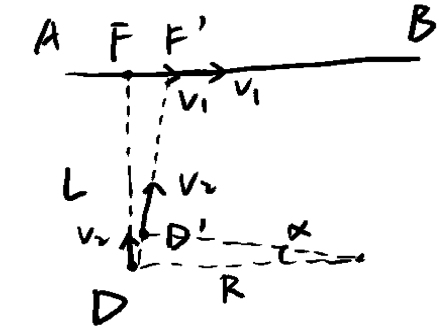
程力

练 2-17



$$a_n = \frac{v_2^2}{R}$$

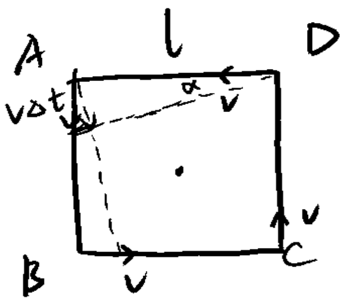
$$v_1 \Delta t = \alpha L$$

$$v_2 \Delta t = \alpha R$$

$$\Rightarrow a_n = \frac{v_1 v_2}{L}$$

$$\left(\dot{\vec{v_2}} = \vec{\omega} \times \vec{v_2} = \frac{\frac{v_1 \Delta t}{L}}{\Delta t} \cdot v_2 \right)$$

2-18



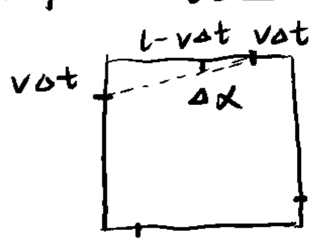
在中心连线上速度分量不变

$$t = \frac{\frac{\sqrt{2}}{2} l}{v \cos 45^\circ} = \frac{l}{v}$$

$$s = vt = l$$

$$a = \frac{v^2}{\rho} = \frac{v^2}{l}$$

in fact. 经过 Δt

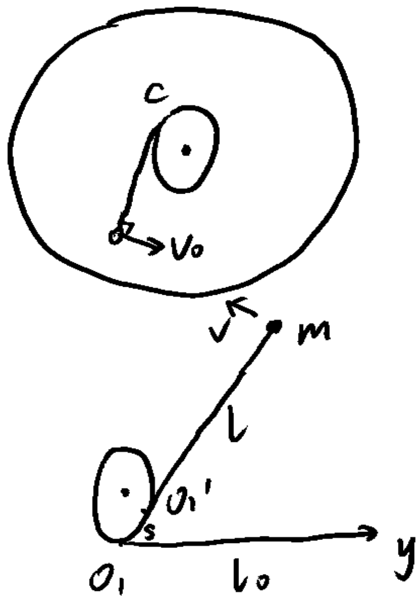


$$\Delta\alpha = \frac{v\Delta t}{l - v\Delta t} = \frac{v\Delta t}{l}$$

$$\rho = \frac{\Delta s}{\Delta \alpha} = \frac{v\Delta t}{\frac{v\Delta t}{l}} = l$$

$$\left(\dot{\vec{v}} = \vec{\omega} \times \vec{v} = \frac{v\Delta t}{l - v\Delta t} \cdot v = \frac{v^2}{l} \right)$$

2-20



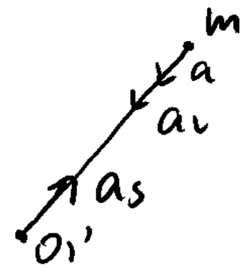
对 O_1' 分析：受到切向 a_s

对 m 分析（在 O_1' 系中受惯性力 ma_s）

绳长改变导致 a_l

向心加速度 a

$T = ma + ma_l + ma_s$



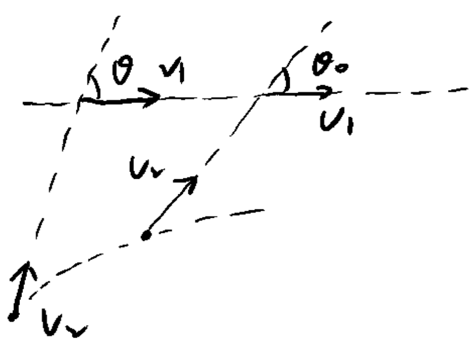
其中 $l + s = l_0 \equiv C$

$\Rightarrow a_l + a_s = 0.$

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$

$\Rightarrow T = \frac{mv_0^2}{l}$

2-21



$v_1 = 5m/s \quad v_2 = 4m/s$

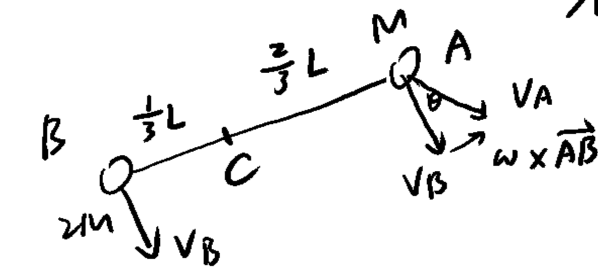
$\boxed{\text{观察速度连线方向}}$

当 $v_2 = v_1 \cos\theta_0$ 时

二者间距最小，为 30m

此时 $\vec{a} = \vec{\omega} \times \vec{v_2} = 4m/s$.

