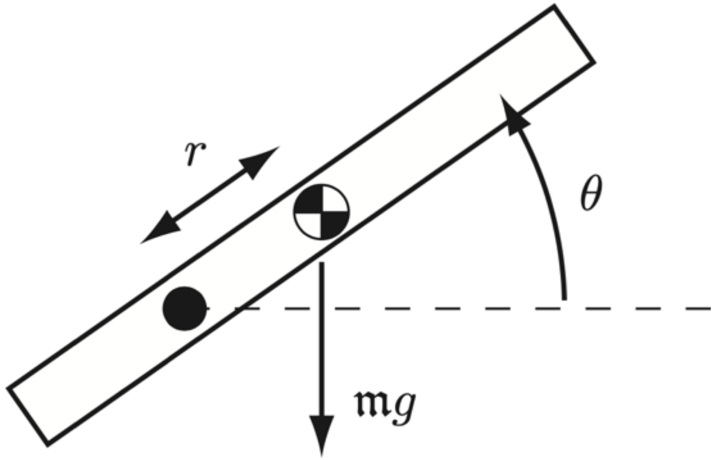
Important concepts, symbols, and equations



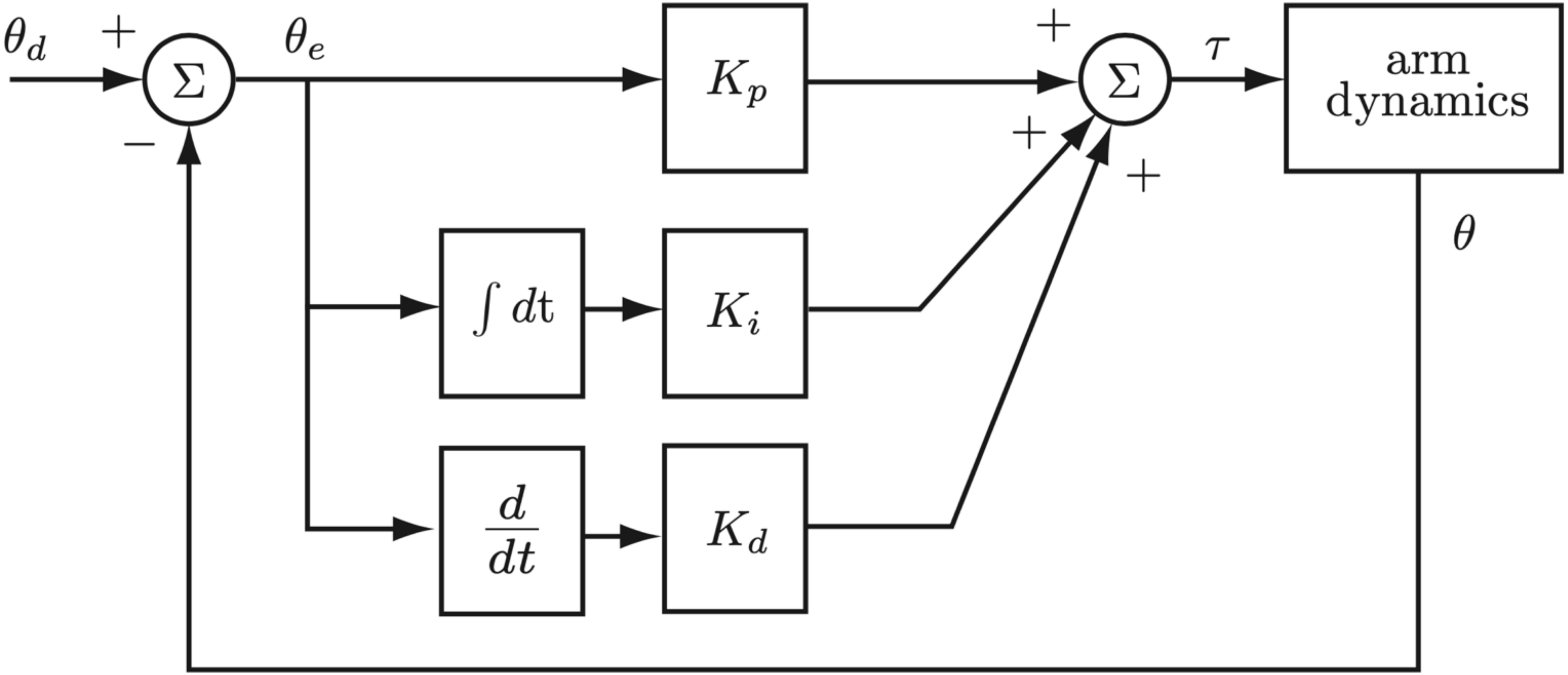
$$\tau = M\ddot{\theta} + \underbrace{\mathfrak{m}gr\cos\theta + b\dot{\theta}}$$

