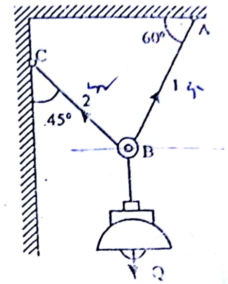
## SOLVED PROBLEMS - 2.3

1. *An electric-light fixture of weight Q = 178 N is supported as shown in fig. Determine the tensile forces $S_1$ and $S_2$ in the wires BA and BC if their angles of inclination are as shown.*



**Soln.** Given data

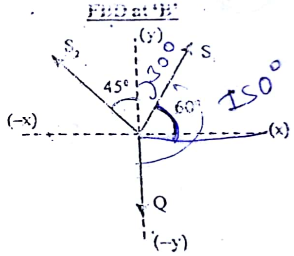
$Q = 178$ N, $S_1 = ?$ $S_2 = ?$

AB is a wire and hence axial force $S_1$ as the tension acts from B making 60° with the horizontal.

BC is a wire and hence an axial force $S_2$ as the tension acts away from the B, making an angle 45° to the vertical.

Since the three forces $S_1$, $S_2$ and Q meet at the concurrent point B, the FBD is drawn at point B as shown in figure.

FBD at 'B'



Applying Lami's theorem

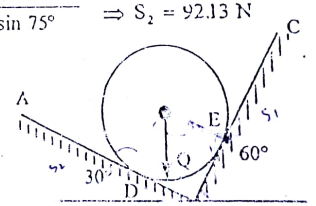
$$\frac{Q}{\sin 75°} = \frac{S_1}{\sin 135°} = \frac{S_2}{\sin 150°}$$

Now; $\frac{Q}{\sin 75°} = \frac{S_1}{\sin 135°}$

$\Rightarrow S_1 = \frac{178 \sin 135°}{\sin 75°} \Rightarrow S_1 = 130.30$ N (Ans.)

Now; $\frac{Q}{\sin 75} = \frac{S_2}{\sin 150°} \Rightarrow S_2 = \frac{178 \sin 150°}{\sin 75°} \Rightarrow S_2 = 92.13$ N (Ans.)

2. *A ball of weight Q = 53.4 N rests in a right-angled through, as shown in fig. Determine the forces exerted on the sides of the through at D and E if all surfaces are perfectly smooth.*



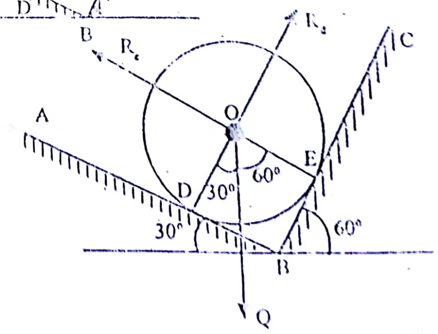
**Soln.** Given data

$Q = 53.4$ N

$R_d = ?$

$R_e = ?$

At E the normal reaction $R_e$ emerges from E & passes towards the body at O, making 60° to the vertical.



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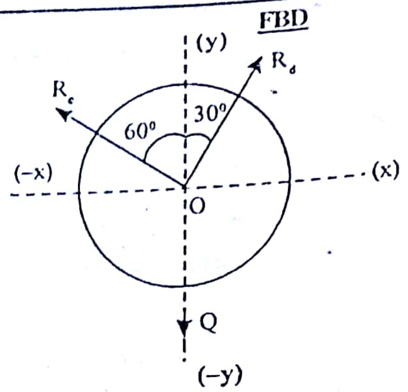
At D the normal reaction $R_d$ emerges from D and passes at the concurrent point 0, making 30° to the vertical. The FBD indicating $R_d$, $R_e$ & Q is shown in the figure.



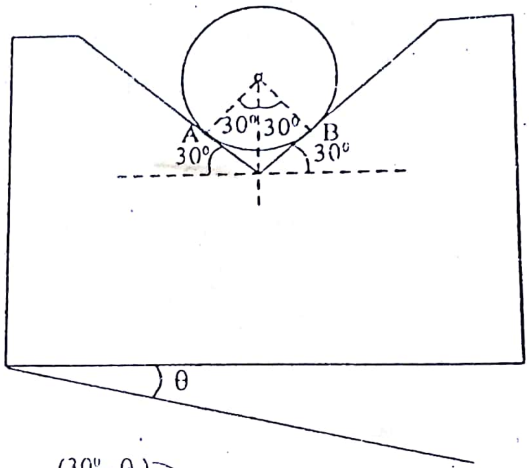

Applying Lami's theorem

$$\frac{Q}{\sin 90°} = \frac{R_e}{\sin 150°} = \frac{R_d}{\sin 120°}$$

Now; $\frac{Q}{\sin 90°} = \frac{R_e}{\sin 150°} \Rightarrow R_e = \frac{53.4 \sin 150°}{\sin 90°} \Rightarrow R_e = 26.7\text{ N}$ **(Ans.)**

Now; $\frac{Q}{\sin 90°} = \frac{R_d}{\sin 120°} \Rightarrow R_d = \frac{53.4 \sin 120°}{\sin 90°} \Rightarrow R_d = 46.24\text{ N}$ **(Ans.)**

3. *A ball rests in a trough as shown in fig. Determine the angle of tilt 'θ' with the horizontal so that the reactive force at B will be one third at A if all surfaces are perfectly smooth.*




Soln. Given data

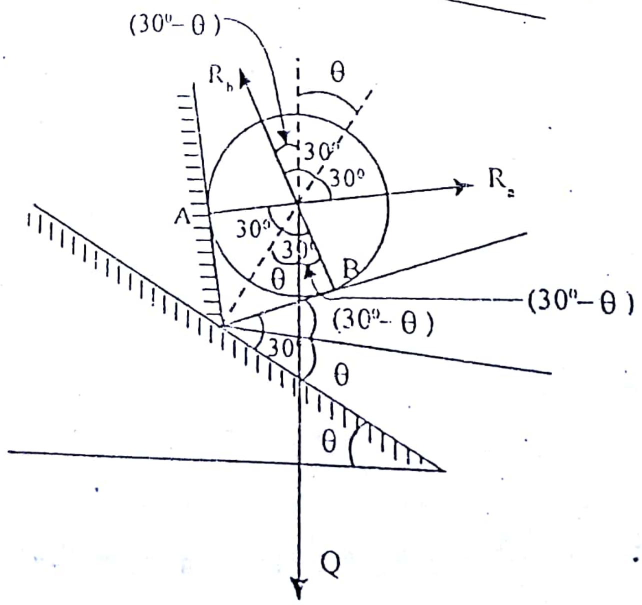
Let $R_b$ be the reaction at B.

Since it will be one third at A, then

$$R_a = \frac{R_b}{3}$$

$\theta = ?$

Let Q is the gravity force of the sphere rest on triangular groove. When the groove is not titled, two reactions $R_a$ and $R_b$ act normal at A and B, making 30° each to the vertical. When the triangular groove tilted at an angle θ, the reaction $R_b$ makes an angle θ with the vertical line of action of Q. The FBD indicating $R_a$, $R_b$ and Q is shown.




Applying Lami's theorem

$$\frac{R_a}{\sin(150+\theta)} = \frac{R_b}{\sin(150-\theta)}$$

or $\dfrac{R_a}{\sin(150+\theta)} = \dfrac{3R_a}{\sin(150-\theta)} \Rightarrow 3\sin(150+\theta) = \sin(150-\theta)$

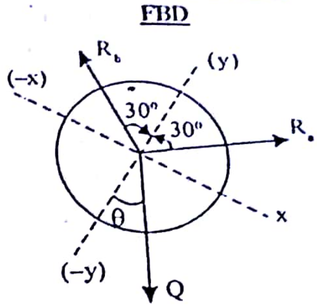
$\Rightarrow 3[\sin 150 \cos\theta + \cos 150.\sin\theta] = \sin 150 \cos\theta - \cos 150 \sin\theta$

$\Rightarrow 3\sin 150 \cos\theta + 3\cos 150 \sin\theta = \sin 150 \cos\theta - \cos 150 \sin\theta$

$\Rightarrow 2\sin 150 \cos\theta + 4\cos 150 \sin\theta = 0$

$\Rightarrow 2 \times \dfrac{1}{2} \times \cos\theta + 4\left(-\dfrac{\sqrt{3}}{2}\right)\sin\theta = 0 \Rightarrow \cos\theta = 2\sqrt{3}\sin\theta$

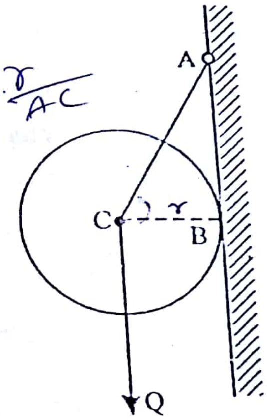
FBD




$\Rightarrow \tan\theta = \dfrac{1}{2\sqrt{3}} \Rightarrow \theta = \tan^{-1}\dfrac{1}{2\sqrt{3}} = 16°6'$ **(Ans.)**

4. *A circular roller of weight $Q = 445$ N and radius $r = 152$ mm hangs by a tie rod $AC = 304$ mm and rests against a smooth vertical wall at B, as shown in fig. Determine the tension S in the tie rod and the force $R_b$ exerted against the wall at B.*

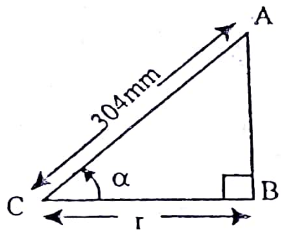
$\cos\alpha = \dfrac{r}{AC}$




**Soln.** Given data

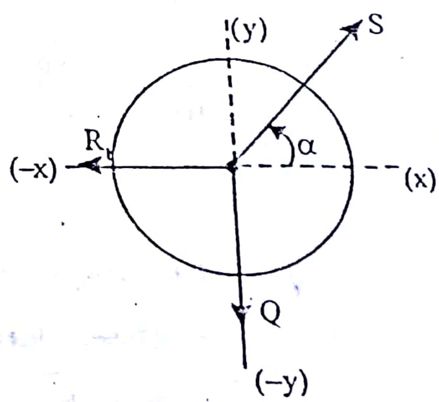
$Q = 445$ N; $r = 152$ mm, $AC = 304$ mm

$S = ?$, $R_b = ?$




AC is a tie rod. Hence an axial force S as the tension act away from joint C. The normal reaction from point B will pass through the concurrent point C. The FBD showing all the forces is shown in the figure. Let α is the angle of inclination of S with respect to x-axis

FBD




From the ΔABC, $\cos\alpha = \dfrac{r}{AC}$

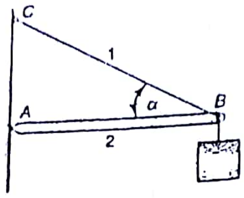
$\Rightarrow \alpha = \cos^{-1}\left(\dfrac{152}{304}\right) \Rightarrow \alpha = 60°$

Applying Lami's theorem $\dfrac{Q}{\sin(180°-\alpha)} = \dfrac{S}{\sin 90°} = \dfrac{R_b}{\sin(90°+\alpha)}$

or $\dfrac{Q}{\sin(180°-\alpha)} = \dfrac{S}{\sin 90°} \Rightarrow S = \dfrac{445 \sin 90°}{\sin(180°-60°)} \Rightarrow S = 513.84$ N (Ans.)

Now; $\dfrac{Q}{\sin(180°-\alpha)} = \dfrac{R_b}{\sin(90°+\alpha)} \Rightarrow R_b = \dfrac{445 \sin(90° + 60°)}{\sin(180°-60°)} \Rightarrow R_b = 256.92$ N (Ans.)

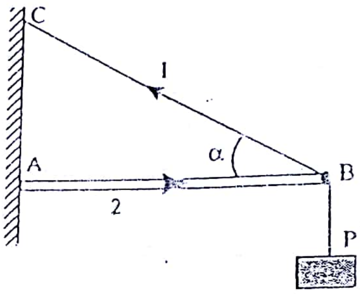
5. *What axial forces does the vertical load P induce in the members of the system shown in fig. ? Neglect the weights of the members themselves and assume an ideal hinge at A and a perfectly flexible string BC.*




**Soln.** BC is a string, hence an axial force $S_1$ as the tension act away from the joint B.

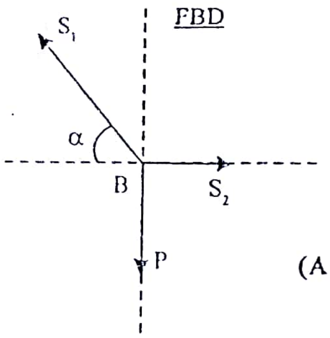
For equilibrium an axial force $S_2$ as the compression act along the rod i.e., from A to B or towards B.

The FBD showing all the forces is shown in the figure.




Applying lamis' theorem

Now; $\dfrac{P}{\sin(180°-\alpha)} = \dfrac{S_1}{\sin 90°} = \dfrac{S_2}{\sin(90+\alpha)}$



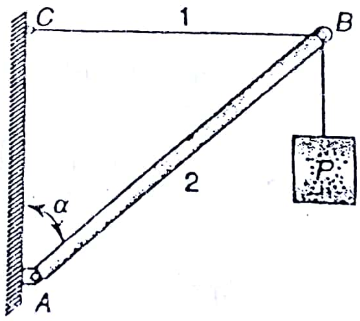

$\dfrac{P}{\sin(180°-\alpha)} = \dfrac{S_1}{\sin 90°}$

$\Rightarrow S_1 = \dfrac{P \sin 90°}{\sin\alpha} = \dfrac{P}{\sin\alpha} \Rightarrow S_1 = P \operatorname{cosec}\alpha$ (Ans.)

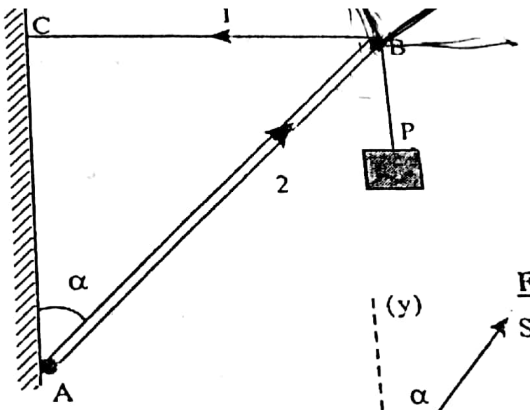
Now; $\dfrac{P}{\sin(180-\alpha)} = \dfrac{S_2}{\sin(90+\alpha)} \Rightarrow \dfrac{P}{\sin\alpha} = \dfrac{S_2}{\cos\alpha}$

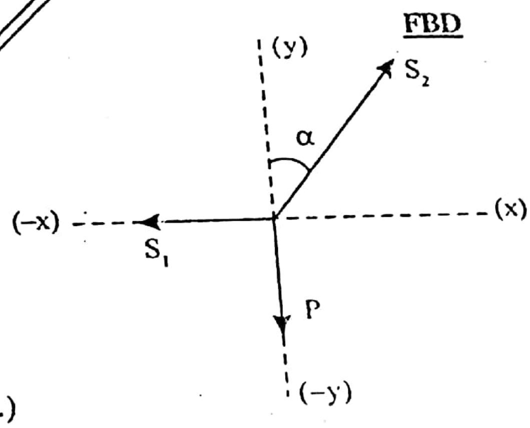
$\Rightarrow S_2 = P \cot\alpha$ (Ans.)

6. *What axial forces does the vertical load P induce in the members of the system show in fig. ? Make the same idealizing assumptions as in prob. 5.*



**Soln.** BC is a tie rod, hence an axial force $S_1$ as the tension acts away from the B. AB is a weight less rod and hence an axial force $S_2$ as the compression acts towards the joint B for equillibrium. The FBD showing all forces is shown in the figure.




Applying Lami's theorem.

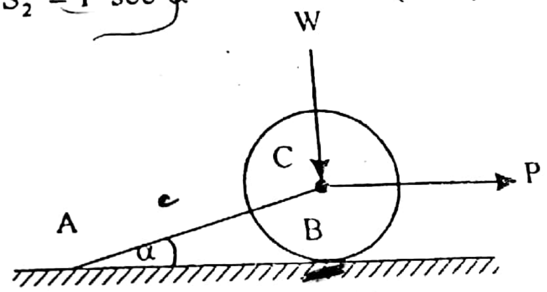
$$\frac{P}{\sin(90+\alpha)} = \frac{S_1}{\sin(180-\alpha)} = \frac{S_2}{\sin 90}$$

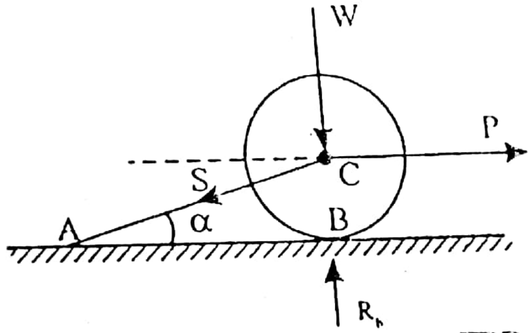
$$\text{Now; } \frac{P}{\sin(90+\alpha)} = \frac{S_1}{\sin(180-\alpha)}$$

$$\Rightarrow S_1 \Rightarrow P\frac{\sin\alpha}{\cos\alpha} \quad \text{or} \quad S_1 = P\tan\alpha \qquad \textbf{(Ans.)}$$

$$\text{Now; } \frac{P}{\sin(90+\alpha)} = \frac{S_2}{\sin 90^\circ} \Rightarrow S_2 = \frac{P}{\cos\alpha} \quad \text{or} \quad S_2 = P\sec\alpha \qquad \textbf{(Ans.)}$$

7. *A right circular roller of weight W rests on a smooth horizontal plane and is held in position by an inclined bar AC as shown in fig. Find the tension S in the bar AC and the vertical reaction $R_b$ at B if there is also a horizontal force P acting at 'C'.*

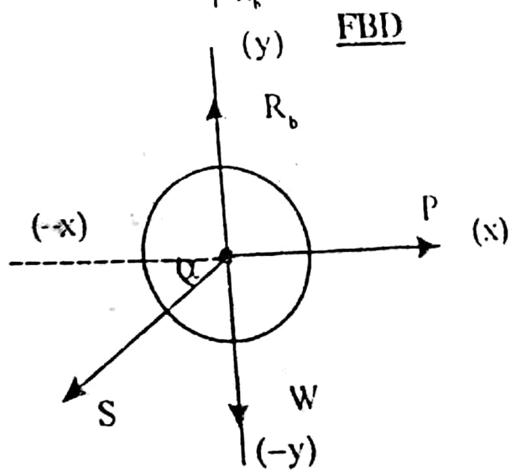



**Soln.** AC is a string

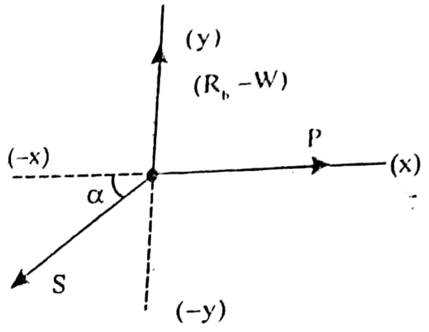
Hence the axial force S as the tension acts away from C inclined at an angle α with x-axis.

The normal reaction $R_b$ acts at the support B and passes towards the cuncurrent point C. The FBD showing all the four forces is shown.




Since the no. of forces are more than three. We cann't applying Lami's theorem directly unless until it being reduced to three numbers.

Since $R_b$ and W are collinear, let $(R_b - W)$ acts vertically up for equilibrium. Now the system remains equillibrium under the action of three forces ($R_b$ – W), S & P as shown in the figure.




Applying lami's theorem

$$\frac{P}{\sin(90+\alpha)} = \frac{(R_b - W)}{\sin(180-\alpha)} = \frac{S}{\sin 90}$$

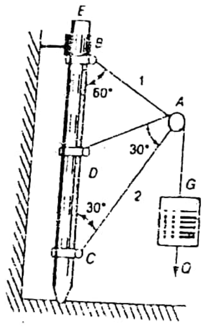
Now; $$\frac{P}{\sin(90^0+\alpha)} = \frac{(R_b - W)}{\sin(180^0-\alpha)}$$

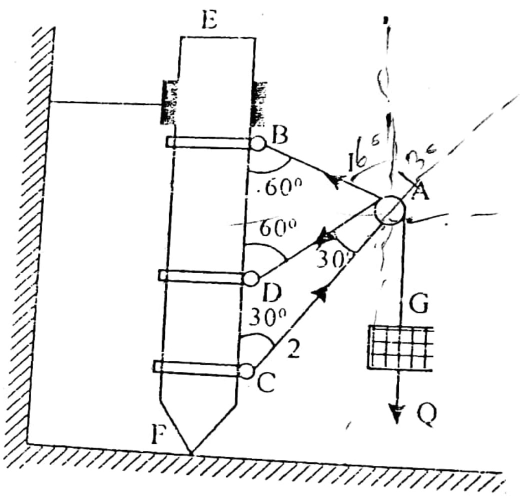
$$\Rightarrow R_b - W = P\frac{\sin\alpha}{\cos\alpha} \quad \Rightarrow R_b = W + P\tan\alpha$$ **(Ans.)**

Now; $$\frac{P}{\sin(90+\alpha)} = \frac{S}{\sin 90^0} \quad \Rightarrow S = \frac{P\sin 90^\circ}{\sin(90+\alpha)}$$

$$\Rightarrow S = \frac{P}{\cos\alpha} \quad \Rightarrow S = P\sec\alpha$$ (Ans.)

8. *A pulley A is supported by two bars AB and AC which are hinged at points B and C to a vertical mast EF. Over the pulley hangs a flexible cable DG which is fastened to the mast at D and carries at the other end G a load Q = 20 kN. Neglecting friction in the pulley, determine the forces produced in the bars AB and AC. The angles between the various members are shown in the figure.*



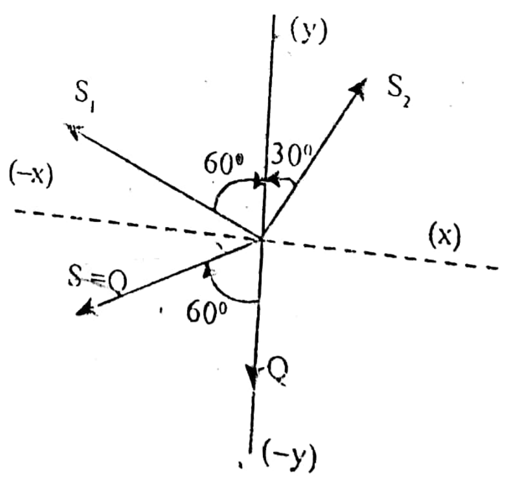

**Soln.** Given data

Q = 20 K.N.

Since DAG is one string passing over the pully, the axial force developed S as the tension is equal to the hanged load Q i.e., S = Q

The bar AB is under tension $S_1$ and the bar AC is under compression $S_2$ due to the gravity pull Q. The FBD at A as shown in the fig.

**FBD**




Resolving horizontally

$\Sigma x = 0$

$$S_2 \sin 30° = S_1 \sin 60° + S \sin 60° \quad .....(i)$$

$\Sigma y = 0$

$$S_2 \cos 30° + S_1 \cos 60° = Q + S \cos 60° \quad .....(ii)$$

–(x)

From equation (i)

$$\frac{S_2}{2} = S_1 \times \frac{\sqrt{3}}{2} + 20 \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow S_2 = \sqrt{3}\, S_1 + 20\sqrt{3} \quad ....(iii)$$

From equation (ii)

$$S_2 \times \frac{\sqrt{3}}{2} + \frac{S_1}{2} = 20 + \frac{20}{2}$$

$$\Rightarrow \sqrt{3}\, S_2 + S_1 = 60 \quad .....(iv)$$

Solving equation (iii) and (iv)

$\sqrt{3}\, S_2 + S_1 = 60$ or $\sqrt{3}\left(\sqrt{3}S_1 + 20\sqrt{3}\right) + S_1 = 60 \Rightarrow 3S_1 + 60 + S_1 = 60$

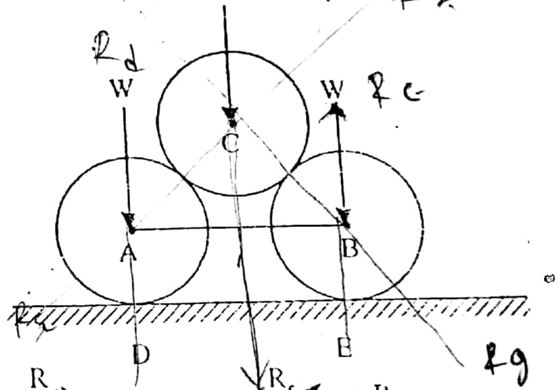
(Ans.)

$\Rightarrow 4S_1 = 0 \quad \Rightarrow \quad S_1 = 0$

$\therefore S_2 = \sqrt{3}\, S_1 + 20\sqrt{3} = \sqrt{3}\,(0) + 20\sqrt{3} = 0 + 20\sqrt{3} = 20\sqrt{3}$

$\therefore S_2 = 34.64$ KN (compression) (Ans.)

9. *Two smooth circular cylinders, each of weight $W = 445N$ and radius $r = 152$ mm, are connected at their centres by a string AB of length $l = 406$ mm and rest upon a horizontal plane, supporting above them a third cylinder of weight $Q = 890N$ and radius $r = 152mm$ as shown in the fig. Find the forces S in the string and the pressures produced on the floor at the points of contact D and E.*



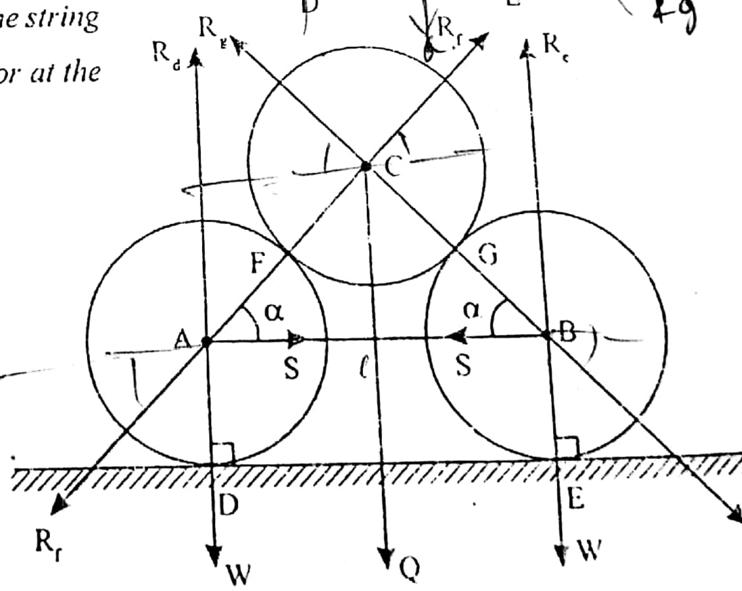

**Soln.** Given data

$W = 445$ N, $Q = 890$ N

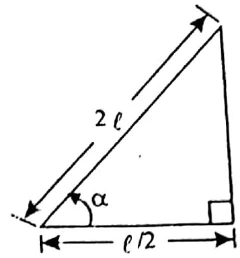
$l = AB = 406$ mm $r = 152$mm

$S = ?$ $R_d = ?$ $R_e = ?$

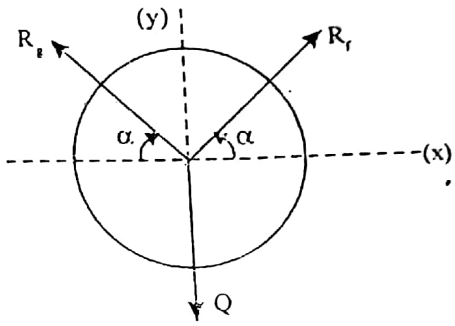
Since AB is a string tied at A & B, equal and opposite tensile forces act away from A and B.



