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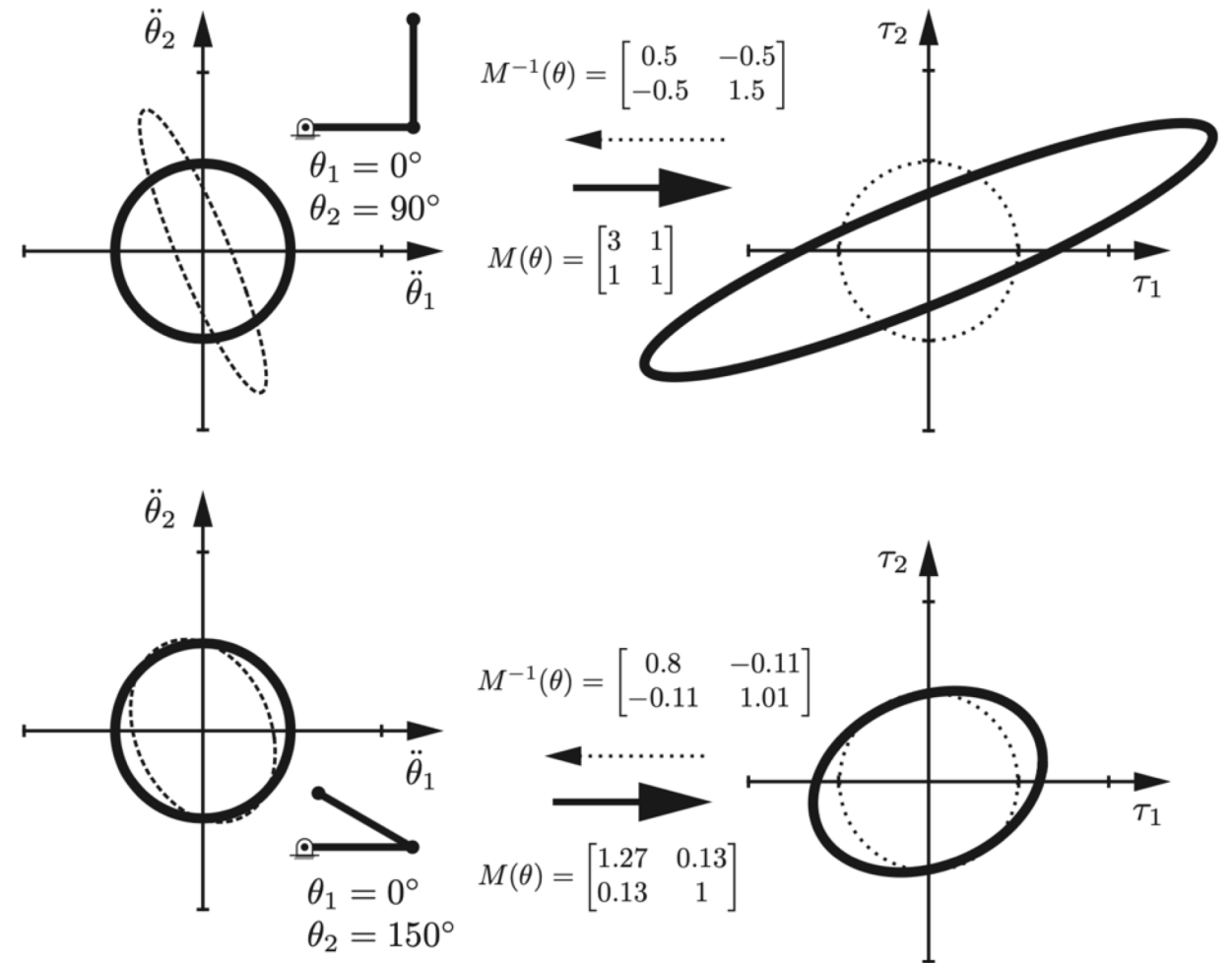
Important concepts, symbols, and equations

$$\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta)$$

kinetic energy of a robot:

$$\mathcal{K}(\theta, \dot{\theta}) = \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta}$$

When $\dot{\theta} = 0$ and $g = 0$,
 $M(\theta)$ maps $\ddot{\theta}$ to τ and
 $M^{-1}(\theta)$ maps τ to $\ddot{\theta}$



Important concepts, symbols, and equations (cont.)

