Engineering Mechanics

Subject Name: Engineering Mechanics Credit -4

L-T-P: 3-1-0

UNIT I: FORCE SYSTEMS AND EQUILIBRIUM

Force moment and couple principle of transmissibility, Varignon’s theorem. Resultant of force system- concurrent and non-concurrent coplanar forces, free body diagram, equilibrium equations and their uses in solving elementary engineering problems.

UNIT II : PROPERTIES OF SURFACE & SOLIDS

Centroid & Centre of Mass-Centroid of lines and areas using Standard formulas. Theorem of Pappus. Area moment of Inertia of plane figures. Hollow section by using Standard formula-Parallel axis theorem and Perpendicular axis theorem-Principle axes of inertia -Mass moment of Inertia for prismatic cylindrical and spherical solids from first principle- Relation to area moments of Inertia

UNIT III: PLANE TRUSSES

Analysis of plane trusses and plane frames (Analytical Method), Method of joints, Method of sections.

UNIT IV: DYNAMICS OF PARTICLES

Displacement, Velocity and acceleration, their relationship -Relative motion -Curvilinear motion- Newton's Law of motion-Work Energy Equation-Impulse and Momentum -Impact of elastic bodies, Simple stress strain for deformable body, Hooke's Law

UNIT V: FRICTION AND ELEMENTS OF RIGID BODY DYNAMICS

Friction force- Laws of friction- equilibrium analysis of simple systems with sliding friction, Translation and Rotation of Rigid Bodies -Velocity of Acceleration-General Plane motion of simple rigid bodies such as cylinder, disc/wheel and sphere.

Text/Reference Books:

1. Timoshenko and Young, *Engineering Mechanics.*

2. R.K. Bansal , *A text Book of Engineering Mechanics.*

Mechanics

It is defined as that branch of science, which describes and predicts the conditions of rest or motion of bodies under the action of forces. Engineering mechanics applies the principle of mechanics to design, taking into account the effects of forces.

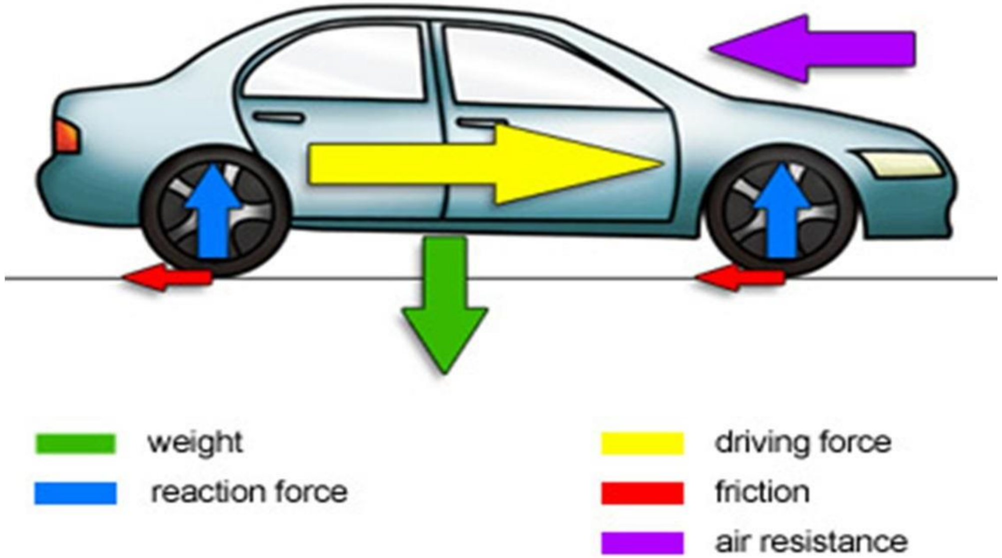
Depending upon the nature of the problems treated, mechanics is divided into Statics and Dynamics .

Statics is the branch of mechanics that is concerned with the analysis of loads (force and torque, or "moment") acting on physical systems that do not experience an acceleration (a=0), but rather, are in static equilibrium with their environment. The application of Newton's second law to a system gives:

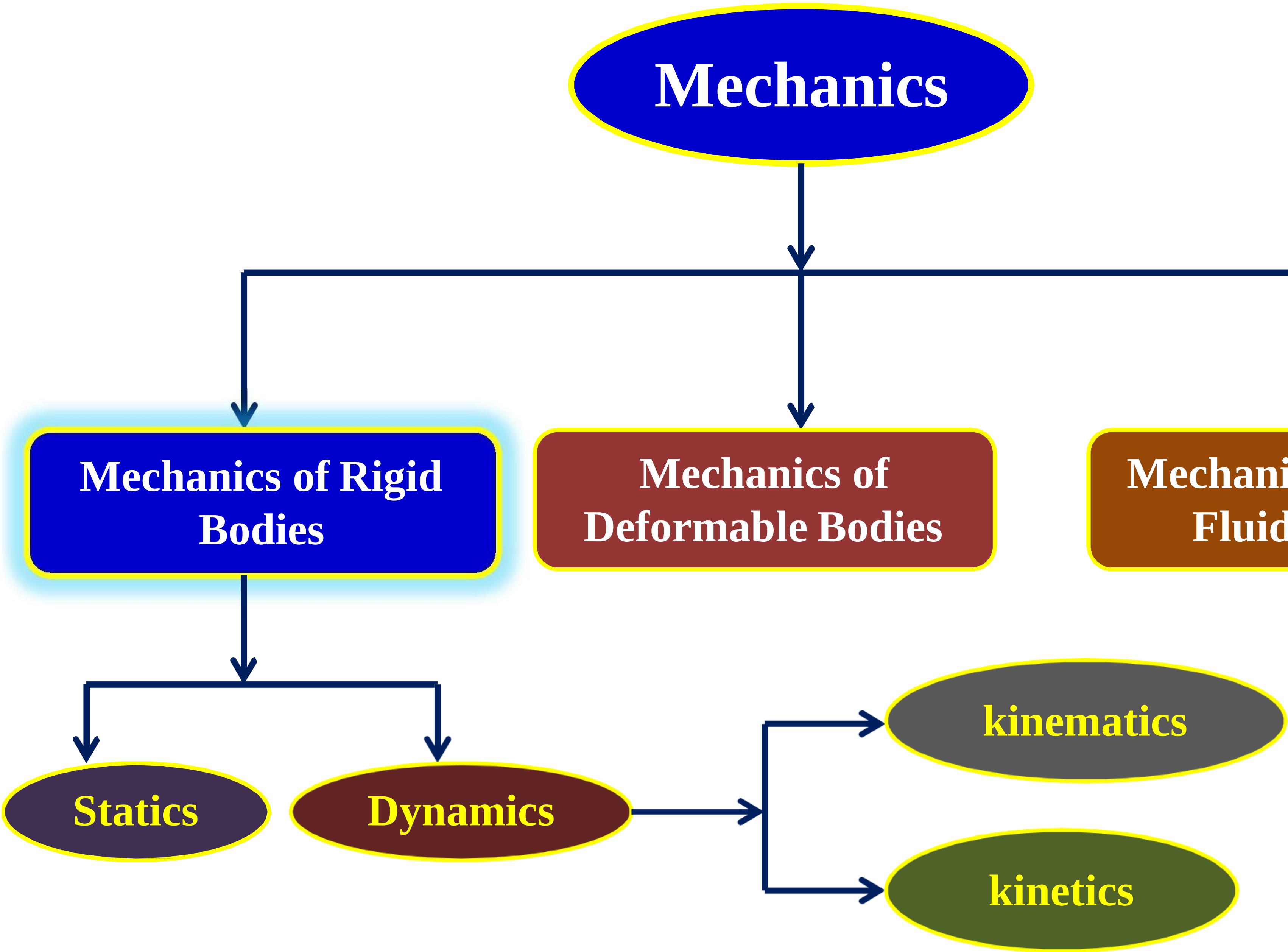
Dynamics is the study of motion of rigid bodies and their correlation with forces causing them.

Dynamics is divided into Kinematics and kinetics.
Kinematics deals with the space time relationship of a given motion of a body and not at all that cause of the motion with the forces.
Kinetics studies the laws of motion of material bodies under the action of forces.

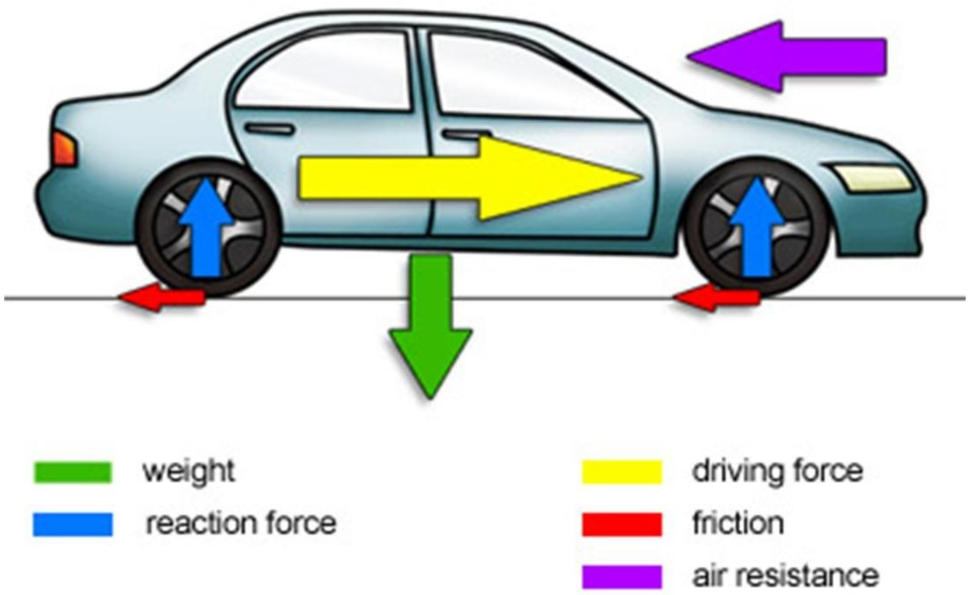
What is the need of knowing MECHANICS?



Mechanics → Deals with forces

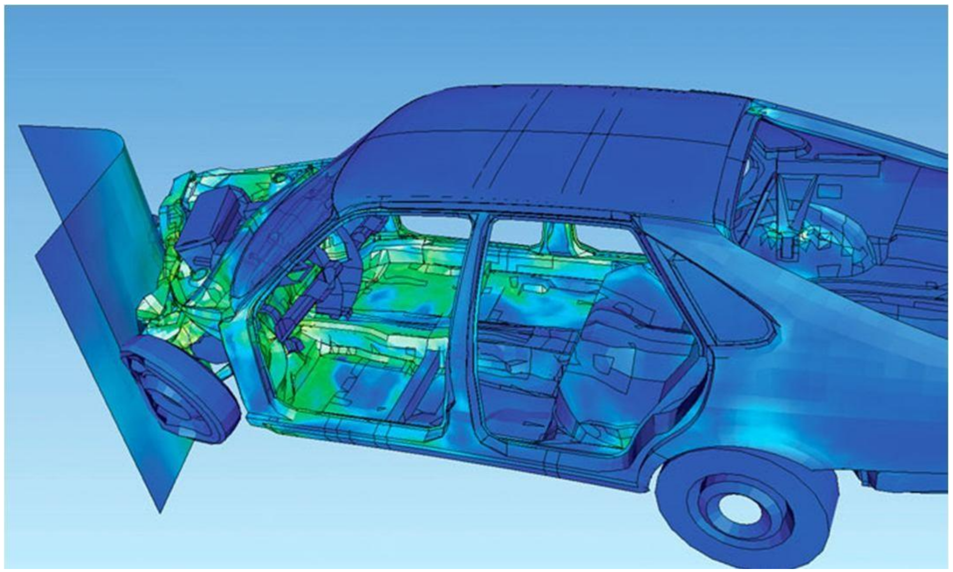

Mechanics
Mechanics of Rigid Bodies
Mechanics of Deformable Bodies
Mechanics of Fluids
Statics
Dynamics
kinematics
kinetics

Rigid body mechanics



Studying External effect of forces on a body such as velocity, acceleration, displacement etc.

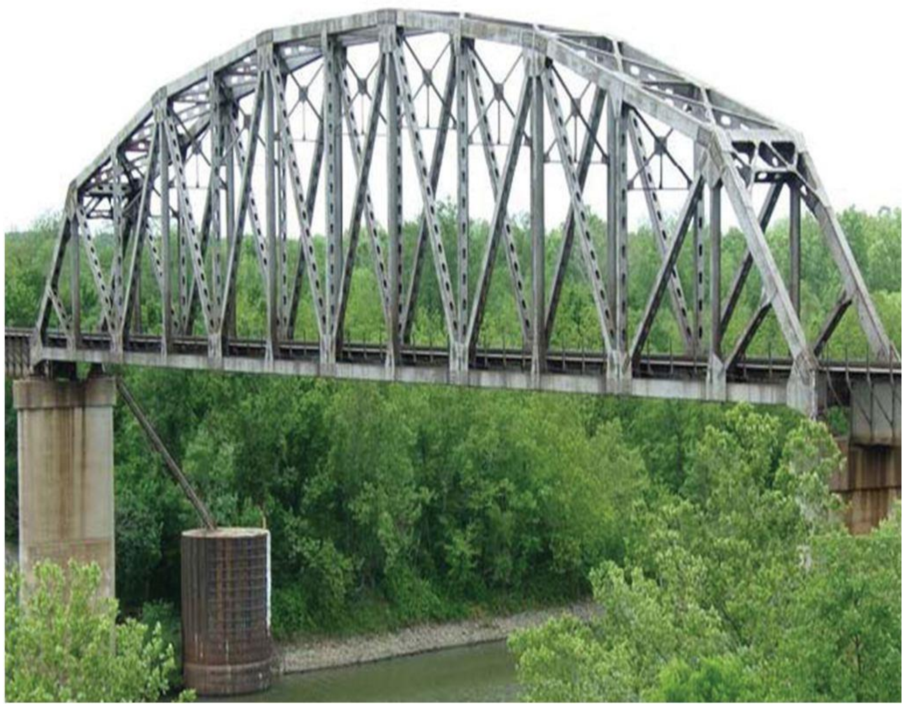
Studying Internal effect of forces on a body such as stresses (internal resistance), change in shape etc.



Deformable body mechanics

Statics

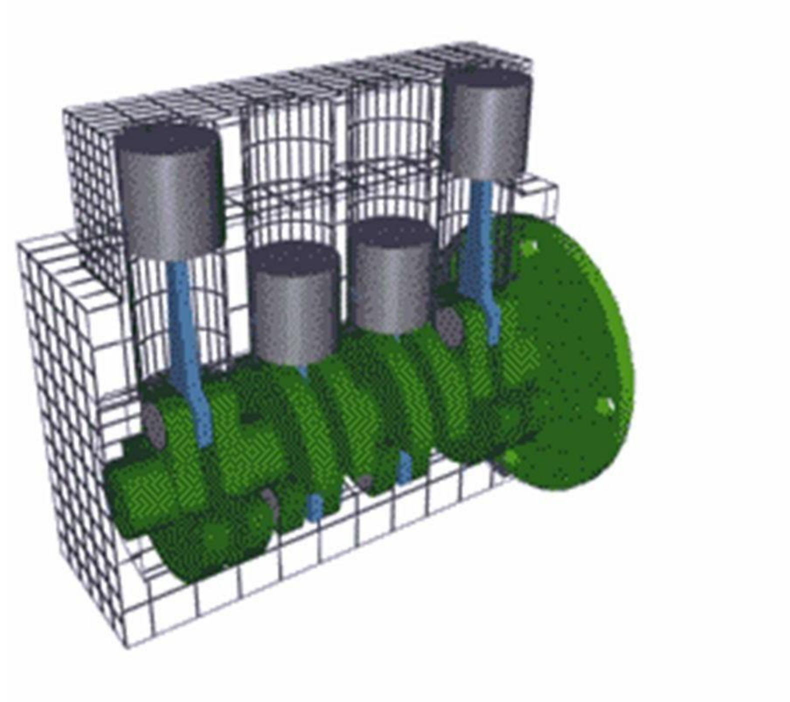
- Deals with forces and its effects when the body is at rest



Truss Bridge

Dynamics

- Deals with forces and its effects when the body is in moving condition

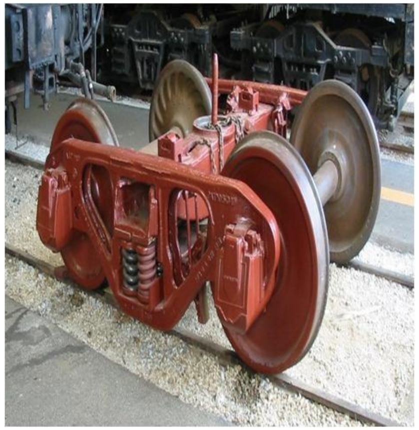


IC Engine

Rigid body mechanics

Negligible deformation (no deformation) under the action of forces. Assuming 100% strength in the materials. Large number of particles occupying fixed positions with each other.

Actual structures and machines are never rigid under the action of external loads or forces.
But the deformations induced are usually very small which does not affect the condition of equilibrium.

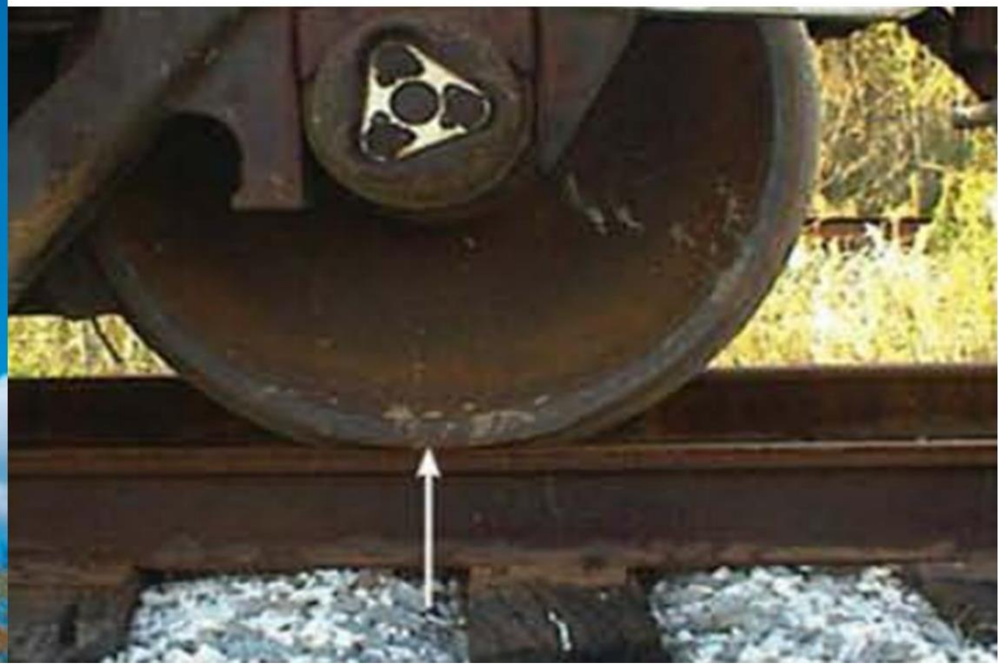



Force ⟹ Action of one body on the other (push or pull)

Point of Application

Direction

Magnitude



Mechanics: Fundamental Concepts:

Length (Space): needed to locate position of a point in space, & describe size of the physical system → Distances, Geometric Properties

Time: measure of succession of events → basic quantity in Dynamics

Mass: quantity of matter in a body → measure of inertia of a body (its resistance to change in velocity)

Force: represents the action of one body on another → characterized by its magnitude, direction of its action, and its point of application.

Mechanics: Fundamental Concepts

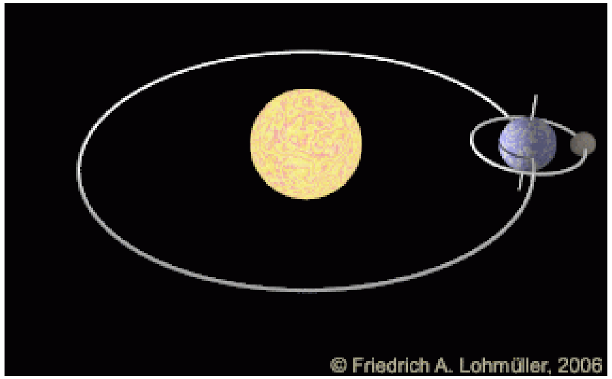
Remember:

- Mass is a property of matter that does not change from one location to another.
- Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located
- Weight of a body is the gravitational force acting on it.

