Important concepts, symbols, and equations

- k independent **holonomic constraints** on $(\theta_1, \ldots, \theta_n)$ reduce an n-dim C-space to $n-k$ dof.

$$g(\theta) = \begin{bmatrix} g_1(\theta_1, \ldots, \theta_n) \\ \vdots \\ g_k(\theta_1, \ldots, \theta_n) \end{bmatrix} = 0$$

- Pfaffian constraints are constraints on velocity: $A(\theta)\dot{\theta} = 0$

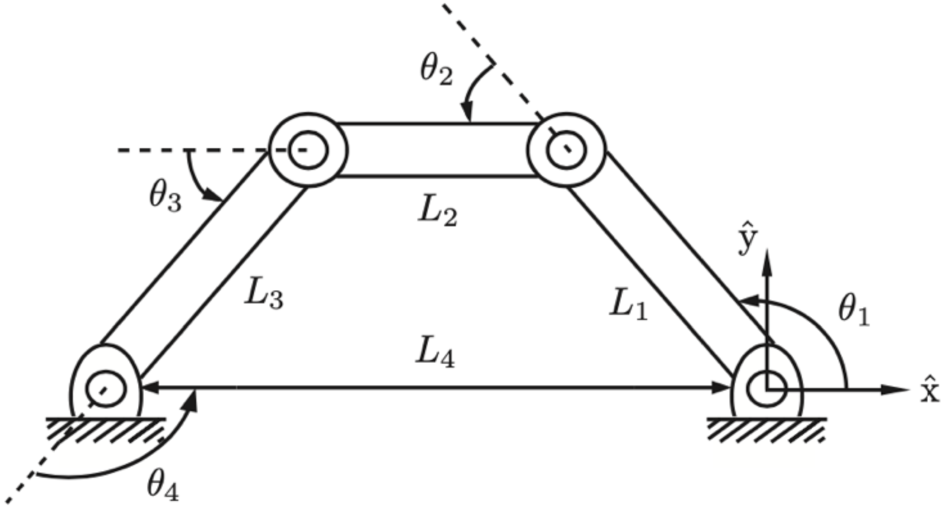
- If velocity constraints can be integrated to equivalent configuration constraints, they are holonomic. If not, they are **nonholonomic**: they reduce the dimension of the feasible velocities, but not the dimension of the C-space.

- Determining if constraints are holonomic or nonholonomic is sometimes difficult (Chapter 13).

Important concepts, symbols, and equations (cont.)

- The **task space** is the space in which a task is most naturally represented. It is independent of a robot.

- The **workspace** is usually a specification of the reachable space by a robot (or its wrist, or end-effector).
  - Often defined in terms of (x,y,z) translational positions only.
  - Sometimes the **dexterous workspace** is the set of translational positions that can be reached with arbitrary orientation.



3R planar robot has its endpoint pinned by a revolute joint, making a four-bar linkage.

$$
\begin{aligned}
L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) + \cdots + L_4 \cos(\theta_1 + \cdots + \theta_4) &= 0, \\
L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) + \cdots + L_4 \sin(\theta_1 + \cdots + \theta_4) &= 0, \\
\theta_1 + \theta_2 + \theta_3 + \theta_4 - 2\pi &= 0.
\end{aligned}
$$

“loop-closure” equations

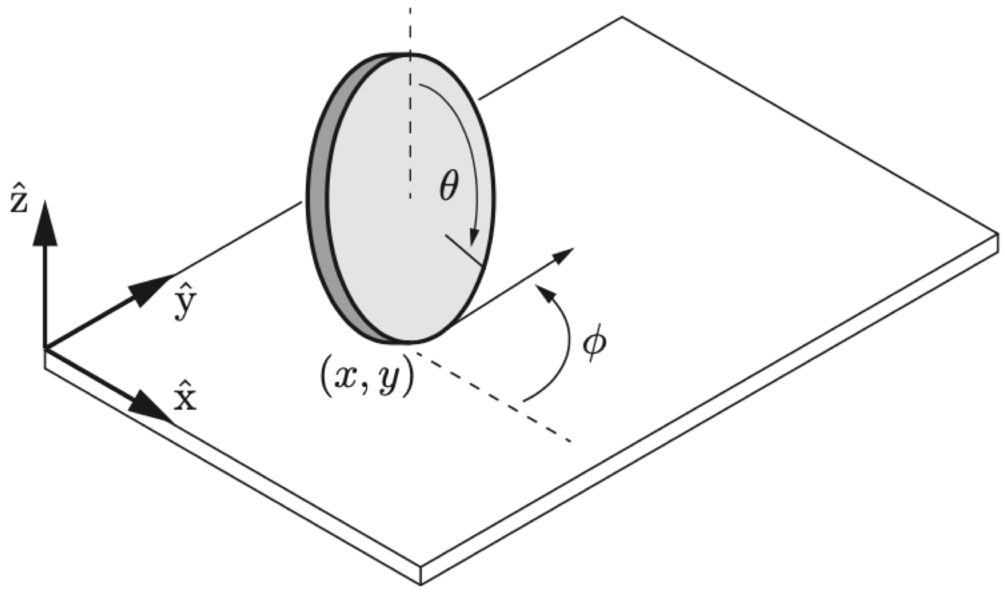
dof?

