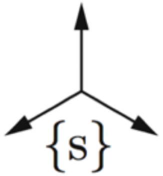
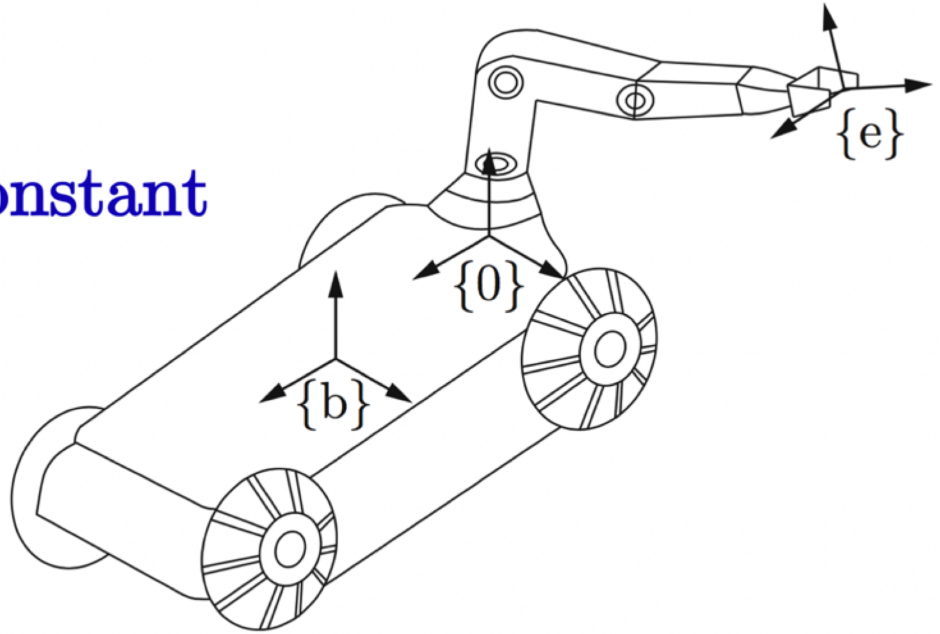
# Important concepts, symbols, and equations

$$T_{sb}(q) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & x \\ \sin\phi & \cos\phi & 0 & y \\ 0 & 0 & 1 & \boxed{z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

constant




$$X(q,\theta) = T_{se}(q,\theta) = T_{sb}(q)\, T_{b0}\, T_{0e}(\theta)$$

# <u>Important concepts, symbols, and equations</u> (cont.)

## Mobile manipulator Jacobian $J_e(\theta)$ (<u>not</u> a function of $q$)

$$\boxed{\mathcal{V}_e = \underbrace{J_e(\theta)}_{6\times(m+n)} \begin{bmatrix} u \\ \dot{\theta} \end{bmatrix} = \overbrace{[\underbrace{J_{\text{base}}(\theta)}_{6\times m} \; \underbrace{J_{\text{arm}}(\theta)}_{6\times n}]}^{J_b(\theta)} \begin{bmatrix} u \\ \dot{\theta} \end{bmatrix}}$$

$$\mathcal{V}_b = Fu, \; F \in \mathbb{R}^{3\times m}$$ (from odometry)

$$F_6 = \begin{bmatrix} 0_m \\ 0_m \\ F \\ 0_m \end{bmatrix} \in \mathbb{R}^{6\times m}$$

$$[\mathrm{Ad}_{T_{eb}(\theta)}]\mathcal{V}_{b6} = \overbrace{\boxed{[\mathrm{Ad}_{T_{0e}^{-1}(\theta)T_{b0}^{-1}}]F_6}}^{J_{\text{base}}(\theta)}u$$