Mechanics: Idealizations

To simplify application of the theory

Particle: A body with mass but with dimensions that can be neglected



Size of earth is insignificant compared to the size of its orbit. Earth can be modelled as a particle when studying its orbital motion

Mechanics: Idealizations

Rigid Body: A combination of large number of particles in which all particles remain at a fixed distance (practically) from one another before and after applying a load.

Material properties of a rigid body are not required to be considered when analyzing the forces acting on the body.

In most cases, actual deformations occurring in structures, machines, mechanisms, etc. are relatively small, and rigid body assumption is suitable for analysis.

ENGINEERING MECHANICS

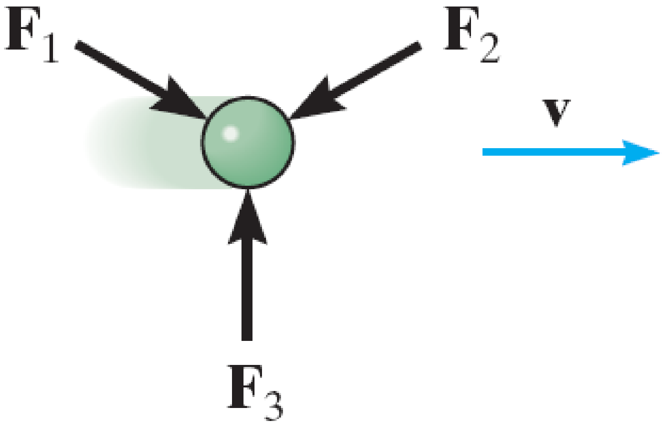
LECTURE 2

Mechanics: Newton's Three Laws of Motion

Basis of formulation of rigid body mechanics.

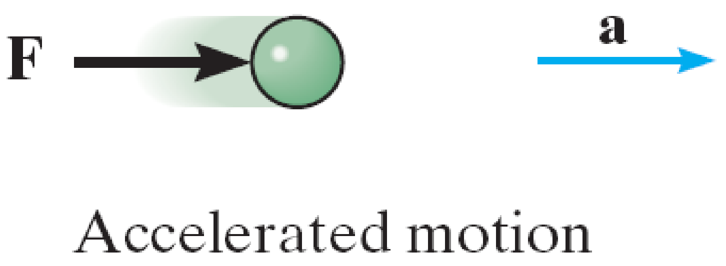
First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.

First law contains the principle of the equilibrium of forces.



Equilibrium

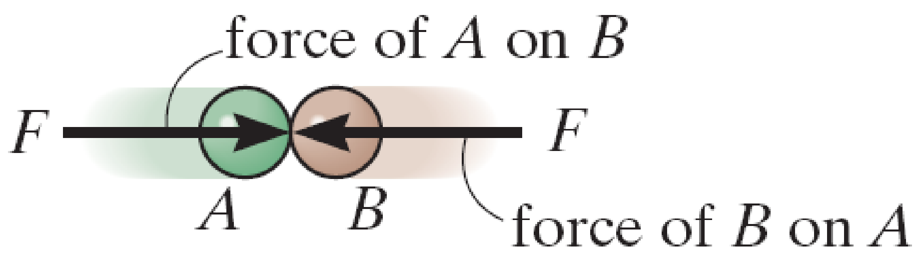
Second Law: A particle of mass "m" acted upon by an unbalanced force "F" experiences an acceleration "a" that has the same direction as the force and a magnitude that is directly proportional to the force.



F = ma

Second Law forms the basis for most of the analysis in Dynamics

Third Law: The mutual forces of action and reaction between two particles are equal, opposite, and collinear.

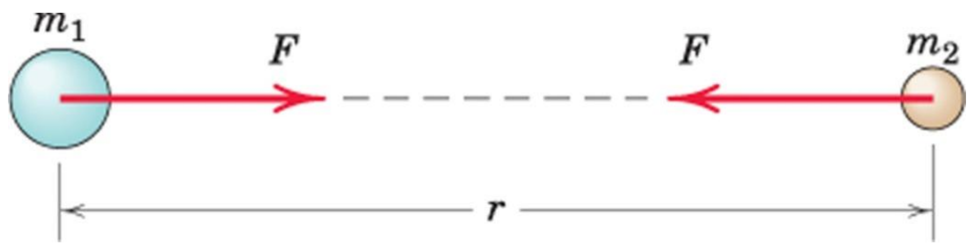


Action – reaction

Third law is basic to our understanding of Force. Forces always occur in pairs of equal and opposite forces.

Weight of a body (gravitational force acting on a body) is required to be computed in Statics as well as Dynamics.
This law governs the gravitational attraction between any two particles.

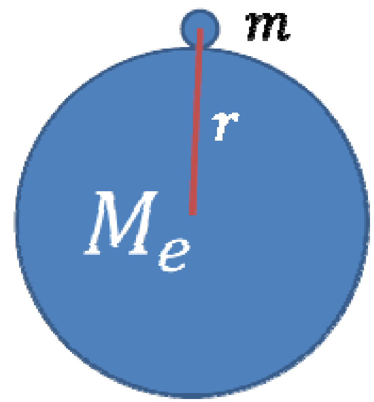
$$F = G\frac{m_1 m_2}{r^2}$$



$\boldsymbol{F}$ = mutual force of attraction between two particles
$\boldsymbol{G}$ = universal constant of gravitation
Experiments G = $6.673\text{x}10^{-11}$ $m^3/(kg.s^2)$
Rotation of Earth is not taken into account
$\boldsymbol{m_1}$, $\boldsymbol{m_2}$ = masses of two particles
$\boldsymbol{r}$ = distance between two particles

Weight of a Body: If a particle is located at near the surface of the earth, the only significant gravitational force is that between the earth and the particle

Weight of a particle having mass $m_1 = m$:



Assuming earth to be a nonrotating sphere of constant density and having mass $m_2 = M_e$

$$W = G\frac{mM_e}{r^2}$$

r = distance between the earth's center and the particle

$$W = mg$$

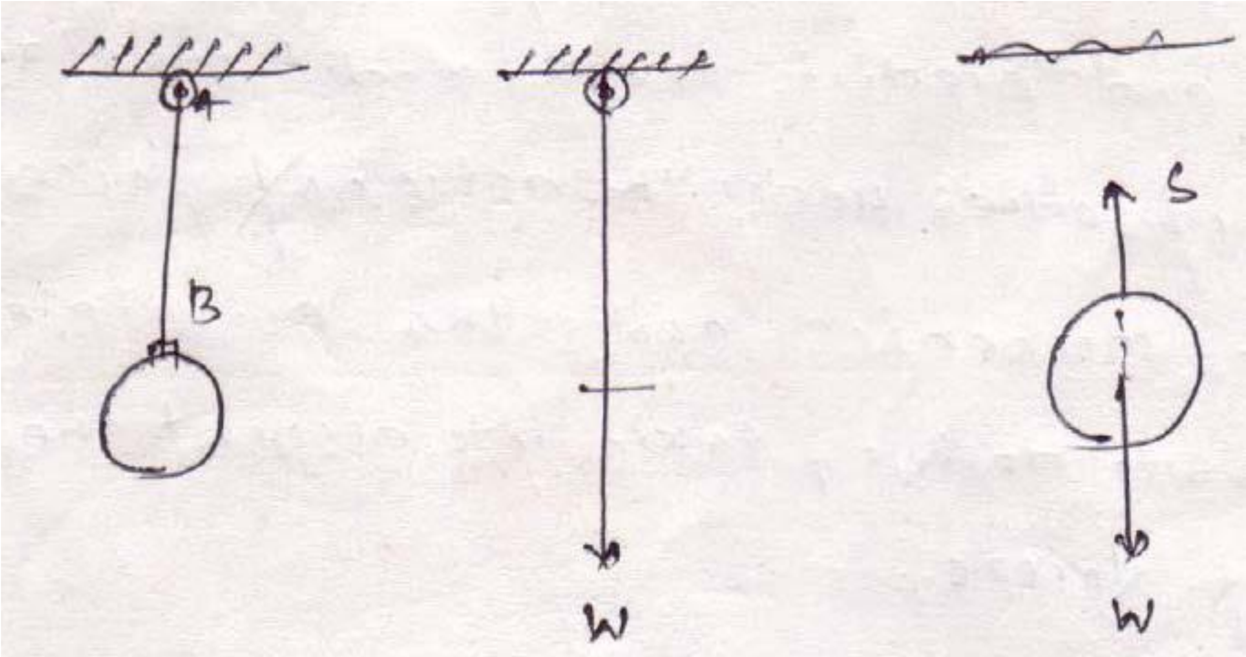
Let $g = G\,M_e/r^2$ = acceleration due to gravity (9.81m/s^2)

Force

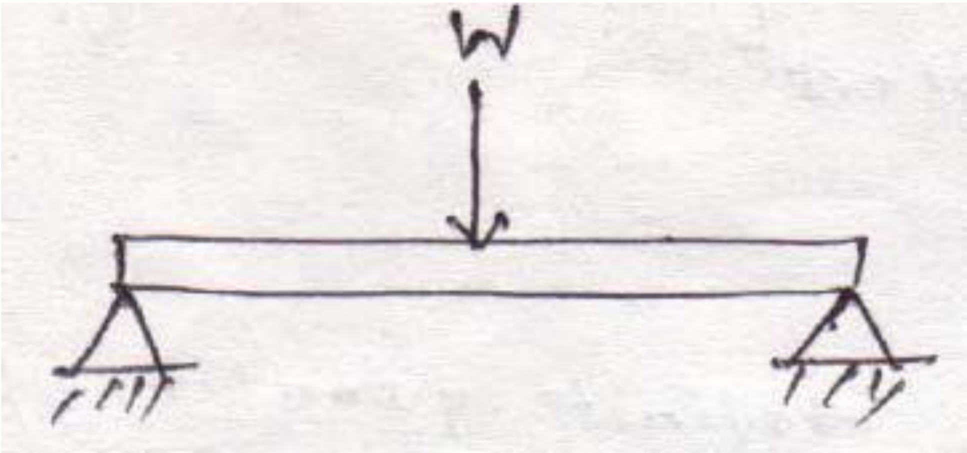
Force may be defined as any action that tends to change the state of rest or motion of a body to which it is applied.

The three quantities required to completely define force are called its specification or characteristics. So the characteristics of a force are:

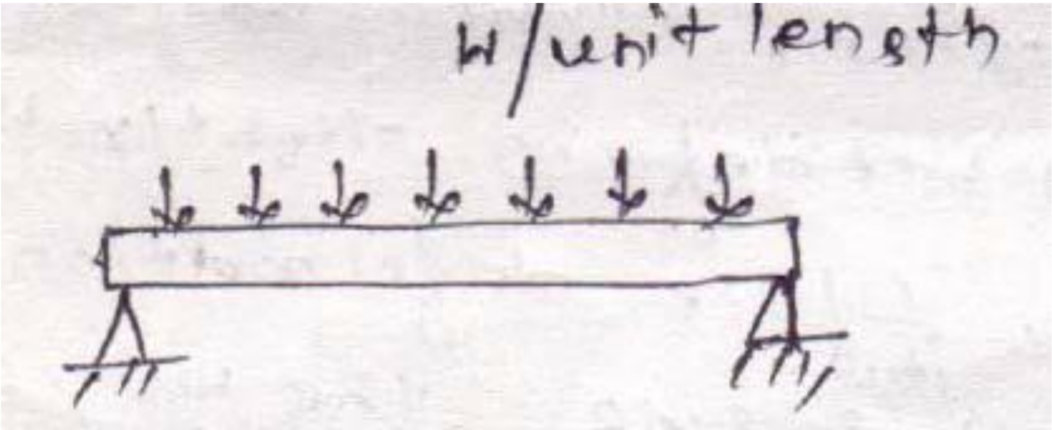
1. Magnitude
2. Point of application
3. Direction of application



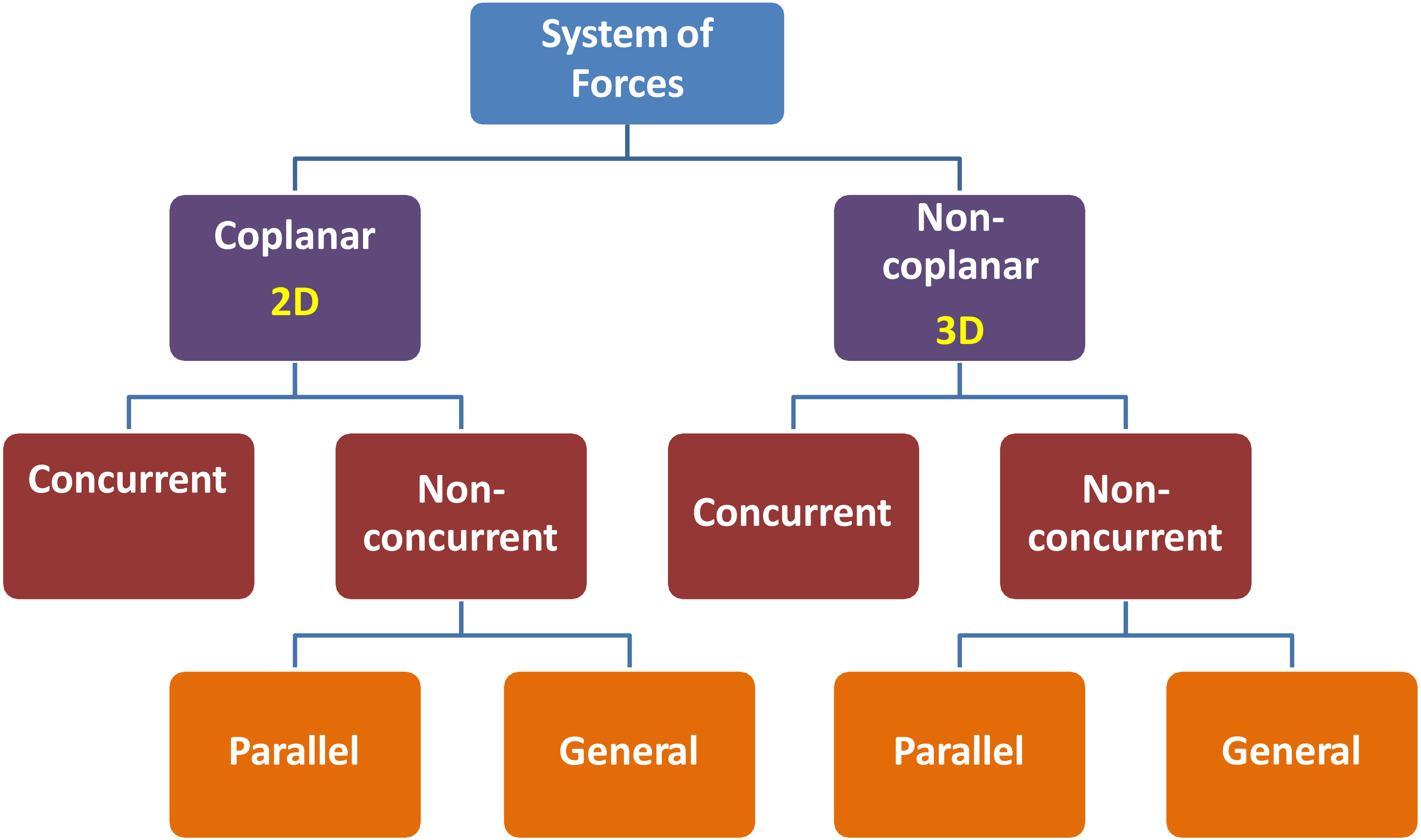
Concentrated force/point load



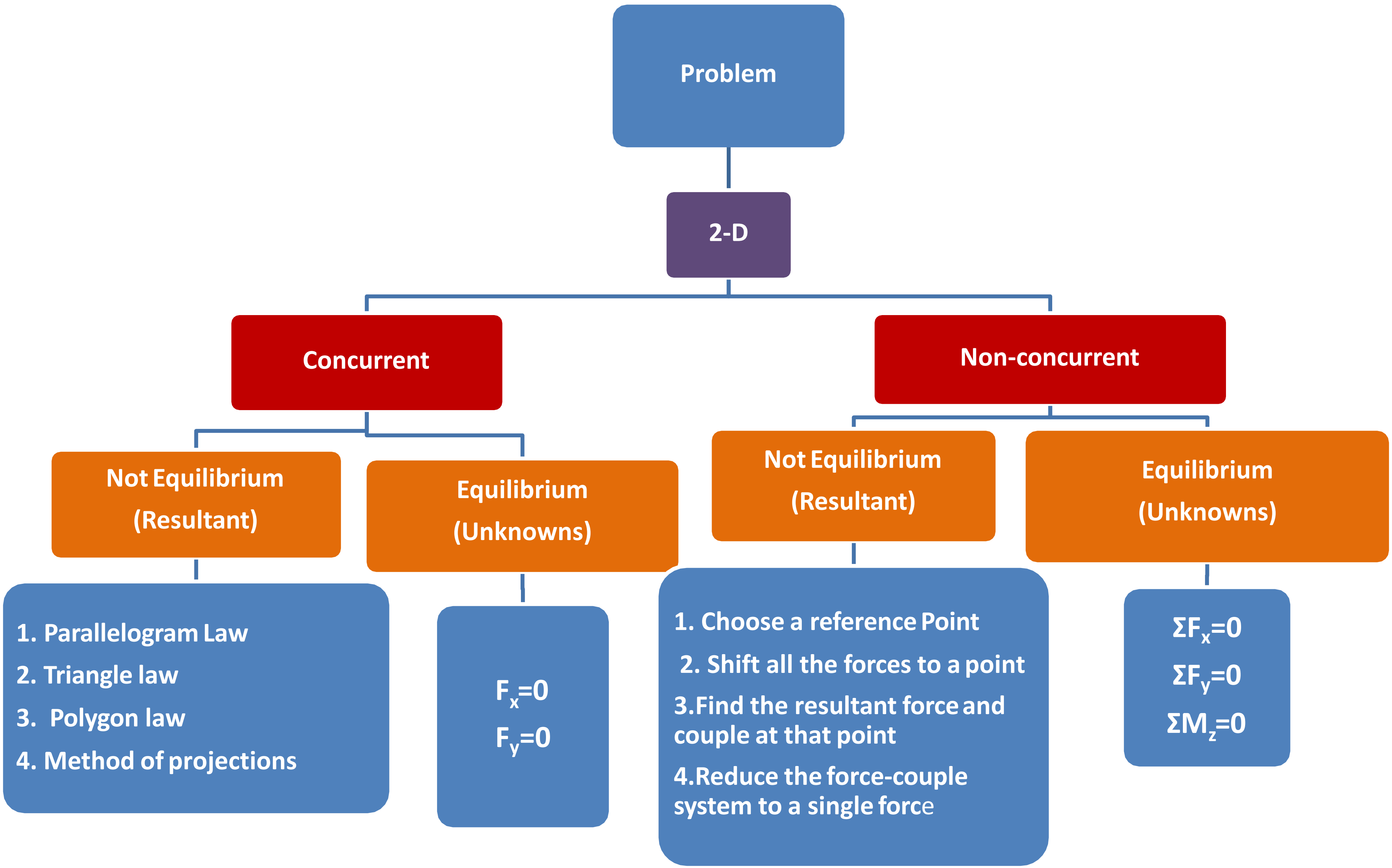
Distributed force



System of Forces: Several forces acting simultaneously upon a body



Method of approach to solve Coplanar (2D) problems

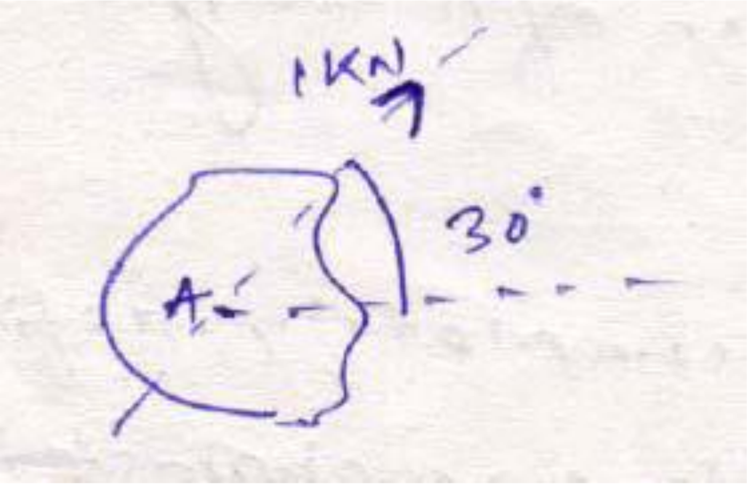


Line of action of force

The direction of a force is the direction, along a straight line through its point of application in which the force tends to move a body when it is applied. This line is called line of action of force.