2-22



某时刻 $v_A=v$, $v_B=2v$

求此刻系统因运动新增张力

合运动 = $\boxed{\text{质心平动} \to \text{绕质心转动}}$

↓ 不会因此新增张力

↓ 产生N

$$N = M\left(\tfrac{2}{3}L\right)\omega^2$$

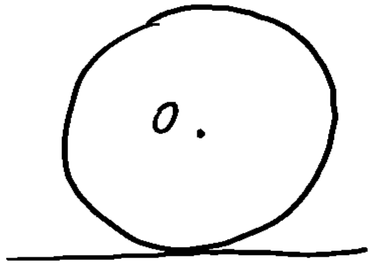
或 $$N = 2M\left(\tfrac{1}{3}L\right)\omega^2$$

θ变化时，$|\omega\times\overrightarrow{AB}| = \omega L \in [v, 3v]$

$\Rightarrow$ N范围

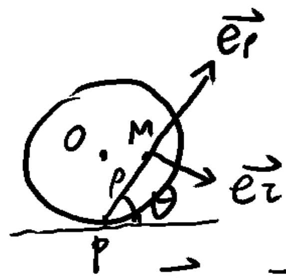
2-24

$\omega \cdot \beta$



(1) v与a方向一致的点集？

(2) v与a方向垂直的点集？



任取一点 $M(\rho, \theta)$

P为瞬心

$$\vec{v_M} = \vec{\omega}\times\overrightarrow{PM} = \omega\rho\vec{e_\tau}$$

$$\vec{a_M} = \vec{a_P} + \vec{\beta}\times\overrightarrow{PM} - \omega^2\overrightarrow{PM}$$

$$\vec{a_P} = \vec{\beta}\times\overrightarrow{OP} + \omega^2\overrightarrow{PO} + a_0$$

$v_0 = \omega r \quad a_0 = \beta r$

$$\Rightarrow \vec{a_0} = -\beta\times\overrightarrow{OP}$$

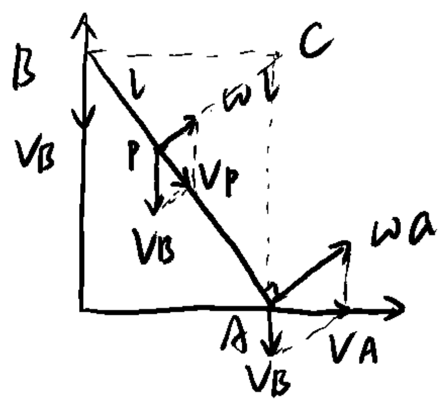
(1) $\vec{a_M}\cdot\vec{e_\rho} = 0 \Rightarrow \rho = r\sin\theta$

以OP为直径的圆

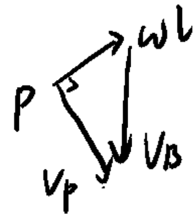
(2) $\vec{a_M}\cdot\vec{e_\tau} = 0 \Rightarrow \rho = \frac{\omega^2 r}{\beta}\cos\theta$

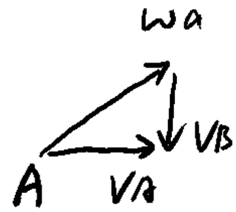
2-26

杆上某一点P速度沿杆



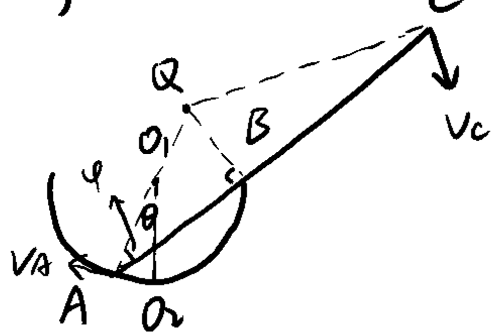
B参考点





in fact，(瞬心C)　$CP \perp AB$

例 2-5



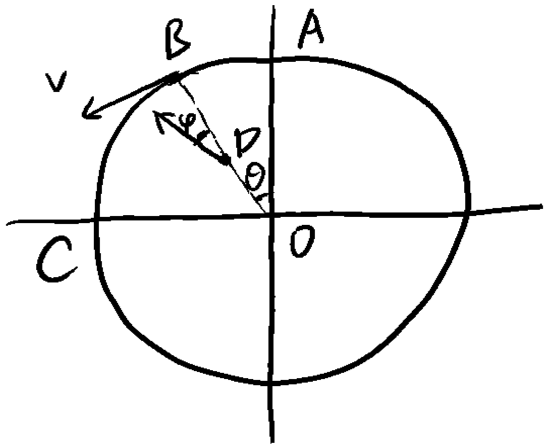
$V_A = \frac{1}{2} V_c$，求 AB:BC

V_B 只能沿杆

从而可以找到瞬心Q

$V_A = \frac{1}{2} V_c \Rightarrow QA = \frac{1}{2} QC$

例 2-11



由 O、A 出发 以匀速 v 追及.

