check week 3 notes for modified Monty Hall problem

Definition of Independence

Two events E and F are defined to be independent if

$$P(E \cap F) = P(E)P(F)$$

If E and F are independent then

$$P(E|F) = P(E)$$

Can A and B be both mutually exclusive and independent?

Naah

I will disprove the proposition "Two events A and B can be both mutually excusive and independent" using proof by contradiction

Assuming that both A and B are possible (i.e. non-zero probability)

Assuming that A and B are mutually exclusive and independent.

We get

$$A \cap B = \phi$$

and

$$P(A \cap B) = 0$$

Since they're also independent, we know:

$$P(A \cap B) = P(A) \cdot P(B)$$

and thus

$$P(A) \cdot P(B) = 0$$

meaning

$P(A) = 0 \vee P(B) = 0$ which contradicts the assumption that both the events are possible.