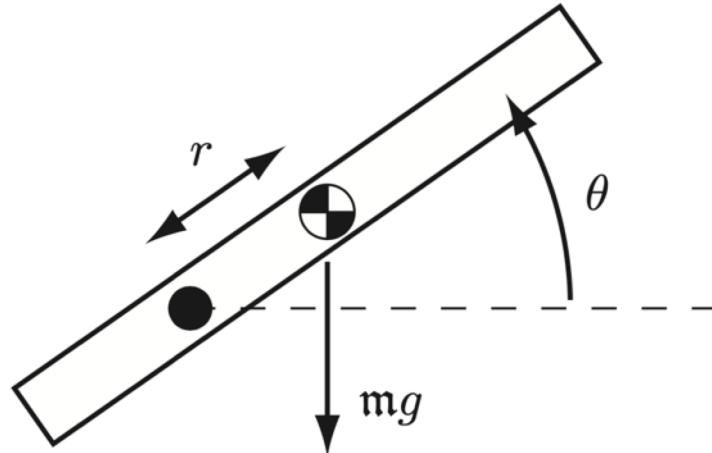


Chapter 2	Configuration Space
Chapter 3	Rigid-Body Motions
Chapter 4	Forward Kinematics
Chapter 5	Velocity Kinematics and Statics
Chapter 6	Inverse Kinematics
Chapter 7	Kinematics of Closed Chains
Chapter 8	Dynamics of Open Chains
Chapter 9	Trajectory Generation
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Chapter 11	Robot Control
	11.1 Control System Overview
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	11.3 Motion Control with Velocity Inputs
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Chapter 12	Grasping and Manipulation
Chapter 13	Wheeled Mobile Robots

Important concepts, symbols, and equations



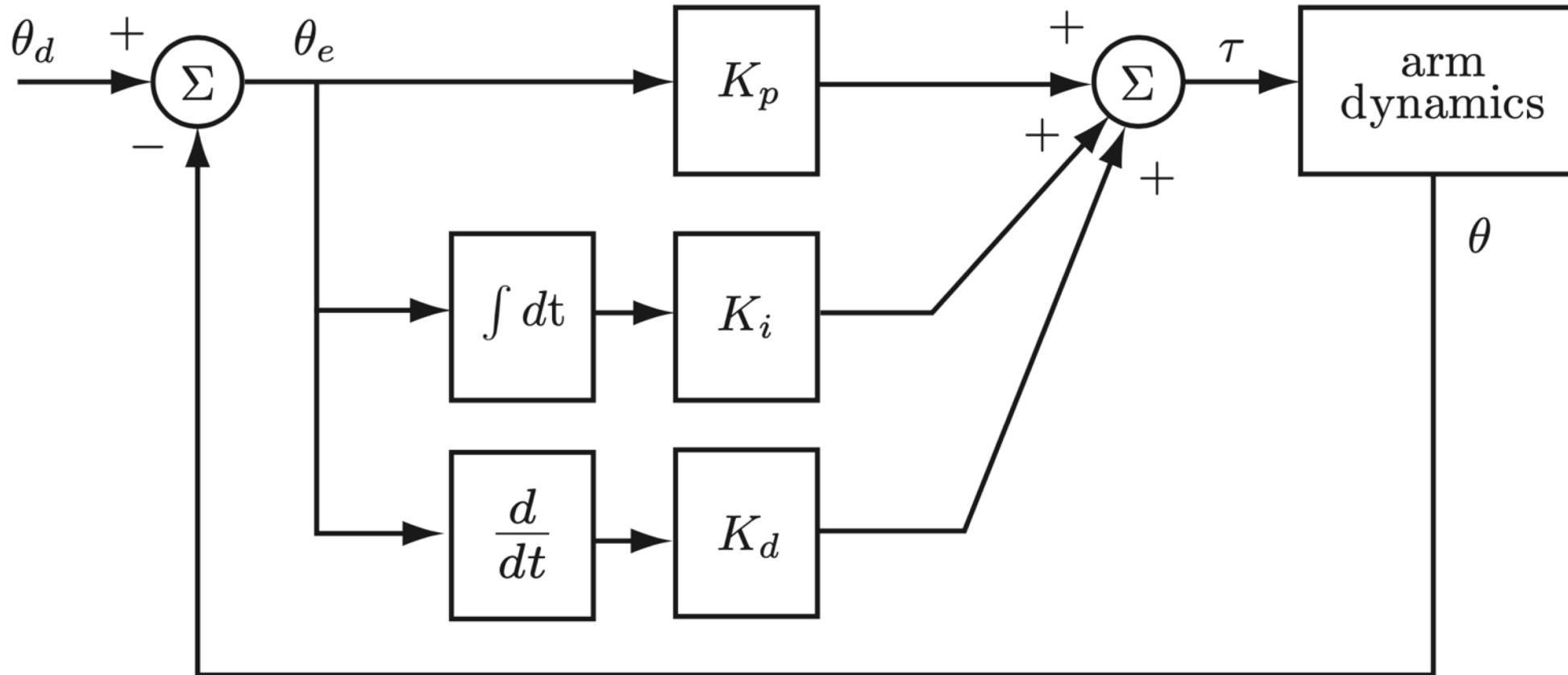
$$\tau = M\ddot{\theta} + \mathfrak{m}gr \cos \theta + b\dot{\theta}$$

