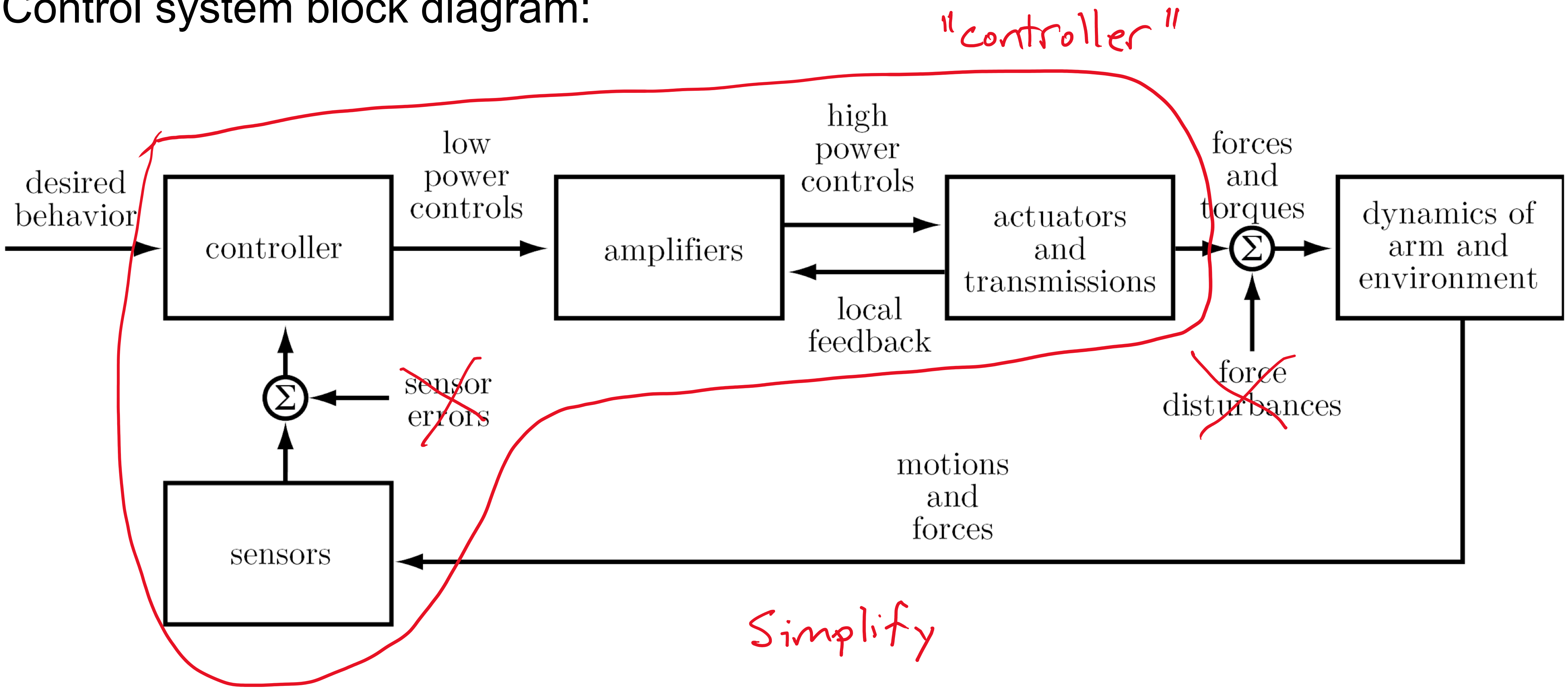
Important concepts, symbols, and equations

Example control objectives:

- **motion control**
- **force control**
- **hybrid motion-force control**
- **impedance control**