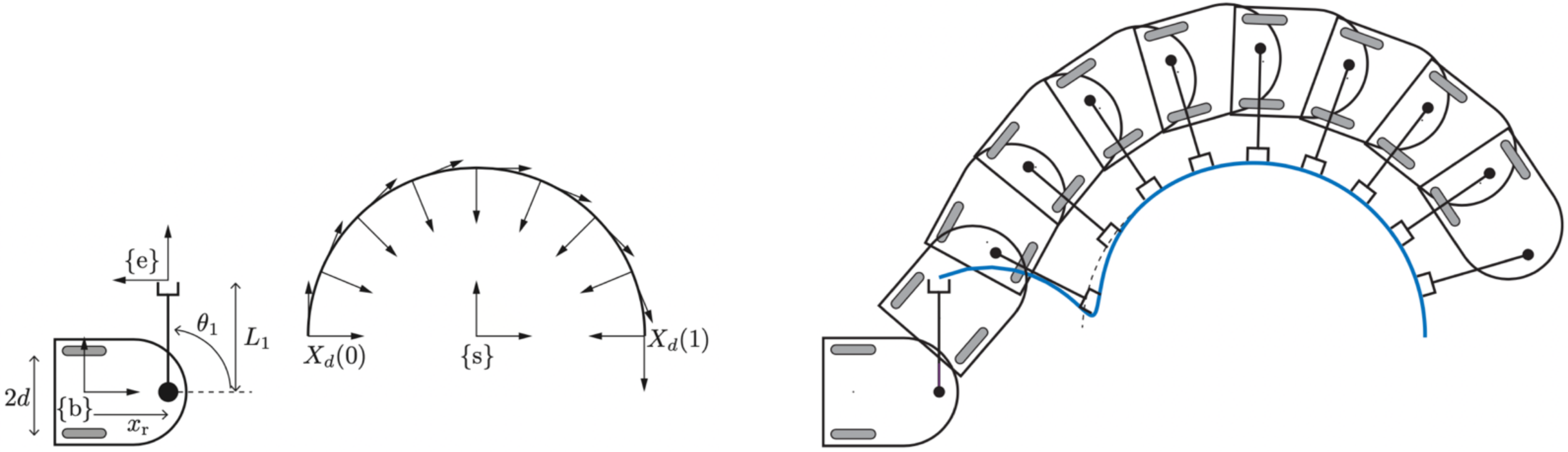
$$\mathcal{V}_{b6} = F_6 u$$

# <u>Important concepts, symbols, and equations</u> (cont.)

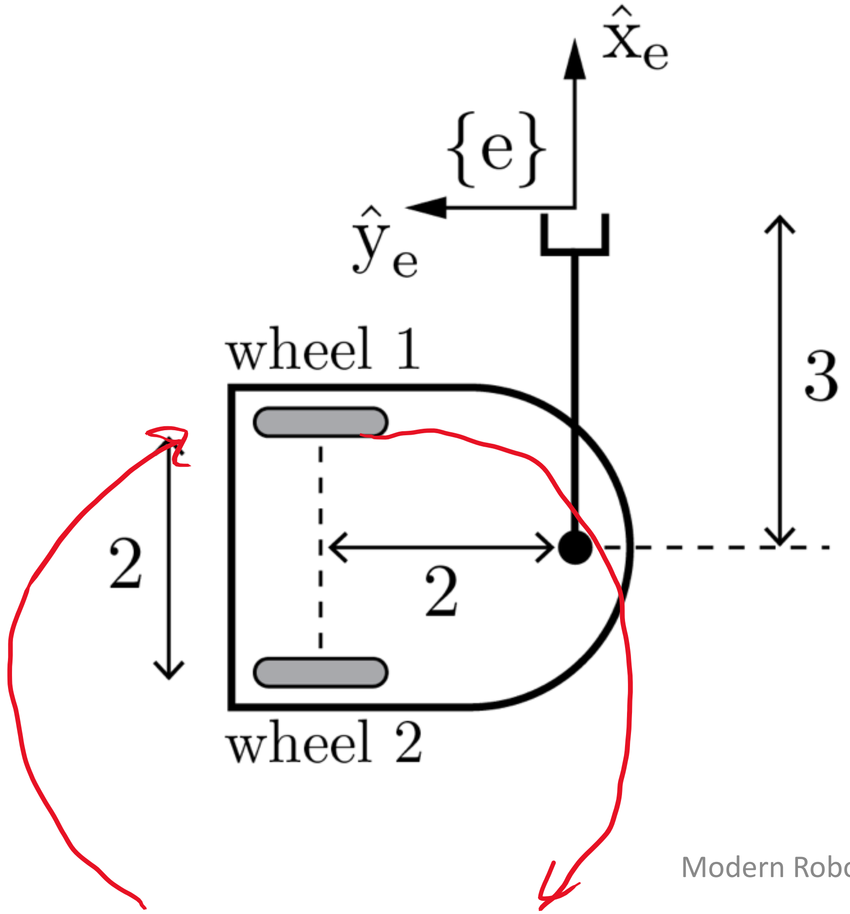
## Task-space feedforward + PI feedback control

$$\mathcal{V}_e(t) = [\mathrm{Ad}_{X^{-1}X_d}]\mathcal{V}_d(t) + K_p X_{\mathrm{err}}(t) + K_i \int_0^t X_{\mathrm{err}}(\mathrm{t})\, dt$$

$$\begin{bmatrix} u \\ \dot{\theta} \end{bmatrix} = J_e^{\dagger}(\theta)\mathcal{V}_e \qquad\qquad [X_{\mathrm{err}}] = \log(X^{-1}X_d)$$



The planar mobile manipulator below has a diff-drive mobile base and a 1R robot arm. Each wheel radius is 0.5. The positive driving direction for each wheel moves the robot forward, and positive rotation for the arm joint is about an axis out of the page. At the configuration below, give the Jacobian $J_e$.