Representation of force

Graphically a force may be represented by the segment of a straight line.

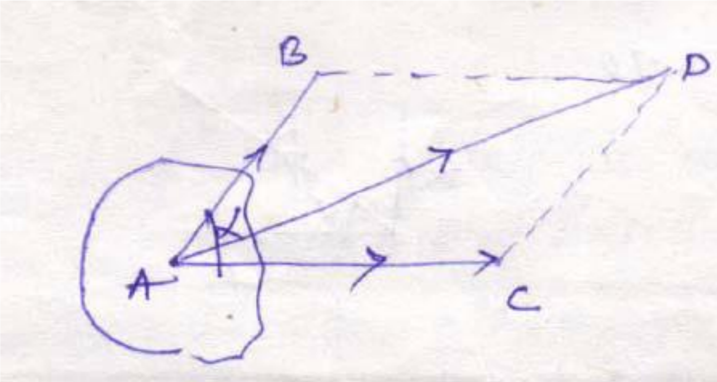


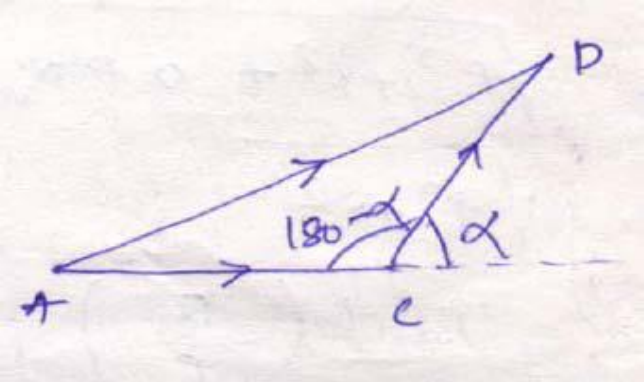
Composition of two forces

The reduction of a given system of forces to the simplest system that will be its equivalent is called the problem of composition of forces.

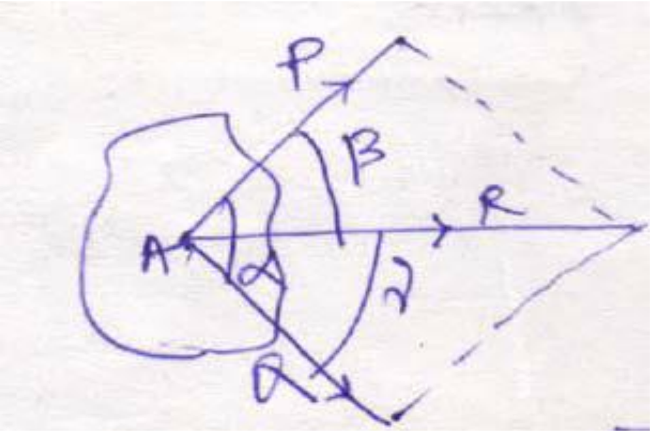
Parallelogram law

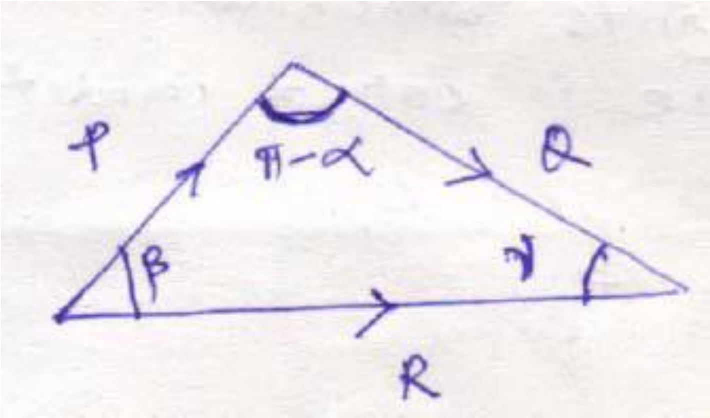
If two forces represented by vectors AB and AC acting under an angle α are applied to a body at point A. Their action is equivalent to the action of one force, represented by vector AD, obtained as the diagonal of the parallelogram constructed on the vectors AB and AC directed as shown in the figure.





Force AD is called the resultant of AB and AC and the forces are called its components.





$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos\alpha\right)}$$

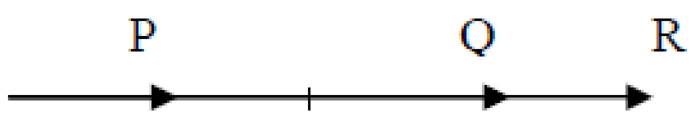
Now applying triangle law

$$\frac{P}{Sin\gamma} = \frac{Q}{Sin\beta} = \frac{R}{Sin(\pi - \alpha)}$$

Special cases

Case-I: If $\alpha = 0^\circ$

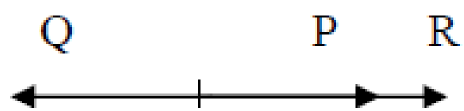
$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos0^\circ\right)} = \sqrt{(P+Q)^2} = P + Q$$



R = P+Q

Case- II: If $\alpha = 180^\circ$

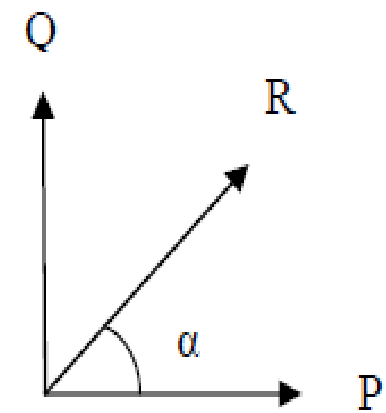
$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos180^\circ\right)} = \sqrt{(P^2 + Q^2 - 2PQ)} = \sqrt{(P-Q)^2} = P - Q$$



Case-III: If $\alpha = 90^\circ$

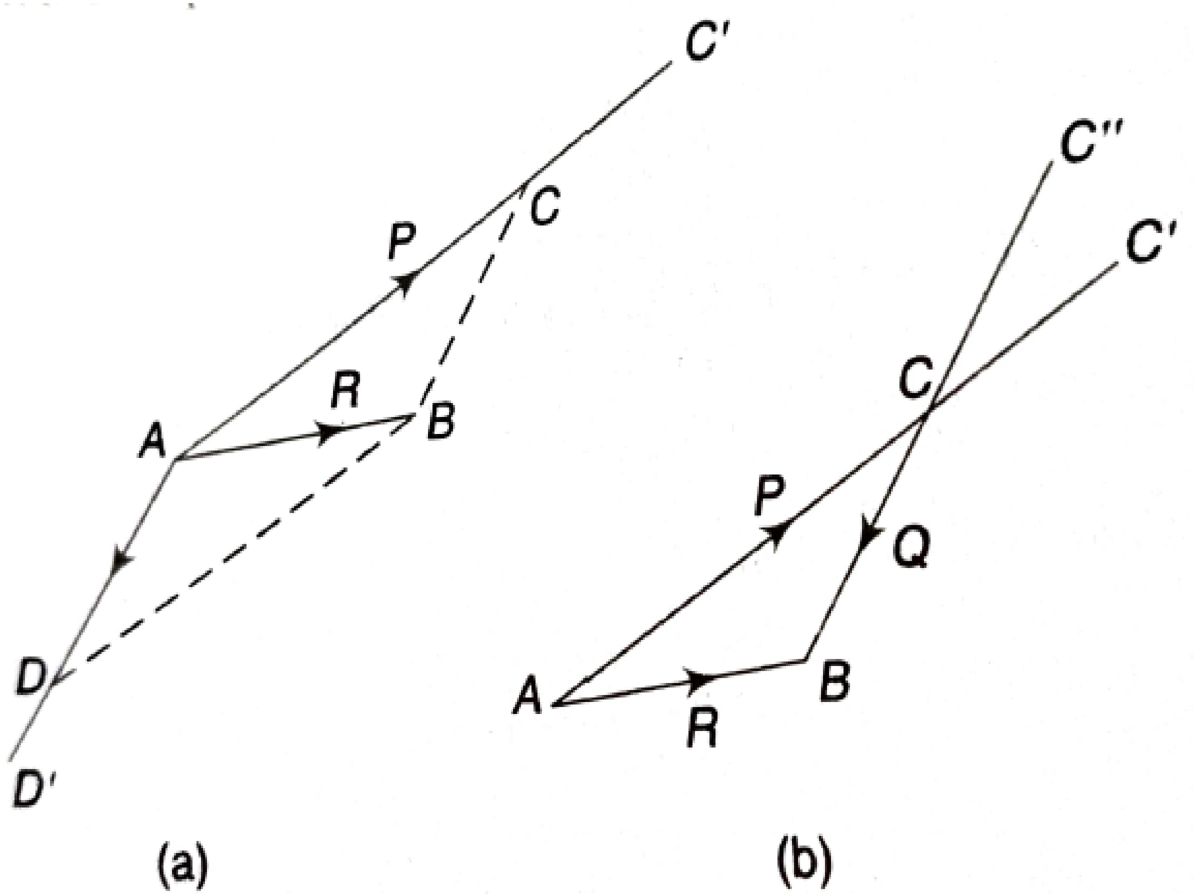
$$R = \sqrt{\left(P^2 + Q^2 + 2PQ \times Cos90^\circ\right)} = \sqrt{P^2 + Q^2}$$

$\alpha = \tan^{-1} (Q/P)$



Resolution of a force

The replacement of a single force by a several components which will be equivalent in action to the given force is called resolution of a force.



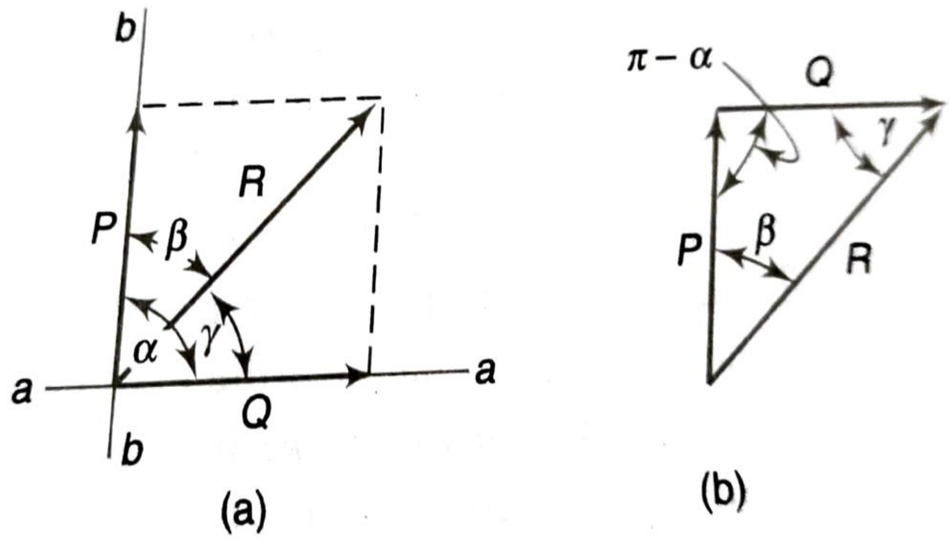


Fig. 2.9

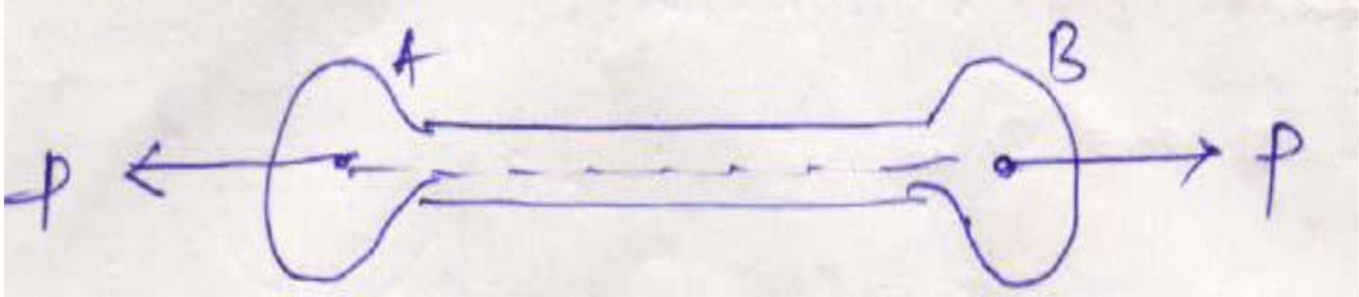
The magnitude of the resultant R, angle β and angle γ being known, we may determine the magnitudes of forces P and Q by using the equations

$$\sin\beta = \frac{Q}{R}\sin\alpha, \qquad \sin\gamma = \frac{P}{R}\sin\alpha$$

... the direction

Equilibrium of colinear forces:

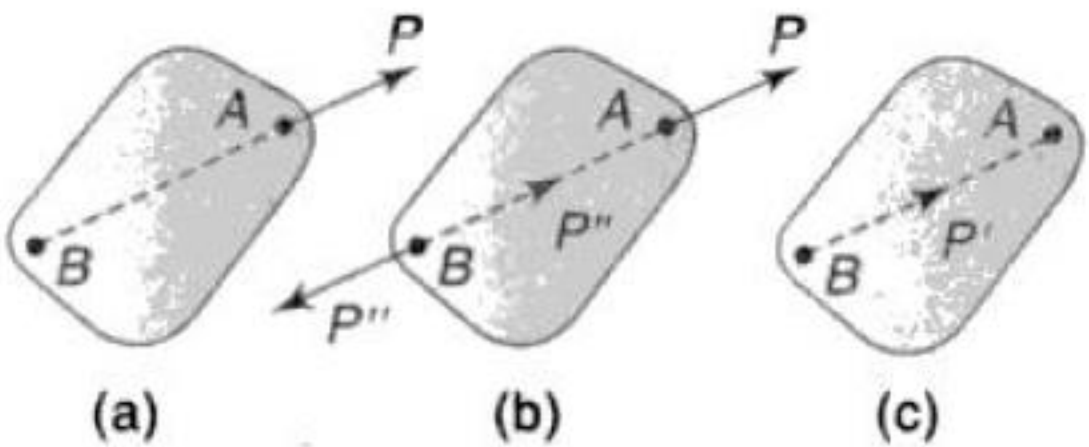
Equllibrium law: Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.



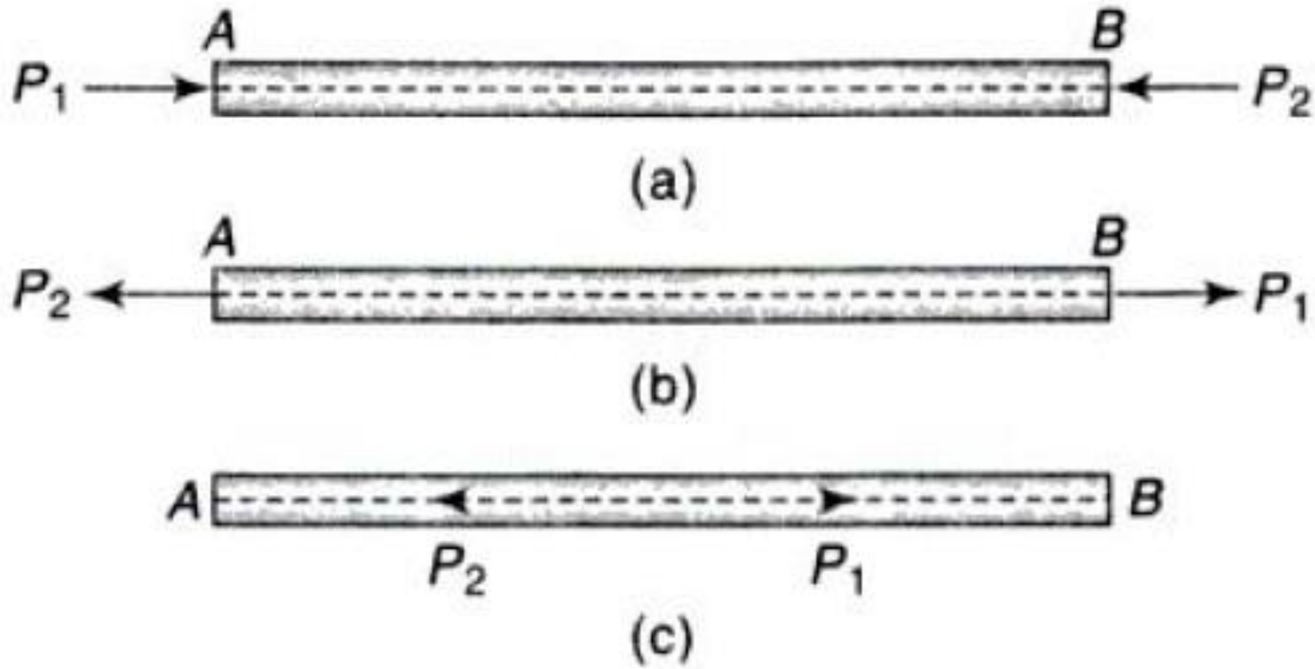
Superposition and Transmissibility When two forces are in equilibrium (equal, opposite and collinear), their resultant is zero and their combined action on a rigid body is equivalent to that of no force at all. A generalization of this observation gives us the third principle of statics, sometimes called the *law of superposition.*

Law of superposition

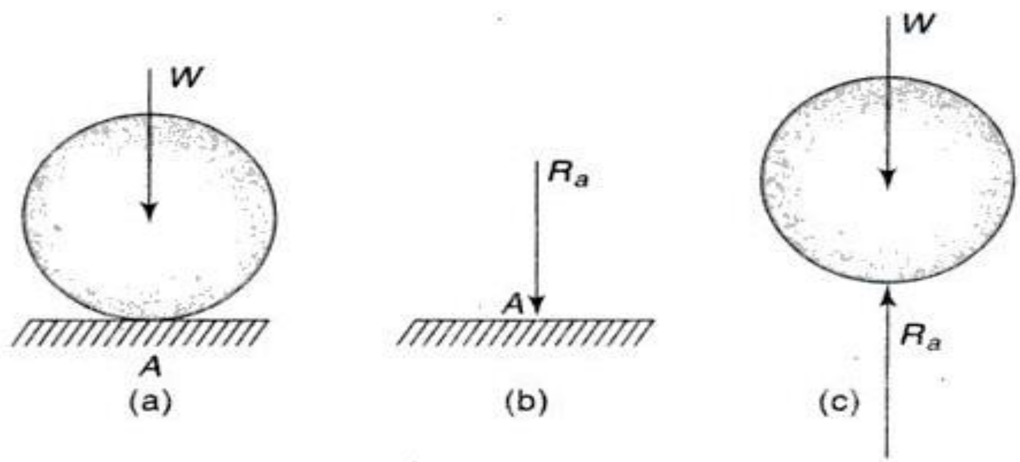
The action of a given system of forces on a rigid body will no way be changed if we add to or subtract from them another system of forces in equilibrium

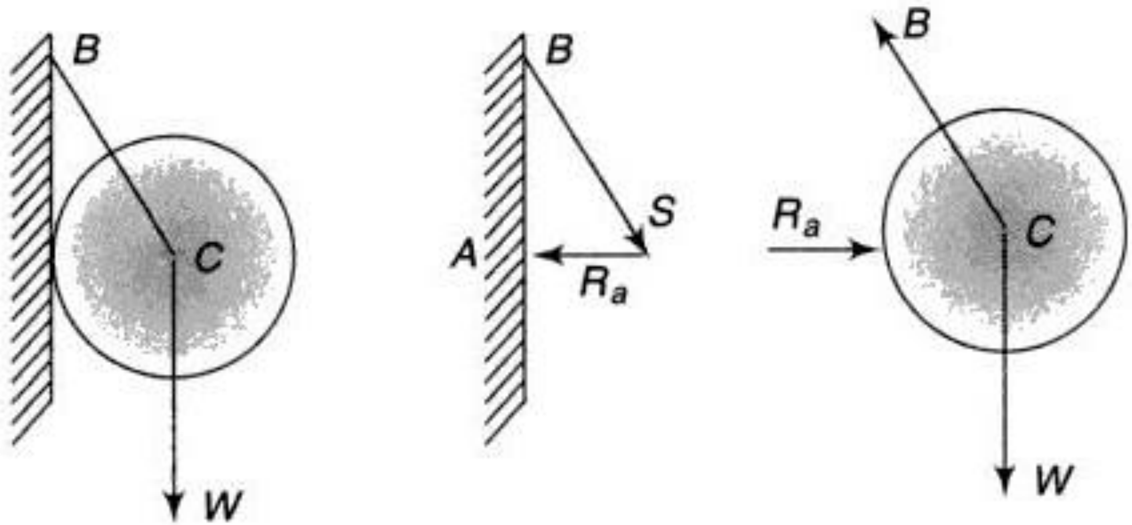


Theorem of transmissibility of a force: *The point of application of a force may be transmitted along its line of action without changing the effect of the force on any rigid body to which it may be applied.*



Law of Action and Reaction *Any pressure on a support causes an equal and opposite pressure from the support so that action and reaction are two equal and opposite forces.* This last principle of statics is of course nothing more than Newton's third law of motion stated in a form suitable for the discussion of problems of statics.