过程中保持 fox dog O 共线

dog 应沿什么轨道追击？

分析 D 点：$v_\theta = r\dot{\theta}$

$$v_r = \sqrt{v^2-(r\dot{\theta})^2} = \dot{r}$$

$$\dot{\theta} = \frac{v}{R}. \qquad \dot{r} = v\cos\varphi$$

$$\Rightarrow \quad r = R\sin\varphi$$

$$\Rightarrow \quad \frac{dr}{d\varphi} = R\cos\varphi.$$

$$r\dot{\theta} = v_\theta = v\sin\varphi$$

$$\dot{\theta} = \frac{d\theta}{dr}\cdot \dot{r} = \frac{d\theta}{dr} v\cos\varphi$$

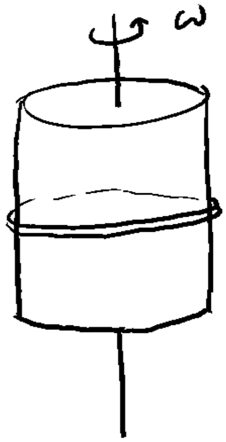
$$\frac{d\theta}{dr} = \frac{1}{r}\tan\varphi = \frac{\tan\varphi}{R\sin\varphi} = \frac{1}{R\cos\varphi}$$

$$\Rightarrow \quad \theta = \varphi$$

$$\Rightarrow \quad r = R\sin\theta$$

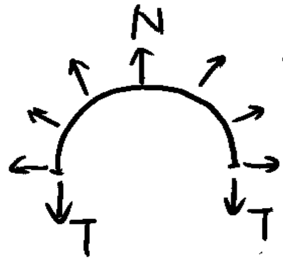
3-11

橡皮圈 (m, r_1, k) 圆柱 $(r_2 > r_1)$



圆柱转动的ω 保持橡皮圈不滑下.

张力 $T = k\cdot(2\pi r_2 - 2\pi r_1)$



$\boxed{\text{对于一半橡皮圈}\quad 2T = N\cdot 2r_2}$

dl: $dm = \dfrac{m}{2\pi r_2} dl$

对于每个 dl. 受惯性离心力 $df = \dfrac{m}{2\pi r_2} dl\cdot\omega^2 r_2$

$$\frac{df}{dl} = \frac{m\omega^2}{2\pi}$$

$$\mu\left(N - \frac{df}{dl}\right) = \frac{mg}{2\pi r_2} \Rightarrow \omega$$

(N、$\frac{df}{dl}$ 均为单位长度受力)

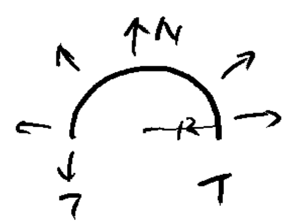
点评: 1、出现切向方向的张力时可取半圆进行分析.

2. 选取一段 dl 进行分析.