When wheel 1 rotates forward with wheel 2 stationary, wheel 1 rolls in a circle about wheel 2 of circumference $C = 2\pi(2) = 4\pi$.

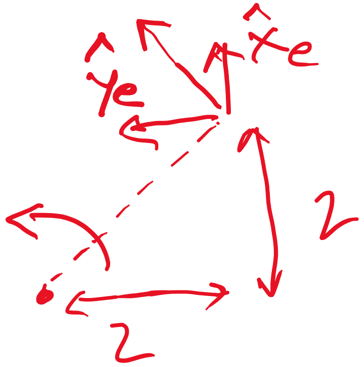
If $\dot{\theta}_1$ for wheel 1 is 1, then the linear velocity at wheel 1 is 0.5. So driving wheel 1 at 1 rad/s follows the circle at $2\pi\left(\frac{0.5}{4\pi}\right)$ rad/s $= 0.25$ rad/s.

Similarly, $\dot{\theta}_2 = 1$ for wheel 2 gives 0.25 rad/s about wheel 1.

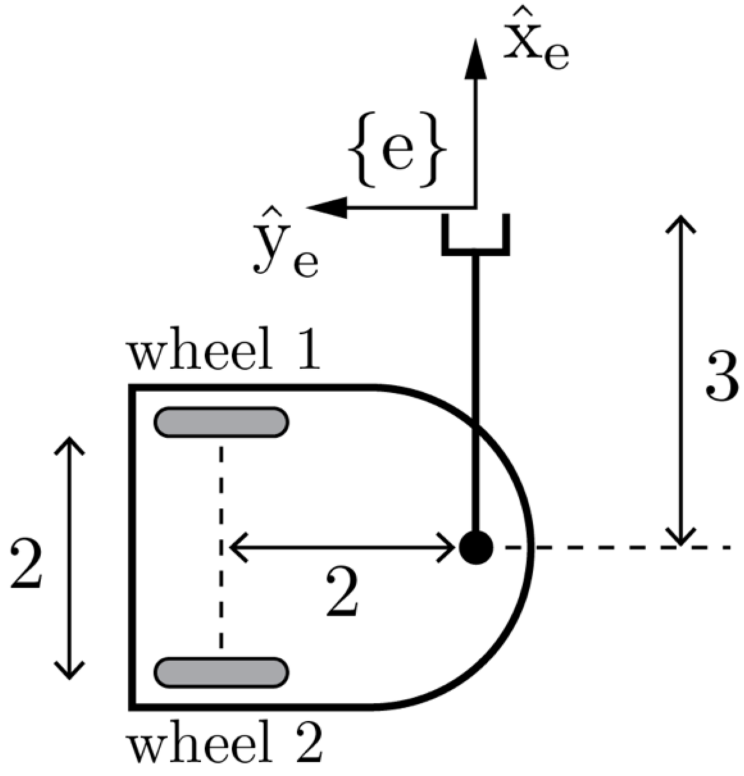
Wheel 1:


$$0.25\begin{bmatrix}-1\\-2\\-4\end{bmatrix} = \begin{bmatrix}-0.25\\-0.5\\-1\end{bmatrix} = \begin{bmatrix}\omega_{ez}\\v_{ex}\\v_{ey}\end{bmatrix}$$

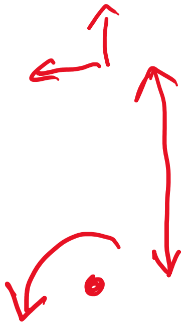
Wheel 2:




$$0.25\begin{bmatrix}1\\2\\2\end{bmatrix} = \begin{bmatrix}0.25\\0.5\\0.5\end{bmatrix} = \begin{bmatrix}\omega_{ez}\\v_{ex}\\v_{ey}\end{bmatrix}$$




arm joint:

