

Chapter 2	Configuration Space
Chapter 3	Rigid-Body Motions
Chapter 4	Forward Kinematics
Chapter 5	Velocity Kinematics and Statics
Chapter 6	Inverse Kinematics
Chapter 7	Kinematics of Closed Chains
Chapter 8	Dynamics of Open Chains
	8.1 Lagrangian Formulation
	8.2 Dynamics of a Single Rigid Body
	8.3 Newton-Euler Inverse Dynamics
	8.4 Dynamic Equations in Closed Form
	8.5 Forward Dynamics of Open Chains
Chapter 9	Trajectory Generation
Chapter 10	Motion Planning
Chapter 11	Robot Control
Chapter 12	Grasping and Manipulation
Chapter 13	Wheeled Mobile Robots

Important concepts, symbols, and equations

Newton-Euler recursive inverse dynamics

Find $\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) + J^T(\theta)\mathcal{F}_{\text{tip}}$

efficiently and numerically, without closed-form expressions or differentiation.

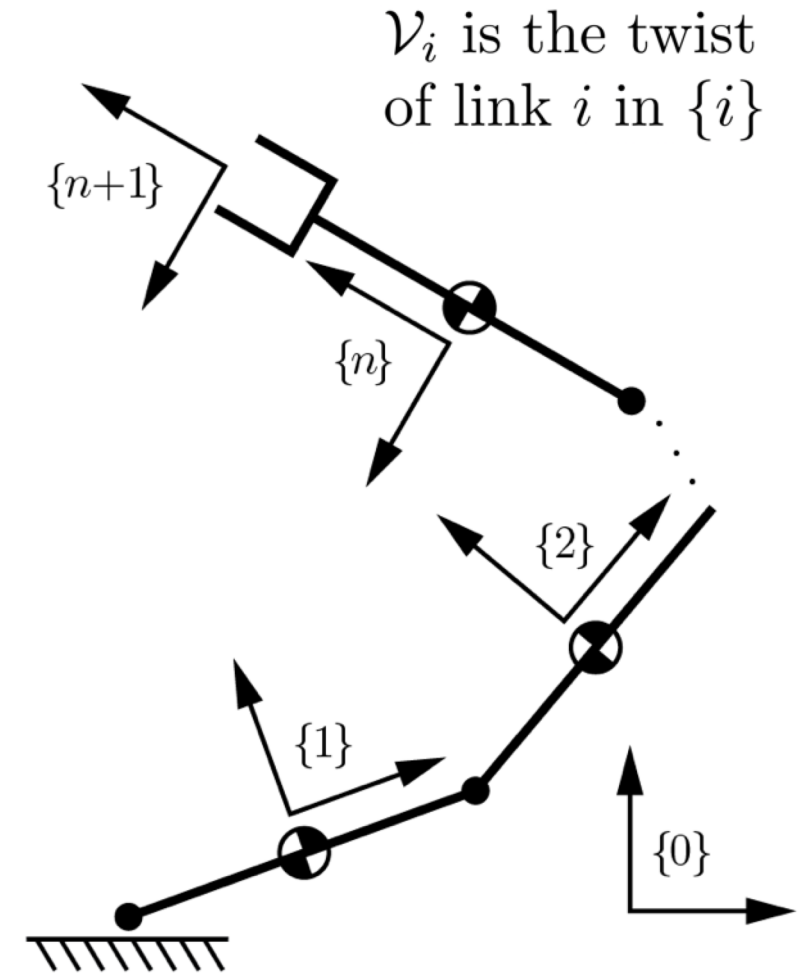
\mathcal{G}_i : spatial inertia matrix of link $\{i\}$ in $\{i\}$

$M_{i,i-1}$: $\{i-1\}$ in $\{i\}$ when $\theta_i = 0$

\mathcal{A}_i : screw axis of joint i in $\{i\}$

\mathcal{F}_{n+1} : wrench \mathcal{F}_{tip} applied by end-effector

$$\dot{\mathcal{V}}_0 = (\dot{\omega}_0, \dot{v}_0) = (0, -\mathbf{g})$$



Important concepts, symbols, and equations (cont.)

