17/01/2020

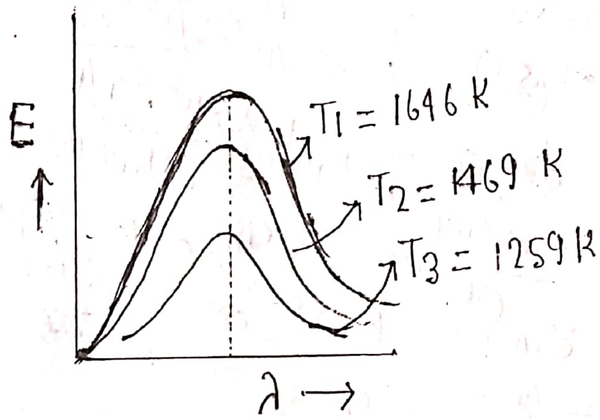
Friday.

PHYSICS.

Black Body Radiation.

Lummer and ~~Pringsheim~~ (Pringsheim) : Scientists

(1.) Wiens Energy Distribution law.



according to "Wiens" the energy density of radiation of wavelength λ and $\lambda + d\lambda$ from a cavity (i.e. black body) of ~~temperature~~ temperature T is

$$\boxed{E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot e^{-hc/K\lambda T} \cdot d\lambda}$$

valid only at the lower wavelength region.
fails to explain B.B.R. in the higher wave-length region.

$$\boxed{\lambda_{max} \propto \frac{1}{T}}$$

λ_{max} = maximum wavelength corresponding to maximum spectral energy.

$$\boxed{\lambda_m T = \text{constant}}$$ or. $\lambda_{max} \cdot T = $ constant.

Wein's displacement law.

(2) Rayleigh - Jeans' law.

The energy density of radiation of wavelength range λ and $\lambda+d\lambda$ from a black body of temp. 'T' is

$$\boxed{E_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda}$$

Valid only at the higher wavelength region.
fails at the lower wavelength region.

$$\boxed{E_\lambda \propto \frac{1}{\lambda^4}}$$

when $\lambda \to 0$, $E_\lambda \to \infty$

But in expt when $\lambda \to 0$, $E_\lambda \to 0$

The contradiction between expt. result of B.B.R. and result by R-J law is known as "Ultra-voilet catastrophe."

Plancks' Radiation Formula.

Postulates of Planck's Formula.

(1) The atom in the walls of the black body radiator behaves like a simple harmonic oscillators and each has a characteristic frequency of oscillation.

(2) The total energy of the oscillator is quantised and therefore they can not have any arbitrary amount of energy i.e. the total energy

$$\boxed{E_n = n \underbrace{h\nu}_{\text{quanta or Photon}}}$$

(3) The oscillators can emit or absorb energy only in the ~~the~~ ~~st~~ discrete unit of quanta $(h\nu)$ only when oscillator jumps from one energy level to another, i.e.

$$\boxed{\Delta E_n = \Delta n h\nu}$$

$$\boxed{E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda KT} - 1} \, d\lambda}$$

or

$$\boxed{E_\nu d\nu = \frac{8\pi h\nu^3}{c^3} \cdot \frac{d\nu}{e^{h\nu/KT} - 1}}$$

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Wien's Radiation Law from Planck's Radiation Law.

Weins energy distribution law.

For the smaller wavelength region.

$\frac{hc}{\lambda KT} >> 1$

$e^{hc/\lambda KT} >> 1$

$$\therefore \boxed{E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda KT}}}$$

Weins displacement law from Planck's Radiation Law

$\lambda_m T =$ constant.

