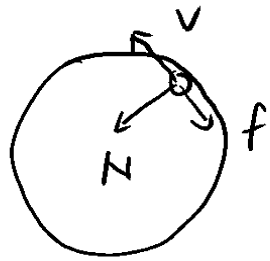
# 物理 题目 运动·力学·刚体

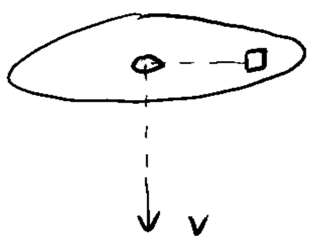
## △ 运动学




$$N = \frac{mv^2}{R}$$

$$f = -m\frac{dv}{dt}$$

$$f = \mu N \quad \Rightarrow -\mu\frac{v^2}{R} = \frac{dv}{dt}$$




物体 $m$ 以 $\omega_0$ 作半径为 $r_0$ 的圆周运动

自 $t=0$ 始，手拉绳子以 $v$ 向下运动

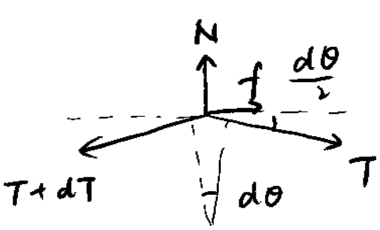
$$\dot{r} = -v \quad \ddot{r} = 0 \quad \dot{\theta} = \omega$$

$$m(\ddot{r} - r\dot{\theta}^2) = -F \quad ①$$

$$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0 \quad ②$$

由② $\quad r\frac{d\omega}{dt} = 2v\omega \Rightarrow \omega(t)$

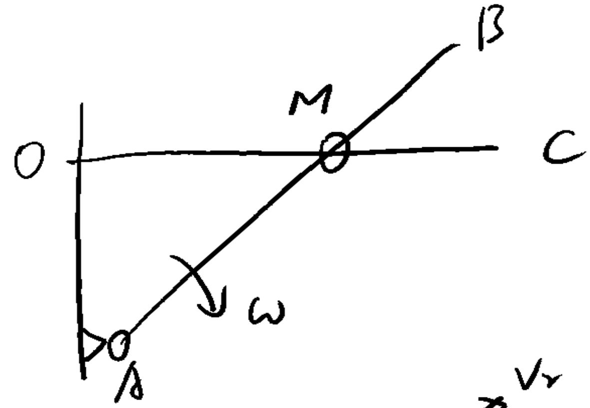
由① $\quad \Rightarrow F(t)$


绳与圆柱间 $\mu$，求绳处于滑动边缘时绳两端张力关系

$$\begin{cases} x: -(T+dT)\cos\frac{d\theta}{2} + T\cos\frac{d\theta}{2} - f = 0 \\ y: -(T+dT)\sin\frac{d\theta}{2} - T\sin\frac{d\theta}{2} + N = 0 \end{cases}$$

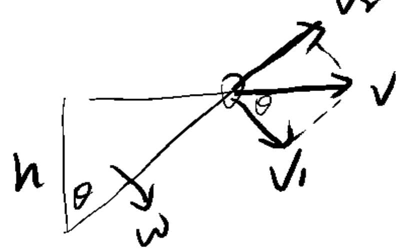
$$f = \mu N$$




分析M:

$\boxed{\text{合运动}}$ M沿OC向右运动

将向右v分解成沿杆/垂直杆




$\boxed{\text{寻找不变量}}$ $h$, $\omega$

$$v_1 = \frac{\omega h}{\cos\theta} \Rightarrow v = \frac{\omega h}{\cos^2\theta}$$

$$a = \frac{dv}{dt} = -2\frac{\omega h}{\cos^3\theta}(-\sin\theta)\,\omega$$

$$= \frac{2\omega^2 h \sin\theta}{\cos^3\theta}.$$

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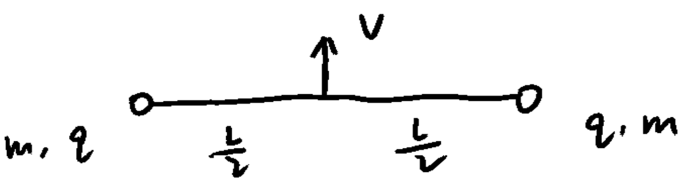



$\boxed{\text{变参考系}}$ 以中点为参考系

则两球圆周运动

运动过程中

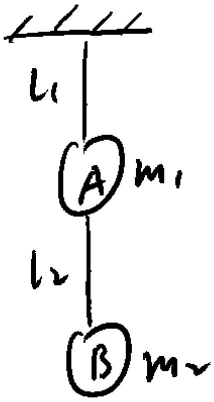
相距最近多远?