(a) (b) (c)

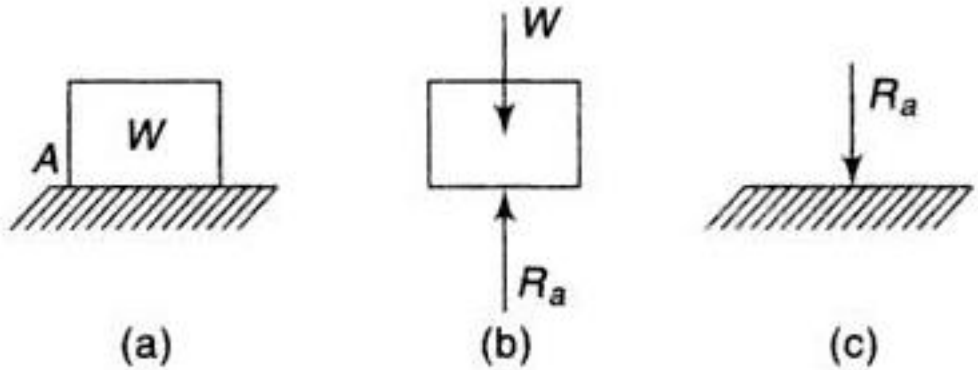
Free body diagram

Free body diagram is necessary to investigate the condition of equilibrium of a body or system. While drawing the free body diagram all the supports of the body are removed and replaced with the reaction forces acting on it.

Free body diagram is a sketch of the isolated body, which shows the external forces on the body and the reactions exerted on it by the removed elements.

The general procedure for constructing a free-body diagram is as follows:

1. A sketch of the body is drawn, by removing the supporting surfaces.
2. Indicate on this sketch all the applied or active forces, which tend to set the body in motion, such as those caused by weight of the body or applied forces; etc.
3. Also indicate on this sketch all the reactive forces, such as those caused by the constraints or supports that tend to prevent motion. (The sense of unknown reaction should be assumed. The correct sense will be determined by the solution of the problem. A positive result indicates that the assumed sense is correct. A negative result indicates that the correct sense is opposite to the assumed sense.)
4. All relevant dimensions and angles, reference axes are shown on the sketch.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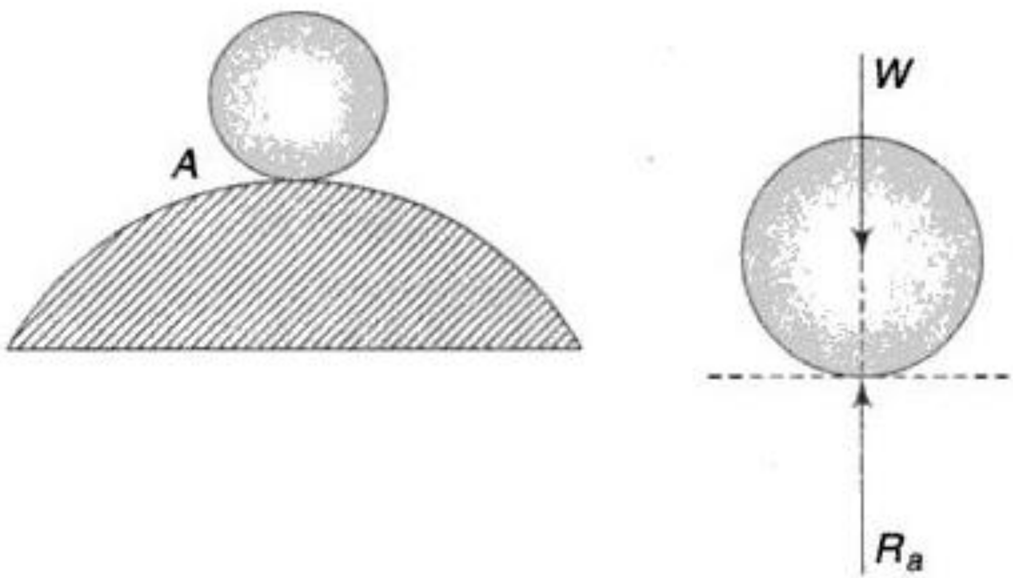
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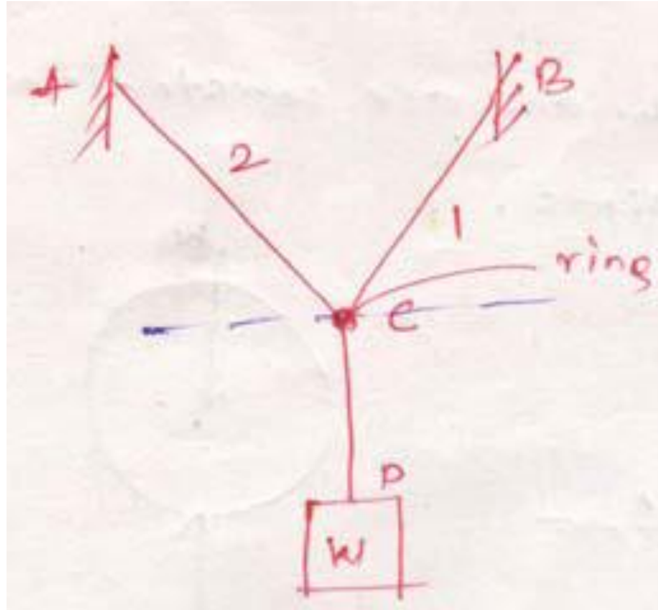
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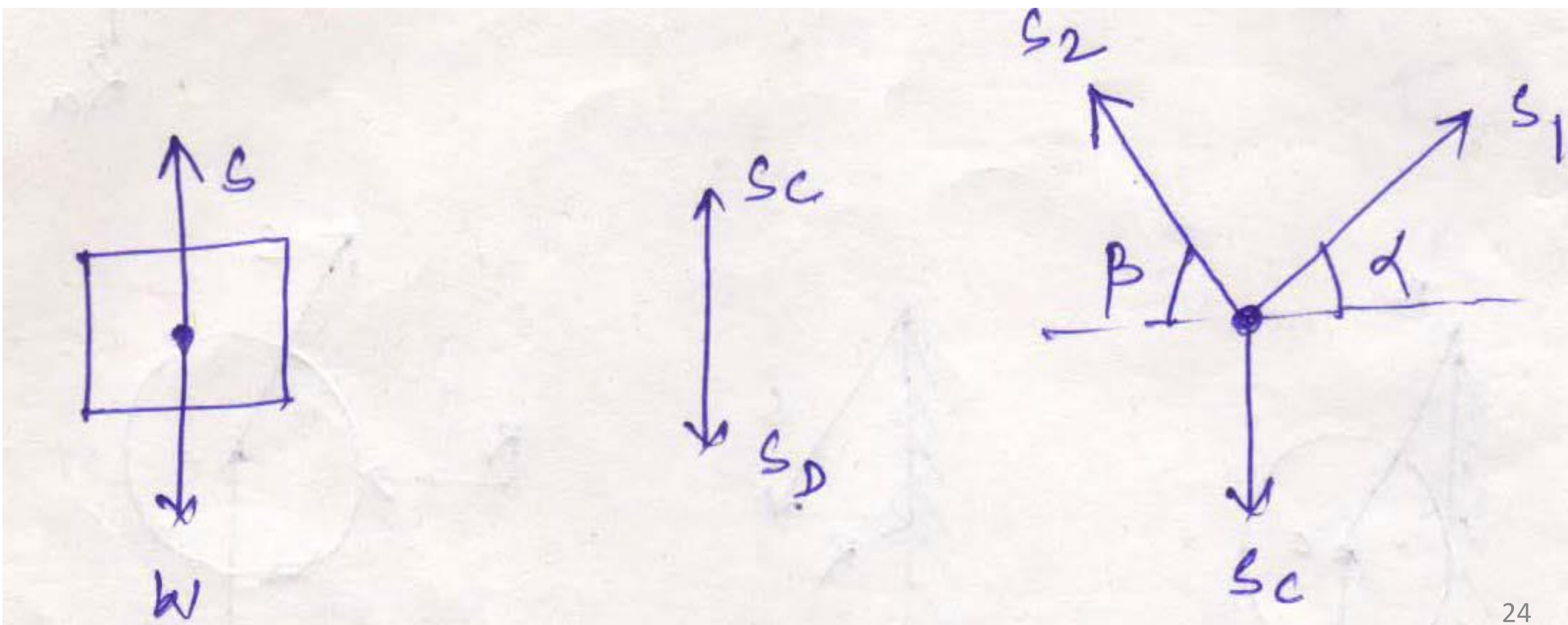
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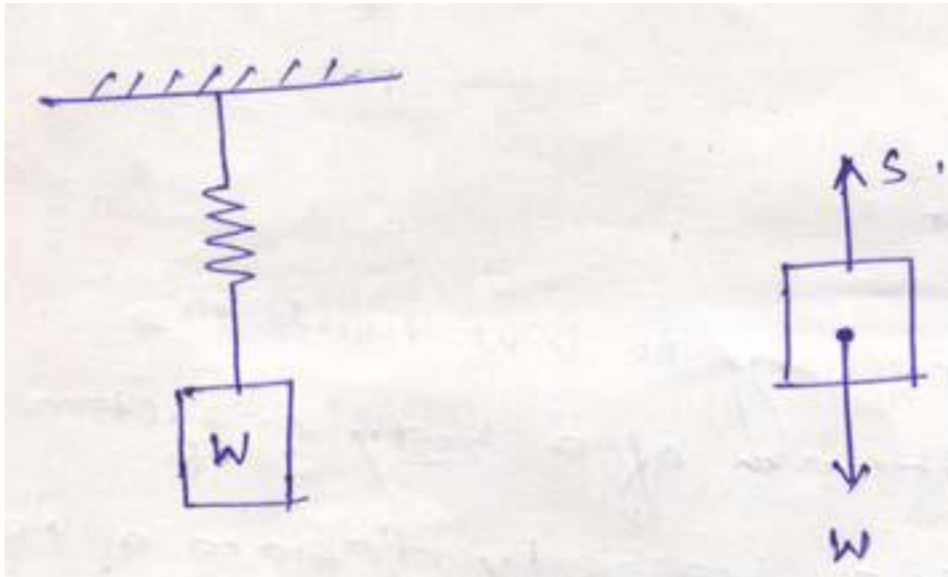
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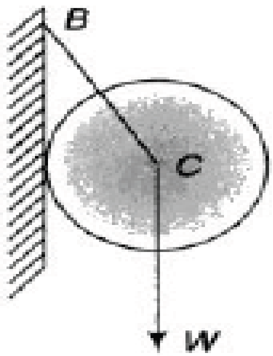
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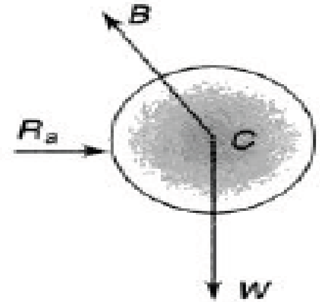
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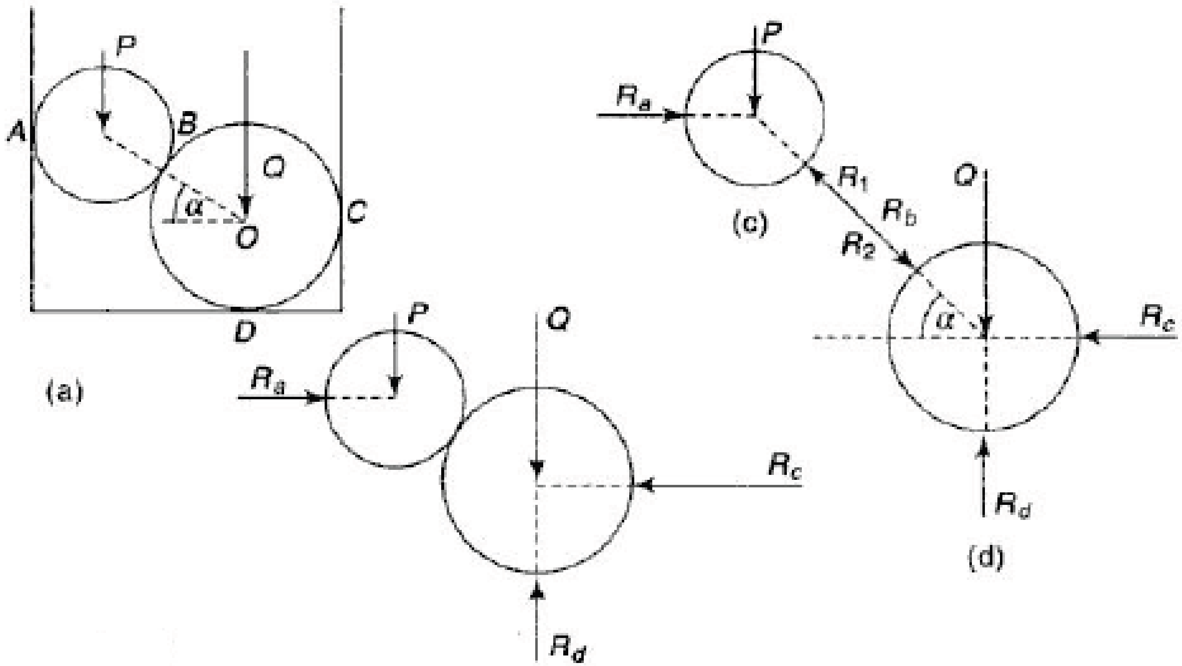
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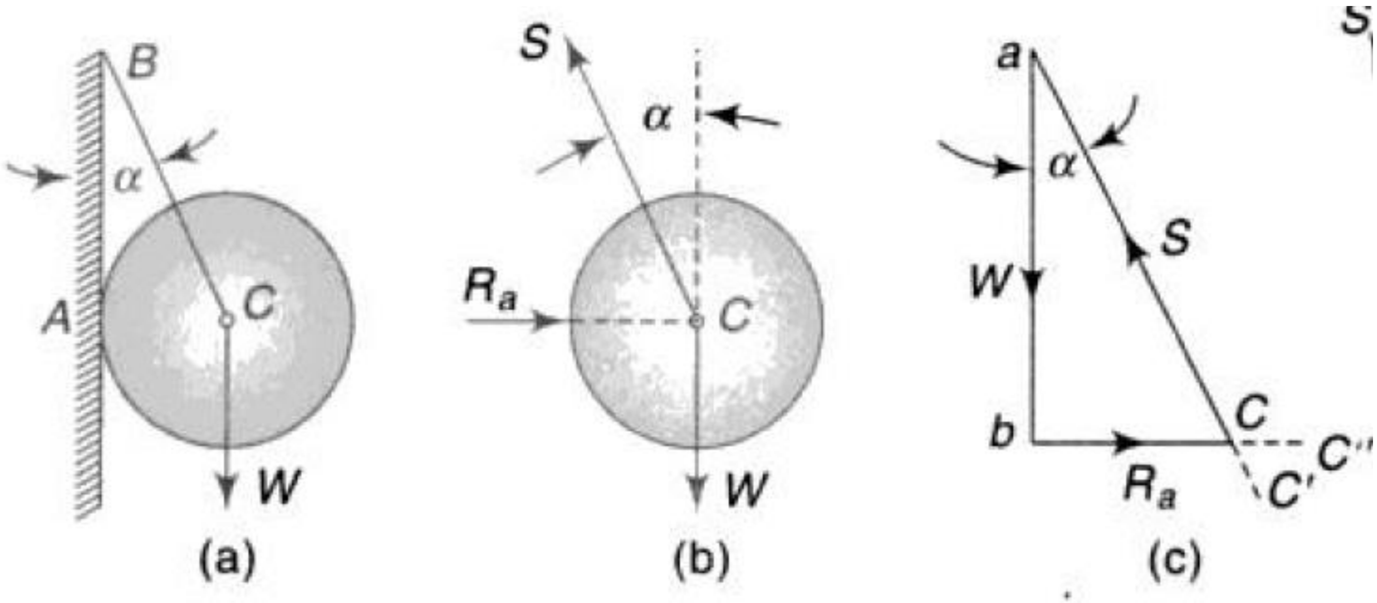
ENGINEERING MECHANICS

LECTURE 3

Equilibrium of concurrent forces in a plane

 If a body known to be in equilibrium is acted upon by several concurrent, coplanar forces, then these forces or rather their free vectors, when geometrically added must form a closed polygon.

 This system represents the condition of equilibrium for any system of concurrent forces in a plane.



$$R_a = W \tan \alpha \qquad S = W \sec \alpha$$

Lami's theorem

If three concurrent forces are acting on a body kept in an equllibrium, then each force is proportional to the sine of angle between the other two forces and the constant of proportionality is same.