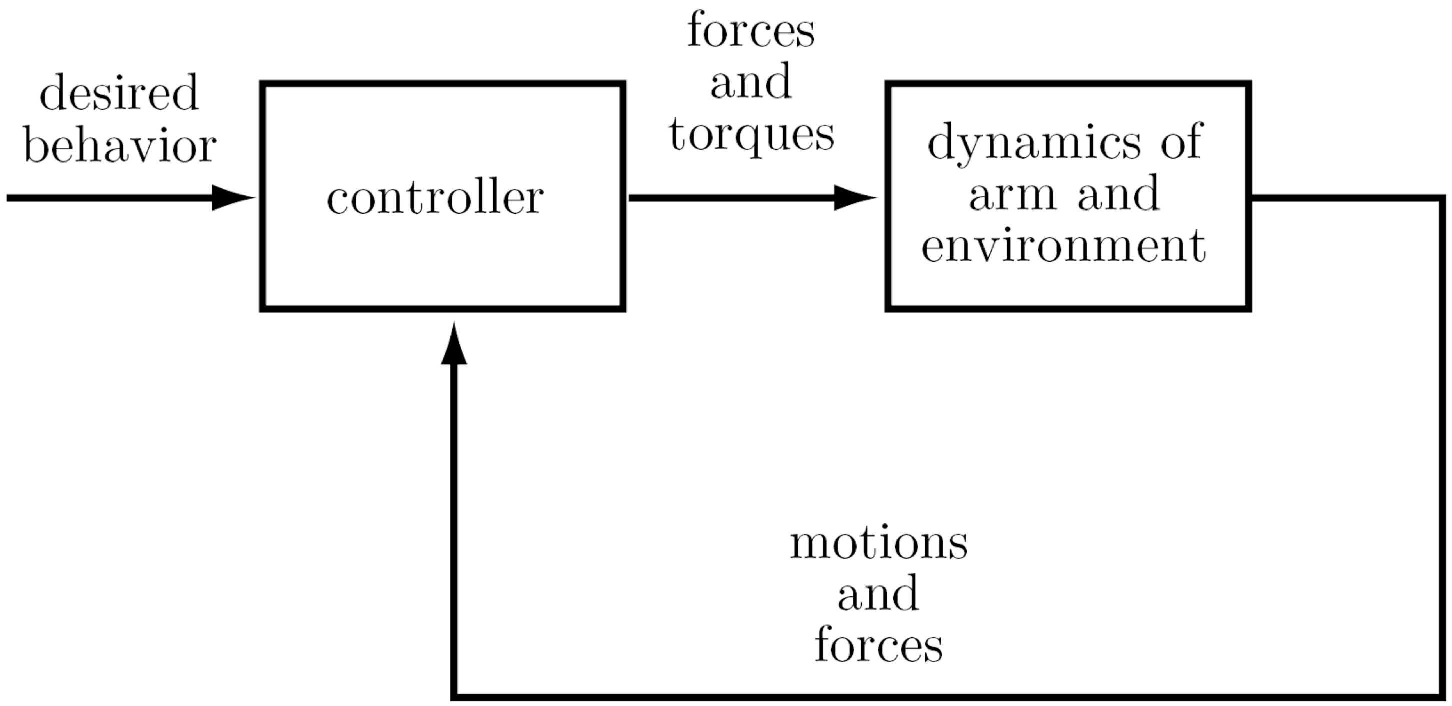
Important concepts, symbols, and equations (cont.)

Control system block diagram:



Important concepts, symbols, and equations (cont.)

Simplified block diagram:



Also assuming continuous-time (not discrete-time) control.

Important concepts, symbols, and equations (cont.)

