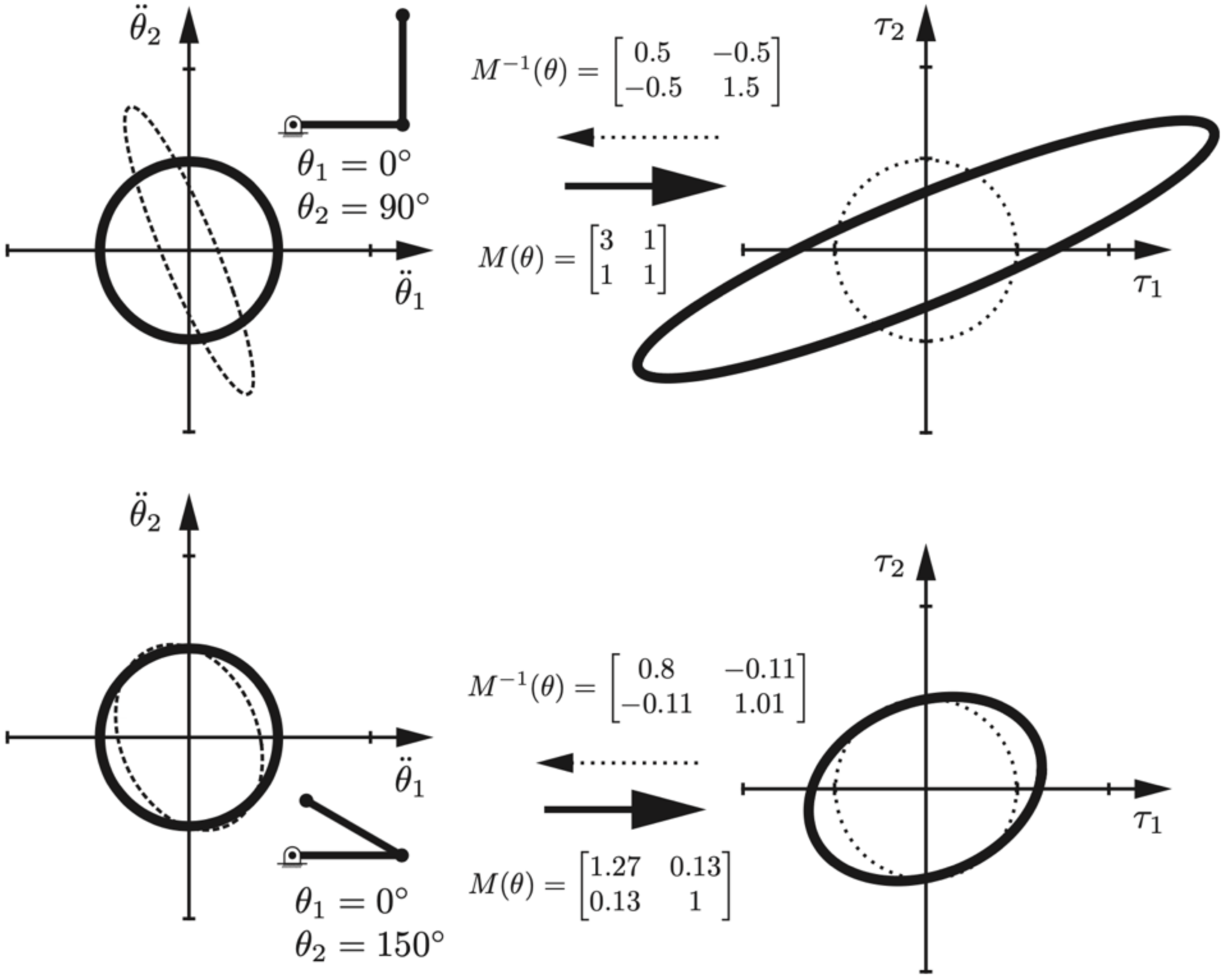
Important concepts, symbols, and equations

$$\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta)$$

kinetic energy of a robot:

$$\boxed{\mathcal{K}(\theta, \dot{\theta}) = \frac{1}{2}\dot{\theta}^{\mathrm{T}} M(\theta)\, \dot{\theta}}$$

When $\dot{\theta} = 0$ and $g = 0$,
$M(\theta)$ maps $\ddot{\theta}$ to τ and
$M^{-1}(\theta)$ maps τ to $\ddot{\theta}$



Important concepts, symbols, and equations (cont.)

