Important concepts, symbols, and equations

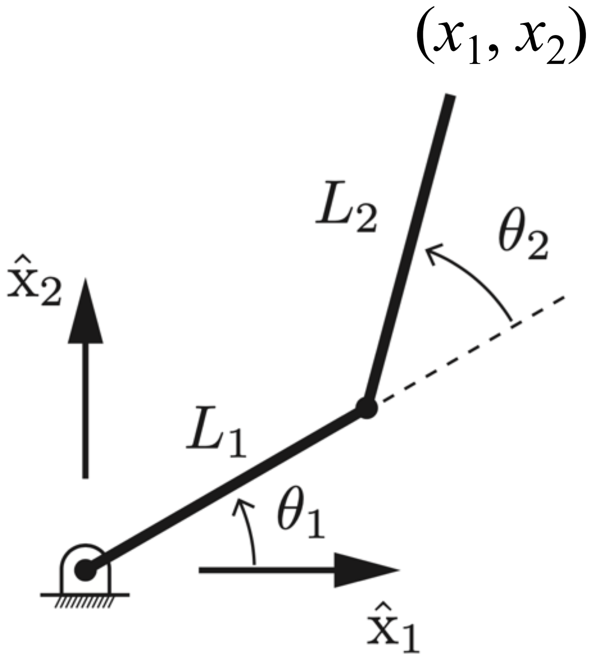
- "Geometric" forward kinematics

$$\theta(t) \in \mathbb{R}^n, \;\; \mathcal{X}(t) = T_{sb}(\theta(t)) \in SE(3) \text{ via PoE}$$

- "Minimum coordinate" forward kinematics

$$\theta(t) \in \mathbb{R}^n, \;\; x(t) = f(\theta(t)) \in \mathbb{R}^m$$

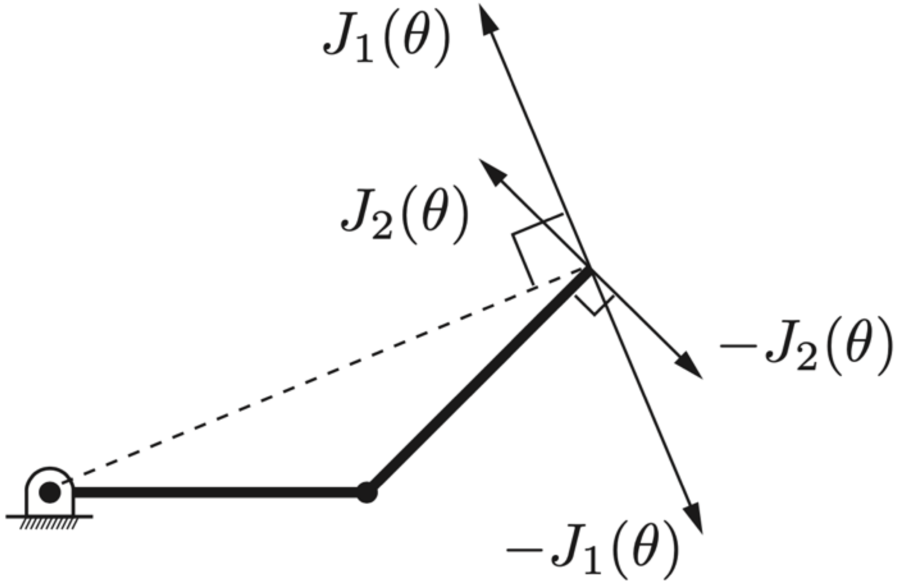
$$\begin{aligned} x_1 &= L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) \\ x_2 &= L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) \end{aligned}$$



Important concepts, symbols, and equations (cont.)

$$\dot{x} = \frac{\partial f(\theta)}{\partial \theta}\frac{d\theta(t)}{dt} = \underbrace{\frac{\partial f(\theta)}{\partial \theta}}_{}\dot{\theta}$$
$$= J(\theta)\dot{\theta}$$