$$\tau = M\ddot{\theta} + h(\theta, \dot{\theta})$$

Proportional-Integral-Derivative (PID) control

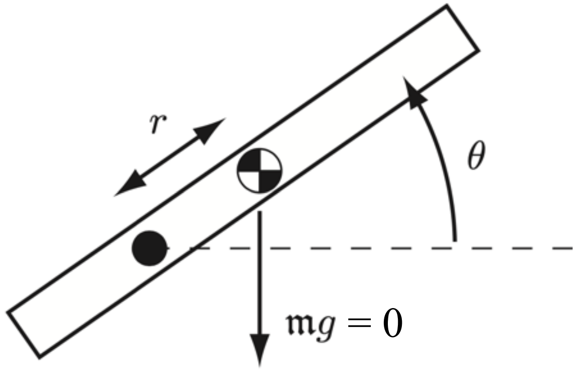
$$\tau = K_p\theta_e + K_i\int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e$$

Important concepts, symbols, and equations (cont.)



Important concepts, symbols, and equations (cont.)

Setpoint PD control, $g = 0$



$$\tau = K_p\theta_e + \cancel{K_i}^{\,0} \int \theta_e(t)dt + K_d\dot{\theta}_e$$

$$\tau = M\ddot{\theta} + \cancel{\mathfrak{m}g}^{\,0} r\cos\theta + b\dot{\theta}$$

$$\dot{\theta}_d = \ddot{\theta}_d = 0$$

$$M\ddot{\theta} + b\dot{\theta} = K_p\theta_e + K_d\dot{\theta}_e$$

$$\ddot{\theta}_e + \frac{b + K_d}{M}\dot{\theta}_e + \frac{K_p}{M}\theta_e = 0$$

$$\zeta = \frac{b + K_d}{2\sqrt{K_p M}} \qquad \omega_n = \sqrt{\frac{K_p}{M}}$$

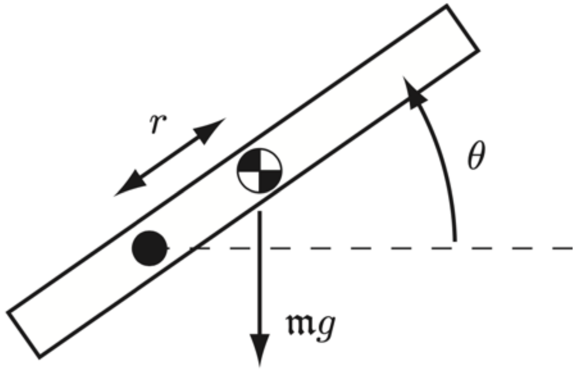
For what gains are the error dynamics stable?

$K_d > -b$

$K_p > 0$

Important concepts, symbols, and equations (cont.)

