

# Recommender Systems

## lecture 2: neighbourhood-based models

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# Recap: taxonomy

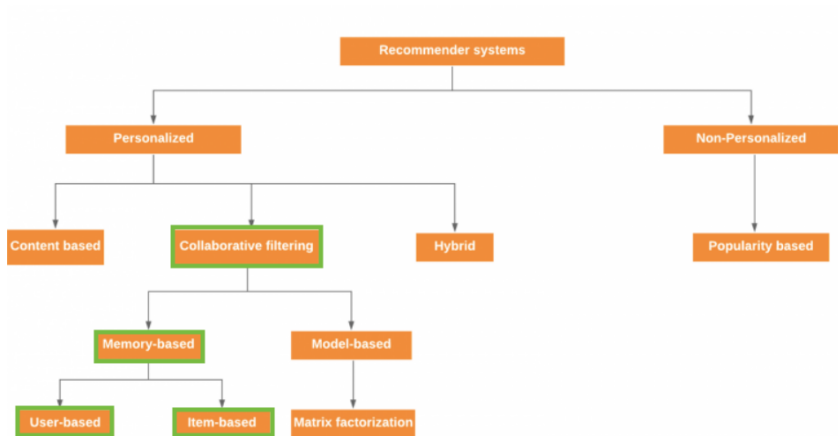
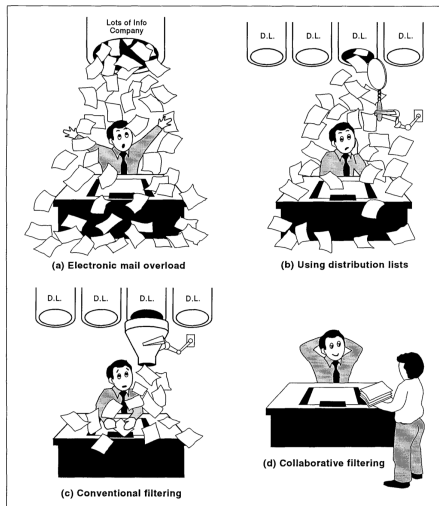


image credit: <https://thingsolver.com/introduction-to-recommender-systems>

# Collaborative filtering



## Idea:

«Collaborative filters help people make choices based on the opinions of other people.»

[Paul Resnick et al. "Grouplens" 1994]

«Collaborative filtering simply means that people collaborate to help one another perform filtering by recording their reactions to documents they read. Such reactions may be that a document was particularly interesting (or particularly uninteresting).

These reactions, more generally called annotations, can be accessed by others' filters.»

[David Goldberg et al. "Tapestry" 1992]

# Memory-based vs Model-based

- memory-based (neighbourhood-based, heuristic-based) algorithms make recommendations based on the entire collection of previously rated items by the users.
- model-based algorithms use the collection of ratings to learn a model which is then used to make recommendations.

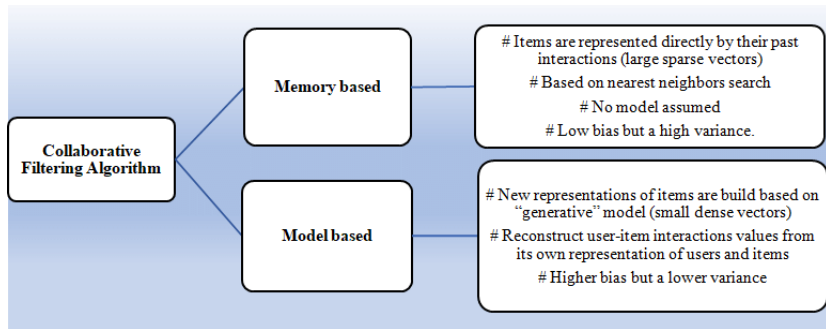


image credit: <https://iopscience.iop.org/article/10.1088/1757-899X/1022/1/012057>

# User-based vs Item-based

- a **User-based:** compute similarity between pairs of **users** based on their past interactions with items
- b **Item-based:** compute similarity between pairs of **items** based on their past interactions with users. The similarity between items is typically more stable than the similarity between the users.