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT}} d\lambda.$$

for the extremum value of λ,

$$\frac{dE_\lambda}{d\lambda} = 0. \qquad E_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT}}$$

$$8\pi hc \left[\frac{1}{\lambda^5} \left(\frac{+hc}{\lambda^2 KT} \right) e^{-hc/\lambda KT} - \frac{5}{\lambda^6} e^{-hc/\lambda KT} \right) = 0.$$

$$e^{-hc/\lambda KT} \left(-1 + \frac{hc}{5\lambda KT} \right) = 0$$

put $\frac{hc}{\lambda KT} = x$.

$$e^{x}\left(-1+\frac{x}{5}\right)=0$$

Roots of x is: $x=0$ & $x=5$.

$$\lambda T=\frac{hc}{xK}=2.89\times10^{-3}\ \text{mK}.$$

$$\boxed{\lambda_m T = \text{constant.}}$$

Rayleigh-Jeans law from Planck's Radiation Law.

for longer wavelength region,

$$\frac{hc}{\lambda KT}\ll 1$$

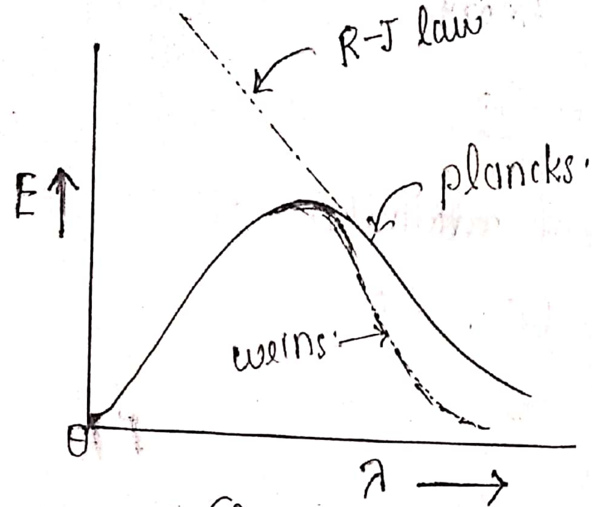
$$e^{hc/\lambda KT}\rightarrow 1+\frac{hc}{\lambda KT}+\frac{1}{2!}\left(\frac{hc}{\lambda KT}\right)^2+\cdots$$

Neglect higher order terms

$$E_\lambda d\lambda=\frac{8\pi hc}{\lambda^5}\cdot\frac{1}{1+\frac{hc}{\lambda KT}-1}\cdot d\lambda$$

$$E_\lambda d\lambda=\frac{8\pi hc}{\lambda^5}\cdot\frac{1}{hc/\lambda KT}\cdot d\lambda.$$

$$\boxed{E_\lambda d\lambda=\frac{8\pi KT}{\lambda^4}d\lambda.}$$



Assignment (1)

Q (1) STEFAN - BOLTZMANN'S LAW from PLANCK LAW. DERIVE

24/01/2020

De Broglie Hypothesis.

wave nature } Dual nature of light.
particle nature }

According to de broglie microparticles like electrons as well as protons which ordinary behave as particles and the certain conditions behave like a train of the waves. The wavelength of this wave upon the momentum which in form depends upon the mass and the velocity of the particle. Such waves associated with a moving small particle is known as a matter wave.

Thus according to de broglie a wave is always associated with every moving micro particle (like electron). and its corresponding wavelength is known as debroglie wavelength $\lambda = h/p$.

According to quantum theory,

$$E = h\nu = \frac{hc}{\lambda} \quad \text{— (1)}$$

According to theory of relativity.

$$E = mc^2 \quad \text{— (2)}$$

$$mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{mc}$$

if we consider a particle of mass 'm' moving with velocity v.

then $\boxed{\lambda = \frac{h}{mv}}$

K.E. of the particle $= \frac{1}{2}mv^2 = E = \frac{1}{2}\frac{(mv)^2}{m} = \frac{p^2}{2m}$

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2mE}}}$$

According to kinetic theory, average kinetic energy of a particle $E = \frac{3}{2}KT$.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m \times \frac{3}{2}KT}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{3KTm}}} \quad \lambda = \frac{h}{\sqrt{3mKT}}.$$

For a charge particle of charge 'e' accelerated by a velocity v, and having potential V and mass m.

$$E = \frac{1}{2}mv^2 = eV \Rightarrow \boxed{v = \sqrt{\frac{2eV}{m}}}$$

$$\lambda = \frac{h}{mv} = \frac{h}{m \times \sqrt{2eV/m}} = \frac{h}{\sqrt{2meV}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2meV}}}$$

For an e^- $\quad e = 1.6 \times 10^{-19}$ C.

$h = 6.63 \times 10^{-34}$ J.sec

$m = 9.1 \times 10^{-31}$ Kg.