If $V = J(\theta) \dot{\theta}$ is the e-e velocity and J is invertible (there exists a unique joint velocity for each e-e velocity):

$$\frac{1}{2} V^T \Lambda(\theta) V = \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta}$$

$$\cancel{\dot{\theta}^T} J^T(\theta) \Lambda(\theta) J(\theta) \cancel{\dot{\theta}} = \cancel{\dot{\theta}^T} M(\theta) \cancel{\dot{\theta}}$$

$$\Lambda(\theta) = J^{-T}(\theta) M(\theta) J^{-1}(\theta)$$

end-effector mass matrix

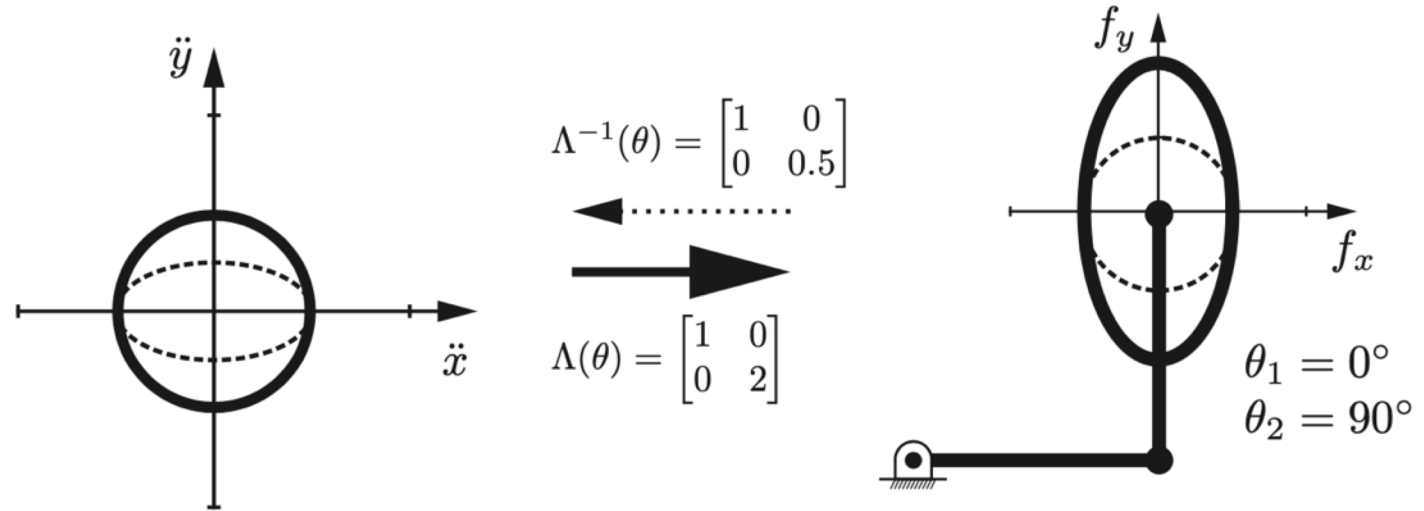
There are some e-e velocities that cannot be achieved by the joints: "Infinitely massive" in these directions.

What if J is tall? wide?

An infinite # of $\dot{\theta}$ correspond to the same \dot{V} .

