TERM PAPER

Cooperative Localization Using Posterior Linearization Belief Propagation

GROUP-9

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STATISTICAL SIGNAL PROCESSING EE602A

November 15, 2018

Contents

1 AIM/OBJECTIVE

To infer the positions of the sensor nodes in cooperative fashion using the posterior linearization belief propagation (PLBP) algorithm with nonlinear measurements.

2 PROBLEM DEFINITION

In cooperative localization, there are some anchor nodes whose positions are known accurately. The remaining nodes infer their positions based on intercommunication between themselves, each node is connected to some other nodes which pass some messages to it for example distance. The measurements are generally non linear. Hence, we linearize the model by using statistical linearization using Sigma Points. Belief Propagation algorithm will be applied on the linearized posteriors to update the node position using Kalman filter updates. Strategies which increases the computational efficiency and noise robust are required.

3 SYSTEM MODEL/METHODOLOGY

A graph $G = (V, E)$ is formed by a collection of vertices/nodes $V = (1, ..., m)$, where m is the number of nodes, and a collection of edges $E \subset V \times V$. Each edge consists of a pair of nodes $(i, j) \in E$. The state of node i is represented by $x_i \in R^{n_x}$

We assume x_i has Gaussian PDF

$$p_i(x_i) = \mathcal{N}(x_i; \overline{x}_i, P_i) \tag{1}$$

With x_i representing the state of the node and $\overline{x}_i, P_i$ are mean and co-variance respectively

1. Defining the system model.

$$z_{i,j} = h_{i,j}(x_i, x_j) + n_{i,j} \tag{2}$$

   $h_{i,j}$ represent the measurement fuction between two nodes i and j. $n_{i,j}$ is the noise measurement.

2. Nonlinear measurement function.

$$h_{i,j}(x_i, x_j) = \sqrt{(p_{x,i} - p_{x,j})^2 + (p_{y,i} - p_{y,j})^2} \tag{3}$$

3. Linearization model of non linear measurements

$$h_{i,j} \approx A^1_{i,j} x_i + A^2_{i,j} x_j + b_{i,j} + e_{i,j} \tag{4}$$

   (a) Select m sigma points $\chi_0, \chi_1, \cdots, \chi_m$.

   (b) Propagate sigma points $Z_j = h(\chi_j)$

   (c) Compute mean and variance.

$\overline{z} = \sum_{j=1}^{m} \omega_j \mathcal{Z}_j (5)$

$$\Psi = \sum_{j=1}^{m} \omega_j \left(\mathcal{X}_j - \overline{x}\right) \left(\mathcal{Z}_j - \overline{z}\right)^T \tag{6}$$

$$\Phi = \sum_{j=1}^{m} \omega_j \left(\mathcal{Z}_j - \overline{z}\right) \left(\mathcal{Z}_j - \overline{z}\right)^T \tag{7}$$

$$\begin{aligned} A^+ &= \Psi^T P^{-1} \\ b^+ &= \overline{z} - A^+ \overline{x} \\ \Omega^+ &= \Phi - A^+ P \left(A^+\right)^T \end{aligned} \tag{8}$$

4. Belief propagation on linearized model.

   (a) The message $\mu_{i \to j}$ from node i to j is given as

$$\mu_{i \to j} \propto \int l_{i,j}(z_{i,j} | x_i, x_j) \mathcal{N}(x_i; \bar{x}_i, P_i) \prod_{p \in n(i) \setminus \{j\}} \mu_{p \to i}(x_i) dx_i \tag{9}$$