$$\tau = M\ddot{\theta} + h(\theta, \dot{\theta})$$

Proportional-Integral-Derivative (PID) control

$$\tau = K_p \theta_e + K_i \int \theta_e(t) dt + K_d \dot{\theta}_e$$

Important concepts, symbols, and equations (cont.)



Important concepts, symbols, and equations (cont.)

Setpoint PD control, $g = 0$

$$\tau = K_p \theta_e + \cancel{K_i} \int \theta_e(t) dt + K_d \dot{\theta}_e$$

$$\tau = M \ddot{\theta} + \cancel{m g r} \cos \theta + b \dot{\theta}$$

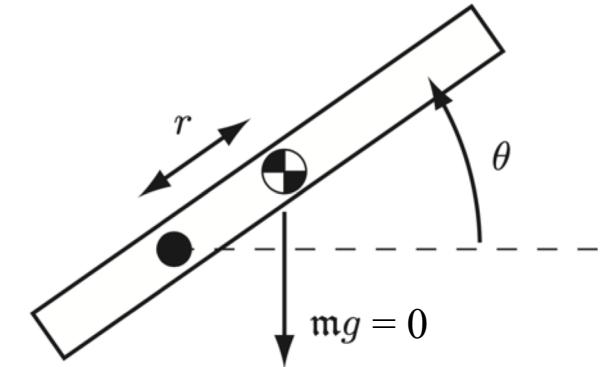
$$\dot{\theta}_d = \ddot{\theta}_d = 0$$

$$M \ddot{\theta} + b \dot{\theta} = K_p \theta_e + K_d \dot{\theta}_e$$

$$\ddot{\theta}_e + \frac{b + K_d}{M} \dot{\theta}_e + \frac{K_p}{M} \theta_e = 0$$

$$\zeta = \frac{b + K_d}{2\sqrt{K_p M}} \quad \omega_n = \sqrt{\frac{K_p}{M}}$$

For what gains
are the error
dynamics stable?



Important concepts, symbols, and equations (cont.)

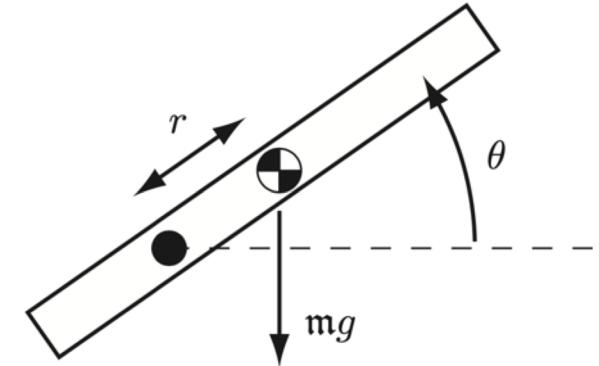
Setpoint PD control, $g \neq 0$

$$\tau = K_p \theta_e + \overset{0}{K_i} \int \theta_e(t) dt + K_d \dot{\theta}_e$$

