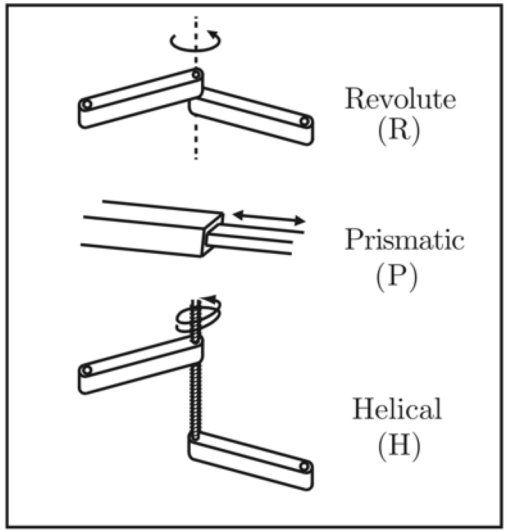
Important concepts, symbols, and equations

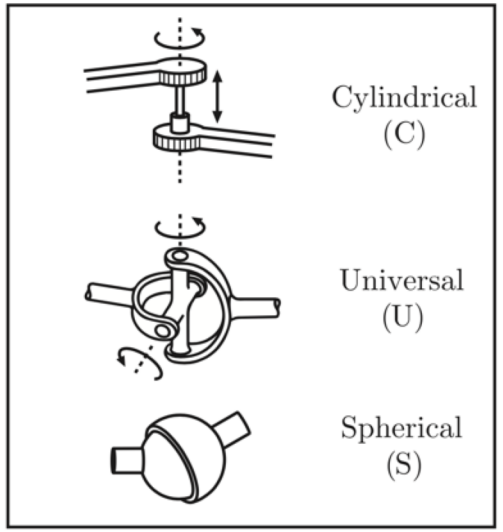
- **configuration**: a specification of the positions of all points of a mechanism
- **degrees of freedom** (dof): # of real #s required to describe a configuration
- **configuration space** (C-space): the dof-dimension space of all configurations
- dof of a planar body: $m = 3$; dof of a spatial body: $m = 6$

Important concepts, symbols, and equations (cont.)

- mechanism dof = Σ (body freedoms) – Σ (independent constraints from joints)

- joint types:



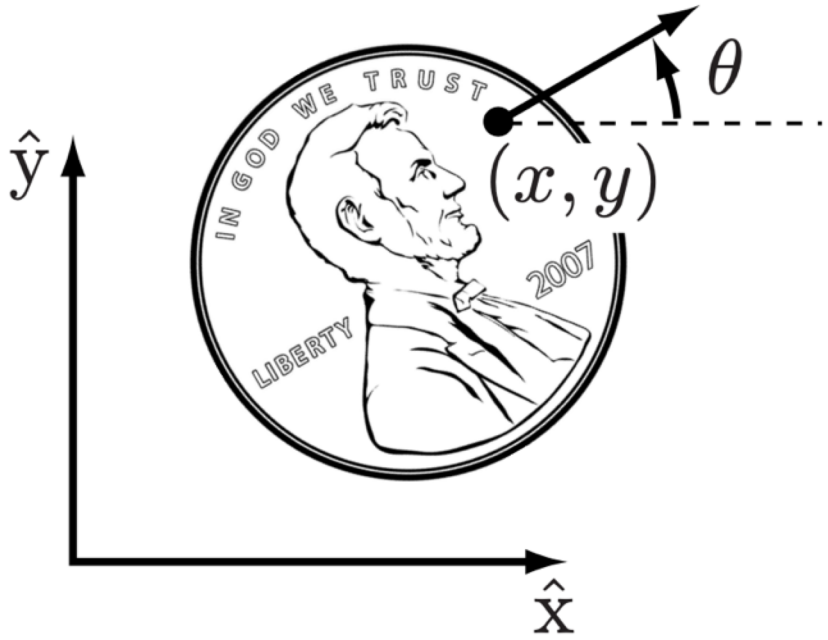


Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

- **Grübler's formula**: $\text{dof} = m(N - 1 - J) + \sum_{i=1}^{J} f_i$

3 or 6 — # bodies — # joints

$\sum_{i=1}^{J} f_i$ → # of freedoms provided by joints

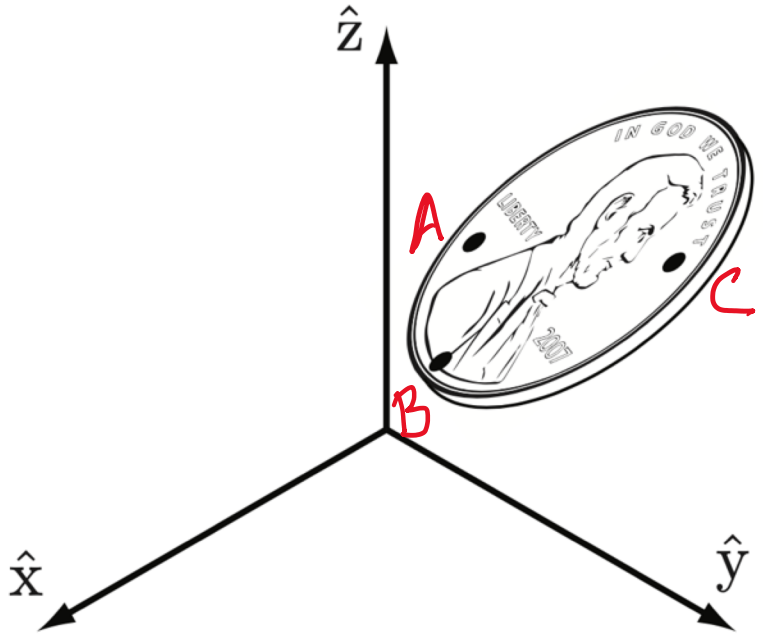


coin on a plane

(x, y, θ) + heads/tails

not a real number

$m = 3$



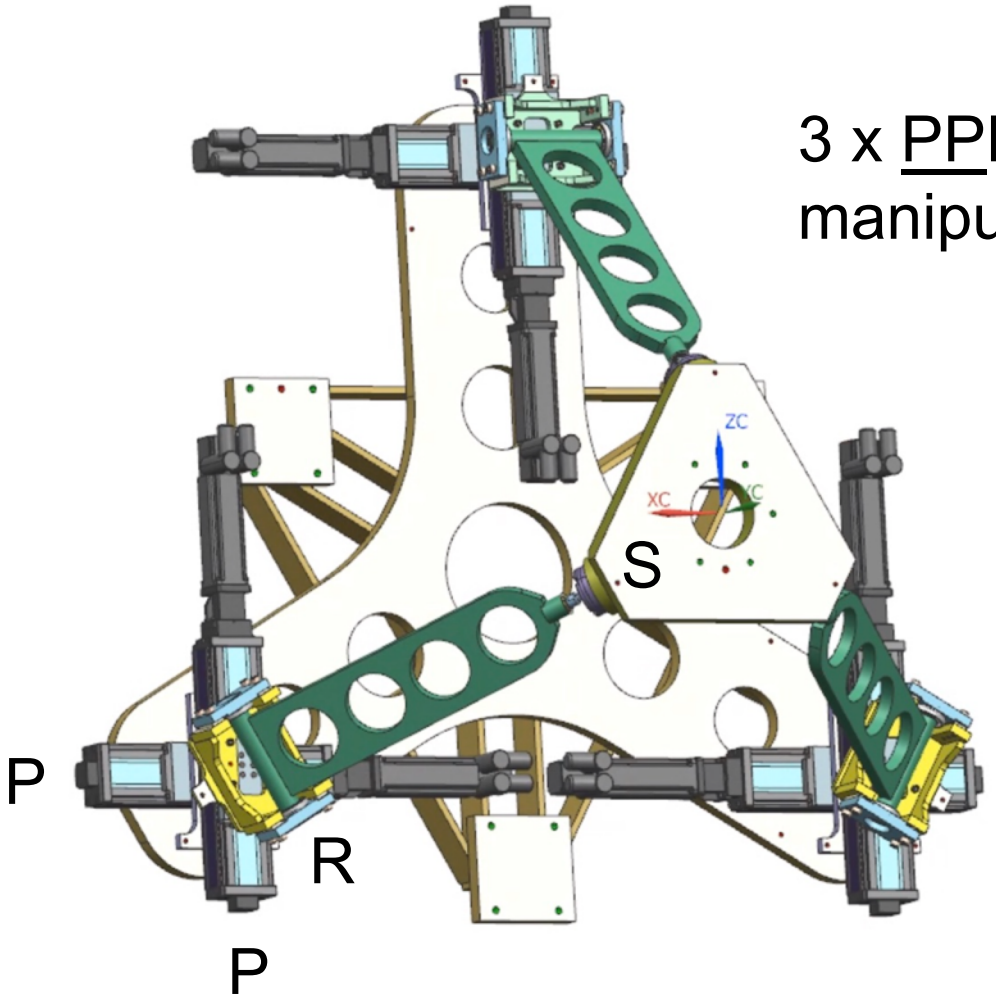
coin in space

A: 3 dof − 0 constraints = 3

B: 3 dof − 1 constraint = 2

C: 3 dof − 2 constraints = 1

$m = 6$



3 x PPRS parallel manipulator

KUKA Systems North America LLC
(patent pending)