Setpoint PD control, $g \neq 0$



$$\tau = K_p\theta_e + \overset{0}{\cancel{K_i}} \int \theta_e(\mathrm{t})d\mathrm{t} + K_d\dot{\theta}_e$$

$$\tau = M\ddot{\theta} + \mathfrak{m}gr\cos\theta + b\dot{\theta}$$

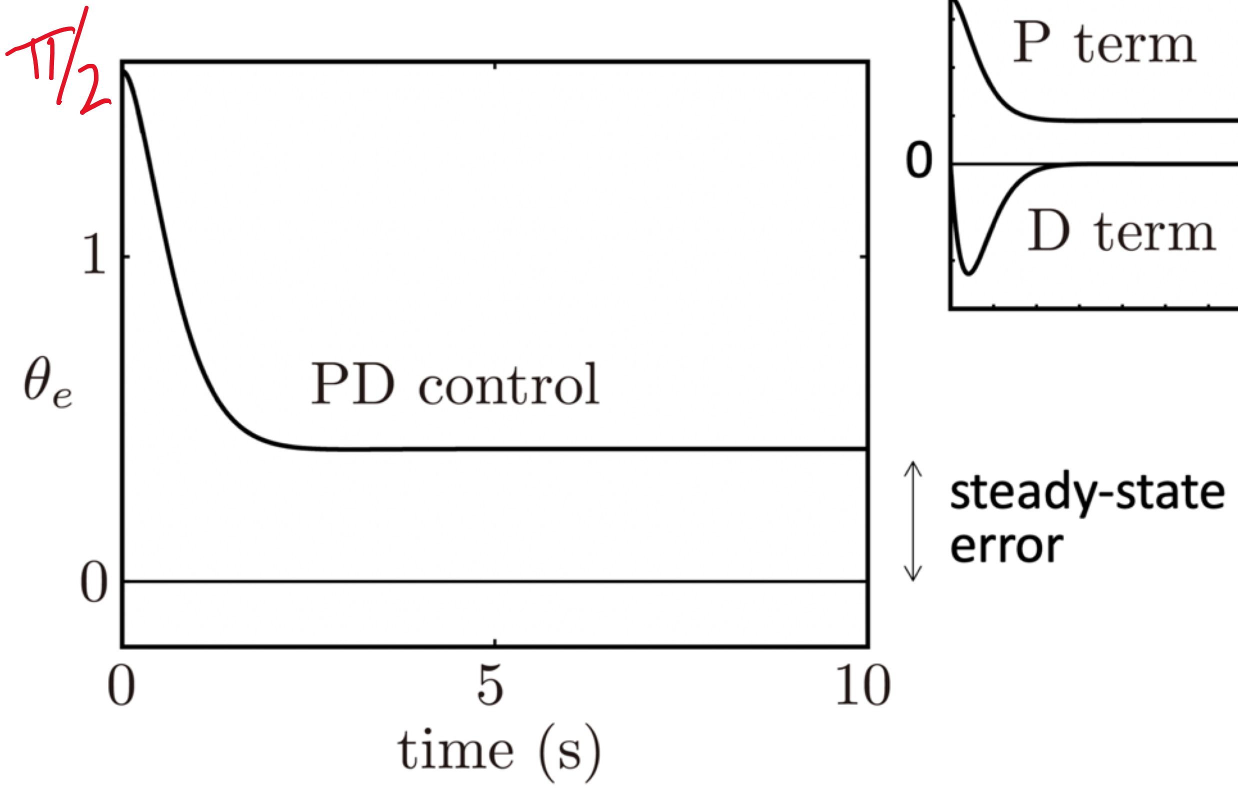
$$\dot{\theta}_d = \ddot{\theta}_d = 0$$

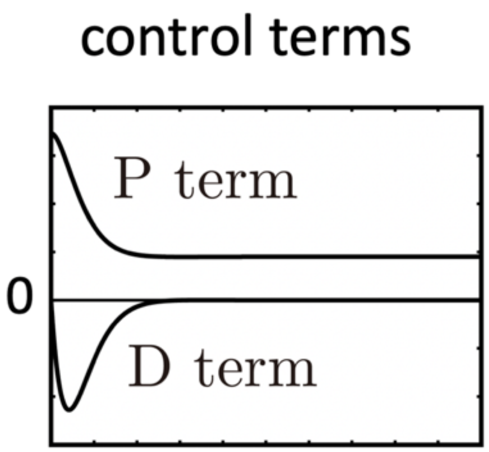
$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e = \mathfrak{m}gr\cos\theta$$

