Important concepts, symbols, and equations

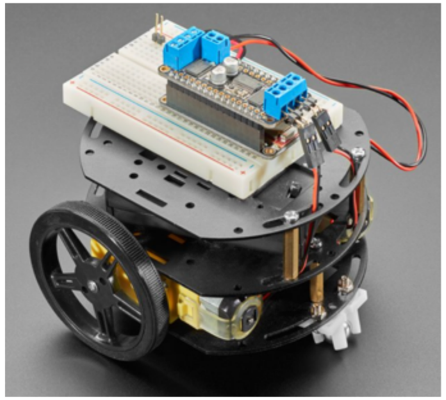
Types of wheels

conventional

omniwheel

mecanum wheel

Kinematic wheeled
mobile robots
(no skidding, slipping)



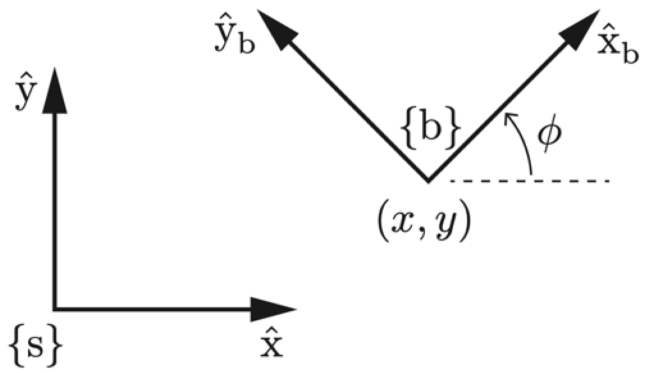
differential drive

car-like

Ackermann steering

omnidirectional

<u>**Important concepts, symbols, and equations**</u> **(cont.)**



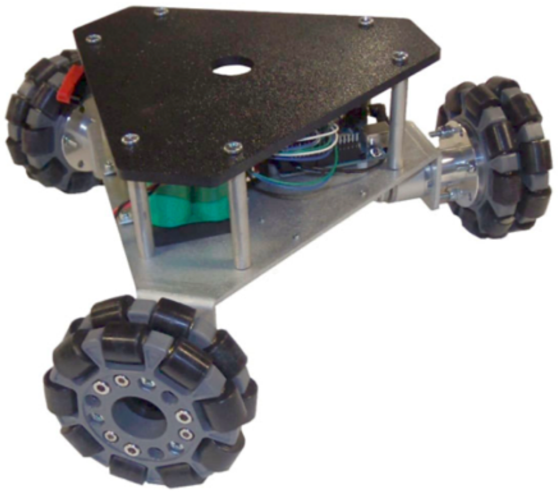
Configuration of the mobile base

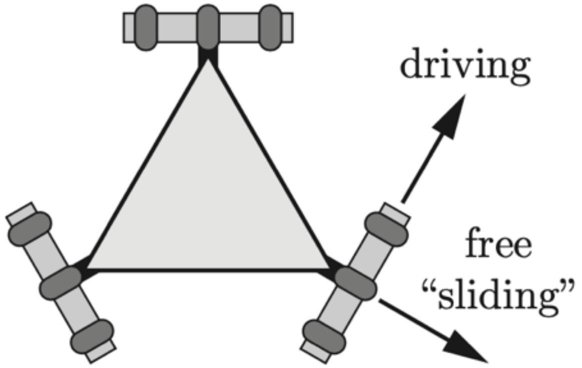
$$T_{sb} = \begin{bmatrix} r_{11} & r_{12} & \cancel{r_{13}} & p_1 \\ r_{21} & r_{22} & \cancel{r_{23}} & p_2 \\ \cancel{r_{31}} & \cancel{r_{32}} & \cancel{r_{33}} & \cancel{p_3} \\ 0 & 0 & \cancel{0} & 1 \end{bmatrix} \in SE(2) \text{ or } q = (\phi, x, y) \in \mathbb{R}^3$$

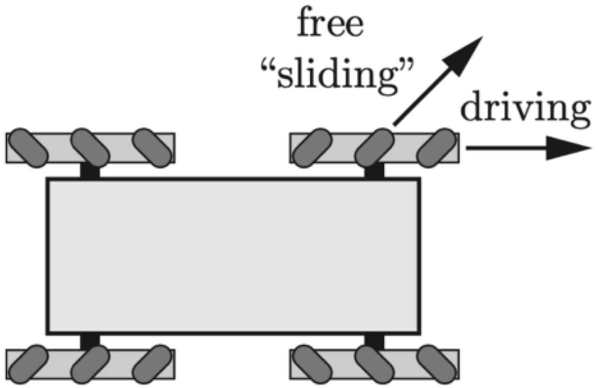
Velocity of the mobile base: $\mathcal{V}_b = (\cancel{\omega_{bx}}, \cancel{\omega_{by}}, \boxed{\omega_{bz}, v_{bx}, v_{by}}, \cancel{v_{bz}})$ or $\dot{q} \in \mathbb{R}^3$

Important concepts, symbols, and equations (cont.)

