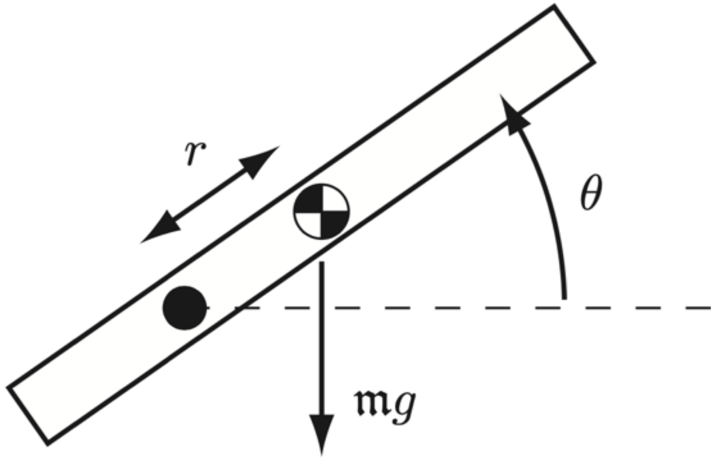
# Important concepts, symbols, and equations



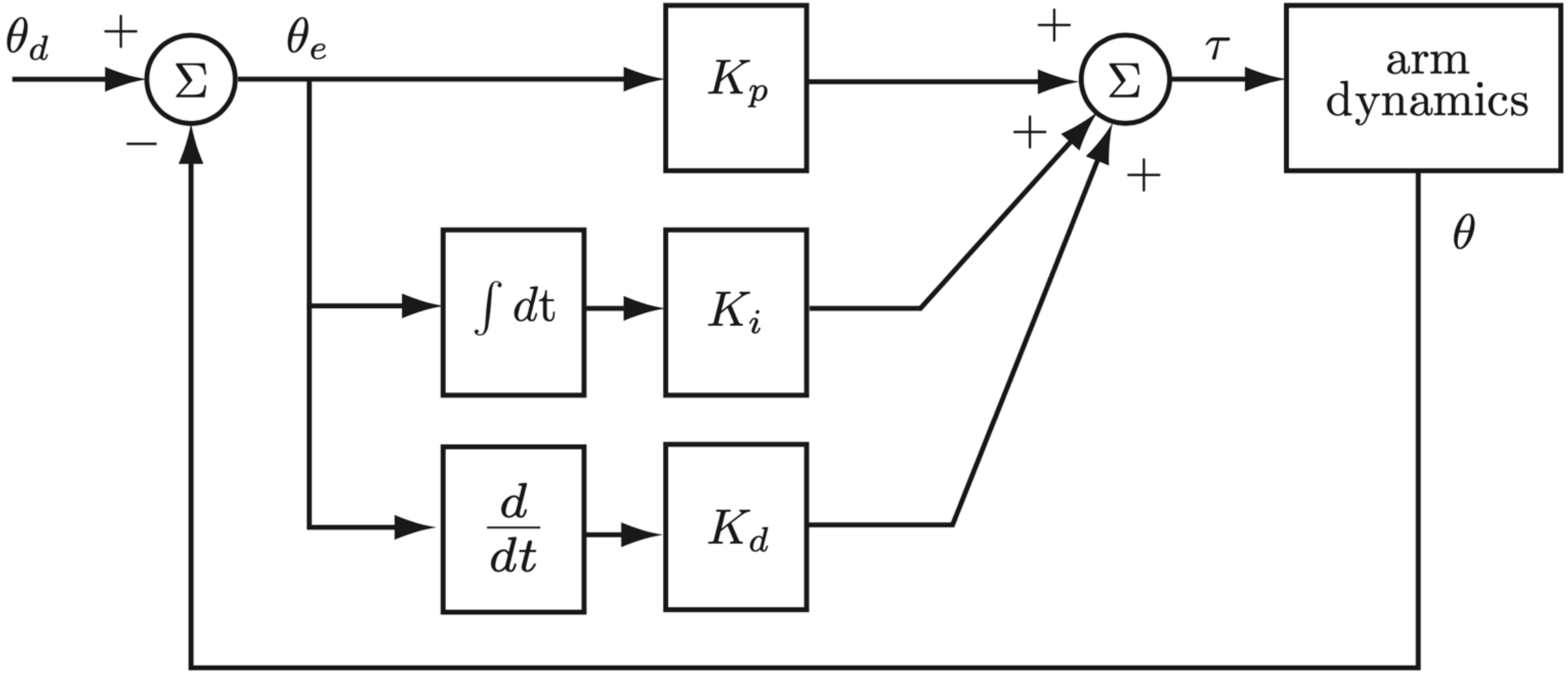

$$\tau = M\ddot{\theta} + \mathfrak{m}gr\cos\theta + b\dot{\theta}$$

$$\tau = M\ddot{\theta} + h(\theta, \dot{\theta})$$

## Proportional-Integral-Derivative (PID) control

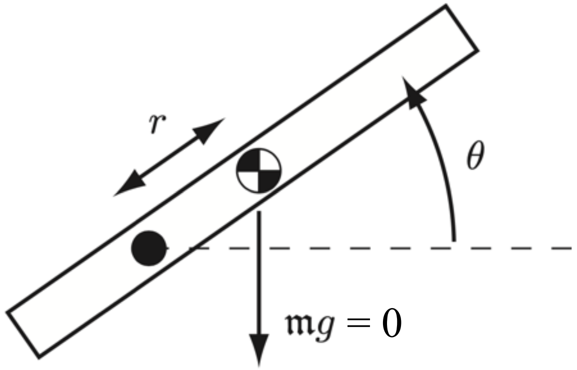
$$\tau = K_p\theta_e + K_i\int\theta_e(\mathrm{t})dt + K_d\dot{\theta}_e$$

# Important concepts, symbols, and equations (cont.)



## Important concepts, symbols, and equations (cont.)

### Setpoint PD control, $g = 0$




$$\tau = K_p\theta_e + \cancelto{0}{K_i} \int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e$$