Normal reaction $R_d$ and $R_e$ act towards A & B
Let three cylinders are in contact at F & G. Hence equal opposite normal reactions $R_f$ and $R_g$ act towards each cylinder.
The minimum no. of forces are acting on cylinder C. Therefore FBD is 1st drawn at C as shown in the fig. Applying Lami's theorem




FBD at 'C'




$$\frac{Q}{\sin(180-2\alpha)} = \frac{R_f}{\sin(90+\alpha)} = \frac{R_g}{\sin(90+\alpha)}$$

$$\therefore R_f = R_g = \frac{Q}{\sin 2\alpha} \times \cos\alpha$$

But from the geometry of the figure

$$\cos\alpha = \frac{\ell}{2}/2r = \frac{\ell}{4r}$$

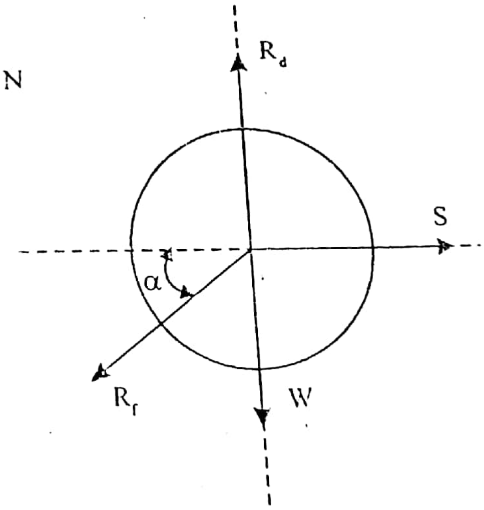
or $\quad \alpha = \cos^{-1}\left(\frac{\ell}{4r}\right) \quad = \cos^{-1}\left(\frac{406}{4 \times 152}\right) = 48° 6'$

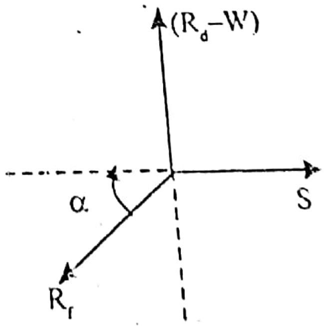
Substituting the value

$$R_f = R_g = \frac{890}{\sin(2 \times 48°6')} \times \cos 48° 6' = 597.86 \text{ N}$$

Now let us draw the FBD at A as shown in the figure. The no. of forces acting on the cylinder are four out of which two forces $R_d$ & W are collinear. Let reduce them to $(R_d - W)$, acts upward to make it equilibrium.
Applying Lami's theorem

FBD at 'A'



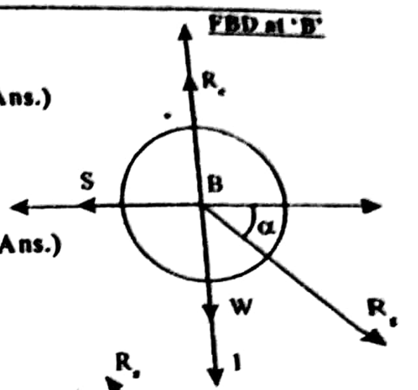

$$\frac{(R_d - W)}{\sin(180° - \alpha)} = \frac{S}{\sin(90+\alpha)} = \frac{R_f}{\sin 90°}$$

Now; $\quad \dfrac{R_d - W}{\sin\alpha} = \dfrac{R_f}{\sin 90^0}$

$$\Rightarrow \frac{R_d - 445}{\sin 48° 6'} = \frac{597.86}{\sin 90°}$$

$\Rightarrow (R_d - 445) = 597.86 \times \sin 48°6'$

$\Rightarrow R_d = (597.86 \times \sin 48°6') + 445 \Rightarrow R_d = 889.99$ N (Ans.)

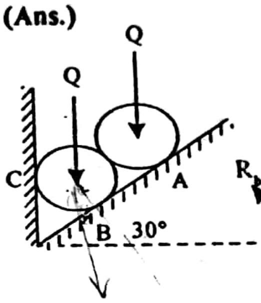



Now ; $\dfrac{S}{\sin(90°+\alpha)} = \dfrac{597.86}{\sin 90°}$

$\Rightarrow S = 597.86 \times \cos 48°6'$ or $S = 399.27$ N (Ans.)

Due to symmetry $R_e = R_d = 889.99$ N (Ans.)

13. *Two identical rollers, each of weight Q = 445N, are supported by an inclined plane and a vertical wall as shown in fig. Assuming smooth surfaces, find the reactions induced at the points of support A, B and C.*




**Soln.** Given data

Q = 445 N

$R_a = ?$ $R_b = ?$ $R_c = ?$

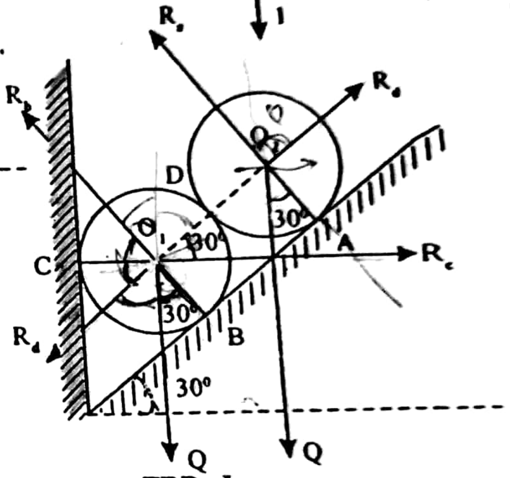
Since the two rollers are in contact at A, B and C

The normal reactions are acting as $R_a$, $R_b$ and $R_c$

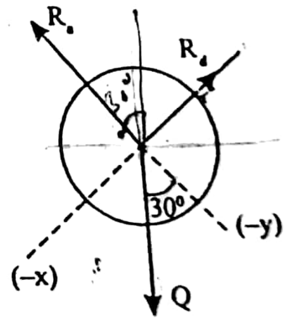
Let 'D' be the contact point between themselves.

The normal reaction $R_d$ acts equally towards each other parallel to the inclined plane.

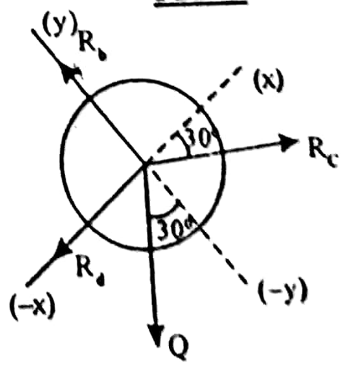
The minimum no. of forces are acting in the upper roller and hence 1st. FBD is drawn as showin in the fig.




FBD -I




FBD-II




Applying Lami's theorem;

$$\frac{Q}{\sin 90°} = \frac{R_d}{\sin(180°-30°)} = \frac{R_a}{\sin(90°+30°)}$$

Now ; $\dfrac{Q}{\sin 90°} = \dfrac{R_d}{\sin(150°)} \Rightarrow R_d = \dfrac{445 \times \sin 150°}{\sin 90°}$

or $R_d = 222.5$ N

Now ; $\dfrac{Q}{\sin 90°} = \dfrac{R_a}{\sin 120°} \Rightarrow R_a = 445 \times \sin 120°$

$R_a = 385.38$ N (Ans.)

The 2nd FBD of the lower roller is shown in the fig.
Applying method of resolution.

$\Sigma x = 0$

$$R_c \cos 30° = R_d + Q \sin 30° \quad \text{.....(i)}$$

$\Sigma y = 0$

$$R_b = Q \cos 30° + R_c \sin 30° \quad \text{....(ii)}$$

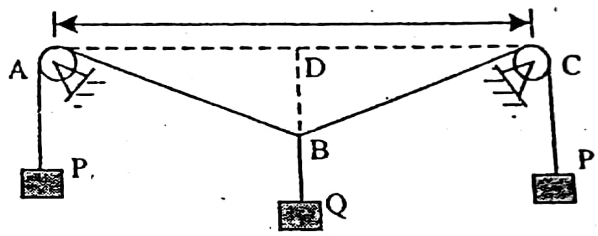
From equation (i)

$$R_c \frac{\sqrt{3}}{2} = 222.5 + 445 \times \frac{1}{2} \Rightarrow R_c = \frac{2}{\sqrt{3}}\left(222.5 + 445 \times \frac{1}{2}\right) = 513.84 \text{ N} \quad \textbf{(Ans.)}$$

From equation (ii)

$$R_b = Q \cos 30° + R_c \sin 30° = 445 \cos 30° + 513.84 \times \frac{1}{2} = 642.15 \text{ N} \quad \textbf{(Ans.)}$$

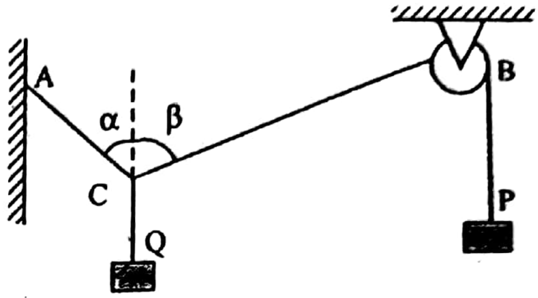
**11.** *A weight Q is suspended from point B of a cord ABC, the ends of which are pulled by equal weights P overhanging small pulleys A and C which are on the same level as shown in fig.. Neglecting the radii of the pulleys, determine the sag BD if l =*



$$\Rightarrow \sin\alpha = \frac{44.5}{2\times 89} \quad \text{or} \quad \alpha = \sin^{-1}\left(\frac{44.5}{2\times 89}\right) \quad \Rightarrow \alpha = 14^\circ\, 28'$$

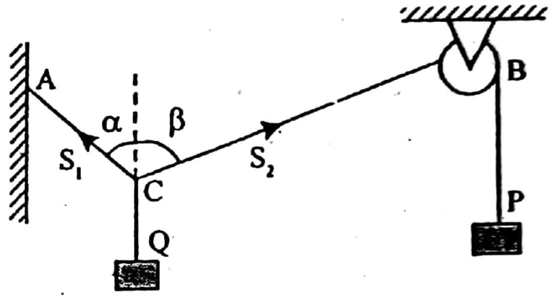
In $\Delta$ ABD ; $\tan\alpha = \dfrac{BD}{AD}$ $\Rightarrow BD = \dfrac{\ell}{2}\times\tan\alpha = \dfrac{3.66}{2}\times\tan 14^\circ 28' = 0.472\text{m}$ **(Ans.)**

12. *A weight Q is suspended from a small ring C, supported by two cords AC and BC as shown in fig.. The cord AC is fastened at A while the cord BC passes over a frictionless pulley at B and carries the weight P as shown. If P = Q and $\alpha = 50^\circ$, find the value of the angle $\beta$.*




**Soln.** Given data

$P = Q$, $\alpha = 50^\circ$, $\beta = ?$

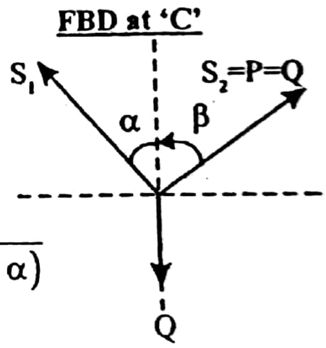



AC and BC are two cords, hence axial forces $S_1$ and $S_2$ are acting away from the C.

The load P = Q is hanged at the free end of the cord CB.

$\therefore S_2 = P = Q$

The FBD at C is shown



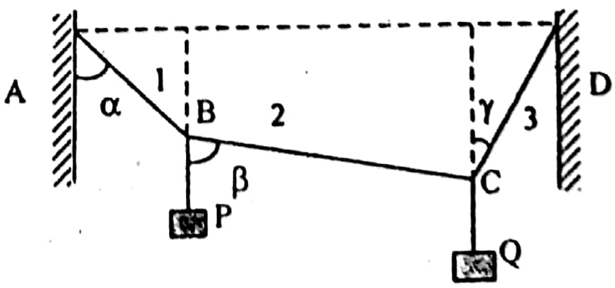

Applying Lamis theorem $\dfrac{Q}{\sin(\alpha+\beta)} = \dfrac{S_1}{\sin(180^\circ-\beta)} = \dfrac{S_2}{\sin(180^\circ-\alpha)}$

Now; $\dfrac{Q}{\sin(\alpha+\beta)} = \dfrac{S_2}{\sin(180^\circ-\alpha)}$ or $\dfrac{Q}{\sin(\alpha+\beta)} = \dfrac{Q}{\sin(180^\circ-\alpha)}$

$\Rightarrow \sin(\alpha+\beta) = \sin(180^\circ-\alpha)$ $\Rightarrow \alpha+\beta = 180^\circ-\alpha$

$\Rightarrow \beta = 180^\circ - 2\alpha$ $\Rightarrow \beta = 180^\circ - 100^\circ$ or $\beta = 80^\circ$ **(Ans.)**

13. *In the given Fig. weights P and Q are suspended in a vertical plane by strings 1,2,3 arranged as shown in fig. M. Find the tension induced in each string if P = 30 kN, Q = 40 kN, $\alpha = 40^\circ$ and $\beta = 50^\circ$. Also find the inclination $\gamma$ of segment CD to the vertical.*

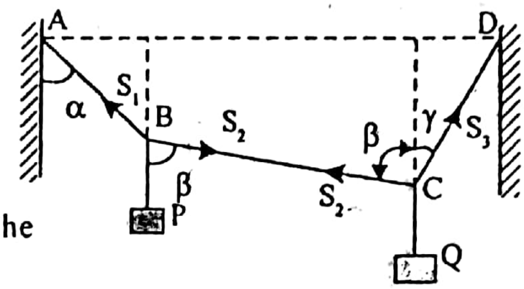


Given data

$P = 30$ KN, $Q = 40$ KN

$\alpha = 40°$, $\beta = 50°$, $\gamma = ?$

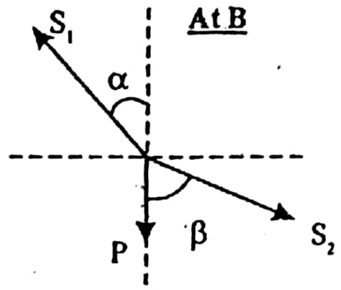
$S_1 = ?$ $S_2 = ?$ $S_3 = ?$




The axial forces $S_1$ and $S_2$ as the tensions along the strings AB and BC act away from the B.

The axial force $S_2$ & $S_3$ as the tensions act away from C.

The FBD at B is shown in the fig.

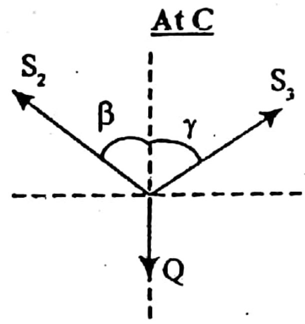



Applying Lami's theorem

$$\frac{P}{\sin(180+\alpha-\beta)} = \frac{S_1}{\sin\beta} = \frac{S_2}{\sin(180°-\alpha)}$$

Now; $\dfrac{P}{\sin(180+\alpha-\beta)} = \dfrac{S_1}{\sin\beta}$

$$\Rightarrow S_1 = \frac{30 \times \sin 50°}{\sin(180° + 40° - 50°)} \Rightarrow S_1 = 132.34 \text{ KN} \quad \textbf{(Ans.)}$$




Now; $\dfrac{P}{\sin(180+\alpha-\beta)} = \dfrac{S_2}{\sin\alpha}$

$$\Rightarrow S_2 = \frac{30 \times \sin 40°}{\sin(180° + 40° - 50°)} \Rightarrow S_2 = 111.04 \text{ KN} \quad \textbf{(Ans.)}$$

Applying Lami's theorem

$$\frac{Q}{\sin(\beta+\gamma)} = \frac{S_2}{\sin(180-\gamma)} = \frac{S_3}{\sin(180-\beta)}$$

Now; $\dfrac{Q}{\sin(\beta+\gamma)} = \dfrac{S_2}{\sin(180-\gamma)} \Rightarrow \dfrac{40}{\sin(50+\gamma)} = \dfrac{111.04}{\sin\gamma}$

$$\Rightarrow 40 \sin\gamma = 111.04 \sin(50°+\gamma) \Rightarrow 40\sin\gamma = 111.04\, \cancel{\sin}\left[\sin 50 \cos\gamma + \cos 50 \sin\gamma\right]$$

$$\Rightarrow 0.36 \sin\gamma = 0.76 \cos\gamma + 0.64 \sin\gamma \Rightarrow \sin\gamma\,(0.36 - 0.64) = 0.76 \cos\gamma$$

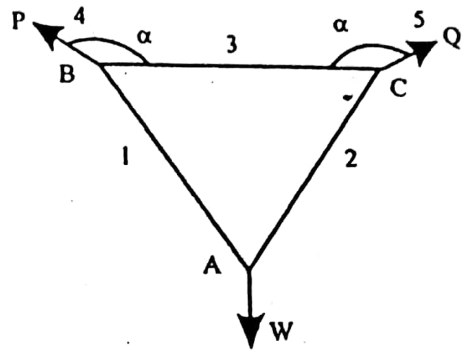
$$\Rightarrow \tan\gamma = \left(\frac{0.76}{0.28}\right) \quad \Rightarrow \gamma = \tan^{-1}\left(-\frac{0.76}{0.28}\right) \quad \Rightarrow \gamma = -69° 46'$$

$\therefore$ Actual $\gamma = 180° + \gamma = 180° - 69° 46' = 110° 22'$ **(Ans.)**

Now; $\dfrac{Q}{\sin(\beta+\gamma)} = \dfrac{S_3}{\sin(180-\beta)}$

$$\Rightarrow \frac{40}{\sin(50 + 110°22')} = \frac{S_3}{\sin 50°} \Rightarrow S_3 = \frac{40 \times \sin 50°}{\sin 160°36} \Rightarrow S_3 = 91.19 \text{ KN} \quad \textbf{(Ans)}$$

**14.** *Three equal inextensible strings of negligible weight are knotted together to form an equilateral triangle ABC and a weight W is suspended from A. If the triangle and weight to be supported with BC horizontal by means of two strings at B and C as shown in fig. each at an angle of $\alpha = 135°$ with BC, find the tension in the string 3.*

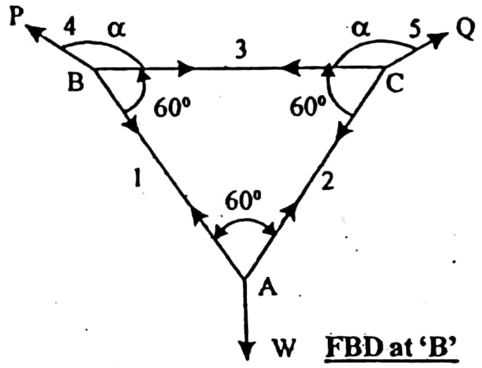



**Soln.** Given data

$\alpha = 135°$

$\Delta$ ABC is an equilateral triangle

The axial forces $S_1$, $S_2$ & $S_3$ act along the strings AB, AC and BC away from their joints. The FBD at 'A' is shown in the figure.



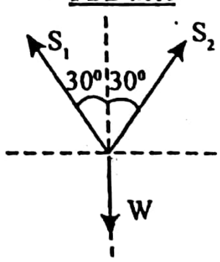

Using Lami's theorem

$$\frac{W}{\sin 60°} = \frac{S_1}{\sin 150°} = \frac{S_2}{\sin 150°}$$

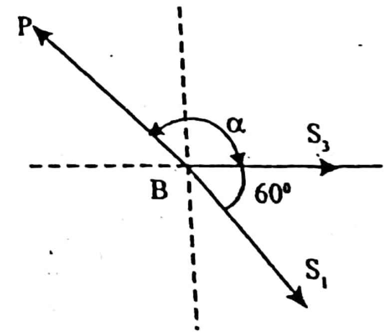
$$\therefore S_1 = S_2 = \frac{W}{\sin 60°} \times \sin 150°$$

$$S_1 = S_2 = 0.577\ W$$

**FBD at A**




**FBD at 'B'**



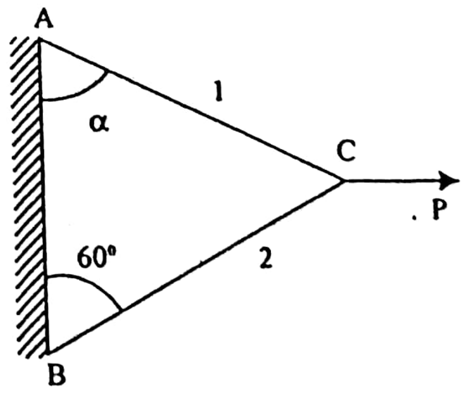

The FBD at B is shown in the fig.

Applying Lami's theorem

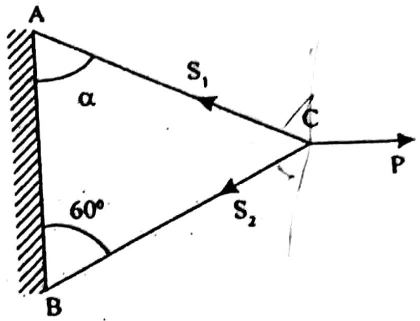
$$\frac{S_1}{\sin \alpha} = \frac{S_3}{\sin(30° + 90° + 180 - \alpha)} \Rightarrow \frac{S_1}{\sin 135°} = \frac{S_3}{\sin(300° - 135°)}$$

$$\Rightarrow \frac{0.577W}{\sin 135°} = \frac{S_3}{\sin 165°} \Rightarrow S_3 = \frac{0.577W \times \sin 165}{\sin 135} = 0.211W \quad \textbf{(Ans.)}$$

**15.** *A force P is applied at point C as shown in fig. Determine the value of angle α° for which the larger of the string tension is as small as possible and the corresponding values of tension in the strings 1 and 2.*



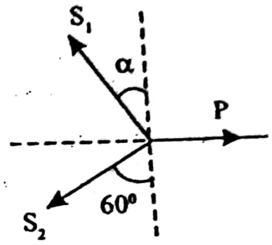
**Soln.** The FBD at 'C' is shown showing the tension $S_1$ making $\alpha°$ with vertical and $S_2$ making 60° to the vertical.




Applying Lami's Theorem

$$\frac{P}{\sin\{180°-(60°+\alpha)\}}=\frac{S_1}{\sin(180°-30°)}=\frac{S_2}{\sin(90+\alpha)}$$

Now; $\dfrac{P}{\sin(60°+\alpha)}=\dfrac{S_1}{\sin(180°-30°)}$




$\therefore\ S_1=\dfrac{P\sin 150°}{\sin(60°+\alpha)}$ .....(i)

Now; $\dfrac{S_1}{\sin 150°}=\dfrac{S_2}{\sin(90°+\alpha)}\ \Rightarrow S_1=\dfrac{S_2\sin 150°}{\sin(90°+\alpha)}$

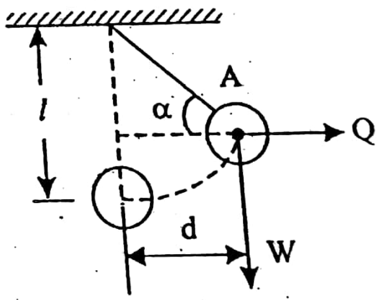
Since the larger of the string tension is as small as possible then,

$S_1=S_2$ or $\sin(90°+\alpha)=\sin 150°$ or $90°+\alpha=150°$ or $\alpha=60°$ **(Ans.)**

Substituting the value, $S_1=\dfrac{P\sin 150°}{\sin(60°+\alpha)}=\dfrac{P\sin 150°}{\sin 120°}=0.577\,P$

$\therefore\ S_1=S_2=0.577\,P$ **(Ans.)**

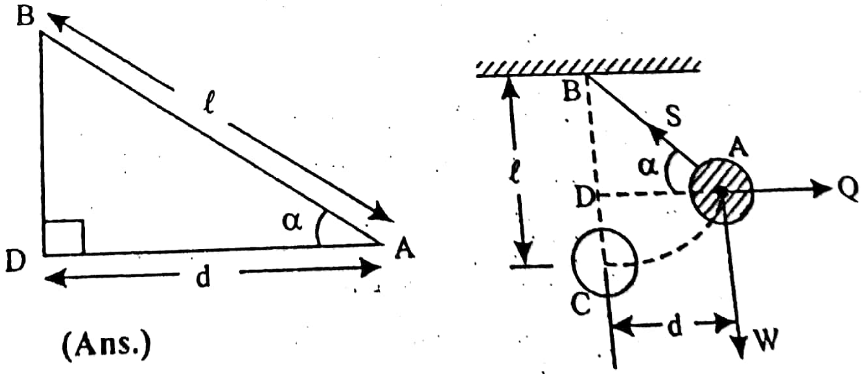
**16.** *A ball of weight W is suspended from a string of length l and is pulled by a horizontal force Q. The weight is displaced by a distance 'd' from the vertical position as shown in fig. Find the angle α, force Q required and the tension S in the string in the displaced position, if the ball is in equilibrium.*




**Soln.** $\alpha=?$

$Q=?$

$S=?$




In the Δ ADB, $\cos\alpha=\dfrac{d}{\ell}$

$\therefore\ \alpha=\cos^{-1}\left(\dfrac{d}{\ell}\right)$ **(Ans.)**

We know that, $\sin^2\alpha+\cos^2\alpha=1$ or $\sin\alpha=\sqrt{1-\cos^2\alpha}$

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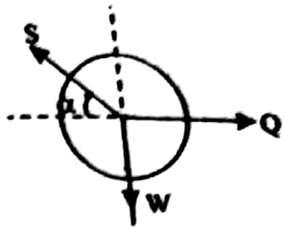
$$\Rightarrow \sin\alpha = \sqrt{1-\left(\frac{d}{\ell}\right)^2} \qquad \Rightarrow \sin\alpha = \frac{1}{\ell}\sqrt{\ell^2-d^2}$$




The FBD at 'A' is drawn as shown in the fig.