$m = 6$ (spatial)

$N = 1 + 1 + 3 \times 3 = 11$

ground, platform, legs, 2 prismatic carriages + link between R + S joints

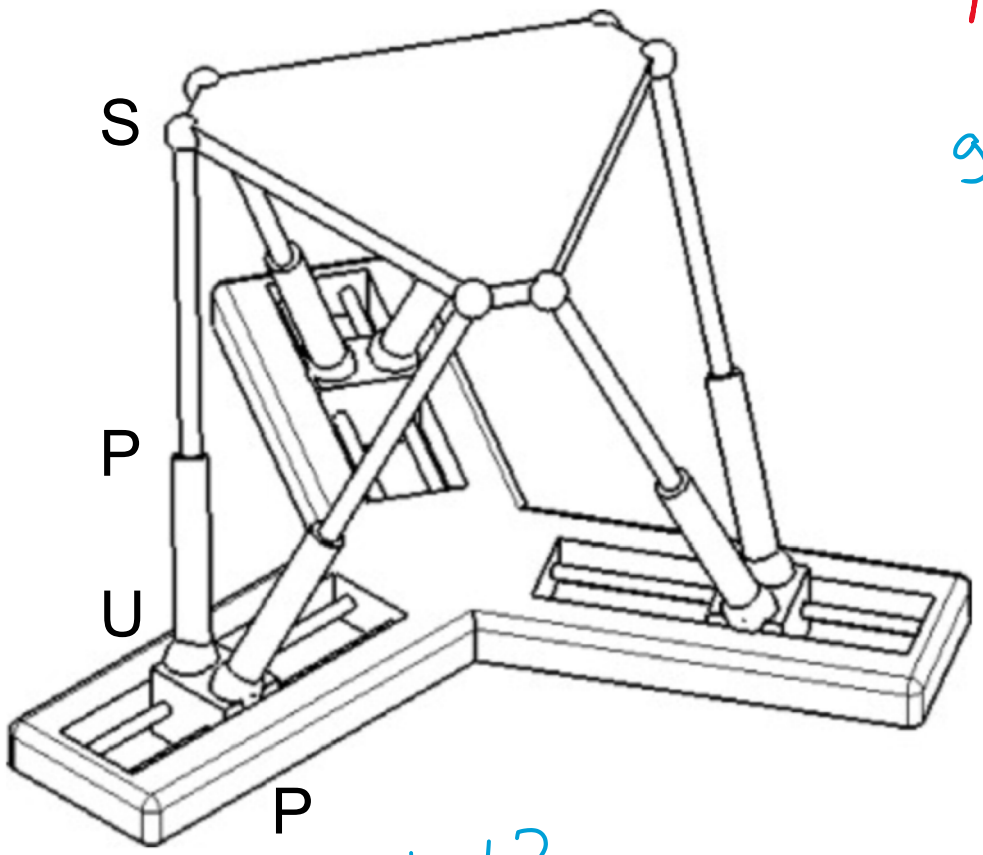
$J = 3 \times 4 = 12$

$\sum f_i = 3 \times (1+1+1+3) = 18$

$dof = 6(11 - 1 - 12) + 18$

$= -12 + 18 = 6$

$m = 6$ (spatial)



underactuated?
redundantly actuated?