$$\tau = M\ddot{\theta} + \cancelto{0}{\mathfrak{m}g} r \cos\theta + b\dot{\theta}$$

$$\dot{\theta}_d = \ddot{\theta}_d = 0$$

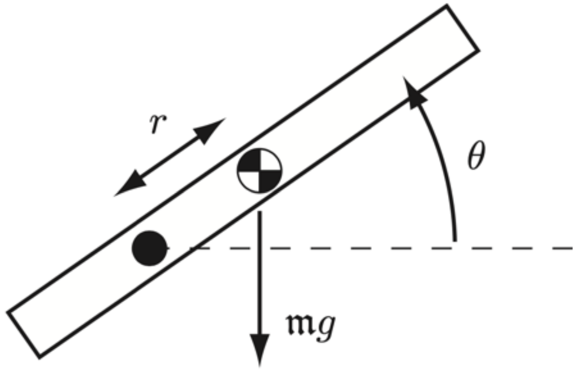
$$M\ddot{\theta} + b\dot{\theta} = K_p\theta_e + K_d\dot{\theta}_e$$

$$\ddot{\theta}_e + \frac{b + K_d}{M}\dot{\theta}_e + \frac{K_p}{M}\theta_e = 0$$

$$\zeta = \frac{b + K_d}{2\sqrt{K_p M}} \quad \omega_n = \sqrt{\frac{K_p}{M}}$$

For what gains are the error dynamics stable?

## Important concepts, symbols, and equations (cont.)




### Setpoint PD control, $g \neq 0$

$$\tau = K_p\theta_e + \cancelto{0}{K_i} \int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e$$