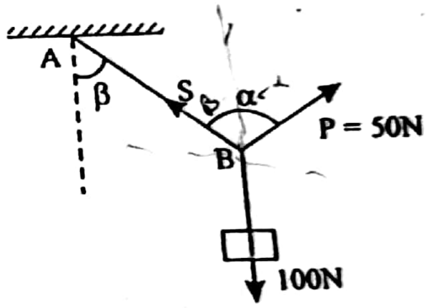
Applying Lami's theorem, $\dfrac{W}{\sin(180°-\alpha)} = \dfrac{Q}{\sin(90°+\alpha)} = \dfrac{S}{\sin 90°}$

Now ; $\dfrac{W}{\sin(180°-\alpha)} = \dfrac{Q}{\sin(90°+\alpha)} \Rightarrow Q = \dfrac{W\cos\alpha}{\sin\alpha}$ or $Q = W \times \left(\dfrac{d}{\ell} \times \dfrac{\ell}{\sqrt{\ell^2-d^2}}\right)$

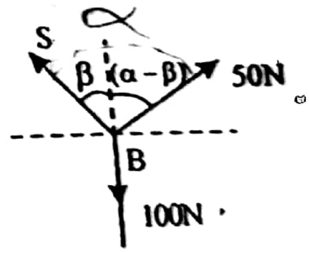
$\Rightarrow Q = \dfrac{Wd}{\sqrt{\ell^2-d^2}}$ **(Ans.)**

Now; $\dfrac{W}{\sin(180°-\alpha)} = \dfrac{S}{\sin 90°} \Rightarrow S = \dfrac{W \times 1}{\sin\alpha} \Rightarrow S = \dfrac{W \times \ell}{\sqrt{\ell^2-d^2}}$ **(Ans.)**

11. *A weight 100 N hangs by an inextensible string from a fixed point A. The string is drawn out of the vertical by applying a force 50 N to the weight at point B as shown in fig. In what direction must this force be applied in order that, in equilibrium, the direction of the string from the vertical may have its greatest value. What is the amount of greatest deflection. Find also the tension in the string.*




**Soln.** The FBD at 'B' is drawn as shown in the fig.




Applying Lami's theorem

$$\frac{100}{\sin\alpha} = \frac{S}{\sin\{180-(\alpha-\beta)\}} = \frac{P}{\sin(180-\beta)}$$

or $\dfrac{100}{\sin\alpha} = \dfrac{S}{\sin(\alpha-\beta)} = \dfrac{P}{\sin\beta}$ .....(i)

The direction of the string from the vertical ($\beta$) may have it's greatest value, when t applied force, P = 50 N is minimum.

Now; $\dfrac{100}{\sin\alpha} = \dfrac{P}{\sin\beta}$ or $P = \dfrac{100\sin\beta}{\sin\alpha}$

P will be minimum when $\sin\alpha$ is maximum.

i.e., $\sin\alpha = 1$ or $\alpha = 90°$ **(Ans.)**

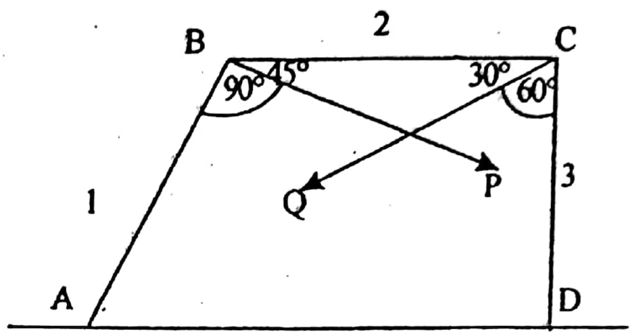
$\therefore$ Substituting the value $P = 100\sin\beta$

or $\sin\beta = \dfrac{50}{100}$ $\Rightarrow \beta = \sin^{-1}\left(\dfrac{1}{2}\right)$ $\Rightarrow \beta = 30°$ **(Ans.)**

Similarly; $\dfrac{100}{\sin\alpha} = \dfrac{S}{\sin(\alpha-\beta)} \Rightarrow \dfrac{100}{\sin 90°} = \dfrac{S}{\sin(90°-30°)}$

or $S = \dfrac{100 \times \sin 60°}{\sin 90°}$ or $S = 86.6$ N **(Ans.)**

**18.** *Three bars in one plane, hinged at their ends as shown in fig. are submitted to the action of a force P = 44.5 N applied at the hinge as shown. Determine the magnitude of the force that it will be necessary to apply at the hinge in order to keep the system of bars in equilibrium if the angles between the bars and the lines of action of the forces are as given in the figure.*

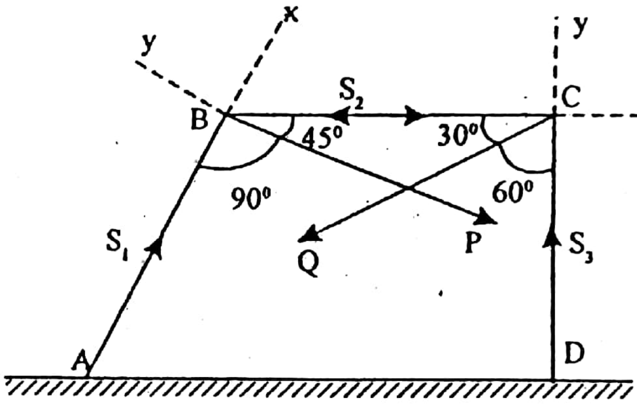



**Soln.** Given data

$P = 44.5$ N

$Q = ?$

The bar AB & BC are pin jointed at B and is acted upon by force P, hence axial forces $S_1$ and $S_2$ as the compression act towards B due to contraction of the bars.

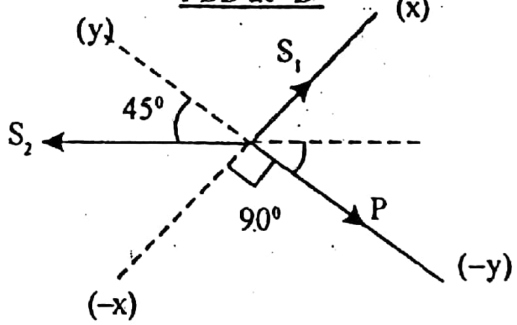



Since the lines of action of $S_1$ and P are right angle, let us assume the line of action of '$S_1$' as the x-axis and line of action of 'P' as the Y-axis.

The FBD at 'B' is shown in the fig.

Applying Lami's theorom

**FBD at 'B'**




$$\frac{P}{\sin 135°} = \frac{S_1}{\sin 135°} = \frac{S_2}{\sin 90°} \qquad \text{Now;}\ \frac{P}{\sin 135°} = \frac{S_1}{\sin 135°}$$

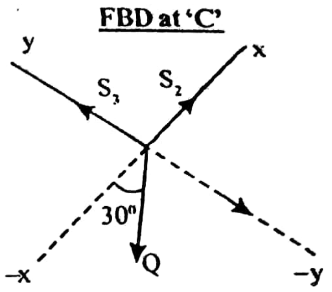
or $S_1 = \dfrac{44.5 \times \sin 135°}{\sin 135°}$ or $S_1 = 44.5$ N (compression)

Now; $\dfrac{P}{\sin 135°} = \dfrac{S_2}{\sin 90°}$ or $S_2 = \dfrac{44.5 \times \sin 90°}{\sin 135°}$

$\Rightarrow S_2 = 62.93$ N (compression)

The axial forces $S_2$ & $S_3$ as the compression act along the bars BC and DC towards 'C' due to the contraction of the bars.

The FBD at C is shown in the figure.



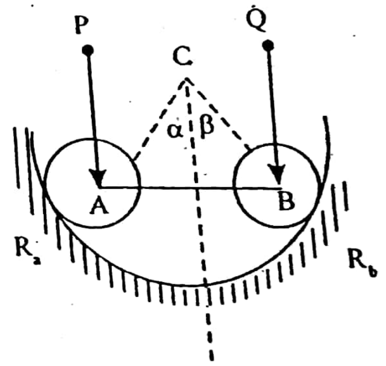

Applying Lami's theorem

$$\frac{S_2}{\sin 120^\circ} = \frac{S_3}{\sin 150^\circ} = \frac{Q}{\sin 90^\circ}$$

Now ; $\dfrac{S_2}{\sin 120^\circ} = \dfrac{Q}{\sin 90^\circ}$ or $Q = \dfrac{62.93 \times \sin 90^\circ}{\sin 120^\circ} \Rightarrow Q = 72.66\text{ N}$ **(Ans.)**

Now; $\dfrac{S_2}{\sin 120^\circ} = \dfrac{S_3}{\sin 150^\circ}$ or $S_3 = \dfrac{62.93 \times \sin 150^\circ}{\sin 120^\circ} \Rightarrow S_3 = 36.33\text{N}$ **(Ans.)**

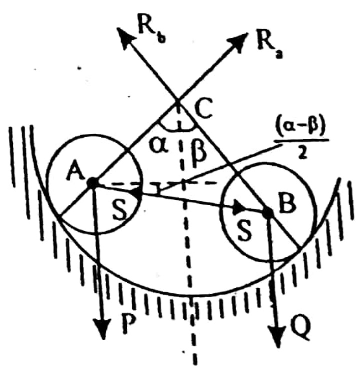
19. *A rigid bar with rollers of weights P = 222.5 N and Q = 445 N at its ends is supported inside a circular ring in a vertical plane as shown in fig. The radius of the ring and the length AB are such that the radii AC and BC form a right angle at C; that is $\alpha + \beta = 90^\circ$. Neglecting friction and the weight of the bar AB, find the configuration of equilibrium as defined by the angle $(\alpha - \beta)/2$ that makes with the horizontal. Find also the reactions $R_a$ and $R_b$ and the compressive force S in the bar AB.*




**Soln.** Given data

P = 222.5 N, Q = 445 N

$\alpha + \beta = 90^\circ$, $\dfrac{(\alpha - \beta)}{2} = ?$




$R_a = ?$ $R_b = ?$ $S = ?$

In $\Delta ABC$, $m\angle ACB = 90^\circ$ & $AC = BC = r$

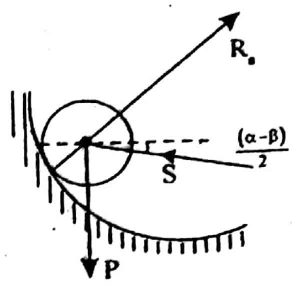
Hence $\Delta ABC$ is a right angle isoscale triangle

$\therefore m\angle CAB = m\angle CBA = 45^\circ$

The normal reactions $R_a$ and $R_b$ act at A & B passing through the center of the circular ring at 'C' AB is a weighless bar. Hence axial force 'S' as the compression acts towards A and B. The FBD at 'A' shown in the figure.

Applying Lami's theorem

$$\frac{P}{\sin 135^\circ} = \frac{S}{\sin(180^\circ-\alpha)} = \frac{R_a}{\sin(45^\circ+\alpha)}$$




Now ; $\frac{P}{\sin 135^\circ} = \frac{S}{\sin(180^\circ-\alpha)} \Rightarrow S = \frac{222.5 \sin\alpha}{\sin 135^\circ}$

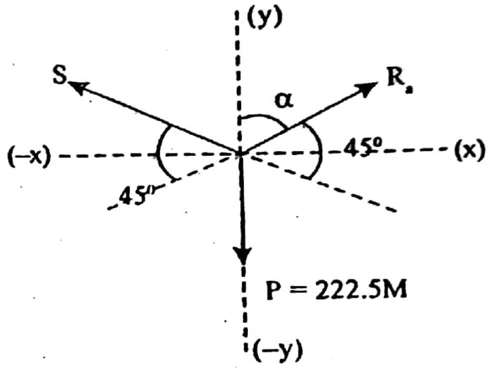
$\Rightarrow S = 314.6 \sin\alpha$ ........(i)

Now ; $\frac{P}{\sin 135^\circ} = \frac{R_a}{\sin(45^\circ+\alpha)}$

$$\Rightarrow R_a = \frac{222.5 \sin(45^\circ+\alpha)}{\sin 135^\circ}$$

$\Rightarrow R_a = 314.6 \sin(45^\circ+\alpha)$ .....(ii)

At 'A'




The FBD at B is shown in the figure.

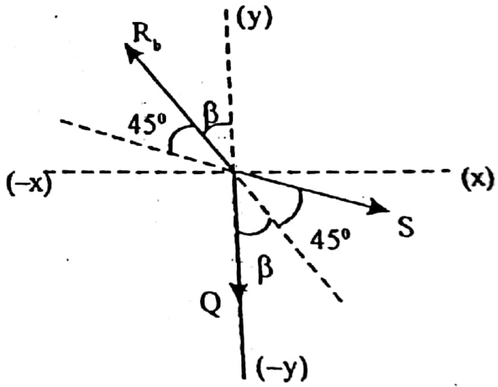
Applying Lami's theorem

$$\frac{Q}{\sin 135^\circ} = \frac{S}{\sin(180^\circ-\beta)} = \frac{R_b}{\sin(45^\circ+\beta)}$$

Now ; $\frac{Q}{\sin 135^\circ} = \frac{S}{\sin(180^\circ-\beta)} \Rightarrow S = \frac{445 \sin\beta}{\sin 135^\circ}$

$\Rightarrow S = 620.32 \sin\beta$ ......(iii)

At 'B'




Now ; $\frac{Q}{\sin 135^\circ} = \frac{R_b}{\sin(45^\circ+\beta)} \Rightarrow R_b = \frac{445 \sin(45^\circ+\beta)}{\sin 135^\circ}$

$\Rightarrow R_b = 629.32 \sin(45^\circ+\beta)$ (iv)

Equating (i) and (iii)

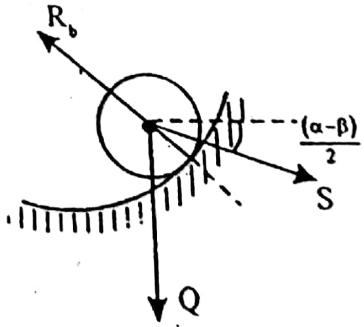
$314.6 \sin\alpha = 629.32 \sin\beta$ (v)

But we know that $\alpha + \beta = 90^\circ$

or $\beta = (90^\circ - \alpha)$ or $\sin\beta = \sin(90^\circ-\alpha) = \cos\alpha$

Substituting the value in equation (v)

$314.6 \sin\alpha = 629.32 \cos\alpha$




$\Rightarrow \tan\alpha = \frac{629.32}{314.6} \Rightarrow \alpha = \tan^{-1}\left(\frac{629.32}{314.6}\right)$

$\Rightarrow \alpha = 63^\circ 26'$

$\therefore \beta = 90^\circ - 63^\circ 26' = 26^\circ 34'$

$$\therefore \frac{\alpha-\beta}{2} = \frac{(63^\circ 26' - 26^\circ 34')}{2} = 18^\circ 26' \qquad \textbf{(Ans.)}$$

From equation (ii)

$$R_a = 314.6 \sin(45^\circ + \alpha) = 314.6 \sin(45^\circ + 63^\circ 26') = 298.45 \text{ N} \qquad \textbf{(Ans.)}$$

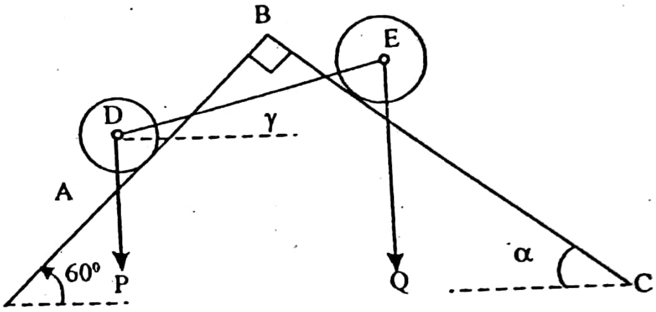
From equation (iv)

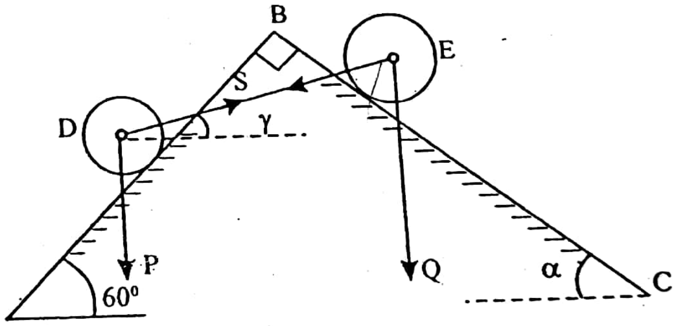
$$R_b = 629.32 \sin(45^\circ + \beta) = 629.32 \sin(45^\circ + 26^\circ 34') = 597.03 \text{ N} \qquad \textbf{(Ans.)}$$

From equation (i)

$$S = 314.6 \sin\alpha = 314.6 \sin 63^\circ 26' = 281.38 \text{ N} \qquad \textbf{(Ans.)}$$

**20.** *Two rollers of weights P and Q are connected by a flexible string DE and rest on two mutually perpendicular planes AB and BC, as shown in fig. Find the tension S in the string and the angle $\gamma$ that it makes with the horizontal when the system is in equilibrium. The following numerical data are given $P = 267$ N, $Q = 445$ N, $\alpha = 30^\circ$. Assume that the string is inextensible and passes freely through slots in the smooth inclined planes AB and BC.*




**Soln.** Given data

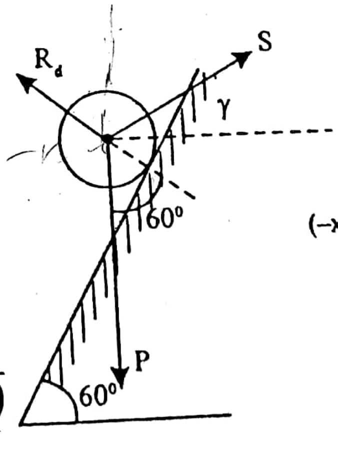
$P = 267$ N, $\alpha = 30^\circ$

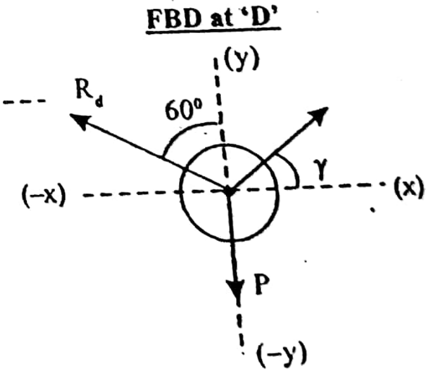
$Q = 445$ N, $S = ?$, $\gamma = ?$

DE is a string, hence an axial force S as the tension acts away from the D & E, each making an angle $\gamma$ with horizontal. The normal reaction $R_d$ and $R_e$ act at D & E making 60° and 30° with the vertical respectively.

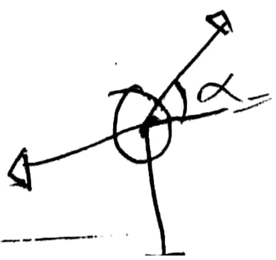
The FBD at D is shown in the figure

Applying Lami's theorem



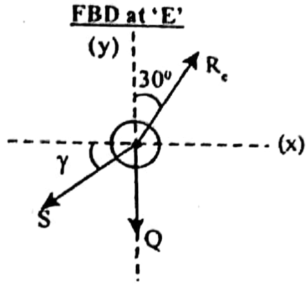

$$\frac{P}{\sin(60+90-\gamma)} = \frac{S}{\sin 120^\circ} = \frac{R_d}{\sin(90+\gamma)}$$



Now $\dfrac{P}{\sin(60+90-\gamma)} = \dfrac{S}{\sin 120^\circ}$

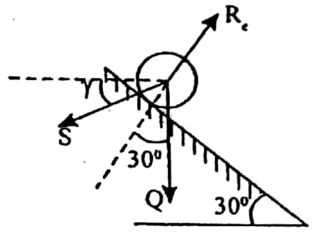
$$\Rightarrow S = \frac{267 \sin 120^\circ}{\sin(150^\circ - \gamma)}$$

$$\Rightarrow S = \frac{231.22}{\sin(150^\circ - \gamma)} \quad \text{.....(i)}$$




The FBD at E is show in the figure.

Applying Lami's theorem




$$\frac{Q}{\sin(120^\circ+\gamma)} = \frac{R_e}{\sin(90^\circ-\gamma)} = \frac{S}{\sin 150^\circ}$$

Now; $\dfrac{Q}{\sin(120^\circ+\gamma)} = \dfrac{S}{\sin 150^\circ}$

$$\Rightarrow S = \frac{445 \times \sin 150^\circ}{\sin(120^\circ + \gamma)}$$

$$\Rightarrow S = \frac{222.5}{\sin(120^\circ + \gamma)} \quad \text{.....(ii)}$$

Equating (i) and (ii)

$$\frac{231.22}{\sin(150^\circ - \gamma)} = \frac{222.5}{\sin(120^\circ+\gamma)} \quad \Rightarrow \frac{231.22}{\sin\{180-(30+\gamma)\}} = \frac{222.5}{\sin\{90+(30+\gamma)\}}$$

$$\Rightarrow \frac{231.22}{\sin(30+\gamma)} = \frac{222.5}{\cos(30+\gamma)} \quad \Rightarrow \tan(30+\gamma) = \frac{231.22}{222.5}$$

$$\Rightarrow 30^\circ + \gamma = \tan^{-1}\left(\frac{231.22}{222.5}\right) \quad \Rightarrow 30^\circ + \gamma = 46^\circ\, 6'$$

$$\Rightarrow \gamma = 46^\circ\, 6' - 30^\circ$$

$$\Rightarrow \gamma = 16^\circ\, 6'$$

**(Ans.)**

Substituting the value of $\gamma$ in equation (ii)

$$S = \frac{222.5}{\sin(120^\circ + v)} = \frac{222.5}{\sin(120^\circ + 16^\circ 6')}$$

$S = 320.88$ N

**(Ans.)**

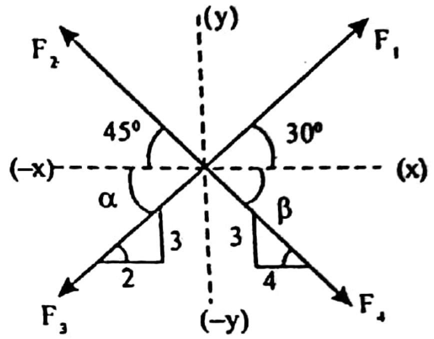
## SOLVED PROBLEMS - 2.4

1. *Using the method of projections, find the magnitude and direction of the resultant R of the four concurrent forces shown in fig. and having the magnitudes $F_1 = 1500N$, $F_2 = 2000N$, $F_3 = 3500N$ and $F_4 = 1000N$.*

**Soln.** Given data

$F_1 = 1500$ N $\quad F_2 = 2000$N

$F_3 = 3500$ N $\quad F_4 = 1000$ N




From the Δ, $\tan\alpha = \frac{3}{2}$

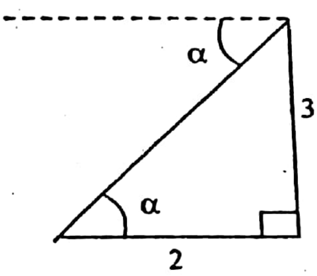
$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{3}{2}\right)$

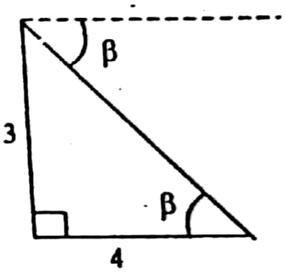
$\Rightarrow \quad \alpha = 56° \; 18'$

$\tan\beta = \frac{3}{4}$

$\Rightarrow \quad \beta = \tan^{-1}\left(\frac{3}{4}\right)$

$\Rightarrow \quad \beta = 36° \; 52'$




Using method of projection and resolving horizontally

$\Sigma x = (1500 \cos 30° + 1000 \cos 36°52' - 2000 \cos 45° - 3500 \cos 56°18') = -1257$ N

Resolving vertically

$\Sigma y = (1500 \sin 30° + 2000 \sin 45° - 3500 \sin 56°18 - 1000 \sin 36°52') = -1347.5$ N

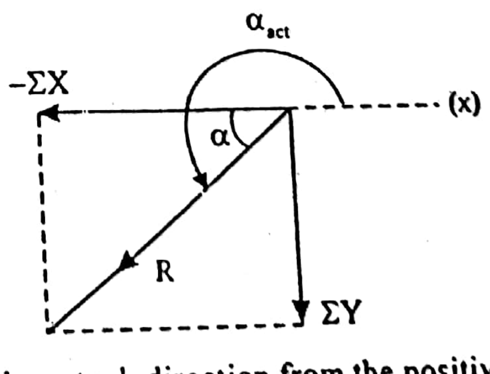
$\therefore$ Resultant $R = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(-1257)^2 + (-1347.5)^2} = 1842.7$ N **(Ans.)**

The direction

$\alpha = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$

$= \tan^{-1}\left(\frac{1347.5}{1257}\right)$

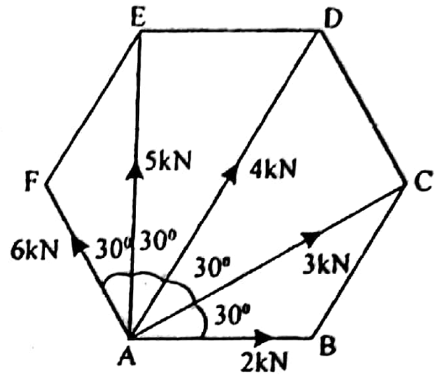
$= 46° \; 59' \simeq 47°$




Since the resultant lies in third quadrant, the actual direction from the positive x-axis is

$\alpha_{act} = (180° + \alpha) = (180° + 47°), \quad \alpha_{act} = 227°$ **(Ans.)**

2. *Forces of 2, 3, 4, 5 and 6 kN are acting at one of the angular points of a regular hexagon towards the other angular points taken in order. Find the resultant of the system of forces.*




**Soln.** Let forces 2, 3, 4, 5 & 6 KN. are acting at 'A' towards B, C, D, E & F of a regular hexagon, as shown.

Resolving horizontally

$$\Sigma x = (2 + 3\cos 30° + 4\cos 60° - 6\sin 30°) = 3.598 \text{ KN}$$

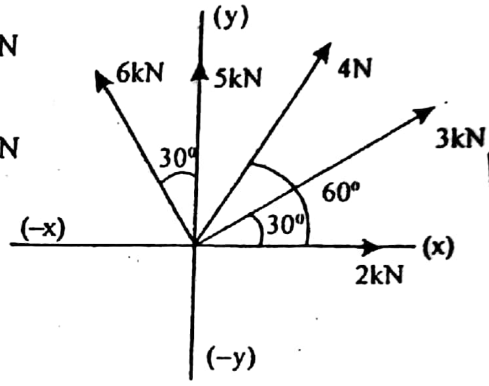
Resolving vertically

$$\Sigma y = (3\sin 30° + 4\sin 60° + 5 + 6\cos 30°) = 15.16 \text{ KN}$$

$$\therefore R = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

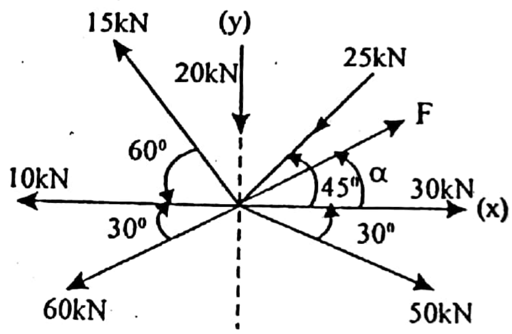
$$= \sqrt{(3.598)^2 + (15.16)^2} = 15.58 \text{ KN}$$

$$\alpha = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}\left(\frac{15.16}{3.598}\right) = 76°\ 38'$$




(Ans.)

