

Chapter 2 Configuration Space
 2.1 DOF of a Rigid Body
 2.2 DOF of a Robot

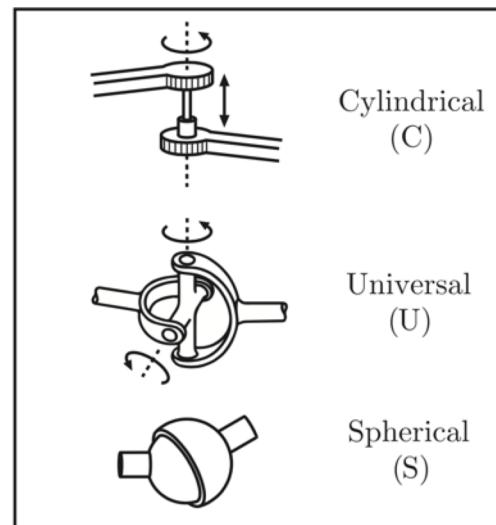
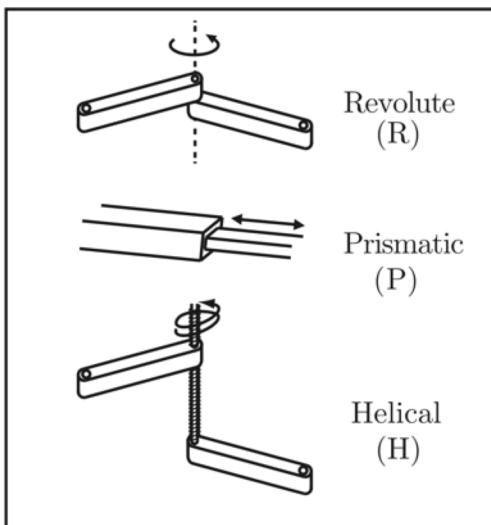
- Chapter 3 Rigid-Body Motions
- Chapter 4 Forward Kinematics
- Chapter 5 Velocity Kinematics and Statics
- Chapter 6 Inverse Kinematics
- Chapter 7 Kinematics of Closed Chains
- Chapter 8 Dynamics of Open Chains
- Chapter 9 Trajectory Generation
- Chapter 10 Motion Planning
- Chapter 11 Robot Control
- Chapter 12 Grasping and Manipulation
- Chapter 13 Wheeled Mobile Robots

Important concepts, symbols, and equations

- **configuration**: a specification of the positions of all points of a mechanism
- **degrees of freedom** (dof): # of real #s required to describe a configuration
- **configuration space** (C-space): the dof-dimension space of all configurations
- dof of a planar body: $m = 3$; dof of a spatial body: $m = 6$

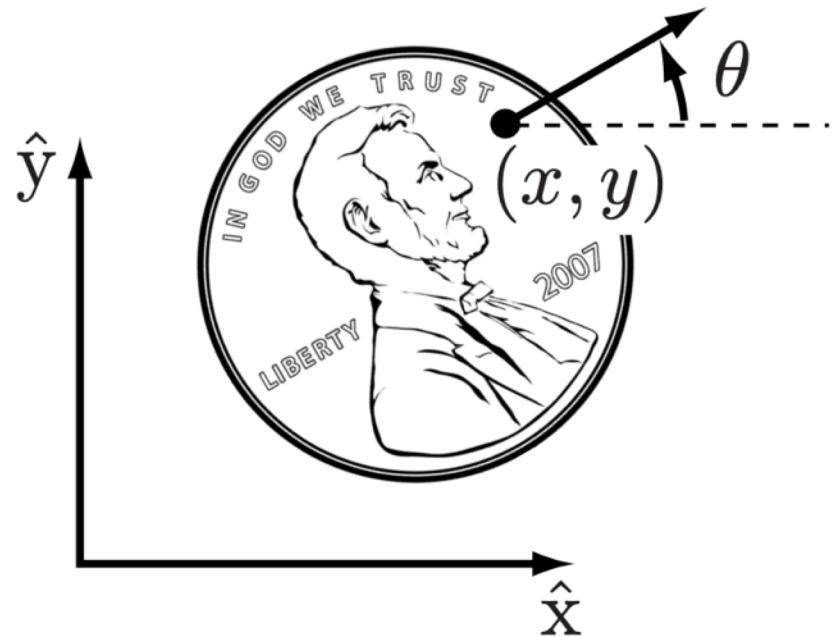
Important concepts, symbols, and equations (cont.)