$$\tau = M \ddot{\theta} + m g r \cos \theta + b \dot{\theta}$$

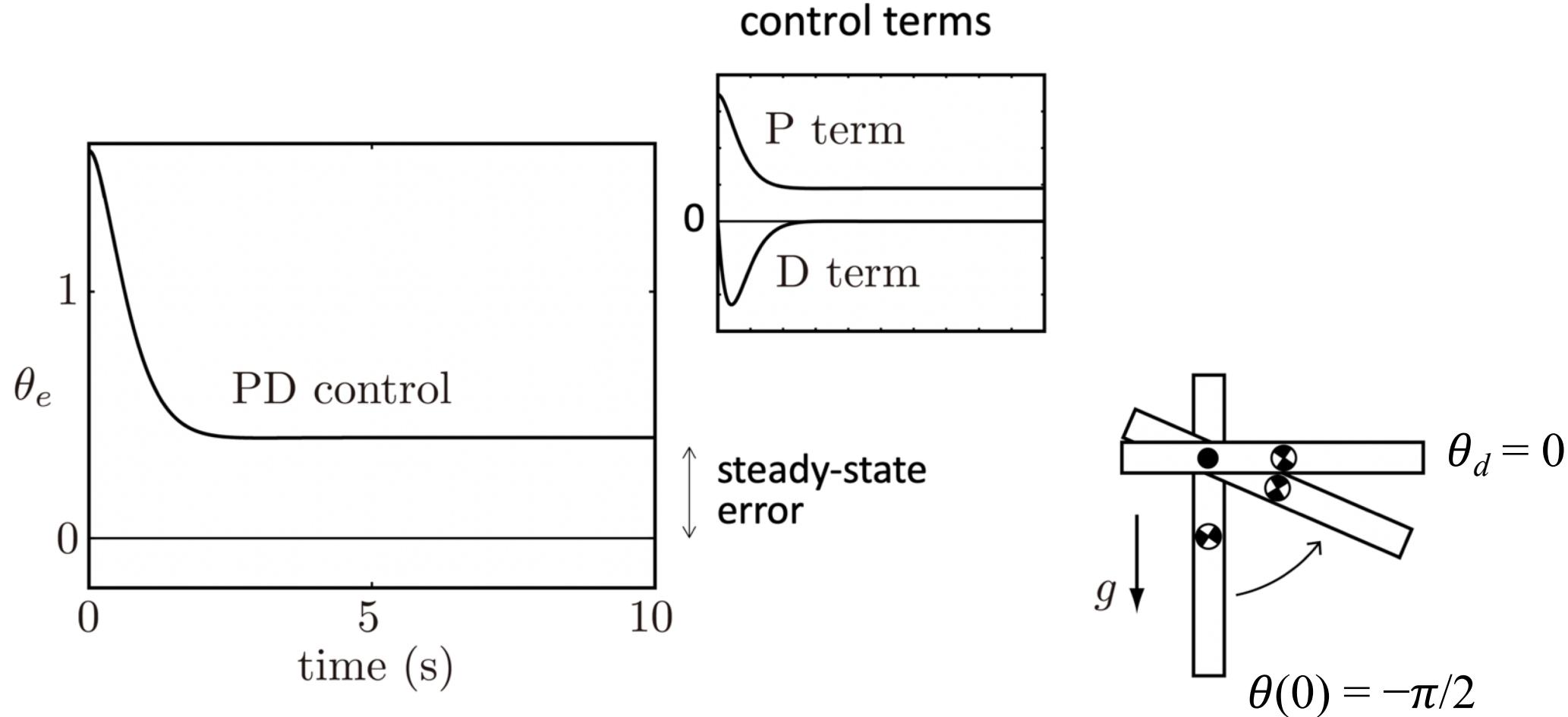
$$\dot{\theta}_d = \ddot{\theta}_d = 0$$

$$M \ddot{\theta}_e + (b + K_d) \dot{\theta}_e + K_p \theta_e = m g r \cos \theta$$



Nonhomogeneous.
What is the steady-state error?

Important concepts, symbols, and equations (cont.)



Important concepts, symbols, and equations (cont.)

