp, and make a normal Q − Q plot of the values, the

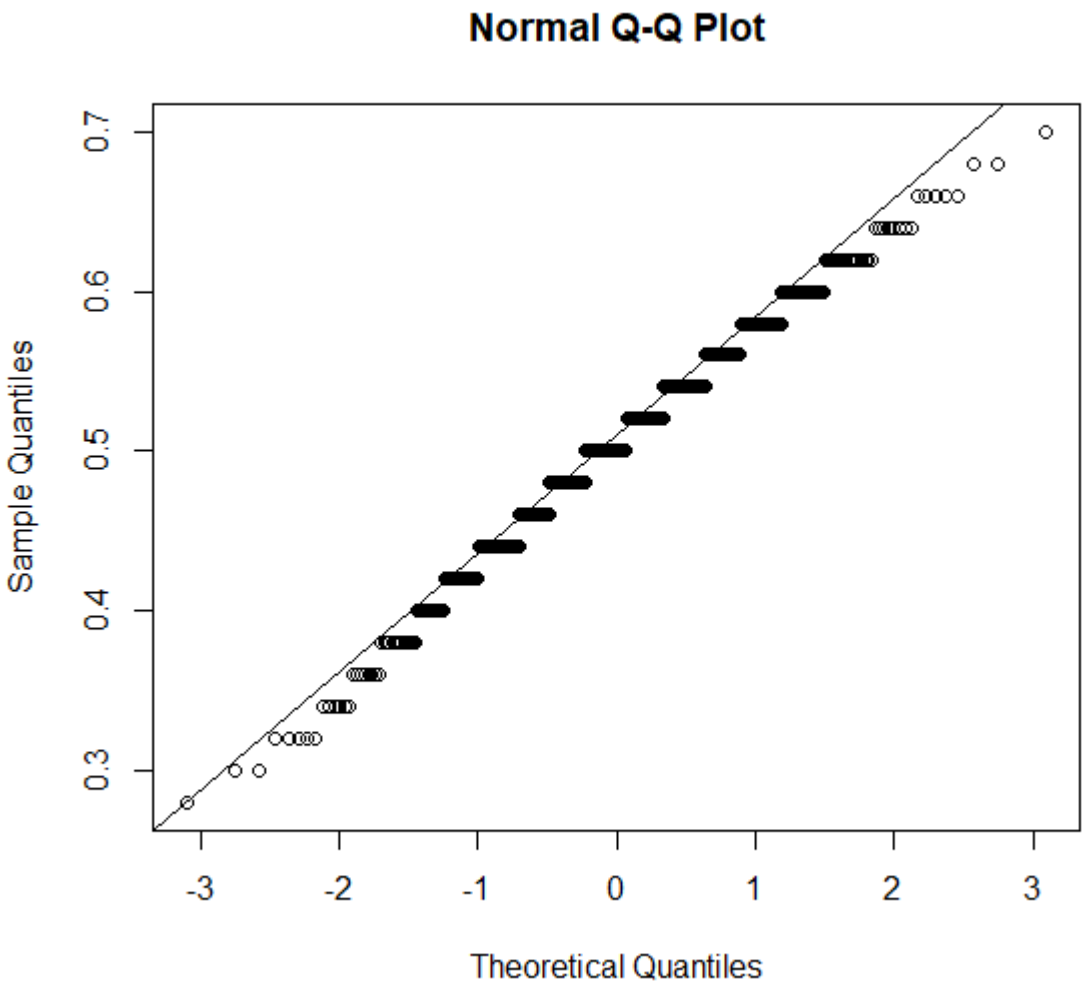
R-Code:

```
p.10k= replicate(500,mean(rbinom(10,1,.1)))

qqnorm(p.10k)
```

qqline(p.10k)

From the graph obtained, it is clear that for a given (n,p) the given distribution does not look like normal distribution, because the value of N is comparably small.



c) Plots for the combination of (n,p) values = (10,(.1,.25,.5,.75,.90))

R-Code:

p.10k= replicate(500,mean(rbinom(10,1,.1)))

qqnorm(p.10k)