- mechanism dof = Σ (body freedoms) – Σ (independent constraints from joints)
- joint types:



Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

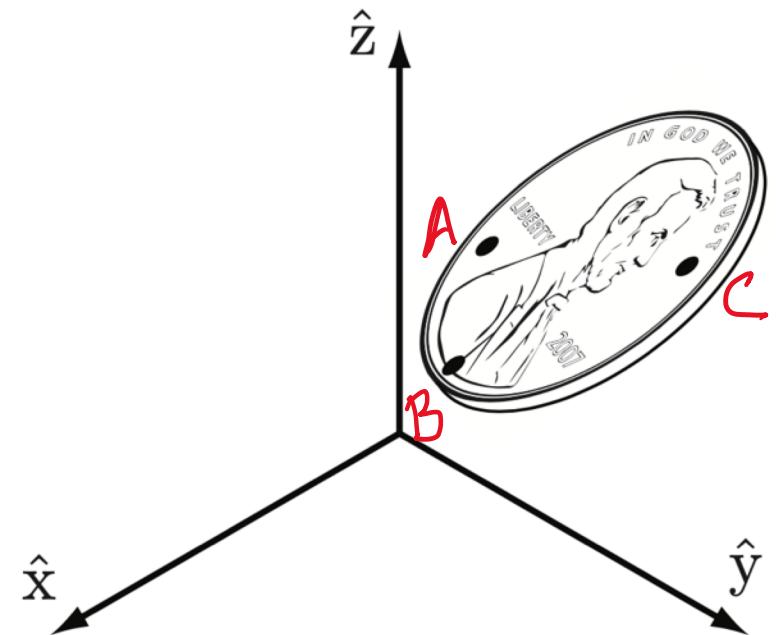
- Grübler's formula:** $dof = m(N - 1 - J) + \sum_{i=1}^J f_i$ → # of freedoms provided by joints
- Annotations for Grübler's formula:
- 3 or 6** (with a red arrow pointing to m)
 - # bodies** (with a red arrow pointing to N)
 - # joints** (with a red arrow pointing to J)