Setpoint PID control, $g \neq 0$

$$\tau = K_p \theta_e + K_i \int \theta_e(t) dt + K_d \dot{\theta}_e$$

$$\tau = M \ddot{\theta} + \text{m}gr \cos \theta + b \dot{\theta}$$

$$\dot{\theta}_d = \ddot{\theta}_d = 0 \quad \tau_{\text{dist}}$$

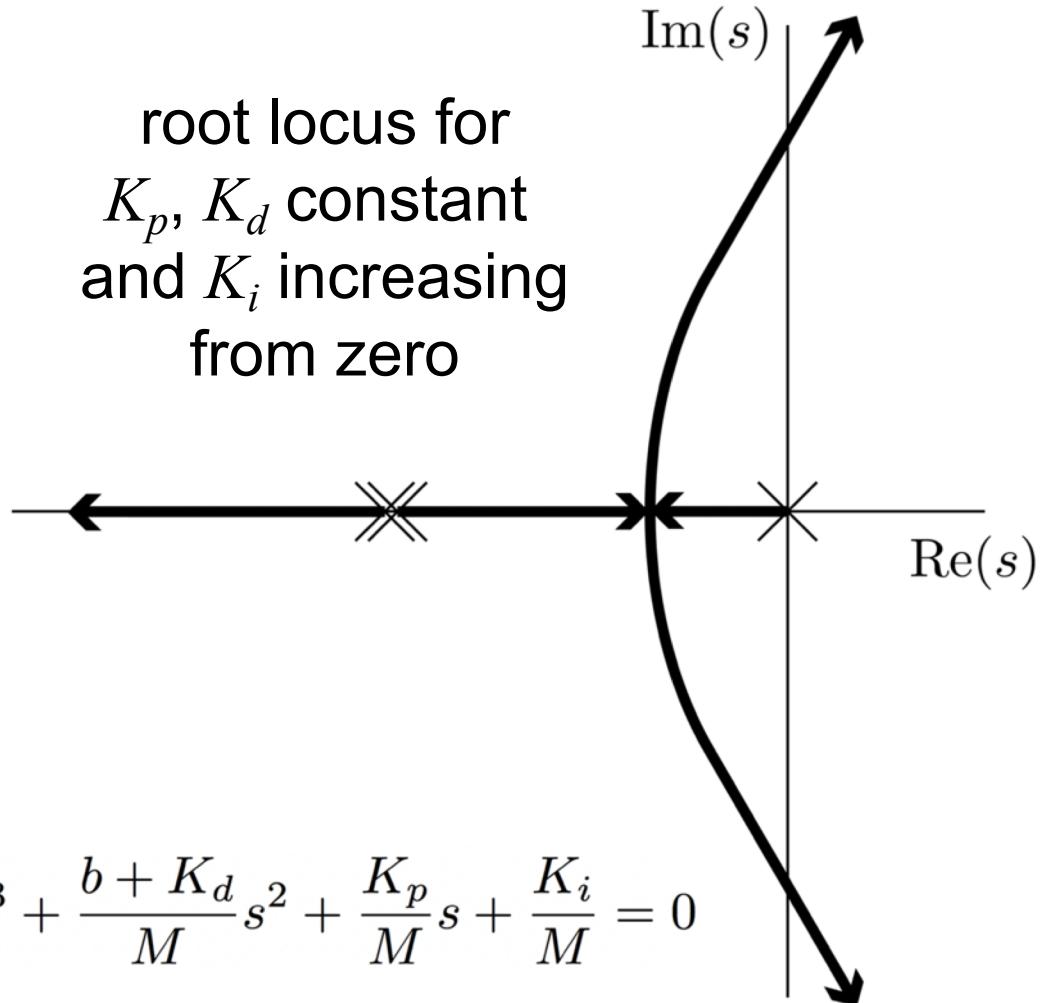
$$M \ddot{\theta}_e + (b + K_d) \dot{\theta}_e + K_p \theta_e + K_i \int \theta_e(t) dt = \tau_{\text{dist}}$$

$$M \theta_e^{(3)} + (b + K_d) \ddot{\theta}_e + K_p \dot{\theta}_e + K_i \theta_e = 0$$

$$s^3 + \frac{b + K_d}{M} s^2 + \frac{K_p}{M} s + \frac{K_i}{M} = 0$$

Important concepts, symbols, and equations (cont.)