3. *Find the magnitude and direction of the force F to be added to the system of coplanar concurrent forces shown in fig. to maintain equilibrium.*




**Soln.** Let A force F is added to the system of coplanner concurrent forces in 1st quadrant making an angle α with the horizontal. Since the system is equillibrium

$\Sigma x = 0$

$$F\cos\alpha + 30° + 50\cos 30° - 15\cos 60° - 10 - 60\cos 30° - 25\cos 45° = 0 \qquad \text{(i)}$$

$$\therefore F\cos\alpha = 15 \times \frac{1}{2} + 10 + 30 \times \frac{\sqrt{3}}{2} + 25 \times \frac{1}{\sqrt{2}} - 30 - 50\frac{\sqrt{3}}{2}$$

$$= 75 + 10 + 51.96 + 17.6 - 30 - 43.3 = 13.76 \text{ N} \qquad \text{(ii)}$$

$\Sigma y = 0$

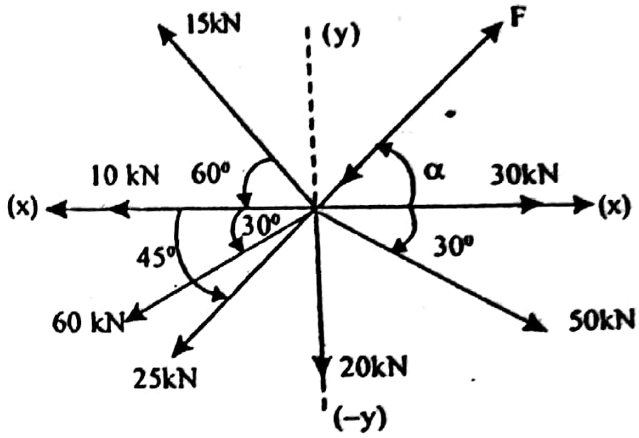
$$F\sin\alpha + 15\sin 60° - 20 - 25\sin 45° - 60\sin 30° - 50\sin 30° = 0 \qquad \text{(iii)}$$

$$F\sin\alpha = 20 + 25\sin 45° + 60\sin 30° + 50\sin 30° - 15\sin 60°$$

$$= 20 + 17.67 + 30 + 25 - 12.99 = 79.68 \text{ N} \qquad \text{(iv)}$$

Dividing equation (iv) (ii)

$$\frac{F \sin \alpha}{F \cos \alpha} = \frac{79.68}{13.76}$$




$\Rightarrow \tan \alpha = 5.8$

$\Rightarrow \alpha = \tan^{-1}(5.8)$

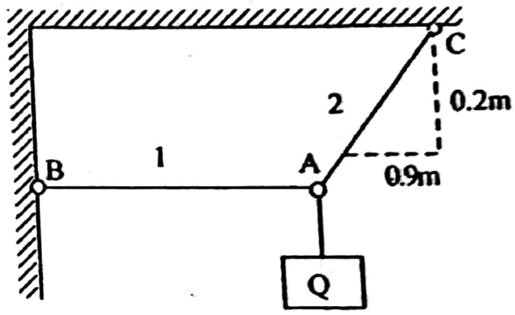
$\Rightarrow \alpha = 80° \, 13'$

From equation (iv)

$F \sin \alpha = 79.68$

$$\Rightarrow F = \frac{79.68}{\sin 80° \, 13'} \qquad \Rightarrow F = 80.85 \text{ N}$$

4. *Referring to fig. calculate the tensions $S_1$ and $S_2$ in the two strings AB and AC that support the lamp of weight $Q$ = 178 N. Use the method of projections.*



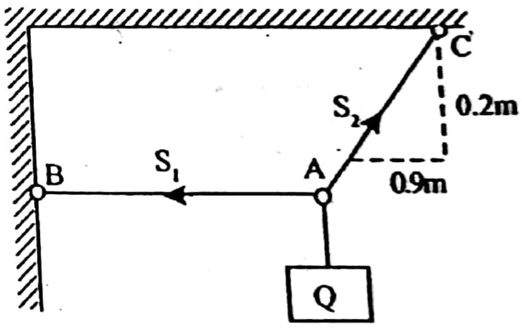

**Soln.** Given data

$Q = 178$ N

$S_1 = ?$ $\quad S_2 = ?$

From the slope of the wire AC

$$\alpha = \tan^{-1}\left(\frac{0.2}{0.9}\right) \Rightarrow \alpha = 12° \, 31'$$



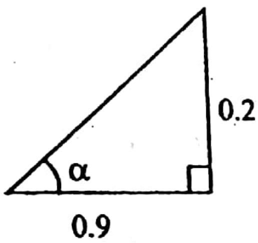

The FBD at 'A' is shown in the figure.

Resolving vertically in to zero

$\Sigma y = 0$

$S_2 \sin \alpha = Q$

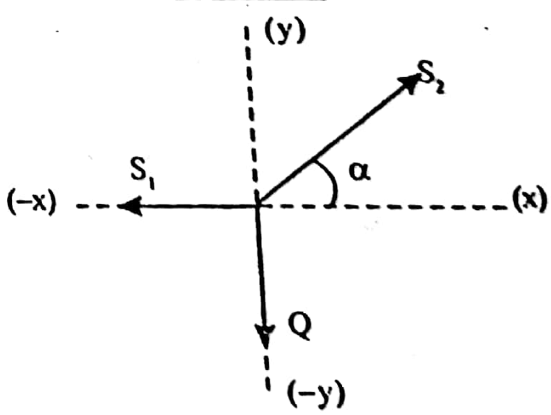
$$\Rightarrow S_2 = \frac{Q}{\sin \alpha}$$




$$\Rightarrow S_2 = \frac{178}{\sin 12°31'}$$

$\Rightarrow S_2 = 821.32$ N **(Ans.)**

FBD at 'A'



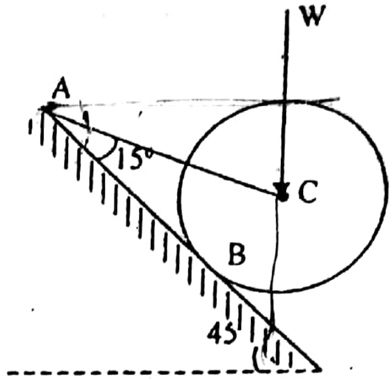

Resolving horizontally in to zero

$\Sigma x = 0$

$S_1 = S_2 \cos \alpha$

$= 821.32 \cos 12° \, 31' = 801.8$ N **(Ans.)**

5. *A roller of weight $W = 4450$ N rests on a smooth inclined plane and is kept from rolling down by a string as shown in fig. Using the method of projections, find the tension S in the string and the reaction $R_b$ at the point of contact B.*




**Soln.** Given data

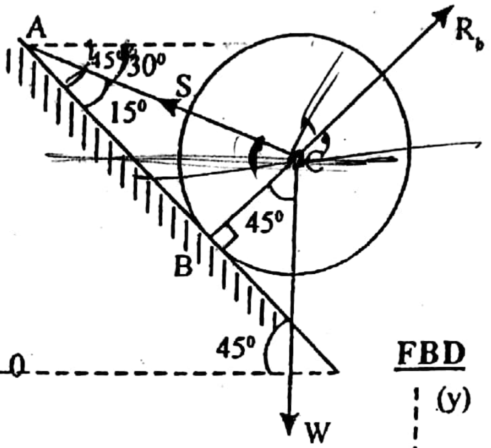
$W = 4450$ KN

$R_b = ?$ $\quad$ $S = ?$

The normal reaction $R_b$ makes 45° to the vertical (y-axis).

The tensile force 's' makes 30° to the horizontal.

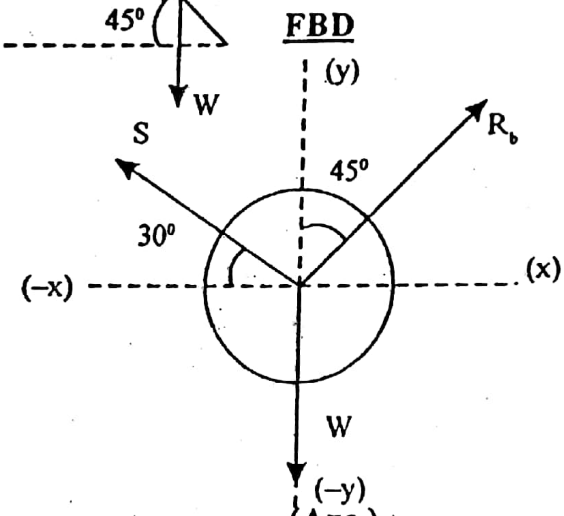
The FBD at 'C' is shown in the figure.




Resolving horizontally in to zero, $\Sigma x = 0$

$R_b \sin 45° = S \cos 30°$

$$R_b = \frac{S \cos 30°}{\sin 45°}$$




$\Rightarrow R_b = 1.22\,S$ ......(i)

Resolving vertically in to zero, $\Sigma y = 0$

$R_b \cos 45° + S \sin 30° = W$

$\Rightarrow 1.22S \times \cos 45° + S \sin 30° = 4450$

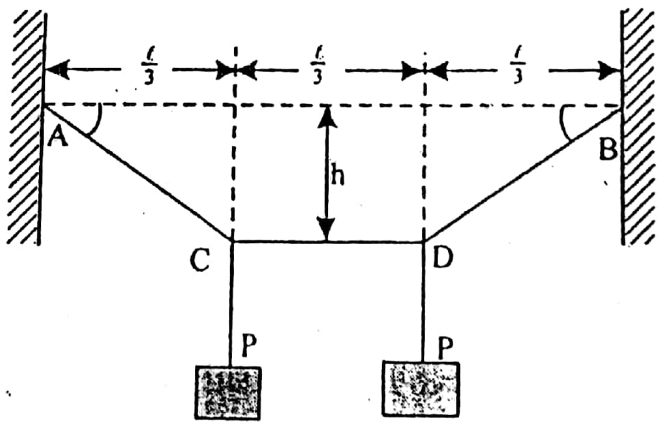
$$\Rightarrow S(0.86 + 0.5) = 4450 \Rightarrow S = \frac{4450}{1.36} = 3272 \text{ N}$$

**(Ans.)**

Substituting the value in equation (1)

$R_b = 1.22S = 1.22 \times 3272 = 3991.84$ N **(Ans.)**

6. *Two loads of equal magnitude P, are supported by a flexible string ACDB, as shown in fig. Determine the tensile forces $S_1$ and $S_2$ in the portions AC and CD, respectively, of the string, if the span $l = 9.144$ m and the sag $h = 1.524$ m. Neglect the weight of the string.*




**Soln.** Given data

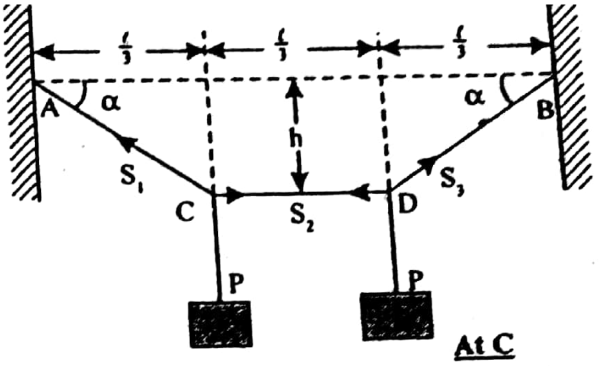
$\ell = 9.144$ m

$h = 1.524$ m

$S_1 = ?$ $S_2 = ?$

The FBD at 'C' is shown in the fig.

Let the line of action of $S_1$ makes an angle $\alpha$ with the horizontal.

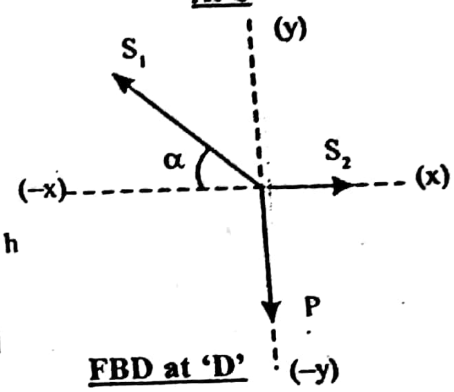



Now; $\alpha = \tan^{-1}(h/\ell/3)$

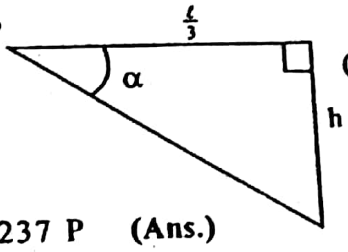
$$\therefore \alpha = \tan^{-1}\left(\frac{3h}{\ell}\right)$$

$$= \tan^{-1}\left(\frac{3 \times 1.524}{9.144}\right) = 26° \, 33'$$

At C




Resolving vertically into zero at 'C'




$\Sigma y = 0$

$S_1 \sin \alpha = P$

$$\Rightarrow S_1 = \frac{P}{\sin 26° \, 33'} \Rightarrow S_1 = 2.237 \text{ P} \quad \textbf{(Ans.)}$$

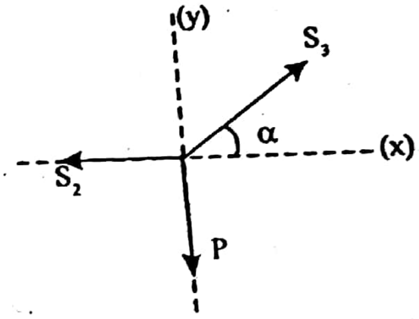
Resolving horizontally into zero at 'C'

$\Sigma x = 0$

$S_2 = S_1 \cos \alpha$

$= 2.237 \text{ P} \times \cos 26° \, 33' = 2.001 \text{ P}$ **(Ans.)**

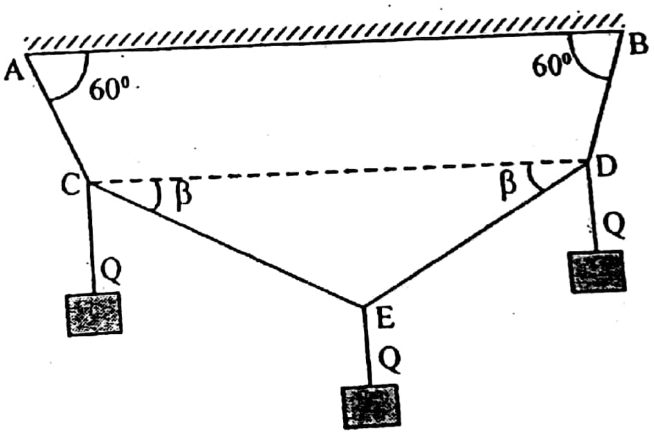
FBD at 'D'




The FBD at D is shown in the figure

Due to symmetry $S_3 = S_1 = 2.237$ P.N. **(Ans.)**

**7.** *On the string ACEDB are hung three equal weights Q symmetrically placed with respect to the vertical line through the mid point E as shown in fig. Determine the value of the angles 'β' if the other angles are as shown in the figure.*




**Soln.** Let the axial forces $S_1$, $S_2$, $S_3$ & $S_4$ are acting along the string AC, CE, ED & DB.

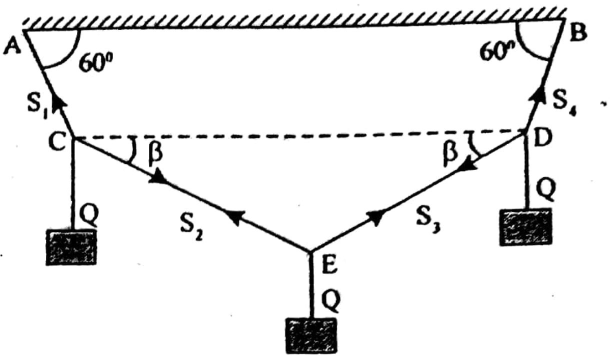
The FBD at 'E' is shown in the figure.

Resolving horizontally in to zero

$\Sigma x = 0$

$S_2 \cos\beta = S_3 \cos\beta$

or $S_2 = S_3$ .....(i)




Resolving vertically into zero $\Sigma y = 0$

$S_2 \sin\beta + S_3 \sin\beta = Q$ or $2S_2 \sin\beta = Q$

or $S_2 = S_3 = \dfrac{Q}{2\sin\beta}$ ......(ii)

The FBD at 'C' is shown in the figure

Resolving horizontally into zero at C $\Sigma x = 0$

$S_1 \cos 60° = S_2 \cos\beta$ or $S_1 \times \dfrac{1}{2} = \dfrac{Q}{2\sin\beta} \times \cos\beta$

$\Rightarrow S_1 = Q\cot\beta$ (iii)

Resolving vertically in to zero $\Sigma y = 0$

$S_1 \sin 60° = Q + S_2 \sin\beta$ (iv)

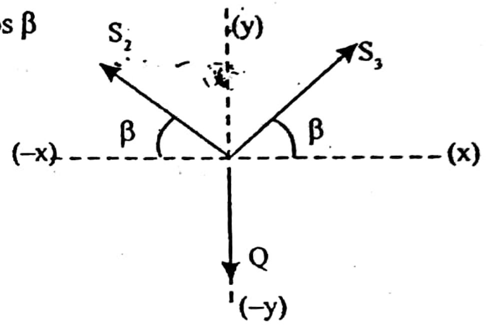
Substituting the values of $S_1$ & $S_2$ in equation (iv);

$$Q\cot\beta \sin 60° = Q + \frac{Q}{2\sin\beta} \times \sin\beta$$

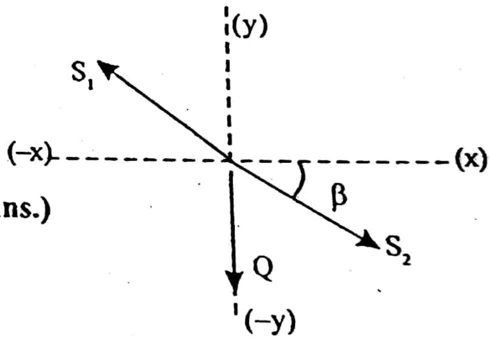
$$\Rightarrow \frac{\cos\beta}{\sin\beta} \times \frac{\sqrt{3}}{2} = \frac{3Q}{2} \text{ or } \tan\beta = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \tan\beta = \frac{1}{\sqrt{3}} \Rightarrow \beta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \beta = 30° \quad \textbf{(Ans.)}$$

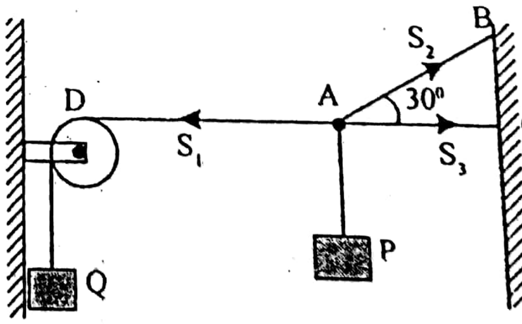
**FBD at E**




**FBD at C**




**8.** *In the given Fig. weights P and Q are suspended in a vertical plane by strings 1, 2, 3, arranged as shown. Find the tension induced in each string if P = 2225 N and Q = 4450N.*




**Soln.** Given data

P = 2225 N

Q = 4450 N

$S_1$ = ?

$S_2$ = ?

$S_3$ = ?

Since the load Q is hanged freely at the end of string '1'

$S_1 = Q = 4450$ N (Ans.)

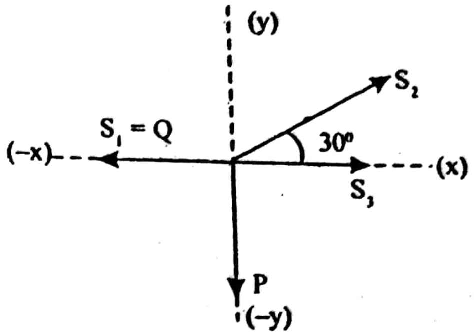
The FBD at A is shown.

Resolving vertically in to zero

$\Sigma y = 0$

$S_2 \sin 30° = P$

$\Rightarrow S_2 = \dfrac{225}{\sin 30°}$ or $S_2 = 4450$ N **(Ans.)**



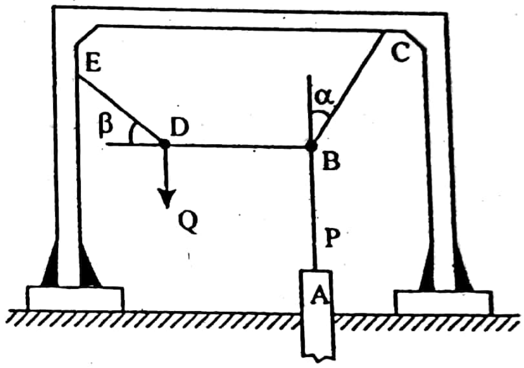

Resolving horizontally into zero

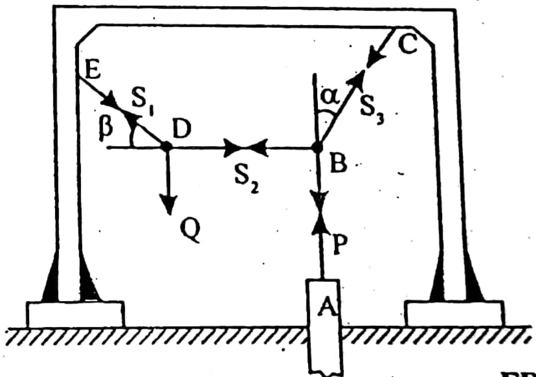
$\Sigma x = 0$

$S_3 + S_2 \cos 30° = S_1$ or $S_3 = S_1 - S_2 \cos 30°$

$\Rightarrow S_3 = 4450 - 4450$ or $S_3 = 596.18$ N **(Ans.)**

9. *To pull up a post, the arrangement shown in fig. is used. A cable ABC is fixed to the post at A and to the frame at C having the portion AB vertical and the portion BC inclined thereto by a small angle $\alpha$. The cable BDE fastened to the ring at B to the frame at E has the portion BD horizontal and the portion DE inclined to the horizontal by the small angle $\beta$. On the ring at D a man pulls vertically downward with his entire weight Q. Determine the vertical pull P applied to the post at A if $\alpha = \beta = 0.1$ radian and Q = 667.5 N.*




**Soln.** Given data

$\alpha = \beta = 0.1 \text{ rad.} = \dfrac{0.1 \times 180°}{\pi} = 5° 43'$

Q = 667° 5N

Let $S_1$, $S_2$ and $S_3$ are tensile forces in the cables ED, DB & BC.

The FBD at "D is shown in the figure.

Rosolving vertically into zero $\Sigma y = 0$

$S_1 \sin \beta = Q \Rightarrow S_1 = \dfrac{667.5}{\sin 5° 43'}$

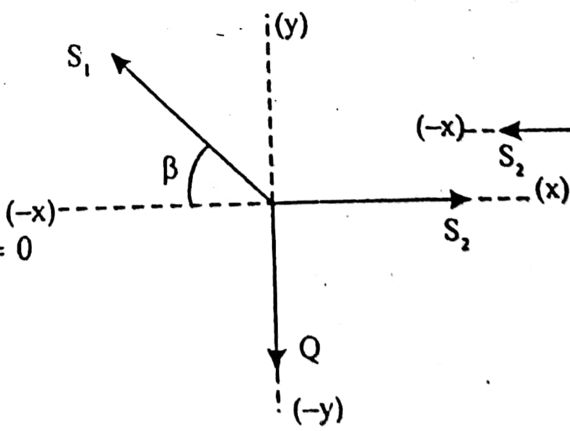
or $S_1 = 6701.18$ N

Resolving horizontally into zero $\Sigma x = 0$

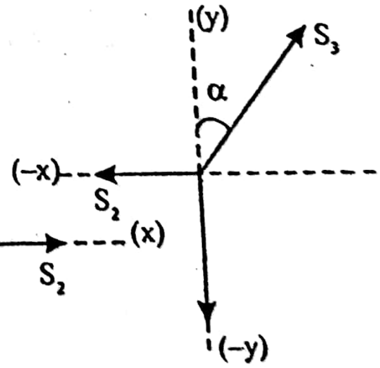
$S_2 = S_1 \cos \beta = 6701.18 \times \cos 5° 43'$

$= 6667.85$ N

**FBD at D**




**FBD at 'B'**




The FBD at 'B' is shown in the figure.

Resolving horizontally into zero at B.

$S_3 \sin\alpha = S_2$

$$\Rightarrow S_3 = \frac{6667.85}{\sin 5°43'} \Rightarrow S_3 = 66940.13\ N$$

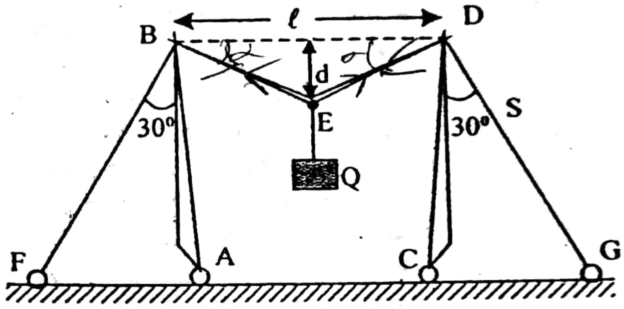
Resolving vertically into zero at B

$\Sigma y = 0$

$S_3 \cos\alpha = P$

$\Rightarrow P = 66940.13 \times \cos 5°43' \Rightarrow P = 66607.21\ N$ **(Ans.)**

10. *Two vertical masts AB and CD are guyed by the wires BF and DG, in the same vertical plane and connected by a cable BD of length l, from the middle point E of which is suspended a load Q as shown in fig. . Find the tensile force S in each of the two guy wires BF and DG if the load Q = 445 N, the length l = 6.1m, and the sag d = 0.305 m.*




**Soln.** Given data

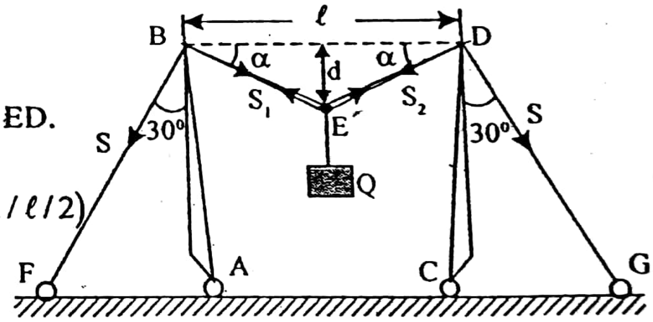
$Q = 445\ N$

$\ell = 6.1\ m$

$d = 0.305\ m$

Let $S_1$ & $S_2$ are the tension in the cable BE & ED.

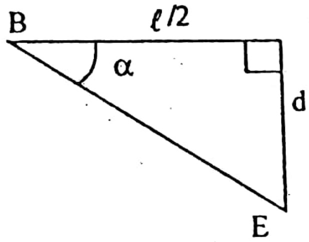
S is the tensile force in the wire BF and DG




From the geometry of the figure $\alpha = \tan^{-1}(d/\ell/2)$

or $\alpha = \tan^{-1}\left(\frac{2d}{\ell}\right) \Rightarrow \alpha = 5°\ 42'$

The FBD at E is shown in the fig..




Resolving horizontally into zero at E

$\Sigma x = 0$

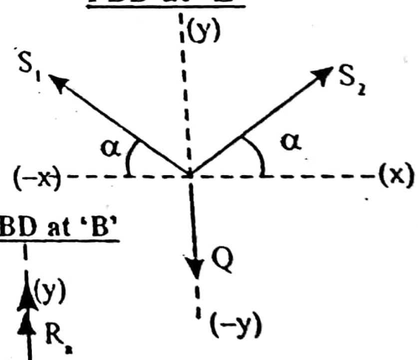
$S_1 \cos\alpha = S_2 \cos\alpha \Rightarrow S_1 = S_2$

Resolving vertically into zero

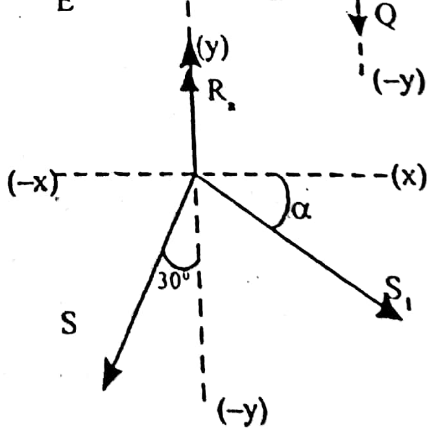
$S_1 \sin\alpha + S_2 \sin\alpha = Q$ or $2S_1 \sin\alpha = Q$

$$\therefore S_1 = S_2 = \frac{Q}{2\sin\alpha} = \frac{445}{2 \times \sin 5°\ 42'} = 2240.23\ N$$

FBD at 'E'




FBD at 'B'




The FBD at 'B' is shown in the fig.