$N = 1 + 1 + 3 + 6 \times 2 = 17$

ground, platform, sliders, legs

$J = 3 + 6 \times 3 = 21$

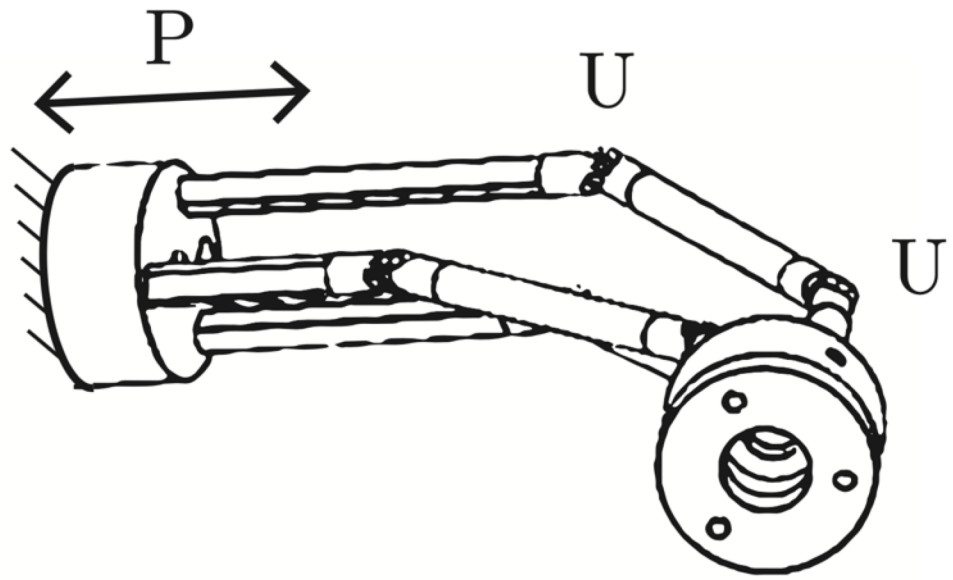
$\Sigma f_i = 3 + 6 \times (2+1+3) = 39$

$dof = 6(17-1-21) + 39$

$= -30 + 39 = \boxed{9}$

Which joints are likely actuated, and which are passive? How many dof does the platform have (not the mechanism)?

3 x PUU miniature surgical parallel manipulator
(National University of Singapore)