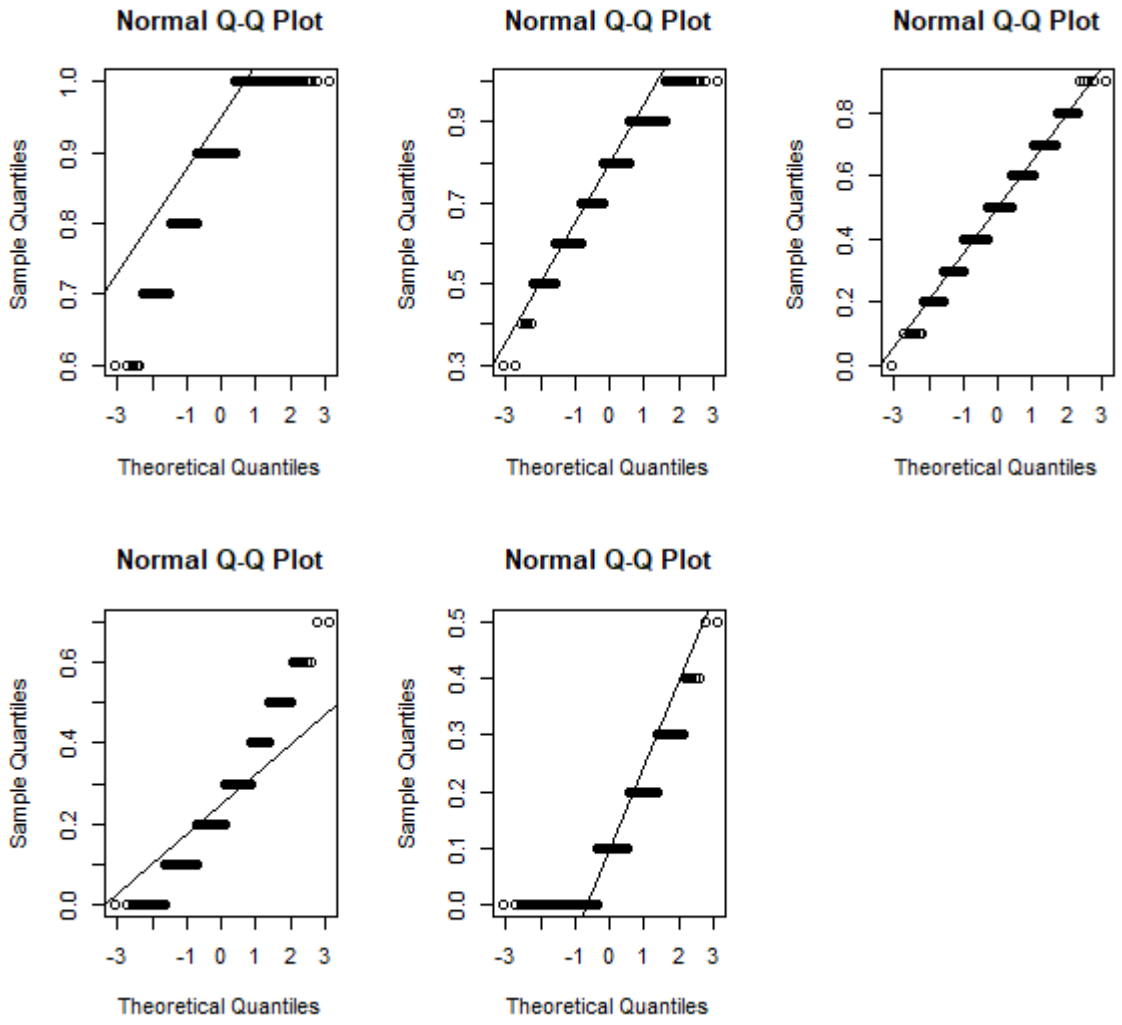
qqline(p.10k)

p.10k= replicate(500,mean(rbinom(10,1,.25)))

qqnorm(p.10k)

```
qqline(p.10k)


p.10k= replicate(500,mean(rbinom(10,1,.50)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(10,1,.75)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(10,1,.90)))
qqnorm(p.10k)
qqline(p.10k)
```



Plots for the combination of (n,p) values = (30,(.1,.25,.5,.75,.90)), (50,(.1,.25,.5,.75,.9))

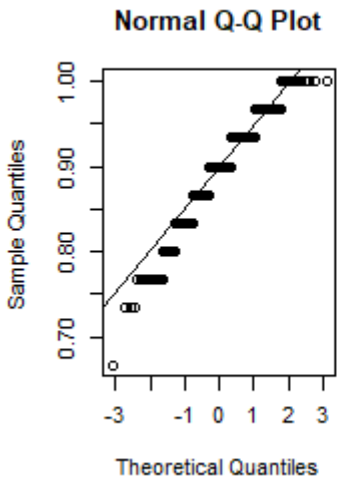
R-Code:

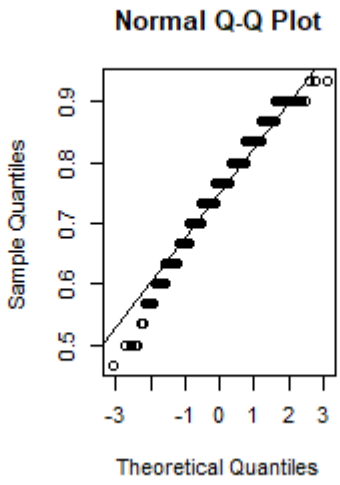
```
p.10k= replicate(500,mean(rbinom(30,1,.1)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(30,1,.25)))
qqnorm(p.10k)
qqline(p.10k)
```

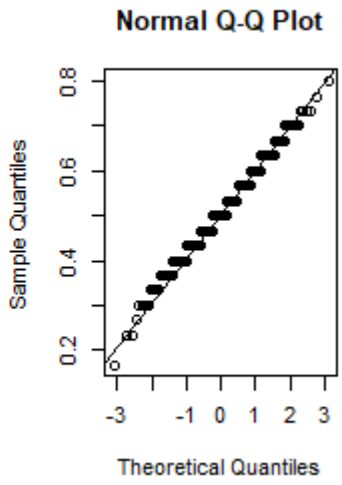
```
p.10k= replicate(500,mean(rbinom(30,1,.50)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(30,1,.75)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(30,1,.90)))
qqnorm(p.10k)
qqline(p.10k)
```

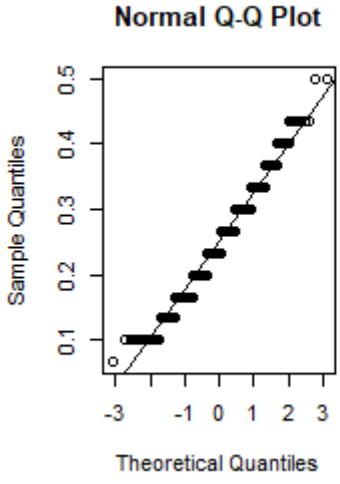
R-Code

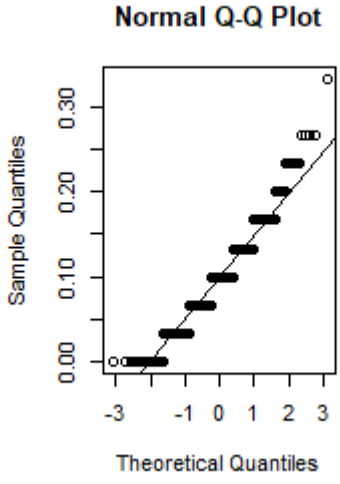
```
p.10k= replicate(500,mean(rbinom(50,1,.1)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(50,1,.25)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(50,1,.50)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(50,1,.75)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(50,1,.90)))
qqnorm(p.10k)
qqline(p.10k)
```