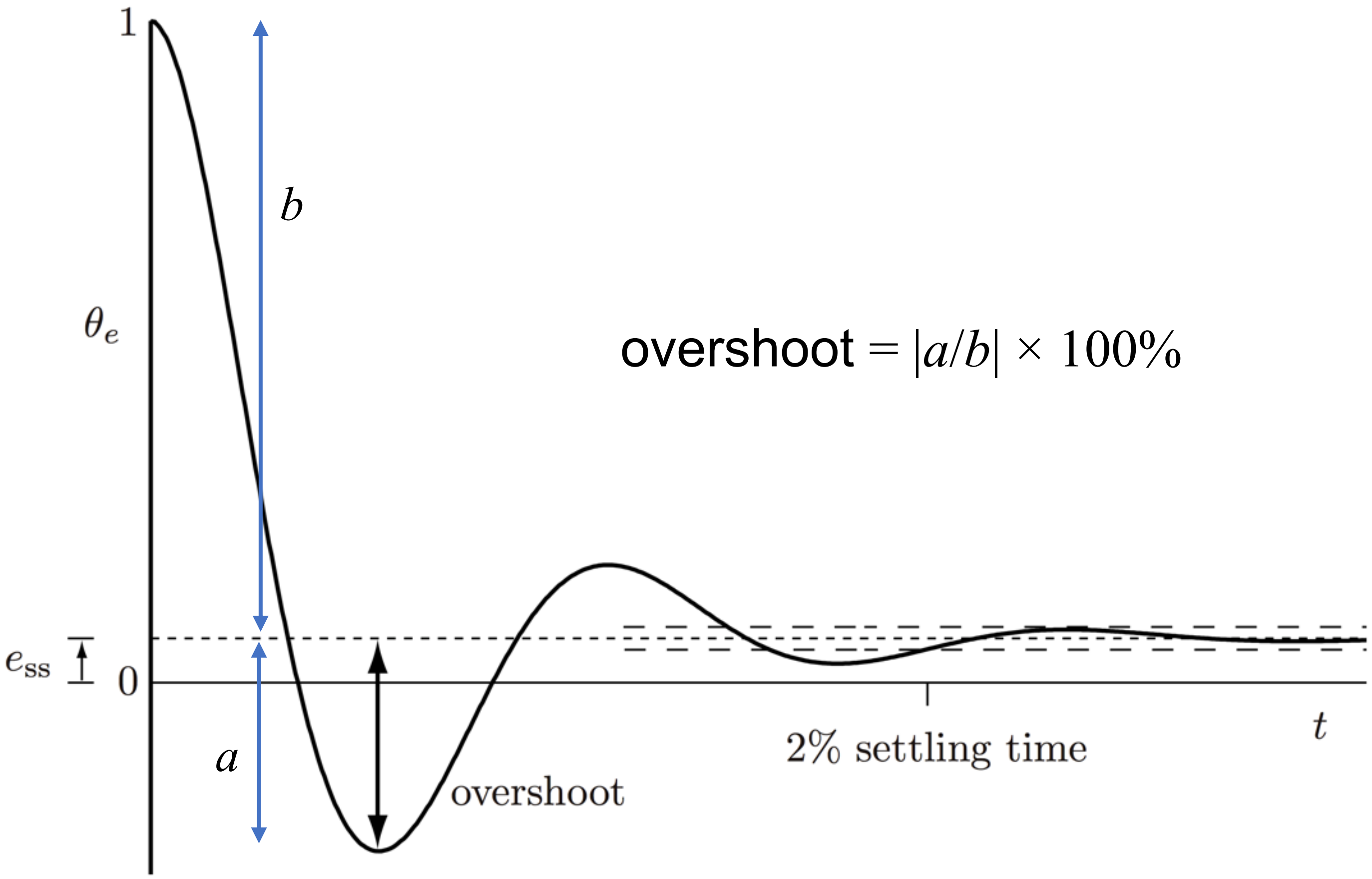
For motion control,

reference: $\theta_d(t)$
actual: $\theta(t)$
error: $\theta_e(t) = \theta_d(t) - \theta(t)$

Unit step error response:
$\theta_e(t)$ starting from $\theta_e(0) = 1$

Steady-state error response: e_{ss}

Transient error response:
overshoot, settling time



overshoot $= |a/b| \times 100\%$

Important concepts, symbols, and equations (cont.)

System dynamics, feedback controllers, and error response are often modeled by **linear ordinary differential equations**.