root locus for
 K_p, K_d constant
and K_i increasing
from zero

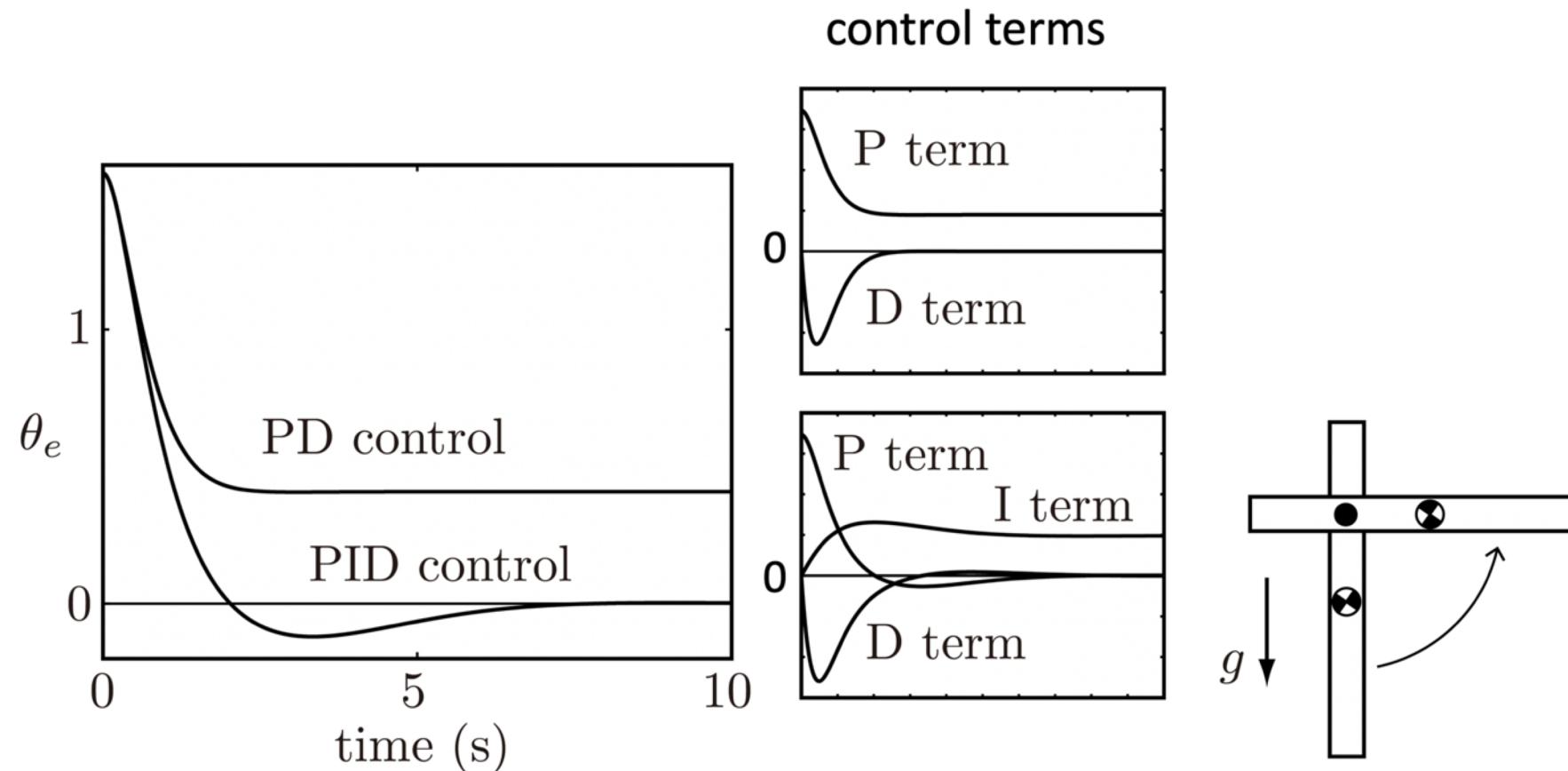


$$\begin{aligned} K_d &> -b \\ K_p &> 0 \end{aligned}$$

$$\frac{(b + K_d)K_p}{M} > K_i > 0$$

K_i improves steady-state response but can worsen the transient response.

Important concepts, symbols, and equations (cont.)



Important concepts, symbols, and equations (cont.)