   (b) under approximation

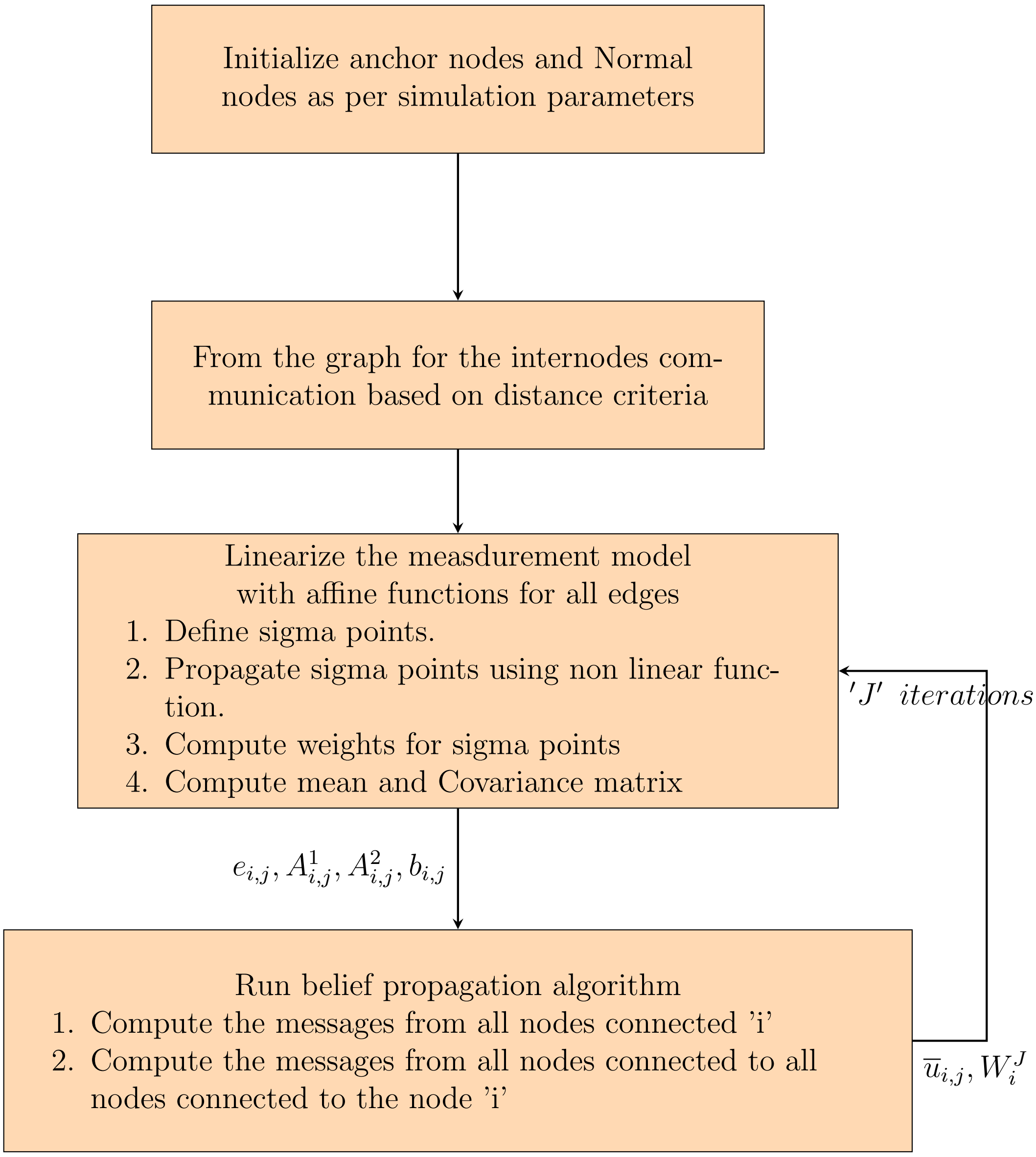
$$\mu_{i \to j}(x_j) \propto \mathcal{N}(\alpha_{i \to j}; H_{i \to j} x_j, \tau_{i \to j}) \tag{10}$$

$$\alpha_{i \to j} = z_{i,j} - A^1_{i,j} \bar{x}_{i \to j} - b_{i,j} \tag{11}$$

$$H_{i \to j} = A^2_{i,j} \tag{12}$$

$$\tau_{i \to j} = R_{i,j} + \Omega_{i,j} + A^1_{i,j} P_{i \to j} (A^1_{i,j})^T \tag{13}$$

3.1 Algorithm flow



4 MATLAB SIMULATIONS AND RESULTS

A MATLAB simulation is performed according to the system model described below.

$$\begin{aligned}
Area: &\quad 100mX100m \\
Number\ of\ Anchor\ nodes: &\quad 13 \\
Number\ of\ Normal\ nodes: &\quad 100 \\
Variance\ of\ Normal\ Nodes: &\quad 100m \\
Variance\ of\ Anchor\ Nodes: &\quad 0.01m \\
Number\ of\ Iterations: &\quad 20 \\
Range\ Measurement\ error: &\quad 1m
\end{aligned}$$

Results for the simulation is shown below. Please find the attached Matlab Code in Appendix at the end of this document.