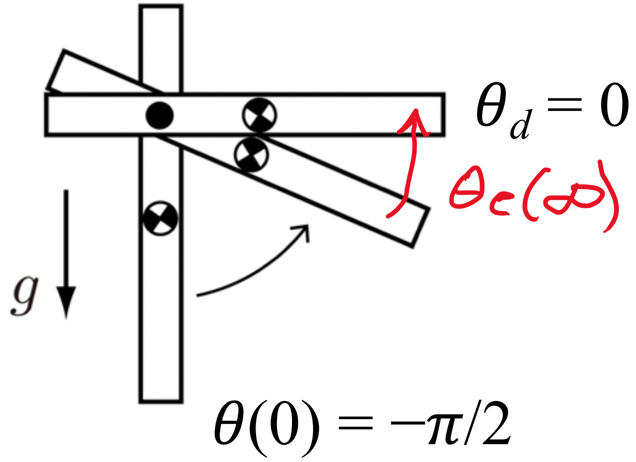
$$\theta_e(t \to \infty) = \frac{mgr\cos\theta}{K_p}$$

Nonhomogeneous. What is the steady-state error?

Important concepts, symbols, and equations (cont.)







Important concepts, symbols, and equations (cont.)

Setpoint PID control, $g \neq 0$

$$\tau = K_p\theta_e + K_i \int \theta_e(\mathrm{t})d\mathrm{t} + K_d\dot{\theta}_e$$

$$\tau = M\ddot{\theta} + \mathfrak{m}gr\cos\theta + b\dot{\theta}$$

$$\dot{\theta}_d = \ddot{\theta}_d = 0 \qquad \tau_{\text{dist}} \longrightarrow$$

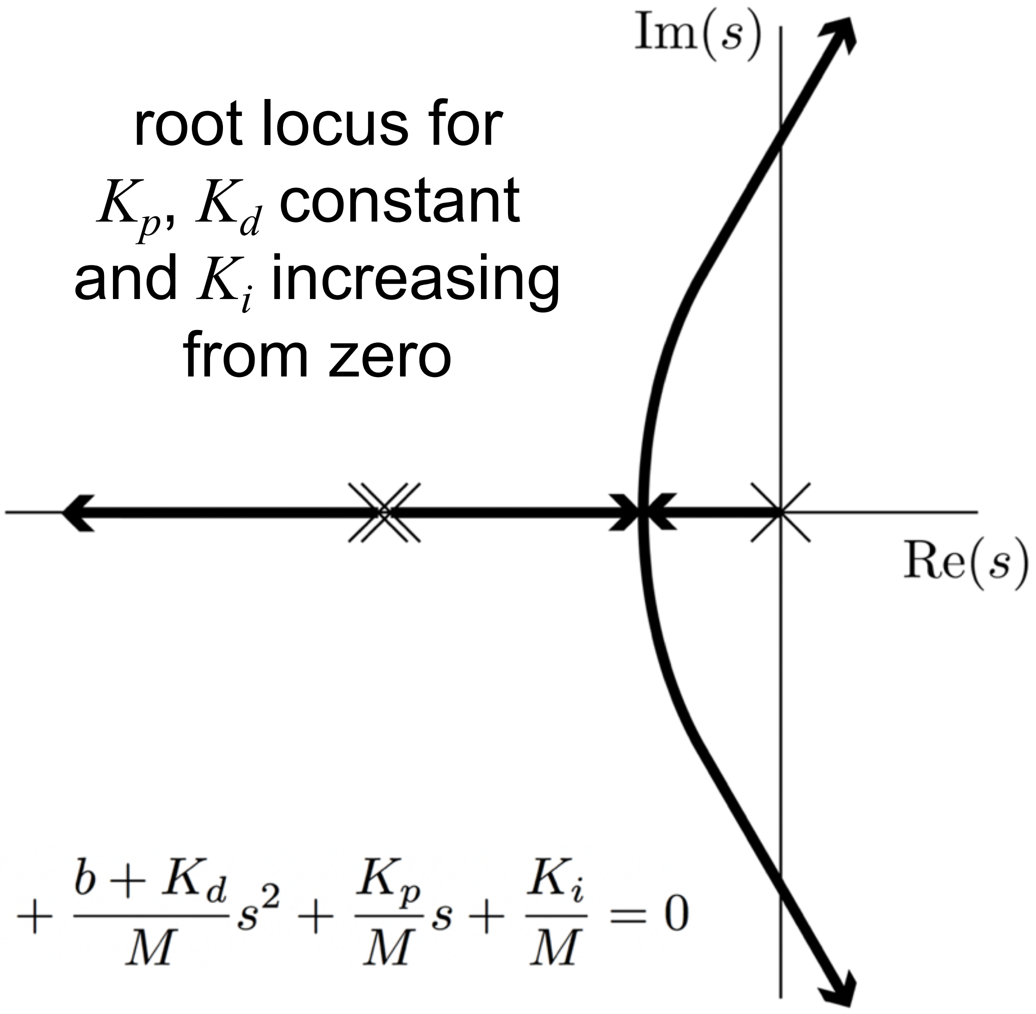
because mgr cos θ is nearly constant near steady state