$\lambda = 12.26/\sqrt{V}$ A°

$$\boxed{\lambda = \frac{12.26}{\sqrt{V}} \text{ A}^\circ}$$

Thus for a proton,

$$\boxed{\lambda = \frac{2.8}{\sqrt{V}} \text{ A}^\circ.}$$

Phase velocity and Particle Velocity.

Particle velocity:
It is the velocity associated with a particle and ~~the~~ indicates the rate of change of displacement of the particle of the medium w.r.t. the time.

Phase velocity.

In a wave every particle begins its vibrations a little latter than its preceders and there is a progressive change of the phase as we travel from one particle to another, It is the phase relationship of this particle that is observed as a phase and the velocity with which the plane of equal phase travel through the medium is known as phase velocity.

Phase velocity $V_p = \frac{\omega}{K}$.

$$V_p = \nu \lambda. \quad — (1)$$

According to the planck quantum theory,

$$E = h\nu \quad — (2)$$

According to theory of relativity

$$E = mc^2 \quad — (3)$$

(2) & (3) gives

$$\nu = \frac{mc^2}{h} \quad — (4)$$

According to de brogele, $\lambda = \frac{h}{mv}$, — (5)

$$V_p = \frac{mc^2}{h} \times \frac{h}{mv} = \frac{c^2}{v}$$

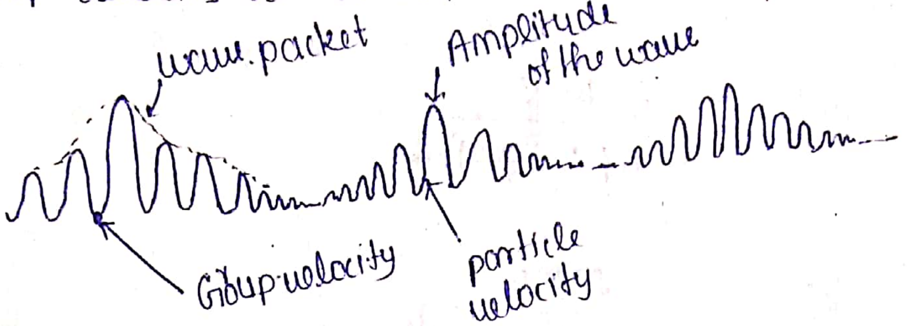
$$V_p > c. \quad — (6)$$

from (6) it can be seen that the phase velocity exceeds the velocity of the light i.e. why the motion of any relativistic particle cannot represent by a single wave but rather with a group of waves.

31/01/2019.

Group Velocity.

Group velocity can be defined in many forms.



For a wave packet the different components move with different phase velocities but the whole group advances through the medium with a single constant velocity.

The constant velocity with which the group of waves move is known as group velocity.

Also it can be define as a velocity with which the maximum amplitude of the resultant group of the waves in the medium.

Relation between Group velocity (V_g) and Particle Velocity (v)

$$\omega = 2\pi\nu = 2\pi \frac{mc^2}{h}$$

$$\omega = \frac{2\pi c^2}{h} \cdot \frac{m_0}{\sqrt{1-v^2/c^2}} \quad \text{--- (1)}$$

m = relativistic mass
v = velocity
$m = m_0 / \sqrt{1-v^2/c^2}$
Theory of relativity
m_0 = mass of the particle at rest

$$K = \frac{2\pi}{\lambda} = \frac{2\pi\, mv}{h} = \frac{2\pi v}{h} \cdot \frac{m_0}{\sqrt{1-v^2/c^2}} \quad \text{--- (2)}$$

According to definition.

$$V_g = \frac{d\omega}{dK} = \frac{d\omega/dv}{dK/dv}$$

$$d\omega/dv = \frac{2\pi m_0 V}{h(1-v^2/c^2)^{3/2}}.$$

$$dK/dv = \frac{2\pi m_0}{h(1-v^2/c^2)^{3/2}}$$

$$V_g = \frac{d\omega}{dK} = \frac{d\omega/dv}{dK/dv} = \frac{2\pi m_0 v / h(1-v^2/c^2)^{3/2}}{2\pi m_0 / h(1-v^2/c^2)^{3/2}}$$

$$\boxed{V_g = v.}$$

That means group velocity is nothing but the particle velocity.