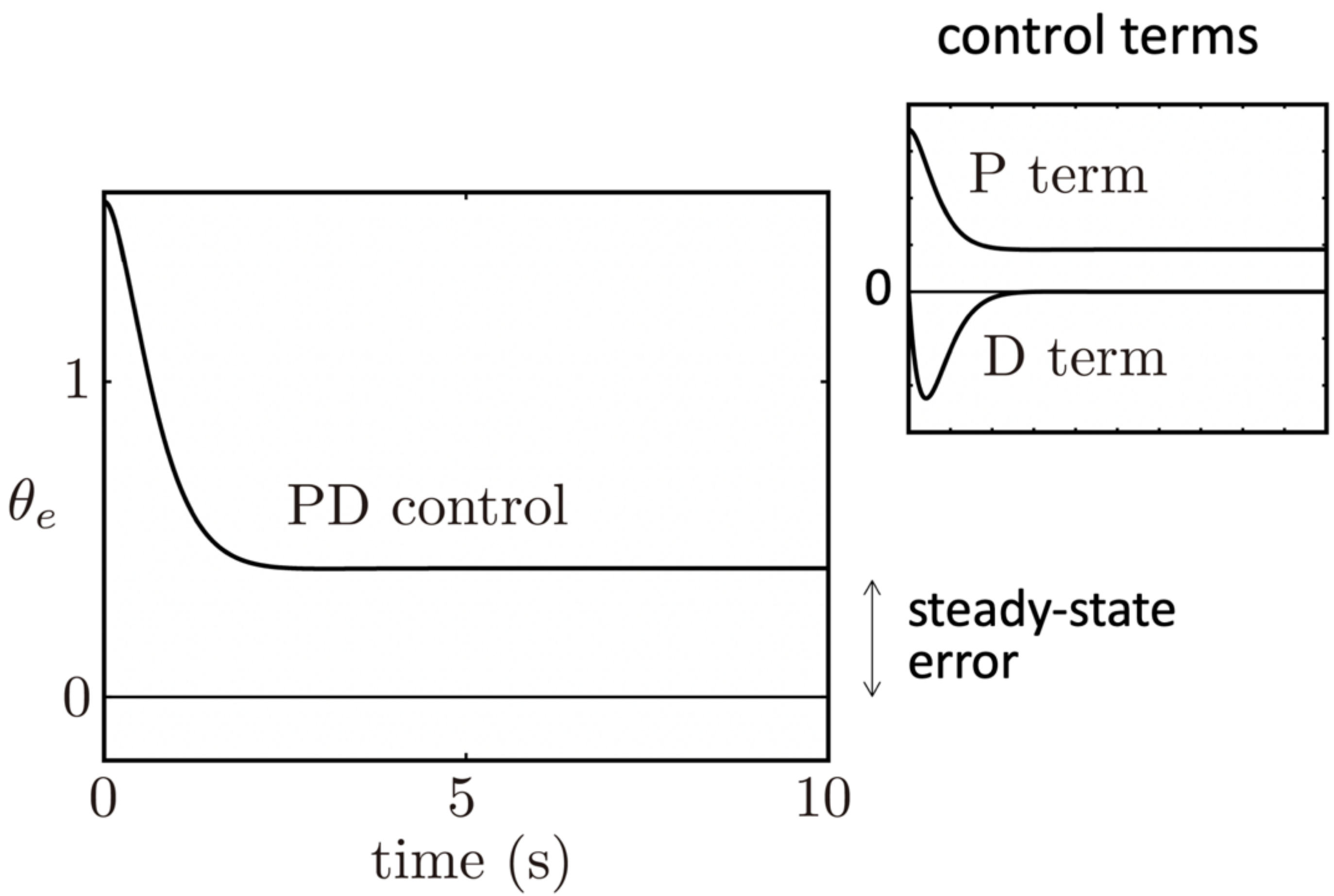
$$\tau = M\ddot{\theta} + \mathfrak{m}gr\cos\theta + b\dot{\theta}$$

