Q1).
$$\frac{\partial u}{\partial t} + c\frac{\partial^2 u}{\partial x^2} = 0$$

{Equation}

$u_x^t \Rightarrow u$ at spatial $\to x$
time $\to t$

Now in the implicit scheme we take the spatial derivative at time $t+1$ and not at t.

so now.

$$\frac{\partial u}{\partial t} = \frac{u_x^{t+1} - u_x^t}{\Delta t}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{x+1}^{t+1} - 2u_x^{t+1} + u_{x-1}^{t+1}}{(\Delta x)^2}.$$

$$\Rightarrow \frac{u_x^{t+1} - u_x^t}{\Delta t} + \frac{c}{(\Delta x)^2}\left\{ u_{x+1}^{t+1} - 2\cdot u_x^{t+1} + u_{x-1}^{t+1} \right\} = 0.$$

collecting terms will give us.
and taking $\frac{c\Delta t}{(\Delta x)^2} = \lambda$ (let)

$$\boxed{\lambda u_{x+1}^{t+1} + (1-2\lambda)u_x^{t+1} + \lambda u_{x-1}^{t+1} = u_x^t.}$$

for $x = 1$. $\lambda u_2^{t+1} + (1-2\lambda)u_1^{t+1} + \lambda u_0^{t+1} = u_1^t.$

as $x = 0$ u is fixed

$x = n-1$ $\lambda u_n^{t+1} + (1-2\lambda)u_{n-1}^{t+1} + \lambda u_{n-2}^{t+1} = u_{n-1}^t$