Relation between V_g and V_p

Dispersive and Nondispersive medium.

If a wavepacket travel through a medium without changing its initial shape over a long distance then the medium is said to be non dispersive.

while A medium in which the wave packet looses its initial shape is known as dispersive medium.

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Relation between V_g and V_p.

$$V_p = \frac{\omega}{k}. \qquad V_g = d\omega/dk.$$

$$\omega = kV_p. \quad — (1)$$

$$V_g = \frac{d\omega}{dk} = V_p + k\frac{dV_p}{dk}. \quad — (2)$$

$$k = \frac{2\pi}{\lambda}$$

$$dk = \frac{-2\pi}{\lambda^2} \cdot d\lambda.$$

$$\frac{k}{dk} = \frac{2\pi/\lambda}{-2\pi/\lambda^2\, d\lambda} = \frac{-\lambda}{d\lambda}$$

$$\frac{k}{dk} = \frac{-\lambda}{d\lambda}.$$

$$\boxed{V_g = V_p - \frac{\lambda}{d\lambda}V_p}$$

kedro

case (1) Dispersive.

V_p is a function of λ.

$$V_g = V_p - \frac{\lambda}{d\lambda}dV_p.$$

$$\boxed{V_g < V_p}$$

case (2) Non dispersive.

v_p is independent of λ.

$$v_g = v_p \Rightarrow \frac{dv_p}{d\lambda} = 0$$

$$\boxed{\lambda \propto \frac{1}{m}}$$

Assignment (2)
Properties of matter waves.

The waves associated with a moving particle are called matter waves.

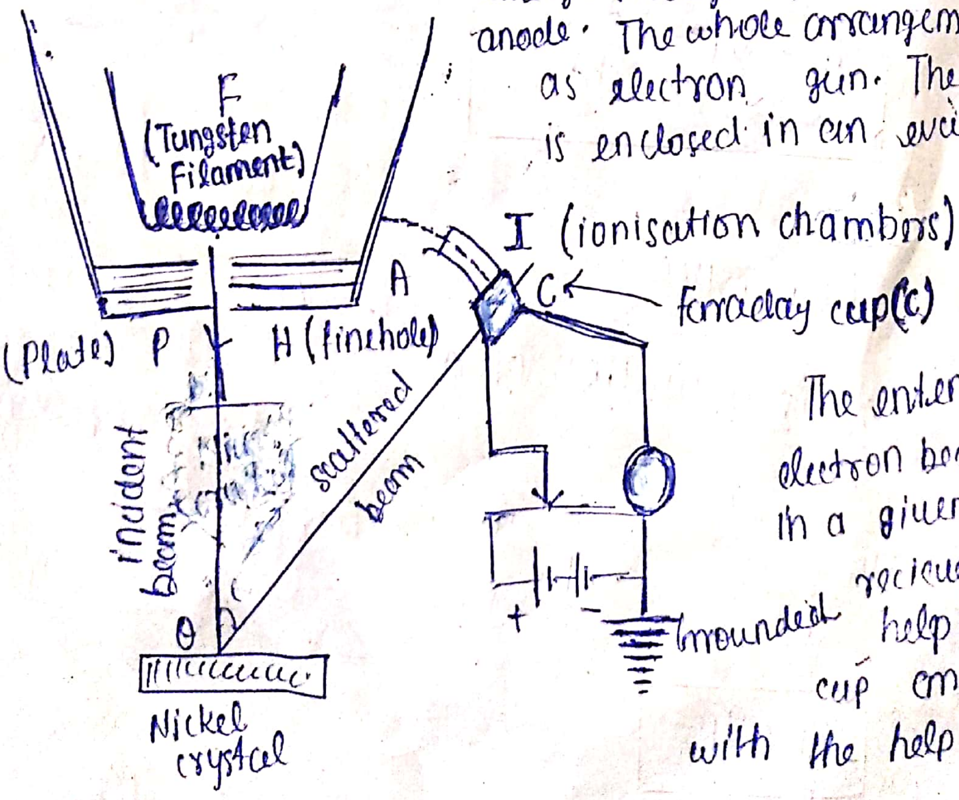
Davisson and Germer experiment

Bragg diffraction law.