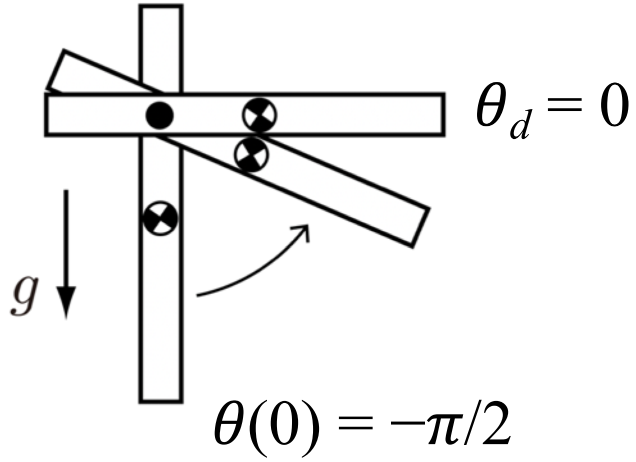
$$\dot{\theta}_d = \ddot{\theta}_d = 0$$

$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e = \mathfrak{m}gr\cos\theta$$

Nonhomogeneous. What is the steady-state error?

# Important concepts, symbols, and equations (cont.)



## Important concepts, symbols, and equations (cont.)

### Setpoint PID control, $g \neq 0$

$$\tau = K_p\theta_e + K_i \int \theta_e(\mathrm{t})d\mathrm{t} + K_d\dot{\theta}_e$$

$$\tau = M\ddot{\theta} + \underbrace{\mathfrak{m}gr\cos\theta}_{\tau_{\text{dist}}} + b\dot{\theta}$$

$$\dot{\theta}_d = \ddot{\theta}_d = 0$$

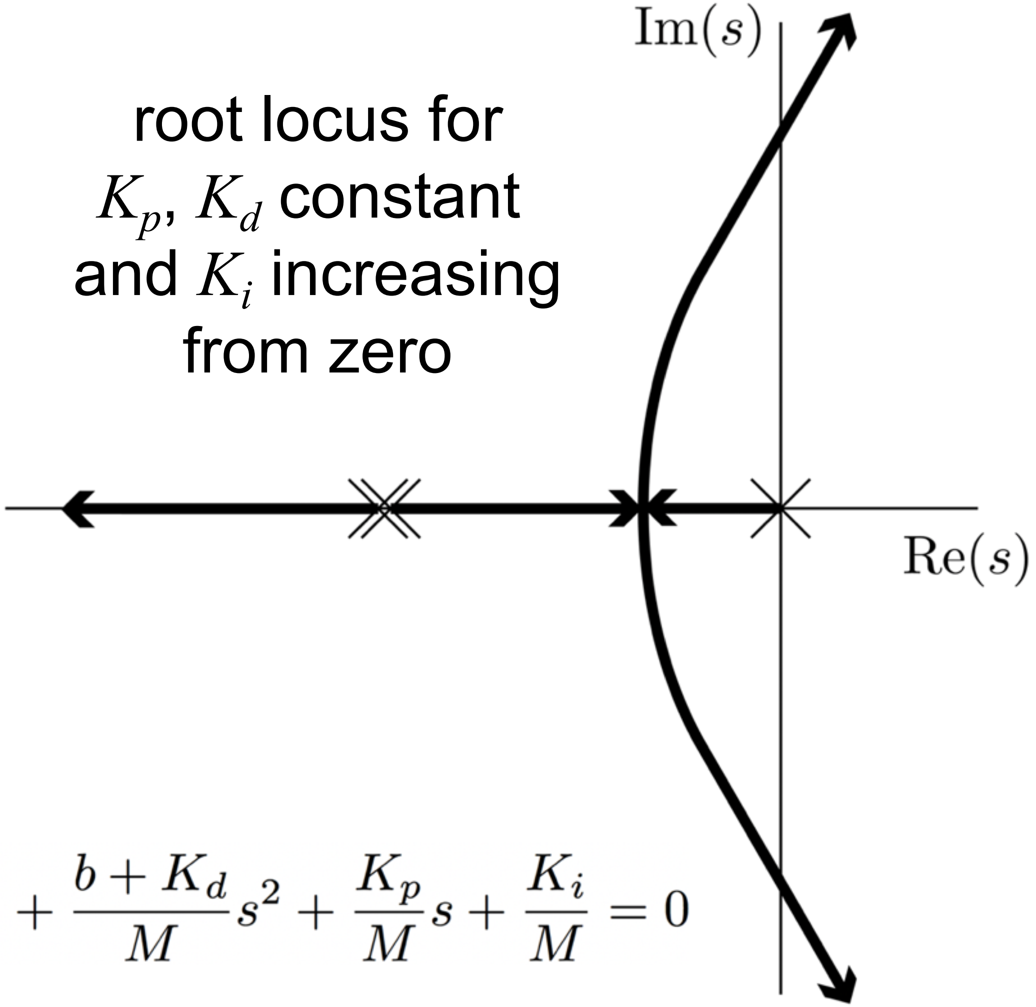
$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e + K_i \int \theta_e(\mathrm{t})d\mathrm{t} = \tau_{\text{dist}}$$