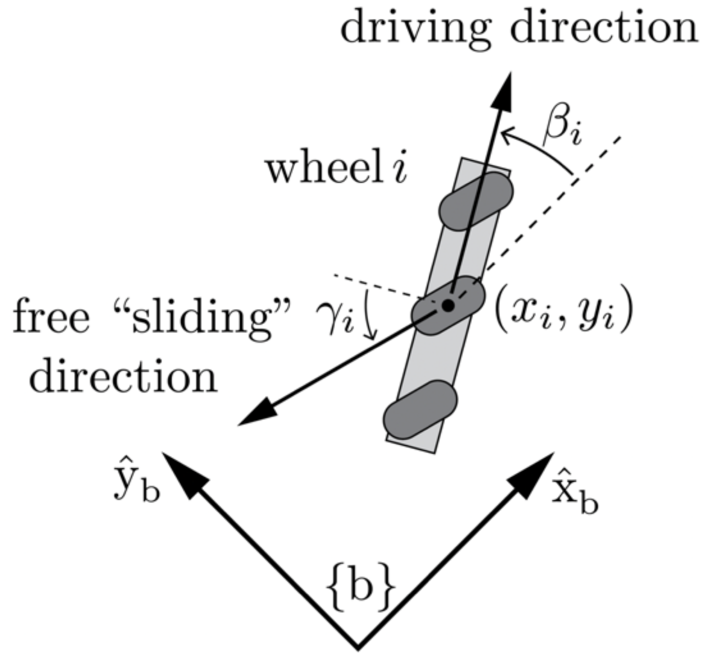
Examples of omnidirectional wheeled mobile robots

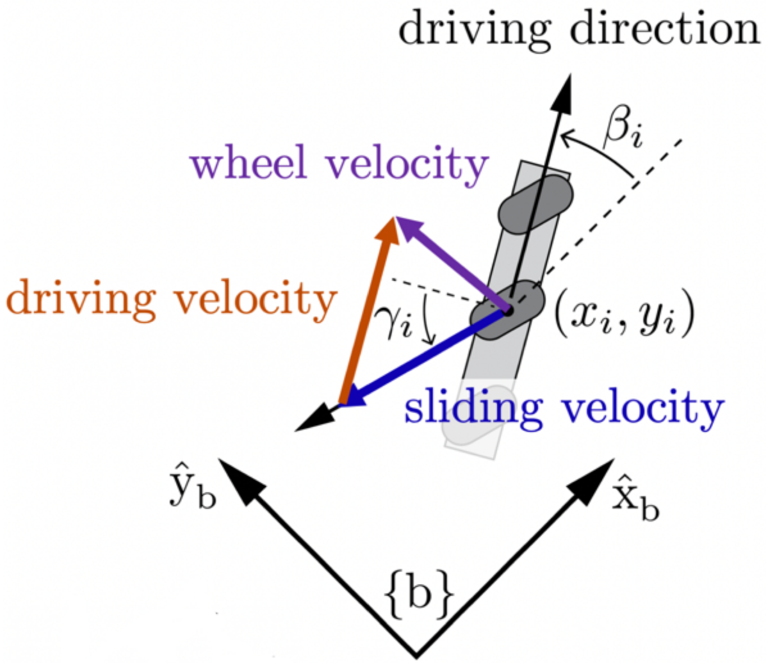






Important concepts, symbols, and equations (cont.)