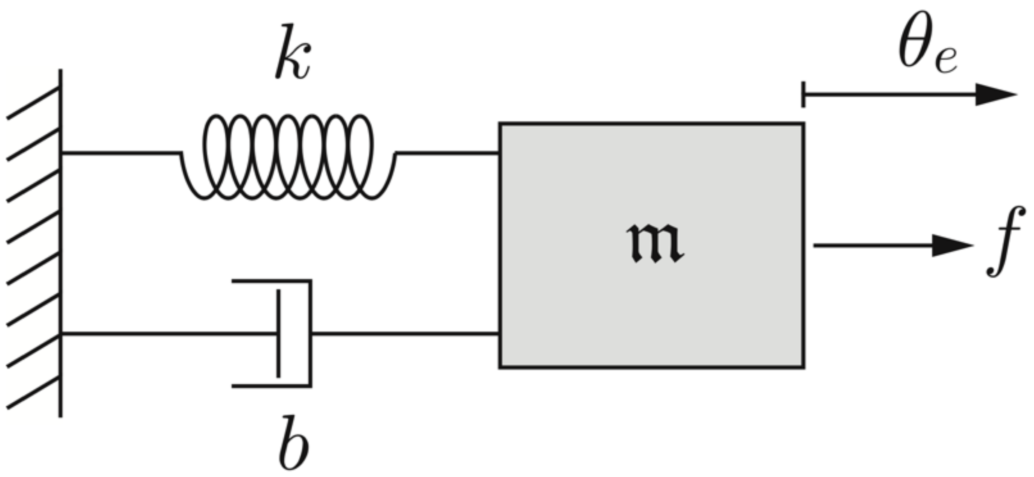
The simplest linear ODE exhibiting overshoot is second order, e.g.,

$$\mathfrak{m}\ddot{\theta}_e + b\dot{\theta}_e + k\theta_e = f$$

or, if $f = 0$,

$$\ddot{\theta}_e + \frac{b}{\mathfrak{m}}\dot{\theta}_e + \frac{k}{\mathfrak{m}}\theta_e = 0$$

mbk model of error dynamics



k and b depend on the control law

Important concepts, symbols, and equations (cont.)

A more general p^{th}-order linear ODE:

$$a_p\theta_e^{(p)} + a_{p-1}\theta_e^{(p-1)} + \cdots + a_2\ddot{\theta}_e + a_1\dot{\theta}_e + a_0\theta_e = c$$ nonhomogenous

$$a_p\theta_e^{(p)} + a_{p-1}\theta_e^{(p-1)} + \cdots + a_2\ddot{\theta}_e + a_1\dot{\theta}_e + a_0\theta_e = 0$$ homogeneous

$$\theta_e^{(p)} + a'_{p-1}\theta_e^{(p-1)} + \cdots + a'_2\ddot{\theta}_e + a'_1\dot{\theta}_e + a'_0\theta_e = 0$$

$$\theta_e^{(p)} = -a'_{p-1}\theta_e^{(p-1)} - \cdots - a'_2\ddot{\theta}_e - a'_1\dot{\theta}_e - a'_0\theta_e$$

Important concepts, symbols, and equations (cont.)

Defining a state vector $x = (x_1, x_2, ..., x_p)$, you can write the p^{th}-order ODE as p first-order ODEs (a vector ODE).

$$
\begin{aligned}
x_1 &= \theta_e, \\
x_2 &= \dot{x}_1 = \dot{\theta}_e, \\
&\vdots \quad \vdots \\
x_p &= \dot{x}_{p-1} = \theta_e^{(p-1)}
\end{aligned}
$$

$$\dot{x}(t) = Ax(t) \rightarrow x(t) = e^{At}x(0)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_0' & -a_1' & -a_2' & \cdots & -a_{p-2}' & -a_{p-1}' \end{bmatrix} \in \mathbb{R}^{p \times p}$$

Important concepts, symbols, and equations (cont.)

$$\dot{x}(t) = Ax(t) \rightarrow x(t) = e^{At}x(0)$$

If $\mathrm{Re}(s) < 0$ for all eigenvalues s of A, then the error dynamics are **stable** (the error decays to zero).

The eigenvalues are the roots of the **characteristic equation**

$$\det(sI - A) = s^p + a'_{p-1}s^{p-1} + \cdots + a'_2 s^2 + a'_1 s + a'_0 = 0$$

the p^{th}-order ODE with s^k replacing $\theta^{(k)}$ for $k = 1..p$.