Consider forces P, Q and R acting at point O as shown in Fig. 2.33(a). Mathematically, Lami's theorem is given by the following equation,

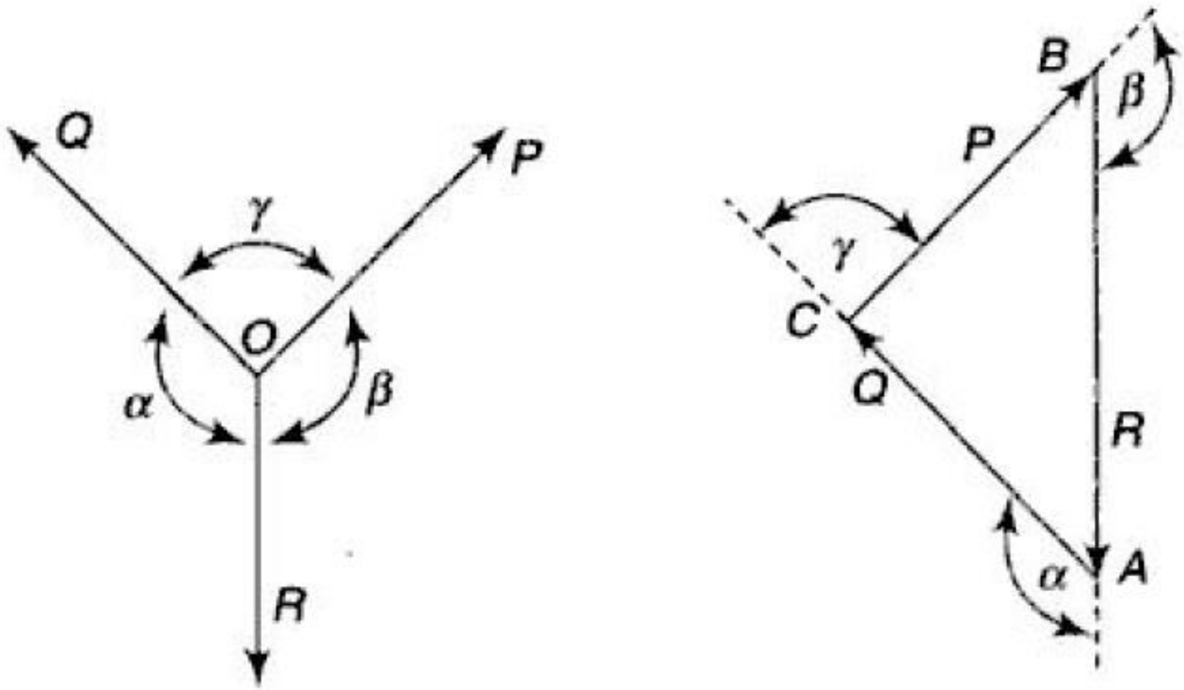
$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma} = k \qquad \text{(b)}$$

Since the forces are in equilibrium, the triangle of forces should close. Draw the triangle of forces ΔABC as shown in Fig. 2.33(b) corresponding to the forces P, Q and R acting at a point O. The angles of triangle are

$$\angle A = \pi - \alpha$$
$$\angle B = \pi - \beta$$
$$\angle A = \pi - \gamma$$

From the sine rule for the triangle, we get

$$\frac{P}{\sin(\pi - \alpha)} = \frac{Q}{\sin(\pi - \beta)} = \frac{R}{\sin(\pi - \gamma)}$$



(a) (b)

$$\because \quad \sin(\pi - \alpha) = \sin \alpha$$
$$\sin(\pi - \beta) = \sin \beta$$
$$\sin(\pi - \gamma) = \sin \gamma$$

Substituting these values into the above equation, it reduces to the Lami's theorem, i.e.,

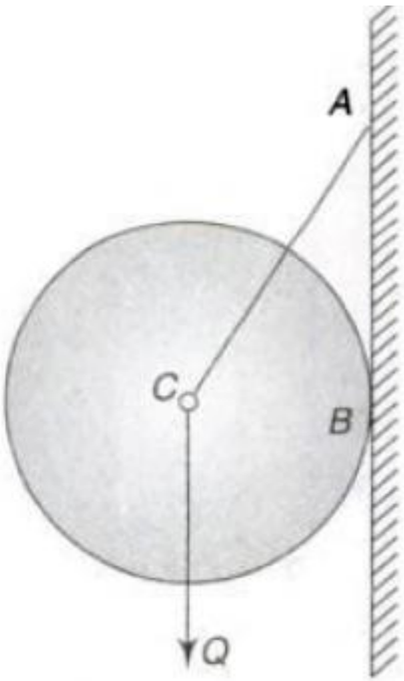
$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

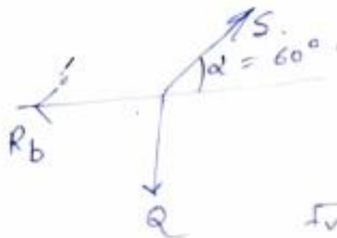
The limitations of Lami's theorem are as follows include only 6 points which must need to be remember before application :-

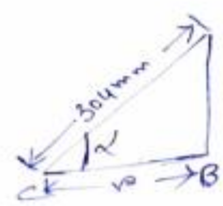
- there should be three forces only .
- those three forces should be coplanar (means should be in single plane).
- those three forces should be **concurrent (means their line of action meeting at point).**
- those three forces should be **non-linear (means their line of action not overlapping on each other).**
- And most important those three forces **must be in equilibrium .**

Solved Problem: 1

A circular roller of weight $Q = 445$ N and radius $r = 152$ mm hangs by a tie rod $AC = 304$ mm and rests against a smooth vertical wall at B, as shown in Fig. Determine the tension S in the tie rod and the force R_b exerted against the wall at B.







$r = 152$ mm

$Q = 445$ N

From, ΔABC,

$$\cos\alpha = \frac{r}{AC}$$

$$\cos\alpha = \frac{152}{304}$$

$$\alpha = 60^\circ$$

For, equilibrium,

$$\Sigma F_x = 0$$

$$R_b = S\cos 60^\circ$$

$$\Sigma F_y = 0$$

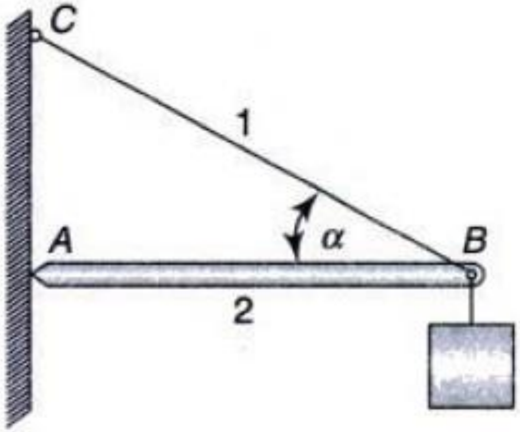
$$Q = S\sin 60^\circ$$

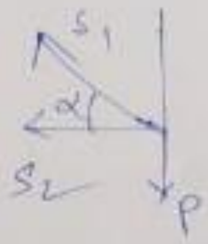
$$445 = S\sin 60^\circ$$

$$\boxed{S = 513.84 \text{ N}}$$

Solved Problem: 2

What axial forces does the vertical load P induce in the members of the system shown in Fig. ? Neglect the weights of the members themselves and assume an ideal hinge at A and a perfectly flexible string BC.





$$\Sigma F_x = 0$$

$$S_1 \cos\alpha + S_2 = 0$$

$$S_2 = -S_1 \cos\alpha$$

$$= -\frac{P\cos\alpha}{\sin\alpha}$$

$$= P\cot\alpha$$

(Compression)

$$\Sigma F_y = 0$$

$$S_1 \sin\alpha = P$$

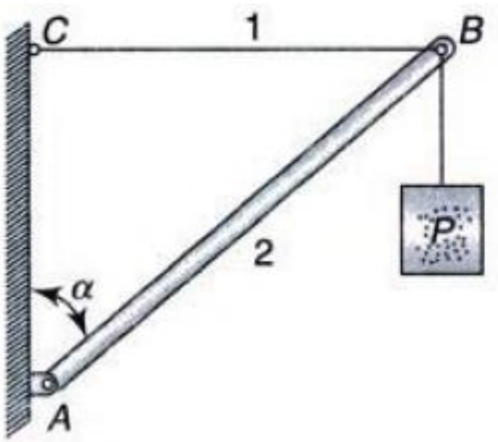
$$S_1 = \frac{P}{\sin\alpha}$$

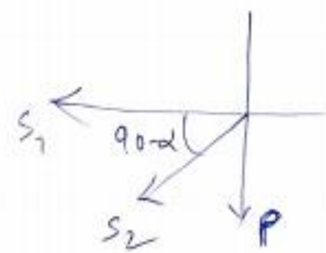
(Tension)

Solved Problem: 3

What axial forces does the vertical load P induce in the members of the system shown in Fig. ? .

Any assumption please refer the previous problem.





$\Sigma F_x = 0$

$S_1 + S_2 \cos(90^\circ - \alpha) = 0$

$S_1 + S_2 \sin\alpha = 0$

$S_1 = -S_2 \sin\alpha$

$\Sigma F_y = 0$

$S_2 \sin(90^\circ - \alpha) + P = 0$

$S_2 \cos\alpha = -P$

$S_2 = \dfrac{-P}{\cos\alpha}$

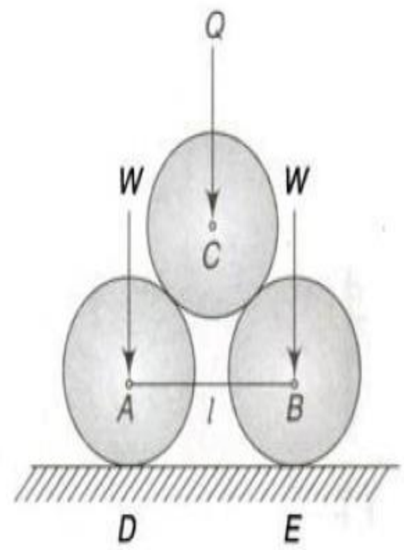
$\boxed{S_2 = P \sec\alpha}$ (Compression)

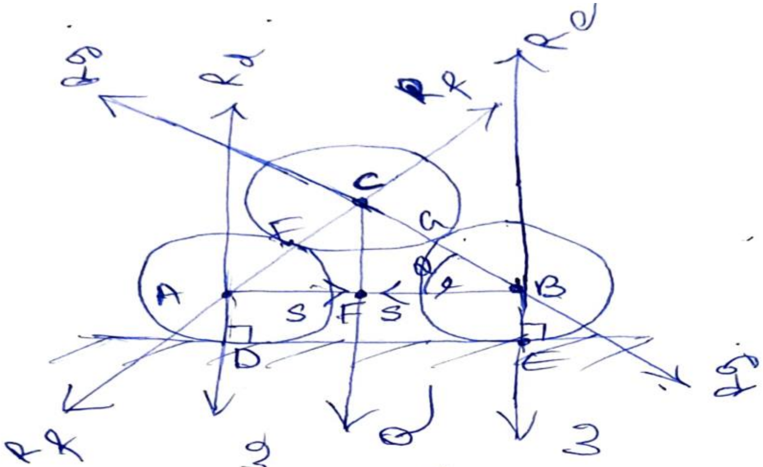
$S_1 = \dfrac{P \sin\alpha}{\cos\alpha}$

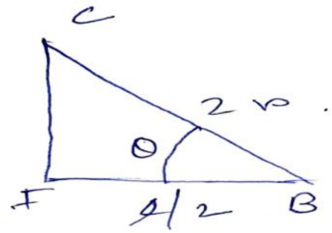
$\boxed{S_1 = P \tan\alpha}$ (Tension)

Solved Problem: 4

Two smooth circular cylinders, each of weight $W = 445$ N and radius $r = 152$ mm, are connected at their centers by a string AB of length $l = 406$ mm and rest upon a horizontal plane, supporting above them a third cylinder of weight $Q = 890$ N and radius $r = 152$ mm (Fig.). Find the forces S in the string and the pressures produced on the floor at the points of contact D and E.







$l = 406$ mm

$r = 152$ mm

$Q = 890$ N

$w = 445$ N

$$\cos\theta = \frac{l/2}{2r} = \frac{l}{4r} = \frac{406}{2 \times 182}$$

$$\cos\theta = 0.66776$$

$$\theta = 48.1^\circ$$

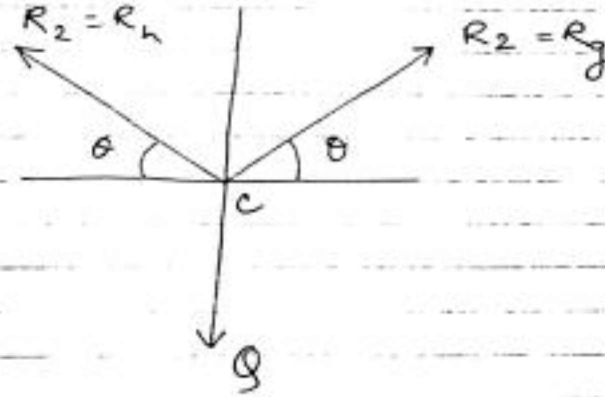
Due to symmetry

$R_d = R_e = R_1$

and $R_g = R_h = R_2$

F.B.D at C

F.B.D at C.



$\Sigma F_x = 0$

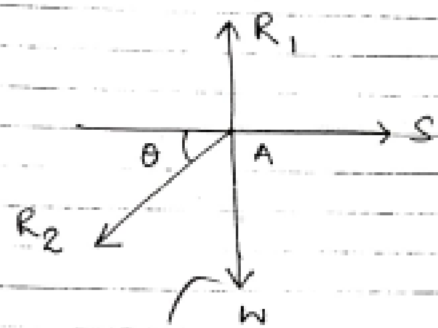
$R_2 \cos\theta = R_2 \cos\theta$

$\Sigma F_y = 0$

$R_2 \sin\theta + R_2 \sin\theta = Q$

$\Rightarrow 2R_2 \sin 48.1 = Q$ —— (1) $\Rightarrow R_2 = 597.86\,N$

F.B.D at A:



$\Sigma F_X = 0$

$S = R_2 \cos\theta = R_2 \cos 48.1$

$\Rightarrow S = 399.27\ N$

$\Sigma F_Y = 0$

$R_1 = W + R_2 \sin 48.1$

$R_1 = 445 + \_\_\_$

$= 889.99\ N$

ENGINEERING MECHANICS

LECTURE 4

METHOD OF PROJECTIONS

Previously, we have handled all problems of composition, resolution and equilibrium of concurrent forces in a plane by using the method of geometric addition of their free vectors. These same problems can also be solved by a method of algebraic addition of the *projections* of the given forces on rectangular coordinate axes x and y taken in the plane of action of the forces. To develop this method, let us consider first the case of two forces F_1 and F_2, applied at point A (Fig. 2.44) and making with the positive directions of the coordinate axes the angles α_1, β_1 and α_2, β_2, respectively. Their resultant R is obtained from the parallelogram of forces, and the angles that it makes with the x and y axes, respectively, will be denoted by α and β. Considering, now, all forces projected onto the x axis, we find for these projections the values $F_1 \cos \alpha_1$, $F_2 \cos \alpha_2$, and $R \cos \alpha$, and we see that

$$R \cos \alpha = F_1 \cos \alpha_1 + F_2 \cos \alpha_1 \qquad \text{(a)}$$

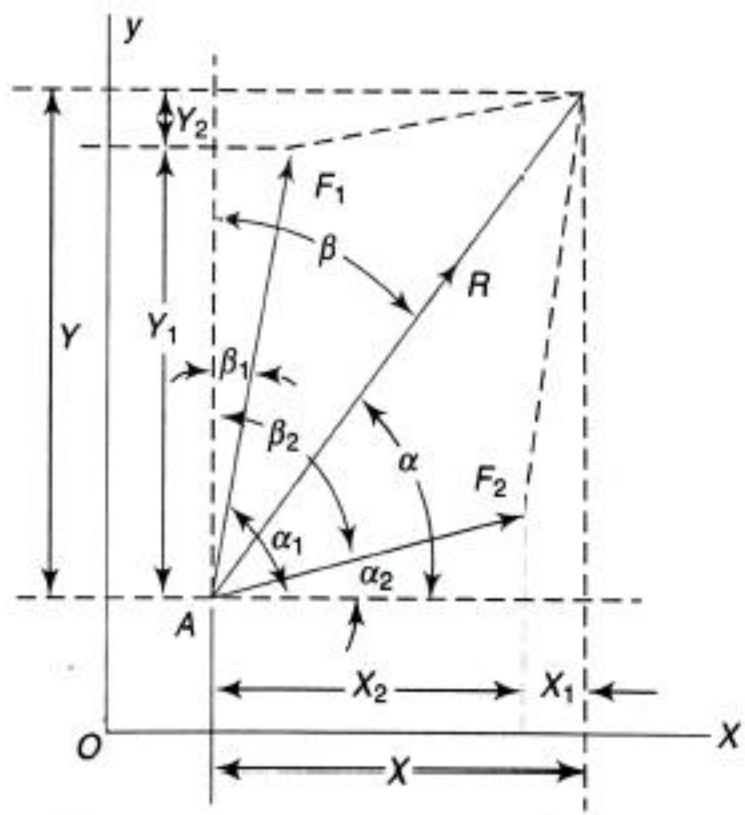


Fig. 2.45

In the same manner, considering all forces projected onto the y-axis, we obtain

$$R \cos \beta = F_1 \cos \beta_1 + F_2 \cos \beta_2 \tag{2.5b}$$

Thus from Eqs (2.5a) and (2.5b) it may be stated that the *projection of the resultant of two forces on any axis is equal to the algebraic sum of the projections of its components on the same axis.*

By successive applications of the principle of the parallelogram of forces, the above conclusion can be obtained for any number of concurrent forces F_1, F_2, ..., F_n in a plane. Using, for the projections of the various forces, the following notations

$$X_i = F_i \cos \alpha_i \quad Y_i = F_i \cos \beta_i$$
$$X = R \cos \alpha \quad Y = R \cos \beta$$

We obtain

$$X = X_1 + X_2 + \ldots + X_n = \Sigma X_i \tag{2.6a}$$
$$Y = Y_1 + Y_2 + \ldots + Y_n = \Sigma Y_i \tag{2.6b}$$

where the summations are understood to include all forces in the system. *Thus, the projections, on the coordinate axes, of the resultant of a system of concurrent forces $F_1, F_2, \ldots, F_n$ acting in one plane are equal to the algebraic sum of the corresponding projections of the components.*

Knowing the magnitudes and directions of the various forces, their projections $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$ on the rectangular coordinate axes x and y, respectively, may be computed and tabulated in systematic order. The algebraic summations, indicated by Eq. (2.6), for determining the projections X and Y of the resultant may then be made and the magnitude and direction of the resultant computed from the following equations:

$$R = \sqrt{X^2 + Y^2}, \qquad \tan\alpha = \frac{Y}{X} \tag{2.7}$$

When the given forces $F_1, F_2, \ldots, F_n$ are in equilibrium, their resultant is zero, and from the first of Eq. (2.7), it is evident that this condition can be satisfied only if we have $X = 0$ and $Y = 0$, which, referring to Eqs (2.6), evidently requires

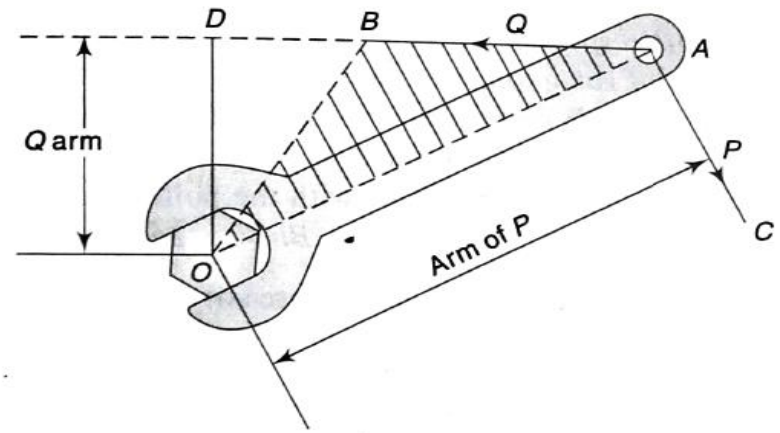
$$\Sigma X_i = 0, \qquad \Sigma Y_i = 0 \tag{2.8}$$

ENGINEERING MECHANICS

LECTURE 6

Method of moments

Moment of a force with respect to a point:



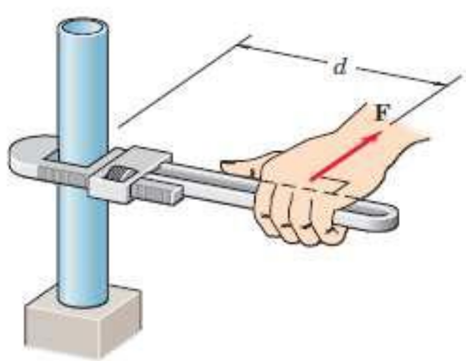
- Considering wrench subjected to two forces P and Q of equal magnitude. It is evident that force P will be more effective compared to Q, though they are of equal magnitude.
- The effectiveness of the force as regards it is the tendency to produce rotation of a body about a fixed point is called the moment of the force with respect to that point.
- Moment = Magnitude of the force × Perpendicular distance of the line of action of force.
- Point O is called moment centre and the perpendicular distance (i.e. OD) is called moment arm.
- Unit is N.m.

Moment is defined as the product of the magnitude of the force and the perpendicular distance of the point from the line of action of the force.

$$M = Fd$$

Sign Convention of Moment

Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for clockwise moments and a minus sign (-) for counter clockwise moments, or vice versa.



(a)

Theorem of Varignon: The moment of the resultant of two concurrent forces with respect to a centre in their plane is equal to the algebraic sum of the moments of the components with respect to the same centre.