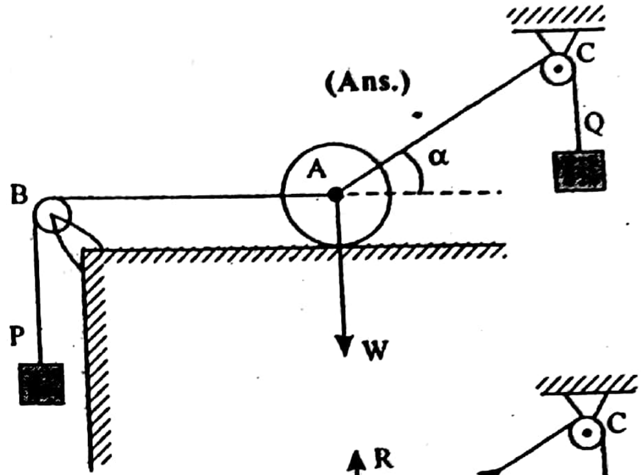
Resolving horizontally into zero

$\Sigma x = 0$

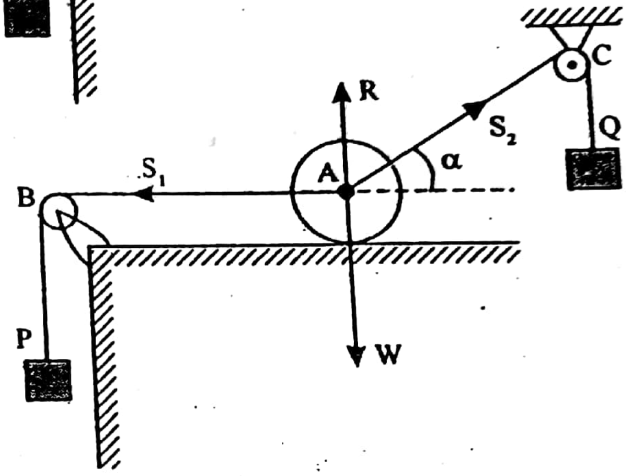
$S_1 \cos\alpha = S \sin 30°$

$\Rightarrow S = 2240.23 \times \cos 5°42' \times 2 = 4458.3\text{N}$ **(Ans.)**

**11.** *A ball of weight W rests upon a smooth horizontal plane and has attached to its centre by two strings AB and AC which pass over frictionless pulleys at B and C and carry loads P and Q, respectively, as shown in fig. If the string AB is horizontal, find the angle α that the string AC makes with the horizontal when the ball is in a position of equilibrium. Also find the pressure R between the ball and the plane.*




**Soln.** Let $S_1$, $S_2$ are the tension in the strings AB and AC act away from A.
R is the normal reaction at A.
The FBD at A is shown in the fig.



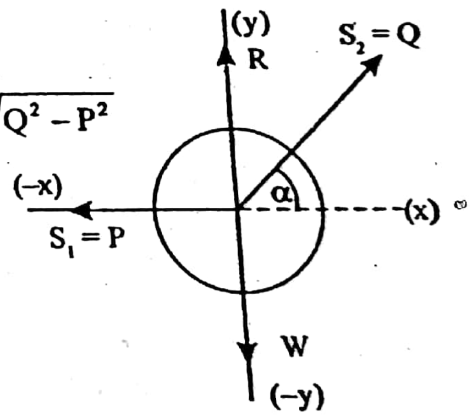

Resolving horizontally into zero $\sum x = 0$

$S_2 \cos\alpha = S_1$

or $Q\cos\alpha = P$ ....(i)

or $\alpha = \cos^{-1}\left(\frac{P}{Q}\right)$ **(Ans.)**

**FBD at A**




From equation (i) $\sin\alpha = \sqrt{(1-\cos^2\alpha)} = \sqrt{1-\left(\frac{P^2}{Q^2}\right)} = \frac{1}{Q}\sqrt{Q^2 - P^2}$

Resolving vertically into zero, $\Sigma y = 0$

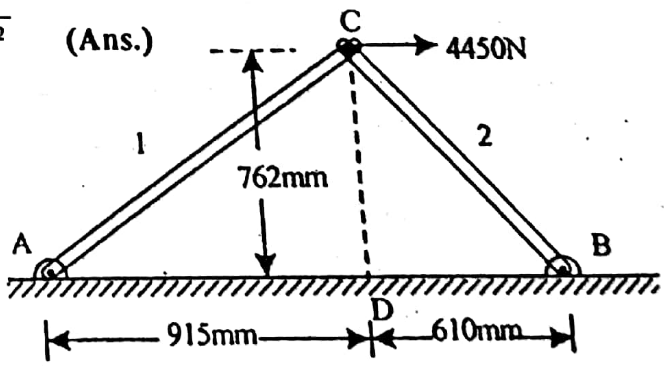
$S_2 \sin\alpha + R = W$

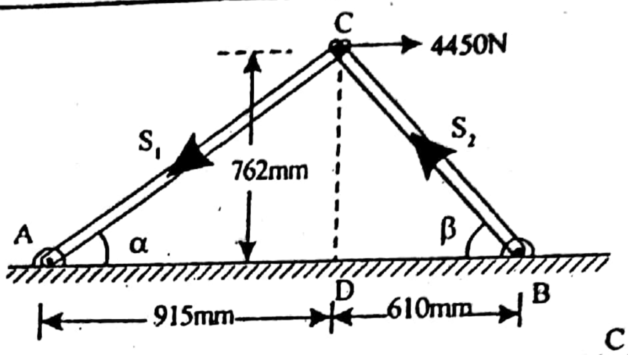
$\Rightarrow R = W - Q\sin\alpha$ .....(ii)

Substituting the value of sin α in equation (ii).

$R = W - \frac{Q}{Q}\sqrt{Q^2 - P^2}$ or $R = W - \sqrt{Q^2 - P^2}$ **(Ans.)**

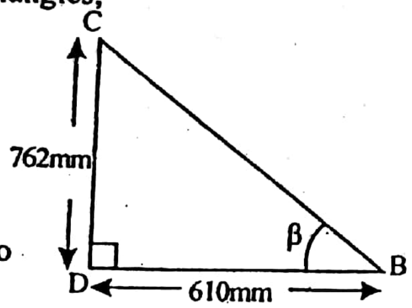
**12.** *Determine the axial forces $S_1$ and $S_2$ induced in the bars AC and BC as shown in fig. due to the action of the horizontal applied load at C. The bars are hinged together at C and to the foundation at A and B.*

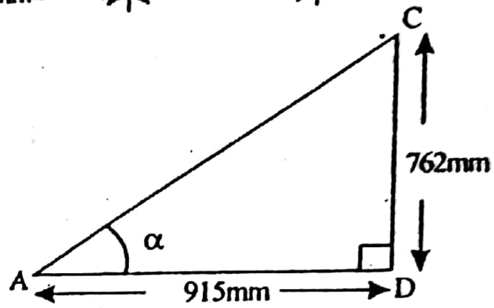





**Soln.** Since the pull 4450N is acting at A, the bar AC will be under extension, hence $S_1$ is the tensile force on it. The bar CB is under contraction, hence compressive force acts towards C on it. The FBD at 'C' is shown in the figure.

From the geometry of the triangles;



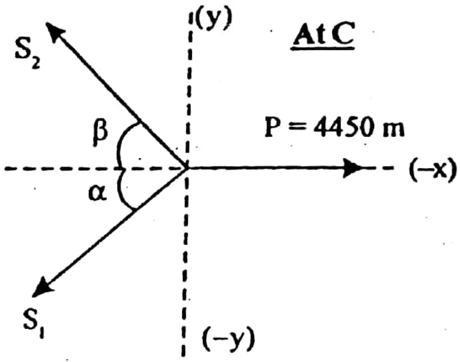




$$\alpha = \tan^{-1}\left(\frac{762}{915}\right) = 39°47'$$

$$\beta = \tan^{-1}\left(\frac{762}{610}\right) = 51°\ 19'$$

Resolving vertically in to zero

$\Sigma y = 0$




$$S_2 \sin\beta = S_1 \sin\alpha$$

$$\Rightarrow S_2 = \frac{S_1 \sin\alpha}{\sin\beta}$$

$$\Rightarrow S_2 = S_1\left(\frac{\sin 39°\ 47'}{51°\ 19'}\right) \text{ or } S_2 = 0.819\ S_1$$

Resolving horizontally in to zero $\Sigma x = 0$

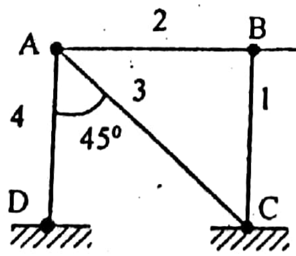
$$S_2 \cos\beta + S_1 \cos\alpha = 4450 \text{ KN}$$

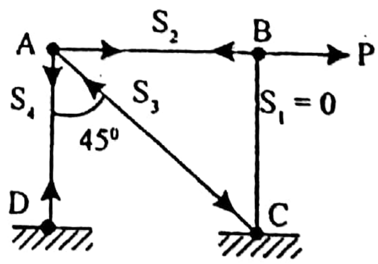
$$\text{or } 0.819\ S_1 \times \cos 51°\ 19' + S_1 \cos 39°47' = 4450$$

$$\Rightarrow S_1 = \frac{4450}{(0.819 \times \cos 51°19' + \cos 39°47')} = 3475 \text{ N (tension) (Ans.)}$$

Substituting the value of 'S', $S_2 = 0.819 \times 3475 = 2847$ N (compression) **(Ans.)**

. *Determine the forces produced in the bars of the system shown in fig. owing to the horizontal force P applied at the hinge B.*




**ln.** We have four concurrent points A, B, C & D. The no. of unknown forces at B are less. Hence the FBD at B is drawn first as shown in fig.

Resolving vertically in to zero

$\Sigma y = 0$

$S_1 = 0$

Resolving horizontally

$\Sigma x = 0$

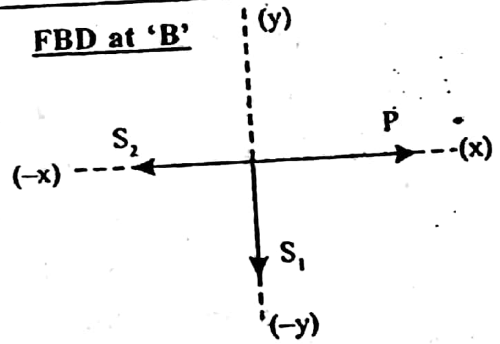
$S_2 = P$ (Tension)

The FBD at A is shown in fig.

Resolving horizontally

$\Sigma x = 0$

FBD at 'B'

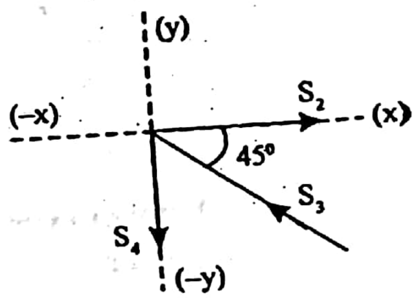



$S_3 \cos 45° = S_2$ or $S_3 = \dfrac{P}{\cos 45} = \sqrt{2}\,P$ (comp.)

Resolving vertically into zero $\Sigma y = 0$

$S_4 = S_3 \sin 45° \Rightarrow S_4 = \sqrt{2}\,P \times \dfrac{1}{\sqrt{2}} \Rightarrow S_4 = P$ (Tension)

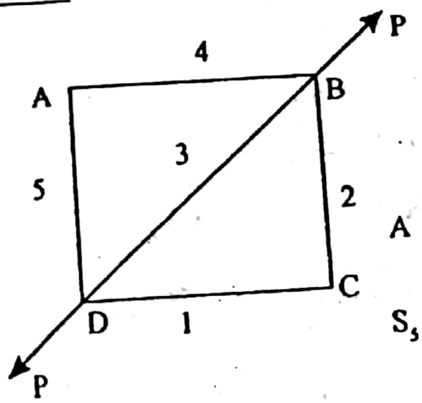
FBD at 'A'

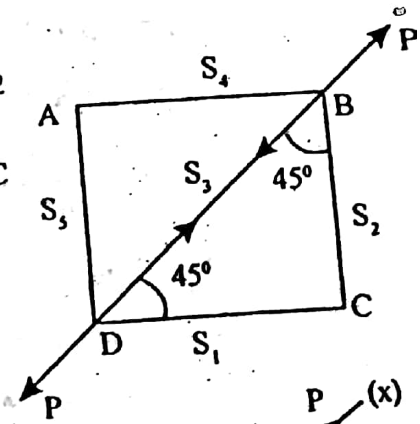



**Answer Table**

| BARS | Axial forces | Magnitude | Nature |
|---|---|---|---|
| BC | $S_1$ | 0 | – |
| AB | $S_2$ | P | T |
| AC | $S_3$ | $\sqrt{2}P$ | C |
| AD | $S_4$ | P | T |

**14.** *A hinged square ABCD as shown in fig. with diagonal BD is submitted to the action of two equal and opposite forces applied as shown. Determine the forces produced in all bars.*




**Soln.** We have four concurrent points ABCD, out of which the no. of unknown forces at A & C are less.

Let us draw the FBD at 'A'

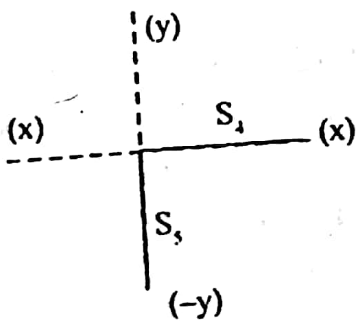
At A, $\Sigma x = 0$

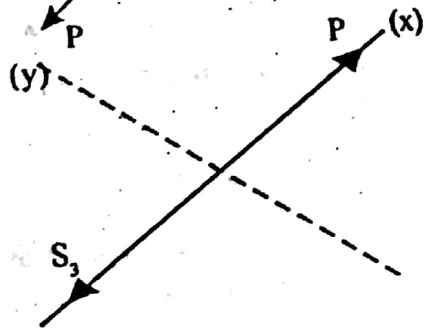
$\therefore S_4 = 0$

$\Sigma y = 0$

$\therefore S_5 = 0$

FBD at 'A'




Due to symmetry at 'C'

$S_1 = S_2 = 0$

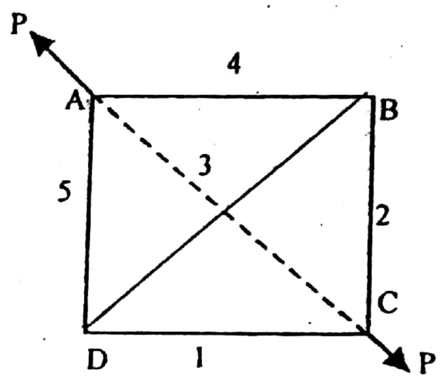
The FBD at B is drawn as shown in the fig.

At B, $\Sigma x = 0 \quad \therefore S_3 = P$ (tens.) (Ans.)

### Answer Table

| BARS | Axial forces | Magnitude | Sense |
|---|---|---|---|
| DC | $S_1$ | 0 | – |
| BC | $S_2$ | 0 | – |
| BD | $S_3$ | P | tens |
| AB | $S_4$ | 0 | – |
| AD | $S_5$ | 0 | – |

**15.** *Determine the forces that will be produced in all bars of the frame ABCD as shown in fig if the external forces are applied in the same manner to the hinges A and C.*




**Soln.** We have four no. of concurrent points A, B, C & D, out of which the no. of unknown forces are less at A & C.

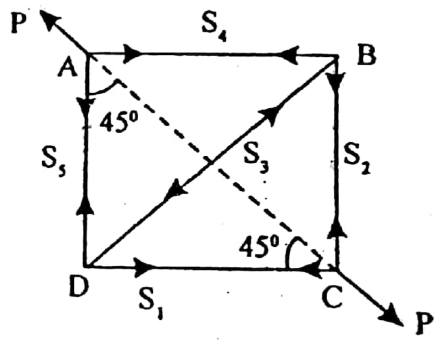
Let us draw FBD at A.

$\Sigma x = 0$

$$S_4 = P\cos 45^\circ \quad \Rightarrow S_4 = \frac{p}{\sqrt{2}} \quad \text{(Tens)}$$

$\Sigma y = 0$

$$S_5 = p\sin 45^\circ \quad \Rightarrow P_5 = \frac{P}{\sqrt{2}} \quad \text{(Tens)}$$



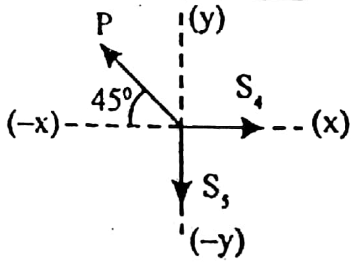

Due to symmetry at 'C'

$$\text{at e} \quad S_1 = S_2 = \frac{P}{\sqrt{2}} \text{ (Tens)}$$

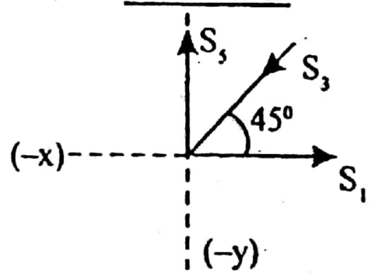
The FBD at D is shown in the figure.

$$S_3\cos 45^\circ = S_1 \quad \Rightarrow S_3 = \frac{P}{\sqrt{2}} \times \sqrt{2} \quad \Rightarrow S_3 = P\,(\text{comp.})$$

**FBD at 'A'**

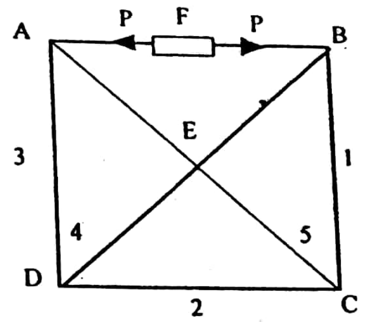



**FBD at 'D**

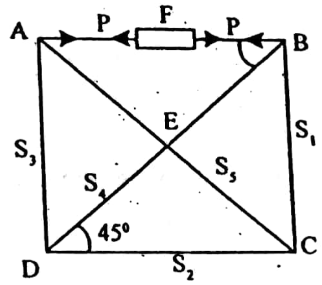



| BARS | Axial forces | Magnitude | Sense |
|---|---|---|---|
| DC | $S_1$ | $\frac{P}{\sqrt{2}}$ | T |
| BC | $S_2$ | $\frac{P}{\sqrt{2}}$ | T |
| BD | $S_3$ | P | C |
| AB | $S_4$ | $\frac{P}{\sqrt{2}}$ | T |
| AD | $S_5$ | $\frac{P}{\sqrt{2}}$ | T |

16. *In the bar of the square frame ABCD as shown in fig a tensile force P is produced by tightening a turnbuckle F. Determine the force produced in the other bars. The diagonals AC and BD pass each other freely at E.*




**Soln.** The no. of concurrent points are 4, out of which the no. of unknown forces are less at A and B.




Let us draw the FBD at A

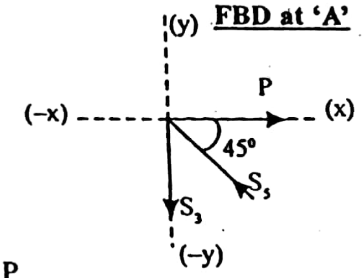
$\Sigma x = 0$

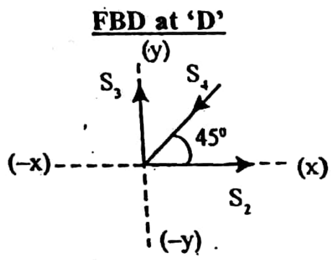
$S_5 \cos 45° = P$ or $S_5 = \sqrt{2}\ P$ (comp.)

$\Sigma y = 0$

$S_3 = S_5 \sin 45° \Rightarrow S_3 = \sqrt{2}\ P \times \dfrac{1}{\sqrt{2}}$

$\Rightarrow S_3 = P(T)$




Due to symmetry at 'B'

$S_4 = S_5 = \sqrt{2}\ P$(comp.)

$S_1 = S_3 = P(T)$.

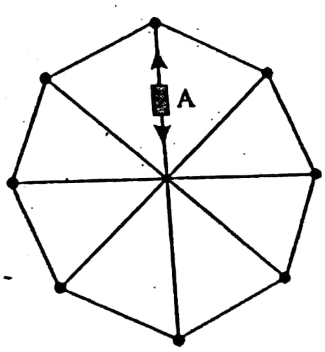
The FBD at 'D' is drawn

$\Sigma x = 0$

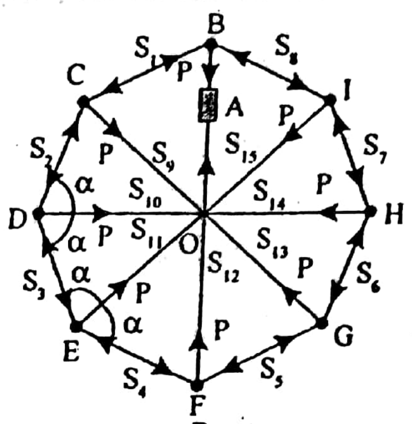
$S_4 \cos 45° = S_2$ or $S_2 = \sqrt{2} \times \dfrac{P}{\sqrt{2}}$ $\Rightarrow S_2 = P(T)$

| BARS | Axial forces | Magnitude | Sense |
|---|---|---|---|
| BC | $S_1$ | P | T |
| CD | $S_2$ | P | T |
| AD | $S_3$ | P | T |
| BD | $S_4$ | P | C |
| AC | $S_5$ | $\sqrt{2}$P | C |

17. *By means of a turnbuckle A a tensile force P is produced in one of the radial bars of the hinged regular octagon shown in fig. Determine the force produced in the other bars of the system.*




Scanned by CamScanner




**Soln.** The FBD at 'B' is 1st drawn, because the no. of unknown forces are less

Since a tensile force P acts at B, the compressive forces $S_1$ and $S_8$ will act on the bar BC and BI towards B, due to contraction.

$$\therefore \alpha = \left(\frac{180° - 45°}{2}\right) = 67.30°$$

Resolving horizontally

$\Sigma x = 0$

$S_1 \sin\alpha = S_8 \sin\alpha$

$\Rightarrow S_1 = S_8$ ........(1)

Resolving vertically

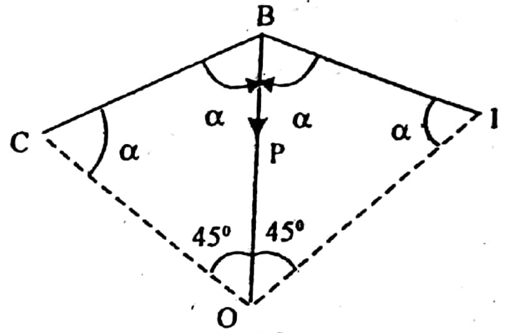
$\Sigma y = 0$

$S_1 \cos\alpha + S_8 \cos\alpha = P$

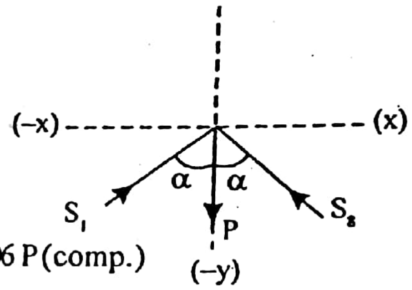
or $2S_1 \cos\alpha = P$

or $S_1 = \dfrac{P}{2\cos\alpha} = \dfrac{P}{2\cos 67°30'}$

or $S_1 = 1.306\ P$ (Comp)



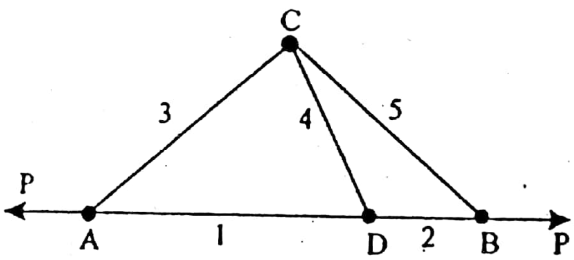

**At B**




Due to symmetry $S_8 = S_7 = S_6 = S_5 = S_4 = S_3 = S_2 = S_1 = 1.306\ P$ (comp.)

Similarly, $S_9 = S_{10} = S_{11} = S_{12} = S_{13} = S_{14} = S_{15} = P$ (tensile)

**18.** *Determine the axial force induced in each bar of the system shown in fig. due to the action of the applied forces P.*




**Soln.** Let $m\angle CAD = \alpha$

$m\angle CDA = \beta$

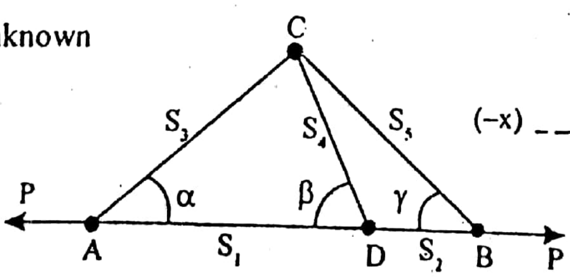
$m\angle CBD = \gamma$

The no. of concurrent points are four out of which the no. of unknown forces are less at 'A' & B.

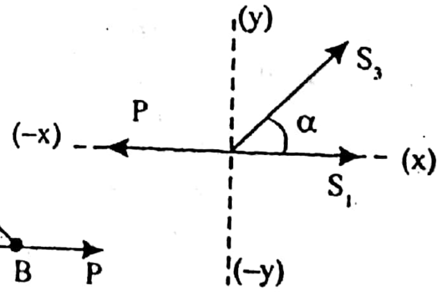
Let us draw FBD at A

$\Sigma y = 0$

$\therefore S_3 \sin\alpha = 0$




**FBD at 'A'**

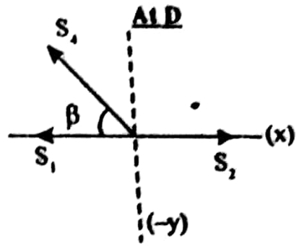


$\therefore S_3 = 0$

$\Sigma x = 0$

$S_1 = P(T)$

The FBD at D is shown in the fig.




$\Sigma y = 0$

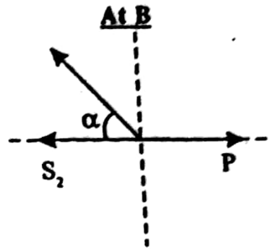
$S_4 \sin \beta = 0$

$\therefore S_4 = 0$

$\Sigma x = 0$

$\therefore S_2 = S_1 = P(T)$

The FBD at B is shown in the fig.