What does the C-space look like embedded in $(\theta_1, \theta_2, \theta_3, \theta_4)$?

What could be an explicit parameterization?

disk rolling upright on a plane

$$q = [q_1 \; q_2 \; q_3 \; q_4]^{\mathrm{T}} = [x \; y \; \phi \; \theta]^{\mathrm{T}}$$

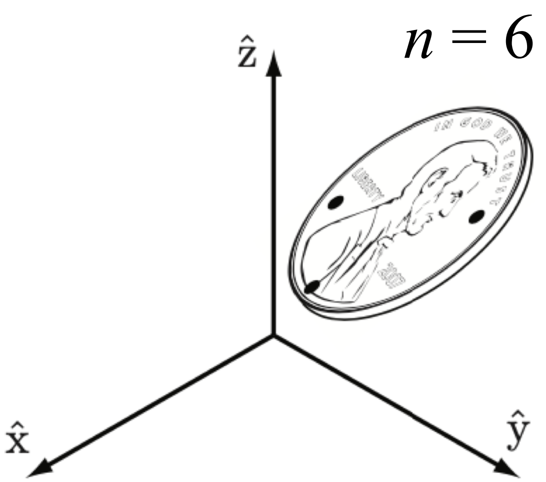


$$\begin{bmatrix} 1 & 0 & 0 & -r\cos q_3 \\ 0 & 1 & 0 & -r\sin q_3 \end{bmatrix} \dot{q} = 0$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = r\dot{\theta} \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

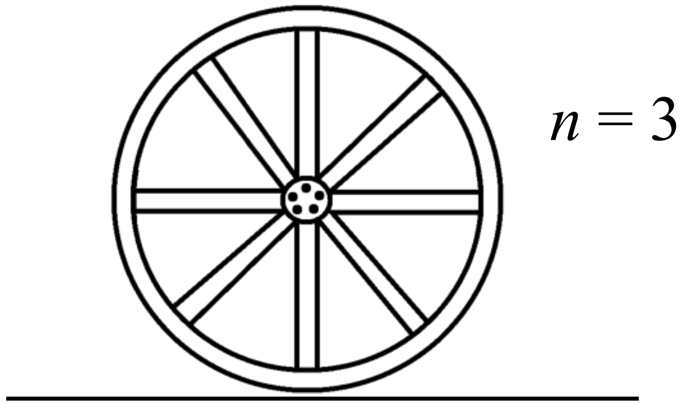
$$A(q)\dot{q} = 0, \; A(q) \in \mathbb{R}^{2\times 4}$$

