Information theory

- “A Mathematical Theory of Communication” - Claude Shannon

Entropy of physics

(in a shitty way)

Entropy: log of the number of microstates and micrscopic configurations

1. If particles in a system have many possible configs then the system has high entropy
2. rigid systems have low entropy

solids have low entropy

gases have high entropy

Entropy and knowledge

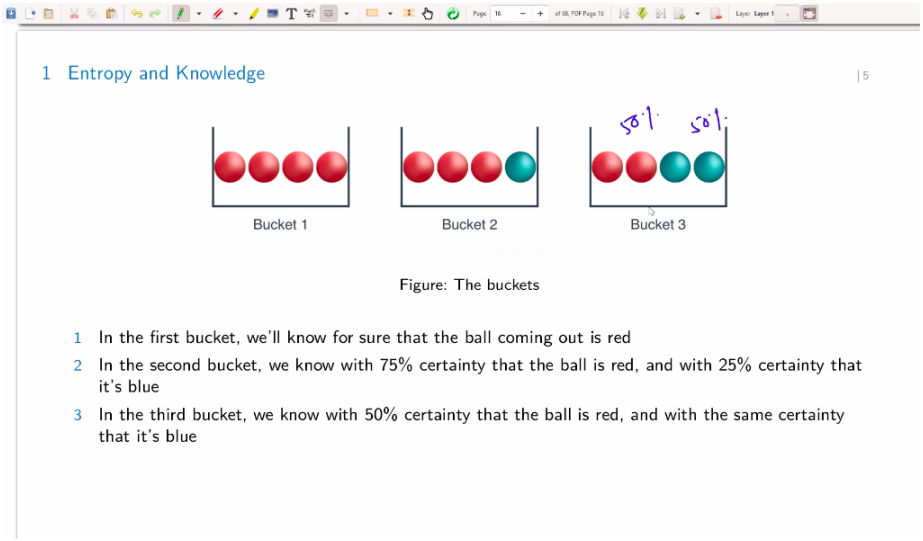

Figure 1: The bucket example

1. Bucket 1 gives us the amount of “knowledge” about the ball we’ll draw cuz we know that the ball will be red

and so on

“Knowledge” is in some way contrary to entropy

Quantifying entropy using probability

From physics we can go on to say that systems with more possible rearrangements corresponds to higher entropy

(and low for low)

Game show timez

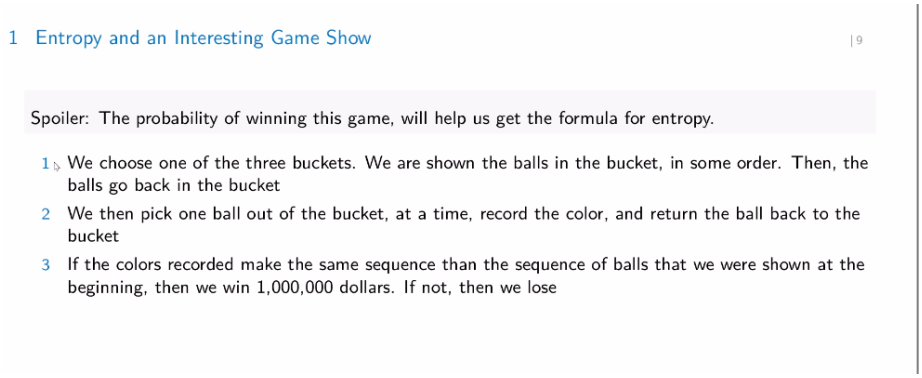

Figure 2: The rules

So in the second case the probability of winning is

$$0.75 \times 0.75 \times 0.75 \times 0.25 = 0.105$$

because the balls get put back, each pick is independent of the other

and the winning sequence is the intersection of all those events (and thus multiplication)

Towards Entropy Formula

We want something opposite of the probability formulae.

Steps:

1. Take log that turns everything to a summation
2. multiply with -1 and we get the nice property we want

thus we have

(for the second case)

$$-\log_2 0.75 - \log_2 0.75 - \log_2 0.75 - \log_2 0.25 = 3.245$$

(this isn't the actual formula now)

The entropy formula:

$$-\sum_{i=1}^{n} p_i \log_2 p_i$$

where n is the number of elementary events

Computer science view

Note: how much can I reduce the number of bits to, to get the same amount of information

Information Theory

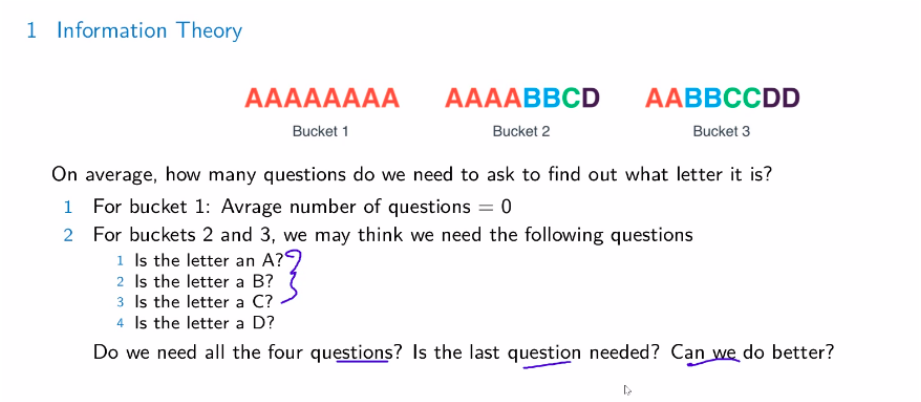

Figure 3: Can we do better?!?

now we ask

1. is the letter A or B?
   a. If answer is yes: is the letter A? if the answer is no: then is it C? (done)
   b. If no: then is it A?

So at most two questions are good enough

The thing looks like a decision tree

For bucket 2

TODO: get something more concrete

1 Average Number of Questions for Bucket 3

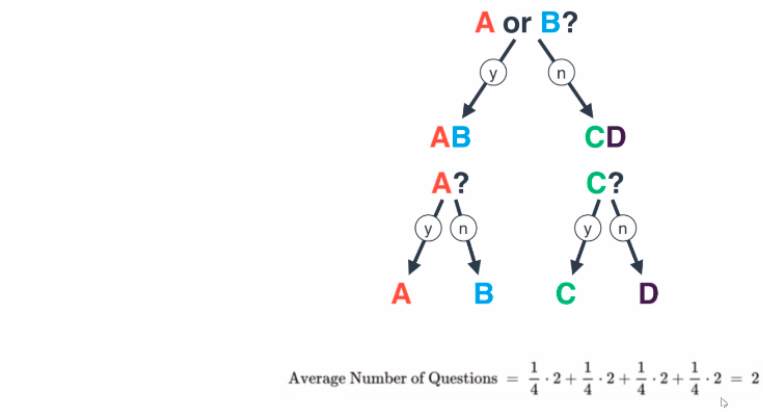

Figure 4: decision_tree