If $V = J(\theta)\,\dot\theta$ is the e-e velocity and J is invertible (there exists a unique joint velocity for each e-e velocity):

$$\frac{1}{2} V^{\mathrm{T}} \Lambda(\theta) V = \frac{1}{2} \dot\theta^{\mathrm{T}} M(\theta)\,\dot\theta$$

$$\cancel{\dot\theta}^{\mathrm{T}} J^{\mathrm{T}}(\theta)\,\Lambda(\theta)\,J(\theta)\,\cancel{\dot\theta} = \cancel{\dot\theta}^{\mathrm{T}} M(\theta)\,\cancel{\dot\theta}$$

$$\boxed{\Lambda(\theta) = J^{-\mathrm{T}}(\theta)\,M(\theta)\,J^{-1}(\theta)}$$

end-effector mass matrix

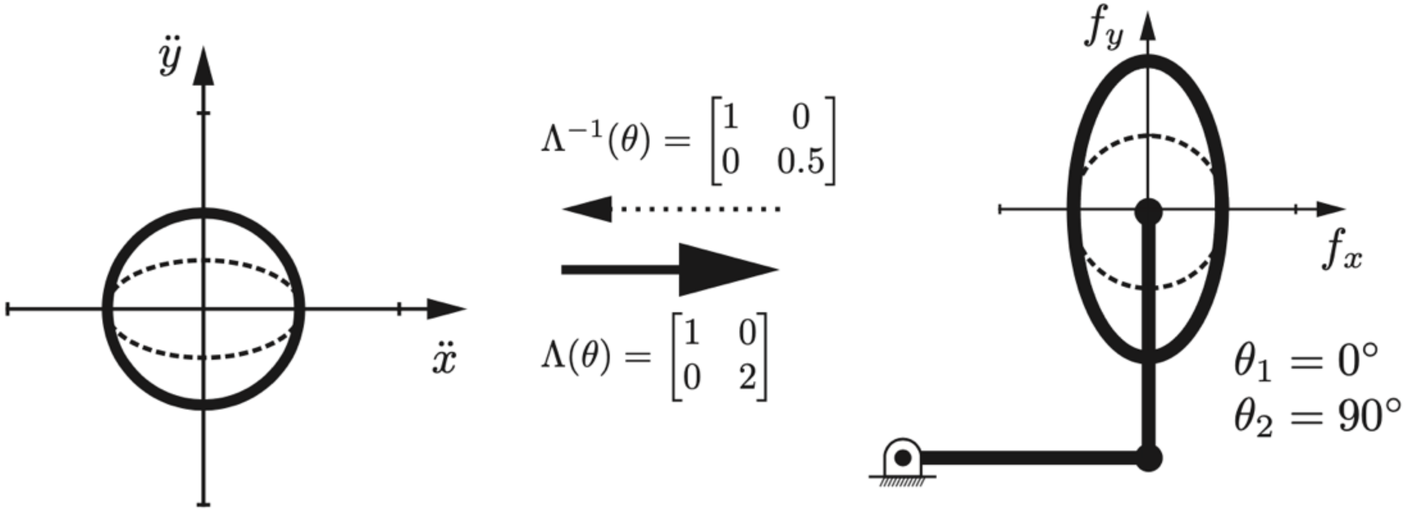
There are some e-e velocities that cannot be achieved by the joints. "Infinitely massive" in these directions.

What if J is tall? wide?