coin on a plane

$(x, y, \theta) + \underbrace{\text{heads/tails}}_{\text{not a real number}}$

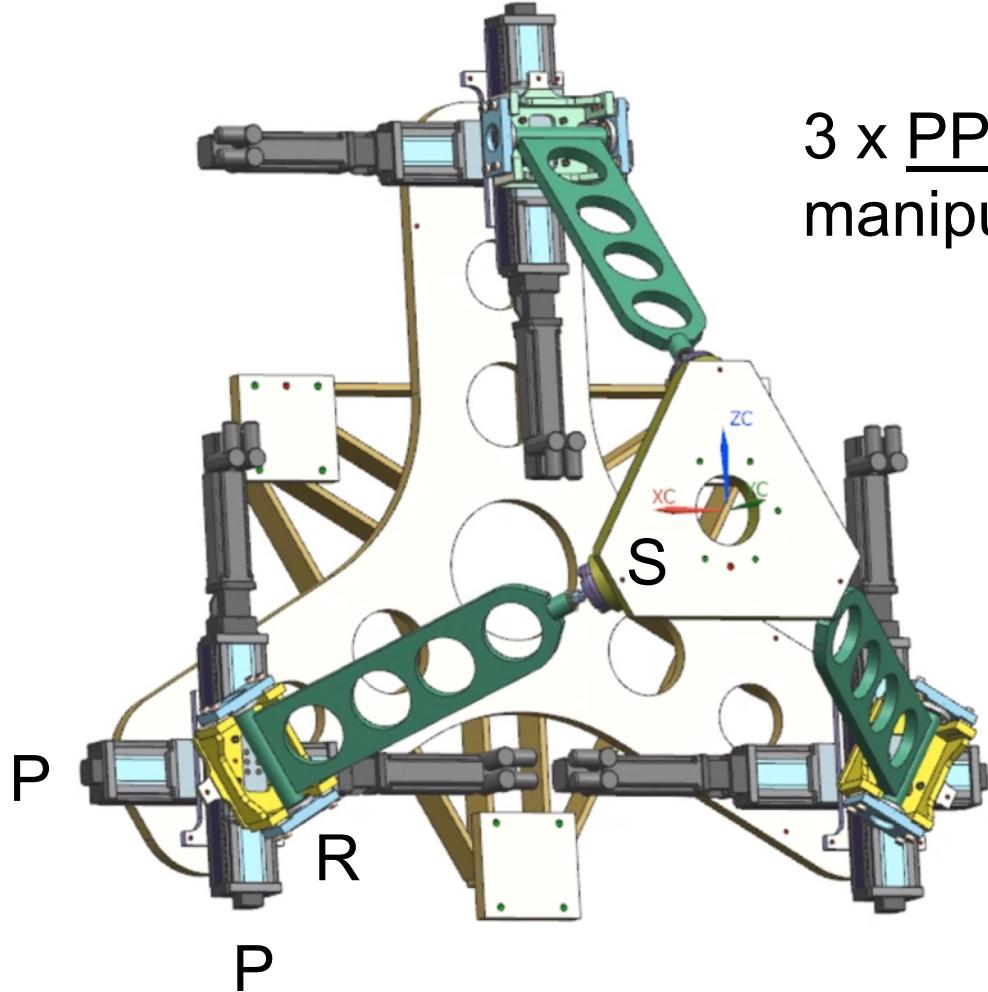
$m = 3$



coin in space

A: 3 dof - 0 constraints = 3
 B: 3 dof - 1 constraint = 2
 C: 3 dof - 2 constraints = 1