$$u_i = h_i(0)\mathcal{V}_b$$

$$H(0) = \begin{bmatrix} h_1(0) \\ \vdots \\ h_m(0) \end{bmatrix} \in \mathbb{R}^{m\times 3}$$

m wheels

$$u = H(0)\mathcal{V}_b$$

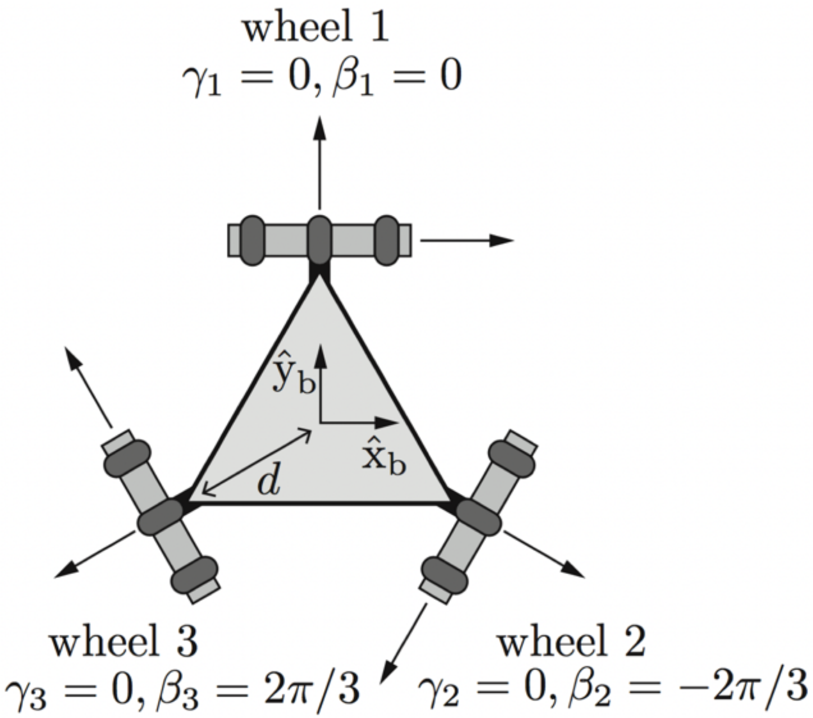
wheel speed vector

wheel driving speed:

$$u_i = \frac{1}{r_i}[1 \;\; \tan\gamma_i] \begin{bmatrix} \cos\beta_i & \sin\beta_i \\ -\sin\beta_i & \cos\beta_i \end{bmatrix} \begin{bmatrix} -y_i & 1 & 0 \\ x_i & 0 & 1 \end{bmatrix} \mathcal{V}_b$$

wheel radius — component in driving direction — linear velocity at wheel, in wheel frame — linear velocity at wheel, in {b}

Important concepts, symbols, and equations (cont.)

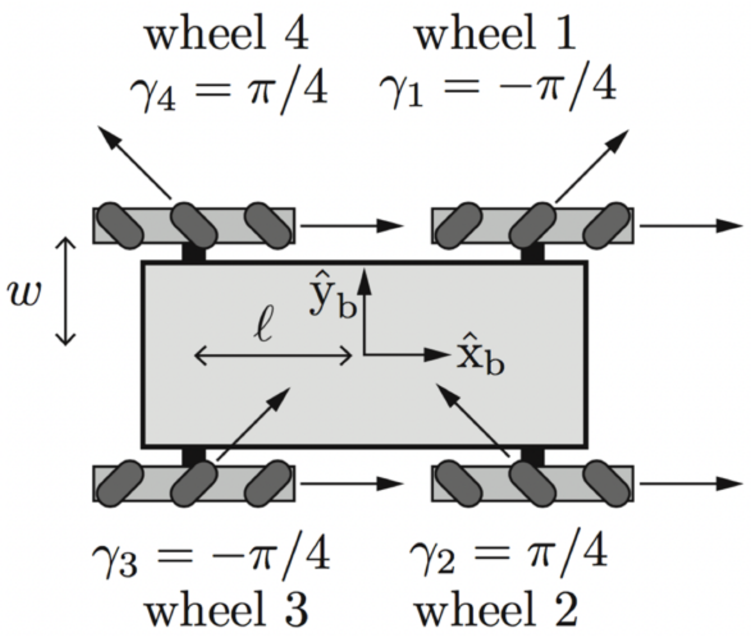


Why are at least three wheels required for omnidirectional motion?

Need at least 3 controls (wheel speeds) to map to all of the 3d twist space

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = H(0)\mathcal{V}_b = \frac{1}{r} \begin{array}{c} \begin{array}{ccc} \text{spin in place} & \text{right} & \text{up} \end{array} \\ \begin{bmatrix} -d & 1 & 0 \\ -d & -1/2 & -\sin(\pi/3) \\ -d & -1/2 & \sin(\pi/3) \end{bmatrix} \end{array} \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix}$$

Important concepts, symbols, and equations (cont.)