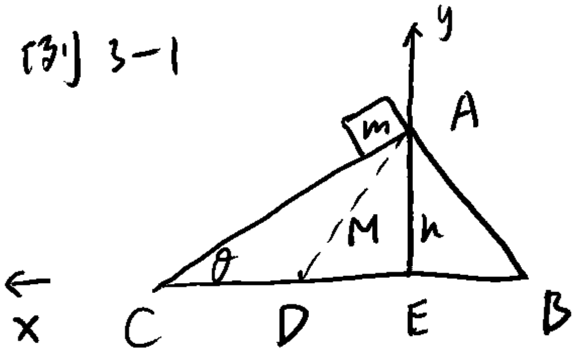
↓

记住此构型

$N\cdot 2R = 2T$



例 3-1



(1) m 从顶端滑到底时 M位移

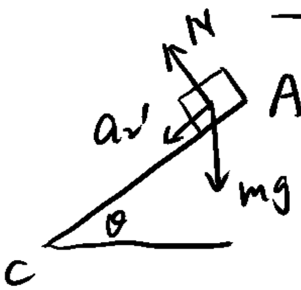
$$m\cdot DE = M\cdot CD$$

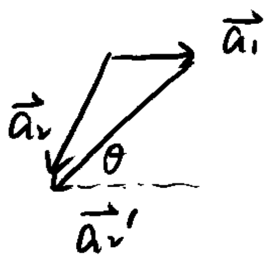
$$CD + DE = h\cot\theta$$

$$\Rightarrow \quad CD = \frac{m}{M+m} h\cot\theta$$

(2) m下滑时，M对地加速度 a_1，m对M：a_2'，m对地 a_2

$$\underline{\vec{a_1} + \vec{a_2}' = \vec{a_2}}$$





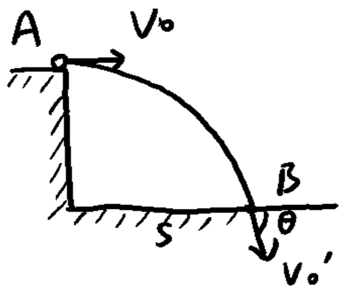
$$\begin{cases} N\sin\theta = m(a_2'\cos\theta - a_1) \\ mg - N\cos\theta = m a_2'\sin\theta \end{cases}$$

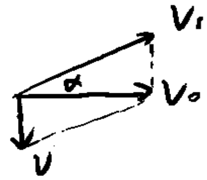
$$N\sin\theta = M a_1$$

$$\Rightarrow a_1、a_2'、a_2、N$$

例3-11

$\boxed{\text{将运动分解 以抵消 } f}$ （类比 $qvB - qE$）



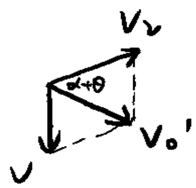


使 $kv=mg$ 从而以v匀速向下.

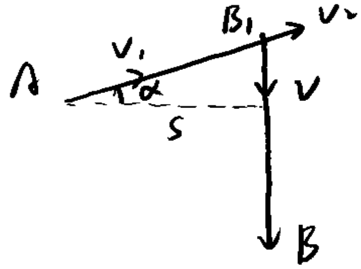
在v_1方向变减速运动 $\vec{f_1} = -k\vec{v_1}$

$\vec{f} = -k\vec{v}$

求 $\vec{v_0'}$, S

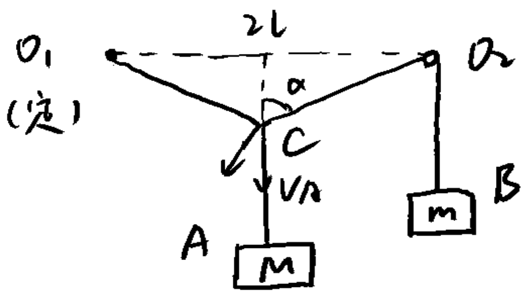


$A \to B = A \to B_1 \to B$.



$S = |AB_1|\cos\alpha$ ✓

★ 例3-12



$\boxed{\text{速度关联}}$ $v_B = 2v_A\cos\alpha$

$\alpha = 60^\circ$时 v_A达到最大

$\Rightarrow \boxed{a_A = 0 \Rightarrow \ddot{r} - r\dot{\theta}^2 = 0}$

对C点: $\ddot{r} = r\dot{\theta}^2 = \frac{(v_A\sin\alpha)^2}{\frac{l}{\sin\alpha}}$

$a_B = 2\ddot{r}$

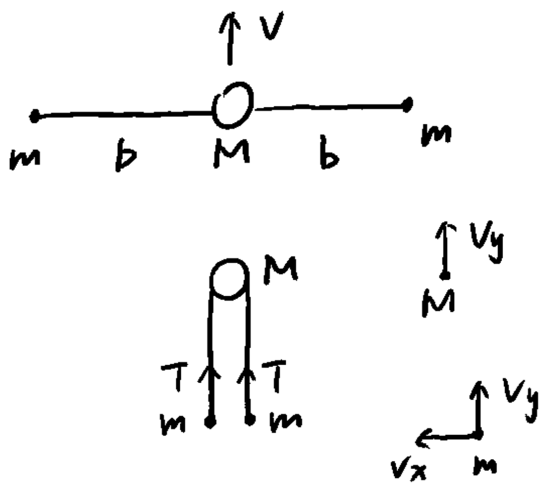
$2T\cos\alpha = Mg$

$T - mg = m a_B$

习题 3-28

M获得初速度v.

求两端小球发生互碰前绳中张力.



