

Chapter 2	Configuration Space
	2.1 DOF of a Rigid Body
	2.2 DOF of a Robot
	2.3 C-space Topology and Representation
	2.4 Configuration and Velocity Constraints
	2.5 Task Space and Workspace
Chapter 3	Rigid-Body Motions
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Chapter 5	Velocity Kinematics and Statics
Chapter 6	Inverse Kinematics
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Chapter 9	Trajectory Generation
Chapter 10	Motion Planning
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Chapter 13	Wheeled Mobile Robots

Important concepts, symbols, and equations

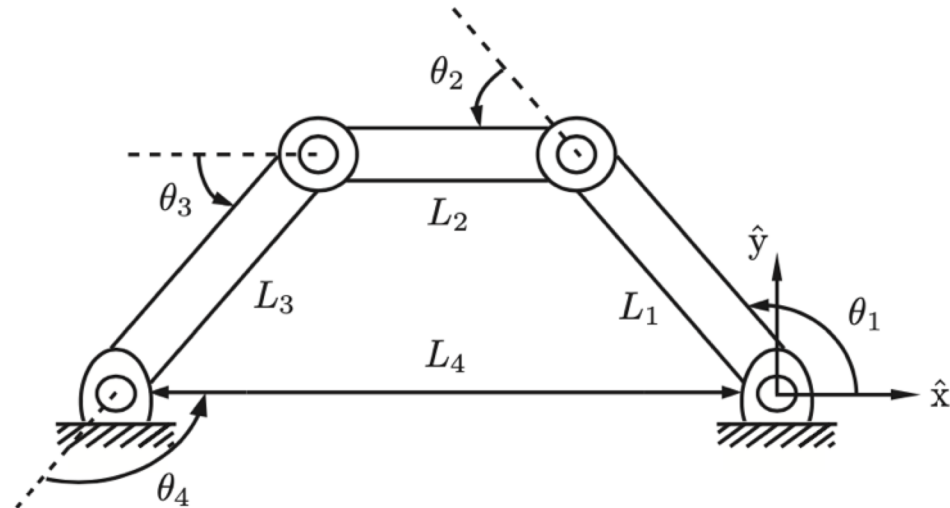
- k independent **holonomic constraints** on $(\theta_1, \dots, \theta_n)$ reduce an n -dim C-space to $n-k$ dof.

$$g(\theta) = \begin{bmatrix} g_1(\theta_1, \dots, \theta_n) \\ \vdots \\ g_k(\theta_1, \dots, \theta_n) \end{bmatrix} = 0$$

- Pfaffian constraints are constraints on velocity: $A(\theta)\dot{\theta} = 0$
- If velocity constraints can be integrated to equivalent configuration constraints, they are holonomic. If not, they are **nonholonomic**: they reduce the dimension of the feasible velocities, but not the dimension of the C-space.
- Determining if constraints are holonomic or nonholonomic is sometimes difficult (Chapter 13).

Important concepts, symbols, and equations (cont.)

- The **task space** is the space in which a task is most naturally represented. It is independent of a robot.
- The **workspace** is usually a specification of the reachable space by a robot (or its wrist, or end-effector).
 - Often defined in terms of (x,y,z) translational positions only.
 - Sometimes the **dexterous workspace** is the set of translational positions that can be reached with arbitrary orientation.



3R planar robot has its endpoint pinned by a revolute joint, making a four-bar linkage.

dof?

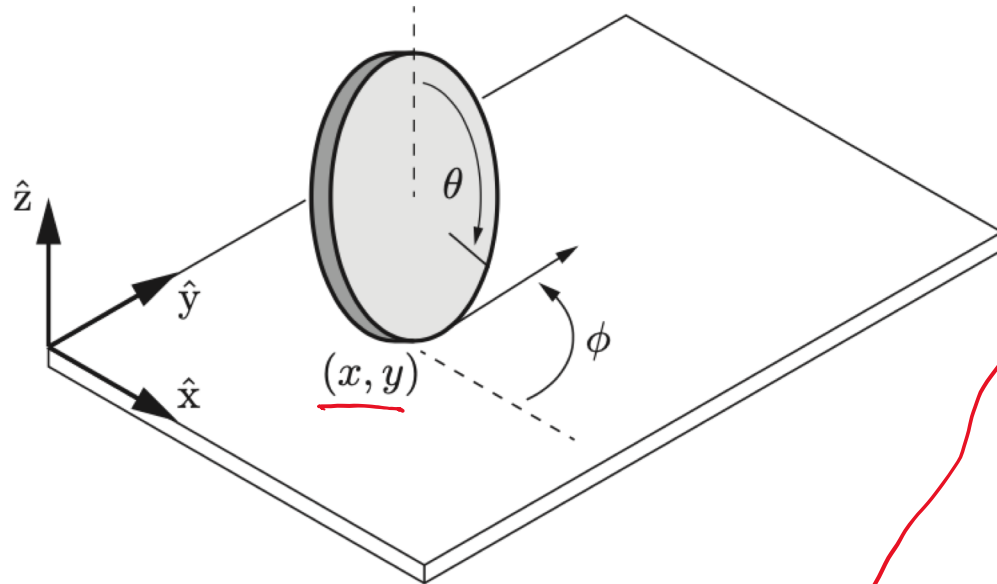
What does the C-space look like embedded in $(\theta_1, \theta_2, \theta_3, \theta_4)$?