$$M\theta_e^{(3)} + (b + K_d)\ddot{\theta}_e + K_p\dot{\theta}_e + K_i\theta_e = 0$$

$$s^3 + \frac{b + K_d}{M}s^2 + \frac{K_p}{M}s + \frac{K_i}{M} = 0$$

## Important concepts, symbols, and equations (cont.)

root locus for
$K_p$, $K_d$ constant
and $K_i$ increasing
from zero

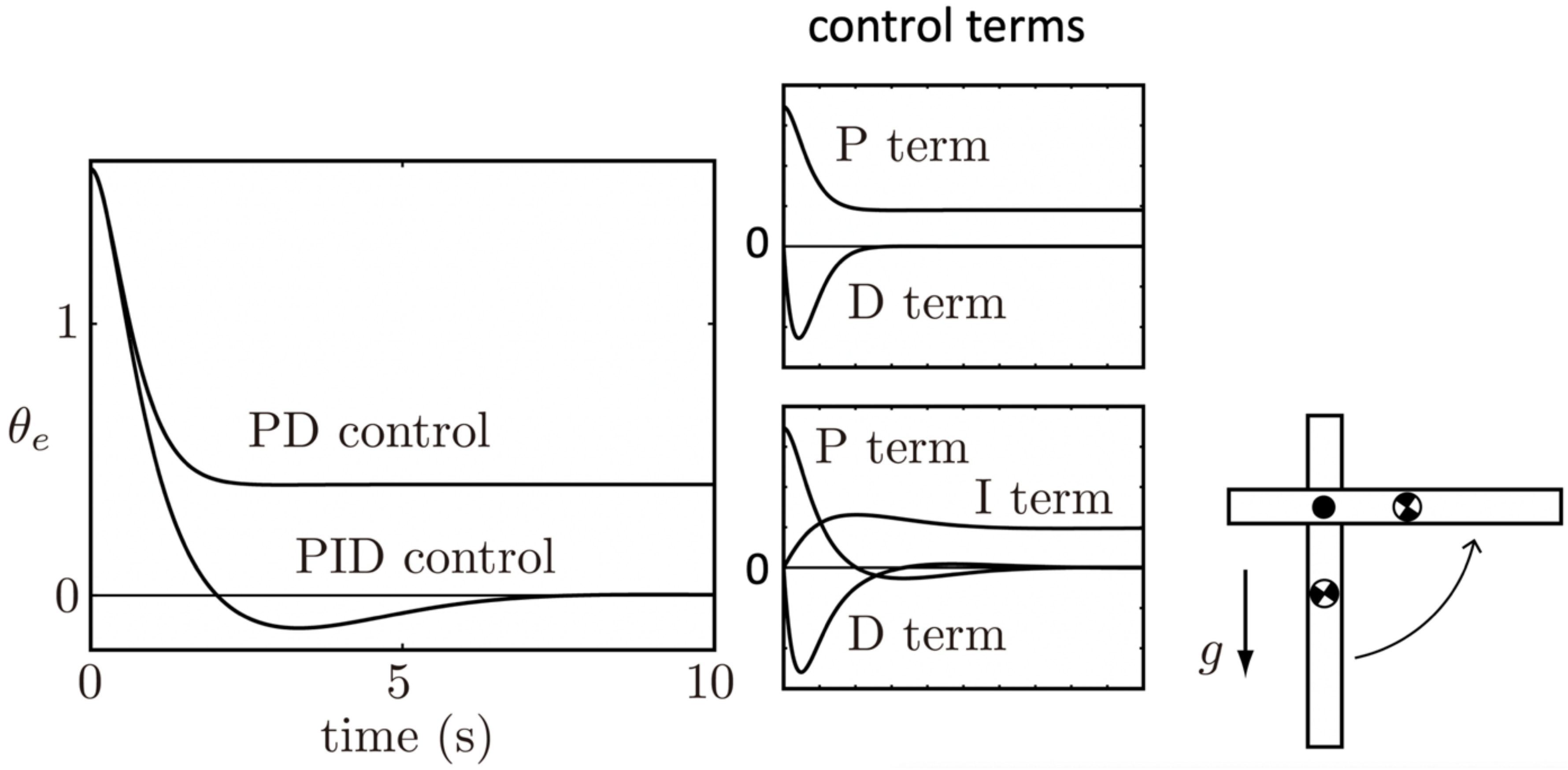



$$s^3 + \frac{b + K_d}{M} s^2 + \frac{K_p}{M} s + \frac{K_i}{M} = 0$$

$$\begin{aligned} K_d &> -b \\ K_p &> 0 \\ \frac{(b + K_d)K_p}{M} > K_i &> 0 \end{aligned}$$

$K_i$ improves steady-state response but can worsen the transient response.