相距最近时, $v=0$, 电势能最大

$$2\cdot\frac{1}{2}mv^2 = k\frac{q^2}{r_{min}} - k\frac{q^2}{L}$$

$$\Rightarrow r_{min} = \frac{kq^2}{mv^2 + \frac{kq^2}{L}}$$

瞬间给A一个向右的v.




$$a_A = m_1\frac{v^2}{l_1}$$

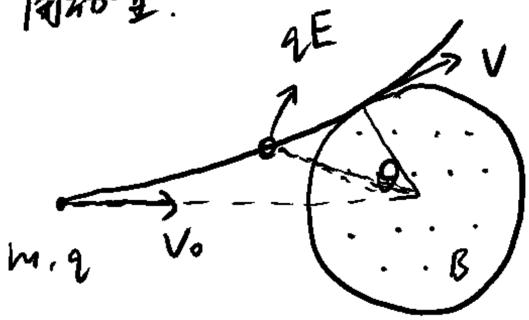
$\boxed{\text{B相对于A}}$ $a_{BA} = m_2\frac{v^2}{l_2}$

$\boxed{\text{B相对于地}}$ $a_B = m_1\frac{v^2}{l_1} + m_2\frac{v^2}{l_2}$

$T_2 - m_2 g = m_2 a_B$ (补非惯性力同理)

$T_1 - m_1 g - T_2 = m_1 a_A$

有关角动量.




涡旋电场 $E = \frac{kR^2}{2r}$

运动方向改变过程中

$M = qE\cdot r = \frac{kqR^2}{2}$ 不变

$$\frac{dB}{dt} = k > 0$$

从而 $\boxed{Mt = mvR}$

$$\boxed{M\theta = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2}$$

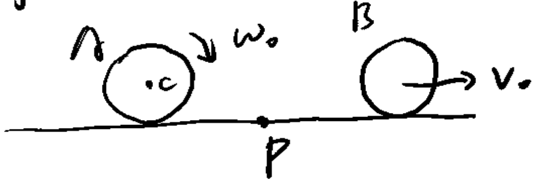
# △刚体

角动量守恒列式

碰后 非纯滚→纯滚

以P为参考点

eg1.

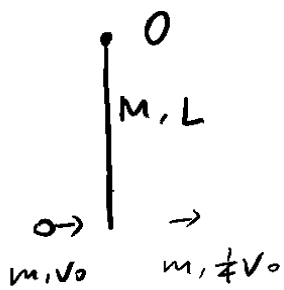



$$\boxed{A.\ I_c\omega + mv_cR = I_c\omega_0}$$

$v_c = \omega R$（纯滚）

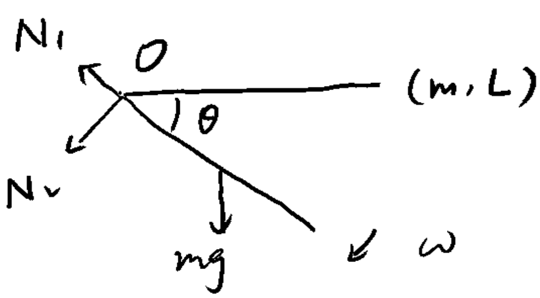
$v_c = \frac{2}{7}v_0 \quad \omega = \frac{2}{7}\frac{v_0}{R}$

eg2.