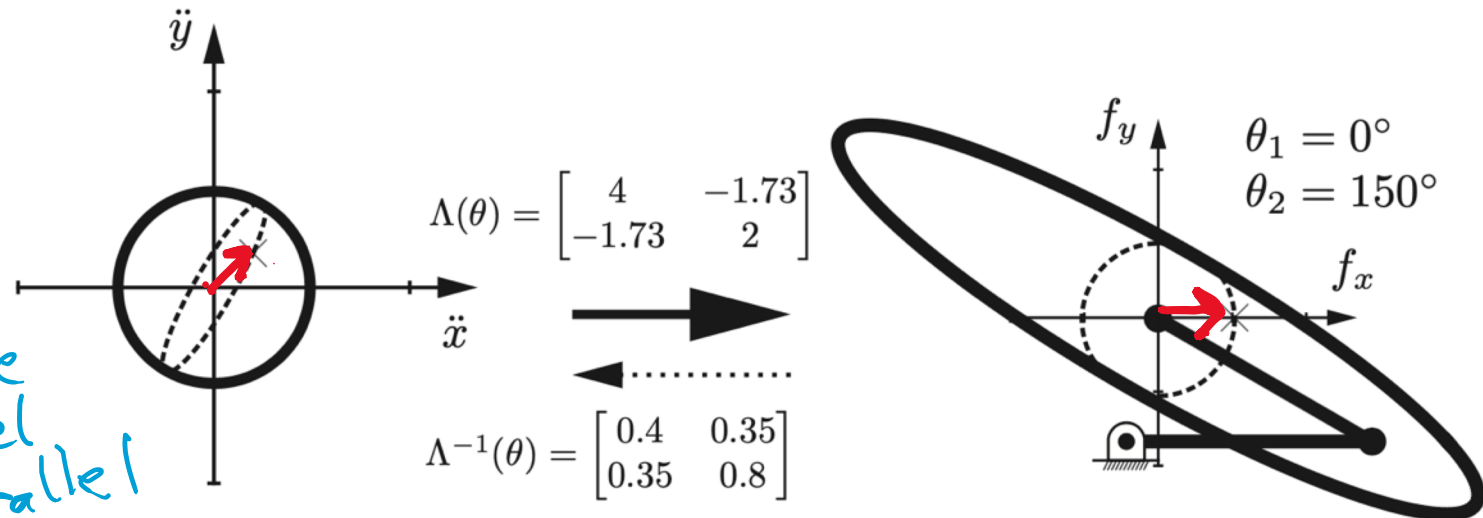
Important concepts, symbols, and equations (cont.)

When $\dot{\theta} = 0$ and $g = 0$,
 $\Lambda(\theta)$ maps \dot{V} to F and
 $\Lambda^{-1}(\theta)$ maps F to \dot{V}



Force and acceleration
are only parallel along
principal axes.

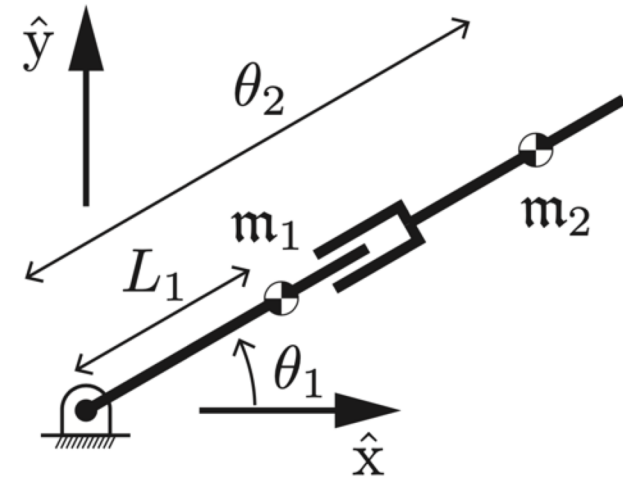
*example where
force and accel
are not parallel*



RP robot

$$\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta)$$

$$M(\theta) = \begin{bmatrix} \mathcal{I}_1 + \mathcal{I}_2 + m_1 L_1^2 + m_2 \theta_2^2 & 0 \\ 0 & m_2 \end{bmatrix}$$