$$Mv = (M+2m) V_y$$

$$a_M = \frac{2T}{M}$$

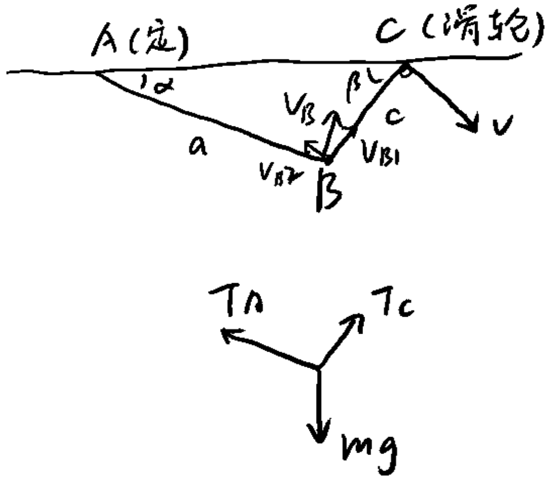
M系中 $T + m a_M = m \frac{V_x^2}{b}$

$$\frac{1}{2}Mv^2 = \frac{1}{2}MV_y^2 + 2\cdot\frac{1}{2}m(V_x^2+V_y^2)$$

$\Rightarrow T$ ✓

习题 3-31

以恒定速率收绳. 求绳中张力



A点. B绕A圆周运动

$\Rightarrow V_B \perp AB$

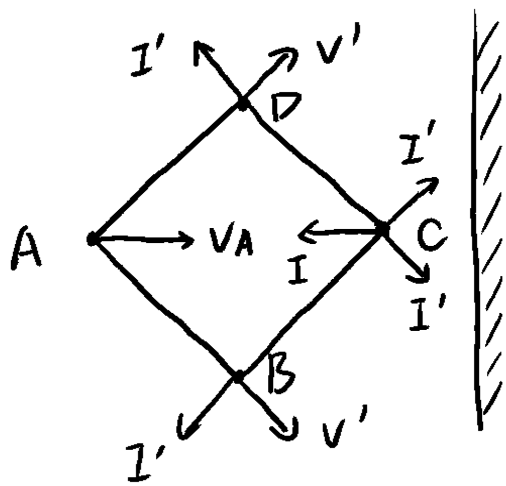
BC方向. $a_{Br} = \ddot{r} - r\dot{\theta}^2$

其中 $\ddot{r} = 0$

$$\dot{\theta} = \frac{V_{B2}}{c}$$

结合受力分析 ✓

4-12 正方形框以 v 向右运动，C与墙壁碰后变为0.



求碰后一瞬间 V_A.

找全冲量	速度关联

V_B、V_D 沿杆

$$\frac{\sqrt{2}}{2}V_A = V_B = V_D = v'$$

C应受到墙壁 I，CD和CB方向 I'

C: $I - \sqrt{2}I' = mv$

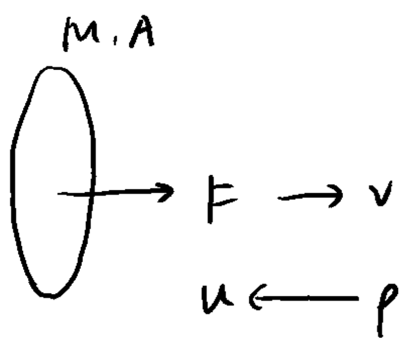
D: $I' = m\frac{\sqrt{2}}{2}v$

* $I = 4mv - (mV_A + 2\cdot mv'\cdot\frac{\sqrt{2}}{2})$

$\Rightarrow V_A = v$

注意速度方向及一些变形.

4-16



M M+dM
v→ v+dv →
dM
←u

$Mdv + dM\cdot v + dM\cdot u = Fdt$

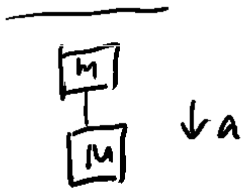
$\Rightarrow Fdt = d(Mv) + dM\cdot u$

$dM = \rho A(v+u)dt$

$M = M_0 + \rho Aut + \rho Ax.$

$Ft = Mv + u(M - M_0)$

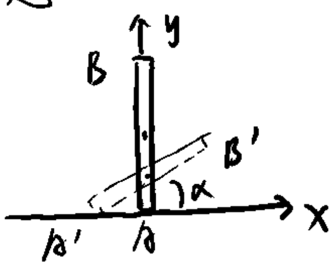
习题 4-3　　注意整体视角



$F_{合} = (M+m)a$

$I = (M+m)a\,(t+t') = mv \Rightarrow v$

习题 4-12　　杆倾倒，求 B' 轨迹。

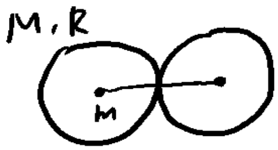


水平方向不受外力，x_c 不变

$\Rightarrow x_B = l\sin\alpha,\quad y_B = 2l\cos\alpha$

$$\frac{x_B^2}{l^2} + \frac{y_B^2}{4l^2} = 1$$

5-11



荷叶不会产生竖直方向运动

(1) 青蛙能一次跳离荷叶，至少应做多少功？

跳起瞬间 青蛙相对地 v_x、v_y

* 荷叶后退 v

$$W = \frac{1}{2}m(v_x^2+v_y^2) + \frac{1}{2}Mv^2.$$

$$mv_x = Mv$$

跳离荷叶 $x' \geq R$ $\quad x' = \frac{2v_y}{g}(v_x+v)$

(2) 由 A 荷叶中央起跳落到 B 荷叶中央，随之一起向前滑动

求机械能损失与青蛙所做功比值

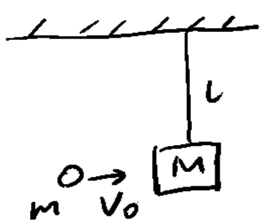
$$x = \frac{2v_y}{g}v_x = 2R$$

注意条件!!!