$M = 6$



3 x PPRS parallel
manipulator

$$m=6 \text{ (spatial)}$$

$$N = 1 + 1 + 3 \times 3 = 11$$

↑ ↑ ↗ ↘
ground platform legs z prismatic
carriages +
link between
R & S joints

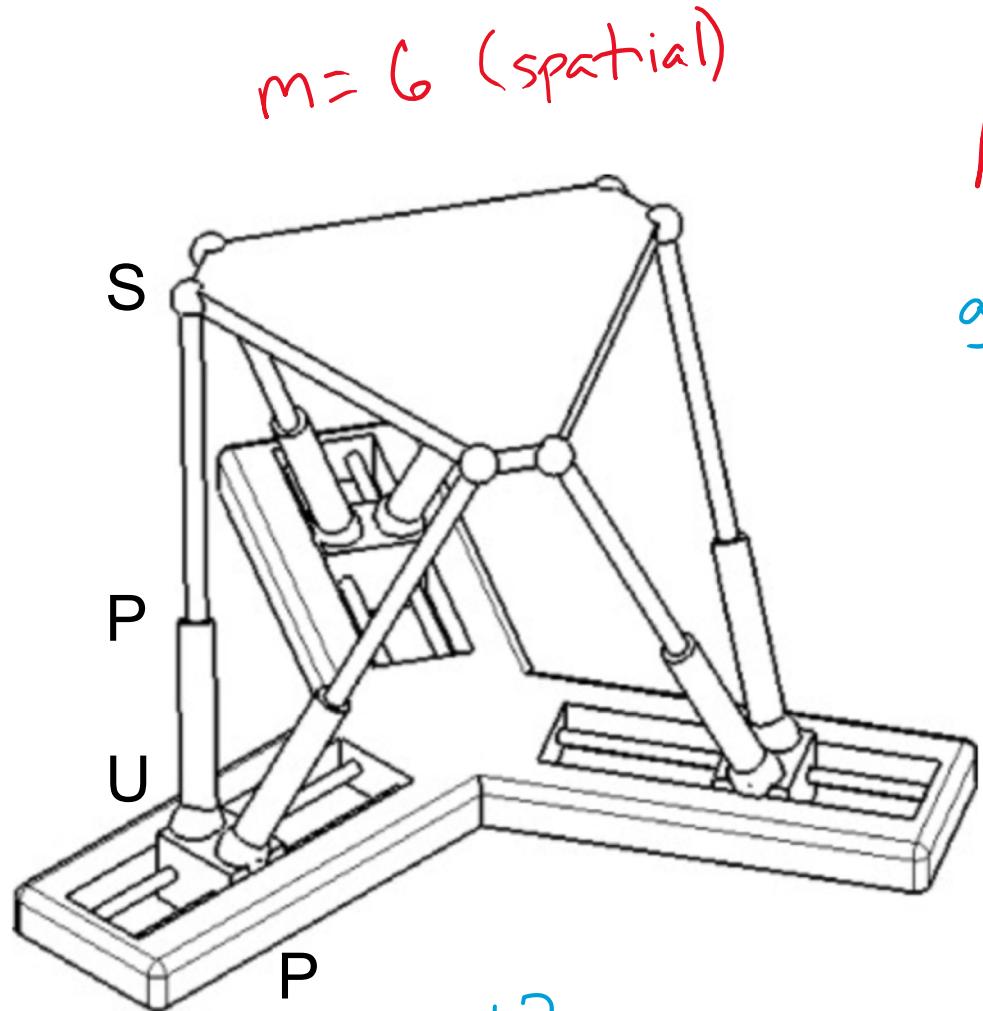
$$\mathcal{J} = 3 \times 4 = 12$$

$$\sum f_i = 3 \times (1+1+1+3) = 18$$

$$dof = 6(11) - 1 - 12 + 18$$

$$= -12 + 18 = 6$$

KUKA Systems North America LLC
(patent pending)



underactuated?
redundantly actuated?