Necessary conditions for stability: each $a'_i > 0$.

These necessary conditions are also **sufficient** for first- and second-order systems.

Higher-order systems: use Routh-Hurwitz criterion.

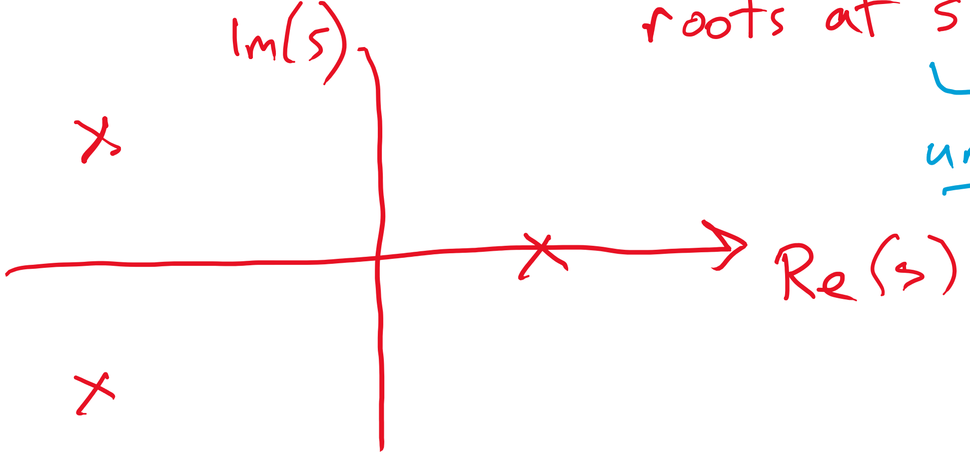
Types of control for the following tasks:

- Shaking hands with a human impedance + "soft" motion control?
- Erasing a whiteboard hybrid
- Spray painting motion
- Back massage hybrid
- Pushing an object across the floor with a mobile robot motion
- Opening a refrigerator door hybrid
- Inserting a peg in a hole impedance (compliance) + motion
- Polishing with a polishing wheel force, maybe
- Folding laundry motion

No single "correct" answer for most.

If the error dynamics characteristic equation is
$(s+3+2j)(s+3-2j)(s-2)=0$, does the error converge to zero?

roots at $\underbrace{s=2}_{\text{unstable}}$ and $s=-3\pm 2j$



Equivalent:

$$s^3+4s^2+s\underbrace{-26}_{\text{does not satisfy necessary condition}}=0$$

Note: if $x_1=$ error and $x=(x_1,x_2,x_3)$, then $\dot{x}=Ax$, where

$x_2=\dot{x}_1$

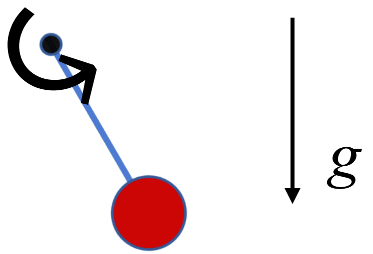
$x_3=\dot{x}_2$

$\dot{x}_3=26x_1-x_2-4x_3$

$$A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 26 & -1 & -4 \end{bmatrix}$$

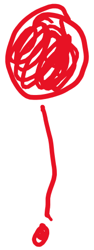
eigenvalues are $2,\ -3\pm 2j$

You can choose a control law to be a virtual spring, a virtual damper, a virtual spring plus damper, or nothing. Which of these could stabilize an actuated pendulum with viscous friction to the upright configuration? To a horizontal configuration? To the downward configuration? Describe the transient and steady-state error response for each.



spring : P control
damper : D control
both : PD

upright



Need P or PD

P has to overcome destabilizing "spring" of gravity.

Natural viscous damping provides stabilization, but D term can allow better trans response.
$e_{ss} = 0$

horizontal

Need P or PD. Will have $e_{ss} \neq 0$ if spring set point is exactly horizontal (due to gravity).

no controller is needed, but can improve trans response. $e_{ss} = 0$

downward