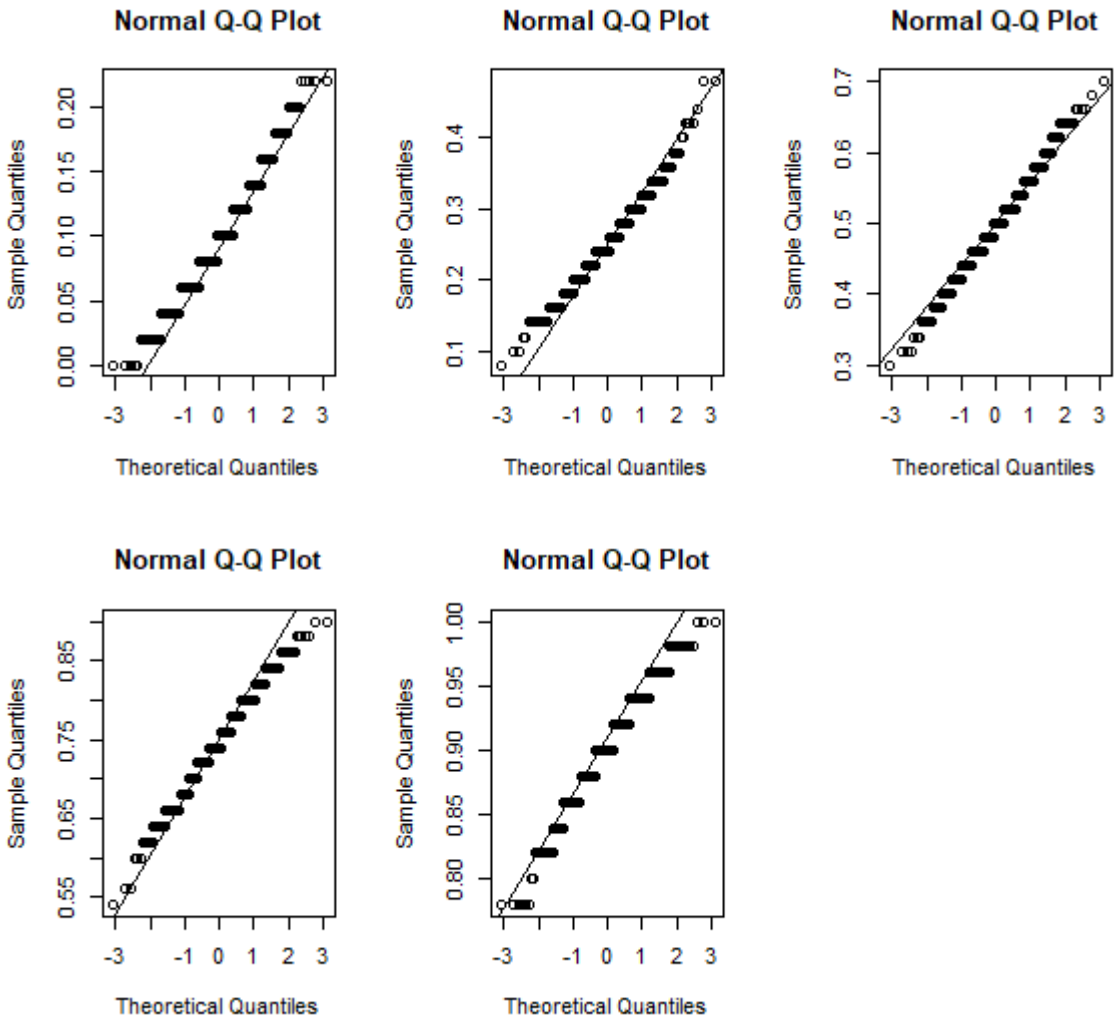

Normal Q-Q Plot
Sample Quantiles
0.70
0.80
0.90
1.00
-3
-1
0
1
2
3
Theoretical Quantiles


Normal Q-Q Plot
Sample Quantiles
0.5
0.6
0.7
0.8
0.9
-3
-1
0
1
2
3
Theoretical Quantiles


Normal Q-Q Plot
Sample Quantiles
0.2
0.4
0.6
0.8
-3
-1
0
1
2
3
Theoretical Quantiles


Normal Q-Q Plot
Sample Quantiles
0.1
0.2
0.3
0.4
0.5
-3
-1
0
1
2
3
Theoretical Quantiles


Normal Q-Q Plot
Sample Quantiles
0.00
0.10
0.20
0.30
-3
-1
0
1
2
3
Theoretical Quantiles