starting with n dof, add k holonomic constraints, m nonholonomic constraints

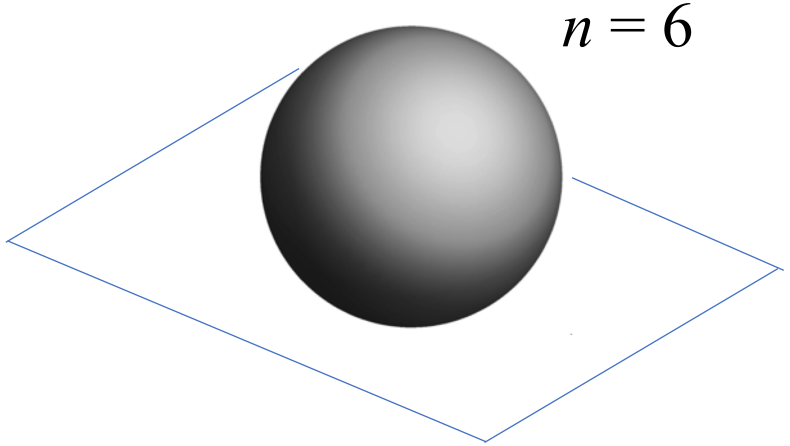


- a coin constrained to stand upright on a plane

- a coin constrained to roll upright on a plane

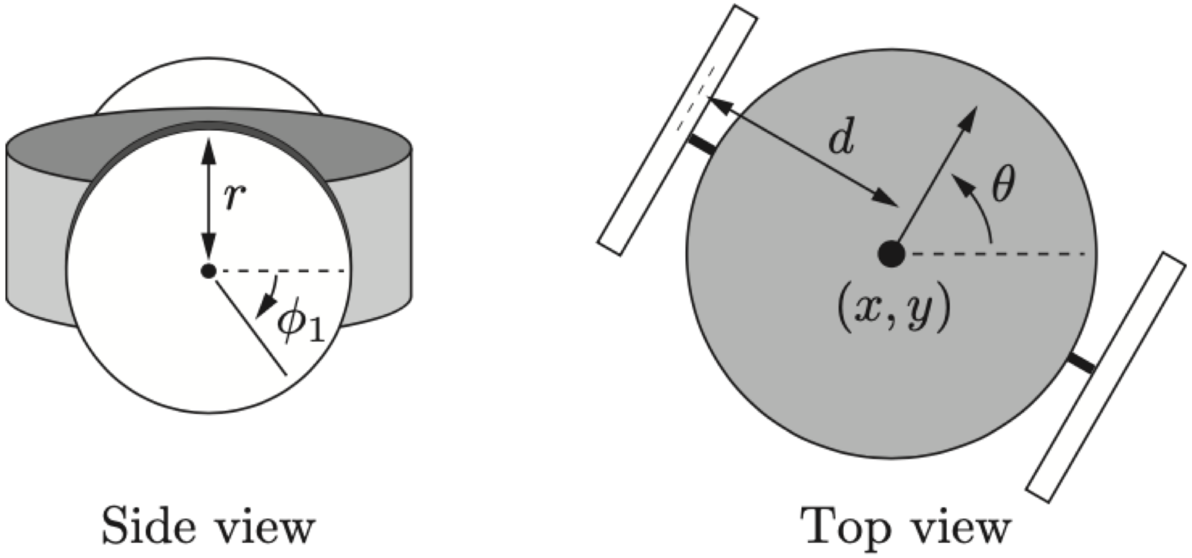


a wheel rolling on a line in the plane of the page



- a sphere touching a plane

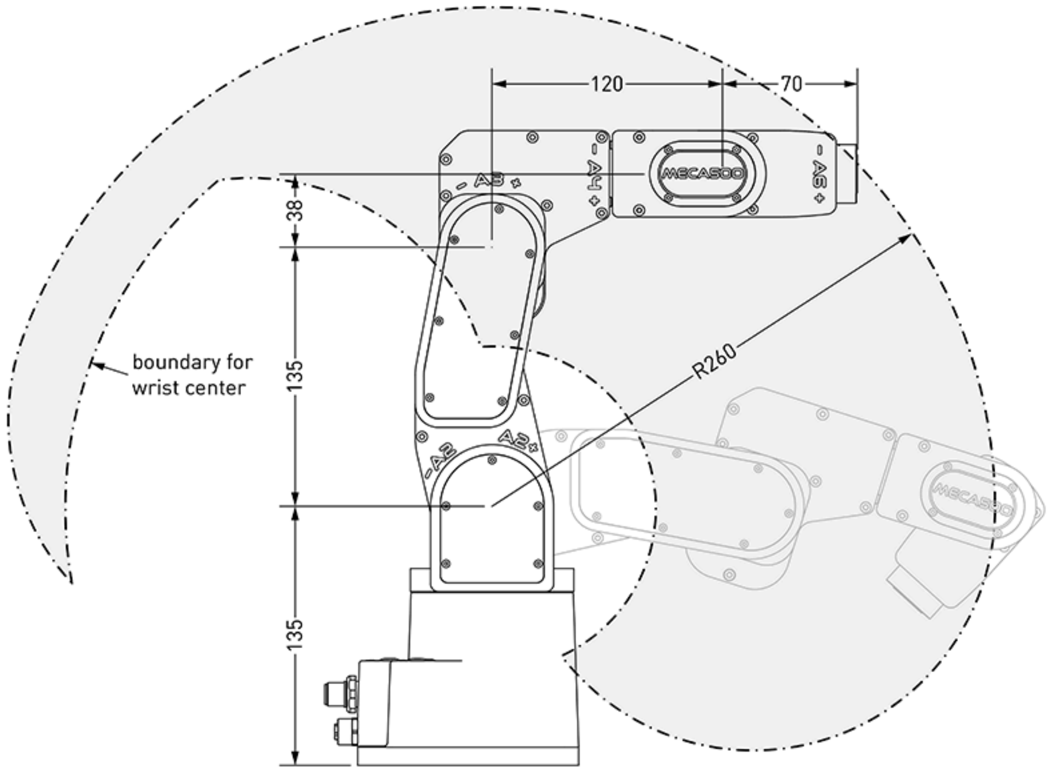
- a sphere rolling on a plane



diff-drive mobile robot,
rolling without slipping

$$q = (x, y, \theta, \phi_1, \phi_2)$$

How many holonomic constraints k and nonholonomic constraints m?



A slice of a position-only workspace for a typical 6R robot (here, the Mecademic Meca500)

Task spaces for:

- manipulating a rigid object?

- operating a laser pointer?

- carrying a tray of glasses to keep them vertical?