Important concepts, symbols, and equations

Newton-Euler recursive inverse dynamics

Find $\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) + J^{\mathrm{T}}(\theta)\mathcal{F}_{\mathrm{tip}}$

efficiently and numerically, without closed-form expressions or differentiation.

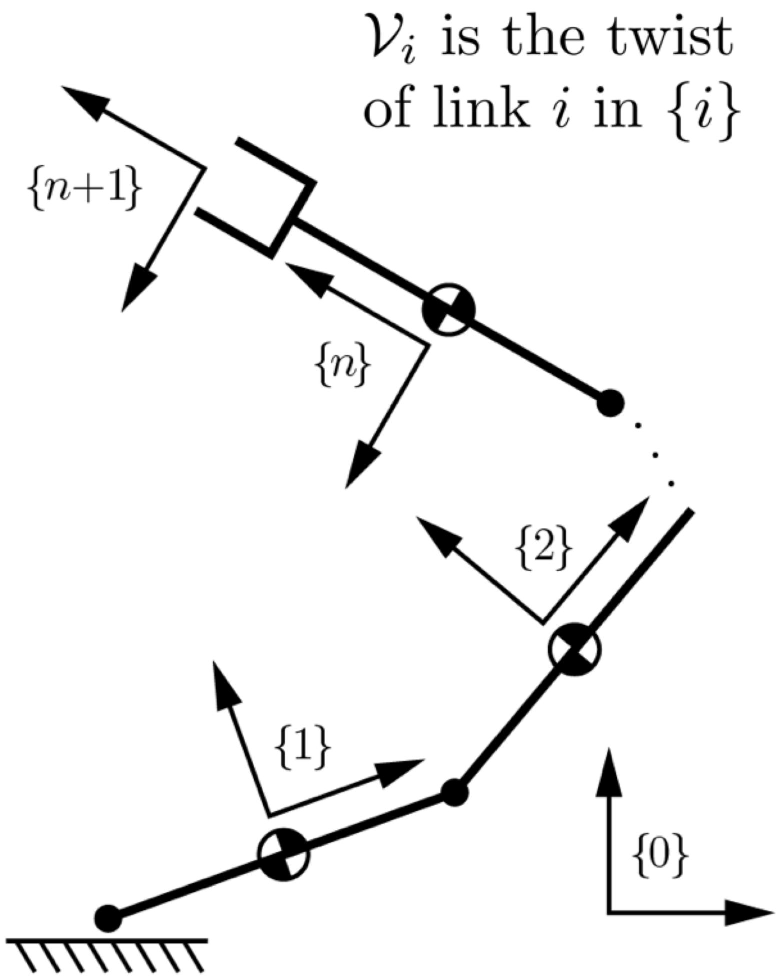
$\mathcal{G}_i$: spatial inertia matrix of link $\{i\}$ in $\{i\}$

$M_{i,i-1}$: $\{i-1\}$ in $\{i\}$ when $\theta_i = 0$

$\mathcal{A}_i$: screw axis of joint i in $\{i\}$

$\mathcal{F}_{n+1}$: wrench $\mathcal{F}_{\mathrm{tip}}$ applied by end-effector

$\dot{\mathcal{V}}_0 = (\dot{\omega}_0, \dot{v}_0) = (0, -\mathfrak{g})$



Important concepts, symbols, and equations (cont.)

Forward iterations

Given $\theta, \dot{\theta}, \ddot{\theta}$, for $i = 1$ to n do:

$$T_{i,i-1} = e^{-[\mathcal{A}_i]\theta_i} M_{i,i-1}$$

$$\mathcal{V}_i = [\mathrm{Ad}_{T_{i,i-1}}]\mathcal{V}_{i-1} + \mathcal{A}_i\dot{\theta}_i$$

$$\dot{\mathcal{V}}_i = [\mathrm{Ad}_{T_{i,i-1}}]\dot{\mathcal{V}}_{i-1} + [\mathrm{ad}_{\mathcal{V}_i}]\mathcal{A}_i\dot{\theta}_i + \mathcal{A}_i\ddot{\theta}_i$$