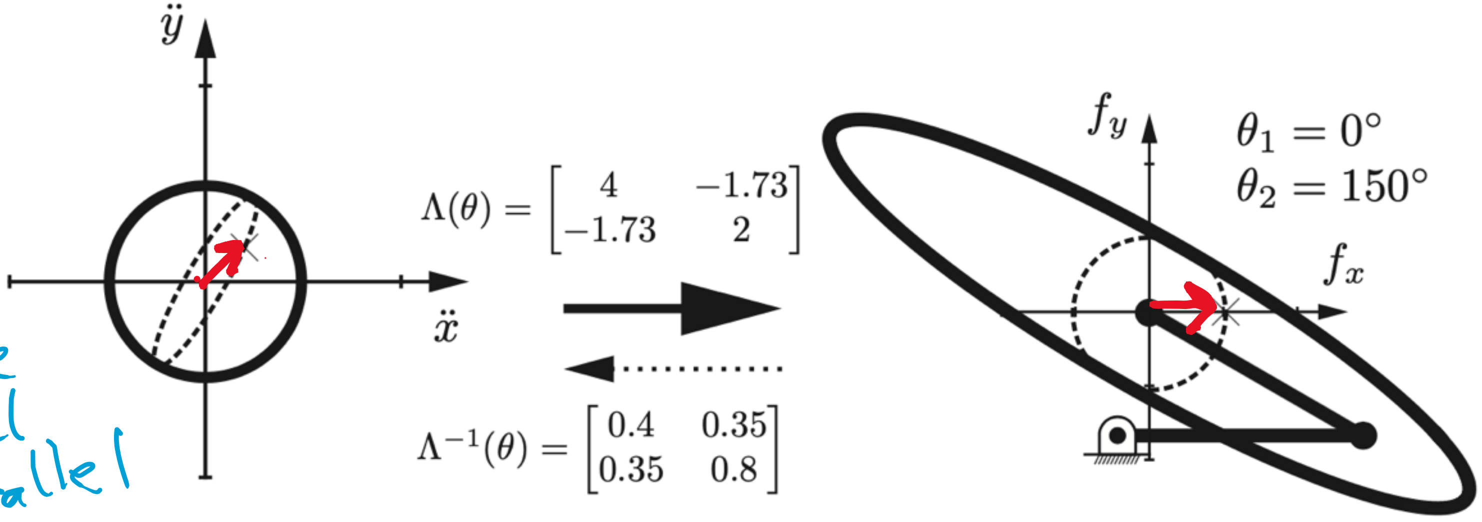
An infinite # of $\dot\theta$ correspond to the same $\dot V$.

Important concepts, symbols, and equations (cont.)

When $\dot{\theta} = 0$ and $g = 0$,
$\Lambda(\theta)$ maps $\dot{V}$ to F and
$\Lambda^{-1}(\theta)$ maps F to $\dot{V}$

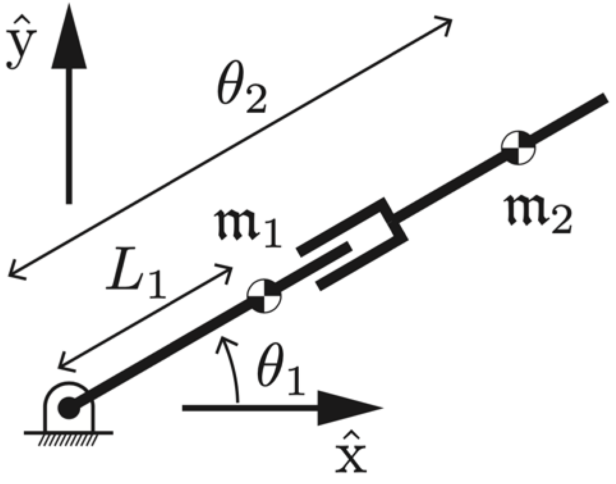


Force and acceleration are only parallel along principal axes.



example where force and accel are not parallel

RP robot



$\dot{\theta} = 0$ and $g = 0$

$$\tau = M(\theta)\ddot{\theta} + c(\theta,\dot{\theta}) + g(\theta)$$

$$M(\theta) = \begin{bmatrix} \mathcal{I}_1 + \mathcal{I}_2 + \mathfrak{m}_1 L_1^2 + \mathfrak{m}_2\theta_2^2 & 0 \\ 0 & \mathfrak{m}_2 \end{bmatrix}$$

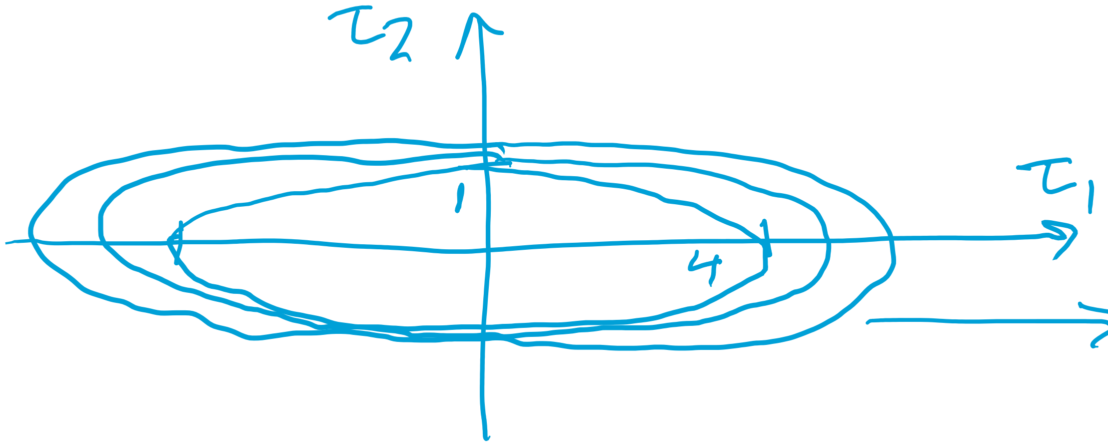
What are the e-vals and e-vecs of M?