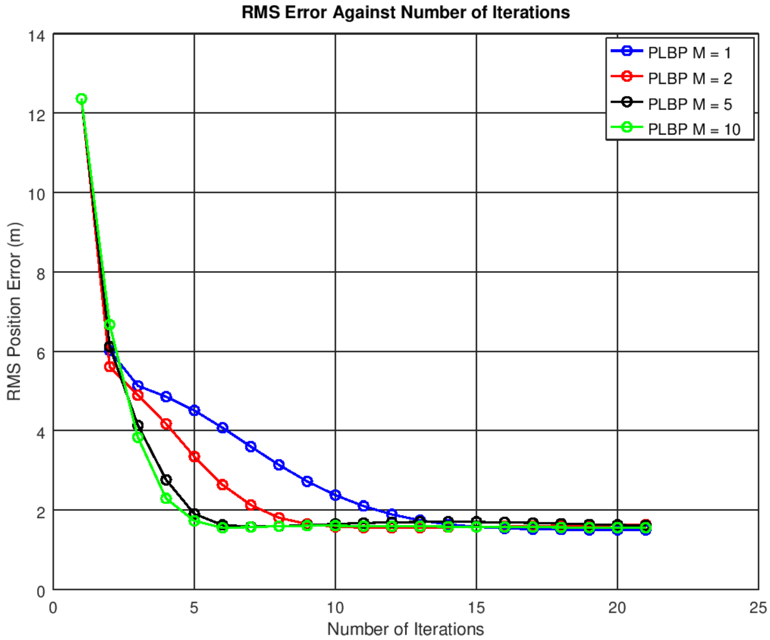


Figure 1: RMS error against number of iterations. Performance improves with M , number of BP iterations per linearisation.

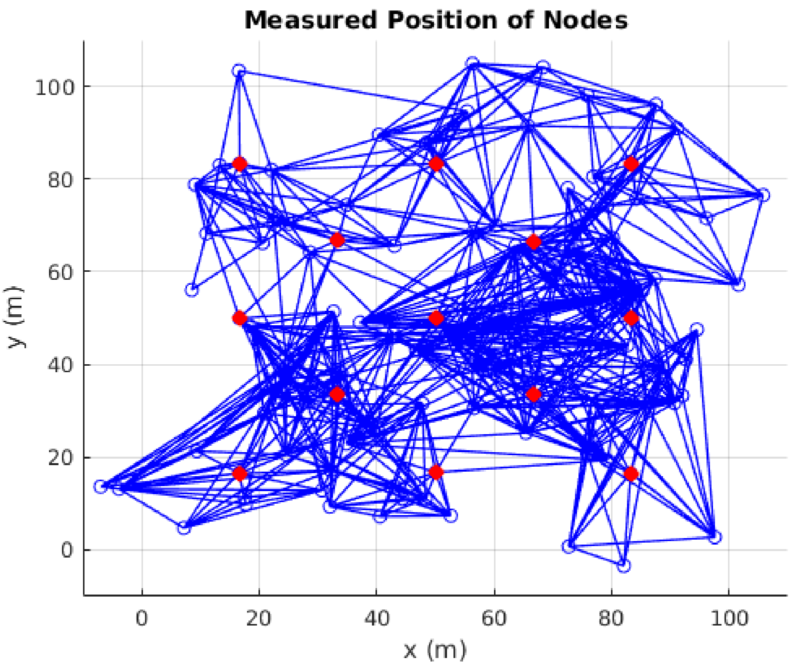


Figure 2: Position of measured nodes i.e. with noise. Red circles indicate the positions of 13 anchor nodes, blue circles the positions of the other 100 nodes and blue lines the edges of the graph. Communication radius is 20 m.