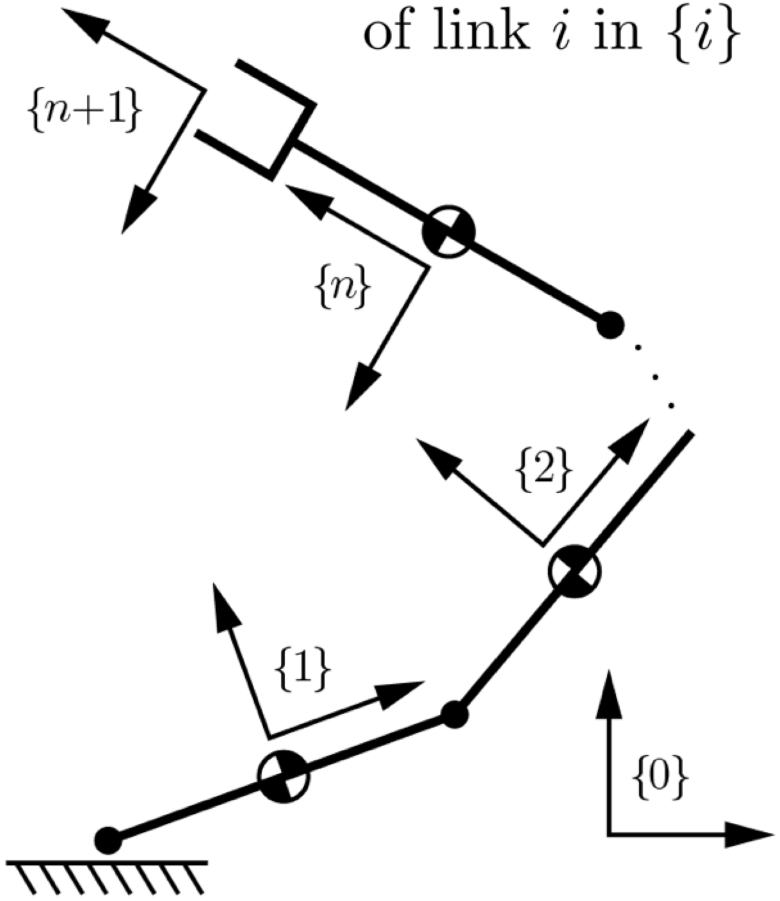
Backward iterations

For $i = n$ to 1 do:

$$\mathcal{F}_i = [\mathrm{Ad}_{T_{i+1,i}}]^{\mathrm{T}}\mathcal{F}_{i+1} + \mathcal{G}_i\dot{\mathcal{V}}_i - [\mathrm{ad}_{\mathcal{V}_i}]^{\mathrm{T}}\mathcal{G}_i\mathcal{V}_i$$

$$\tau_i = \mathcal{F}_i^{\mathrm{T}}\mathcal{A}_i$$

$\mathcal{V}_i$ is the twist of link i in $\{i\}$



Important concepts, symbols, and equations (cont.)

Forward dynamics: Solve $M(\theta)\ddot{\theta} = \tau - c(\theta, \dot{\theta}) - g(\theta) - J^{\mathrm{T}}(\theta)\mathcal{F}_{\mathrm{tip}}$ for $\ddot{\theta}$.

Use $n+1$ calls of N-E inverse dynamics to get

- $c(\theta, \dot{\theta}) + g(\theta) + J^{\mathrm{T}}(\theta)\mathcal{F}_{\mathrm{tip}}$ by setting $\ddot{\theta} = 0$
- $M(\theta) = [M_1(\theta) \ \cdots \ M_n(\theta)]$, where $\tau = M_i(\theta)$ if $\ddot{\theta}_i = 1$, $\ddot{\theta}_j = 0$ for all $j \neq i$, $\dot{\theta} = 0$, $\mathfrak{g} = 0$, and $\mathcal{F}_{\mathrm{tip}} = 0$.

Use any efficient algorithm to solve $M\ddot{\theta} = b$ for $\ddot{\theta}$.

Important concepts, symbols, and equations (cont.)

Euler integration for simulation:

$$
\begin{aligned}
\ddot{\theta}[k] &= ForwardDynamics(\theta[k], \dot{\theta}[k], \tau(k\delta t), \mathcal{F}_{\text{tip}}(k\delta t)) \\
\theta[k+1] &= \theta[k] + \dot{\theta}[k]\delta t \quad + \frac{1}{2}\ddot{\theta}[k]\,\delta t^2 \\
\dot{\theta}[k+1] &= \dot{\theta}[k] + \ddot{\theta}[k]\delta t
\end{aligned}
$$

Could add a second-order correction to the position calculation.

Explain each term in the equations below.

Forward iterations

Given $\theta, \dot{\theta}, \ddot{\theta}$, for $i = 1$ to n do:

$$T_{i,i-1} = e^{-[\mathcal{A}_i]\theta_i} M_{i,i-1}$$

$$\mathcal{V}_i = [\mathrm{Ad}_{T_{i,i-1}}]\mathcal{V}_{i-1} + \mathcal{A}_i\dot{\theta}_i$$

link i-1 twist plus added twist due to joint i

$$\dot{\mathcal{V}}_i = [\mathrm{Ad}_{T_{i,i-1}}]\dot{\mathcal{V}}_{i-1} + [\mathrm{ad}_{\mathcal{V}_i}]\mathcal{A}_i\dot{\theta}_i + \mathcal{A}_i\ddot{\theta}_i$$

link i-1 accel

velocity-product term

added accel due to joint i

Backward iterations

For $i = n$ to 1 do:

outboard wrench to support

$$\mathcal{F}_i = [\mathrm{Ad}_{T_{i+1,i}}]^{\mathrm{T}}\mathcal{F}_{i+1} + \underbrace{\mathcal{G}_i\dot{\mathcal{V}}_i - [\mathrm{ad}_{\mathcal{V}_i}]^{\mathrm{T}}\mathcal{G}_i\mathcal{V}_i}_{\text{rigid-body dynamics}}$$

$$\tau_i = \mathcal{F}_i^{\mathrm{T}}\mathcal{A}_i$$

component of the wrench that does work

$\mathcal{V}_i$ is the twist of link i in $\{i\}$