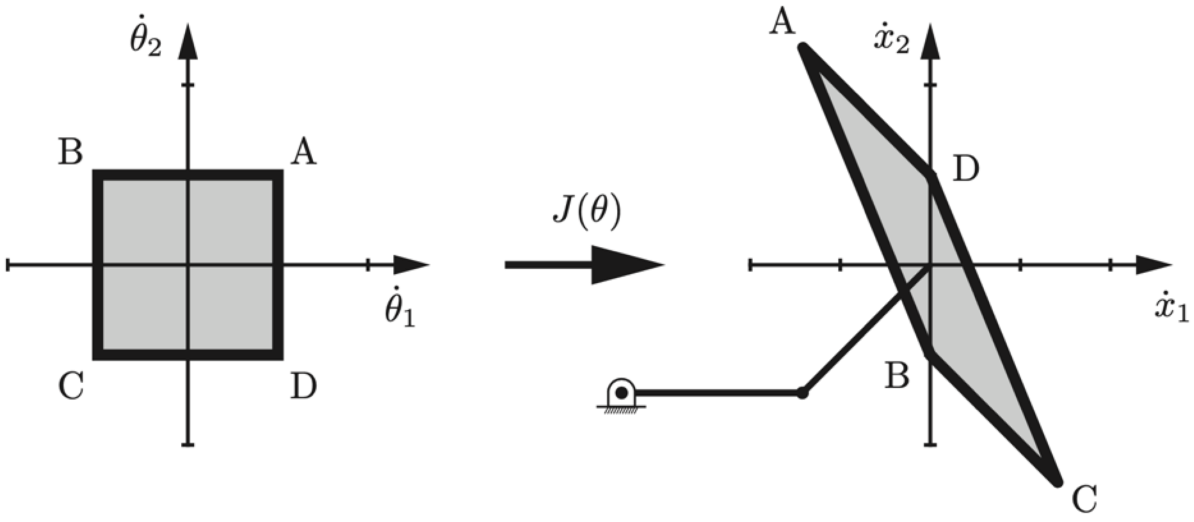
(coordinate)
Jacobian



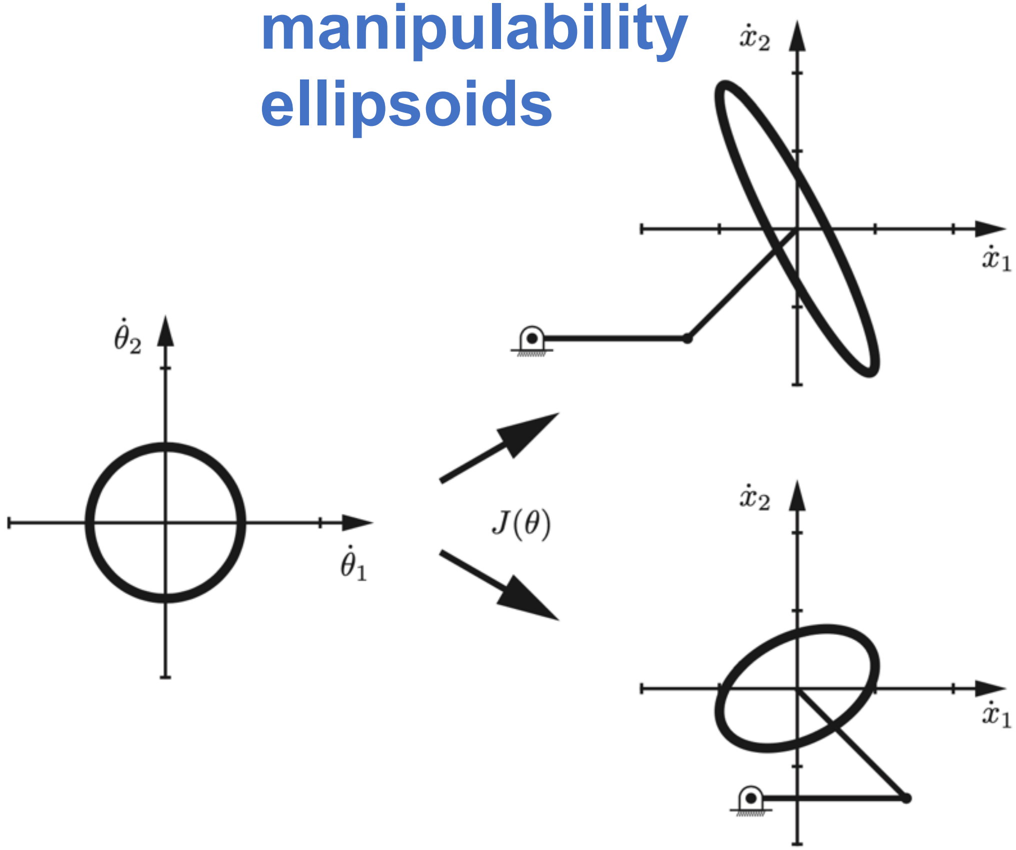
$$\dot{x} = [\, J_1(\theta) \;\; J_2(\theta) \,] \; [\, \dot{\theta}_1 \;\; \dot{\theta}_2 \,]^{\mathrm{T}} = J_1(\theta)\,\dot{\theta}_1 + J_2(\theta)\,\dot{\theta}_2$$