What could be an explicit parameterization?

$$\begin{aligned} L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) + \cdots + L_4 \cos(\theta_1 + \cdots + \theta_4) &= 0, \\ L_1 \sin \theta_1 + L_2 \sin(\theta_1 + \theta_2) + \cdots + L_4 \sin(\theta_1 + \cdots + \theta_4) &= 0, \\ \theta_1 + \theta_2 + \theta_3 + \theta_4 - 2\pi &= 0. \end{aligned}$$

“loop-closure” equations

disk rolling upright on a plane



$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = r\dot{\theta} \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix}$$

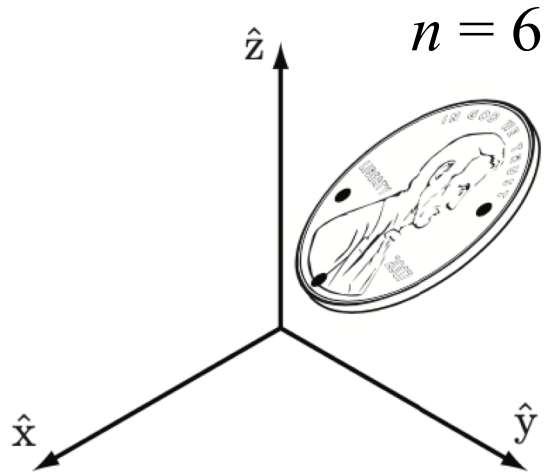
$$q = [q_1 \ q_2 \ q_3 \ q_4]^T = [x \ y \ \phi \ \theta]^T$$

$$\begin{bmatrix} 1 & 0 & 0 & -r \cos q_3 \\ 0 & 1 & 0 & -r \sin q_3 \end{bmatrix} \dot{q} = 0$$

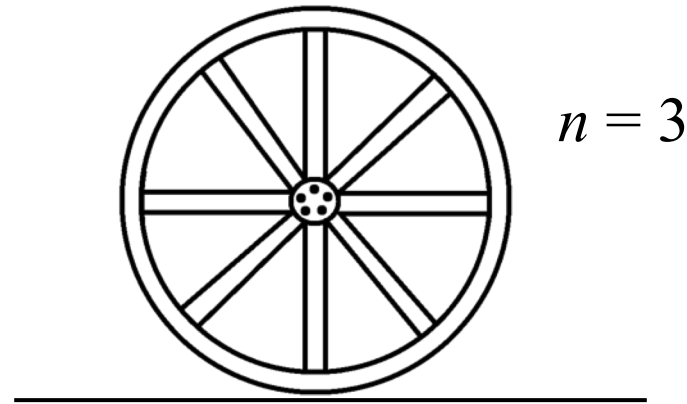
$$A(q)\dot{q} = 0, \ A(q) \in \mathbb{R}^{2 \times 4}$$

nonholonomic constraints

starting with n dof, add k holonomic constraints, m nonholonomic constraints

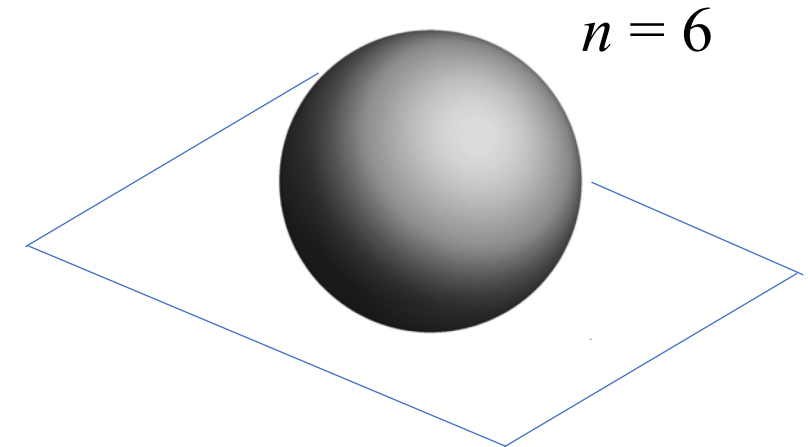


- a coin constrained to stand upright on a plane
- a coin constrained to roll upright on a plane