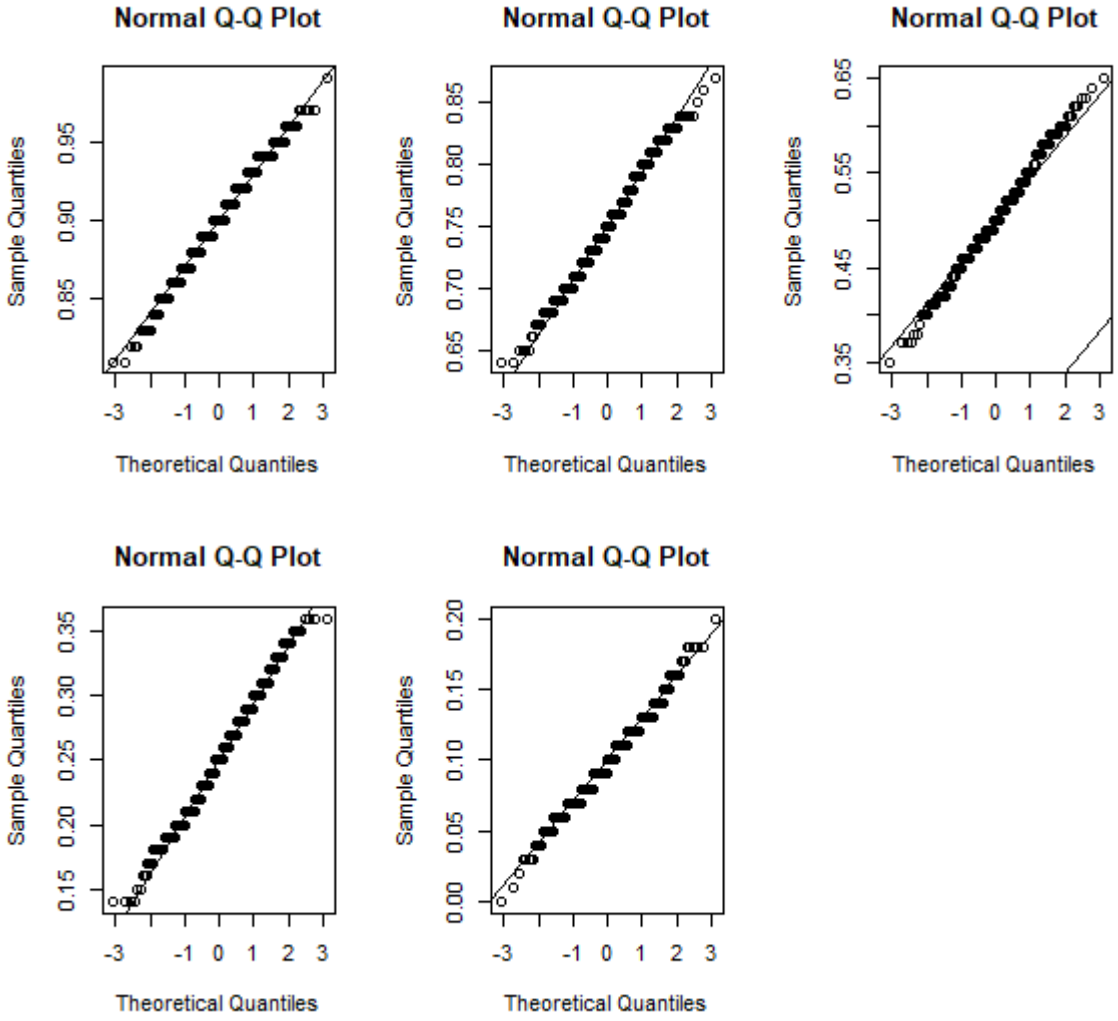


Plots for the combination of (n,p) values = (30,(.1,.25,.5,.75,.90)), (50,(.1,.25,.5,.75,.9))

R-Code:

```
p.10k= replicate(500,mean(rbinom(100,1,.1)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(100,1,.25)))
qqnorm(p.10k)
qqline(p.10k)
```

```
p.10k= replicate(500,mean(rbinom(100,1,.50)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(100,1,.75)))
qqnorm(p.10k)
qqline(p.10k)
p.10k= replicate(500,mean(rbinom(100,1,.90)))
qqnorm(p.10k)
qqline(p.10k)
```



d) From the observations in (c), it is evident that the distribution is normal as the value of N increases following LLN. The value of N can be calculated by the calculating $N\sim= Z^2_{\alpha/2}$ *p(1-p)/epsilon^2

Here, the conclusion depends on the value of p. So we have to replace p value with a good guess say p*.

So, to get the max value of N we have to replace p(1-p) with max value ~1/4.

- ⇨ $N\sim= Z^2_{\alpha/2}$ *1/4/Epsilon^2
- ⇨ Suppose desired accuracy is, (Epsilon, α)=(0.02,0.05)
- ⇨ Then $N\sim= 1.96*2/4/0.03^2$
- ⇨ N~1068(larger value of N)

  From above statements, we can conclude that the value of N should be around 1068 for good approximation also it depends on value of p.

  Graph for p.10k=replicate(500,mean(rbinom(1068,1,.25/.1/.75/.5))) shows that as N=1068, the approximation is best fit for the distribution.