Column i is the end-effector velocity when $\dot{\theta}_i = 1$.

Important concepts, symbols, and equations (cont.)

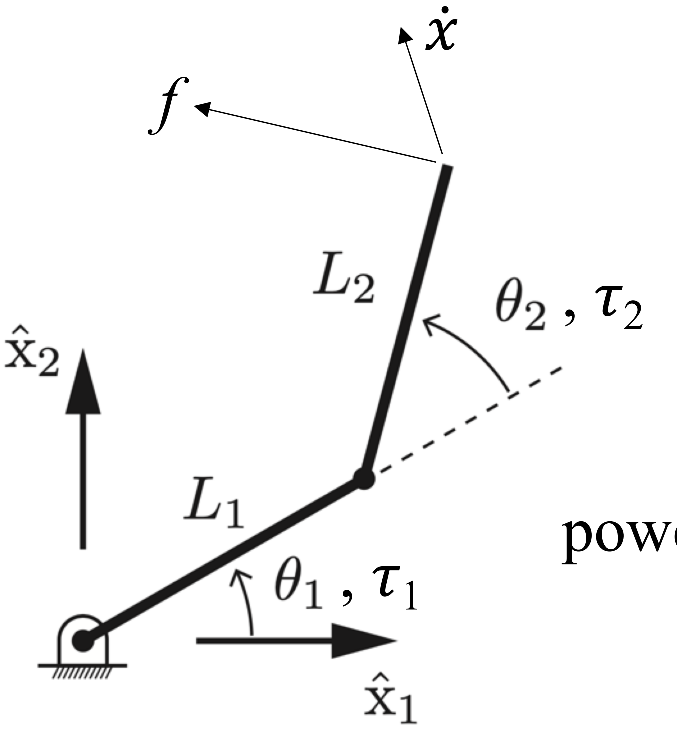


manipulability ellipsoids



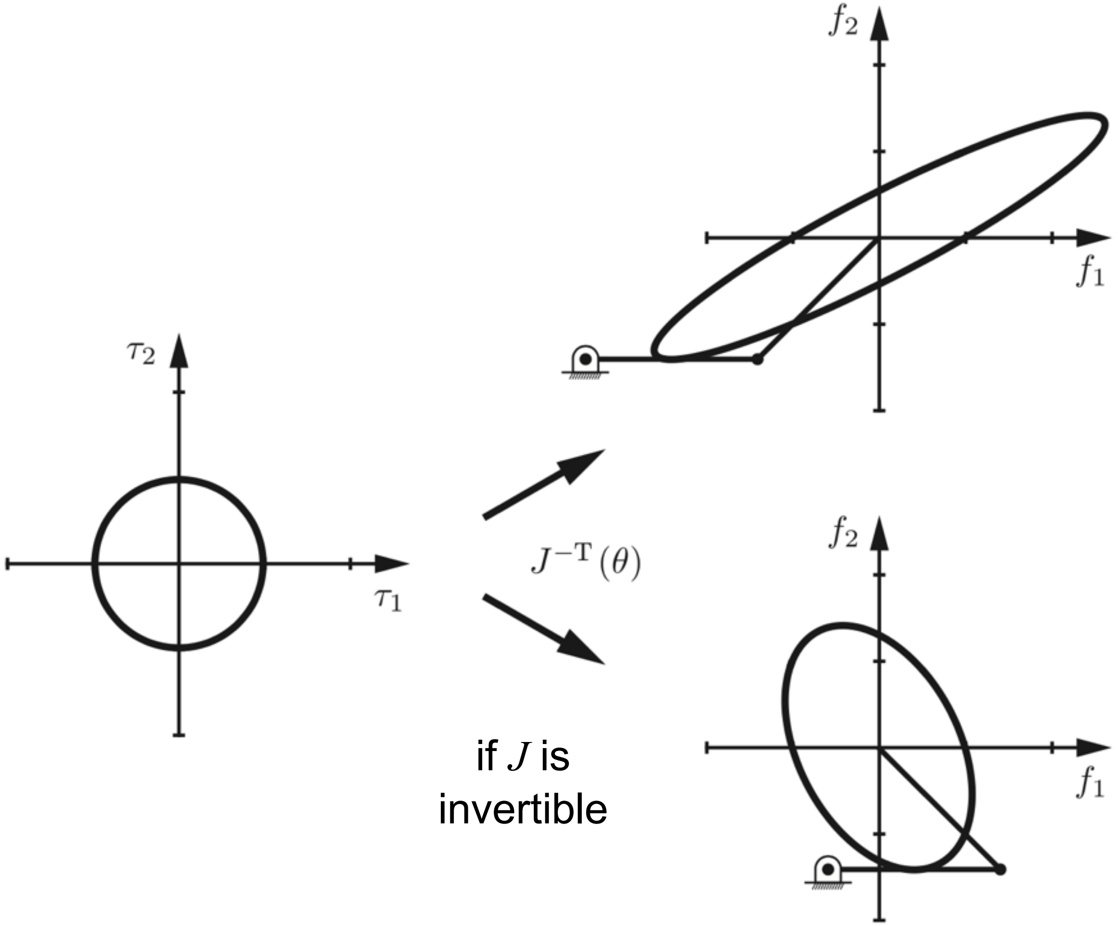
mapping joint velocity limits to end-effector velocity limits

Important concepts, symbols, and equations (cont.)



$$
\begin{aligned}
\text{power} \;=\; \dot{\theta}^{\mathrm{T}}\tau &= \dot{x}^{\mathrm{T}} f \\
\dot{\theta}^{\mathrm{T}}\tau &= (J\,\dot{\theta})^{\mathrm{T}} f \\
\cancel{\dot{\theta}^{\mathrm{T}}}\,\tau &= \cancel{\dot{\theta}^{\mathrm{T}}}\,J^{\mathrm{T}} f
\end{aligned}
$$