What about tracking general trajectories, not just setpoint control?

$$\tau = M \left(\overline{\ddot{\theta}_d + K_p \theta_e + K_i \int \theta_e dt + K_d \dot{\theta}_e} \right) + h(\theta, \dot{\theta})$$

commanded $\ddot{\theta}$

$$\ddot{\theta}_e = \ddot{\theta}_d - \ddot{\theta}$$

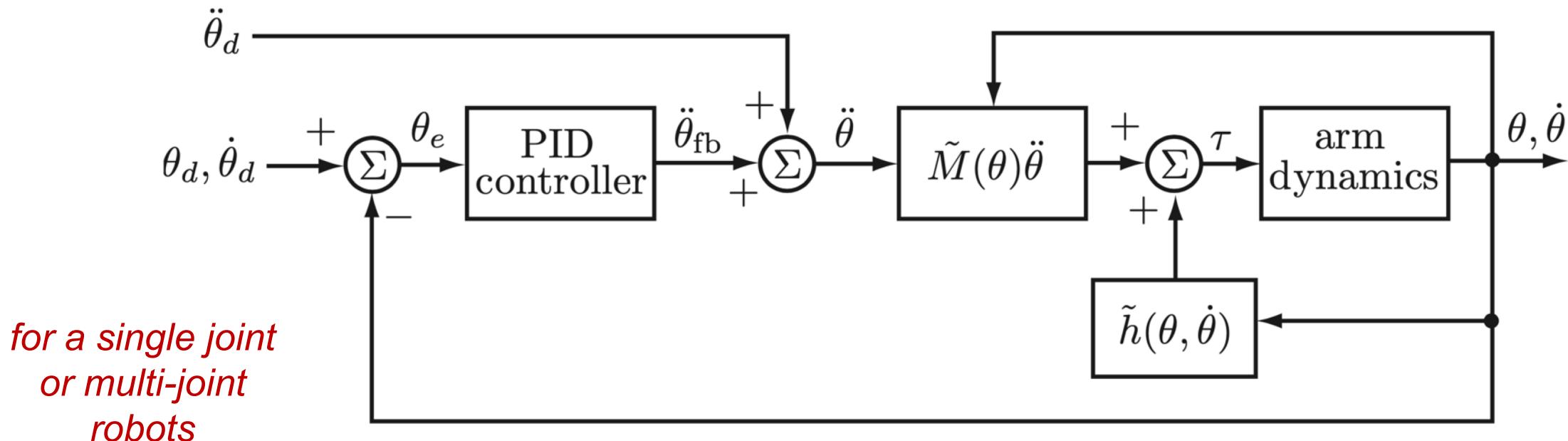
$$\ddot{\theta}_e = -K_d \dot{\theta}_e - K_p \theta_e - K_i \int \theta_e dt$$