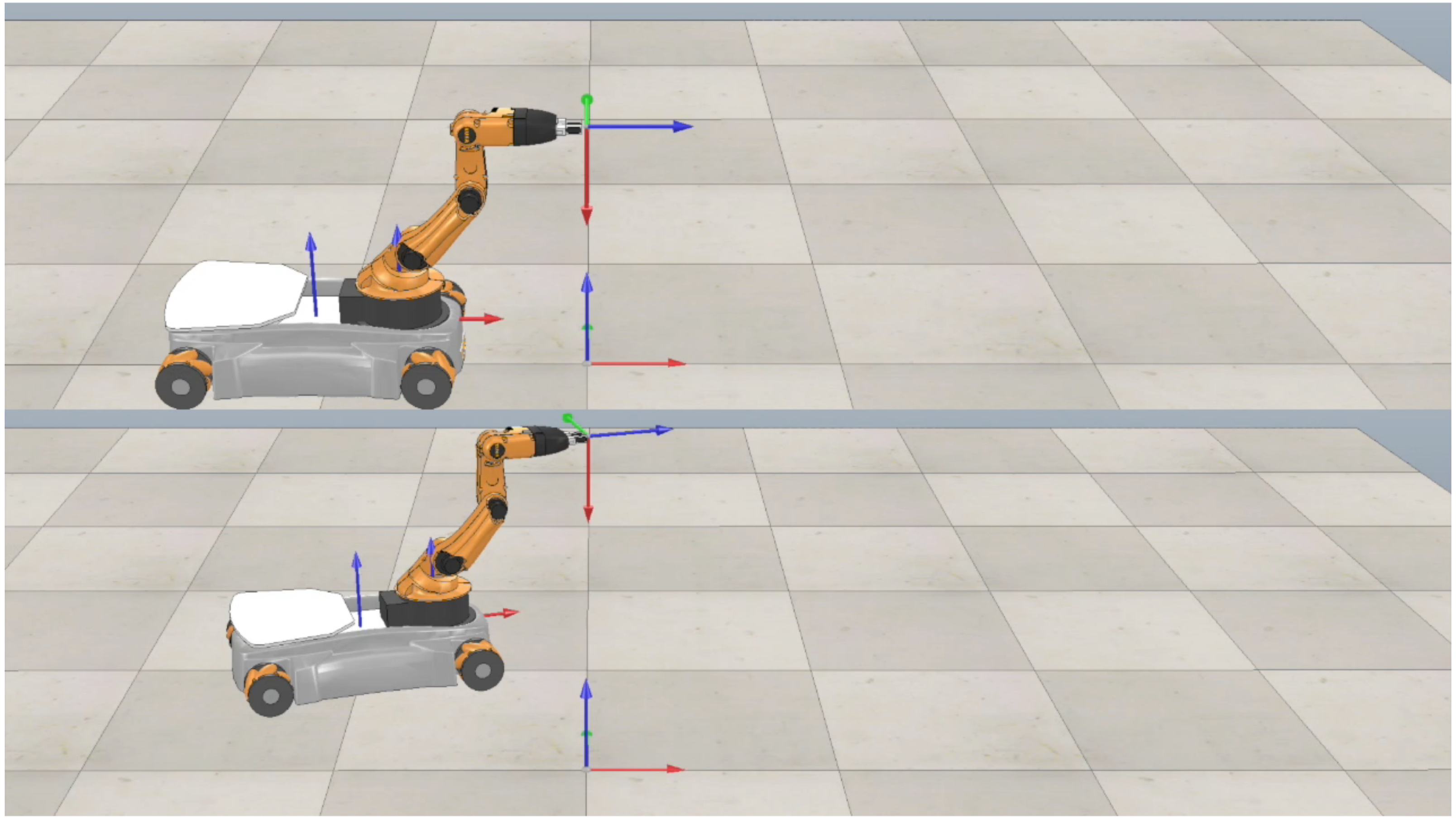



$$\begin{bmatrix}1\\0\\3\end{bmatrix} = \begin{bmatrix}\omega_{ez}\\v_{ex}\\v_{ey}\end{bmatrix}$$

so $J_e = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\-0.25 & 0.25 & 1\\-0.5 & 0.5 & 0\\-1 & 0.5 & 3\\0 & 0 & 0\end{bmatrix}$

(a) Why does the arm stretch out before the wheels move?
(b) Would anything change if the wheels had a larger radius?

$$\begin{bmatrix} u \\ \dot{\theta} \end{bmatrix} = J_e^{\dagger}(\theta)\mathcal{V}_e$$



(a) pinv finds solution minimizing the 2-norm, and the arm joints are more effective than the wheels, initially, at moving the hand along the trajectory.

(b) Larger wheels could make them more effective.