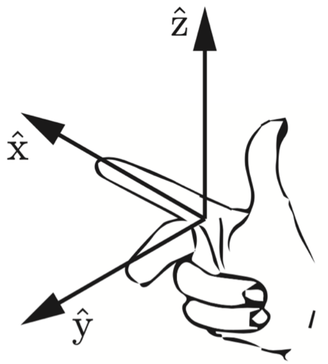
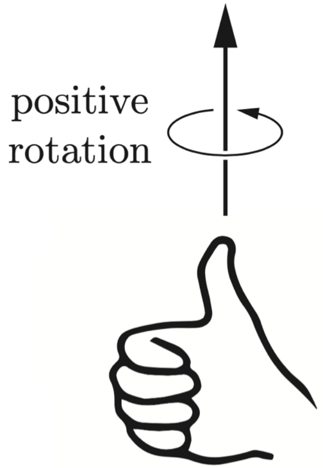
Important concepts, symbols, and equations

- We often define a fixed **space frame** $\{s\}$ and a **body frame** $\{b\}$ attached to some body of interest. All frames are *instantaneously stationary*.

- Right-handed frames, and right-hand rule for positive rotation.





orthogonal special

- **Special orthogonal group** $SO(3)$: matrices R in $\mathbb{R}^{3\times 3}$ where $R^{\mathrm{T}}R = I$, $\det R = 1$. R is a **rotation matrix**. Implicit representation with 9 numbers for 3 dof.

Important concepts, symbols, and equations (cont.)

- A **group** is a set of elements $G = \{a, b, c \ldots\}$ and a binary operation $\bullet$ satisfying

closure	$a \bullet b \in G$ for all $a, b \in G$
associativity	$(a \bullet b) \bullet c = a \bullet (b \bullet c)$
identity element exists	there is an $I \in G$ such that $a \bullet I = I \bullet a = a$ for each $a \in G$
inverse exists	for each $a \in G$, there exists $a^{-1} \in G$ such that $a \bullet a^{-1} = a^{-1} \bullet a = I$

Integers under addition? Nonnegative integers under addition? Square real matrices under multiplication? What is a **Lie group**?

Important concepts, symbols, and equations (cont.)

- $SO(3)$ is a **matrix (Lie) group** (the group operation is matrix multiplication).

closure:	$R_1R_2 \in SO(3)$
associative:	$(R_1R_2)R_3 = R_1(R_2R_3)$ (*not commutative!* $R_1R_2 \neq R_2R_1$ generally)
identity:	identity matrix I
inverse:	matrix inverse

$R^{\mathrm{T}}R = I$, so $R^{-1} = R^{\mathrm{T}}$.

For $x \in \mathbb{R}^3$, $\|x\| = \|Rx\|$.

Important concepts, symbols, and equations (cont.)

- Uses of a rotation matrix:

  1. Represent an orientation. R_{ab} represents orientation of $\{b\}$ in $\{a\}$.
  2. Change the reference frame of a vector or frame.

     subscript cancellation:

     $$R_{ab}R_{bc} = R_{a\cancel{b}}R_{\cancel{b}c} = R_{ac}$$

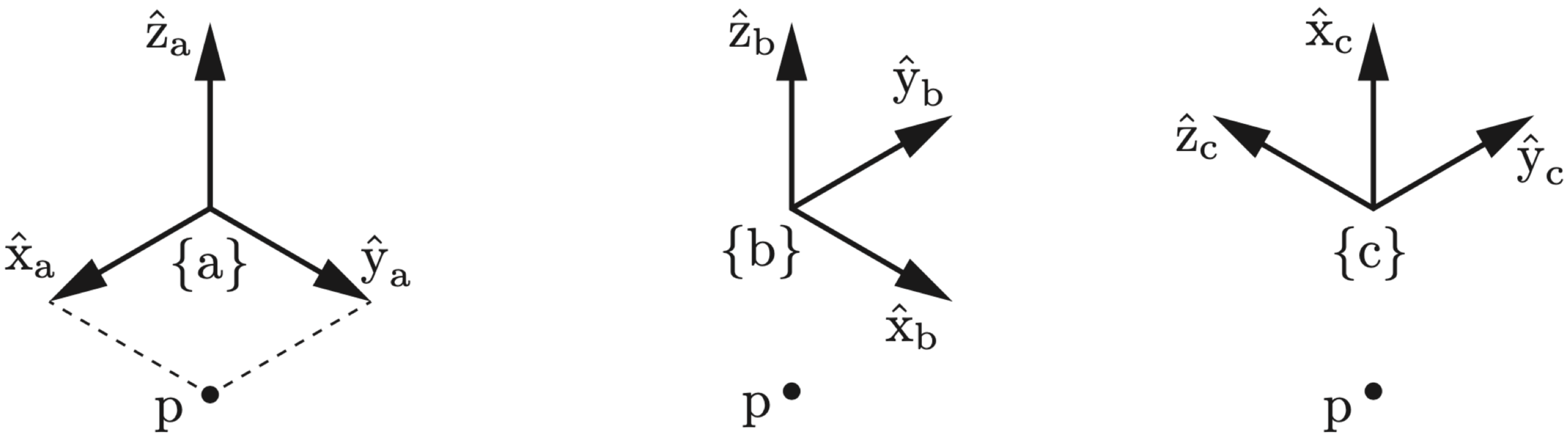
     $$R_{ab}p_b = R_{a\cancel{b}}p_{\cancel{b}} = p_a$$