Wheel velocities are 4d.
Chassis twist is 3d.
Implications?

All u not in the 3d space { H(0) $\mathcal{V}_b$ for all $\mathcal{V}_b$ } mean that the wheels must be skidding

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = H(0)\mathcal{V}_b = \frac{1}{r} \begin{array}{c} \begin{array}{ccc} \text{spin in place} & \text{forward} & \text{sideways} \end{array} \\ \begin{bmatrix} -\ell - w & 1 & -1 \\ \ell + w & 1 & 1 \\ \ell + w & 1 & -1 \\ -\ell - w & 1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix}$$

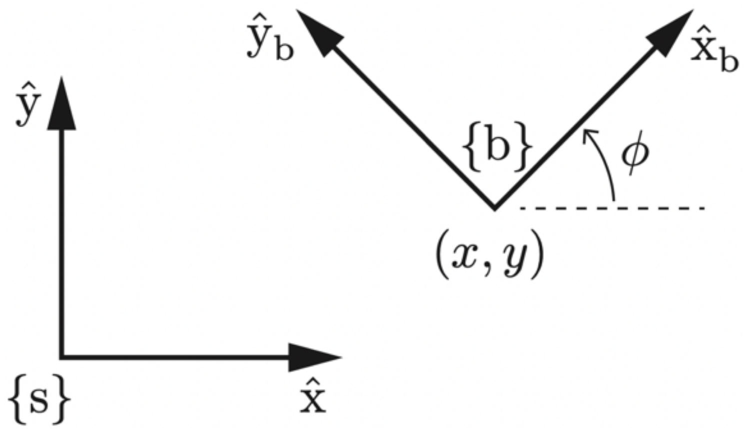
Important concepts, symbols, and equations (cont.)

Wheel speeds in terms of $\dot{q}$:

$$u = H(0)\mathcal{V}_b \,, \quad H(0) \in \mathbb{R}^{m\times 3}$$

$$u = \underbrace{H(0)\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}}_{H(\phi)} \underbrace{\begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{y} \end{bmatrix}}_{\dot{q}}$$

$$u = H(\phi)\dot{q}$$



Important concepts, symbols, and equations (cont.)

Feedforward + PI feedback stabilization of a planned trajectory:

$$\dot{q}(t) = \dot{q}_d(t) + K_p(q_d(t) - q(t)) + K_i \int_0^t (q_d(\mathrm{t}) - q(\mathrm{t}))\, d\mathrm{t}$$

$$u = H(\phi)\dot{q}$$

$K_p, K_i > 0$
for stability

Setpoint control:
$e_{ss} = 0$ for P, PI

Constant velocity reference:
$e_{ss} = 0$ for PI

Stability and steady-state error for different control laws and desired trajectories?

Important concepts, symbols, and equations (cont.)

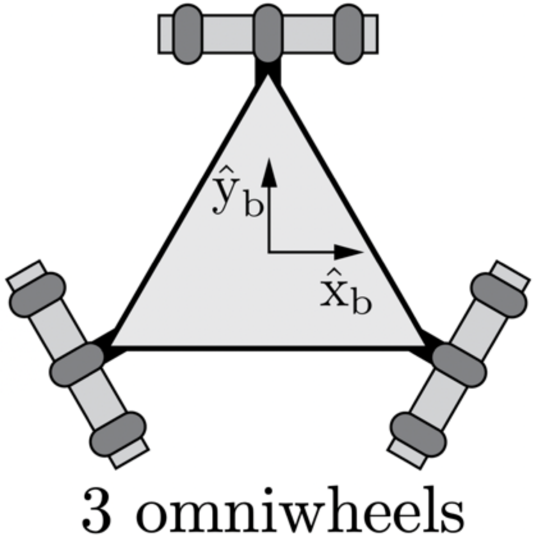
Given wheel velocity limits, the chassis' feasible twists lie inside a $2m$-sided convex polyhedron:

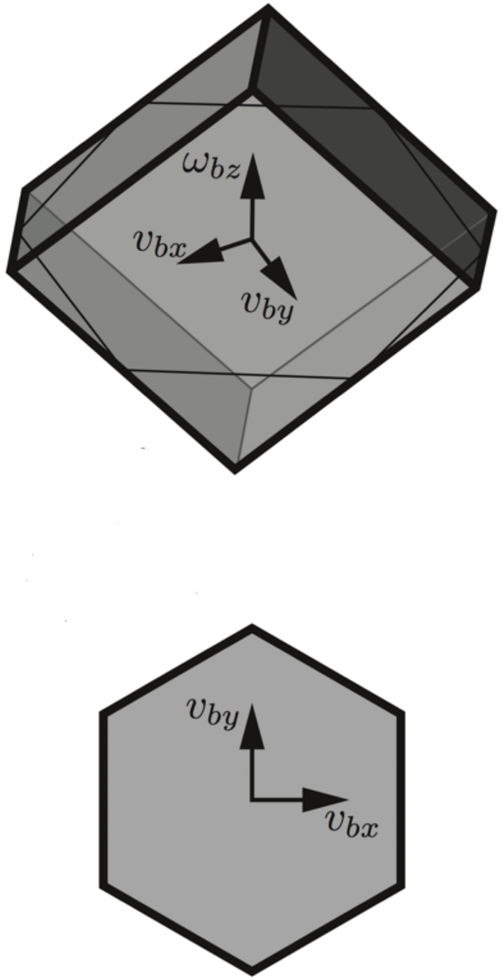
$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = H(0)\mathcal{V}_b = \begin{bmatrix} h_1(0) \\ \vdots \\ h_m(0) \end{bmatrix} \mathcal{V}_b$$

$$-u_{i,\max} \leq u_i = h_i(0)\mathcal{V}_b \leq u_{i,\max}$$

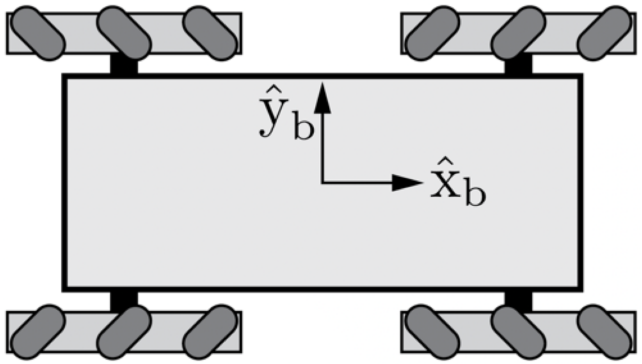
$$\left.\begin{aligned} -u_{i,\max} &= h_i(0)\mathcal{V}_b \\ u_{i,\max} &= h_i(0)\mathcal{V}_b \end{aligned}\right\} \text{define two parallel bounding planes in twist space}$$

Important concepts, symbols, and equations (cont.)

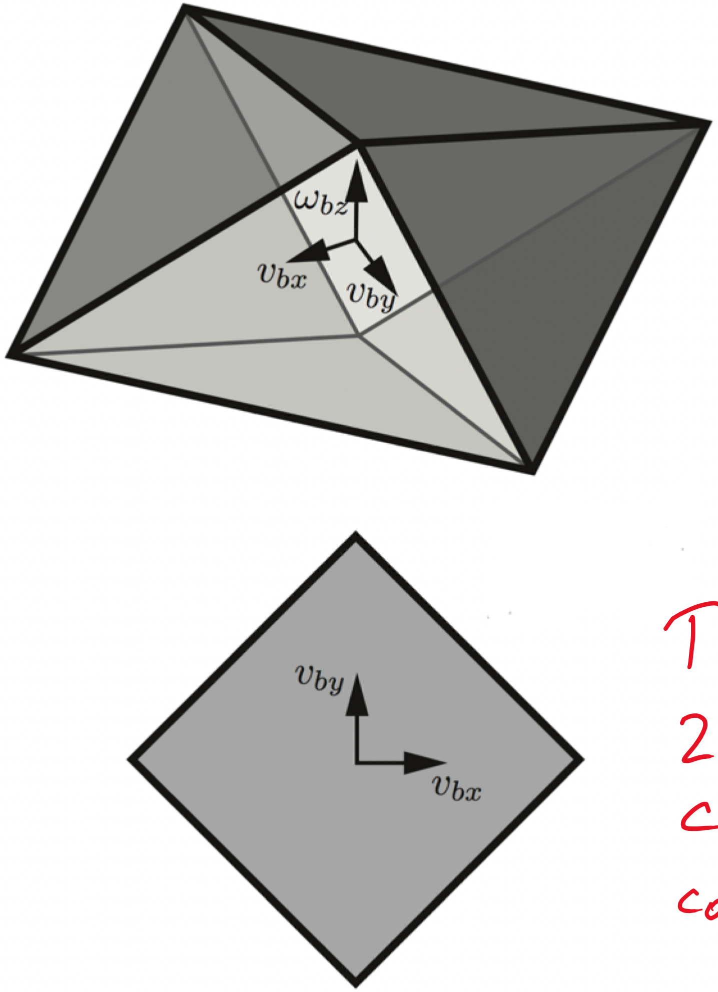




Important concepts, symbols, and equations (cont.)