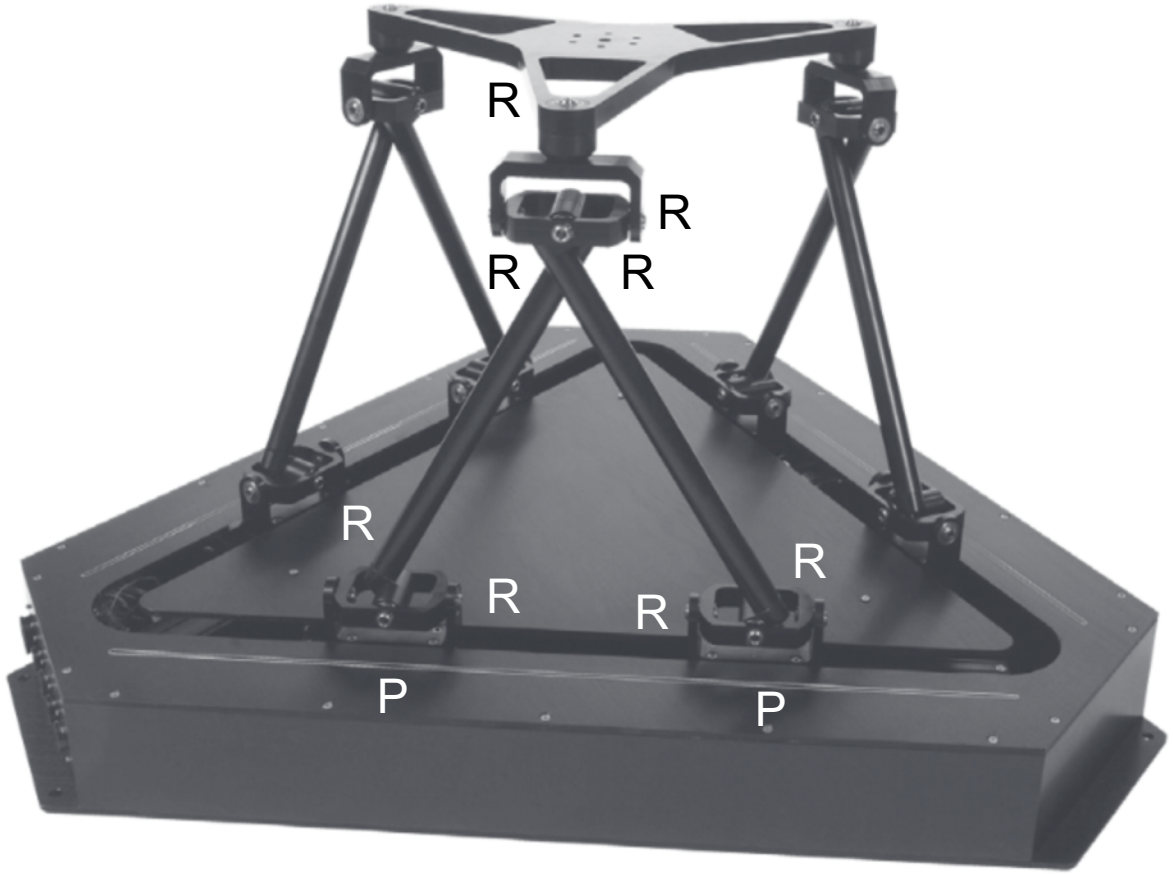


How many dof?

Discussion

underline of P indicates it is actuated. U joints are unactuated.

Quanser Hexapod



https://www.youtube.com/watch?v=AyVu4AE25DM

$m = 6$

$J = 3 \times 10 = 30$

$N = 1 + 1 + 6 \times 3 + 3 \times 2 = 26$

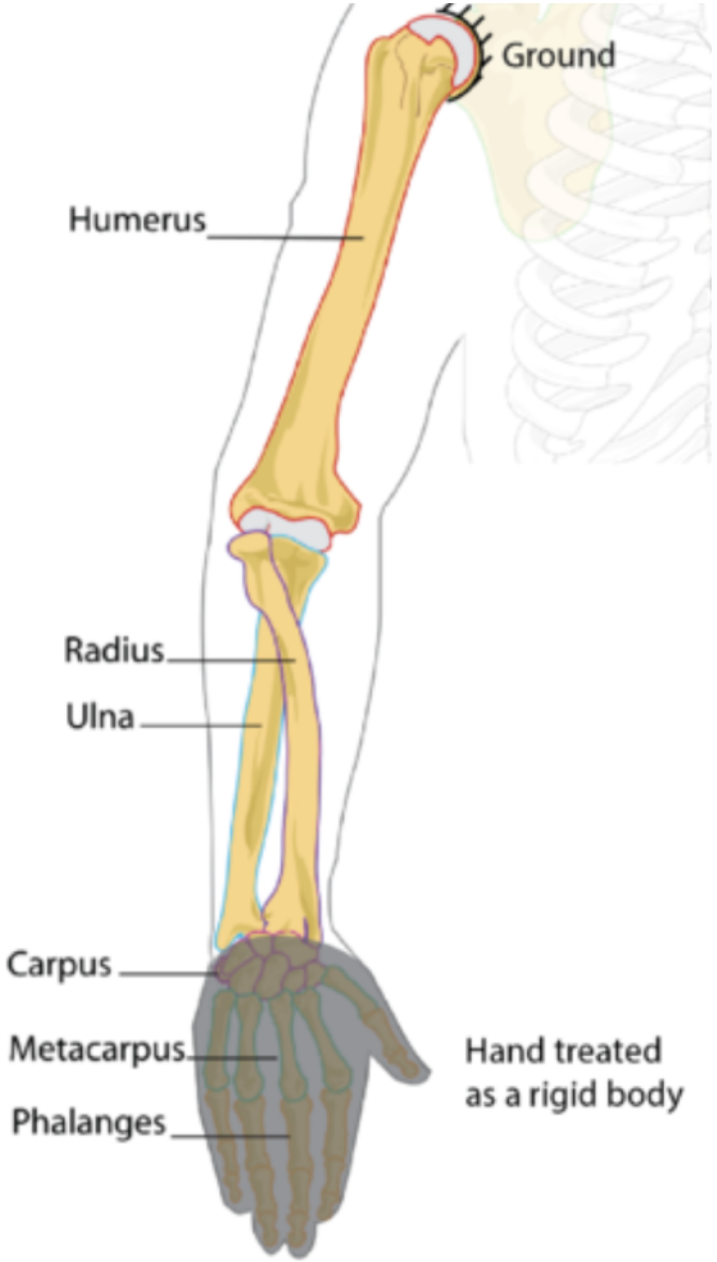
$\Sigma f_i = 30 \times 1 = 30$

$dof = 6(26 - 1 - 30) + 30$

$= 0$?

Clearly the platform has 6 dof.

Did we do something wrong?



How many dof does the human arm have?

Method 1: add dof of joints (shoulder, elbow, wrist)
Method 2: fully constrain hand's position

Discussion

How many total constraints are imposed by the joints?

Discussion