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## Important concepts, symbols, and equations

### Newton-Euler recursive inverse dynamics

Find  $\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) + J^T(\theta)\mathcal{F}_{\text{tip}}$

efficiently and numerically, without closed-form expressions or differentiation.

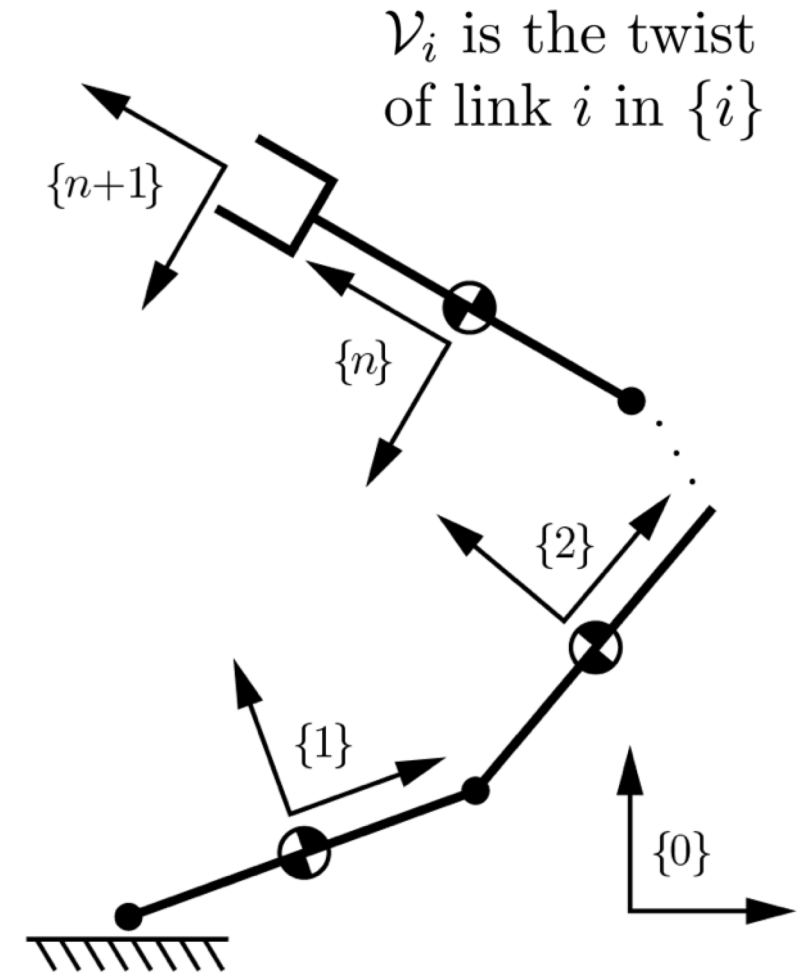
$\mathcal{G}_i$  : spatial inertia matrix of link  $\{i\}$  in  $\{i\}$

$M_{i,i-1}$  :  $\{i-1\}$  in  $\{i\}$  when  $\theta_i = 0$

$\mathcal{A}_i$  : screw axis of joint  $i$  in  $\{i\}$

$\mathcal{F}_{n+1}$  : wrench  $\mathcal{F}_{\text{tip}}$  applied by end-effector

$$\dot{\mathcal{V}}_0 = (\dot{\omega}_0, \dot{v}_0) = (0, -\mathbf{g})$$



## Important concepts, symbols, and equations (cont.)

### Forward iterations

Given  $\theta, \dot{\theta}, \ddot{\theta}$ , for  $i = 1$  to  $n$  do:

$$T_{i,i-1} = e^{-[\mathcal{A}_i]\theta_i} M_{i,i-1}$$

$$\mathcal{V}_i = [\text{Ad}_{T_{i,i-1}}] \mathcal{V}_{i-1} + \mathcal{A}_i \dot{\theta}_i$$