$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e + K_i \int \theta_e(\mathrm{t})d\mathrm{t} = \tau_{\text{dist}}$$

$$M\theta_e^{(3)} + (b + K_d)\ddot{\theta}_e + K_p\dot{\theta}_e + K_i\theta_e = 0$$

$$s^3 + \frac{b + K_d}{M}s^2 + \frac{K_p}{M}s + \frac{K_i}{M} = 0$$

Important concepts, symbols, and equations (cont.)



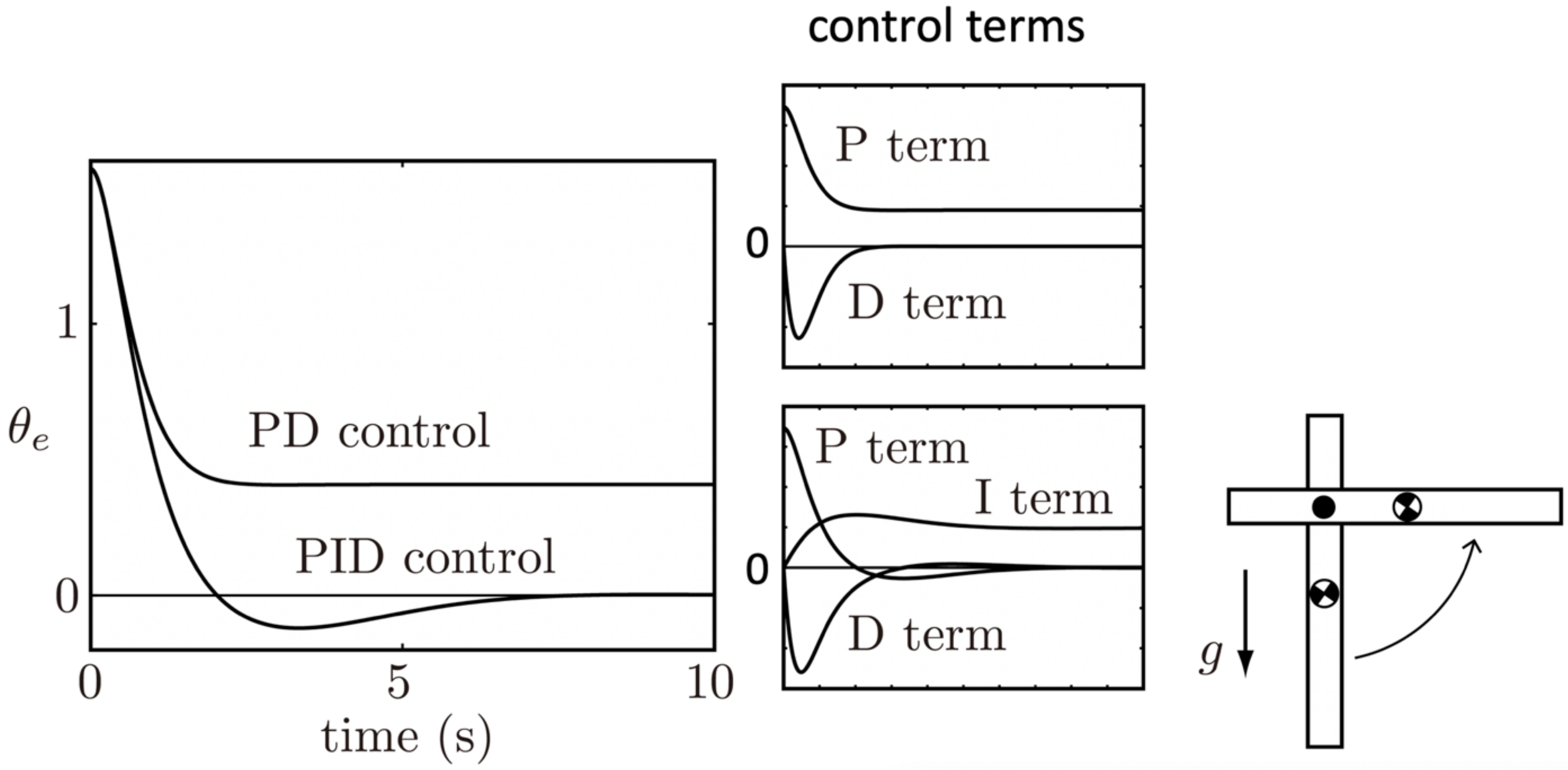
root locus for K_p, K_d constant and K_i increasing from zero