Draw the ellipse of τ corresponding to a unit circle of $\ddot{\theta}$ as θ_2 increases from zero and $I_1 = I_2 = \mathfrak{m}_1 = \mathfrak{m}_2 = L_1 = 1$.

$$M(\theta) = \begin{bmatrix} 3+\theta_2^2 & 0 \\ 0 & 1 \end{bmatrix}$$

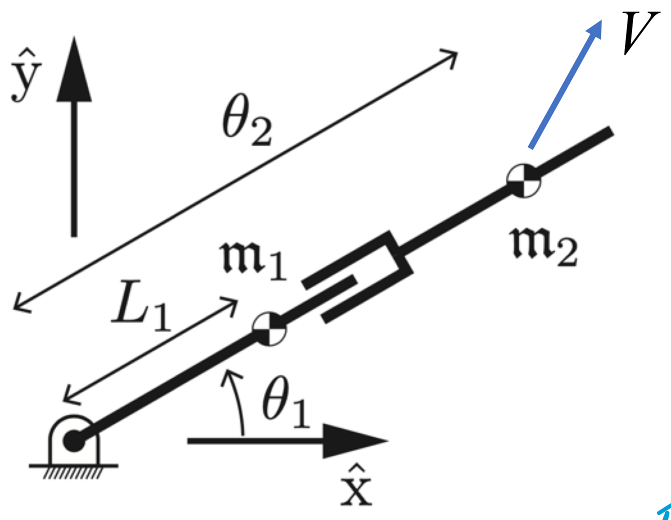
$\lambda_1 = 3+\theta_2^2, \; v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\lambda_2 = 1, \; v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

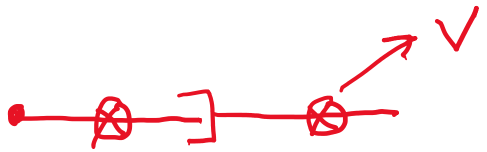


$\rightarrow$ θ_2 increases

RP robot



$\dot{\theta} = 0$ and $g = 0$



At $\theta_1 = 0$, the e-e mass matrix is

$$\Lambda(\theta) = \begin{bmatrix} \mathfrak{m}_2 & 0 \\ 0 & (\mathcal{I}_1 + \mathcal{I}_2 + \mathfrak{m}_1 L_1^2 + \mathfrak{m}_2 \theta_2^2)/\theta_2^2 \end{bmatrix}$$

Draw the ellipse of F corresponding to a unit circle of $\dot{V}$ as θ_2 increases from zero and $I_1 = I_2 = \mathfrak{m}_1 = \mathfrak{m}_2 = L_1 = 1$. How does it change as θ_1 changes?

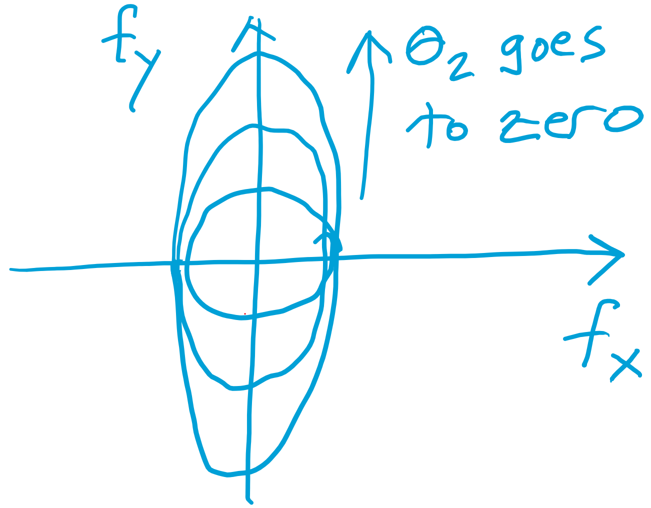
$F = \Lambda(\theta)\dot{V}$ $\quad \Lambda = \begin{bmatrix} 1 & 0 \\ 0 & (3+\theta_2^2)/\theta_2^2 \end{bmatrix}$

if $\theta_2 = \infty$: $\lambda_1 = 1,\ v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\lambda_2 = 1,\ v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

if $\theta_2 = 0$: $\lambda_1 = 1,\ v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\lambda_2 = \infty,\ v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



As θ_1 rotates, so do the force ellipses.