LESSON PLAN-02

2)

cent respectively. An item is drawn at random from the day's production. An item is found to *be defective. What is probability that it comes from the output of*

(i) Machine I, *(ii) Machine II,* *(iii) Machine (iii) ?*

Solution. Let E_1, E_2 and E_3 denote the events that the output is produced by machines I, II and III respectively and let A denote the event that the output is defective. Then we have :

$$P(E_1) = \frac{3000}{10,000} = 0{\cdot}30, \quad P(E_2) = \frac{2500}{10,000} = 0{\cdot}25, \quad P(E_3) = \frac{4500}{10,000} = 0{\cdot}45$$

$$P(A \mid E_1) = 1\% = 0{\cdot}01, \quad P(A \mid E_2) = 1{\cdot}2\% = 0{\cdot}012, \quad P(A \mid E_3) = 2\% = 0{\cdot}02$$

The probability that an item selected at random from day's production is defective is given by :

$$P(A) = \sum_{i=1}^{3} P(E_i \cap A) = \sum_{i=1}^{3} P(E_i) \cdot P(A \mid E_i)$$

$$= 0{\cdot}30 \times 0{\cdot}01 + 0{\cdot}25 \times 0{\cdot}012 + 0{\cdot}45 \times 0{\cdot}02 = 0{\cdot}015$$

By Baye's rule, the required probabilities are given by :

(i) $$P(E_1 \mid A) = \frac{P(E_1) \cdot P(A \mid E_1)}{P(A)} = \frac{0{\cdot}003}{0{\cdot}015} = \frac{1}{5}$$

(ii) $$P(E_2 \mid A) = \frac{P(E_2) \cdot P(A \mid E_2)}{P(A)} = \frac{0{\cdot}003}{0{\cdot}015} = \frac{1}{5}$$

(iii) $$P(E_3 \mid E) = \frac{P(E_3) \cdot P(A \mid E_3)}{P(A)} = \frac{0{\cdot}009}{0{\cdot}015} = \frac{3}{5}$$

The probabilities in *(i)*, *(ii)* and *(iii)* are known as posterior probabilities of events E_1, E_2 and E_3 respectively.

...bilities can be obtained elegantly in a tabular form as

3)

Let us define the following events.

E_1 : Disease X is diagnosed correctly by doctor A.

E_2 : Disease X is not diagnosed correctly by doctor A.

B : A patient (of doctor A) who has disease X dies.

Then, we are given, $P(E_1) = 0.6, P(E_2) = 1 - P(E_1) = 1 - 0.6 = 0.4$

and $P\left(\frac{B}{E_1}\right) = 0.4$, $P\left(\frac{B}{E_2}\right) = 0.7$

By Bay's Theorem

$$P\left(\frac{E_1}{B}\right) = \frac{P(E_1)P\left(\frac{B}{E_1}\right)}{P(E_1)P\left(\frac{B}{E_1}\right) + P(E_2)P\left(\frac{B}{E_2}\right)}$$

$$= P\left(\frac{E_1}{B}\right) = \frac{0.6 \times 0.4}{0.6 \times 0.4 + 0.4 \times 0.7} = \frac{0.24}{0.24 + 0.28} = \frac{0.24}{0.52} = \frac{6}{13}$$

Ans-4) Let E_1 : Student ~~Kn~~ guess the Ans

let E_2 : Student Knows the Ans.

A : Given Answer is correct

A.T.Q

$$P(E_1) = 1-P$$

$$P(E_2) = P$$

$$P\left(\frac{A}{E_1}\right) = \frac{1}{5}, \quad P\left(\frac{A}{E_2}\right) = 1$$

By using Baye's theorem:-

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

$$P\left(\frac{E_2}{A}\right) \Rightarrow \frac{P\cdot(1)}{(1-P)\cdot\left(\frac{1}{5}\right) + P\cdot(1)}$$

$$\Rightarrow \frac{P}{\frac{1}{5}(1-P) + P} = \frac{5P}{(1-P)+5P}$$

$$= \frac{5P}{1+4P}$$

5) The white was drawn hence there may be the following possible combinations -

1) 1w 3b - probability = (3C1 X 5C3) / 8C4 = 30/8C4

2) 2w 2b - probability = (3C2 X 5C2) / 8C4 = 30/8C4

3) 3w 1b - probability = (3C3 X 5C1) / 8C4 = 5/8C4

Therefore, the probability of case 3

= (5)/(30+30+5)

= 1/13