Enrolment No. | | | | | | | |

S_3 (All): MA

B. Tech. 3rd SEMESTER END TERM EXAMINATION 2022-23

SUBJECT NAME: Engineering Mathematics-III/Mathematics-III

SUBJECT CODE: UCE03B14/UME03C14/UEE03C17/UCS03C16/UEC03C15/ UEE13C01/UPE03C16/UCH03C18/UBE03C03

Full Marks: 50 **Time: 2 Hours**

Symbols used here have their usual meanings

Group –A

Answer all the following questions: $[5 \times 2 = 10]$

1. If X and Y are random variables each with expectation 10 and variance 1 and 4 respectively. The correlation coefficient of X and Y is $r(X,Y) = 0$. Obtain the numerical values of $Cov(X+3, Y+3)$.
2. If X is a Poisson variate such that $P(X=2) = 9P(X=4) + 90P(X=6)$, then find the mean of X.
3. Define moment generating function.
4. Define linear partial differential equation with suitable example.
5. Solve the partial differential equation $\left(4D^2 + 12DD' + 9D'^2\right)z = 0$

Group –B

Answer all the following questions: $[4 \times 5 = 20]$

1. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of the time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. Find his chance of having the claim (i) accepted, (ii) rejected, when he does have the ability he claims.

2. The mean yield for one-acre plot is 662 kilos with standard deviation 32 kilos. Assuming normal distribution, how many one acre plots in a batch of 1,000 plots would you expect to have yield.

   (i) Over 700 kilos

   (ii) Below 650 kilos

   (iii) What is the lowest yield of the best 100 plots?

3. The random variable X and Y are jointly normally distributed and U and V are defined by $U = X\cos\alpha + Y\sin\alpha$, $V = Y\cos\alpha - X\sin\alpha$. Show that U and V will be uncorrelated if $\tan 2\alpha = \frac{2r\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}$, Where $r = corr.(X,Y)$; $\sigma_x^2 = Var(X)$ and $\sigma_y^2 = Var(Y)$.

4. Find the most likely price in Mumbai corresponding to the price of Rs. 70 at Kolkata from the following:

	Kolkata	Mumbai
Average Price	65	67
Standard Deviation	2.5	3.5

Correlation coefficient between the prices of commodities in the two cities is 0.8.

Group- C

Answer all the following questions: [2 × 10 = 20]

1. (a) Solve $\left(D^2 - 3DD' + 2D'^2\right) z = e^{2x-y} + e^{x+y} + \cos(x+2y)$

   (b) Find the Fourier series of $f(x) = \begin{cases} \pi x, 0 \le x < 1 \\ 0, \quad x = 1 \\ \pi(x-2), 1 < x \le 2 \end{cases}$

[5+5]

2. (a) Find the general solution of the equation $(y+zx)p - (x+yz)q + y^2 - x^2 = 0$.

   (b) Find a complete integral of $\left(\frac{\partial z}{\partial x}\right)^2 x(x-1) + 2\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}xy + \left(\frac{\partial z}{\partial y}\right)^2 y(y-1) - 2\left(\frac{\partial z}{\partial x}\right)xz - 2\left(\frac{\partial z}{\partial y}\right)yz + z^2 = 0$

[4+6]

The following table gives the shaded area in the diagram, *viz.*, $P(0 < Z < z)$ for different values of z.

TABLE OF AREAS

↓Z→	0	1	2	3	4	5	6	7	8	9
·0	·0000	·0040	·0080	·0120	·0160	·0199	·0239	·0279	·0319	·0359
·1	·0398	·0438	·0478	·0517	·0557	·0596	·0636	·0675	·0714	·0759
·2	·0793	·0832	·0871	·0910	·0948	·0987	·1026	·1064	·1103	·1141
·3	·1179	·1217	·1255	·1293	·1331	·1368	·1406	·1443	·1480	·1517
·4	·1554	·1591	·1628	·1664	·1700	·1736	·1772	·1808	·1844	·1879
·5	·1915	·1950	·1985	·2019	·2054	·2088	·2123	·2157	·2190	·2224
·6	·2257	·2291	·2324	·2357	·2389	·2422	·2454	·2486	·2517	·2549
·7	·2580	·2611	·2642	·2673	·2703	·2734	·2764	·2794	·2823	·2852
·8	·2881	·2910	·2939	·2967	·2995	·3023	·3051	·3078	·3106	·3133
·9	·3159	·3186	·3212	·3238	·3264	·3289	·3315	·3340	·3365	·3389
1·0	·3413	·3438	·3461	·3485	·3508	·3531	·3554	·3577	·3599	·3621
1·1	·3643	·3655	·3686	·3708	·3729	·3749	·3770	·3790	·3810	·3830
1·2	·3849	·3869	·3888	·3907	·3925	·3944	·3962	·3980	·3997	·4015
1·3	·4032	·4049	·4066	·4082	·4099	·4115	·4131	·4147	·4162	·4177
1·4	·4192	·4207	·4222	·4236	·4251	·4265	·4279	·4292	·4306	·4319
1·5	·4332	·4345	·4357	·4370	·4382	·4394	·4406	·4418	·4429	·4441
1·6	·4452	·4463	·4474	·4484	·4495	·4505	·4515	·4525	·4535	·4545
1·7	·4554	·4564	·4573	·4582	·4591	·4599	·4608	·4616	·4625	·4633
1·8	·4641	·4649	·4656	·4664	·4671	·4678	·4686	·4693	·4699	·4706
1·9	·4713	·4719	·4726	·4732	·4738	·4744	·4750	·4756	·4761	·4767
2·0	·4772	·4778	·4783	·4788	·4793	·4798	·4803	·4808	·4812	·4817
2·1	·4821	·4826	·4830	·4834	·4838	·4842	·4846	·4850	·4854	·4857
2·2	·4861	·4864	·4868	·4871	·4875	·4678	·4881	·4884	·4887	·4890
2·3	·4893	·4896	·4898	·4901	·4904	·4906	·4909	·4911	·4913	·4916
2·4	·4918	·4920	·4922	·4925	·4927	·4929	·4931	·4932	·4934	·4936
2·5	·4938	·4940	·4941	·4943	·4945	·4946	·4948	·4959	·4951	·4952
2·6	·4953	·4955	·4956	·4957	·4959	·4960	·4961	·4962	·4963	·4964
2·7	·4965	·4966	·4967	·4968	·4969	·4970	·4971	·4972	·4973	·4974
2·8	·4974	·4975	·4976	·4977	·4977	·4978	·4979	·4979	·4980	·4981
2·9	·4981	·4982	·4982	·4983	·4984	·4984	·4985	·4985	·4986	·4986
3·0	·4987	·4987	·4987	·4988	·4988	·4989	·4989	·4989	·4990	·4990
3·1	·4990	·4991	·4991	·4991	·4992	·4992	·4992	·4992	·4993	·4993
3·2	·4993	·4993	·4994	·4994	·4994	·4994	·4994	·4995	·4995	·4995
3·3	·4995	·4995	·4995	·4996	·4996	·4996	·4996	·4996	·4996	·4997
3·4	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4998
3·5	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998
3·6	·4998	·4998	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999
3·7	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999
3·9	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000

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