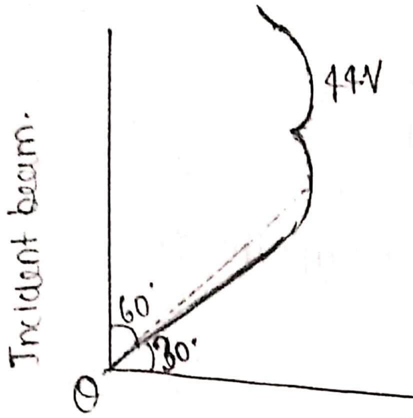
$$2d\sin\theta = n\lambda.$$

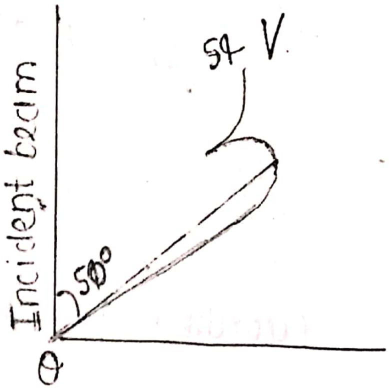
$$\lambda = h/mv.$$

It consists of a tungsten filament F coated with ZnO. A beam of electrons from a heated filament F are accelerated by a potential difference V between anode (A) and Filament (F). After collimination of electron, a beam of electrons emerge through a fine narrow hole of the anode. The whole arrangement is known as electron gun. The apparatus is enclosed in an evacuated chamber.

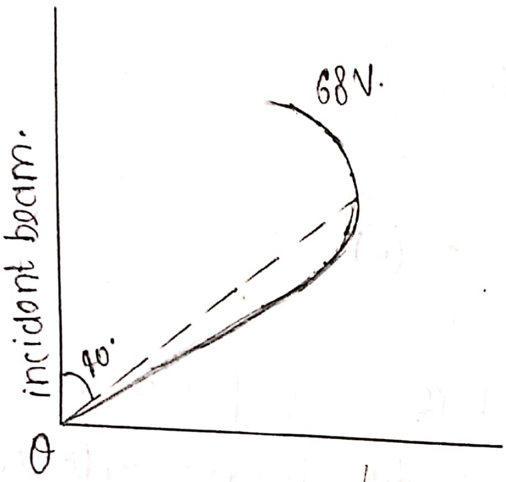


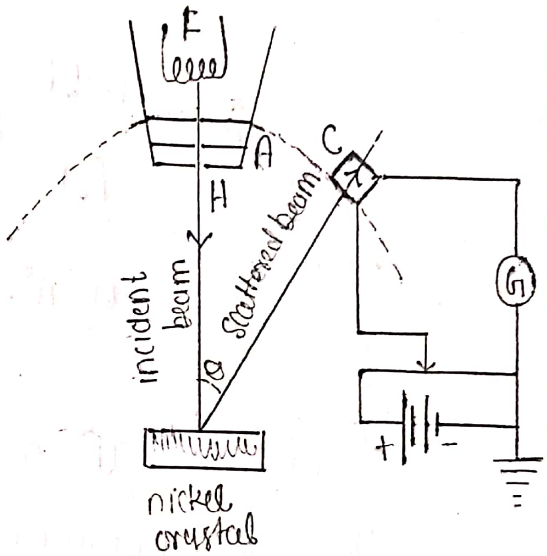
The intensity of the electron beam scattered in a given direction is recieved with the help of Faraday cup and is measured with the help of Galvanometer.





Intensity of scattered beam.

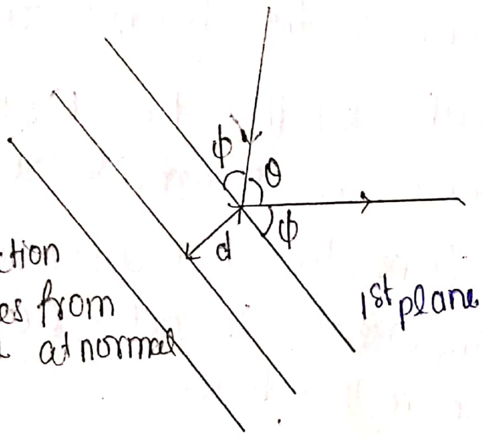




An Experimental Arrangement of Dawisson and Germer Exp.