$$\boxed{mv_0L = m\frac{1}{4}v_0L + I\omega}$$

$\Rightarrow \omega$



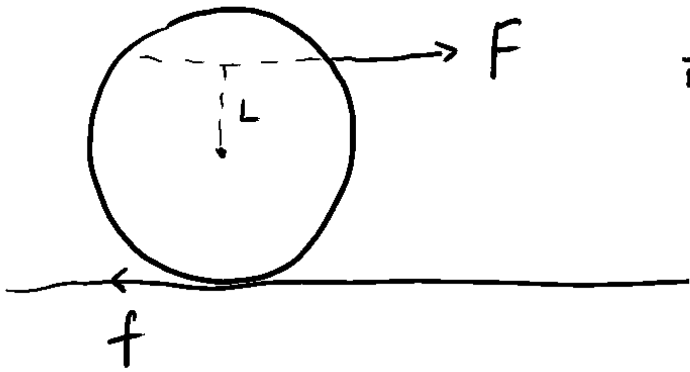

$mg\frac{L}{2}\sin\theta = \frac{1}{2}I\omega^2$

$$\boxed{力矩:\ mg\frac{L}{2}\cos\theta = I\beta}$$

$N_1 - mg\sin\theta = m\omega^2\frac{L}{2}$

$N_2 + mg\cos\theta = m\beta\frac{L}{2}$

力矩列式



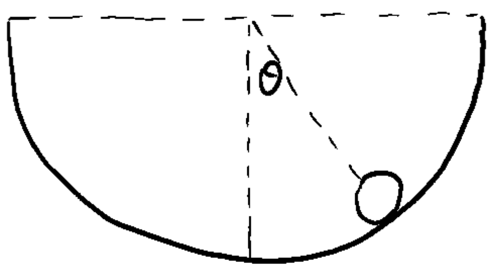

$(m, R)$

求质心加速度. 所受 $f$

$$\boxed{FL + fR = I_c\beta}$$

$a_c = \beta R$ (纯滚)

