Assignment -2022
Engineering Mathematics – III

1. Ram and Sham are two weak students of statistics and their chances of solving a problem in statistics correctly are $^1/_6$ and $^1/_8$ respectively. If the probability of their making a common error is $^1/_{525}$ and they obtain the same result, find the probability that their answer is correct?

2. The chances that doctor A will diagnose a disease X correctly is 60%. The chances that a patient will die by his treatment after correct diagnosis is 40% and the chances of death by wrong diagnosis is 70%. A patient of doctor A, who has disease X, died. What is the probability that his disease was diagnosed correctly?

3. Two dice are rolled. Let X denote the random variable which counts the total number of points on the upturned faces, Construct a table giving the non-zero values of the probability mass function and draw the probability chart. Also find the distribution function of X?

4. Suppose that the life in hours of a certain part of radio tube is a continuous random variable X with p.d.f. given by:

$$f(x) = \begin{cases} \frac{100}{x^2}, when\ x \geq 100 \\ 0, \qquad elsewhere \end{cases}$$

(i) What is the probability that all of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation?

(ii) What is the probability that none of three of the original tubes will have to be replaced during that first 150 hours of operation?

(iii) What is the probability that a tube will last less than 200 hours if it is known that the tube is still functioning after 150 hours of service?

5. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of the time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. Find his chance of having the claim (i) accepted, (ii) rejected, when he does have the ability he claims.

6. The probability of a man hitting a target is ¼

(i) If he fires 7 times, what is the probability of his hitting the target at least twice?

(ii) How many times he fires so that the probability of his hitting at least once is greater than $\frac{2}{3}$?

7. X is a normal variate with mean 30 and standard deviation 5. Find the probabilities of the following:

(i) $26 \leq X \leq 40$

(ii) $X \geq 45$

(iii) $|X - 30| > 5$

8. The mean yield for one-acre plot is 662 kilos with a s.d. 32 kilos. Assuming normal distribution, how many one acre plots in a batch of 1,000 plots would you expect to have yield.

(i) Over 700 kilos
(ii) Below 650 kilos
(iii) What is the lowest yield of the best 100 plots?

9. Calculate the correlation coefficient for the following heights (in inches) of father (X) and their sons (Y)

X:	**65**	**66**	**67**	**67**	**68**	**69**	**70**	**72**
Y:	**67**	**68**	**65**	**68**	**72**	**72**	**69**	**71**

10. The random variable X and Y are jointly normally distributed and U and V are difined by **U=Xcosα + Ysinα, V=Ycosα – Xsinα**. Show that U and V will be uncorrelated if $\mathbf{\tan 2\alpha = \frac{2r\sigma_x\sigma_y}{\sigma_x^2-\sigma_y^2}}$, Where **r = corr.(X, Y); σ_x^2 = Var(X) and σ_y^2 = Var(Y)**.

11. If X and Y are uncorrelated random variables with means zero and varience σ_1^2 and σ_2^2 respectively. Show that **U=Xcosα + Ysinα, V=Xsinα – Ycosα.** Have a correlation coefficient ρ given by

$$\rho = \frac{\boldsymbol{\sigma_1^2 - \sigma_2^2}}{\sqrt{[(\boldsymbol{\sigma_1^2 - \sigma_2^2})^2 + 4\boldsymbol{\sigma_1^2\sigma_2^2 cosec^2 2\alpha}]}}$$

12. Two random variables X and Y have zero means, the same variance σ^2 and zero correlation. Show that $U = Xcos\alpha + Ysin\alpha$ and $V = Xsin\alpha + Ycos\alpha$, have the same variance σ^2 and zero correlation.

13. X, Y and Z are random variables each with expectation 10 and variance 1, 4 and 9 respectively. The correlation coefficients are $r(X,Y) = 0; r(Y,Z) = r(Z,X) = 1.4$. Obtain the numerical values of:

(i) $E(X + Y - 2Z)$
(ii) $Cov(X + 3, Y + 3)$
(iii) $V(X - 2Z)$
(iv) $Cov(3X, 5Z)$

14. Obtain the equation of two lines of regression for the following data. Also obtain the estimate of X for Y=70.

X:	**65**	**66**	**67**	**67**	**68**	**69**	**70**	**72**
Y:	**67**	**68**	**65**	**68**	**72**	**72**	**69**	**71**

15. Find the most likely price in Mumbai corresponding to the price of Rs. 70 at Kolkata from the following:

	Kolkata	**Mumbai**
Average Price	**65**	**67**
Standard Deviation	**2.5**	**3.5**

Correlation coefficient between the prices of commodities in the two cities is 0.8.

****