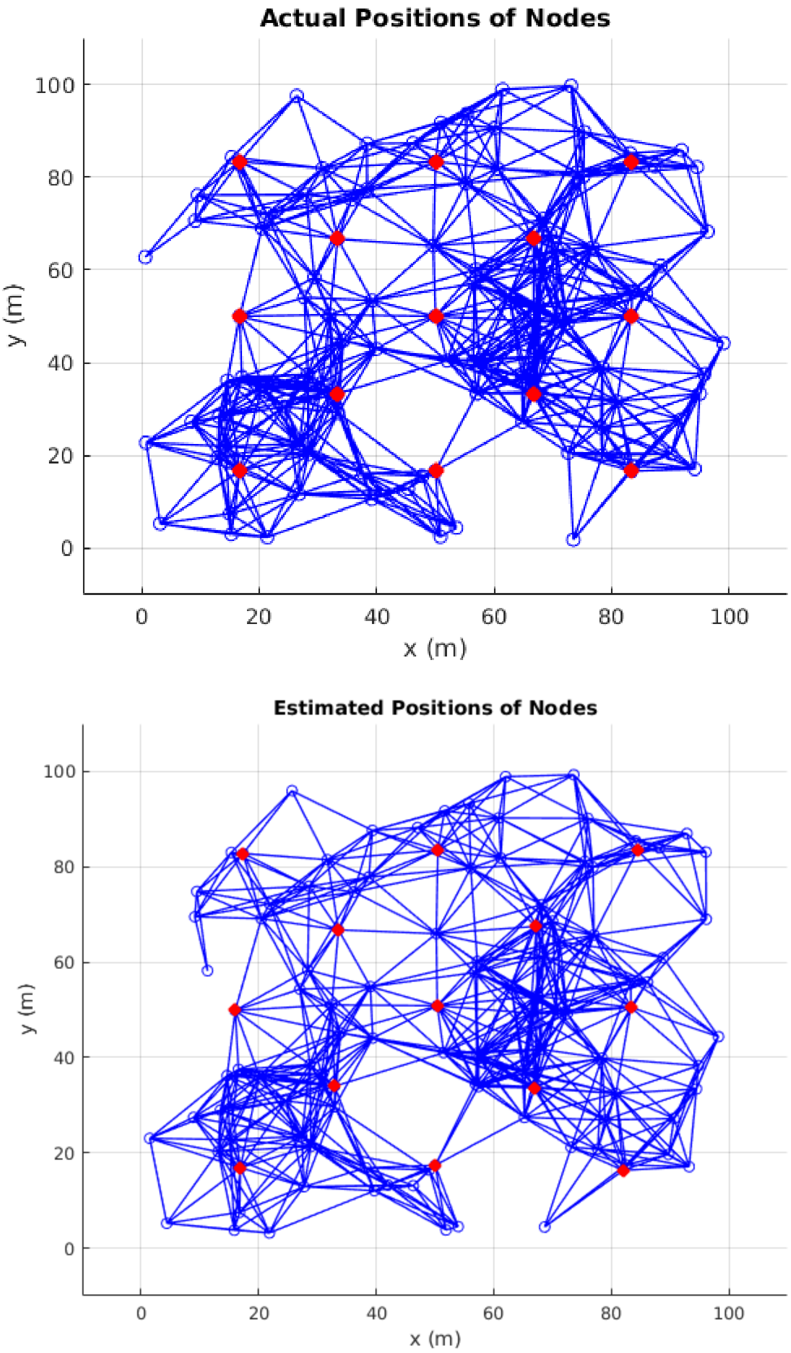


Figure 3: Comparison of actual node position and estimated node position using PLBP algorithm.

5 CONCLUSION

Posterior Linearizaation Belief propagation algorithm is used to infer the positions of the unknown nodes in a cooperative manner in a wireless sensor network with less computational complexity.

6 References

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Appendix