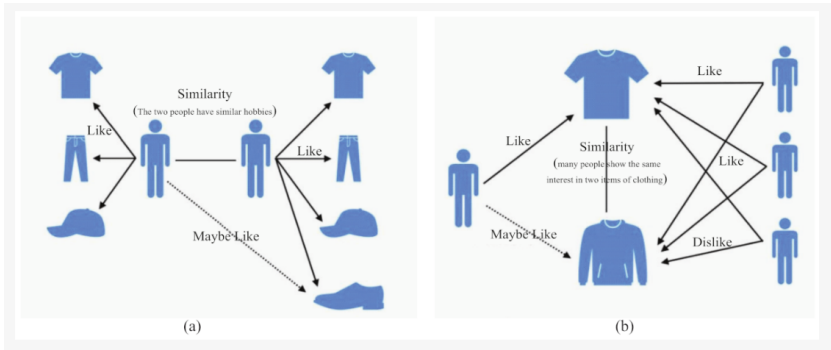


image credit: <https://www.mdpi.com/2076-3417/11/20/9554>

# Recap: problem statement

## Given:

- $U = \{u_j | j \in 1, \dots, n_{users}\}$  — set of users
- $I = \{i_j | j \in 1, \dots, n_{items}\}$  — set of items.
- $R = ||r_{ui}||$  — relation matrix of shape  $n_{users} \times n_{items}$ ,

$$r_{ui} \in \begin{cases} \text{typically } \{0, 1\} \text{— implicit feedback} \\ \text{typically } \{1, 2, 3, 4, 5\} \text{— explicit feedback} \end{cases}$$

## Possible tasks:

### Rating prediction

- Predict unknown  $r_{ui}$  — regression or (multi-class) classification

### Top-k ranking

- Rank top-k recommendations for users — **item2user (course focus)**

# Neighbourhood-based recommendations

$$r_{ui} = \begin{cases} \text{aggr } r_{u'i} & \text{— user-based} \\ & u' \in U_i \\ \text{aggr } r_{ui'} & \text{— item-based} \\ & i' \in I_u \end{cases}$$

Next we will focus on **item-based** (if not specified explicitly).  
Examples of the aggregation functions are:

$$r_{ui} = \frac{1}{N} \sum_{i' \in I_u} r_{ui'}$$

$$r_{ui} = k \sum_{i' \in I_u} \text{sim}(i, i') \times r_{ui'}$$

$$r_{ui} = \bar{r}_i + k \sum_{i' \in I_u} \text{sim}(i, i') \times (r_{ui'} - \bar{r}_i),$$

$k$  serves as a normalization factor and usually used as  $k = \frac{1}{\sum_{i' \in I_u} |\text{sim}(i, i')|}$

# Similarity weight computation

Possible variants for  $\text{sim}(i, j)$ :

$$\text{Cosine}(i, j) = \frac{\sum_{u \in U_{ij}} r_{ui} \cdot r_{uj}}{\sqrt{\sum_{u \in U_{ij}} r_{ui}^2} \sqrt{\sum_{u \in U_{ij}} r_{uj}^2}}$$

$$\text{PC}(i, j) = \frac{\sum_{u \in U_{ij}} (r_{ui} - \bar{r}_i)(r_{uj} - \bar{r}_j)}{\sqrt{\sum_{u \in U_{ij}} (r_{ui} - \bar{r}_i)^2} \sqrt{\sum_{u \in U_{ij}} (r_{uj} - \bar{r}_j)^2}}$$

For user-based (additional variants):

$$\text{MSD}(u, v) = \frac{|I_{uv}|}{\sum_{i \in I_{uv}} (r_{ui} - r_{vi})^2}; \quad \text{SRC}(u, v) = \frac{\sum_{i \in I_{uv}} (k_{ui} - \bar{k}_i)(k_{vi} - \bar{k}_i)}{\sqrt{\sum_{i \in I_{uv}} (k_{ui} - \bar{k}_i)^2} \sqrt{\sum_{i \in I_{uv}} (k_{vi} - \bar{k}_i)^2}},$$

where  $k_u$  is the average rank of items rated by  $u$



# Additional weighting (optional)

IDF, BM25, etc.

To reduce the influence of popular items in the similarity measure and to give more weight to less popular items that are more relevant to a particular user, one may use common weighting from NLP. For example, reweight cosine similarity with IDF:

$$\lambda_u = \log \frac{|I|}{|I_u|}; \quad \text{Cosine}(i, j) = \frac{\sum_{u \in U_{ij}} \lambda_u r_{ui} \cdot r_{uj}}{\sqrt{\sum_{u \in U_{ij}} \lambda_u r_{ui}^2} \sqrt{\sum_{u \in U_{ij}} \lambda_u r_{uj}^2}}$$

Shrink coefficient

When weight is computed using only a few ratings, one may use shrinkage where a weak or biased estimator can be improved if it is “shrunk” toward a null-value (typical value for  $\beta$  is 100):

$$\text{sim}'(i, j) = \frac{|U_{ij}|}{|U_{ij}| + \beta} \text{sim}(i, j)$$

$$r_{ui} = \frac{\sum_{i' \in J} \text{sim}(i, i') \times r_{ui'}}{\sum_{i' \in J} \text{sim}(i, i')},$$

where  $J$  — set of  $k$  most similar items to  $i$ .