半径为 $r$ 的小球在半径为 $R$ 的半球形内纯滚动



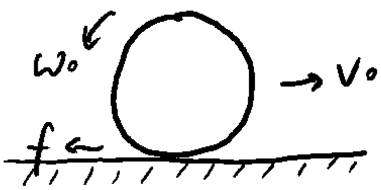

$$mg(R-r)(1-\cos\theta) + \frac{1}{2}mV_c^2 + \frac{1}{2}I\omega^2 = E_0$$

纯滚. $V_c = \omega r$

$$V_c = (R-r)\dot{\theta}$$

与牛二联系：

初速度 $v_0$、$\omega_0$，此后运动状态？



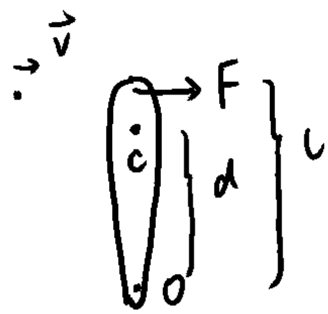

$$\underline{v_t = v_0 - \mu g t}$$

$$fR = I\beta$$

$$\underline{\omega_t = \omega_0 - \beta t}$$

纯滚 $v_t = \omega_t R \Rightarrow t,\ v_t$

$v_t = \frac{2}{3}v_0 - \frac{2}{3}\omega_0 R$ $\underline{<0}$ 能回来




球撞击棒，手握O处，l多大时感受到力最小

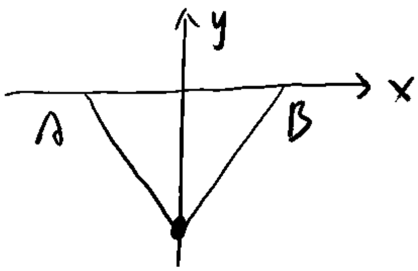
$$F(l-d) + F_0 d = I_c \beta$$

$$\underline{F - F_0 = m\beta d}$$

感受到力最小 $\Rightarrow$ $\underline{F_0 = 0}$ $\Rightarrow l$

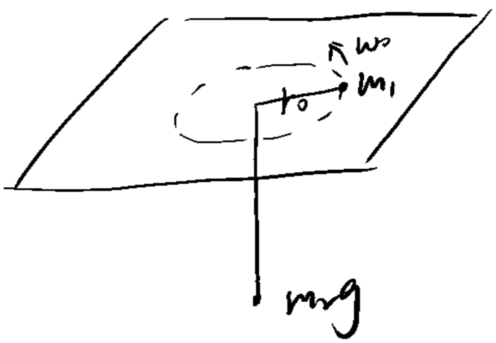
△ 振动

3-19




$AB = 3r$，绳 AB 长 $5r$，绳上一小球 $(m, r)$ 左右微振动，$T$？

$$2\pi\sqrt{\frac{15r}{8g}}$$




$m_1$ 与小孔

$r(t) = r_0 + \delta(t)$

证 $\delta$ 随 t 简谐振动

$$m_1 \omega r^2 = m_1 r_0^2 \omega_0$$

$$\Rightarrow \omega = \frac{r_0^2 \omega_0}{r^2}$$

① 能量. $\frac{1}{2}(m_1 + m_2) v_r^2 + \frac{1}{2} m_1 v_\theta^2 + m_2 g (r - r_0) = E_0$

$$\xrightarrow{\text{对t导}} (m_1 + m_2)\ddot{r} + m_1 r \omega \left(\omega - \frac{2 r_0^2 \omega_0}{r^2}\right) + m_2 g = 0$$

$$(m_1 + m_2)\ddot{r} - m_1 \frac{r_0^4 \omega_0^2}{r^3} + m_2 g = 0 \qquad \frac{r_0^3}{r^3} = 1 - 3\frac{\delta}{r_0}$$

$r(t) = r_0 + \delta(t)$

$$(m_1 + m_2)\ddot{\delta} - m_1 r_0 \omega_0^2 \cdot \left(1 - 3\frac{\delta}{r_0}\right) + m_2 g = 0$$

$$\ddot{\delta} + \frac{3 m_1}{m_1 + m_2} \omega_0^2 \delta = 0$$

$$\omega_\delta = \sqrt{\frac{3 m_1}{m_1 + m_2}}\, \omega_0$$

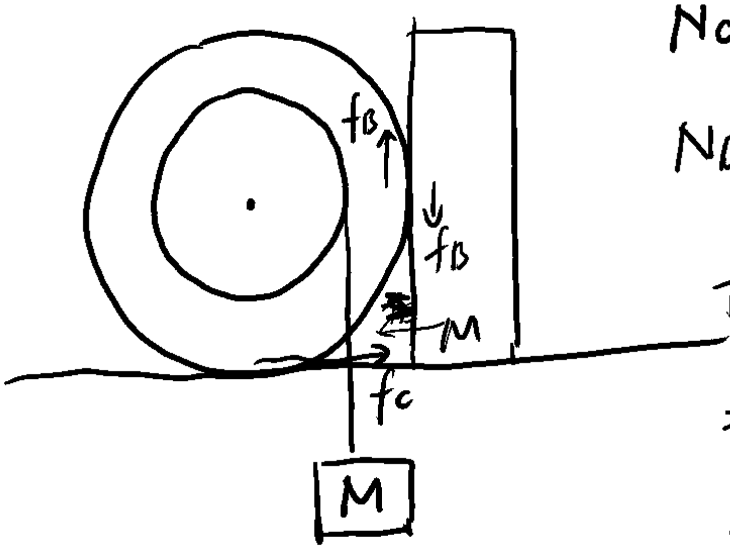
② 牛二

对 $m_2$: $N - m_2 g = m_2 \ddot{r}$

$m_1$: $N = -m_1 a_r$

$= -m_1(\ddot{r} - r\dot{\theta}^2)$

$\Rightarrow (m_1 + m_2)\ddot{r} - m_1 r\omega^2 + m_2 g = 0$




$N_C = 2Mg - f_B$ ✓

$N_B = f_B + Mg$.

$f_B \cdot 2r + f_C \cdot 2r = Mgr$ ✓

$\Rightarrow f_B + f_C = \frac{1}{2}Mg$

$\Rightarrow \underline{N_C > N_B}$.