Forward iterations

Given $\theta, \dot{\theta}, \ddot{\theta}$, for $i = 1$ to n do:

$$T_{i,i-1} = e^{-[\mathcal{A}_i]\theta_i} M_{i,i-1}$$

$$\mathcal{V}_i = [\text{Ad}_{T_{i,i-1}}] \mathcal{V}_{i-1} + \mathcal{A}_i \dot{\theta}_i$$

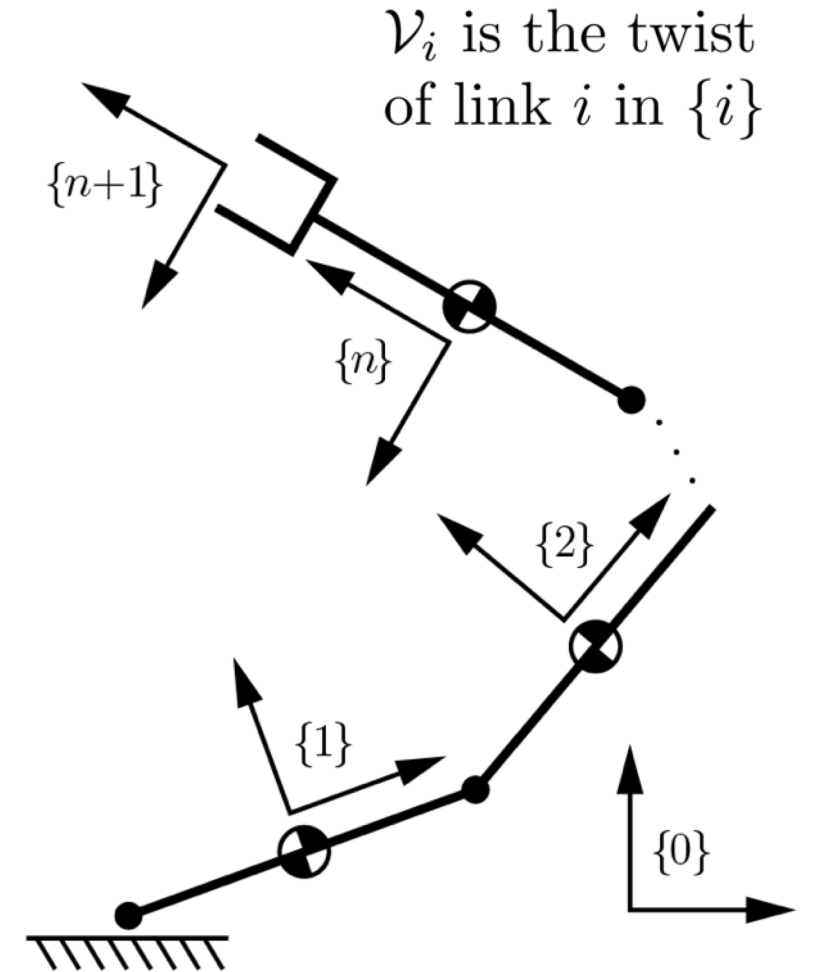
$$\dot{\mathcal{V}}_i = [\text{Ad}_{T_{i,i-1}}] \dot{\mathcal{V}}_{i-1} + [\text{ad}_{\mathcal{V}_i}] \mathcal{A}_i \dot{\theta}_i + \mathcal{A}_i \ddot{\theta}_i$$

Backward iterations

For $i = n$ to 1 do:

$$\mathcal{F}_i = [\text{Ad}_{T_{i+1,i}}]^T \mathcal{F}_{i+1} + \mathcal{G}_i \dot{\mathcal{V}}_i - [\text{ad}_{\mathcal{V}_i}]^T \mathcal{G}_i \mathcal{V}_i$$

$$\tau_i = \mathcal{F}_i^T \mathcal{A}_i$$



Important concepts, symbols, and equations (cont.)