$d = 0.91\,A^{0}.$

= interplanner spacing for the Nickel crystal.



Bragg reflection of electron waves from Nickel crystal at normal incidence

ϕ = angle b/w plane and incident wave.

$\theta = 50'$

$\phi + \theta + \phi = 180$

$\phi = 65^{\circ}$

if $n = 1$.

According to Bragg's law

$2d\sin\phi = n\lambda.$

$$\lambda = \frac{2 \times 0.91 \sin 65}{1}$$

$$\lambda = 1.65 \ A^\circ \quad —(1)$$

According to the de broglie hypothesis.

$$\lambda = \frac{12.26}{\sqrt{V}} \ A^\circ$$

$$\lambda = \frac{12.26}{\sqrt{54}} \ A^\circ$$

$$\lambda = 1.66 \ A^\circ . \quad —(2)$$

from eqn (1) and (2) it is found that the λ obtained by the Braggs relation and the λ obtained by the De Broglie hypothesis is same. This agreement b/w λ as found from the two approaches confirm the existence of the debroglie wave and hence debroglie hypothesis.

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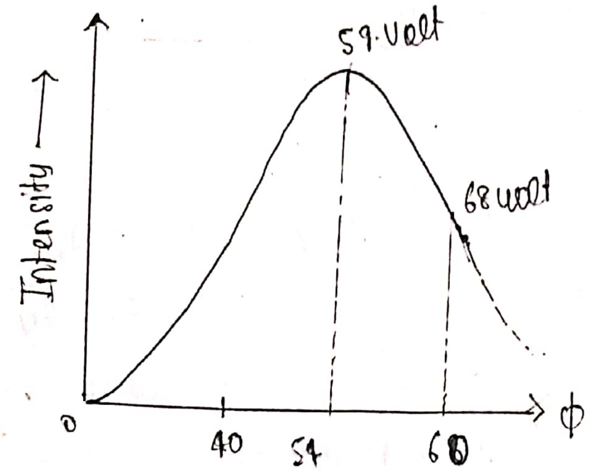
03/02/2020.



HEISENBERG UNCERTAINITY PRINCIPLE.

$$\lambda = \frac{h}{p}$$

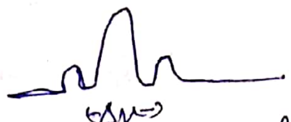
Δx

$\Delta p = h/\Delta x$

$\Delta p \sim h/\Delta x$

$\Delta p \cdot \Delta x \geq \hbar/2$

where $\hbar = h/2\pi$.



According to heisenberg it is impossible to know both the exact position and the exact momentum of a small particle simultaneously.

Therefore in any simultaneous determination of the position and momentum of a small particle the product of the uncertainities in the measurement of position and momentum is always equal to or greater than a constant. i.e. $\Delta x \cdot \Delta p_x \geq \hbar/2$

more precisely $\Delta x \Delta p_x \geq \hbar/2$.

Physical Interpretation of heisenberg Uncertainity Principle.

(1) If we can measure the position of the particle (if $\Delta x = 0$) accurately then the uncertainity in the measurement of momentum i.e ΔP_x at the same instant of time becomes infinite

(2) If ($\Delta P_x = 0$) we can measure the momentum of the particle accurately i.e. $\Delta P_x = 0$ then the uncertainity in the measurement of position i.e. Δx at the same instant of time becomes Infinite.

(3) if a particle of mass m moving with velocity v the position momentum uncertainity relation can be written as

$$\Delta x \, \Delta P_x \geq h/2\pi \times \frac{1}{2} \geq 1/2 \times h/2\pi$$

$$\Delta x \cdot \Delta v \geq h/4m\pi \geq h/4m\pi$$

for a heavier object $m \to \infty$ $h/2m \to 0$

$$\therefore \Delta x \cdot \Delta v \sim 0$$

from the uncertainity relation. we can measure

$$\Delta E \cdot \Delta t \geq \hbar/2.$$

energy accurately only if the measurement is made over an infinite period of time

that means:

$$\Delta E = 0 \quad \text{iff} \quad \Delta t = \infty$$

$$\Delta E \cdot \Delta t \geq \bar{h}/2.$$

$$\Delta \theta \cdot \Delta L \geq \bar{h}/2.$$

uncertainity in angular displacement

~ momentum

$\boxed{\Delta\theta \cdot \Delta L \geq h/2}$ assignment

$\Delta E \cdot \Delta t \geq \hbar/2.$

proof

electron mass of m moving with velocity v

$E = \frac{1}{2} m v^2$

if m = constant.