$f_B \leq \mu f_C$

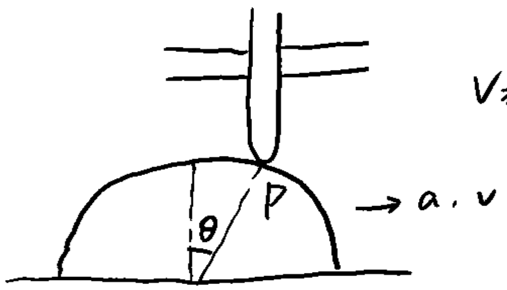
B和地面间先达到 $f_{max}$

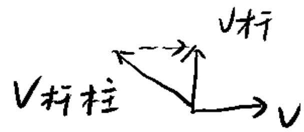
$\Rightarrow f_C = \mu(f_B + Mg)$

$f_B = \frac{1-2\mu}{2(1+\mu)} mg$

$f_C = \frac{3\mu}{2(1+\mu)} mg$.

1、

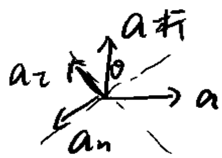



$V_{杆} = v\tan\theta .$

$a\tan\theta + v\frac{1}{\cos^2\theta}\omega$

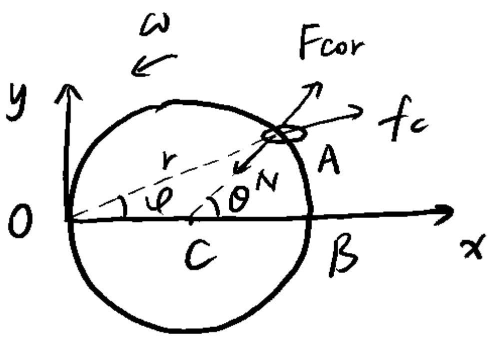
$$a_n = \frac{V_{杆柱}^2}{R} = \frac{v^2}{R\cos^2\theta}$$




$a_{杆}\cos\theta = a\sin\theta - a_n$

$$a_{杆} = a\tan\theta - \frac{v^2}{R\cos^3\theta}$$

2、




质量为m的小圆环套在R上，
后者在水平面内以ω绕O转动
小环在大环上运动时的切向
加速度和水平面内所受约束力

小环受力：N（法向）

$f_c = m\omega^2 \cdot r$ （沿 $\vec{OA}$）

$F_{cor} = 2m\vec{v}' \times \vec{\omega}$ （法向）

$v' = R\frac{d\theta}{dt}$ （切向）

$a_\tau = R\ddot{\theta} = -\frac{1}{m} f_c \sin\varphi$

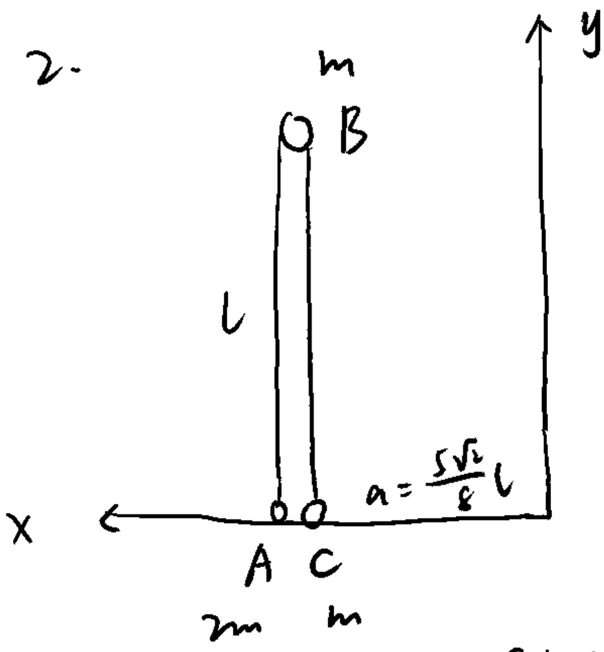
$= -\omega^2 R\sin\theta$

（小环以B为平衡点来回摆动）

$N - F_{cor} - f_c\cos\varphi = m a_n = \frac{m v'^2}{R}$

$\Rightarrow N$

2.




(1) C刚好与墙碰撞时 如图(1)

由于碰前 A.C距质心 3:5不变

⇒ $V_A:V_C=3:5$ 不变.