$$\theta_e^{(3)} + K_d \ddot{\theta}_e + K_p \dot{\theta}_e + K_i \theta_e = 0$$

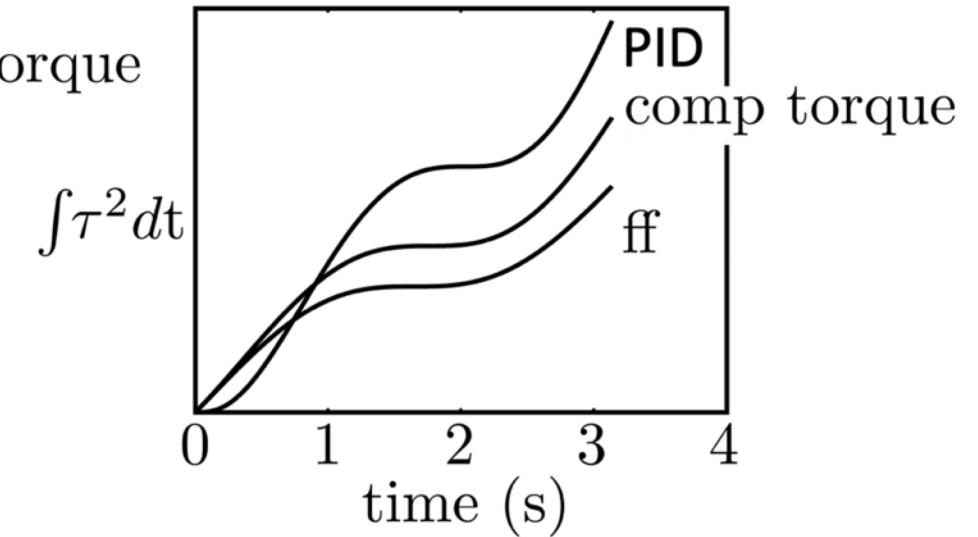
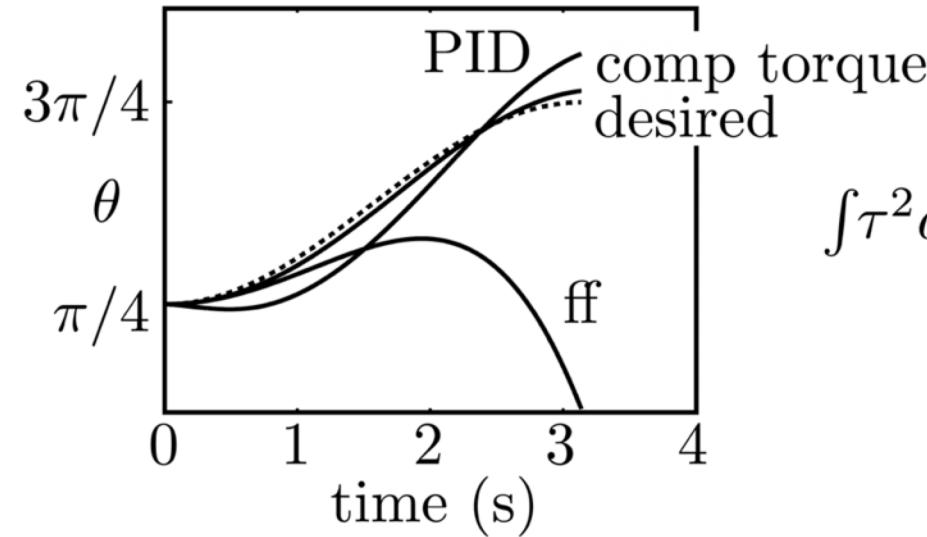
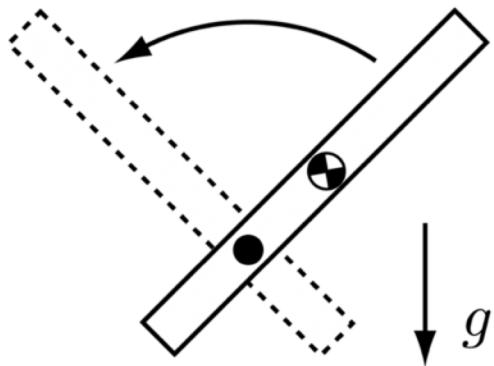
Important concepts, symbols, and equations (cont.)

Computed torque control (feedback linearization)

$$\tau = \tilde{M}(\theta) \left(\ddot{\theta}_d + K_p \theta_e + K_i \int \theta_e(t) dt + K_d \dot{\theta}_e \right) + \tilde{h}(\theta, \dot{\theta})$$



Important concepts, symbols, and equations (cont.)



Important concepts, symbols, and equations (cont.)

Task-space computed torque control

$$\mathcal{F}_b = \Lambda(\theta) \dot{\mathcal{V}}_b + \eta(\theta, \mathcal{V}_b)$$

dynamic model: $\{\tilde{\Lambda}, \tilde{\eta}\}$

$$\tau = \tilde{M}(\theta) \left(\ddot{\theta}_d + K_p \theta_e + K_i \int \theta_e dt + K_d \dot{\theta}_e \right) + \tilde{h}(\theta, \dot{\theta})$$

$$\mathcal{F}_b = \tilde{\Lambda}(\theta) \left(\dot{\mathcal{V}}_d + K_p X_e + K_i \int X_e dt + K_d \mathcal{V}_e \right) + \tilde{\eta}(\theta, \mathcal{V}_b)$$

$$[X_e] = \log(X^{-1} X_d)$$

$$\mathcal{V}_e = [\text{Ad}_{X^{-1} X_d}] \mathcal{V}_d - \mathcal{V}_b$$

$$\tau = J_b^T(\theta) \mathcal{F}_b$$

What if your dynamic model is poor?

The characteristic equation of the error dynamics are

$$s^5 + 2s^4 + s^3 + 2s^2 + 4s + 2 = 0$$

Write the error dynamics in the form $\dot{x} = Ax$. Determine if the system is stable. (Use any software you want.)