ΔE = uncertainity in the measurement of E

$\Delta E = \Delta(\frac{1}{2} m v^2) = m v \Delta v = v \Delta(mv)$

$\Delta E = v \Delta p.$

now velocity $v = \frac{\Delta x}{\Delta t}.$

$\Delta E = \frac{\Delta x}{\Delta t} \cdot \Delta p \Rightarrow \Delta E \cdot \Delta t = \Delta x \cdot \Delta p.$

we know. $\Delta x \cdot \Delta p \geq \hbar/2.$

hence $\Delta E \cdot \Delta t \geq \hbar/2.$

APPLICATION.

(I) Non - Existence of Electrons inside the nucleus.

Δx = radius of the nucleus $= 10^{-14}$ m

According to uncertainity principle

$\Delta x \Delta p_x \geq h/2.$

$\Delta p_x \geq \frac{6.6 \times 10^{-34}}{4 \times 10^{-14} \times 3.14}$

$\Delta p_x \geq 5.3 \times 10^{-21}$ kg m/sec.

i.e. the total momentum of e^- should be of the order of minimum 5.3×10^{-21} kg m sec^{-1} if it is to exists inside the nucleus.

According to theory of relativity

$$E^2 = p^2c^2 + m_0^2c^4.$$

$m_0 = 9.1 \times 10^{-31}$ kg

$c = 3 \times 10^8$ m/sec.

$p = 5.3 \times 10^{-21}$ kg m/sec.

After putting all values, we will find $E^2 = (5.3 \times 10^{-21} \times 3 \times 10^8)^2 + (9.1 \times 10^{-31})^2 (3 \times 10^8)^4.$

$$E = \frac{16 \times 10^{-13}}{1.6 \times 10^{-19}} \text{ eV.}$$

$$E = 10^7 \text{ eV.}$$

$$E = 10 \text{ MeV.}$$

$$E \sim 10 \text{ MeV.}$$

Thus the kinetic energy of the free electron confined within the nucleus should have minimum value of 10 MeV however from the experimental data the highest value of an electron emitted during radioactive decay found to be 3-4 MeV since these value is much much less than the calculated value of the minimum energy the electrons can not exist inside the nucleus:

(2) existence of proton and α-particle inside the nucleus

Assignment-

(3) Binding Energy of an electron in H- atom.

r = radius of the H-atom $= \Delta x$,
$r = 10^{-10}$ m $= \Delta x$.

According to uncertainity principle

$$\Delta x \cdot \Delta p = \hbar/2,$$

$$\Delta p = \hbar/2\Delta x.$$

if we take the minimum value of the momentum of e^- in its orbits as the uncertainity in its momentum i.e. $p \sim \Delta p = \frac{\hbar}{2\Delta x} = \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 10^{-10}}$

$$\sim 5.2 \times 10^{-25} \text{ kg m/sec.}$$

$$K.E = K = \frac{p^2}{2m} = \frac{(5.2 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}}.$$

$$K \simeq 1 \text{ eV};$$

if P = Potential energy of the electron with atom no Z

$$V = \frac{-Ze^2}{4\pi\varepsilon_0 \cdot r} = Z \frac{(1.6 \times 10^{-19})^2}{4 \times 3.14 \times 8.85 \times 10^{-12} \times 10^{-10}}$$

$$V = -14.4 \text{ eV}.$$

Total energy $= K + V$
$= (1 - 14.4 \cdot Z)$ eV.

for H-atom

$\therefore E = (1 - 14.4)$ eV $= -13.4$ eV for He-atom $z = 2$.

$$E = (1 - 2 \times 14.4)\ eV$$

$$E = -27.8\ eV.$$

The two values of the binding energy of H and He agree very closely with a experimentaly observed values of the binding energies of the two atoms.