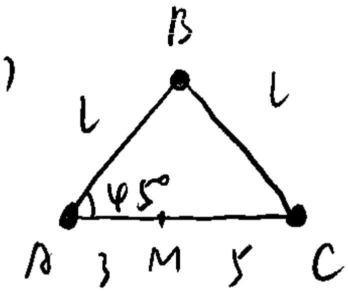
沿杆 $V_{BC}=\frac{\sqrt{2}}{2}V_C$ $V_{BA}=\frac{\sqrt{2}}{2}V_A$

$$V_B=\sqrt{V_{BC}^2+V_{BA}^2}$$

$$mg\left(l-\frac{\sqrt{2}}{2}l\right)=\frac{1}{2}mV_B^2+\frac{1}{2}mV_C^2+mV_A^2$$

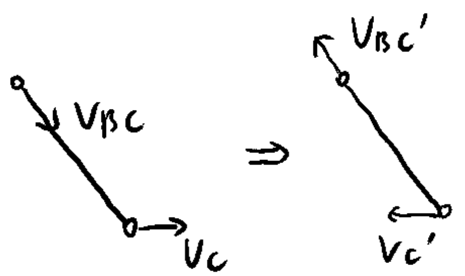
C与墙碰撞前不受外力

(1)

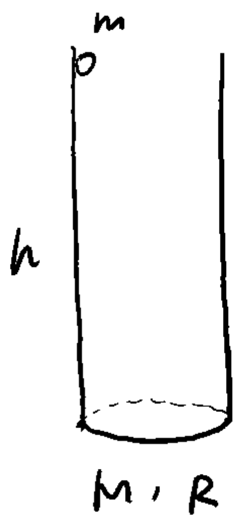



$\boxed{质心}$M的水平位置不变

(2) C球碰墙后 $V_C$ 反向 $\boxed{质心}$拥有向左的速度并不变



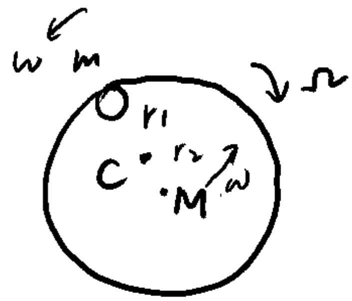
3.




圆筒 $I_0 = MR^2$ 系统角动量守恒

$$m\omega r_1^2 + M\omega r_2^2 = I_0 \Omega$$

相对圆筒角速度 $\omega' = \omega + \Omega$




$t \to t + dt$

$dl = \omega r_1 dt$

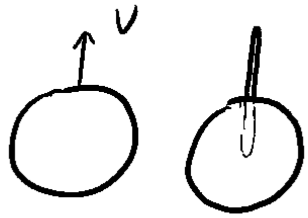
$dl' = \omega' R dt$

$\Rightarrow dl, dl'$ 关系

$dl' = 2\pi R$

$S = \sqrt{l^2 + h^2}$

4、以第一宇宙速度上抛，回到抛点 t




将轨迹视为退化椭圆

$$-\frac{GMm}{R}+\frac{1}{2}m\frac{GM}{R}=-\frac{GMm}{R+h}$$

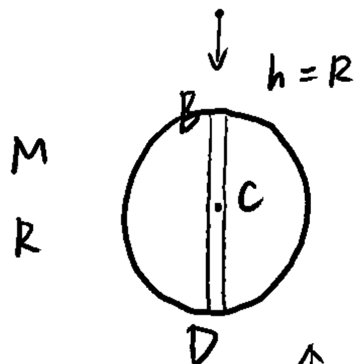
$$\Rightarrow h=R \qquad \Rightarrow a=R$$

