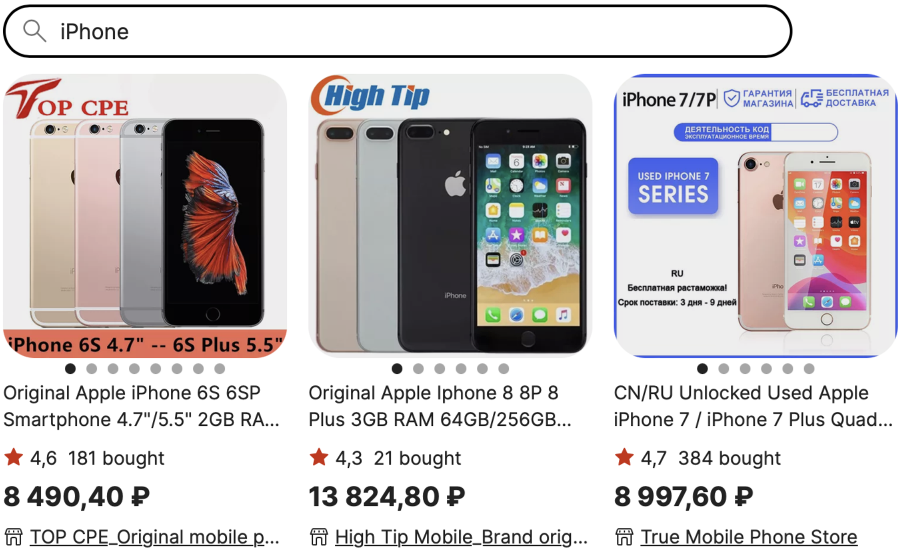
1 (1.5 + 1* point) Multi-armed bandits

Consider 3-armed bandit problem as described in picture (action is choosing particular item, reward is a rating received).
You have information about mean reward $\mathcal{D} = \{(1, 4.6), (2, 4.3), (3, 4.7)\}$ and number of clicks for each arm.



Here and further you may use $[p_1, p_2, p_3]^T$ notation for policy.

1. (0.5 point) Compute ε-greedy policy π_ε (set $\varepsilon = 0.01$).
2. (1 point) Compute UCB policy π_{UCB} (set α by yourself, you may choose from $\{0.1, 0.5, 1\}$).
   Note: Hoeffding inequality works not only for bernoulli rewards, but for arbitrary $r \in [0, 1]$, so you can scale reward into [0, 1] to apply formulas from lecture.
3. (1* point) Explain what is required to use Thompson Sampling here.

2 (2.5 points) Counterfactual evaluation

Using problem setup from task 1:

1. compute estimation of logging policy π_0
2. evaluate policy $\pi_1 = [0.3, 0.04, 0.66]^T$
   (get expected mean rating from running π_1: $\hat{V}(\pi_1, \mathcal{D}) = \mathbb{E}_{p(x)\pi_1(a|x)p(r|x,a)}[r]$)
3. evaluate policy $\pi_2 = [0.3, 0.66, 0.04]^T$
4. choose 1 most promising policy from task 1 and evaluate it.
5. Analyze results.
   Is it possible to evaluate policies from 3 previous subtasks with adequate precision? If yes describe how, otherwise explain why.

3 (1 point) Unbiasedness of IPS

1. (0.5 point) Prove that IPS estimator is unbiased, e.g.

$$\mathbb{E}_{\mathcal{D}}\left[\hat{V}_{\text{IPS}}(\pi; \mathcal{D})\right] = V(\pi) = \mathbb{E}_{p(x)\pi(a|x)p(r|x,a)}[r]$$

2. (0.5 point) Under which conditions unbiasedness holds?