$$N = 1 + 1 + 3 + \underbrace{6 \times 2}_{\text{legs}} = 17$$

ground platform sliders legs

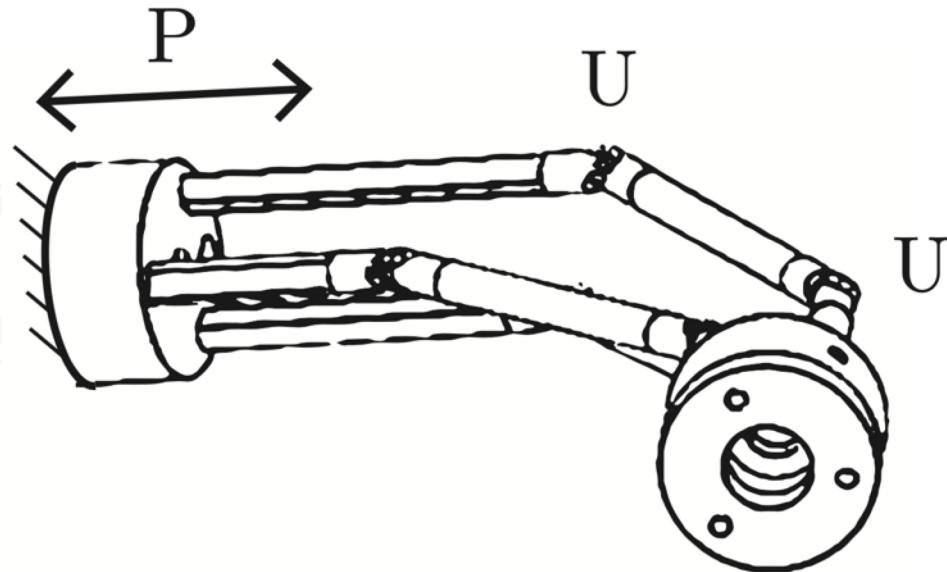
$$J = 3 + 6 \times 3 = 21$$

$$\sum f_i = 3 + 6 \times (2+1+3) = 39$$

$$\begin{aligned} \text{dof} &= 6(17 - 1 - 21) + 39 \\ &= -30 + 39 = 9 \end{aligned}$$

Which joints are likely actuated,
and which are passive? How many
dof does the platform have (not the
mechanism)?

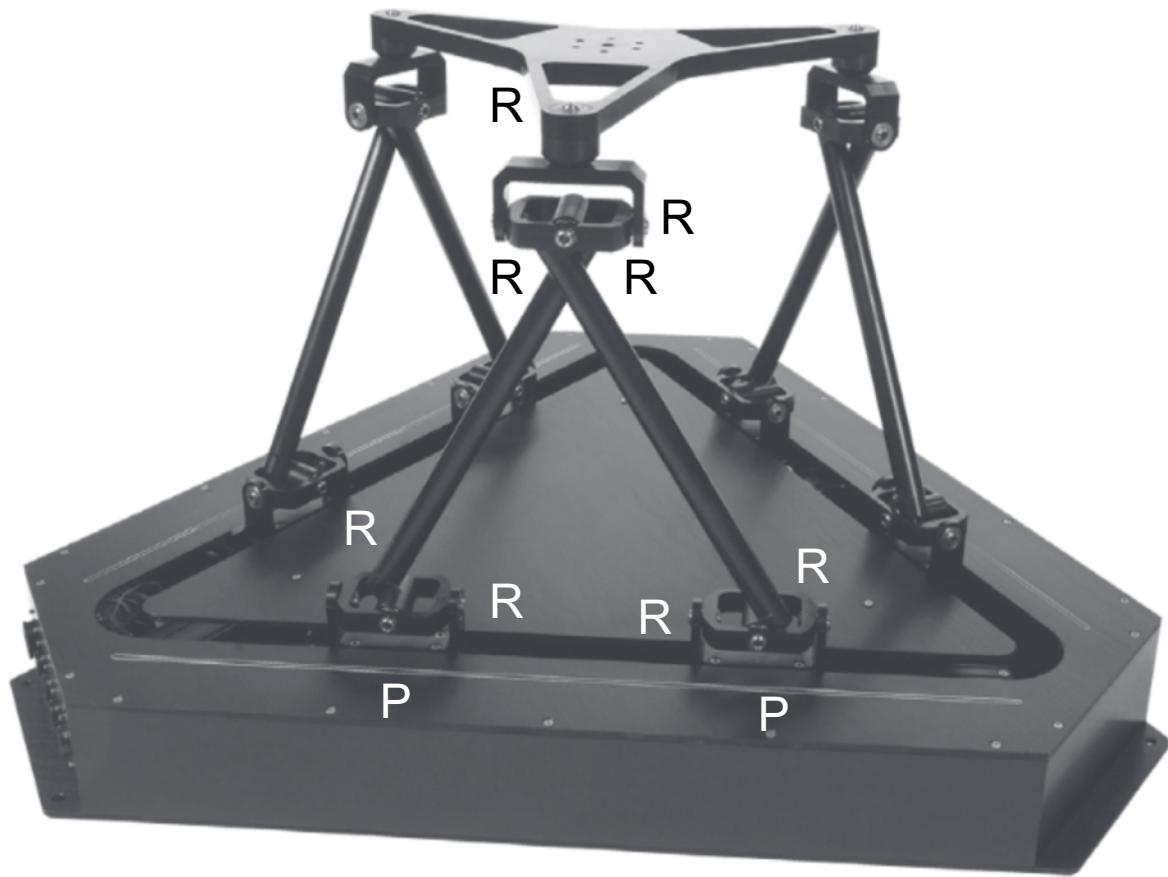
$3 \times \underline{P}UU$ miniature surgical parallel manipulator (National University of Singapore)



How many dof?
Discussion

underline of P indicates it is actuated. U joints are unactuated.