焦点

$$t=\frac{S}{\pi ab}T \qquad \frac{a^3}{T^2}=\frac{GM}{4\pi^2}\Rightarrow T$$

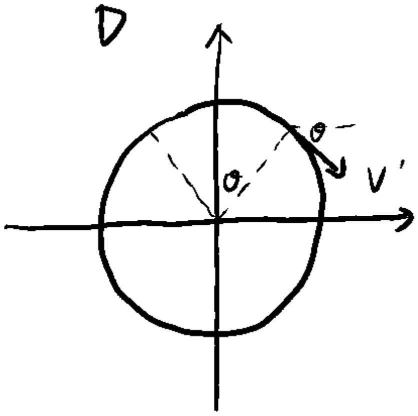
$$S=\frac{1}{2}\pi ab+ab$$

$$\Rightarrow t=(\pi+2)\sqrt{\frac{R}{g}}$$




B处速度为v. BD段为部分简谐

$$\frac{GMmr}{R^3}=-m\ddot{r}=kr$$

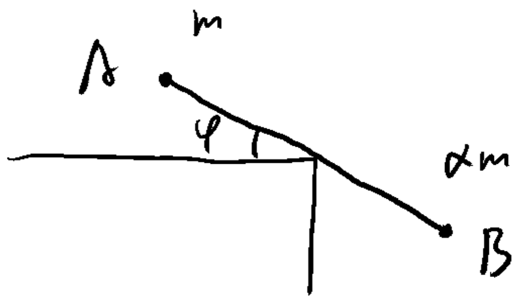



$$\frac{1}{2}mv^2=\frac{1}{2}mv_C^2-\frac{1}{2}kR^2$$

$$\Rightarrow v_C=\sqrt{2}\,v$$

$$\theta=45^\circ. \Rightarrow t$$

5.

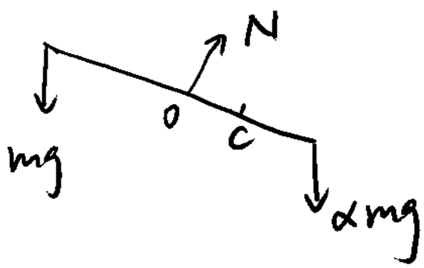



由水平开始旋转至 $\varphi_0$ 时中点开始滑离桌面刚棱，求杆与桌之间 $\mu$

质心并非在中点 …

$$\alpha mg\frac{l}{2}\sin\alpha - mg\frac{l}{2}\sin\alpha = \frac{1}{2}(\alpha+1)mv^2$$

$$\Rightarrow v$$




$$\overline{OC} = \frac{\alpha-1}{\alpha+1}l \Rightarrow v_c = \frac{OC}{OB}v$$

$$v_c^2 = 2\left(\frac{\alpha-1}{\alpha+1}\right)^3 gl\sin\varphi$$

$$a_{cn} = \frac{v_c^2}{OC}$$

$$\frac{v}{l} = \omega$$

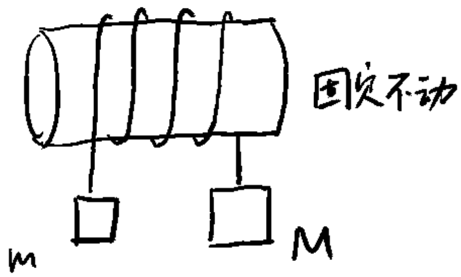
$$2v_c a_{c\tau} = 2\left(\frac{\alpha-1}{\alpha+1}\right)^3 gl\cos\varphi\,\omega$$

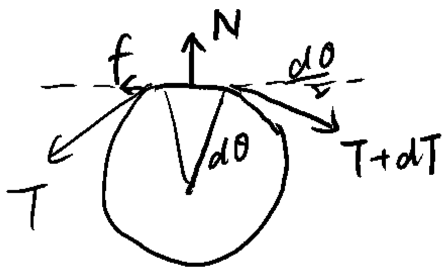
$$\begin{cases}(\alpha+1)mg\cos\varphi - N = (\alpha+1)ma_{c\tau} \\ f - (\alpha+1)mg\sin\varphi = ma_{cn}\end{cases}$$

$\varphi = \varphi_0$ 时 $f = \mu N$

6、 为使两桶静止不动，绳至少绕多少圈？

绳与圆柱 $\mu$



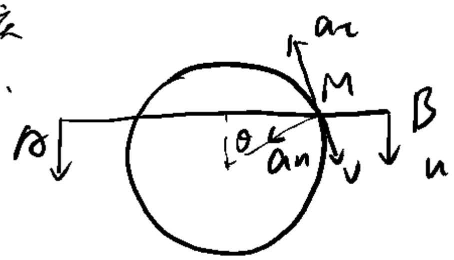

$$(T+dT)\cos\frac{d\theta}{2} = T\cos\frac{d\theta}{2} + \mu N$$

$$N = T\sin\frac{d\theta}{2} + (T+dT)\sin\frac{d\theta}{2}$$

$$dT = \mu T d\theta$$

$$\frac{dT}{T} = \mu d\theta.$$

圆周

1. 