(4) Zero-Point Energy of a Particle Assignment.

10/02/2020.

(1) What voltage must be applied to an electron microscope to produce electrons of wave length $\lambda = 1\ A^\circ$

(2) Calculate the debroglie wavelength of an electron moving with velocity $3c/5$? (c = velocity of light)

Ans: $$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \cdot v} = \frac{h\sqrt{1-v^2/c^2}}{m_0 v}.$$

$$\lambda = \frac{h\sqrt{1-(3c/5)^2/c^2}}{m_0 \cdot 3c/5} = \frac{h \cdot 4/5}{3m_0c/5} = \frac{4h}{3m_0c} = \frac{4 \times 6.63 \times 10^{-34}}{3 \times 9.1 \times 10^{-31} \times 3 \times 10^8}$$

$$\lambda = \frac{6.63 \times 4 \times 10^{-11}}{9 \times 9.1} = \frac{2.21 \times 4 \times 10^{-11}}{3 \times 9.1} = 0.324 \times 10^{-11}$$

$$\lambda = 0.0324\ A^\circ$$

(3) Find the ratio of wavelength of Deuterium and Proton accelerated through same potential difference.

Ans. $\lambda = \dfrac{h}{\sqrt{2meV}}$

$$\frac{\lambda_D}{\lambda_P} = \sqrt{\frac{m_P}{m_D}} = \sqrt{\frac{m_P}{2m_P}} = \frac{1}{\sqrt{2}}.$$

(4) An electron moves in the x-direction with speed of 4×10^6 m/sec we can measure its speed precision of 1%. with what precision can its position simultaneously be measured

Ans: $P = mv = 9.1\times10^{-31}\times4\times10^6$

$$\Delta P = \frac{9.1\times10^{-31}\times4\times10^6}{100}$$

$\Delta P\,\Delta x \geq h/4\pi$

$$\Delta x \geq \frac{6.6\times10^{-34}\times100}{4\times3.14\times9.1\times10^{-31}\times4\times10^6}$$

$$\Delta x = \frac{6.63\times10^{-7}}{16\times9.1\times3.14}$$

$\Delta x = 0.0145\times10^{-7}$

$\Delta x = 1.45\times10^{-89}$

$\Delta x = 1.45$ nm

(5) An electron has a momentum 5.4×10^{-26} kg m/sec with an accuracy of 0.05%. find the minimum uncertainty in the location of electron

Ans. $\Delta P = 5.4\times10^{-26}\times0.05/100 = 2.70\times10^{-29}$

$$\Delta x = \frac{h}{4\pi\Delta P} = \frac{6.63\times10^{-34}}{4\times3.14\times2.7\times10^{-29}} = \frac{6.63\times10^{-5}}{4\times3.14\times2.7}$$

$\Delta x = 0.196\times10^{-5} = 1.95\times10^{-6}$ m $= 1.95\,\mu$m.

(6) Calculate the uncertainity in the momentum and velocity of an electron confined in a box of length of 1 A°

Ans.

$\Delta x = 1\ A°$

$$\Delta P = \frac{h}{4\pi \Delta x} = \frac{6.63\times10^{-34}}{4\times3.14\times10^{-10}} = 0.53\times10^{-24} = 5.3\times10^{-25}\ kgm/sec$$

$$\delta V = \frac{h}{4\pi m \Delta x} = \frac{6.63\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times10^{-10}} = 0.058\times10^{7} = 5.8\times10^{5}\ m/sec$$

Assignment.

(1) Calculate the energy of a photon having de broglie wavelength 0.5 Fm = 0.5×10^{-15} m.

(2) What is the effect of increasing the electron energy on the scattering angle in dawison & germer experiment.

(3) Which has a shorter wavelength 10 eV photon or 10 meV electron explain.

(4.) The average life time of an excited atomic state is 10^{-8} sec. If the wavelength of an spectral line associated with a transition 4000 A° what is the width of line

(5) An experiment design to know the position of a proton can result in changing the proton kinetic energy by at most 1 keV. Find the maximum accuracy in the position of the proton.