Quanser Hexapod



$$m = 6$$

$$J = 3 \times 10 = 30$$

$$N = 1 + 1 + 6 \times 3 + 3 \times 2 = 26$$

$$\sum f_i = 30 \times 1 = 30$$

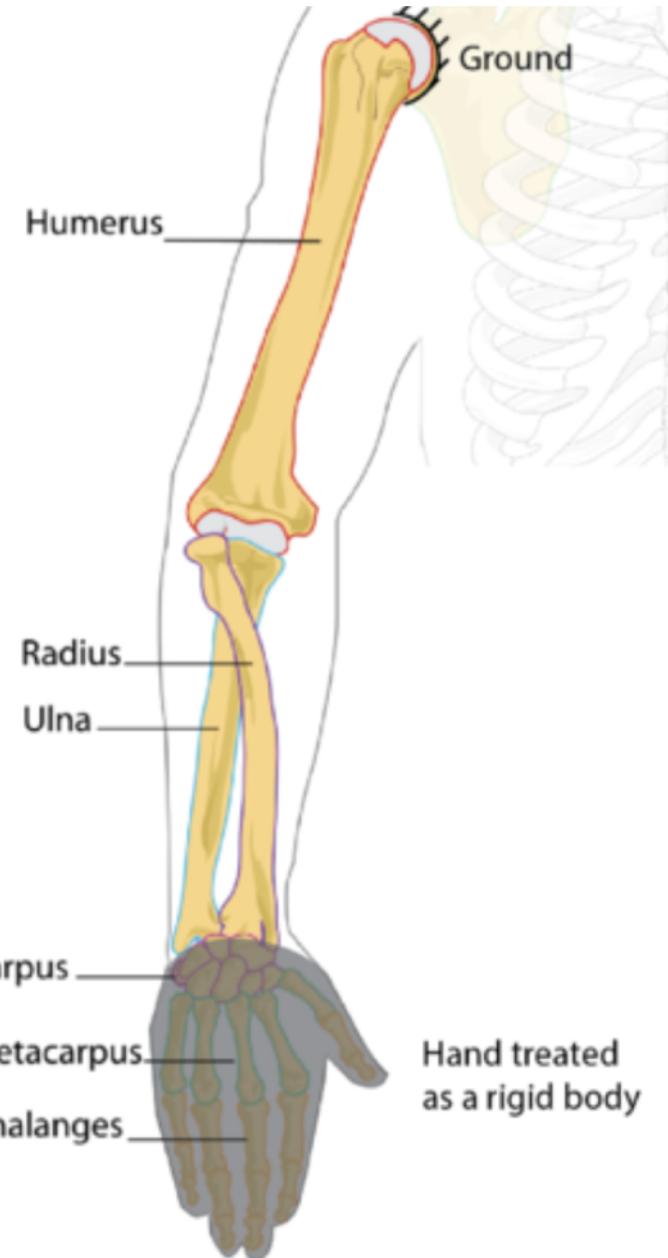
$$dof = 6(26 - 1 - 30) + 30$$

$$= 0 ?$$

Clearly the platform has 6 dof.

Did we do something wrong?

<https://www.youtube.com/watch?v=AyVu4AE25DM>



How many dof does the human arm have?

Method 1: add dof of joints (shoulder, elbow, wrist)

Method 2: fully constrain hand's position

Discussion

How many total constraints are imposed by the joints?

Discussion