$$mv_x = (m+M)v_x'$$

$$\Delta E = \frac{1}{2}m(v_x^2+v_y^2) + \frac{1}{2}Mv^2 - \frac{1}{2}Mv^2 - \frac{1}{2}(M+m)v_x'^2$$

5-12



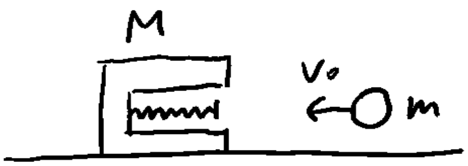
$\boxed{两体问题}$ 总动能 $E_k = \frac{1}{2}mv_0^2$

质心系中系统动能 $E_k' = \frac{1}{2}\mu v_0^2$

$$E_{kc} = \frac{1}{2}mv_0^2 - \frac{1}{2}\mu v_0^2.$$

E_k' 将被完全损耗，仅剩 E_{kc}

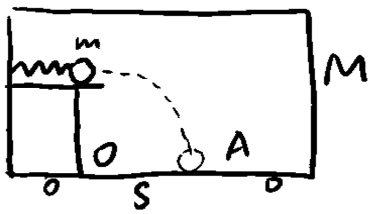
5-13　弹簧(k)的最大压缩量?



$E_k' = \frac{1}{2}\mu v_0^2 = \frac{1}{2}kx^2$

E_k' 可被转移，剩余 E_{kc}

5-15　弹簧解除压缩后 运动至点A

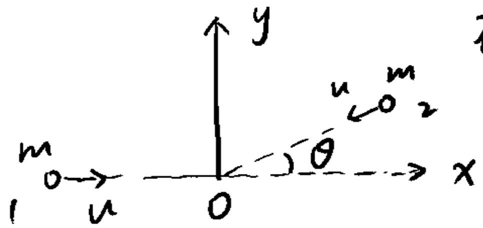


小车固定. $W = \frac{1}{2}mv^2$

小车不固定: $W = \frac{1}{2}\mu V_{相}^2$ (质心系中 $V_c = 0$)

$\frac{S_2}{S_1} = \frac{V_{相}}{V_1}$

5-17　1,2具有相同质量 相同能量 作碰撞.



在某个以v运动的质心系中看1,2作对心正碰

$u_1 = u\vec{i}$　$u_2 = -u\cos\theta\,\vec{i} - u\sin\theta\,\vec{j}$

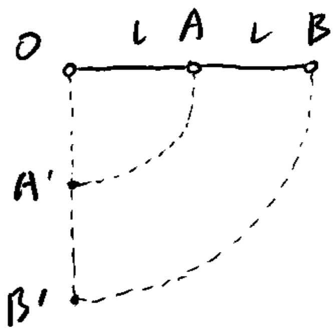
$\vec{v} = \frac{m_1\vec{u_1} + m_2\vec{u_2}}{m_1+m_2} \Rightarrow |\vec{v}|$

碰前质心系中的总动能: $E_k' = \frac{1}{2}\mu V_{相}^2$

地系中的总动能. $E_k = mu^2$

$\left(E_{kc} + E_k' = \frac{1}{2}\cdot 2m\cdot v^2 + \frac{1}{2}\mu V_{相}^2 = E_k = mu^2 \right)$

5-26



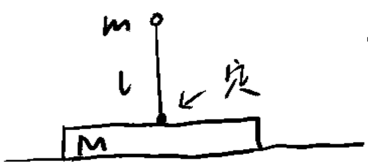
$$mgl + 2mgl = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$$

$$\Rightarrow v_A = \sqrt{\frac{6gl}{5}} \quad v_B = 2\sqrt{\frac{6gl}{5}}$$

而若仅存在A/B $v_A' = \sqrt{2gl}$, $v_B' = \sqrt{4gl}$

$$\Rightarrow W_A = \frac{1}{2}mv_A^2 - \frac{1}{2}mv_A'^2 \ . \ W_B$$

5-27



接近地面(板)时小球速度. 杆中张力

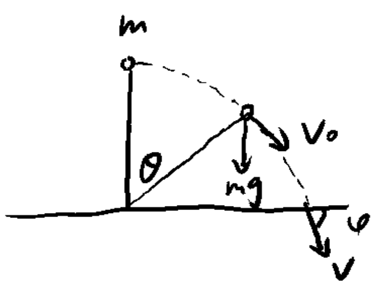
① $mgl = \frac{1}{2}mv^2$

$T = ma_m = Ma_M$

$a_m = a_{m\to M} + a_M = \frac{v^2}{l} + a_M$

② $\boxed{\text{两体问题}}$ $mgl = \frac{1}{2}mv^2$

$$T = \frac{mv^2}{l}$$



杆与地面间有足够大的静摩擦力

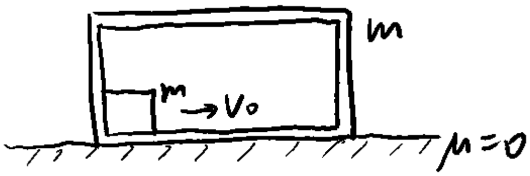
当 $T=0$ 时 杆球分离. $v = \sqrt{2gl}$

由 v_0 后续的斜抛运动确定 v 方向.