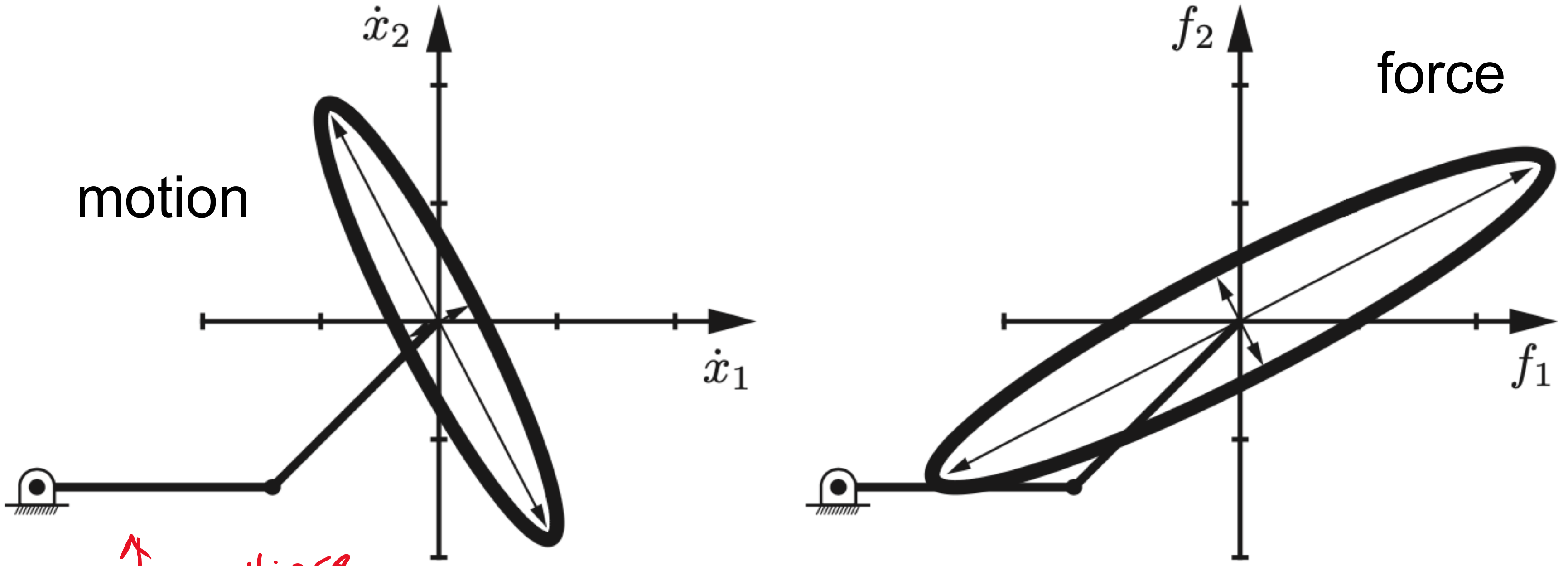
$$\boxed{\tau = J^{\mathrm{T}} f}$$



Important concepts, symbols, and equations (cont.)

A configuration θ is **singular** if $J(\theta)$ loses rank.

- e-e motion capability becomes zero in one or more directions
- e-e can resist infinite force in one or more directions



ellipse collapses to line segment

approaching a singularity...

infinite force can be resisted

Important concepts, symbols, and equations (cont.)

- **Space Jacobian**

No derivatives!

$$\mathcal{V}_s = J_s(\theta)\dot{\theta} \quad \text{where} \begin{cases} J_{s1} = \mathcal{S}_1 \\ J_{si}(\theta) = \mathrm{Ad}_{e^{[\mathcal{S}_1]\theta_1} \cdots e^{[\mathcal{S}_{i-1}]\theta_{i-1}}}(\mathcal{S}_i) \end{cases} \quad i = 2, \ldots, n$$