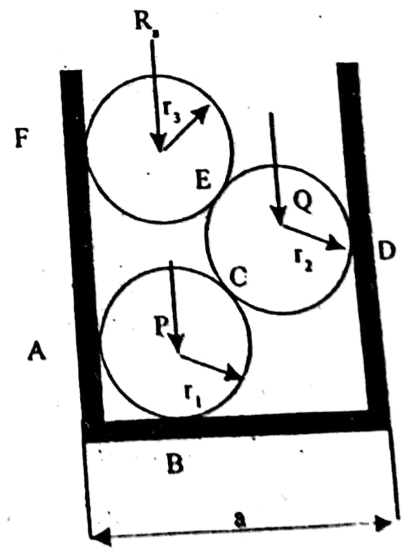

$\Sigma y = 0$

$S_5 \sin \gamma = 0$

$\therefore S_5 = 0$

| BARS | Axaial forces | Magnitude | Sense |
|---|---|---|---|
| AD | $S_1$ | P | T |
| BD | $S_2$ | P | T |
| AC | $S_3$ | O | – |
| CD | $S_4$ | O | – |
| CB | $S_5$ | O | – |

**19.** *The smooth cylinders rest in a horizontal channel having vertical walls, the distance between which is 'a' as shown in fig. Find the pressures exerted on the walls and floor at the points of contact A, B, D and F. The following numerical data are given : P = 200N, Q = 400N, R = 300N, $r_1$ = 120 mm, $r_2$ = 180 mm, $r_3$ = 150 mm and a = 540 mm.*




**Soln.** Given data

P = 200 N

Q = 400 N

R = 300N

$r_1$ = 120 mm

$r_2$ = 180 mm

$r_3$ = 150 mm

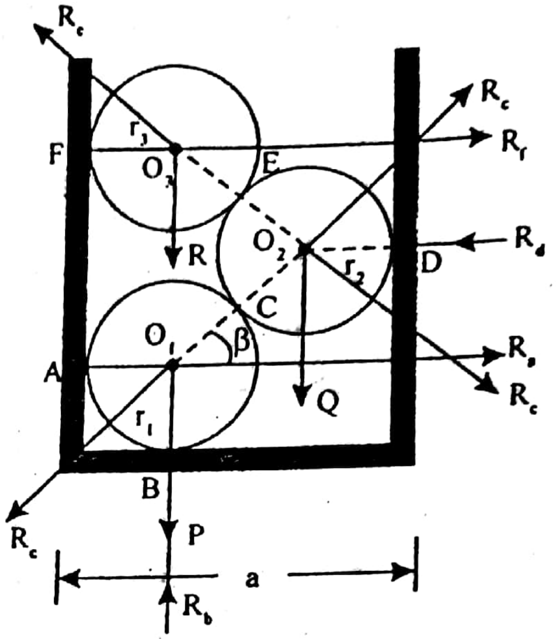
a = 540 mm

$R_a$ = ? $R_b$ = ? $R_d$ = ? $R_f$ = ?

Let $R_c$ & $R_e$ are the equal and opposite reactions towards each other, between the cylinders at C & E.

The FBD at $O_3$ is shown in the figure

From the geometry of the triangle



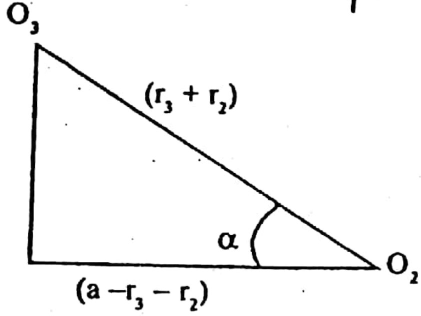

$$\cos\alpha = \frac{(a - r_2 - r_3)}{(r_2 + r_3)} = \frac{(540 - 180 - 150)}{(180 + 150)} = 0.636$$

$$\Rightarrow \alpha = \cos^{-1}(0.636) = 50°28'$$

$$\Sigma y = 0$$

$$R_e \sin\alpha = R$$

$$\Rightarrow R_e = R/\sin\alpha = \frac{300}{\sin 50°28'}$$




$$\Rightarrow R_e = 388.97 \text{ N}$$

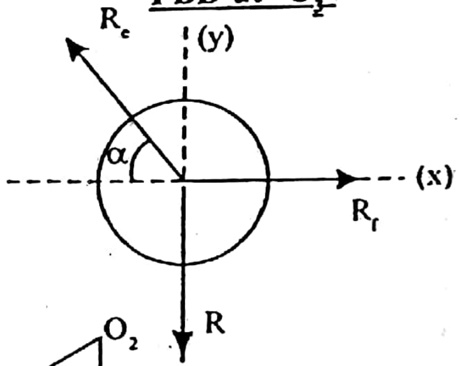
$$\Sigma x = 0$$

$$R_e \cos\alpha = R_f$$

$$\Rightarrow R_f = 388.97 \times \cos 50°\ 28'$$

$$\Rightarrow R_f = 247.58 \text{ N}$$ **(Ans.)**

**FBD at '$O_2$'**

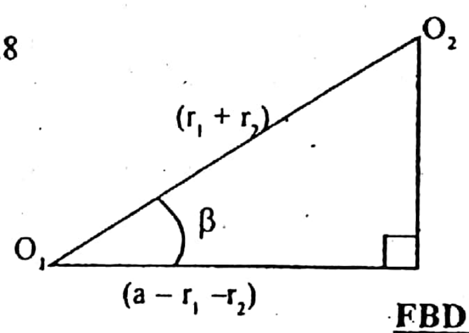



The FBD at $O_2$ is shown in the fig.

From the geometry of the triangle

$$\cos\beta = \left(\frac{a - r_1 - r_2}{r_1 + r_2}\right) = \frac{(540 - 120 - 180)}{(120 + 180)} = 0.8$$

$$\Rightarrow \beta = \cos(0.8) = 36°\ 52'$$

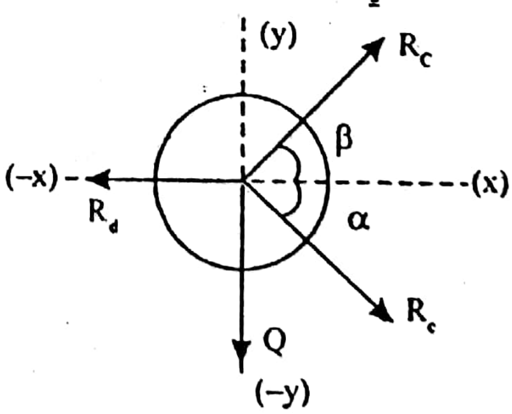



$$\Sigma y = 0$$

$$R_c \sin\beta = Q + R_e \sin\alpha$$

$$\Rightarrow R_c = \frac{(400 + 388.97 \sin 50°\ 28')}{(\sin 36°\ 52')} = 1166.74 \text{ N}$$

**FBD at '$O_2$'**




$$\Sigma x = 0$$

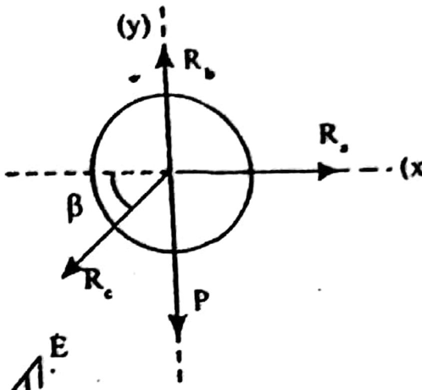
$$R_c \cos\beta + R_e \cos\alpha = R_d$$

or $R_d = (1166.74 \cos 36°52' + 388.97 \times \cos 50°28')$

$$\Rightarrow R_d = 1181.02\text{N}$$ **(Ans.)**

The FBD at $O_1$ is shown in the fig.

FBD at '$O_1$'



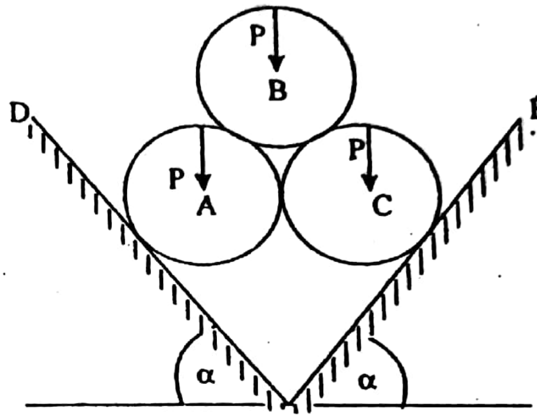

$\Sigma x = 0$

$R_a = R_c \cos\beta = 1166.74 \times \cos 36°52' = 933.43$ N

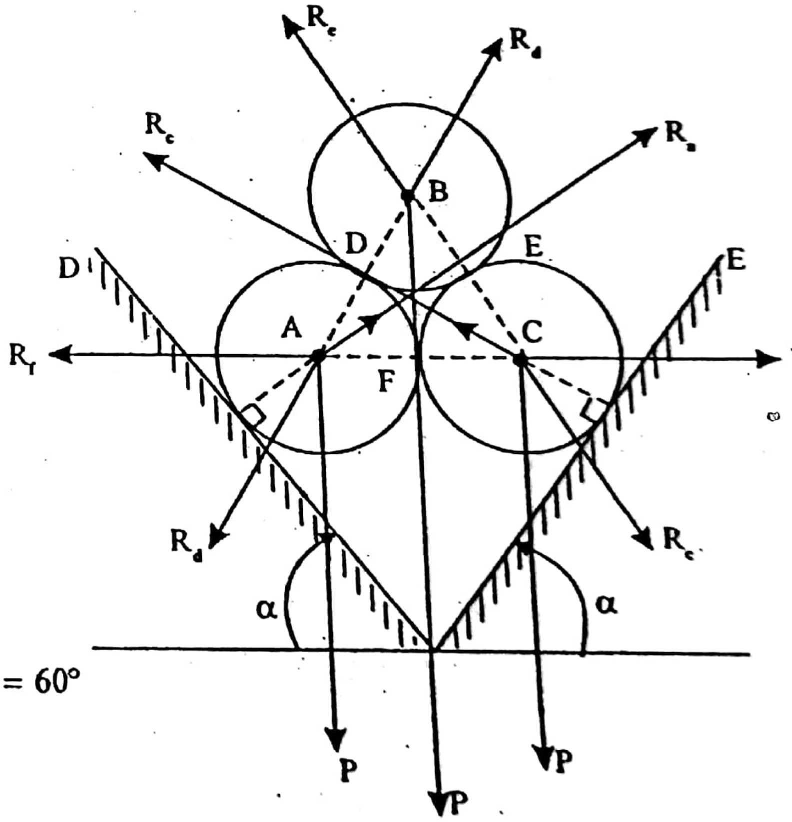
$\Sigma y = 0$

$R_b = P + R_c \sin\beta = 200 + (1166.74 \times \sin 36°52) = 900$ N (Ans.)

**20.** *In the given fig. three smooth right circular cylinders, each of radius r and weight P, are arranged on smooth inclined surfaces as shown. Determine the least value of angle α that will present the arrangement from slipping.*




**Soln.** The reaction $R_d$ acts equal opposite at the point of contact D towards the cylinders A & B. The reaction $R_e$ acts at the contact point E towards each other of cylinders B & C. The reactions $R_a$ & $R_c$ act normal to the inclined plane making an angle α each with vertical.




The FBD at ''B is shown in the fig.

From the geometry of the triangle ABC;

$\beta = m\angle BAC = m\angle BCA = m\angle ABC = 60°$

$\Sigma x = 0$

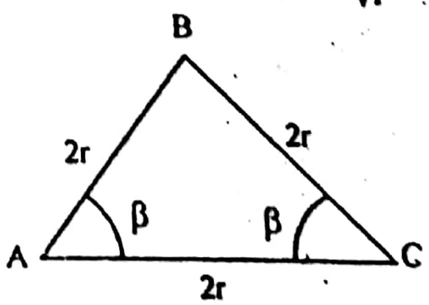
$R_d \cos 60° = R_e \cos 60°$

$\therefore \quad R_d = R_e$ ....(i)

$\Sigma y = 0$

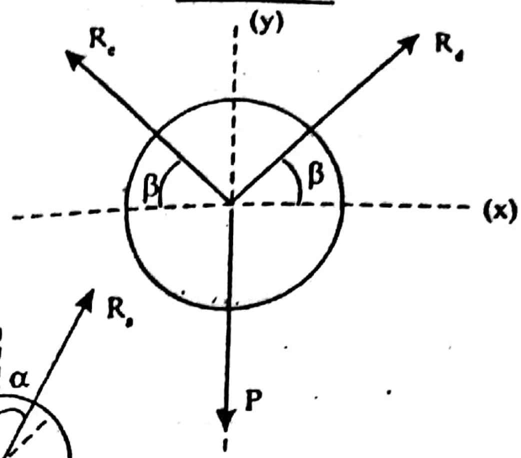
$R_d \sin 60° + R_e \sin 60° = P$

or $\quad 2 R_d \sin 60° = P$



or $\quad R_d = R_e = \dfrac{P}{2\sin 60^\circ} = \dfrac{P}{\sqrt{3}}$ ........(ii)

FBD at 'B'



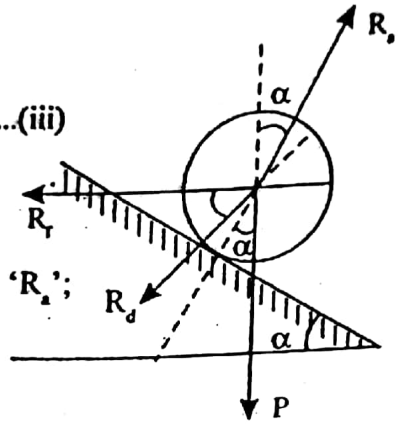

The FBD at 'A' is shown in the fig.

$$\Sigma y = 0$$

$$R_a \cos\alpha = P + R_d \sin 60^\circ$$

$$\therefore\ R_a \cos\alpha = P + \frac{P}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow R_a = 3P/2\cos\alpha \qquad \text{.........(iii)}$$




$$\Sigma x = 0$$

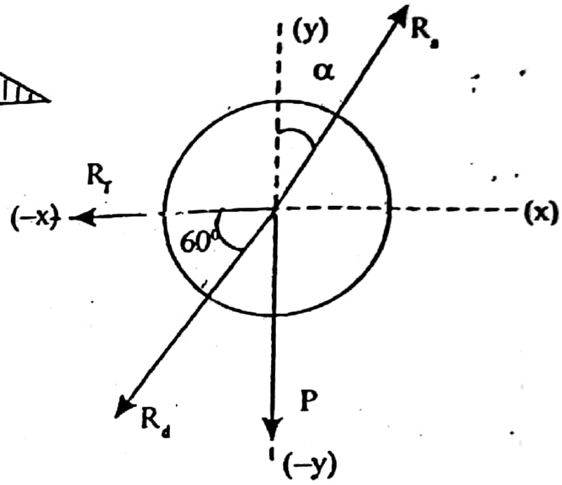
$$R_f + R_d \cos 60^\circ = R_a \sin\alpha$$

Substituting the value of '$R_d$' and '$R_a$';

$$R_f = \frac{3P}{2\cos\alpha} \times \sin\alpha - \frac{P}{\sqrt{3}}\,\frac{1}{2}$$

or $\quad R_f = \dfrac{3}{2} P \tan\alpha - \dfrac{P}{2\sqrt{3}}$ .......(iv)

FBD at A



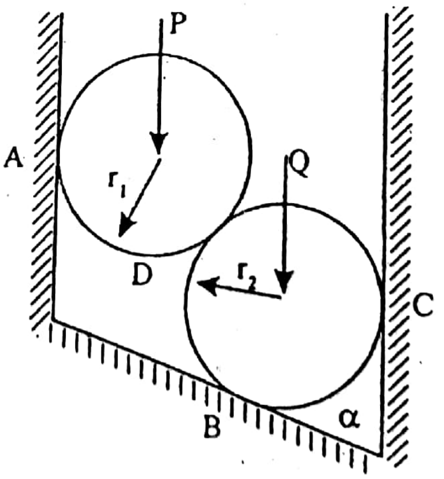

If there will be no slipping between the cylinders A and C, then the normal reaction $R_f$ between themselves is zero. $\quad$ i.e., $R_f = 0$

$\therefore$ The above equation (iv) reduces to ;

$$\frac{3}{2} P \tan\alpha - \frac{P}{2\sqrt{3}} = 0 \quad \text{or} \quad \frac{3}{2} P \tan\alpha = \frac{P}{2\sqrt{3}}$$

$$\Rightarrow \tan\alpha = \frac{1}{3\sqrt{3}} \qquad \Rightarrow \alpha = \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right) \qquad \Rightarrow \alpha = 10^\circ\ 53'$$ **(Ans.)**

21. *Two smooth cylinders of weights P and Q are placed in a smooth trough as shown in fig. Determine the reactions at contact surfaces A, B and C. The following numerical data are given P = 200N and Q = 800N; $r_1$ = 100 mm, $r_2$ = 200mm and a a = 400 mm ; α = 45°.*




**Soln.** Given data

$P = 200$N $\qquad Q = 800$ N

$r_1 = 100$ mm $\qquad r_2 = 200$ mm

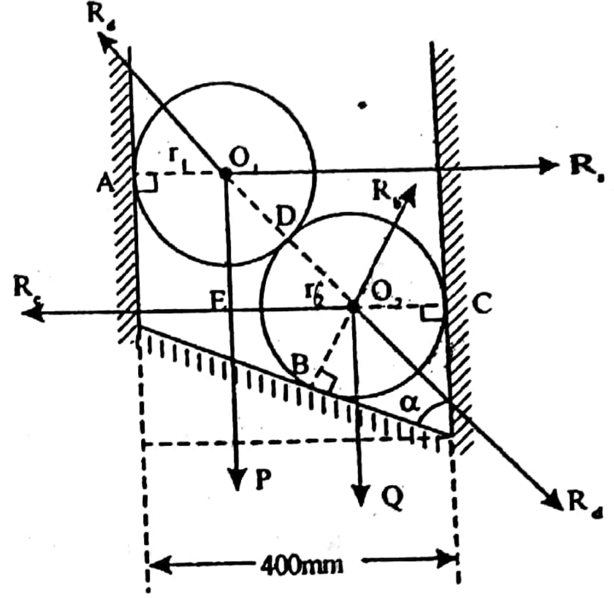
$a = 400$ mm $\qquad \alpha = 45^\circ$

The normal reactions

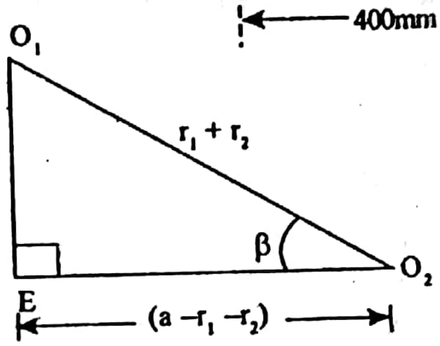
$R_a$, $R_b$ and $R_c$ act at A, B and C.

The reaction $R_d$ acts towards each sphere at common point of contact 'D'.

The FBD at $O_1$ is shown in the figure



From the geometry of the triangle $O_1$, $O_2E$;



$$\beta = \cos^{-1}\left(\frac{a - r_1 - r_2}{r_1 + r_2}\right)$$

$$\Rightarrow \beta = \cos^{-1}\left(\frac{400 - 100 - 200}{100 + 200}\right)$$

$$= \cos^{-1}(0.3) = 70^\circ\ 31'$$

Resolving vertically

$$\Sigma y = 0$$

$$R_d \sin \beta = P$$

$$\Rightarrow R_d = 200/\sin 70^\circ\ 31' = 212.14 \text{ N}$$

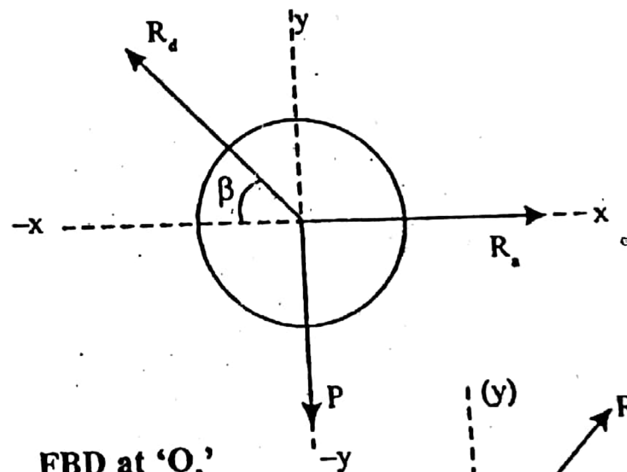
Resolving horizontally $\Sigma x = 0$

$$R_a = R_d \cos \beta$$

$$\Rightarrow R_a = 212.14 \times \cos 70^\circ 31$$

$$\Rightarrow R_a = 70.756 \text{ N} \qquad \textbf{(Ans.)}$$

**FBD at '$O_1$'**



The FBD at $O_2$ is shown in the figure

Resolving vertically

$$\Sigma y = 0$$

$$R_b \sin \alpha = Q + R_d \sin \beta$$

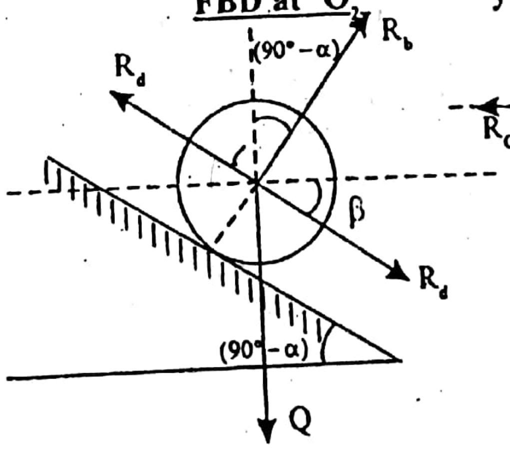
$$\Rightarrow R_b \sin \alpha = (800 + 212.14 \times \sin 70^\circ 31')$$

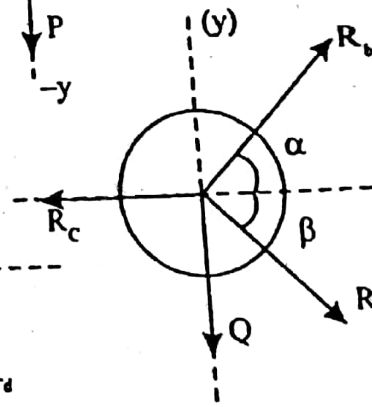
$$\Rightarrow R_b \sin \alpha = 999.86$$

$$\Rightarrow R_b = \frac{999.86}{\sin 45^\circ}$$

$$\Rightarrow R_b = 1414.01 \text{ N} \qquad \textbf{(Ans.)}$$

**FBD at '$O_2$'**



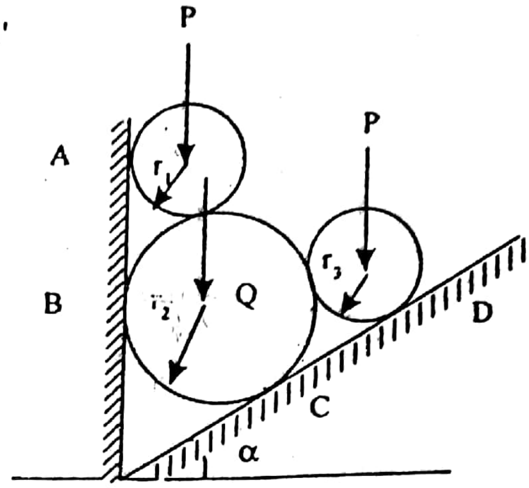


Resolving horizontally

$$R_c = R_b \cos \alpha + R_d \cos \beta$$

$= \quad 1414.01 \times \cos 45° + 212.14 \cos 70°31'$

$= \quad 1070.61$ N (Ans.)




22. *Three smooth spheres of weights P, P and Q are placed in a smooth trench as shown in fig. Find the pressure exerted on the walls and floor at the points of contact A, B, C and D. The following numerical data are given; P = 0.3 kN, Q = 0.6 kN and R = 0.3 kN; $r_1 = 0.4$ m, $r_2 = 0.6$ m and $r_3 = 0.4$m; $\alpha = 30°$.*

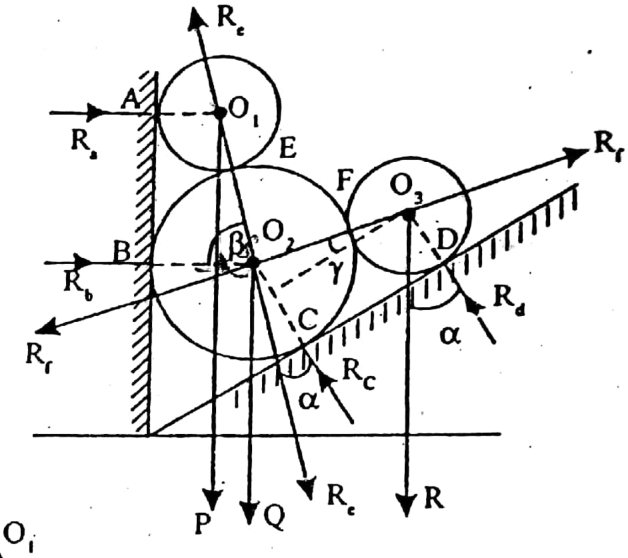
**Soln.** Given data

$P = 0.3$ KN $\quad Q = 0.6$ KN

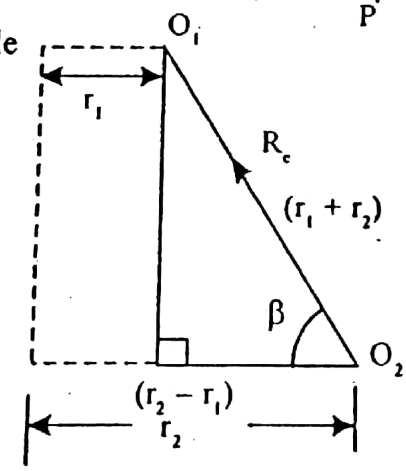
$R = 0.3$ KN $\quad r_1 = 0.4$ m

$r_2 = 0.6$ m $\quad r_3 = 0.4$ m

$\alpha = 30°$




The normal reacations $R_a$, $R_b$, $R_c$ & $R_d$ act at point A, B, C & D normal to their planes. The reaction $R_e$ acts at common point 'E' inbetween two cylinders $O_1$ & $O_2$, making an angle $\beta$ to the horizontal.

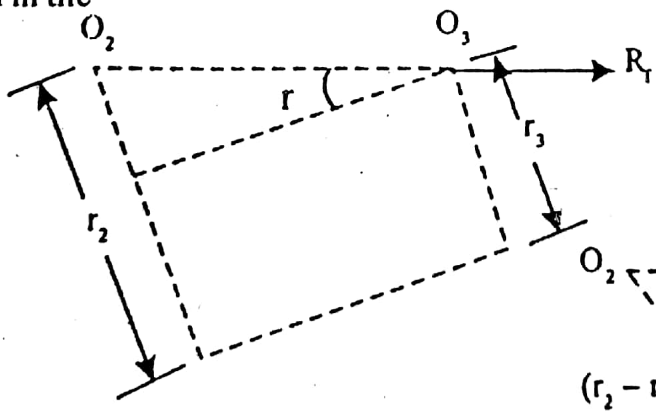


The reaction $R_f$ acts at the common point of contact F in between two cylinders $O_2$ & $O_3$ making an angle. 'γ' with the inclined plane.