$\boxed{\text{tips}}$ 动量定理中 v 为绝对速度

$\frac{v^2}{R}$ 中需度为相对速度

★ 5-37　　恢复系数 e　　$\boxed{\text{两体问题}}$



$$E_0=\frac{1}{2}mv_0^2 \qquad v_{相0}=v_0 \qquad \underline{v_c=\frac{1}{2}v_0}$$

始终不变

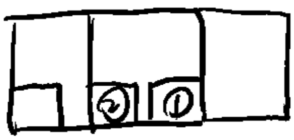
经过 n 次碰撞 $\underline{v_{相n}=e^n v_0}$

从而 $E_k'=\frac{1}{2}\mu v_0^2-\frac{1}{2}\mu(e^n v_0)^2$ 被损耗

从滑块开始运动到刚完成 4 次碰撞期间，箱子平均速度？

第 i 次碰撞时刻 t_i，每一段时间内相对位移为 L.

$$t_1-0=\frac{L}{v_0} \qquad t_2-t_1=\frac{L}{ev_0} \qquad t_3-t_2=\frac{L}{e^2v_0} \qquad t_4-t_3=\frac{L}{e^3v_0}$$



偶数次碰撞前
箱子相对质心位移均为0

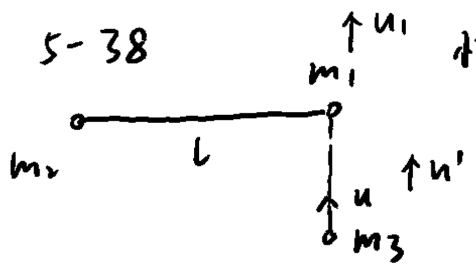
$$\underline{u_{箱}=u_c+u_{箱c}}$$

$$u_0=0$$
$$u_1=\frac{1}{2}v_0+\frac{1}{2}ev_0$$
$$u_2=\frac{1}{2}v_0-\frac{1}{2}e^2v_0$$
$$u_3=\frac{1}{2}v_0+\frac{1}{2}e^3v_0$$

$$\bar{u}=\frac{u_0\cdot\Delta t_0+u_1\Delta t_1+u_2\Delta t_2+u_3\Delta t_3}{\Delta t_{总}}=\frac{v_0}{2}$$

此外，$\bar{u}=\dfrac{\text{质心位移}+\overset{=0}{\text{相对质心位移}}}{\text{时间}}=\dfrac{\text{质心位移}}{\text{时间}}=v_c=\dfrac{v_0}{2}$

5-38 恢复系数e，求碰后瞬间绳上张力



$$\begin{cases} m_3 u = m_1 u_1 + m_3 u' \\ u_1 - u' = eu \end{cases} \Rightarrow u_1$$

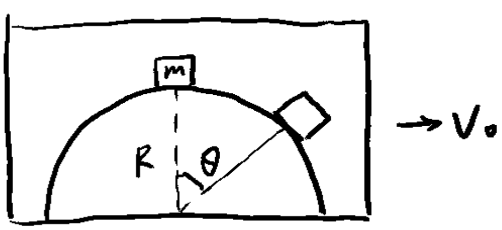
① $T = \mu \dfrac{u_1^2}{l}$ □

② 在质心系中考虑 (m_1、m_2系统)

$$u_c = \frac{m_1 u_1}{m_1+m_2} \qquad l_1 = \frac{m_2}{m_1+m_2} l.$$

$$T = m_1 \frac{(u_1-u_c)^2}{l_1}$$ □

SOME TIPS (e.g. 5-4)



位于车厢参考系 v'为相对车厢速度

$$mgR(1-\cos\theta) = \frac{1}{2}mv'^2$$

地面系 v为m相对地速度

$\vec{v} = \vec{v'} + \vec{v_0}$ 与圆柱面不垂直

需考虑脱离圆柱面

从而存在支持力做功

$$\underline{\frac{1}{2}mv_0^2} + W_N + mgR(1-\cos\theta) = \frac{1}{2}mv^2$$

本就存在的向右V_0的速度.