Jacobian column i depends only on $\mathcal{S}_i$ and joints between it and {s}

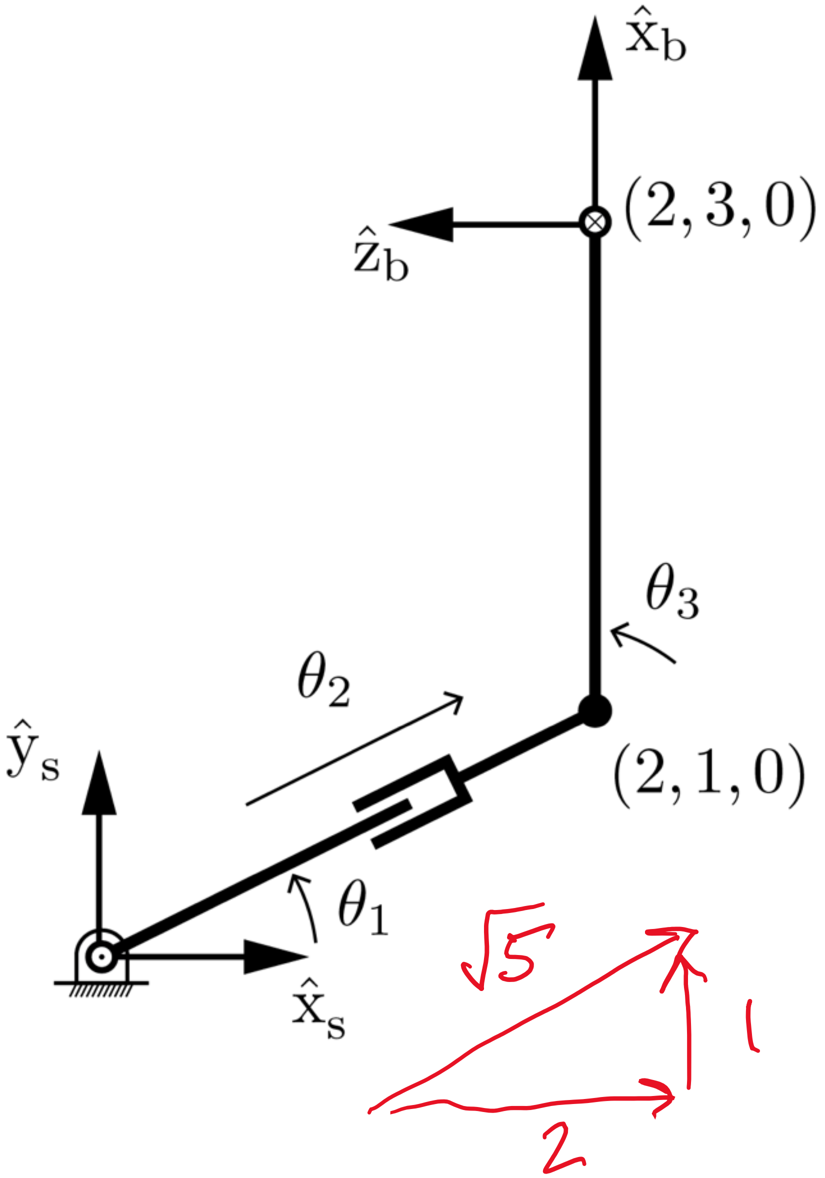
- **Body Jacobian**

$$\mathcal{V}_b = J_b(\theta)\dot{\theta} \quad \text{where} \begin{cases} J_{bi}(\theta) = \mathrm{Ad}_{e^{-[\mathcal{B}_n]\theta_n} \cdots e^{-[\mathcal{B}_{i+1}]\theta_{i+1}}}(\mathcal{B}_i) & i = 1, \ldots, n-1 \\ J_{bn} = \mathcal{B}_n \end{cases}$$

column i depends only on $\mathcal{B}_i$ and joints between it and {b}

$$\begin{aligned} J_s(\theta) &= [\mathrm{Ad}_{T_{sb}}]J_b(\theta) \\ J_b(\theta) &= [\mathrm{Ad}_{T_{bs}}]J_s(\theta) \end{aligned}$$

Column i is the end-effector twist when $\dot{\theta}_i = 1$.



$$J_s(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 2/\sqrt{5} & 1 \\ 0 & 1/\sqrt{5} & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_b(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 0 \\ 2 & 1/\sqrt{5} & 0 \\ 0 & 0 & 0 \\ 3 & -2/\sqrt{5} & 2 \end{bmatrix}$$