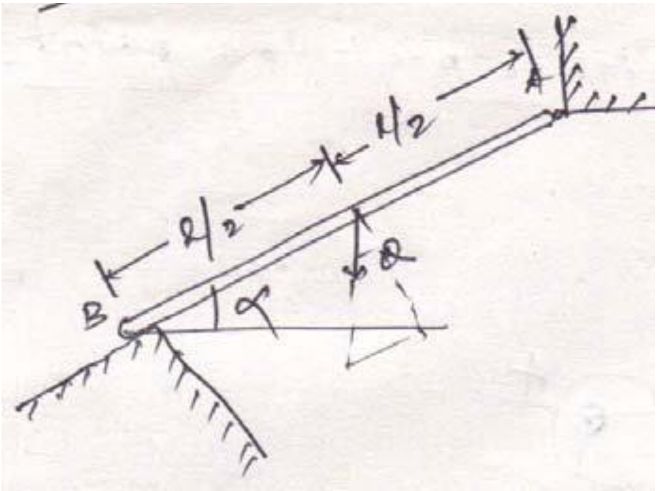
Problem 1:

A prismatic clear of AB of length l is hinged at A and supported at B. Neglecting friction, determine the reaction R_b produced at B owing to the weight Q of the bar.

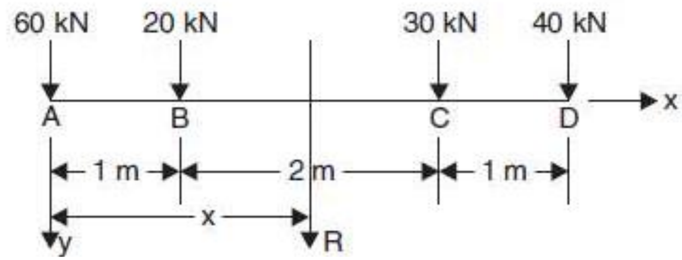
Taking moment about point A,

$$R_b \times l = Q\cos\alpha . \frac{l}{2}$$

$$\Rightarrow R_b = \frac{Q}{2}\cos\alpha$$



Example 2.13 Determine the resultant of four parallel forces acting on the axle of a vehicle as shown in Fig. 2.26.



Solution: Let x and y axes be selected as shown in the Figure

$$R_x = \Sigma F_x = 0$$

$$R_y = \Sigma F_y = 60 + 20 + 30 + 40 = 150 \text{ kN}$$

$$\therefore \qquad R = \sqrt{0^2 + 150^2} = \mathbf{150\ kN}$$

Taking clockwise moment as +ve,

$$\Sigma M_A = 60 \times 0 + 20 \times 1 + 30 \times 3 + 40 \times 4$$

$$= \mathbf{270\ kN\text{-}m}$$

$\therefore$ Distance of resultant from A

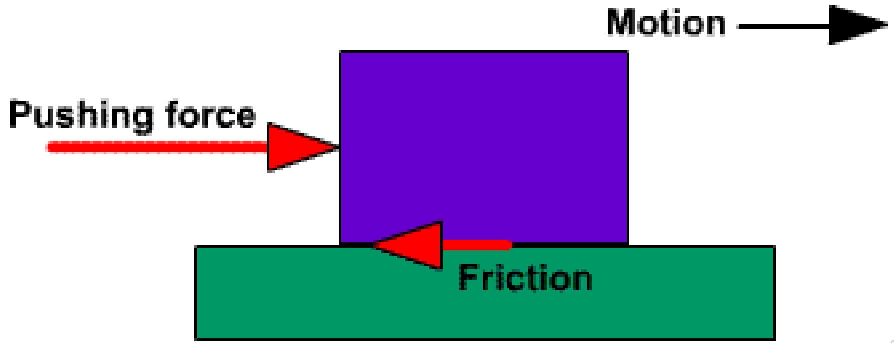
$$x = \frac{270}{150} = \mathbf{1.8\ m} \text{ as shown in the figure.}$$

Engineering Mechanics

Friction

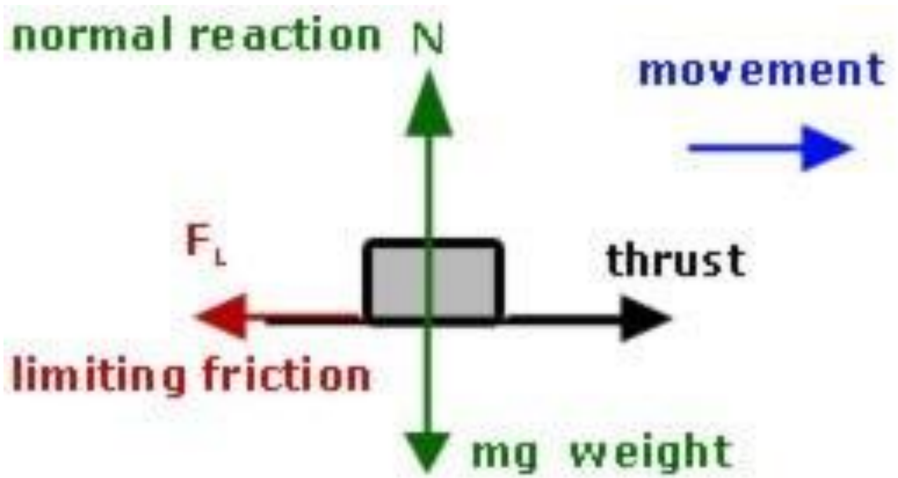
What Is Friction?

- When a body slide or tends to slide on a surface on which it is resting, a resisting force opposing the motion is produced at the contact surface. This resisting force is called friction or friction force.



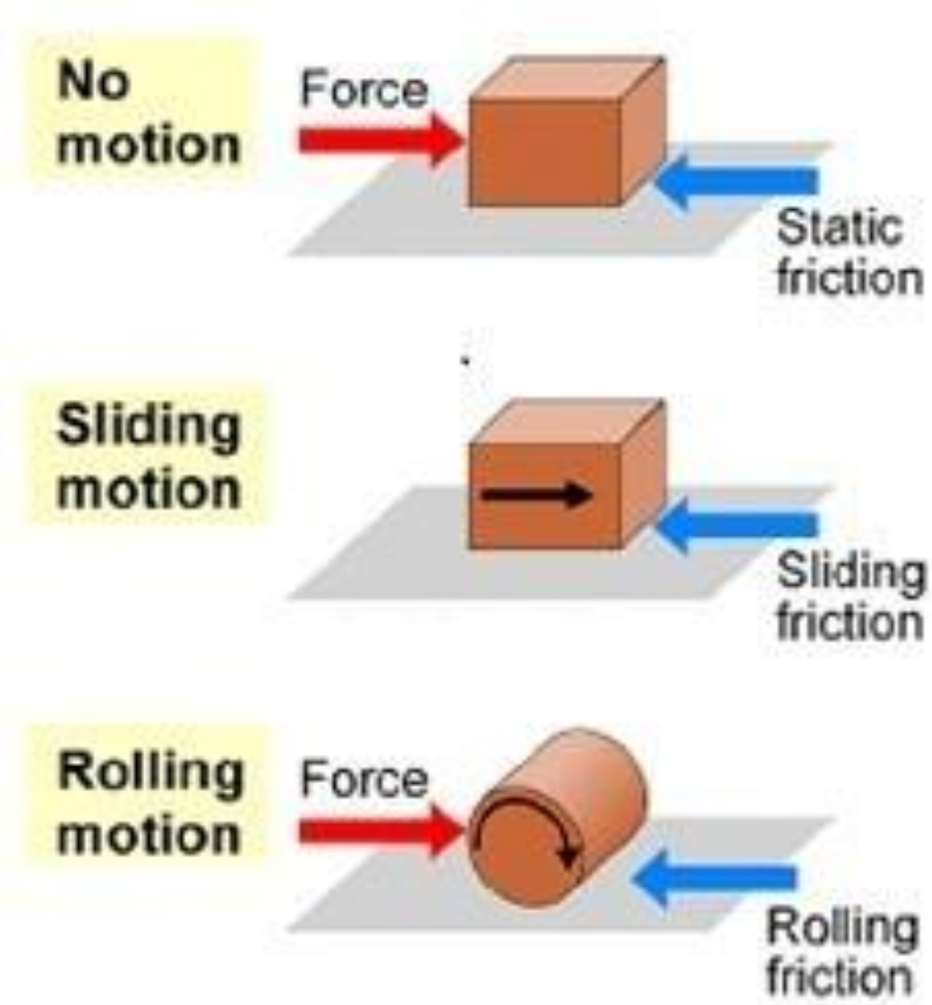
Limiting Friction

- The maximum friction force that can be developed at the contact surface, when body is just on the point of moving is called limiting force of friction.



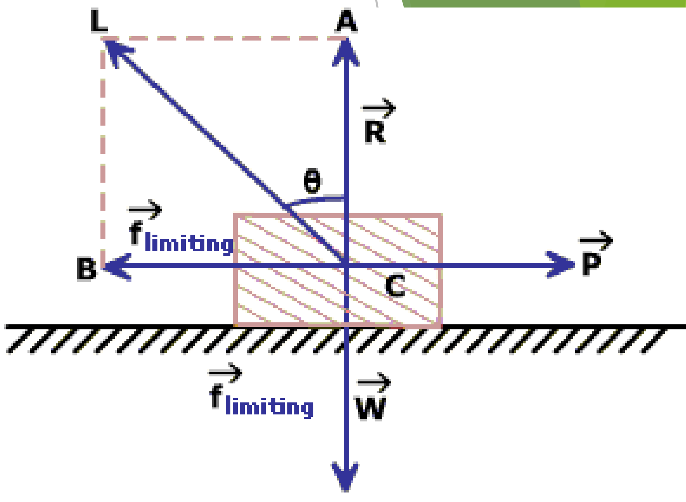
Types Of Friction

- Static Friction:-
  - Friction experienced by a body when it is at rest is called static friction.
- Dynamic Friction:-
  - Friction experienced by a body when it is motion is called dynamic friction.
- Sliding Friction:-
  - Friction experienced by a body when it slides over another body, is called sliding friction.
- Rolling Friction:-
  - Friction experienced by a body when it rolls over another body is called rolling friction.


No motion
Force
Static friction
Sliding motion
Sliding friction
Rolling motion
Force
Rolling friction

Angle of Friction

- The angle between normal reaction (A) and resultant force (L) is called angle of friction.
- It is also called limiting angle of friction.



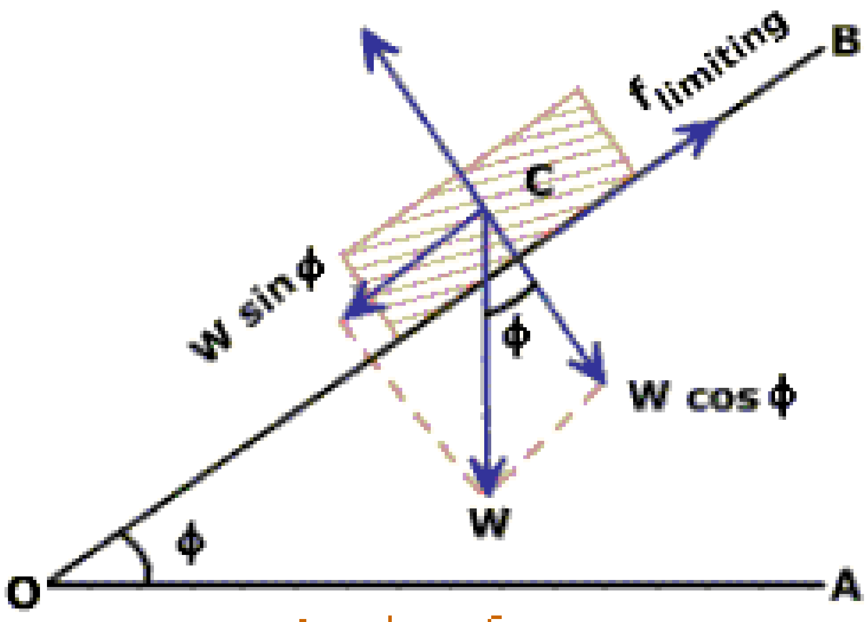
Angle of friction

Coefficient of friction

- The ratio of limiting friction (F) and normal reaction (N) is called coefficient of friction.
- $\mu = F / N$
  - F = Friction Force
  - N = Normal Reaction

Angle Of Repose

- With increase in angle of the inclined surface, the maximum angle at which body starts sliding down is called angle of friction.



Laws Of friction

- Laws of static friction
  - The friction force always acts in a direction, opposite to that in which the body tends to move.
  - The magnitude of friction force is equal to the external force.

    F=P
  - The ratio of limiting friction (F) and normal reaction (N) is constant.

    F/N=μ
  - The friction force does not depends upon the area of contact between the two surfaces.
  - The friction force depends upon the roughness of the surfaces.

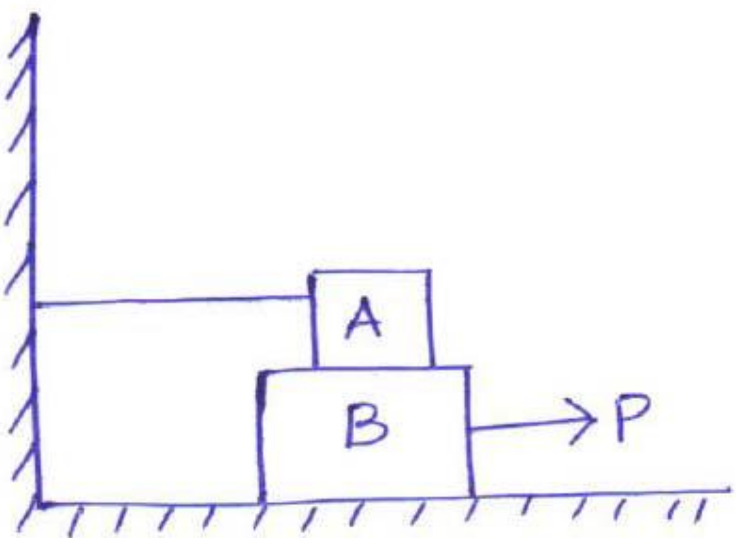
Laws of dynamic friction

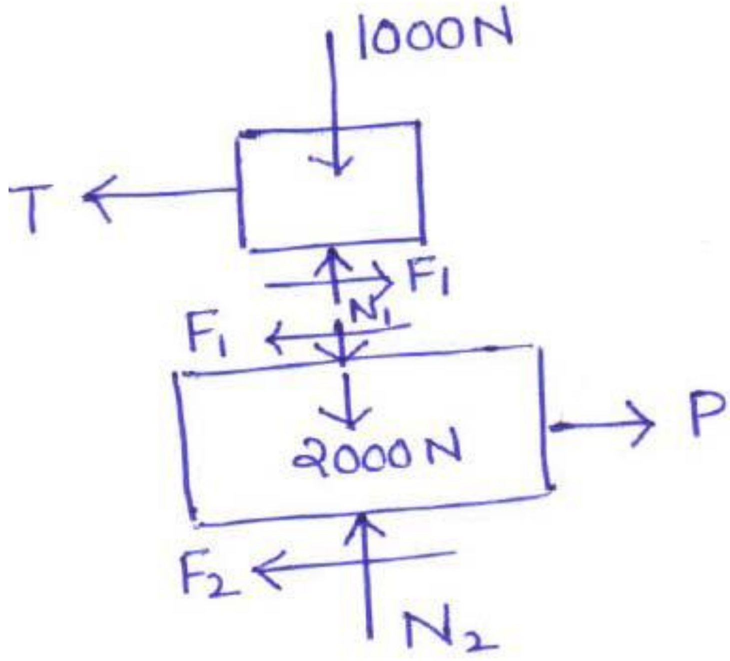
- The friction force always acts in a direction, opposite that in which the body is moving.
- The ratio of limiting friction (F) and normal reaction (N) is constant & it is known as coefficient of friction (μ).
- For moderate speeds, the friction force remains constant. But it decreases slightly with the increase of speed.

Problem 1: Block A weighing 1000N rests over block B which weighs 2000N as shown in figure. Block A is tied to wall with a horizontal string. If the coefficient of friction between blocks A and B is 0.25 and between B and floor is 1/3, what should be the value of P to move the block (B), if

(a) P is horizontal.

(b) P acts at 30° upwards to horizontal.





Considering block A,

$$\sum V = 0$$
$$N_1 = 1000N$$

Since F_1 is limiting friction,

$$\frac{F_1}{N_1} = \mu = 0.25$$
$$F_1 = 0.25N_1 = 0.25 \times 1000 = 250N$$

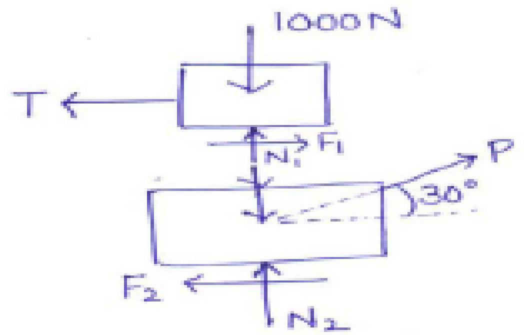
$$\sum H = 0$$
$$F_1 - T = 0$$
$$T = F_1 = 250N$$

Considering equilibrium of block B,

$$\sum V = 0$$
$$N_2 - 2000 - N_1 = 0$$
$$N_2 = 2000 + N_1 = 2000 + 1000 = 3000N$$

$$\frac{F_2}{N_2} = \mu = \frac{1}{3}$$
$$F_2 = 0.3N_2 = 0.3 \times 1000 = 1000N$$

$$\sum H = 0$$
$$P = F_1 + F_2 = 250 + 1000 = 1250N$$



(b) When P is inclined:

$$\sum V = 0$$
$$N_2 - 2000 - N_1 + P.\sin 30 = 0$$
$$\Rightarrow N_2 + 0.5P = 2000 + 1000$$
$$\Rightarrow N_2 = 3000 - 0.5P$$

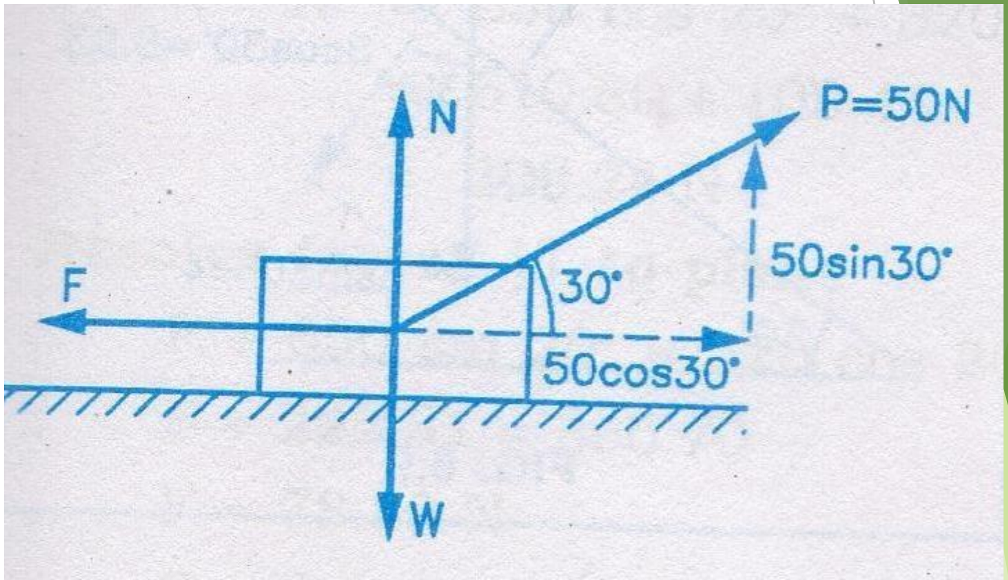
From law of friction,

$$F_2 = \frac{1}{3}N_2 = \frac{1}{3}(3000 - 0.5P) = 1000 - \frac{0.5}{3}P$$

$$\sum H = 0$$
$$P\cos 30 = F_1 + F_2$$
$$\Rightarrow P\cos 30 = 250 + \left(1000 - \frac{0.5}{3}P\right)$$
$$\Rightarrow P\left(\cos 30 + \frac{0.5}{3}P\right) = 1250$$
$$\Rightarrow P = 1210.43N$$

A Pull of 50 N inclined at 30° to the horizontal is necessary to move a wooden block on horizontal table. If coefficient of friction is 0.20, find the weight of wooden block.