$\dot{\theta} = 0$ and $g = 0$

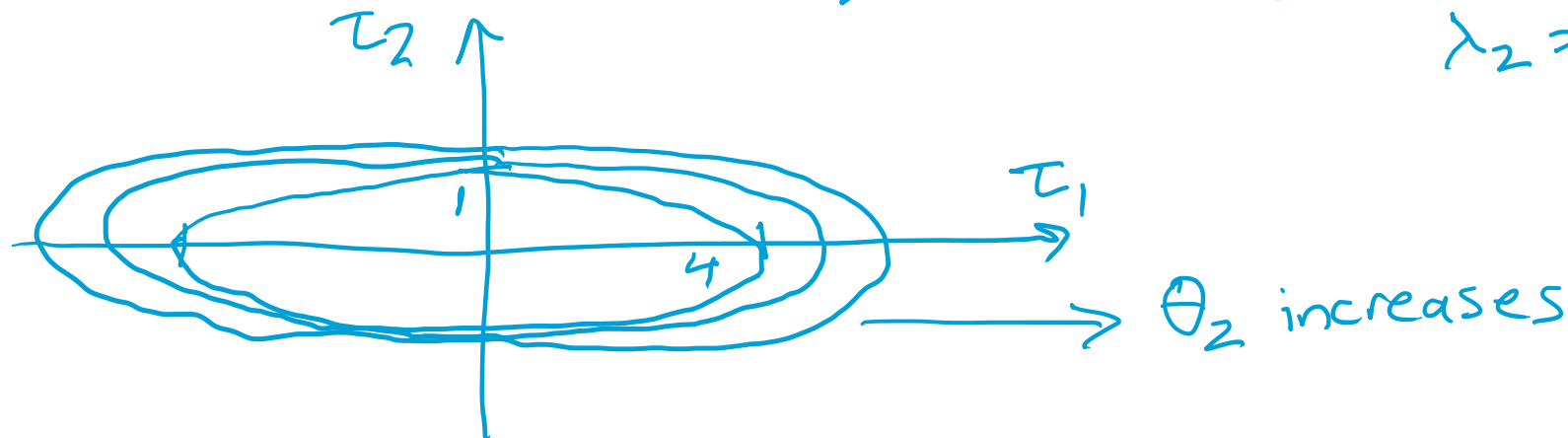
What are the e-vals and e-vecs of M ?

Draw the ellipse of τ corresponding to a unit circle of $\ddot{\theta}$ as θ_2 increases from zero and $I_1 = I_2 = m_1 = m_2 = L_1 = 1$.

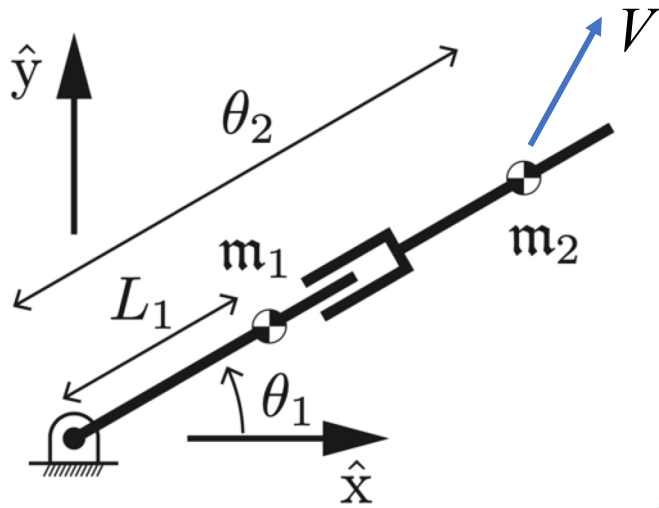
$$M(\theta) = \begin{bmatrix} 3 + \theta_2^2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\lambda_1 = 3 + \theta_2^2, v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

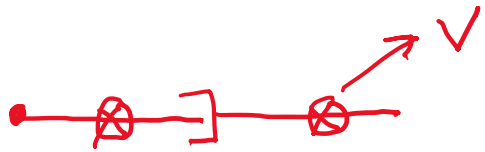
$$\lambda_2 = 1, v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



RP robot



$\dot{\theta} = 0$ and $g = 0$



At $\theta_1 = 0$, the e-e mass matrix is

$$\Lambda(\theta) = \begin{bmatrix} m_2 & 0 \\ 0 & (I_1 + I_2 + m_1 L_1^2 + m_2 \theta_2^2) / \theta_2^2 \end{bmatrix}$$

Draw the ellipse of F corresponding to a unit circle of \dot{V} as θ_2 increases from zero and $I_1 = I_2 = m_1 = m_2 = L_1 = 1$. How does it change as θ_1 changes?

$$F = \Lambda(\theta) \dot{V}$$

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & (3 + \theta_2^2) / \theta_2^2 \end{bmatrix}$$

if $\theta_2 = \infty$: $\lambda_1 = 1, v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
 $\lambda_2 = 1, v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

if $\theta_2 = 0$: $\lambda_1 = 1, v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
 $\lambda_2 = \infty, v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

As θ_1 rotates,
so do the force ellipses.