  3. Rotate a vector or frame. $R = R_{cd} = \mathrm{Rot}(\hat{w}, \theta)$, axis $\hat{w}$ expressed in $\{c\}$.

     $p'_c = R_{cd}\, p_c$ (no subscript cancellation)

     $R_{ab'} = RR_{ab}$ (after rotating about axis in $\{a\}$)

     $R_{ab''} = R_{ab}R$ (after rotating about axis in $\{b\}$)



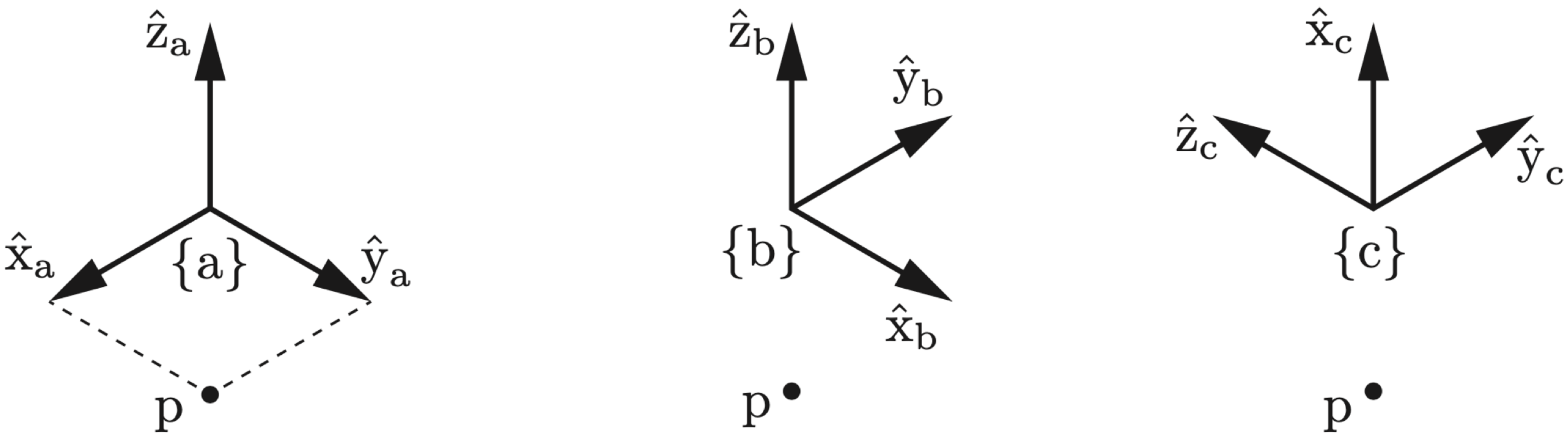
$\hat{z}_a$
$\hat{x}_a$
$\{a\}$
$\hat{y}_a$
p
$\hat{z}_b$
$\hat{y}_b$
$\{b\}$
$\hat{x}_b$
p
$\hat{x}_c$
$\hat{z}_c$
$\hat{y}_c$
$\{c\}$
p

$R_{ab} =$

$p_b =$

Given $R_1 = R_{ab}$, $R_2 = R_{bc}$, and $R_3 = R_{ad}$,
write R_{dc} in terms of R_1, R_2, and R_3 (no inverses!).

Given p_b, what is p_d in terms of R_1, R_2, and R_3 (no inverses)?



$R = R_{ba} = \mathrm{Rot}(\hat{w}, \theta)$: $\theta = \pi/2$, axis $\hat{w} =$

$R_{bc'} = RR_{bc} =$

$R_{bc''} = R_{bc}R =$

orientation representation	# nums	imp/exp?	pros	cons
Euler angles, roll-pitch-yaw				
Unit quaternions				
Rotation matrices				