Forward dynamics: Solve $M(\theta)\ddot{\theta} = \tau - c(\theta, \dot{\theta}) - g(\theta) - J^T(\theta)\mathcal{F}_{\text{tip}}$
for $\ddot{\theta}$.

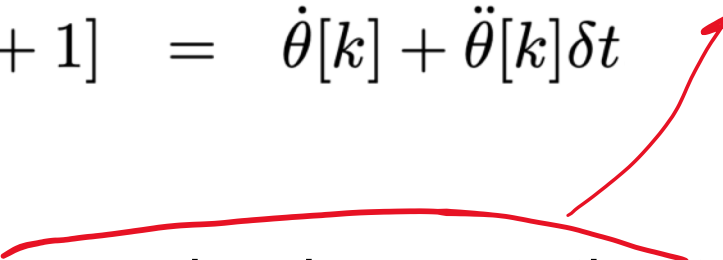
Use $n + 1$ calls of N-E inverse dynamics to get

- $c(\theta, \dot{\theta}) + g(\theta) + J^T(\theta)\mathcal{F}_{\text{tip}}$ by setting $\ddot{\theta} = 0$
- $M(\theta) = [M_1(\theta) \ \cdots \ M_n(\theta)]$, where $\tau = M_i(\theta)$ if $\ddot{\theta}_i = 1$, $\ddot{\theta}_j = 0$ for all $j \neq i$, $\dot{\theta} = 0$, $\mathfrak{g} = 0$, and $\mathcal{F}_{\text{tip}} = 0$.

Use any efficient algorithm to solve $M\ddot{\theta} = b$ for $\ddot{\theta}$.

Important concepts, symbols, and equations (cont.)

Euler integration for simulation:

$$\begin{aligned}\ddot{\theta}[k] &= \textit{ForwardDynamics}(\theta[k], \dot{\theta}[k], \tau(k\delta t), \mathcal{F}_{\text{tip}}(k\delta t)) \\ \theta[k+1] &= \theta[k] + \dot{\theta}[k]\delta t + \frac{1}{2} \ddot{\theta}[k] \delta t^2 \\ \dot{\theta}[k+1] &= \dot{\theta}[k] + \ddot{\theta}[k]\delta t\end{aligned}$$


Could add a second-order correction to the position calculation.

Explain each term in the equations below.

Forward iterations

Given $\theta, \dot{\theta}, \ddot{\theta}$, for $i = 1$ to n do:

$$T_{i,i-1} = e^{-[\mathcal{A}_i]\theta_i} M_{i,i-1}$$

$$\mathcal{V}_i = [\text{Ad}_{T_{i,i-1}}] \mathcal{V}_{i-1} + \mathcal{A}_i \dot{\theta}_i$$

link i-1 twist plus added twist due to joint i

$$\dot{\mathcal{V}}_i = [\text{Ad}_{T_{i,i-1}}] \dot{\mathcal{V}}_{i-1} + [\text{ad}_{\mathcal{V}_i}] \mathcal{A}_i \dot{\theta}_i + \mathcal{A}_i \ddot{\theta}_i$$

link i-1 accel *velocity-product term* *added accel due to joint i*

Backward iterations

For $i = n$ to 1 do:

$$\mathcal{F}_i = [\text{Ad}_{T_{i+1,i}}]^T \mathcal{F}_{i+1} + \underline{\mathcal{G}_i \dot{\mathcal{V}}_i - [\text{ad}_{\mathcal{V}_i}]^T \mathcal{G}_i \mathcal{V}_i}$$

outboard wrench to support

$$\tau_i = \mathcal{F}_i^T \mathcal{A}_i$$

rigid-body dynamics

component of the wrench that does work