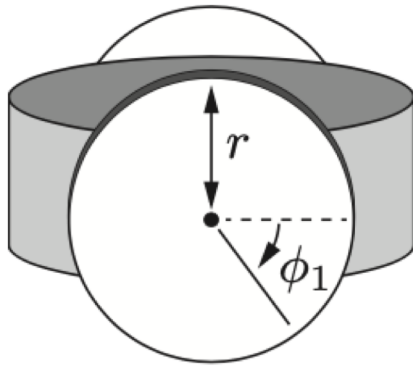


a wheel rolling on a line
in the plane of the page

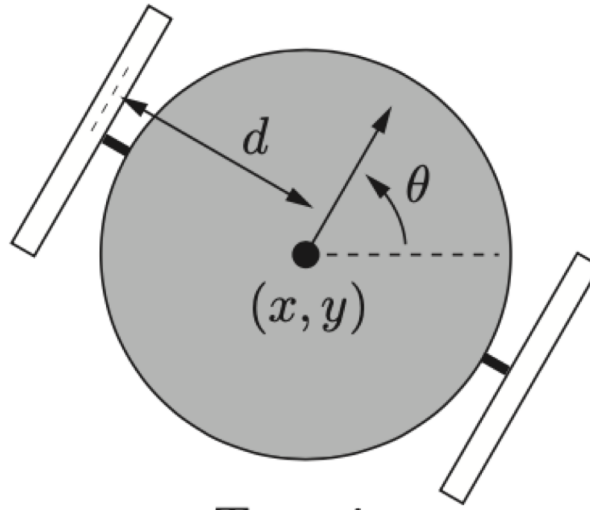
1 holonomic
constraint
($z = 0$)
2nd holonomic constraint
due to rolling



- a sphere touching a plane
1 hol const
- a sphere rolling on a plane
+ 2 nonhol constraints



Side view



Top view

diff-drive mobile robot,
rolling without slipping

$$q = (x, y, \theta, \phi_1, \phi_2)$$

How many holonomic constraints k and nonholonomic constraints m ?

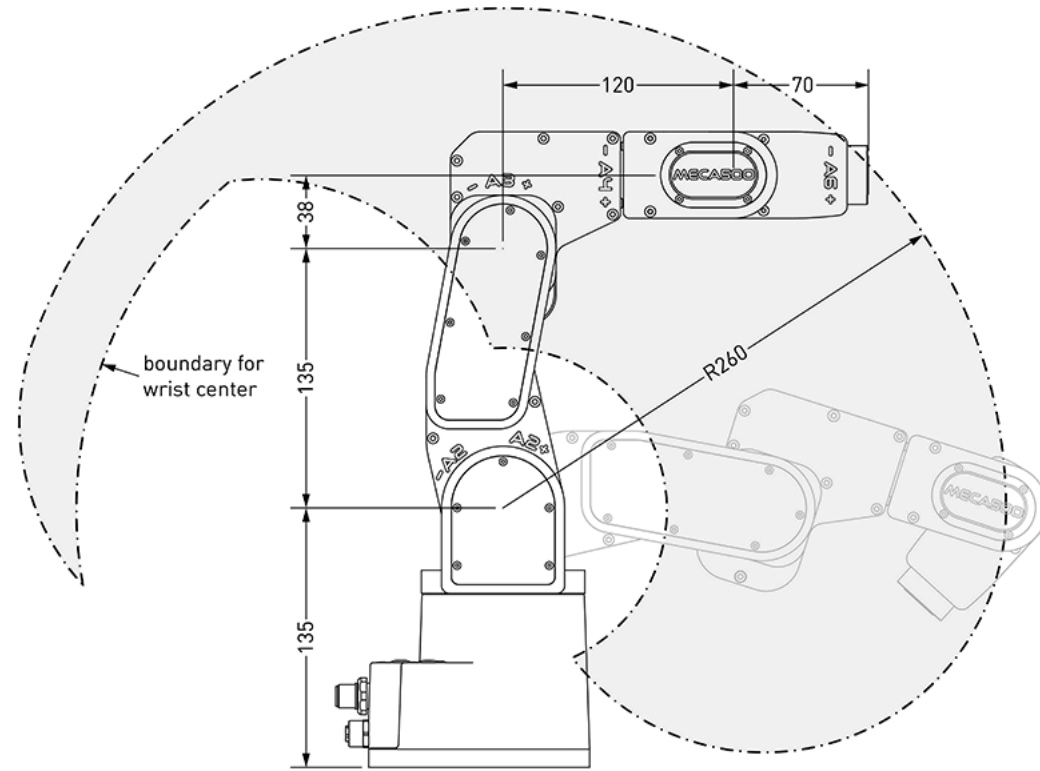
2 controls: spinning velocity of wheels

feasible velocities are 2-dimensional

3 velocity constraints

2 are nonholonomic, 1 is holonomic

reachable C-space
is 4d



A slice of a position-only workspace for a typical 6R robot (here, the Meca500)

Task spaces for:

- manipulating a rigid object?

6 dof

- operating a laser pointer?

5 dof

- carrying a tray of glasses to keep them vertical?

4 dof planar body + height