此位置M加速度？

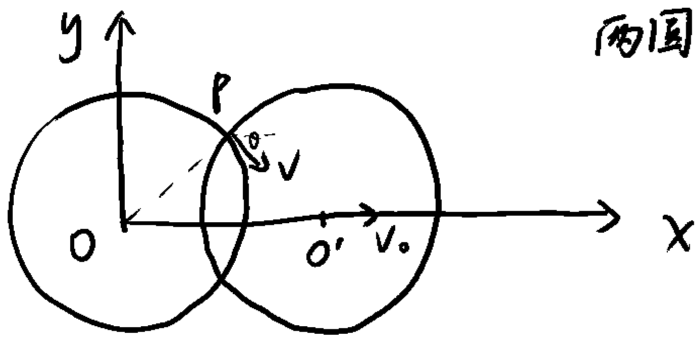
M位于杆上 ⇒ y方向上M无加速度

∴ $a_n \cos\theta = a_\tau \sin\theta$.

2. 由长度关联找到速度关联

O'以$v_0$向右运动

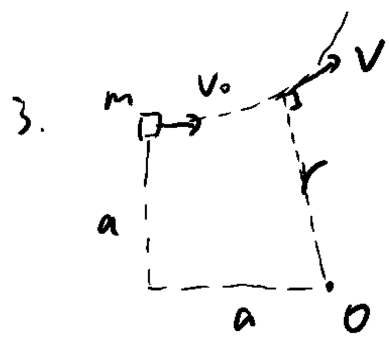
两圆交点P的速度v. $a_n(t)$？




$$\boxed{x_p = \frac{1}{2} x_{O'}}$$

$$v_{px} = \frac{1}{2} v_0$$

$$v = \frac{v_{px}}{\cos\theta} \qquad a_n = \frac{v^2}{R}.$$

3. 



m在运动中受O点斥力 $F_0 = \alpha r\ (\alpha > 0)$

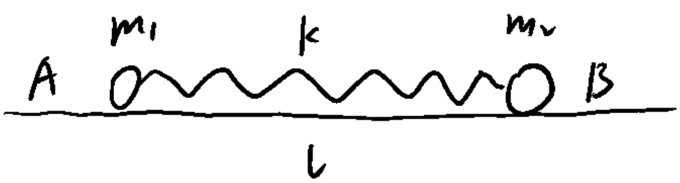
应视作存在势能 $E_p = \int_r^0 \alpha r\,dr$

$= -\frac{1}{2}\alpha r^2$

∴ $mv_0 a = mvr$

$$\frac{1}{2}mv_0^2 - \frac{1}{2}\alpha\cdot 2a^2 = \frac{1}{2}mv^2 - \frac{1}{2}\alpha\cdot r^2$$

4. (1) 给球以向右初速度，求B的加速度 $a(t)$




A参考系中，B质量以 $\mu$ 代替

$$\omega=\sqrt{\frac{k}{\mu}}$$

$$\frac{1}{2}\mu v_0^2=\frac{1}{2}kA^2$$

$$\Rightarrow x=\sqrt{\frac{\mu}{k}}v_0\sin\sqrt{\frac{k}{\mu}}t$$

换至地面系中 $a=\frac{F}{m_2}=\frac{-kx}{m_2}$

(2) 给A电量 $q_1$，B $q_2$（同号）将弹簧压缩L后释放

求弹簧在原长时B相对A的速度 $v$.

$$\frac{1}{2}kL^2+\frac{kq_1q_2}{l_0-L}=\frac{kq_1q_2}{l_0}+\frac{1}{2}\mu v^2$$

考前 8. $\frac{m}{2}$ 相对圆环以不变的速率 $v$ 沿圆环运动

(1) 圆环转动的初始角速度 $\Omega_0$