$$s^3 + \frac{b + K_d}{M} s^2 + \frac{K_p}{M} s + \frac{K_i}{M} = 0$$

$$\begin{aligned} K_d &> -b \\ K_p &> 0 \\ \frac{(b + K_d)K_p}{M} > K_i &> 0 \end{aligned}$$

K_i improves steady-state response but can worsen the transient response.

Important concepts, symbols, and equations (cont.)



<u>Important concepts, symbols, and equations</u> (cont.)

What about tracking general trajectories, not just setpoint control?

$$\tau = M\Big(\overbrace{\underbrace{\ddot{\theta}_d}_{ff} + \underbrace{K_p\theta_e + K_i\int\theta_e dt + K_d\dot{\theta}_e}_{fb}}^{\text{commanded } \ddot{\theta}}\Big) + h(\theta,\dot{\theta})$$

$$\ddot{\theta}_e = \ddot{\theta}_d - \ddot{\theta}$$

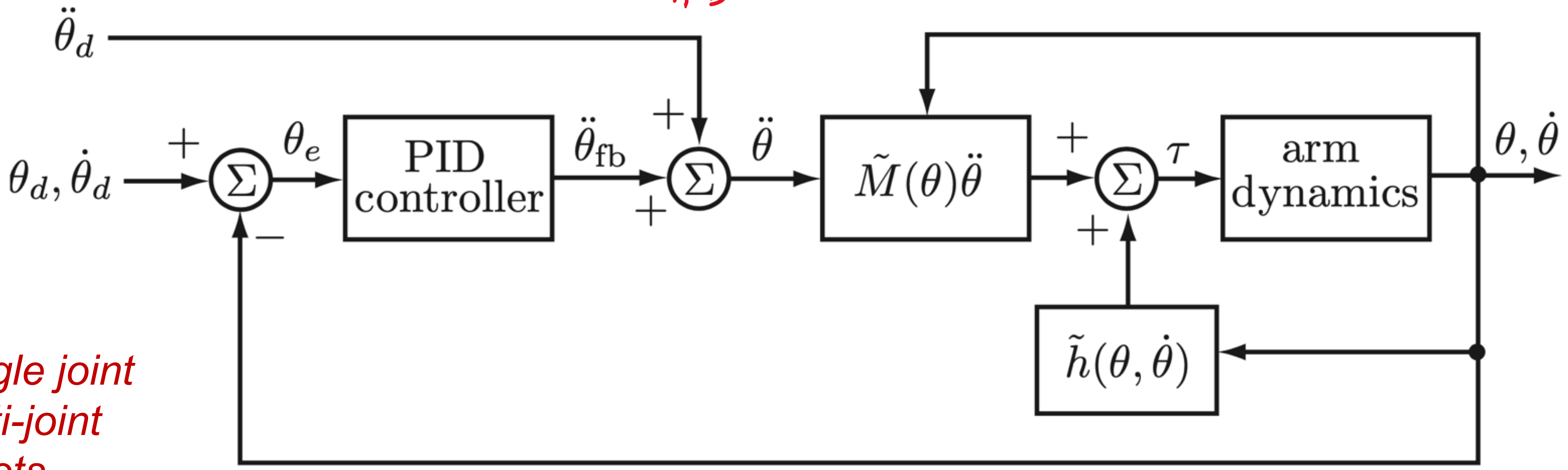
$$\ddot{\theta}_e = -K_d\dot{\theta}_e - K_p\theta_e - K_i\int\theta_e dt$$

$$\theta_e^{(3)} + K_d\ddot{\theta}_e + K_p\dot{\theta}_e + K_i\theta_e = 0$$

Important concepts, symbols, and equations (cont.)

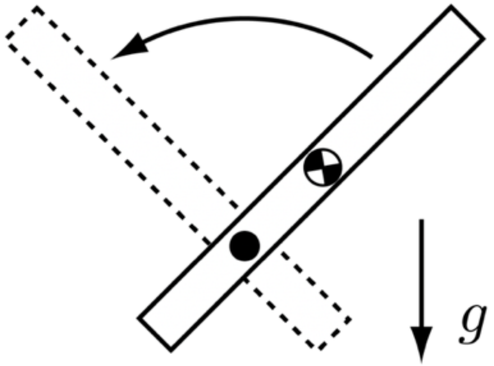
Computed torque control (feedback linearization)

$$\tau = \underset{\text{model}}{\tilde{M}(\theta)} \left(\underset{\text{ff}}{\ddot{\theta}_d} + \underbrace{K_p\theta_e + K_i \int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e}_{\text{fb}}\right) + \underset{\text{model}}{\tilde{h}(\theta, \dot{\theta})}$$

