

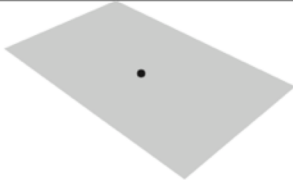

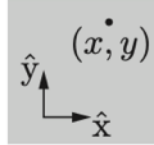
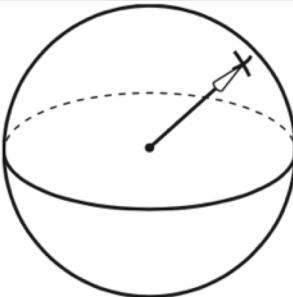

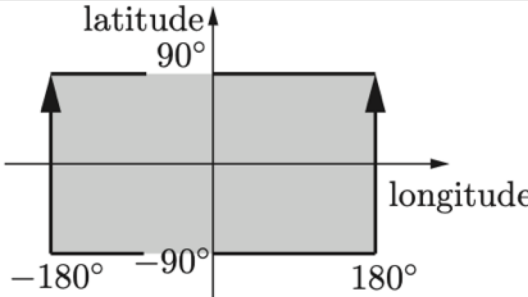
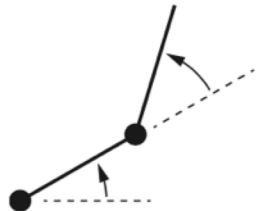

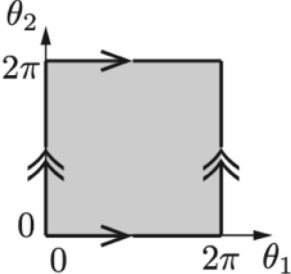
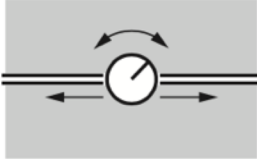

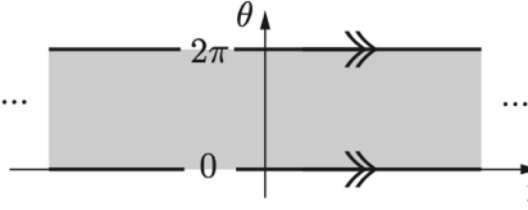
Chapter 2	Configuration Space
	2.1 DOF of a Rigid Body
	2.2 DOF of a Robot
	2.3 C-space Topology and Representation
Chapter 3	Rigid-Body Motions
Chapter 4	Forward Kinematics
Chapter 5	Velocity Kinematics and Statics
Chapter 6	Inverse Kinematics
Chapter 7	Kinematics of Closed Chains
Chapter 8	Dynamics of Open Chains
Chapter 9	Trajectory Generation
Chapter 10	Motion Planning
Chapter 11	Robot Control
Chapter 12	Grasping and Manipulation
Chapter 13	Wheeled Mobile Robots

Important concepts, symbols, and equations

- Two C-spaces may have the same dof but differ in other ways. The **topology** (“shape”) of a space is independent of how we **represent** it.
- Two spaces are **topologically equivalent** if one can be continuously deformed to the other without cutting or pasting.
- Some spaces are **Cartesian products** of spaces of lower dimension, e.g.,

$$\text{(1d)} \mathbb{E}, S = T \quad \text{(2d)} \mathbb{E} \times \mathbb{E} = \mathbb{E}^2, S \times S = T^2, S^2, \mathbb{E} \times S \quad \text{(higher)} \mathbb{E}^k \times S^m \times T^n$$

- Represent Euclidean (“flat”) spaces \mathbb{E}^n as \mathbb{R}^n . For curved spaces, choose
 - minimum-parameter **explicit parameterizations** (choose between **singularities** or an **atlas** of **coordinate charts**), OR
 - **implicit representation** (use more numbers subject to constraints).

system	topology	sample representation
 point on a plane	 \mathbb{E}^2	 \mathbb{R}^2
 spherical pendulum	 S^2	 $[-180^\circ, 180^\circ] \times [-90^\circ, 90^\circ]$
 2R robot arm	 $T^2 = S^1 \times S^1$	 $[0, 2\pi) \times [0, 2\pi)$
 rotating sliding knob	 $\mathbb{E}^1 \times S^1$	 $\mathbb{R}^1 \times [0, 2\pi)$

Any value in an atlas of coordinate charts?
 An implicit representation?

Any value in an atlas of coordinate charts?
 An implicit representation?

Any value in an atlas of coordinate charts?
 An implicit representation?



hexrotor with two 5-DOF arms

<https://www.prodrone.com/archives/1420/>

each arm joint $\in (0, \pi)$

$$\mathbb{R}^3 \times S^2 \times S^1 \times T^6 \times \mathbb{R}^{10} = \mathbb{R}^3 \times S^2 \times T^7$$

C-space topology, with and without
arm joint limits, rotor angles?
Implicit/explicit representations?
Grübler's formula?

$$\text{body: } \mathbb{R}^3 \times S^2 \times S^1$$

$$\text{rotors: } S^1 \times S^1 \dots = T^6$$

6 times

$$\text{arm: } T^5 \text{ (no joint limits)}$$

$$\mathbb{R}^3 \times S^2 \times S^1 \times T^6 \times T^5 \times T^5 =$$

$$\mathbb{R}^3 \times T^{17} \times S^2$$



KUKA youBot
 mecanum-wheel omnidirectional base
 moving on flat ground
 plus 5-DOF robot arm + gripper

C-space topology and representation?
 Include gripper, wheel angles?

$$\underbrace{\mathbb{R}^2 \times S^1}_{\text{chassis}} \times \underbrace{\mathbb{R}^5}_{\text{arm}} \times \underbrace{T^4}_{\text{wheels}}$$

or, if joint limits

$$[a, b]^5$$

$$I^5$$