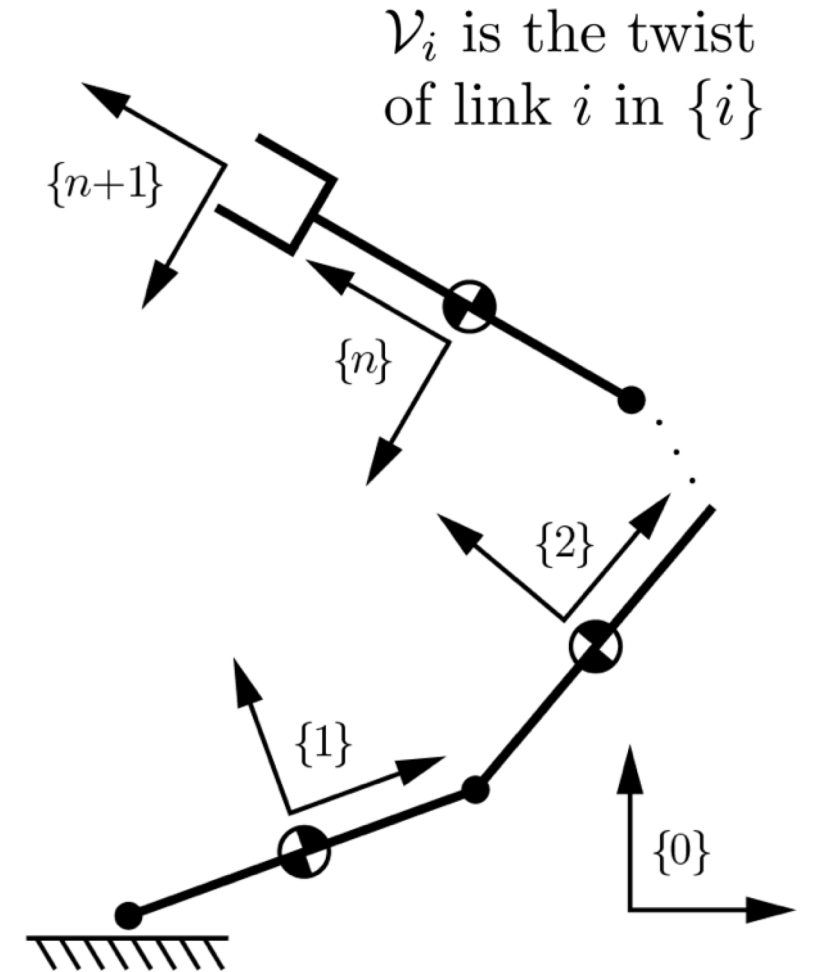
$$\dot{\mathcal{V}}_i = [\text{Ad}_{T_{i,i-1}}] \dot{\mathcal{V}}_{i-1} + [\text{ad}_{\mathcal{V}_i}] \mathcal{A}_i \dot{\theta}_i + \mathcal{A}_i \ddot{\theta}_i$$

### Backward iterations

For  $i = n$  to 1 do:

$$\mathcal{F}_i = [\text{Ad}_{T_{i+1,i}}]^T \mathcal{F}_{i+1} + \mathcal{G}_i \dot{\mathcal{V}}_i - [\text{ad}_{\mathcal{V}_i}]^T \mathcal{G}_i \mathcal{V}_i$$

$$\tau_i = \mathcal{F}_i^T \mathcal{A}_i$$



## Important concepts, symbols, and equations (cont.)

Forward dynamics: Solve  $M(\theta)\ddot{\theta} = \tau - c(\theta, \dot{\theta}) - g(\theta) - J^T(\theta)\mathcal{F}_{\text{tip}}$   
for  $\ddot{\theta}$ .

Use  $n + 1$  calls of N-E inverse dynamics to get

- $c(\theta, \dot{\theta}) + g(\theta) + J^T(\theta)\mathcal{F}_{\text{tip}}$  by setting  $\ddot{\theta} = 0$
- $M(\theta) = [M_1(\theta) \ \cdots \ M_n(\theta)]$ , where  $\tau = M_i(\theta)$  if  $\ddot{\theta}_i = 1$ ,  $\ddot{\theta}_j = 0$  for all  $j \neq i$ ,  $\dot{\theta} = 0$ ,  $\mathfrak{g} = 0$ , and  $\mathcal{F}_{\text{tip}} = 0$ .

Use any efficient algorithm to solve  $M\ddot{\theta} = b$  for  $\ddot{\theta}$ .

## Important concepts, symbols, and equations (cont.)

Euler integration for simulation:

$$\begin{aligned}\ddot{\theta}[k] &= \textit{ForwardDynamics}(\theta[k], \dot{\theta}[k], \tau(k\delta t), \mathcal{F}_{\text{tip}}(k\delta t)) \\ \theta[k+1] &= \theta[k] + \dot{\theta}[k]\delta t \\ \dot{\theta}[k+1] &= \dot{\theta}[k] + \ddot{\theta}[k]\delta t\end{aligned}$$

Could add a second-order correction to the position calculation.

Explain each term in the equations below.

## Forward iterations

Given  $\theta, \dot{\theta}, \ddot{\theta}$ , for  $i = 1$  to  $n$  do:

$$T_{i,i-1} = e^{-[\mathcal{A}_i]\theta_i} M_{i,i-1}$$

$$\mathcal{V}_i = [\text{Ad}_{T_{i,i-1}}] \mathcal{V}_{i-1} + \mathcal{A}_i \dot{\theta}_i$$

$$\dot{\mathcal{V}}_i = [\text{Ad}_{T_{i,i-1}}] \dot{\mathcal{V}}_{i-1} + [\text{ad}_{\mathcal{V}_i}] \mathcal{A}_i \dot{\theta}_i + \mathcal{A}_i \ddot{\theta}_i$$

## Backward iterations

For  $i = n$  to 1 do:

$$\mathcal{F}_i = [\text{Ad}_{T_{i+1,i}}]^T \mathcal{F}_{i+1} + \mathcal{G}_i \dot{\mathcal{V}}_i - [\text{ad}_{\mathcal{V}_i}]^T \mathcal{G}_i \mathcal{V}_i$$

$$\tau_i = \mathcal{F}_i^T \mathcal{A}_i$$