Solution :



$P = 50$ N

$\mu = 0.20$

Resolve || to plane : F =

$50 \cos 30^\circ$

F = 43.30 N

μ = F/N

$0.20 = 43.30/N$

N = 216.5

Resolve (perpendicular) to plane

$N + 50 \sin 30^\circ = W$

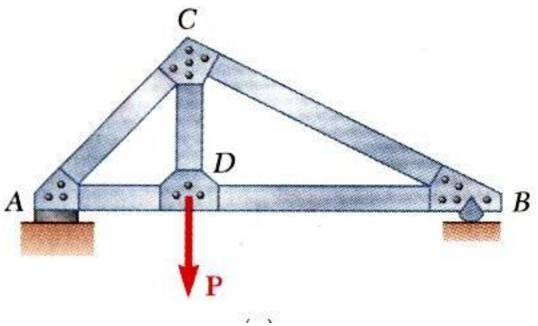
$216.5 + 25 = W$

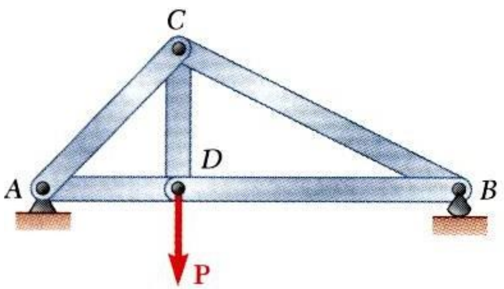
$W = 241.5$ N

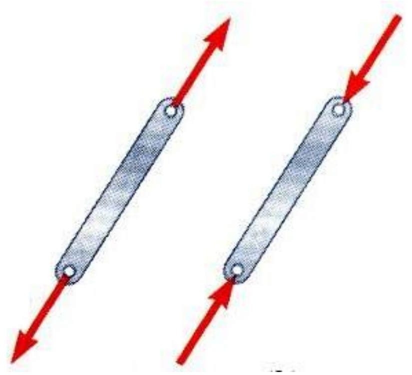
Engineering Mechanics

Truss

Definition of a Truss







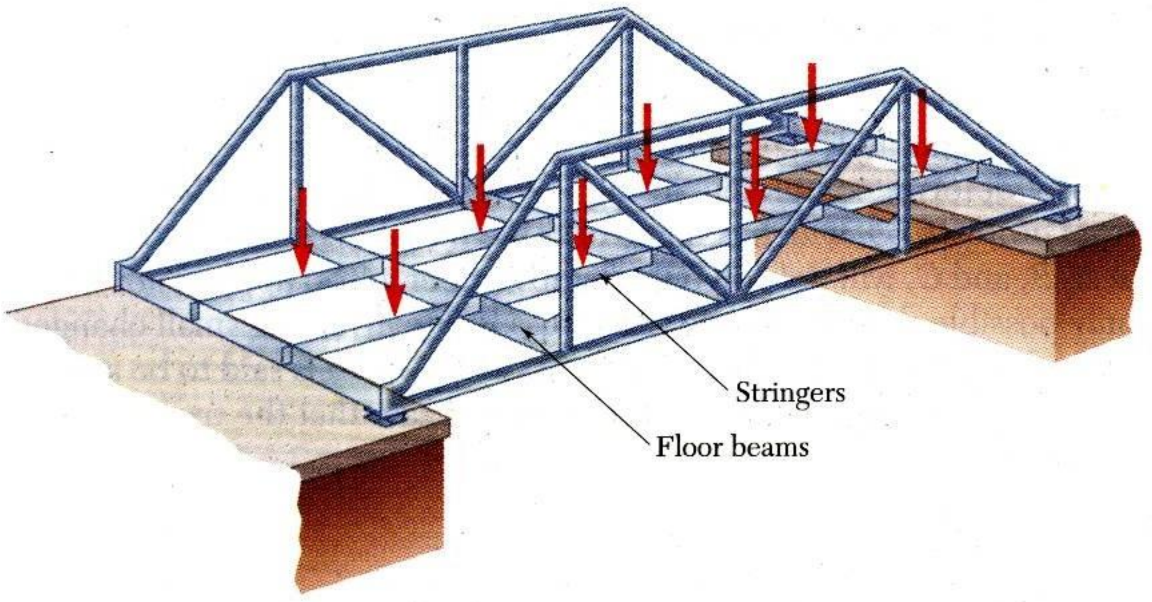
- A truss is an assembly of straight members connected at joints. **No member is continuous through a joint.**

- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.

- **Bolted or welded connections are assumed to be pinned together**. Forces acting at the member ends reduce to a single force and no couple. Only two-force members are considered.

- When forces tend to pull the member apart, it is in tension. When the forces tend to compress the member, it is in compression.

Definition of a Truss

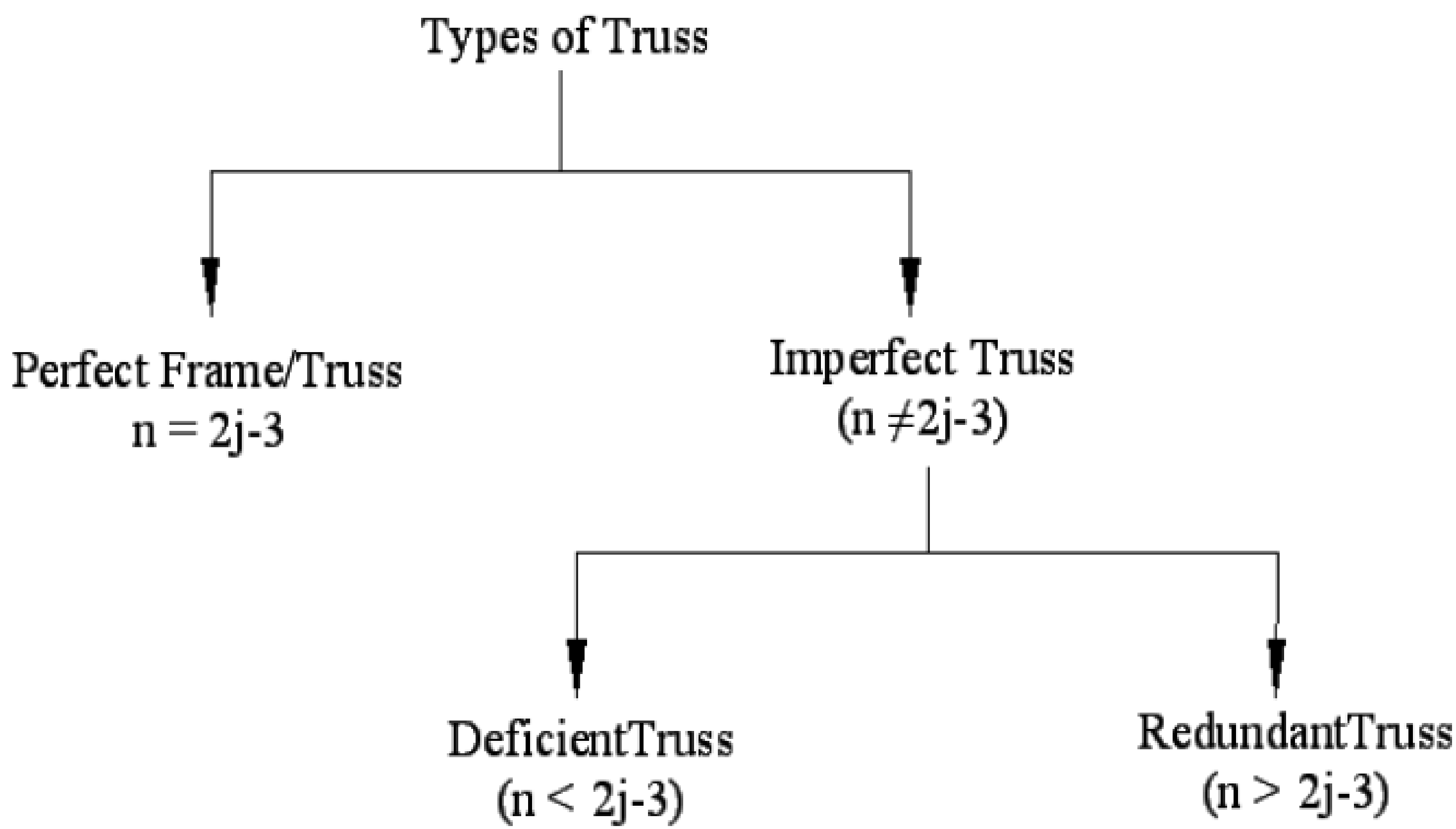


Members of a truss are slender and not capable of supporting large lateral loads. **Loads must be applied at the joints.**

- **Weights are assumed to be distributed to joints.**
- **External distributed loads transferred to joints via stringers and floor beams.**

Truss

Types of Trusses:



Perfect Truss:

A perfect truss is that, which is made up of members just sufficient to keep it in equilibrium, when loaded, without any change in its shape. Such structure satisfies following equation.

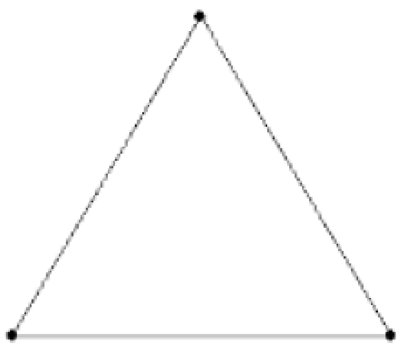
$$n = 2j - 3$$

where,

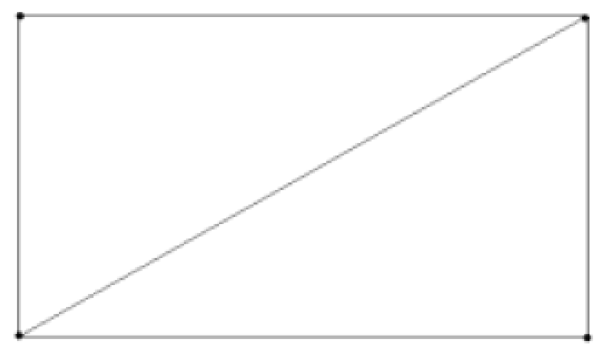
n = number of members

j = number of joints

Eg.



Basic Perfect Truss
Here $n = 3$ and $j = 3$
$3 = (2 \times 3) - 3$
$3 = 3$

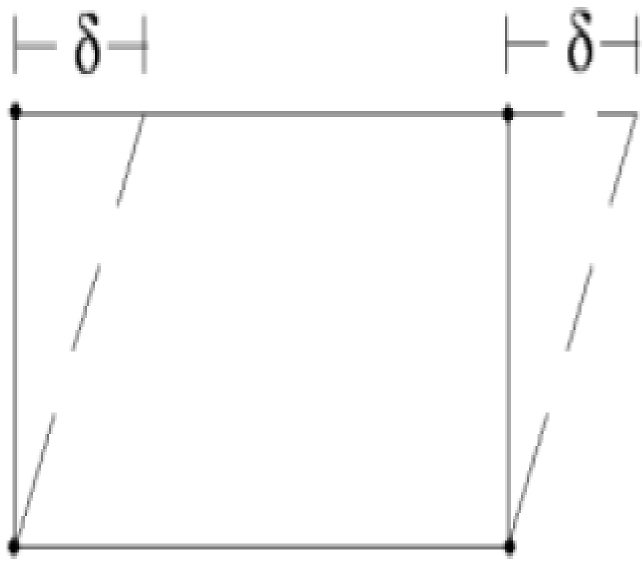


Perfect Truss
Here $n = 5$ and $j = 4$
$5 = (2 \times 4) - 3$
$5 = 5$

Deficient Truss:

In such truss the numbers of members are less than (2j-3). Such trusses are unable to carry any loads. So such trusses are unstable which undergo deformation.

Eg.



Here n = 4 and j = 4

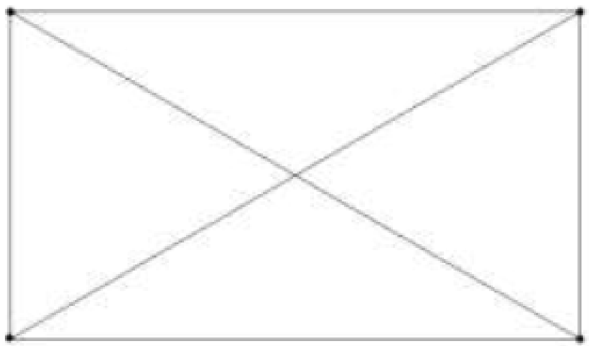
4 = (2 x 4) – 3

4 < 5

Redundant Truss:

In such truss the numbers of members are more than (2j-3). In such trusses the members are more than required which is sufficient to carry loads. They don't undergo any deformation.

Eg.



Here n = 6 and j = 4

6 = (2 x 4) – 3

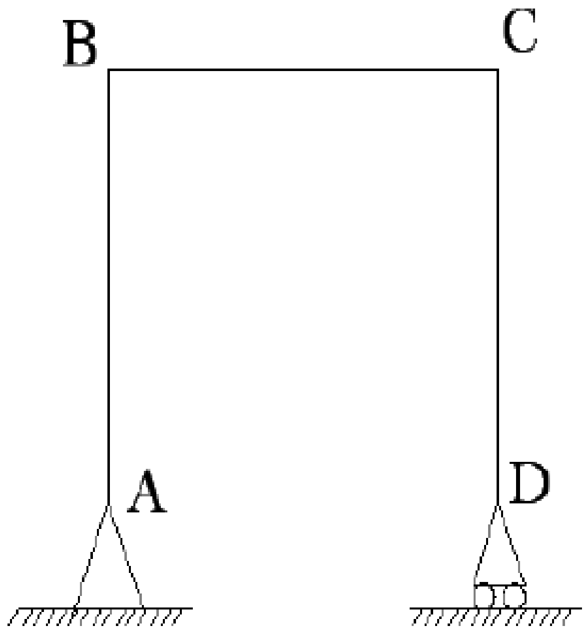
6 > 5

Member:

The straight component bars of the trusses joined at the ends by the pins are known as members.

Frame:

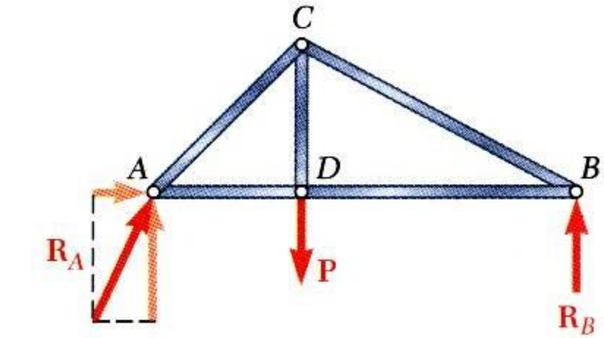
A frame is structure of combination of two force members and three force members or multi-force members as shown in Fig. 2.

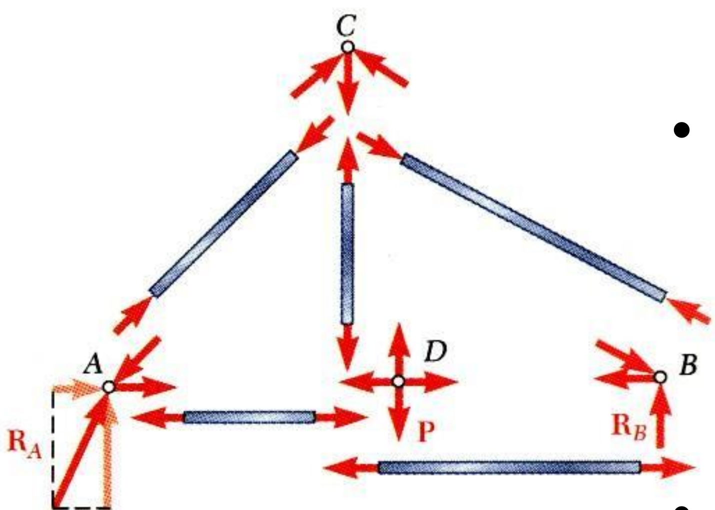


Assumption made for the analysis of truss

1) All the members are pin jointed.
2) All the members are assumed to two force members.
3) The truss is loaded at the joints only.
4) The self weight of the truss is considered as negligible in comparison with the other external forces acting on a truss.
5) The cross section of the members of trusses is uniform.

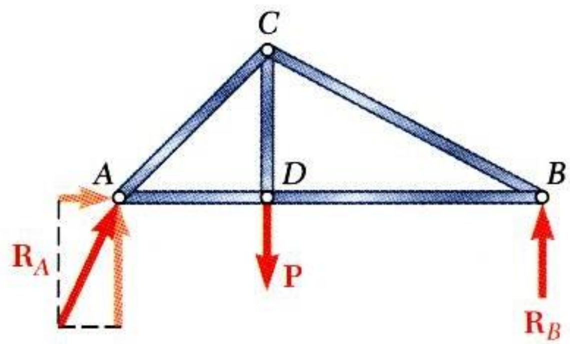
Analysis of Trusses by the Method of Joints



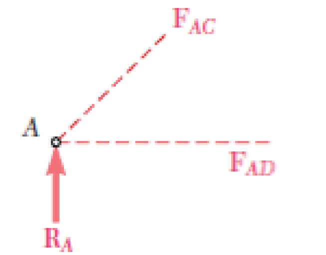
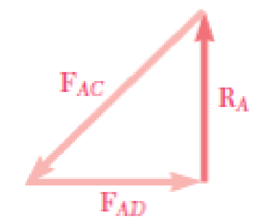
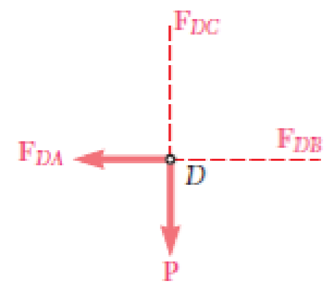
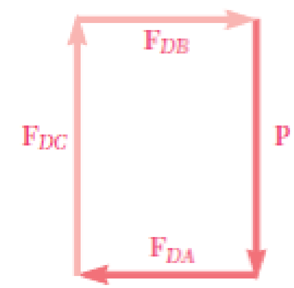
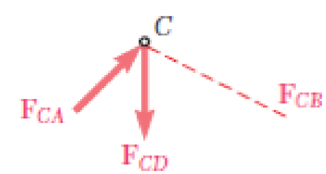
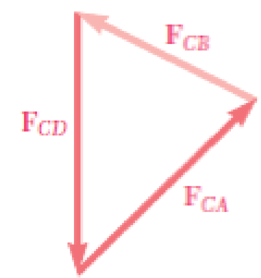
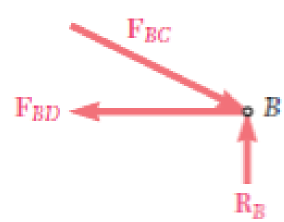
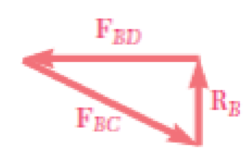


- Dismember the truss and create a free body diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide 2j equations for 2j unknowns. For a simple truss, 2j = m + 3. May solve for m member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

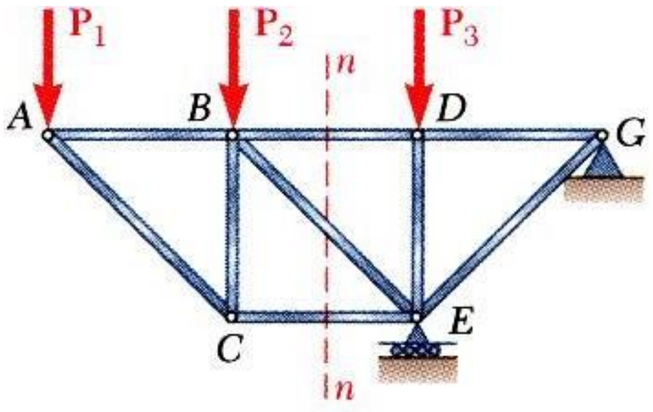
Analysis of Trusses by the Method of Joints

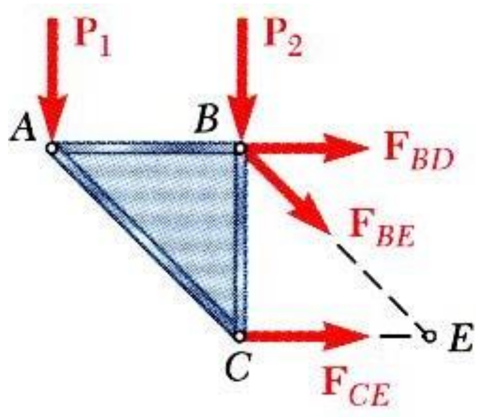


- Use conditions for equilibrium for the entire truss to solve for the reactions R_A and R_B.