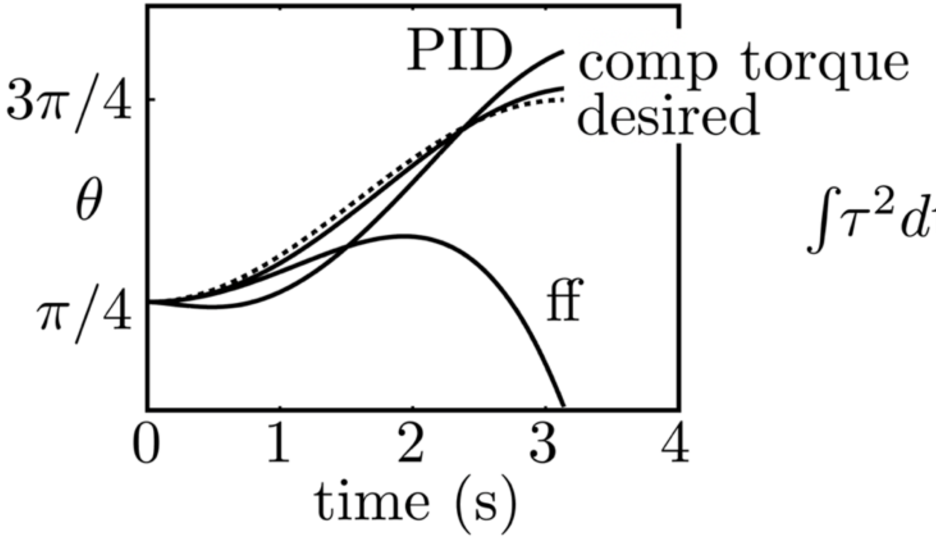


for a single joint or multi-joint robots

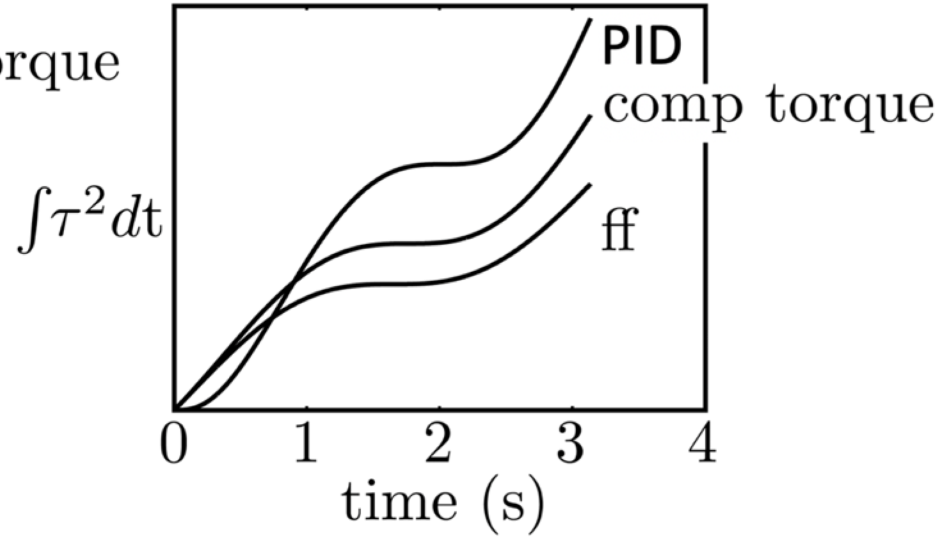
Important concepts, symbols, and equations (cont.)



assume the
model has some
error



better tracking
than ff or PID



less "effort" than PID

Important concepts, symbols, and equations (cont.)

Task-space computed torque control

What if your dynamic model is poor?

$$\mathcal{F}_b = \Lambda(\theta)\dot{\mathcal{V}}_b + \eta(\theta, \mathcal{V}_b)$$

dynamic model: $\{\tilde{\Lambda}, \tilde{\eta}\}$

$$\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int \theta_e \mathrm{d}t + K_d\dot{\theta}_e\right) + \tilde{h}(\theta, \dot{\theta})$$

comp torque is appropriate if model is "reasonable" or better, not if it's poor

