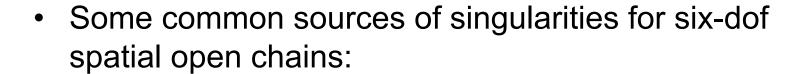
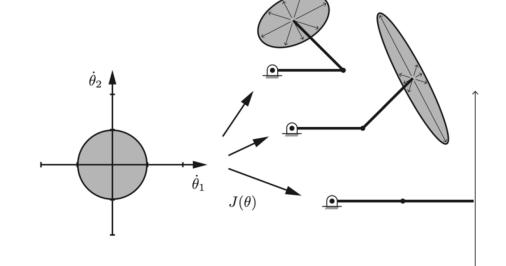
Chapter 2 Configuration Space
Chapter 3 Rigid-Body Motions
Chapter 4 Forward Kinematics
Chapter 5 Velocity Kinematics and Statics
5.1 Manipulator Jacobian
5.2 Statics of Open Chains
5.3 Singularity Analysis
5.4 Manipulability

Chapter 6 **Inverse Kinematics** Chapter 7 Kinematics of Closed Chains **Dynamics of Open Chains** Chapter 8 Chapter 9 **Trajectory Generation** Chapter 10 Motion Planning **Robot Control** Chapter 11 Chapter 12 Grasping and Manipulation Chapter 13 Wheeled Mobile Robots

- A configuration $\theta \in \mathbb{R}^n$ is **singular** if rank $(J(\theta))$ is less than its maximum rank over all θ .
- $rank(J_s(\theta)) = rank(J_b(\theta))$; singularities are independent of frame.

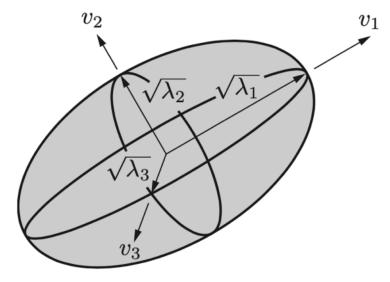


- i. two collinear revolute joint axes
- ii. three coplanar and parallel revolute joint axes
- iii. four revolute joint axes intersecting at a common point
- iv. four coplanar revolute joints
- v. six revolute joint axes intersecting a common line



- A fat or wide Jacobian (or matrix generally) has more columns than rows.
 A skinny or tall Jacobian has more rows than columns.
- A robot is **redundant** for a task if there exists a family of $\dot{\theta}$ satisfying $v = J(\theta) \dot{\theta}$ at most θ , where v could be a twist v or a coordinate velocity. (wide Jacobians)
- A robot is **kinematically deficient** for a task if there are required velocities v for which there is no $\dot{\theta}$ satisfying $v = J(\theta) \dot{\theta}$ at most θ . (tall Jacobians)
- Redundancy and deficiency depend on the robot and the task (e.g., dimensions of the Jacobian); singularities depend on θ .

- The manipulability of a robot at a configuration θ measures how close it is to being singular.
- The manipulability ellipsoid defines the ease of motion in different directions.
- The principal axes of the ellipsoid are given by the eigenvectors and square roots of the eigenvalues of the square matrix JJ^{T} .
- Manipulability measures include
 - 1. the ratio of the largest to smallest principal axis half-lengths
 - 2. the ratio of the largest to smallest eigenvalues
 - 3. the volume of the ellipsoid (proportional to $\sqrt{\lambda_1 \lambda_2 \dots}$).



 Split the body Jacobian into angular and linear components to to get angular velocity and linear velocity ellipsoids.

$$J_b(\theta) = \begin{bmatrix} J_{b\omega}(\theta) \\ J_{bv}(\theta) \end{bmatrix} \in \mathbb{R}^{6 \times n}$$

$$J_{b\omega}(\theta) \in \mathbb{R}^{3 \times n} \to \text{angular velocity/moment ellipsoids}$$

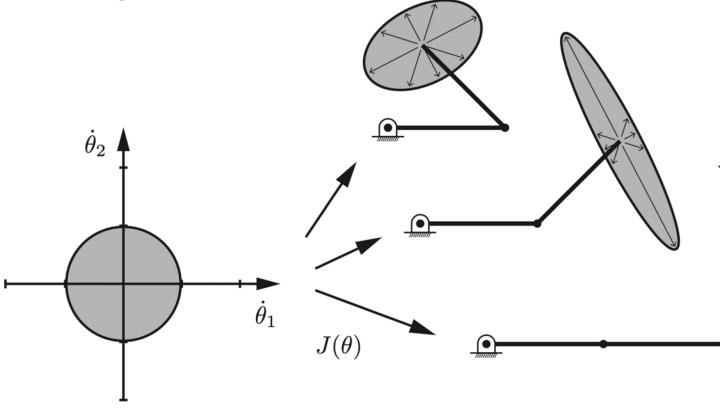
$$J_{bv}(\theta) \in \mathbb{R}^{3 \times n} \to \text{linear velocity/force ellipsoids}$$

Modern Robotics, Lynch and Park, Cambridge University Press

- The principal axes of the force (wrench) ellipsoid are given by the eigenvectors and square roots of the eigenvalues of $(JJ^{T})^{-1}$.
- The force ellipsoid has the same principal axes as the manipulability ellipsoid, and the principal axis half-lengths are the reciprocal of those of the manipulability ellipsoid.
- The product of the volumes of the force and manipulability ellipsoids is constant over θ .

Modern Robotics, Lynch and Park, Cambridge University Press

Approximately draw the corresponding force ellipsoids at the three configurations.

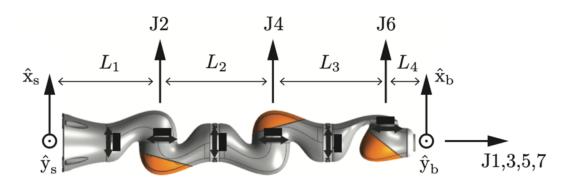


Why use J_b instead of J_s for manipulability ellipsoids?



Adept SCARA RRRP robot





KUKA LBR iiwa 7R robot

If using a twist to represent the e-e velocity, what are the dimensions of each Jacobian?

For the task of manipulating a rigid body, which of these is redundant or deficient?

What is the rank of the Jacobian of the iiwa at its home configuration (shown)?