Important concepts, symbols, and equations

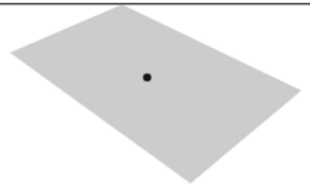
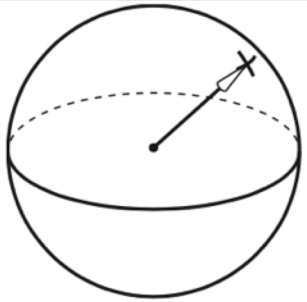
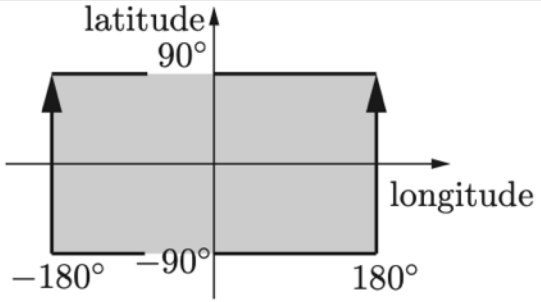
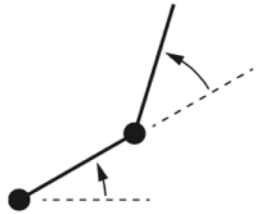
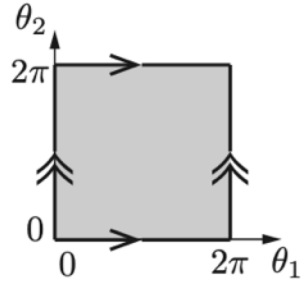
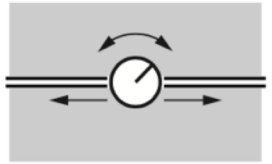
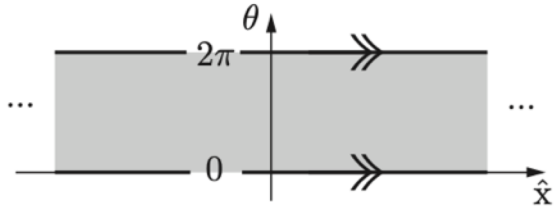
- Two C-spaces may have the same dof but differ in other ways. The **topology** ("shape") of a space is independent of how we **represent** it.

- Two spaces are **topologically equivalent** if one can be continuously deformed to the other without cutting or pasting.

- Some spaces are **Cartesian products** of spaces of lower dimension, e.g.,

(1d) $\mathbb{E}$, $S = T$ **(2d)** $\mathbb{E} \times \mathbb{E} = \mathbb{E}^2$, $S \times S = T^2$, S^2, $\mathbb{E} \times S$ **(higher)** $\mathbb{E}^k \times S^m \times T^n$

- Represent Euclidean ("flat") spaces $\mathbb{E}^n$ as $\mathbb{R}^n$. For curved spaces, choose
  - minimum-parameter **explicit parameterizations** (choose between **singularities** or an **atlas** of **coordinate charts**), OR
  - **implicit representation** (use more numbers subject to constraints).

system	topology	sample representation
 point on a plane	 $\mathbb{E}^2$	 $\mathbb{R}^2$
 spherical pendulum	 S^2	 $[-180°, 180°) \times [-90°, 90°]$
 2R robot arm	 $T^2 = S^1 \times S^1$	 $[0, 2\pi) \times [0, 2\pi)$
 rotating sliding knob	 $\mathbb{E}^1 \times S^1$	 $\mathbb{R}^1 \times [0, 2\pi)$

Any value in an atlas of coordinate charts?
An implicit representation?

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hexrotor with two 5-DOF arms

https://www.prodrone.com/archives/1420/

C-space topology, with and without arm joint limits, rotor angles? Implicit/explicit representations? Grübler's formula?

body: $\mathbb{R}^3 \times S^2 \times S^1$

rotors: $S^1 \times S^1 \ldots = T^6$ (6 times)

arm: T^5 (no joint limits)

$\mathbb{R}^3 \times S^2 \times S^1 \times T^6 \times T^5 \times T^5 =$

$\mathbb{R}^3 \times T^{17} \times S^2$

each arm joint $\in (0, \pi)$

$\mathbb{R}^3 \times S^2 \times S^1 \times T^6 \times \mathbb{R}^{10} \simeq \mathbb{R}^3 \times S^2 \times T^7$

KUKA youBot
mecanum-wheel omnidirectional base
moving on flat ground
plus 5-DOF robot arm + gripper

C-space topology and representation?
Include gripper, wheel angles?

$\mathbb{R}^2 \times S^1 \times \mathbb{R}^5 \times T^4$

chassis arm wheels

or, if joint limits

$[a, b]^5$

I^5