# Important concepts, symbols, and equations (cont.)



# Important concepts, symbols, and equations (cont.)

What about tracking general trajectories, not just setpoint control?

$$\tau = M\Big(\overbrace{\ddot{\theta}_d + K_p\theta_e + K_i\int\theta_e \mathrm{d}t + K_d\dot{\theta}_e}^{\text{commanded } \ddot{\theta}}\Big) + h(\theta,\dot{\theta})$$

$$\ddot{\theta}_e = \ddot{\theta}_d - \ddot{\theta}$$

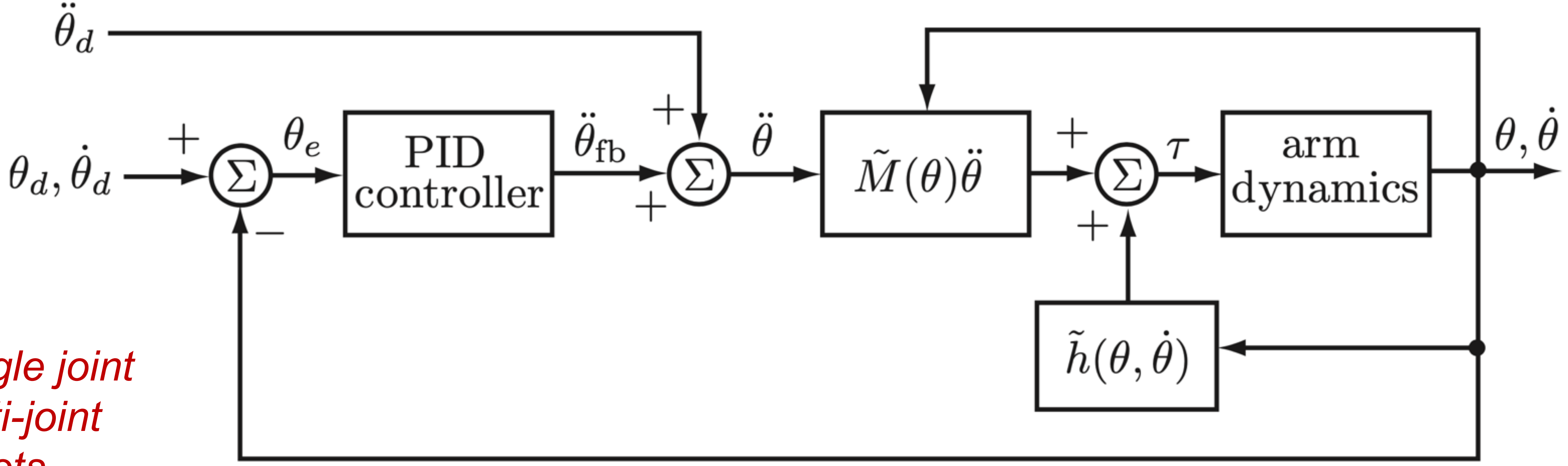
$$\ddot{\theta}_e = -K_d\dot{\theta}_e - K_p\theta_e - K_i\int\theta_e dt$$