例 5-6　　原长 a，由 b（b>a）释放

求两物体相碰时相对速度大小

质心系　设相碰时相对质心为 V_1、V_2

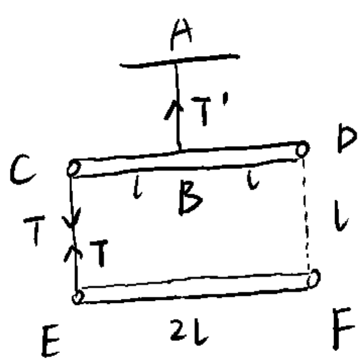
$$\frac{1}{2}k(b-a)^2-\mu m_1 g l_1-\mu m_2 g l_2=\frac{1}{2}m_1V_1^2+\frac{1}{2}m_2V_2^2$$

l_1、l_2 为相对质心的位移　$m_1l_1=m_2l_2$

$l_1+l_2=b$

质心系中动量始终为0，两物位移具有确定关系　$m_1V_1=m_2V_2$ ✓

6-6　　突然剪断 DF，剪断后瞬间 AB 中张力



E　刚剪断时杆 EF 对其无作用

$$mg-T=ma_E$$

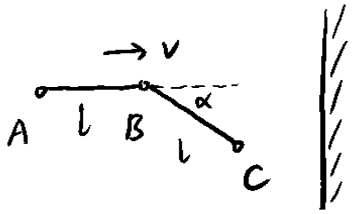
CD　刚剪断时绕 B 点运动

对 B 轴　$\boxed{Tl=\frac{\Delta(mlv+mlV)}{\Delta t}=2mla}$

$$T'=T+2mg$$

6-7

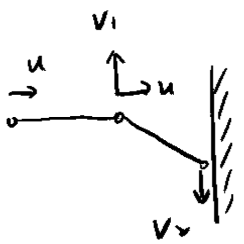
C与完全非弹性墙壁相碰时，求受冲量 I.



$$I = 3mv - 2mu$$

$$v_1 = v_2$$

$$u\cos\alpha - v_1\sin\alpha = v_2\sin\alpha$$

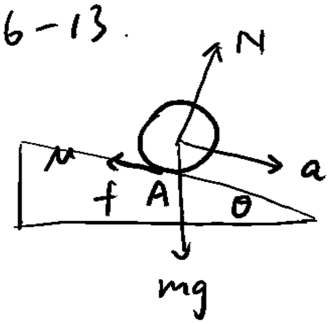


冲量作用点在C点 ⇒ 碰撞中相对于过C的轴的冲量矩为0 ⇒ 碰撞前后相对C轴角动量守恒

$$2mul\sin\alpha + mv_1l\cos\alpha = 2mvl\sin\alpha$$

本题也可通过设每根杆上冲量解出.

6-13.



对于 $\theta < \theta_c$ 条件下，圆柱将无滑动滚下.

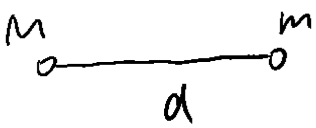
纯滚动: $mg\sin\theta - f = ma$

$mg\cos\theta = N$ $\quad f = \mu N$

$fR = I\beta$ $\quad a = \beta r$

另: 点A为瞬心, $mgR\sin\theta = I_A\beta$ 即可求出

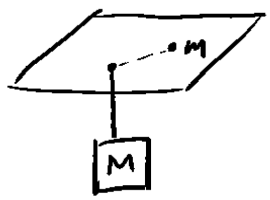
6-28　　在万有引力作用下相向运动，经t后碰撞



$$\frac{(\frac{1}{2}d)^3}{T^2}=\frac{G(M+m)}{4\pi^2}$$

$t=\frac{1}{2}T$（认为M·m同时充当焦点、顶点）

习题　6-16　　m在与孔连线方向振动，与孔最近b.最远a

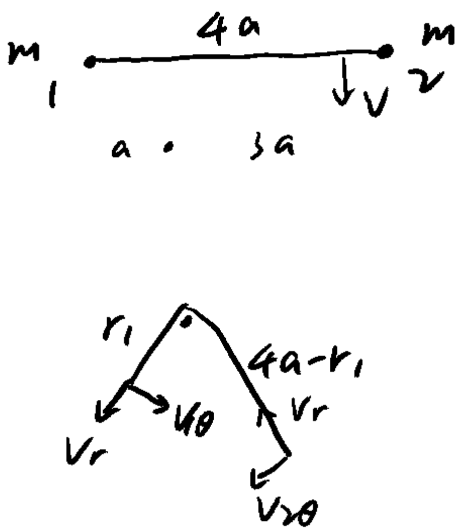


受有心力 ⇒ 角动量守恒

$$mv_1 b = mv_2 a$$

$$Mg(a-b)=\frac{1}{2}mv_1^2-\frac{1}{2}mv_2^2$$

6-17　　绳子运动过程中碰到钉子，求1与钉子最大距离



速度分解

与钉子距离最远 ⇒ $v_r=0$

$$2\cdot\frac{1}{2}mv^2=(\quad)+(\quad)$$

$$mva=mv_{1\theta}r_1$$

$$mv\cdot 3a=mv_{2\theta}(4a-r_1)$$