$$\mathcal{F}_b = \tilde{\Lambda}(\theta)\left(\dot{\mathcal{V}}_d + K_p X_e + K_i\int X_e \mathrm{d}t + K_d\mathcal{V}_e\right) + \tilde{\eta}(\theta, \mathcal{V}_b)$$

$$[X_e] = \log(X^{-1}X_d)$$

$$\mathcal{V}_e = [\mathrm{Ad}_{X^{-1}X_d}]\mathcal{V}_d - \mathcal{V}_b$$

$$\tau = J_b^{\mathrm{T}}(\theta)\mathcal{F}_b$$

The characteristic equation of the error dynamics are

$$s^5 + 2s^4 + s^3 + 2s^2 + 4s + 2 = 0$$

Write the error dynamics in the form $\dot{x} = Ax$. Determine if the system is stable. (Use any software you want.)

$x_1 = \theta_e$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = x_4$

$\dot{x}_4 = x_5$

$\dot{x}_5 = -2x_1 - 4x_2 - 2x_3 - x_4 - 2x_5$

$\mathrm{eig}(A) = -1, -1, -1.26,$

$\underbrace{0.68 \pm 1.09j}_{\text{unstable due to positive real component}}$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -2 & -4 & -2 & -1 & -2 \end{bmatrix}$$