$\cos B = r_2 -$

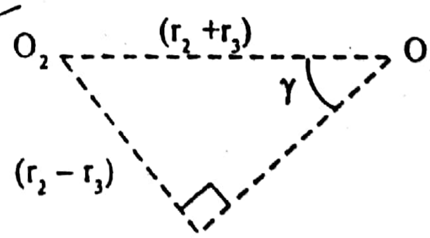
$B = 78°28$

The FBD at '$O_1$' is shown in the figure.



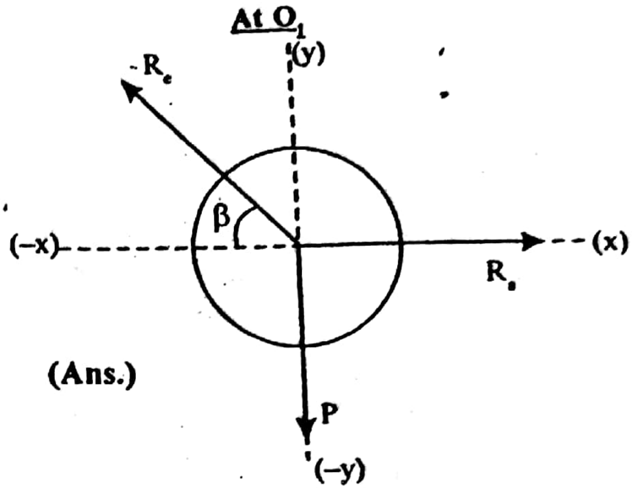
From the trigonmetry

$$\cos\beta = \left(\frac{r_2 - r_1}{r_1 + r_2}\right)$$



$$\Rightarrow \beta = \cos^{-1}\left(\frac{0.6 - 0.4}{0.4 + 0.6}\right)$$

$\Rightarrow \beta = 78° 27'$




Resolving vertically $\Sigma y = 0$

$$R_e \sin\beta = P$$

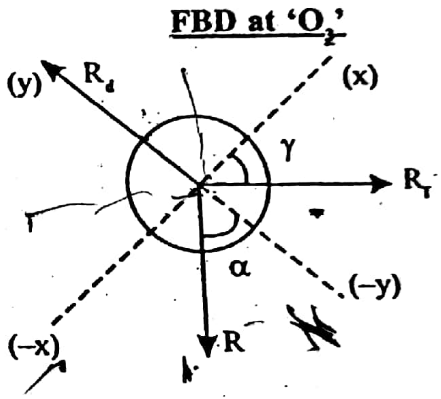
$\Rightarrow R_e = (0.3/\sin 78° 27') \Rightarrow R_e = 0.306$ K.N.

Resolving horizontally $\Sigma x = 0$

$R_a = R_e \cos\beta$

$= 0.306 \times \cos 78° 27' = 0.061$ KN (Ans.)

The FBD at $O_3$, is shown in the fig. From the trigonmetry between the cylinders $O_2$ & $O_3$




$$\sin\gamma = \left(\frac{r_2 - r_3}{r_2 + r_3}\right) \Rightarrow \gamma = \sin^{-1}\left(\frac{0.6 - 0.4}{0.6 + 0.4}\right)$$

$\Rightarrow \gamma = 11° 32'$

Resolving horizontally $\Sigma x = 0$

$R_f \cos\gamma = R \sin\alpha$

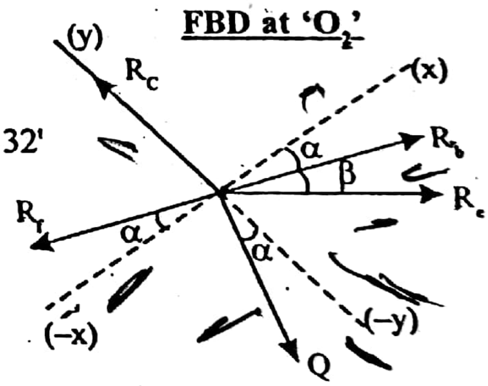
or $\quad R_f = \dfrac{0.3 \sin 30°}{\cos 11° 32'} = 0.153$ KN

Resolving vertically

$$\Sigma y = 0$$

$R_d = R\cos\alpha + R_f \sin\gamma = 0.3\cos 30° + 0.153 \sin 11°32'$

$= 0.291$ KN (Ans.)




FBD at $O_2$ is shown in the fig. resolving horizontally

$\Sigma x = 0$

$R_b \cos\alpha + R_e \cos(\alpha+\beta) = R_f \cos\gamma + Q\sin\alpha$

$\Rightarrow R_b \cos\alpha = 0.153 \cos 11°32' + 0.6 \sin 30° – 0.306 \cos(30°+78°27') \Rightarrow R_b \cos\alpha = 0.546$

$\Rightarrow R_b = \dfrac{0.529}{\cos 30°} \Rightarrow R_b = 0.632$ KN (Ans.)

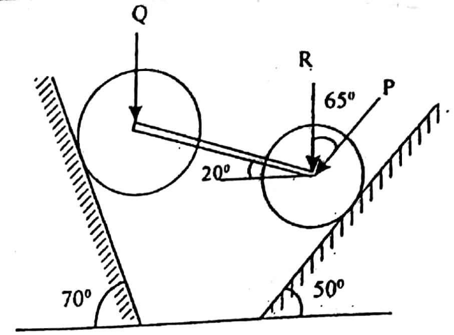
Resolving vertically $\qquad \Sigma y = 0$

$$R_c = Q\cos\alpha + R_e \sin(\alpha+\beta) + R_b \sin\alpha - R_f \sin\gamma$$

$$= 0.6\cos 30° + 0.306 \sin(30 + 78°27') + 0.631 \sin 30° – 0.153 \sin 11°32'$$

$$R_c = 0.519 + 0.290 + 0.316 - 0.030 = 1.095 \text{ KN}$$

23. *Two cylinders of weights Q and R are interconnected by a bar of negligible weight hinged to each cylinder at its geometric centre by ideal pins. Determine the magnitude of P applied at the centre of cylinder R to keep the cylinders in equilibrium in the position shown in fig. The following numerical data are given : Q = 2000N and R = 1000N*




**Soln.** Given data

$Q = 2000$ N

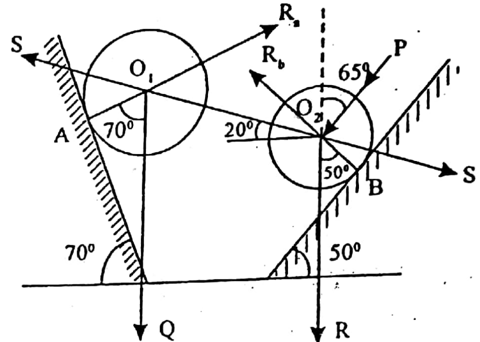
$R = 1000$ N

$P = ?$

The reaction $R_a$ & $R_b$ act at A & B normal to the inclined planes.

An axial force 'S' as the compression act along the bar towards each cylinder.

The FBD at $O_1$ is shown in the figure.

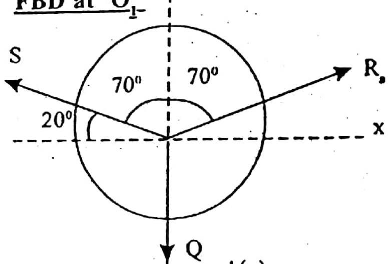



Resolving horizontally, $\Sigma x = 0$

$R_a \sin 70° = S \cos 20° \quad \Rightarrow R_a = \dfrac{S \cos 20°}{\sin 70°}$

$\Rightarrow R_a = S$ ......(i)

**FBD at '$O_1$'**



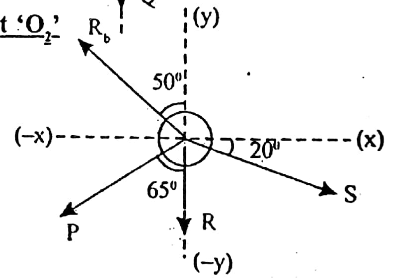

Resolving vertically, $\Sigma y = 0$

$R_a \cos 70° + S \cos 70° = Q$

$\Rightarrow 2R_a \cos 70° = 2000$

$\Rightarrow R_a = S = \dfrac{2000}{2 \cos 70°} = 2923.8$ N

**FBD at '$O_2$'**




The FBD at $O_2$ is shown in the figure

Resolving vertically $\quad \Sigma y = 0$

$R_b \cos 50° = R + S \sin 20° + P \cos 65°$

$\Rightarrow R_b \cos 50° = 1000 + 2923.8 \sin 20° + P \cos 65°$

$\Rightarrow R_b = 3111.44 + 0.657$ P ....(ii)

Resolving horizontally $\Sigma x = 0$

$R_b \sin 50° + P \sin 65° = S \cos 20°$

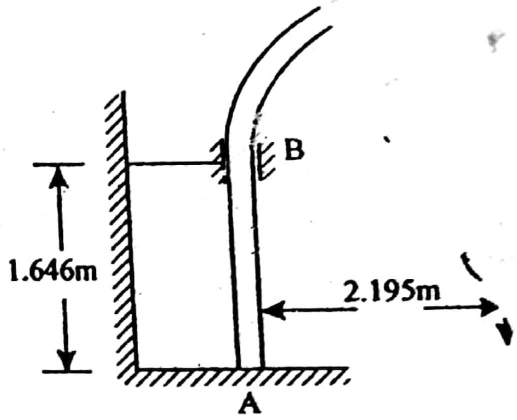
$\Rightarrow \sin 50° (3111.44 + 0.657 P) + P \sin 65° = 2923.8 \cos 20°$

$\Rightarrow 2383.5 + 0.5033 P + 0.906 P = 2747.473$

$\Rightarrow P(0.5033 + 0.906) = 363.973 \Rightarrow P = 258.26$ N **(Ans.)**

# SOLVED PROBLEMS - 2.5

1. *A boat is suspended on two identical davits like ABC which is pivoted at A and supported by a guide at B as shown in the fig. Determine the reactions $R_a$ and $R_b$ at the points of support A and B if the vertical load transmitted to each davit at C is 4272 N. Friction in the guide at B should be neglected.*

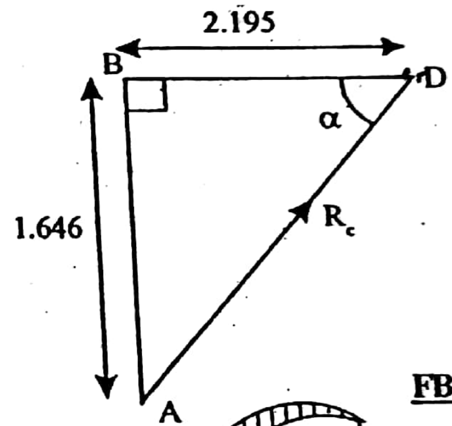



**Soln.** Given data

$Q = 4272$ N

$R_a = ?$ $R_b = ?$

The reaction $R_b$ acts at B horizontally towards left because the davit leans towards right due to the weight Q.

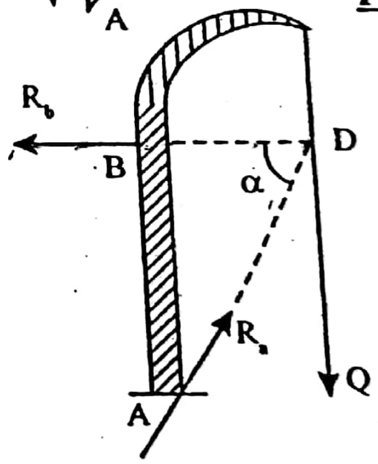



Let the line of actions of $R_b$ and Q meet at D, which is the cuncurrent point

The hinge reaction $R_a$ acts at A and passes through D from A to D for equillibrium.

The FBD is shown in the figure

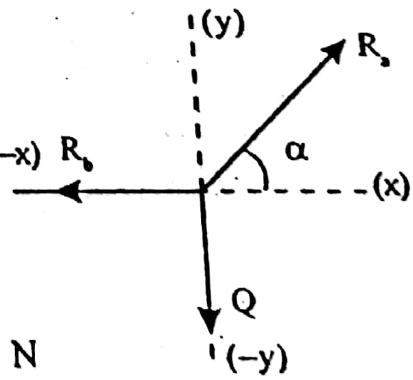
FBD




From the trigonmetry $\tan\alpha = \dfrac{1.664}{2.195}$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{1.646}{2.195}\right) \Rightarrow \alpha = 36^\circ\, 51'$$

Resolving vertically $\Sigma y = 0$



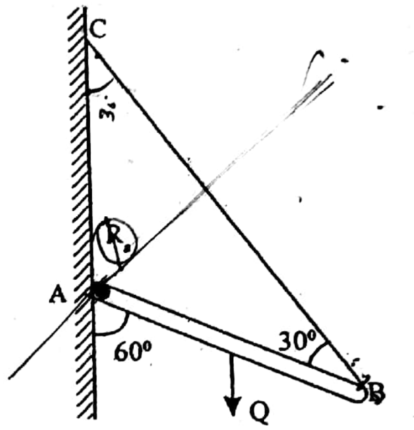

$R_a \sin\alpha = Q$

$$\Rightarrow R_a = \frac{4272}{\sin 36^\circ\, 51'} \Rightarrow R_a = 7123.29 \text{ N}$$

Resolving horizontally $\Sigma x = 0$

$R_b = R_a \cos\alpha = 7123.29 \times \cos 36^\circ 51' = 5700.12$ N

2. A prismatic bar AB of weight $Q = 17.8$ kN is hinged to a vertical wall at A and supported at B by a cable BC as shown in the fig.. Determine the magnitude and direction of the reaction $R_a$ at the hinge A and the tensile force S in the cable BC. The directions of the bar and the cable are as shown in the figure.




**Soln.** Given data

$Q = 17.8$ K.N.

$S = ?$ $\quad$ $Ra = ?$

BC is a cable, hence tensile force S act along BC away from B.

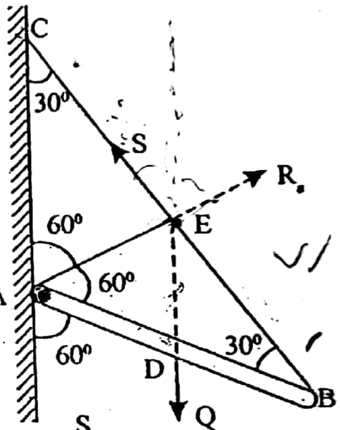
Let the line of actions of Q and S meet at E.

Hence the hinge reaction $R_a$ will pass from A to E for equillibrium.

Since D is the mid point of AB, E must be the mid point of BC

The line AE devide m $\angle CAB$ into equal angles of 60°

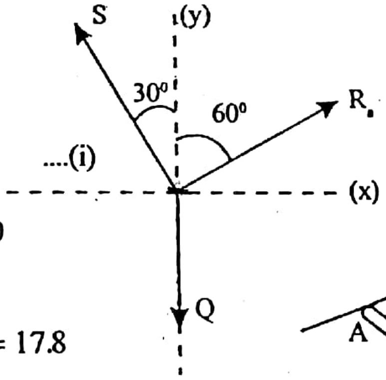
The FBD is shown in the figure.




Resolving horizontally $\Sigma x = 0$

$R_a \sin 60° = S \sin 30°$

$\therefore R_a = S \dfrac{\sin 30°}{\sin 60°} = 0.577S$ ....(i)




Resolving vertically $\quad \Sigma y = 0$

$R_a \cos 60° + S \cos 30° = Q$

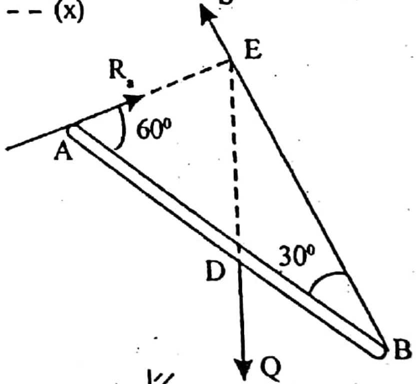
or $0.577S \cos 60° + S \cos 30° = 17.8$

$\Rightarrow S (0.577 \cos 60° + \cos 30°) = 17.8$

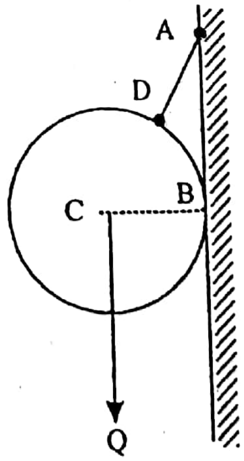
$\Rightarrow 1.154\, S = 17.8 \qquad \Rightarrow S = \dfrac{17.8}{1.154}$

$\Rightarrow S = 15.42$ KN (Ans.)

$R_a = 15.42 \dfrac{\sin 30°}{\sin 60°} = 8.89$ KN (Ans.)




3. *A ball of weight Q and radius r is attached by a string AD to a vertical wall AB, as shown in fig. Determine the tensile force S in the string and the pressure $R_b$ against the wall at B if $Q = 35.6$ N, $r = 75$mm, $AB = 100$mm. Neglect friction at wall.*



**Soln.** Given data

$Q = 35.6$ N

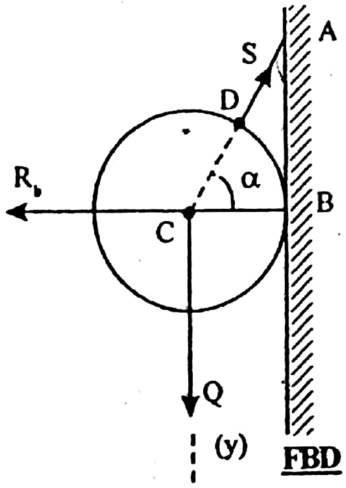
$R = 75$ mm

$AB = 100$ mm

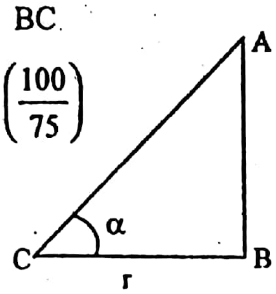
$S = ?$

$R_b = ?$

The FBD at C is shown in the figure

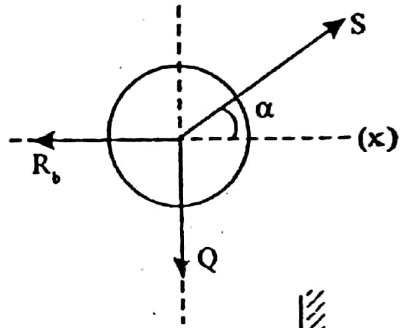



From the geometry of the figure; $\tan\alpha = \dfrac{AB}{BC}$




$$\tan\alpha = \frac{100}{75} \quad \text{or} \quad \alpha = \tan^{-1}\left(\frac{100}{75}\right)$$

$\alpha = 53° \ 7'$




Resolving vertically $\Sigma y = 0$

$S \sin\alpha = Q$

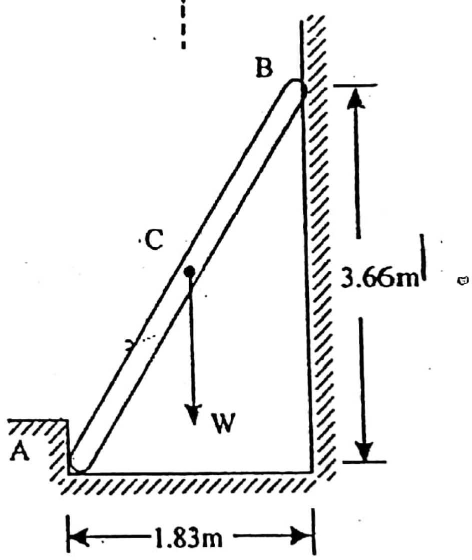
$\Rightarrow S = 35.6 / \sin 53° \ 7' \Rightarrow S = 44.5$ N **(Ans.)**

Resolving horizontally $\Sigma x = 0$

$R_b = S \cos\alpha$

$\Rightarrow R_b = 44.5 \times \cos 53° \ 7' = 26.7$ N **(Ans.)**

**4.** *A 667.5 N man stands on the middle rung of a 222.5 N ladder, as shown in fig. Assuming a smooth wall at B and a stop at A to prevent slipping, find the reactions at A and B.*




**Soln.** Given data

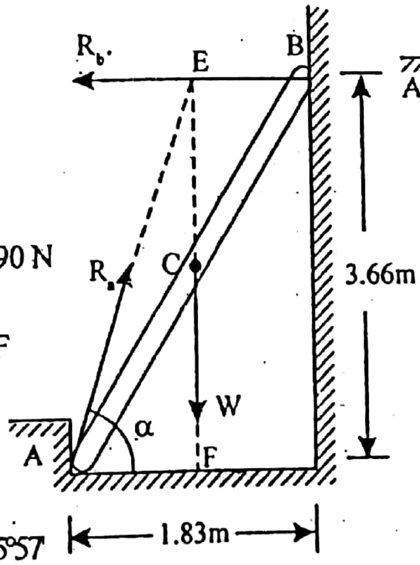
$W_1$ (man) $= 667.5$ N

$W_2$ (ladder) $= 222.5$ N

The total weight act at the middle rung of the ladder,

$W = W_1 + W_2 = 667.5 + 222.5 = 890$ N

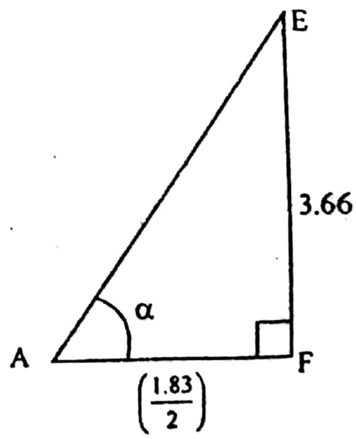
The FBD is shown in the figure.




From the geometry of the Δ AEF

$$\tan\alpha = \frac{EF}{AF}$$

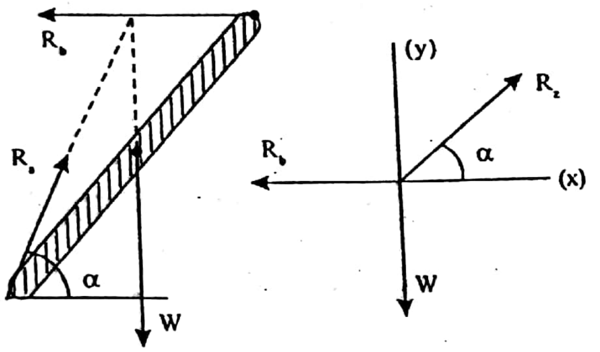
$$\Rightarrow \alpha = \tan^{-1}\left(\frac{3.66 \times 2}{1.83}\right) \Rightarrow \alpha = 75°57'$$




Resolving vertically $\Sigma y = 0$

$R_a \sin \alpha = W$

**FBD**




$$\Rightarrow R_a = \frac{890}{\sin 75° 57'}$$

$$\Rightarrow R_a = 917.44 \text{ N} \qquad \textbf{(Ans.)}$$

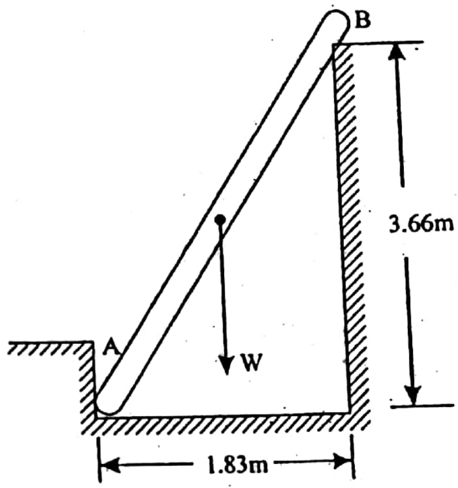
Resolving horizontally $\Sigma x = 0$

$$R_b = R_a \cos \alpha$$

$$= 917.44 \times \cos 75° 57'$$

$$= 222.72 \text{ N} \qquad \textbf{(Ans.)}$$

**5.** *A 667.5 N man stands on the middle rung of a 222.5 N ladder, as shown in fig. Assuming the end B rests on the corner of a wall and a stop at A to prevent slipping, find the reactions at A and B.*




**Soln.** Given data

$W_1$ (Man) = 667.5 N

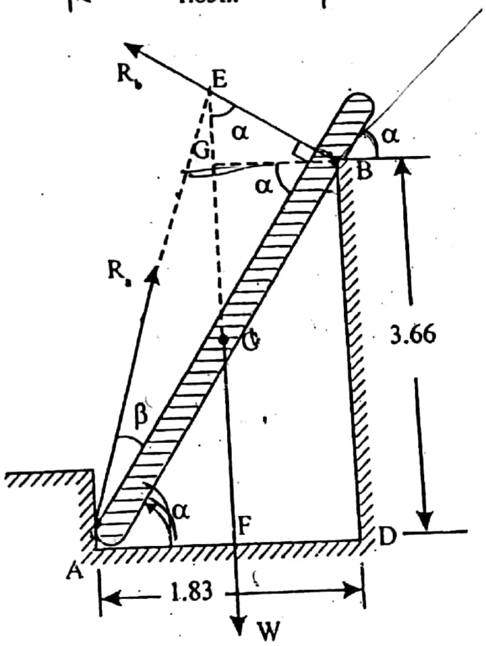
$W_2$(Ladder) = 222.5 N

The total weight act at the middle rung of the ladder.

$W = W_1 + W_2 = 667.5 + 222.5 = 890$ N

The ladder is in contact at B and hence the reaction $R_b$ acts at B normal to it. The reaction $R_a$ will pass at the E, which is the cuncurrent point of W & $R_b$

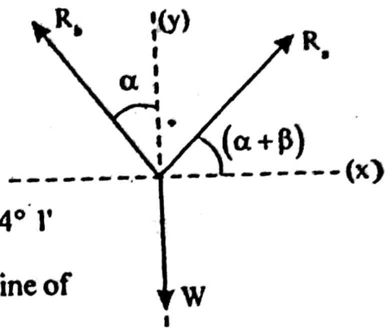
Let, $m\angle BAD = \alpha$




$$\therefore \alpha = \tan^{-1}\left(\frac{3.66}{1.83}\right) = 63°26'$$

$$AB = \sqrt{(1.83)^2 + (3.66)^2} = 4.09 \text{ m}$$

$$\therefore AC = BC = \frac{4.09}{2} = 2.045 \text{m}$$

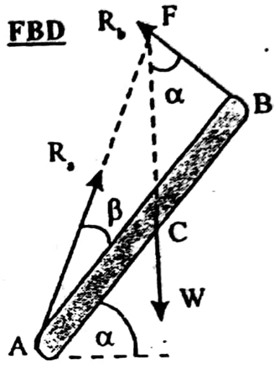



In the triangle Δ BCE

$$\tan\alpha = \frac{BC}{BE} \Rightarrow BE = \frac{2.045}{\tan 63°26'} \Rightarrow BE = 1.022 \text{ m}$$

In the Δ ABE $\quad \tan\beta = \frac{BE}{AB} \Rightarrow \beta = \tan^{-1}\left(\frac{1.022}{4.09}\right) \Rightarrow \beta = 14°\ 1'$

The line of action of $R_b$ makes an angle α with the vertical and the line of action of $R_a$ makes an angle (α + β) with horizontal.