$$\theta_e^{(3)} + K_d\ddot{\theta}_e + K_p\dot{\theta}_e + K_i\theta_e = 0$$

# Important concepts, symbols, and equations (cont.)

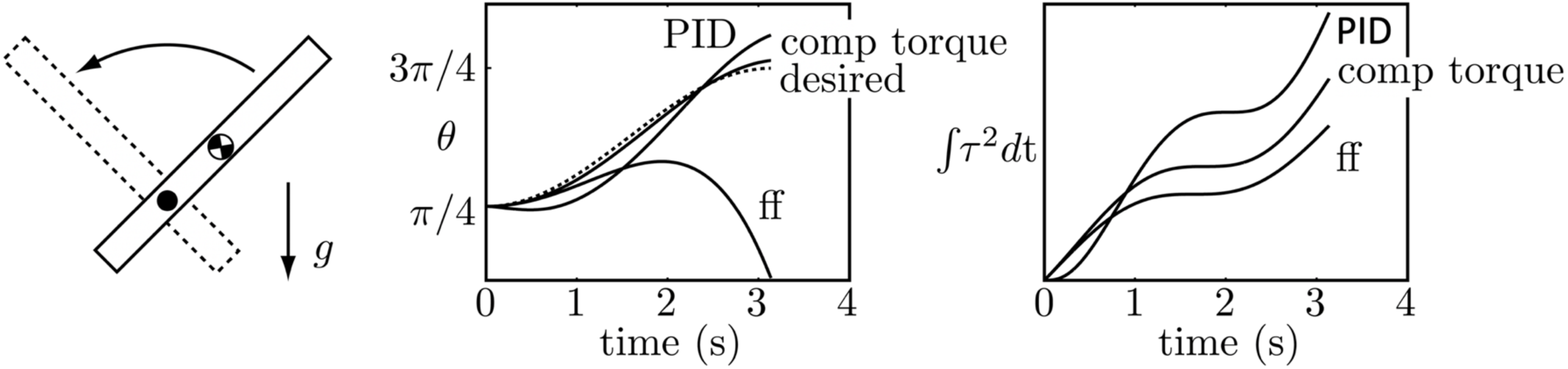
## Computed torque control (feedback linearization)

$$\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e\right) + \tilde{h}(\theta,\dot{\theta})$$




*for a single joint or multi-joint robots*

## Important concepts, symbols, and equations (cont.)



## Important concepts, symbols, and equations (cont.)

### Task-space computed torque control

What if your dynamic model is poor?

$$\mathcal{F}_b = \Lambda(\theta)\dot{\mathcal{V}}_b + \eta(\theta, \mathcal{V}_b)$$

dynamic model: $\{\tilde{\Lambda}, \tilde{\eta}\}$

$$\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int \theta_e \mathrm{d}t + K_d\dot{\theta}_e\right) + \tilde{h}(\theta, \dot{\theta})$$

$$\mathcal{F}_b = \tilde{\Lambda}(\theta)\left(\dot{\mathcal{V}}_d + K_pX_e + K_i\int X_e \mathrm{d}t + K_d\mathcal{V}_e\right) + \tilde{\eta}(\theta, \mathcal{V}_b)$$

$$[X_e] = \log(X^{-1}X_d)$$

$$\mathcal{V}_e = [\mathrm{Ad}_{X^{-1}X_d}]\mathcal{V}_d - \mathcal{V}_b$$

$$\tau = J_b^{\mathrm{T}}(\theta)\mathcal{F}_b$$

The characteristic equation of the error dynamics are

$$s^5 + 2s^4 + s^3 + 2s^2 + 4s + 2 = 0$$

Write the error dynamics in the form $\dot{x} = Ax$. Determine if the system is stable. (Use any software you want.)