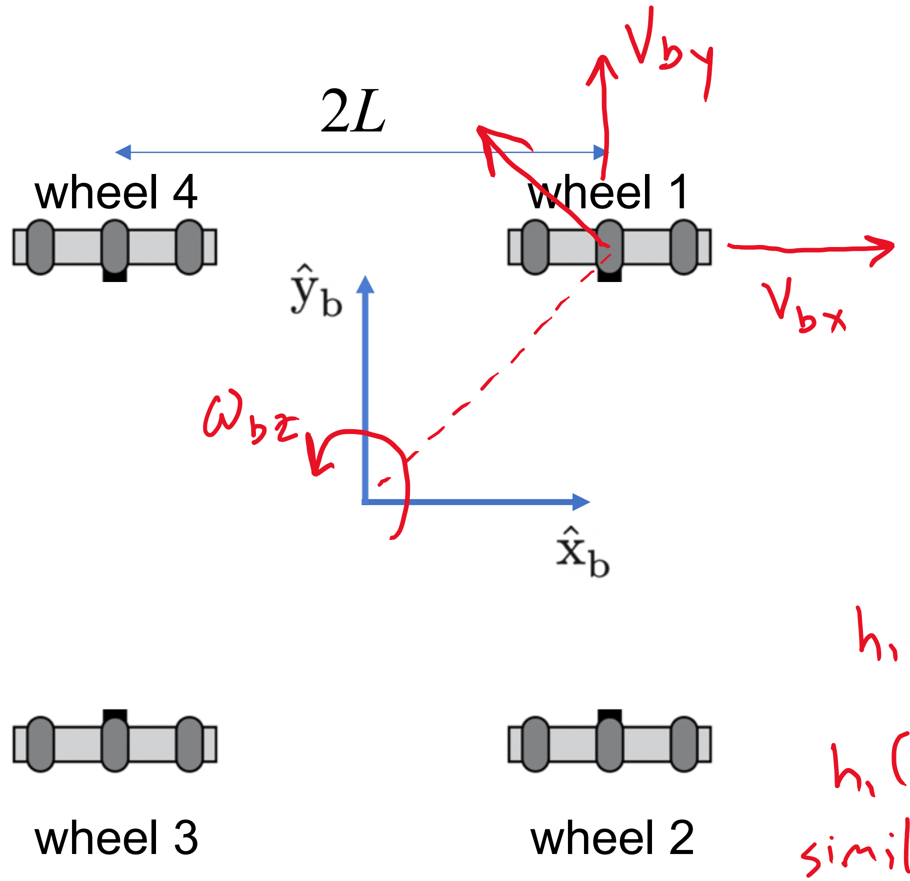


4 mecanum wheels



What does it mean if the convex polyhedron is unbounded?

The $H(0)$ matrix has rank 2 or less. Some twists cannot be obtained by controlling wheel speeds.



The centers of the omniwheels of a mobile robot are at the corners of a square a distance $2L$ from each other. The radius of the wheels is r, and the forward driving direction for each wheel is to the right. What is $H(0)$?

$h_1(0)$ first. Find driving speed for $\begin{bmatrix}\omega_{bz}\\ v_{bx}\\ v_{by}\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$ or $\begin{bmatrix}0\\1\\0\end{bmatrix}$ or $\begin{bmatrix}0\\0\\1\end{bmatrix}$

$h_1(0) = (-L, 1, 0)$

similarly

$$H(0) = \begin{bmatrix} h_1(0)\\ h_2(0)\\ h_3(0)\\ h_4(0)\end{bmatrix} = \frac{1}{r}\begin{bmatrix} -L & 1 & 0\\ L & 1 & 0\\ L & 1 & 0\\ -L & 1 & 0\end{bmatrix}$$

only rank 2

no way to achieve $\begin{bmatrix}0\\0\\v_{by}\end{bmatrix} = \mathcal{V}_b$