	Free-body diagram	Force polygon
Joint *A*		
Joint *D*		
Joint *C*		
Joint *B*		

Analysis of Trusses by the Method of Sections





- When the force in only one member or the forces in a very few members are desired, the method of sections works well.
- To determine the force in member BD, pass a section through the truss as shown and create a free body diagram for the left side (or right side).
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including F_{BD}.

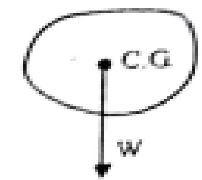
Engineering Mechanics

Centre of Gravity

3.3.1 INTRODUCTION

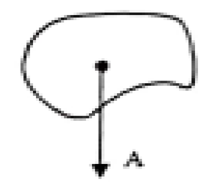
A body consists of lots of particles which are attracted towards the centre of earth due to gravity. Hence force of attraction on each particle in the body constitute a system of parallel forces. The point on the bdoy at which the resultant of all these forces (weight) acts is called centre of gravity.

In other word it is the point on the body through which the weight acts and the sum moments of weighs of all particles constituting the body about that point is zero.



3.3.2 CENTROID OR CENTRE OF AREA

It is the point through which the whole area of a plane figure concentrates and the sum of moments of all areas constituting the figure about that point is zero.



$\Sigma xdA = 0, \ \Sigma ydA = 0$

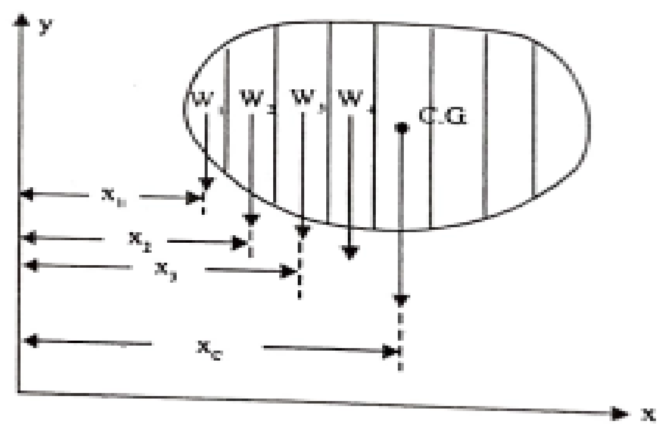
3.3.3 CENTRE OF MASS

It is the point at which the mass of a body acts and the sum of the moment of each particle constituting the body about that point is zero. $\Sigma dm.x = 0, \ \Sigma dm.y = 0$

3.3.4 LOCATION OF C.G.

(Using Method of Moments)

Let a body consists of lots of particles, W_1, W_2, W_3 ..... which are at co-ordinates of (x_1, y_1) (x_2, y_2), (x_3, y_3) ..... from the x–y co-ordinate axes.



Similarly let (x_c, y_c) be the co-ordinates of centre of gravity throuh which the whole weight W acts.

Since W_1, W_2, W_3 .... constitute a parallel system of forces, their resultant will be $W = w_1 + w_2 + w_3 + \ldots\ldots\ldots$

According to Varignon's principle, if the moment is taken about y-axis, then

$$W \times x_c = w_1 \times x_1 + w_2 \times x_2 + w_3 \times x_3 + \ldots$$

or $x_c = \dfrac{w_1 \times x_1 + w_2 \times x_2 + w_3 \times x_3 + \ldots}{W}$ or $x_c = \dfrac{w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots\ldots}{w_1 + w_2 + w_3 + \ldots}$

or $x_c = \dfrac{\Sigma wx}{\Sigma w}$

Similarly we can write $y_c = \dfrac{w_1 y_1 + w_2 y_2 + w_3 y_3 + \ldots\ldots}{w_1 + w_2 + w_3 + \ldots\ldots}$ or $y_c = \dfrac{\Sigma wy}{\Sigma w}$

The co-ordinate (x_c, y_c) gives the location of centre of gravity.

Note : If we consider the moment of W about its C.G., then it is zero, because $x_c = 0$. Hence

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots\ldots = 0 \quad \text{or} \quad \Sigma wx = 0$$

which gives the real definition of C.G i.e. it is the point at which the sum of moments of all weighs comprising the body is zero.

3.3.5 LOCATION OF CENTROID

In the same way the location of the centroid can be found out by applying method of moments.

i.e., $Ax = A_1 x_1 + A_2 x_2 + A_3 x_3 + \ldots\ldots$

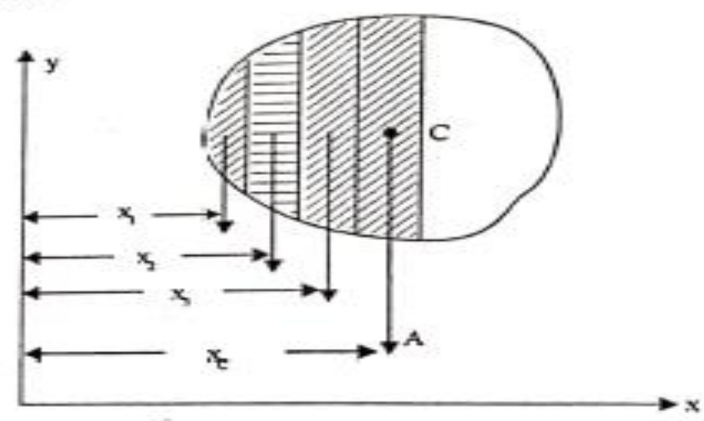
where $A = A_1 + A_2 + A_3 + \ldots\ldots$

or $x_c = \dfrac{A_1 x_1 + A_2 x_2 + A_3 x_3 + \ldots}{A_1 + A_2 + A_3 + \ldots\ldots}$

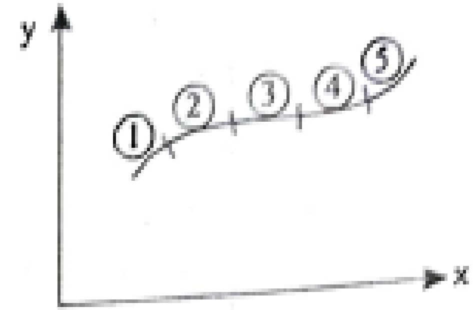
or $x_c = \dfrac{\Sigma Ax}{\Sigma A}$

Similarly $y_c = \dfrac{A_1 y_1 + A_2 y_2 + A_3 y_3 + \ldots\ldots}{A_1 + A_2 + A_3 + \ldots}$

or $y_c = \dfrac{\Sigma Ay}{\Sigma A}$



3.3.6 CENTRE OF GRAVITY



Centre of length

For one dmensional bodies or for line segments, the centre of length is located instead of C.G. i.e., it is the point where the entire length of a segment assumed to be concentrated.

i.e $\qquad \sum d\ell \,.\, x = 0 , \quad \sum d\ell \,.\, y = 0$

Location of centre of length

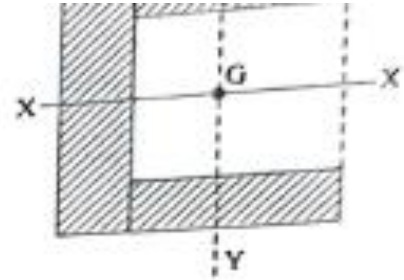
Let x_c & y_c be the coordinates of the centre of segment of length 'L'

$$\therefore \; x_c = \frac{L_1x_1 + L_2x_2 + L_3x_3}{L_1 + L_2 + L_3} , \quad y_c = \frac{L_1y_1 + L_2y_2 + L_3y_3}{L_1 + L_2 + L_3}$$

or $$x_c = \frac{\sum Lx}{\sum L} \quad \text{and} \quad y_c = \frac{\sum Ly}{\sum L}$$

Where (x_1, y_1), (x_2, y_2), (x_3, y_3) .... are the coordinates of elemental segments of lengths L_1, L_2, L_3 etc.

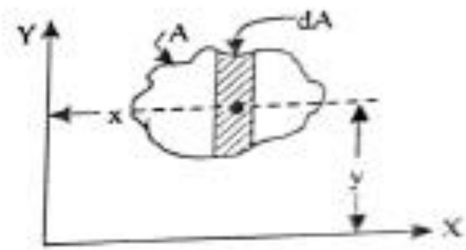
as shown in the figure.



3.3.8 Location of Centroid by Method of Integration

Consider an infinite small element of area dA is at distance of x and y from Y and X axes.

Let x_c and y_c be the coordinates of the centroid of a given section of area A.



According to the principle moment;

$$y_c \times \int dA = \int y\, dA \quad \text{or} \quad y_c = \frac{\int y\, dA}{\int dA}$$

Similarly, $$x_c = \frac{\int x\, dA}{\int dA}$$

But the centroid is the point through which the sum of moments of all areas comprising the sector is zero.

i.e., $\int y\, dA = 0,\ \int x\, dA = 0$

For centre of length

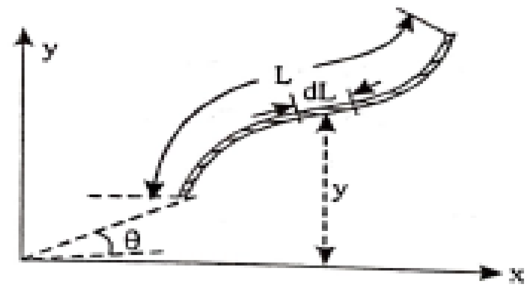
Similarly, $$x_c = \frac{\int x\, dL}{\int dL},\quad y_c = \frac{\int y\, dL}{\int dL}$$

3.3.12 PAPUS THEOREM - I

The area of the plane surface generated by rotating any plane curve about a non-intersecting axis is equal to the product of "the length, angle of rotation and centroidal distance of that curve" from that non-intersecting axis.

i.e., $A_x = L \times \theta \times y_c$

and $A_y = L \times \theta \times x_c$



Proof Consider an infinite small element "dL" of a curve of length 'L', revolves at angle 'θ' about non intersecting axis. x, y be the centroidal distance of 'dL' from y-axis and x-axis.

The surface area 'dA' generated by the rotation of 'dL' is ; $dA = dL \cdot \theta \cdot y$

Therefore the total area generated by the segment 'L' be

$$A_x = \int dL \cdot \theta \cdot y \qquad \text{...(i)}$$

But; $\int dL \cdot y = L\, y_c$

Hence the equation (i) can be written as; $A_x = L\, \theta\, y_c$

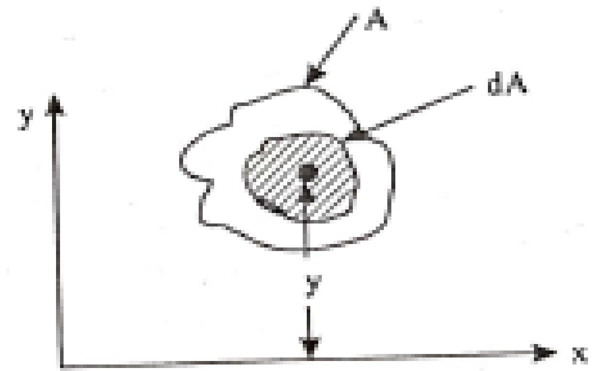
Similarly, area generated about y-axis; $A_y = L\, \theta\, x_c$

3.3.13 Papus Theorem - II

The volume of the solid generated by rotating any plane figure about a non intersecting axis is equal to the product of area of the figure, angle of rotation and centroidal distance of that figure from that non intersecting axis.

i.e., $V_x = A \times \theta \times y_c$

and $V_y = A \times \theta \times x_c$



Proof

Consider an infinite small area dA, is at a distance of 'y' from x-axis, rotates at an angle 'θ'.

The volume generated dV by the rotation of dA is;

$$dV = dA \, . \, \theta \, . \, y$$

Therefore the total volume of the solid generated;

$$V_x = \int dA \, . \, \theta \, . \, y \qquad \text{.......(i)}$$

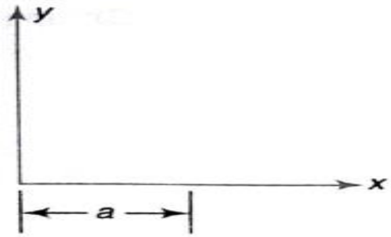
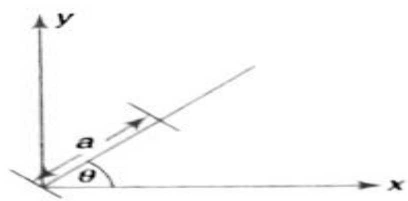
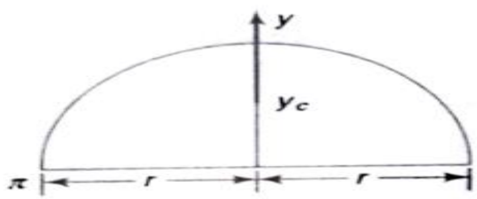
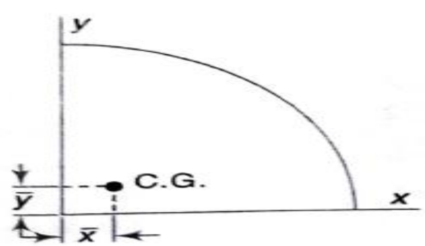
But $\int dA \, . \, y = Ay_c$

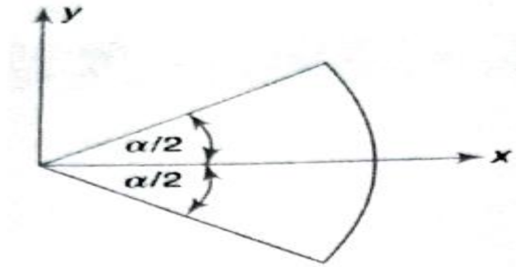
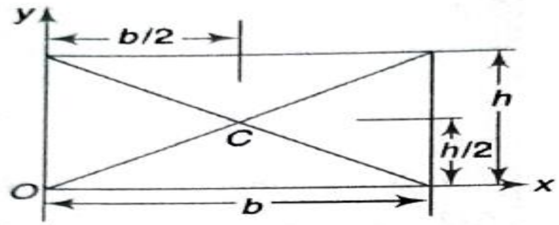
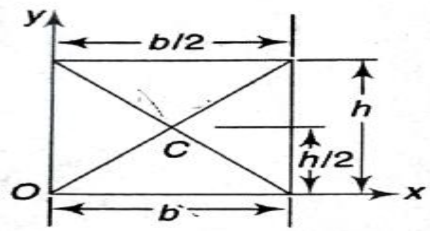
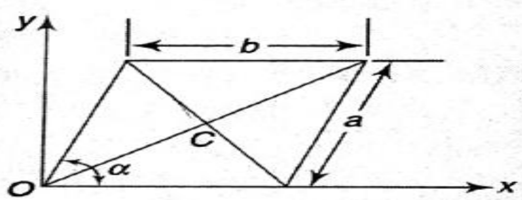
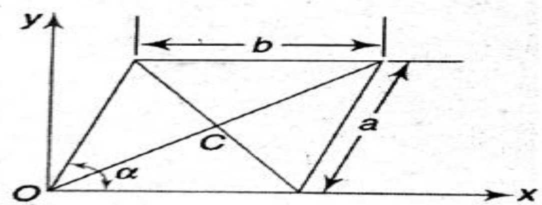
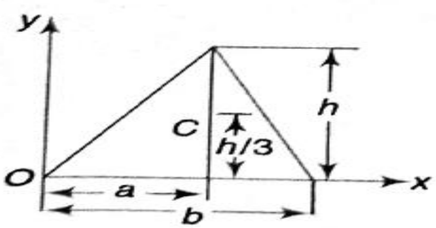
Hence; $V_x = A \, . \, \theta \, . \, y_c$

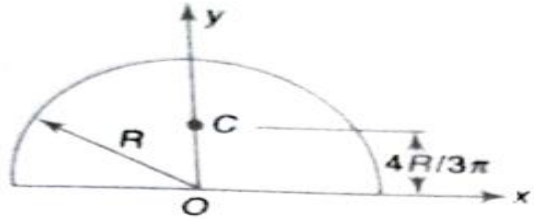
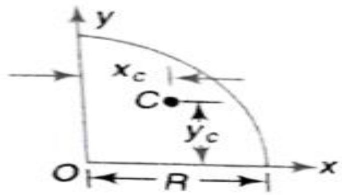
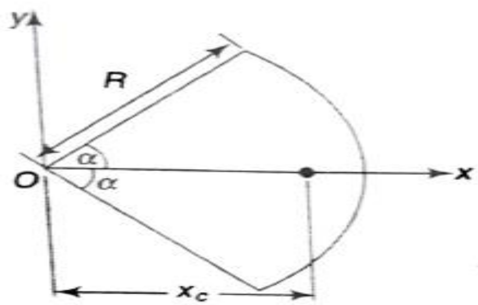
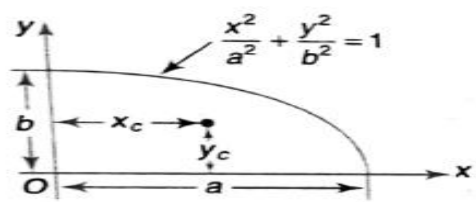
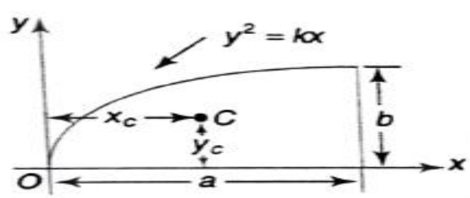
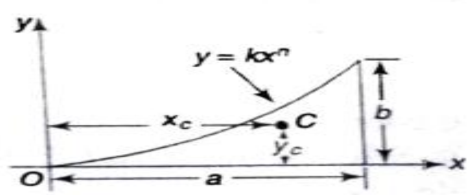
Similarly the volume generated about y-axis;

$V_y = A \, . \, \theta \, . \, x_c$

Various Centroids of Curves

Description	*Shape*	L	x_c	y_c
Horizontal line		a	$a/2$	0
Vertical line		a	0	$a/2$
Incline line with θ		a	$\left(\frac{a}{2}\right)\cos\theta$	$\left(\frac{a}{2}\right)\sin\theta$
Semicircular arc		πr	0	$\frac{2r}{\pi}$
Quarter-circular arc		$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$

Circular arc		αr	$\dfrac{2r \sin(\alpha/2)}{\alpha}$	0
Rectangle		bh	$b/2$	$h/2$
Square ($h = b = a$)		a^2	$a/2$	$a/2$
Parallelogram		$ab \sin \alpha$	$\dfrac{b + a \cos \alpha}{2}$	$\dfrac{a \sin \alpha}{2}$
Rectangle ($a = \pi/2$)		ab	$b/2$	$a/2$
Triangle		$1/2bh$	$1/3(a + b)$	$h/3$

Semicircle		$\dfrac{\pi R^2}{2}$	0	$\dfrac{4R}{3\pi}$ $= 0.424\,R$
Quarter circle		$\dfrac{\pi R^2}{4}$	$\dfrac{4R}{3\pi}$ $= 0.424\,R$	$\dfrac{4R}{3\pi}$ $= 0.424\,R$
Sector of a circle		$R^2\alpha$	$\dfrac{2}{3}\,\dfrac{R\sin\alpha}{\alpha}$	0
Quarter ellipse		$\dfrac{4ab}{4}$	$\dfrac{4a}{3\pi}$	$\dfrac{4b}{3\pi}$
Quarter parabola		$\dfrac{3ab}{3}$	$\dfrac{3}{5}a$	$\dfrac{3}{5}b$
General spandrel		$\dfrac{ab}{3}$	$\dfrac{3a}{4}$	$\dfrac{3b}{4}$