## Note

Consider  $W \in \mathbb{R}^{|I| \times |I|}$  — item-item weight-matrix,  
 $R_{u,\cdot}$  — row  $u$ ;  $W_{\cdot,i}$  — column  $i$ .

$r_{ui}$  might also be computed using  $R_{u,\cdot}$  and  $W_{\cdot,i}$

## Exercise

The UserKNN algorithm also exists. Derive its formulas.

$r_{ui}$  is calculated as a sparse aggregation of items, user  $u$  interacted with:

$$r_{ui} = R_{u,\cdot} \cdot W_{\cdot,i}$$

## Optimization problem

Learning  $W$  for SLIM:

$$\begin{aligned} & \underset{W}{\text{minimize}} && \|R - RW\|_F^2 + \frac{\beta}{2} \|W\|_F^2 + \lambda \|W\|_1 \\ & \text{subject to} && W \geq 0 \\ & && \text{diag}(W) = 0, \end{aligned}$$

where  $\|W\|_1 = \sum_{i=1}^n \sum_{j=1}^n |w_{ij}|$  is the entry-wise  $l_1$ -norm of  $W$ ,  
and  $\|\cdot\|_F$  is the matrix Frobenius norm.

$r_{ui}$  is calculated similar to SLIM:

$$r_{ui} = R_{u,\cdot} \cdot W_{\cdot,i}$$

### Optimization problem

Learning  $W$  for EASE:

$$\begin{aligned} & \underset{W}{\text{minimize}} && ||R - RW||_F^2 + \lambda ||W||_F^2 \\ & \text{subject to} && \text{diag}(W) = 0 \end{aligned}$$

- Square loss allows for a closed-form solution (next slide). Training with other loss functions, however, might result in improved ranking accuracy.
- The constraint of a zero diagonal,  $\text{diag}(W) = 0$ , is crucial for ranking accuracy (in contrast to  $W \geq 0$ ).

# EASE: closed-form solution derivation

We start by forming the Lagrangian:

$$L = \|R - RW\|_F^2 + \lambda \|W\|_F^2 + 2 \cdot \gamma^T \cdot \text{diag}(W),$$

where  $\gamma = (\gamma_1, \dots, \gamma_{|I|})^T$  — Lagrangian multipliers.

Next, set its derivative to zero, which yields the estimate of the weight matrix:

$$W = (R^T R + \lambda E)^{-1} (R^T R - \text{diagMat}(\gamma))$$

Defining (for sufficiently large  $\lambda$ )

$$P = (R^T R + \lambda E)^{-1}$$

this can be substituted into the previous equation:

$$\begin{aligned} W &= (R^T R + \lambda E)^{-1} (R^T R - \text{diagMat}(\gamma)) \\ &= P (P^{-1} - \lambda E - \text{diagMat}(\gamma)) \\ &= E - P (\lambda E + \text{diagMat}(\gamma)) \\ &= E - P \cdot \text{diagMat}(\tilde{\gamma}), \text{ (where } \tilde{\gamma} = \lambda \vec{1} + \gamma) \end{aligned}$$

# EASE: closed-form solution

$$0 = \text{diag}(W) = \vec{1} - \text{diag}(P) \odot \tilde{\gamma}$$

$$\Rightarrow \tilde{\gamma} = \vec{1} / \text{diag}(P),$$

where  $/$  — elementwise division.

Substituting into previous slide gives

$$W = E - P \cdot \text{diagMat}(\vec{1} / \text{diag}(P))$$

## EASE closed-form solution

$$P = (R^T R + \lambda E)^{-1}$$

$$W_{ij} = \begin{cases} 0, & \text{if } i = j \\ -\frac{P_{ij}}{P_{jj}}, & \text{otherwise} \end{cases}.$$

- ① *F. Ricci, L. Rokach, B. Shapira.* (2011). Recommender Systems Handbook.
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- ④ *X. Ning and G. Karypis* (2011). SLIM: Sparse Linear Methods for Top-N Recommender Systems, ICDM
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