Resolving horizontal $\Sigma x = 0$; $R_a \cos(\alpha+\beta) = R_b \sin\alpha$

$$\Rightarrow R_a = R_b \frac{\sin 63°\ 26'}{\cos(63°\ 26' + 14°\ 1')}$$

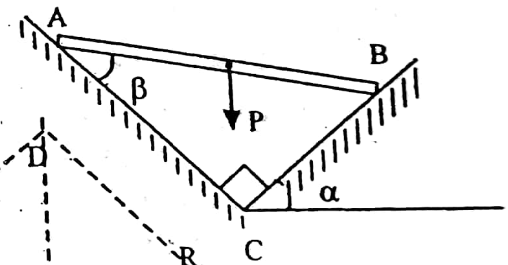
$$\Rightarrow R_a = R_b \left(\frac{\sin 63°\ 26'}{\cos 77°\ 27'}\right) \Rightarrow R_a = 4.11 R_b \qquad \text{......(i)}$$

Resolving vertically $\qquad \Sigma y = 0 \;\; R_a \sin(\alpha+\beta) + R_b \cos\alpha = W$

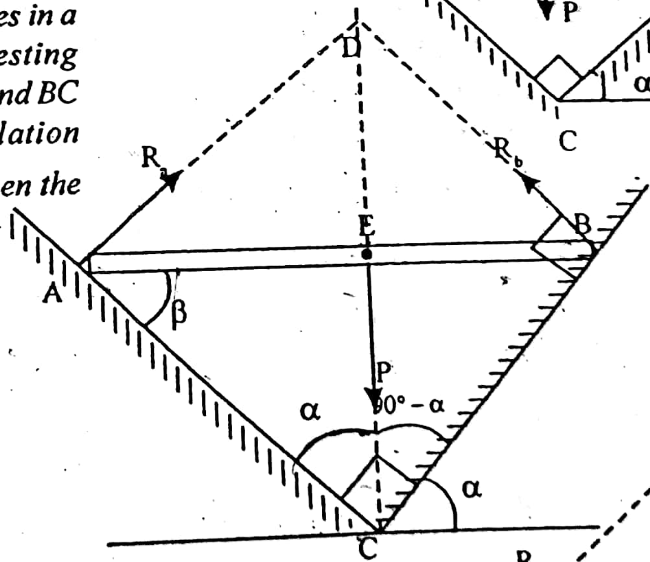
$$\Rightarrow 4.11\, R_b \sin 77°\ 27' + R_b \cos 63°26' = 890 \qquad \Rightarrow R_b(4.11 \sin 77°27' + \cos 63°26') = 890$$

$$\Rightarrow R_b \times 4.459 = 890 \Rightarrow R_b = \frac{890}{4.459} = 199.59\text{ N} \quad \textbf{(Ans.)}$$

$$\Rightarrow R_a = 4.11 \times 199.59 \Rightarrow R_a = 820.31\text{ N}$$




6. *A prismatic bar AB of weight P lies in a vertical plane with its ends resting against the smooth surfaces AC and BC as shown in fig. Find the relation between the angles* α and β, *when the bar is in equilibrium.*




**Soln.** The reactions $R_a$ and $R_b$ and the gravity force P meet at D then, ACBD form a rectangle in which

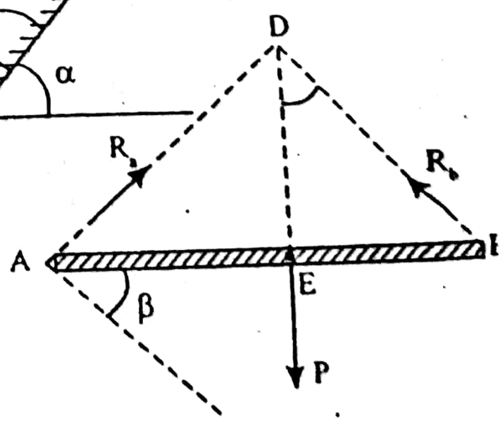
CB || AD & AC || BD

E is the midpoint of AB & CD

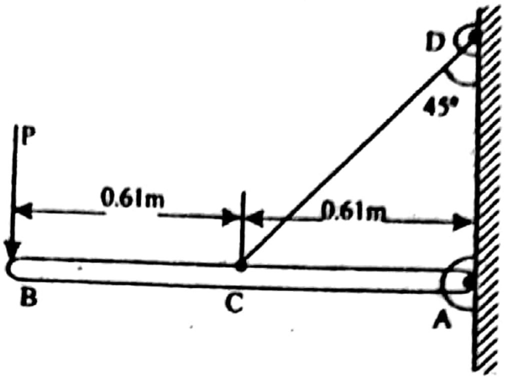
Also AE = ED = CE = BE

∴ m∠ECA = m∠EAC

∴ α = β $\qquad$ **(Ans.)**



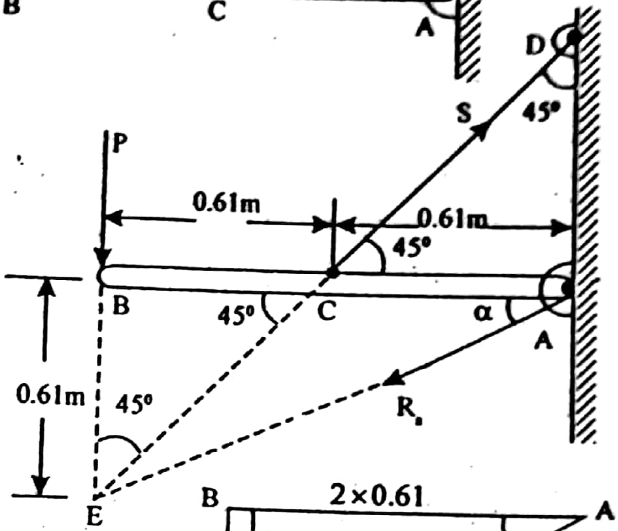
7. ***A horizontal beam AB is hinged to a vertical wall at A and supported at its midpoint C by a tie rod CD as shown in Fig. Find the tension S in the tie rod and the reaction at A due to a vertical load P applied at B.***




**Soln.** Given data

$S_{CD} = ?$

$R_a = ?$

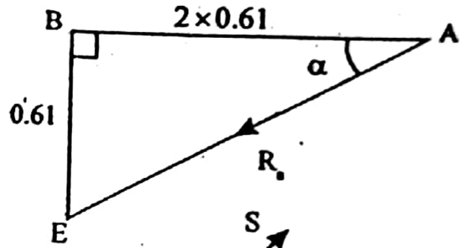



The line of actions of P and S meet at point E at which $R_a$ passes from A to E

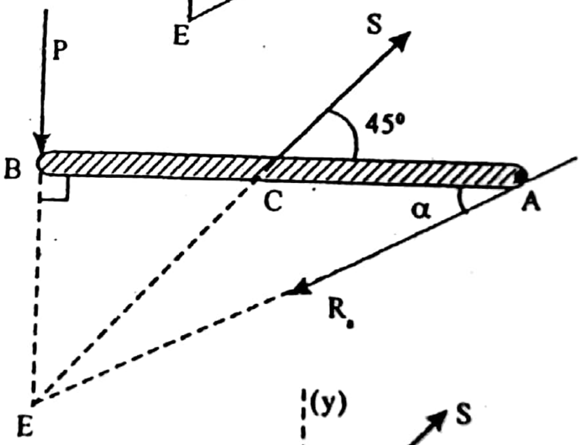
From the geometry;

$$\alpha = \tan^{-1}\left(\frac{0.61}{2 \times 0.61}\right)$$

$$= 26° \, 33'$$




The FBD is shown in the figure.




Resolving horizontally $\sum x = 0$

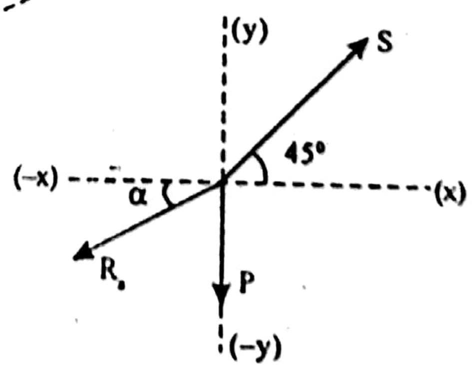
$$S \cos 45° = R_a \cos \alpha \quad \Rightarrow S = \frac{\cos \alpha}{\cos 45°} R_a$$

$$\Rightarrow S = 1.265 \, R_a \qquad .....(i)$$

Resolving vertically $\sum y = 0$

$$S \sin 45° = P + R_a \sin \alpha$$

$$\Rightarrow 1.265 \, R_a \times \sin 45° = P + Ra \sin 26°3'$$

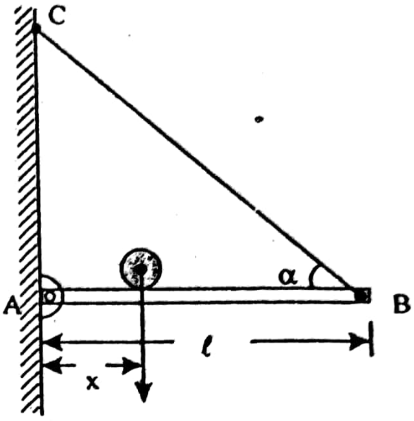



$$\Rightarrow R_a \, (0.894 - 0.446) = P \quad \Rightarrow P = 0.448 \, R_a$$

$$\Rightarrow R_a = 2.232 \, P \qquad \textbf{(Ans.)}$$

$$S = 1.265 \, R_a = 1.265 \times 2.232 \, P = 2.823 \, P \qquad \textbf{(Ans.)}$$

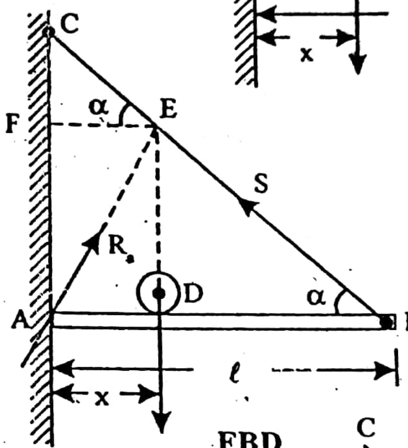
**8.** ***A horizontal prismatic bar AB, of negligible weight and length l, is hinged to a vertical wall at A and supported at B by a tie rod BC that makes the angle α with the horizontal as shown in fig. A weight P can have any position along the bar as defined by the distance x from the wall. Determine the tensile force S in the tie bar.***




**Soln.** Given data

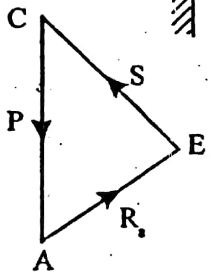
S = ?

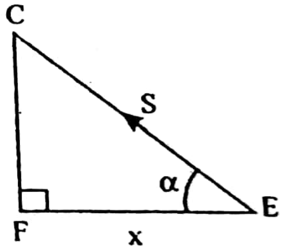
The line of actions of 'P' and 'S' meet at point 'E' at which '$R_a$' passes from A to E. From the above fig. the Δ ACE is equivalent to three forces P, S and $R_a$.




FBD

Applying triangle law.

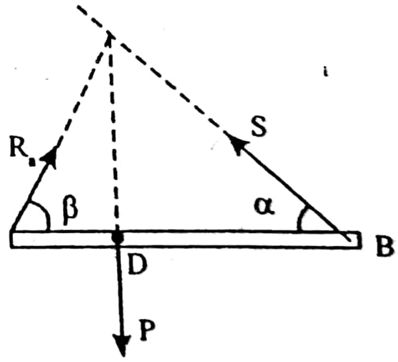



$$\frac{P}{AC} = \frac{S}{CE} = \frac{R_a}{AE}$$

$$\because \frac{P}{AC} = \frac{S}{CE} \Rightarrow S = \frac{P \times CE}{AC}$$

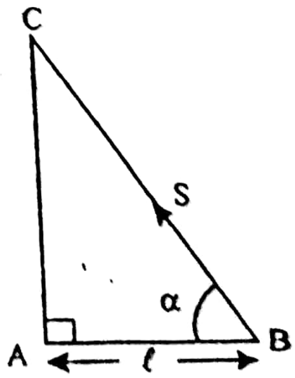
From the triangle EFC; $\cos\alpha = \dfrac{x}{CE}$ or $CE = \dfrac{x}{\cos\alpha}$




Also from the triangle ACB; $\tan\alpha = \dfrac{AC}{AB}$

or $AC = \ell \tan\alpha$ substituting the value.

$$\Rightarrow S = \frac{P \times \dfrac{x}{\cos\alpha}}{\ell \tan\alpha} \quad \Rightarrow S = \frac{P.x}{\ell \cos\alpha - \tan\alpha} \quad \Rightarrow S = \frac{Px}{\ell \sin\alpha}$$

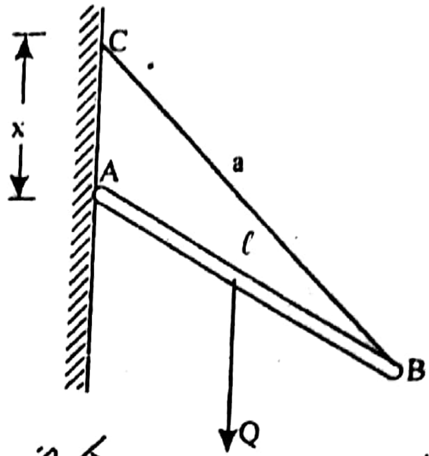



***Note :*** By using method of moment we can solve it in one line.

i.e., $\Sigma M_A = 0$

$$P.x = s \sin\alpha \times \ell \Rightarrow S = \frac{Px}{\ell \sin\alpha} \quad \textbf{(Ans.)}$$

9. *A prismatic bar AB, of weight Q and length l, is supported at one end B by a string CB of length a and rests at A, vertically below C, against a perfectly smooth vertical wall as shown in the fig. Find the position of the bar, as defined by the length x, for which equilibrium will be possible.*

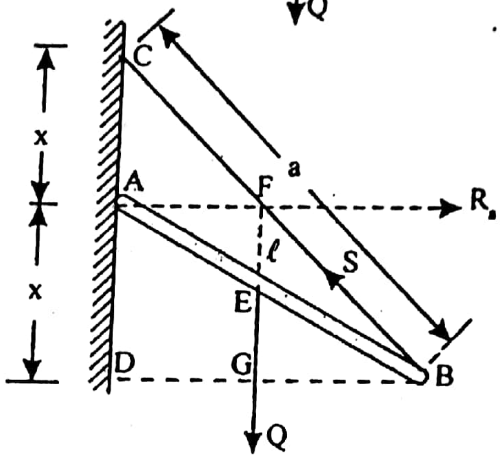



**Soln.** Given data

$AB = \ell, \quad AC = x, \quad BC = a$

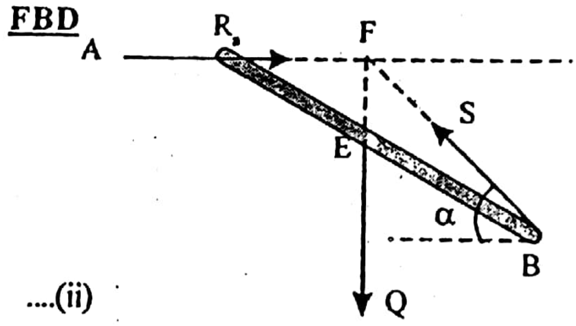
The line of actions of Q & S meet at point F at which $R_a$ passes from A to F

Moreover, the wall is smooth and the bar = AB rests at A. hence the reaction $R_a$ is normal.




Since E is the mid point of AB, the vertical line EF devide BC into two equal halves i.e., CF = BF.

The horizontal line AF devides CD in to two equal halfs i.e., AC = AD = x




In the triangle ADB,

$AB^2 - AD^2 = BD^2$ or $BD^2 = \ell^2 - x^2$ ......(i)

In Δ BCD

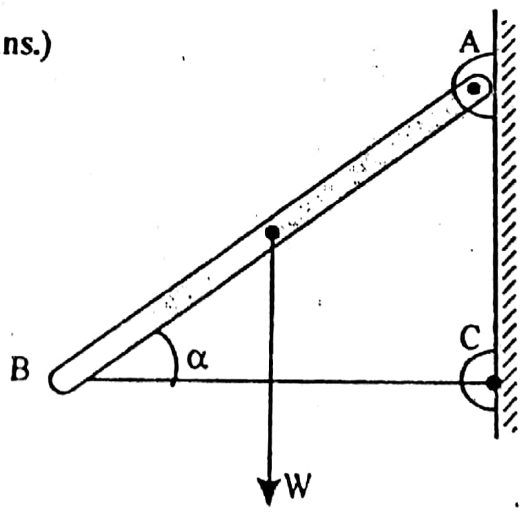
$BD^2 = BC^2 - CD^2 = a^2 - (2x)^2$ or $BD^2 = a^2 - 4x^2$ ....(ii)

Equating (i) and (ii)

$\ell^2 - x^2 = a^2 - 4x^2 \Rightarrow 4x^2 - x^2 = a^2 - \ell^2 \quad \Rightarrow 3x^2 = a^2 - \ell^2$

$$\Rightarrow x = \sqrt{\left(\frac{a^2 - \ell^2}{3}\right)}$$ (Ans.)

10. *A Prismatic bar AB of weight W = 62.3 N and length l = 2.44m is hinged to a vertical wall at A and supported at its other end B by a horizontal strut BC, as shown in Fig. Find the compressive force S induced in the strut and the reaction $R_a$ at A if α = 25°.*



**Soln.** Given data

$\alpha = 25°$

$W = 62.3$ N

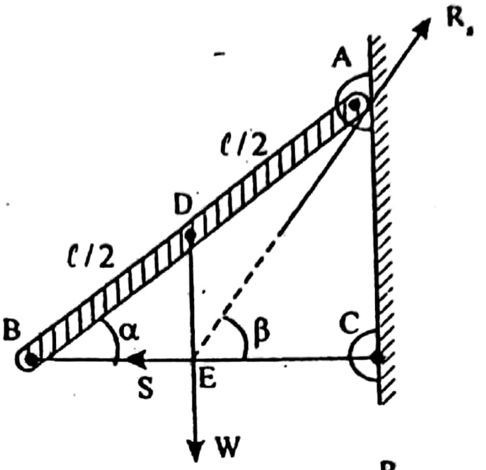
$AB = \ell = 2.44$ m

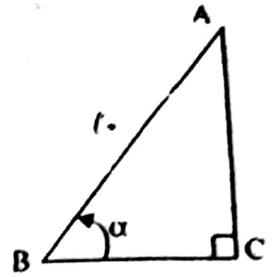
$S = ?$

$R_a = ?$

$AC = \ell \sin \alpha$

$BC = \ell \cos \alpha$

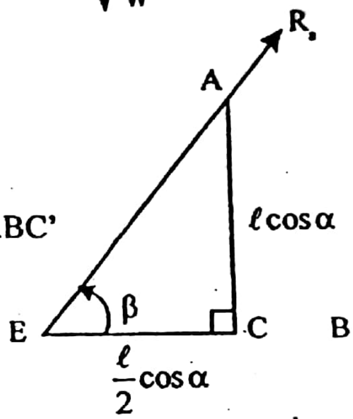


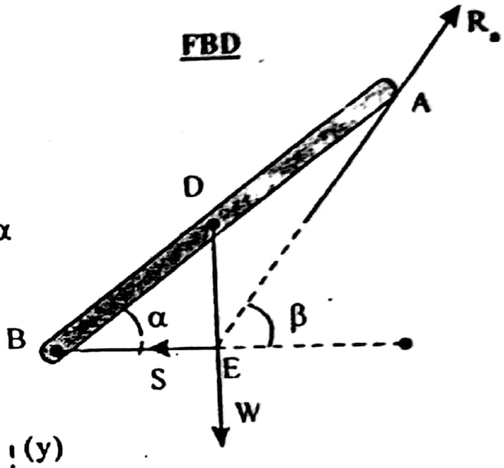




$\therefore\ BE = CE = \dfrac{\ell \cos \alpha}{2}$

The line of actions of compressive force S and gravity force W meet at E, at which $R_a$ passes from E to A.

From the geometry of the triangle 'ABC'



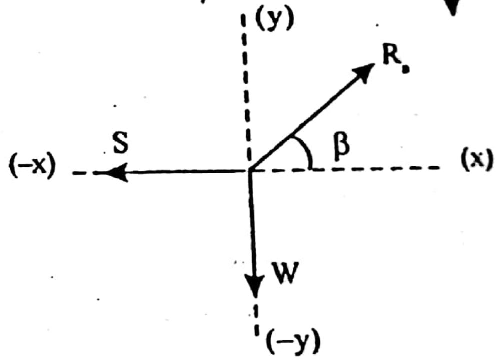




$$\tan \beta = \frac{\ell \sin \alpha}{\ell/2 \cos \alpha} = 2 \tan \alpha$$

$$\beta = \tan^{-1}(2 \tan \alpha)$$

$$= \tan^{-1}(2 \times \tan 25°) = 43°$$

The FBD is shown in the figure.




Resolving vertically $\sum y = 0$

$$R_a \sin \beta = W$$

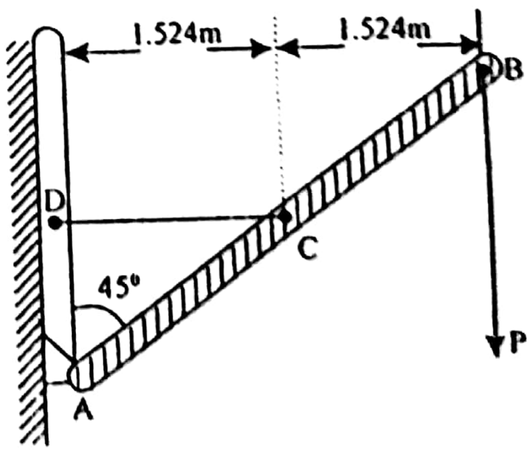
$$\Rightarrow R_a = \frac{62.3}{\sin 43°}$$

$$\Rightarrow R_a = 91.349 \text{ N}$$

Resolving horizontally $\sum x = 0$

$$S = R_a \cos \beta = 91.349 \times \cos 43° = 66.8 \text{ N (comp.)}$$

11. *A weighless bar AB is supported in a vertical plane by a hinge at 'A' and a tie bar DC, as shown in fig. Determine, the axial force S induced in the tie bar by the action of a vertical load P applied at B.*

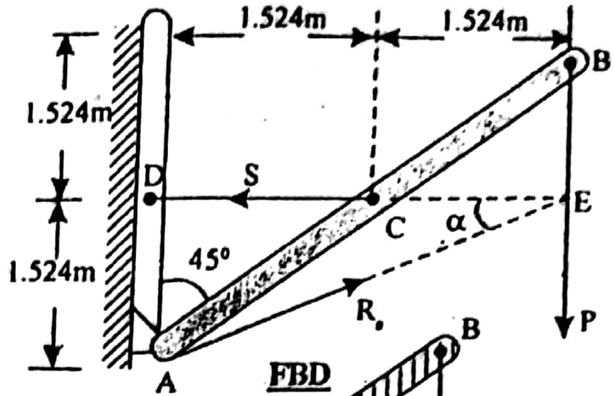



**Soln.** Since $m\angle DAC = 45°$

$m\angle ACD = 45°$

$\therefore\ CD = AD = 1.524$ m

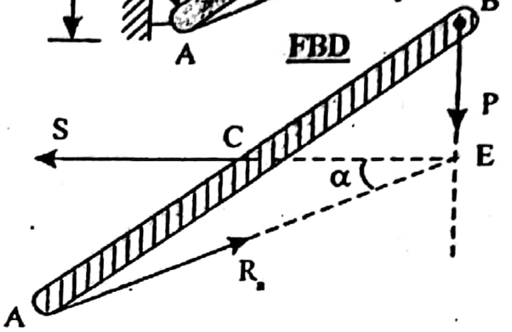
The line of actions of S & P meet at E, at which $R_a$ passes from A to E.




Let $R_a$ makes an angle $\alpha$ with horizontal.

From the geometry of the triangle AED;

$$\tan \alpha = \frac{AD}{DE}$$

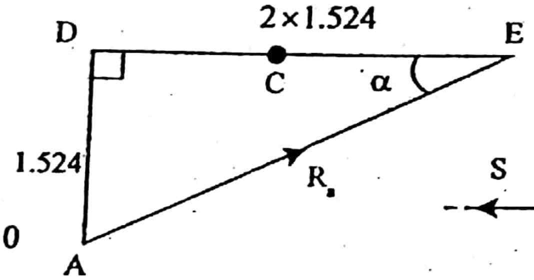


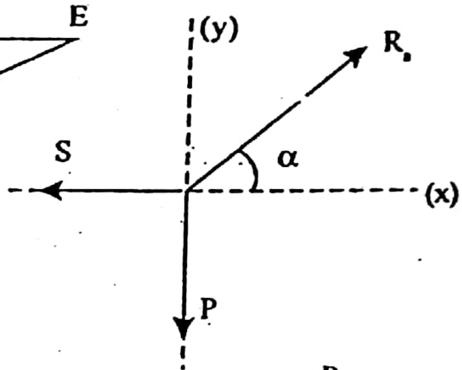

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{1.524}{2 \times 1.524}\right) = 26° \, 33'$$

The FBD is shown in the figure.

Resolving vertically $\sum y = 0$

$R_a \sin \alpha = P$




$$\Rightarrow R_a = \frac{P}{\sin 26° \, 33'}$$

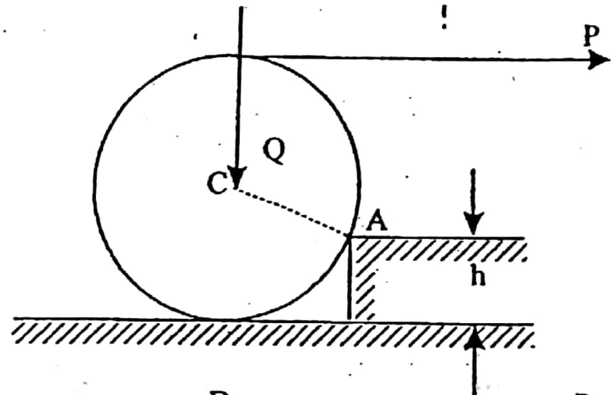
$\Rightarrow R_a = 2.237\,P$ (Ans.)

Resolving horizontally $\sum x = 0$

$S = R_a \cos \alpha$

$= 2.237\ P \times \cos 26° \, 33' = 2P$ (Ans.)

12. *A roller of radius $r = 304.8$ mm. and weight $Q = 2225N$ is to be pulled over a curb of height $h = 152.4mm$ by a horizontal force P applied to the end of a string wound around the circumference of the roller, as shown in the fig. Find the magnitude of P required to start the roller over the curb.*



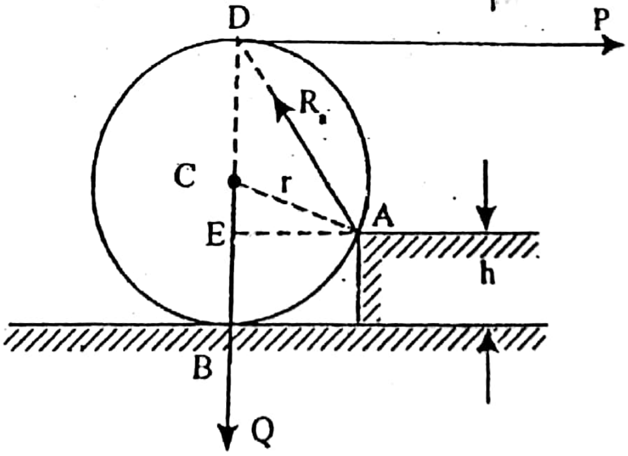

Soln. Given data

$r = 304.8$ mm

$Q = 2225$ N

$h = 152.4$ mm

$P = ?$



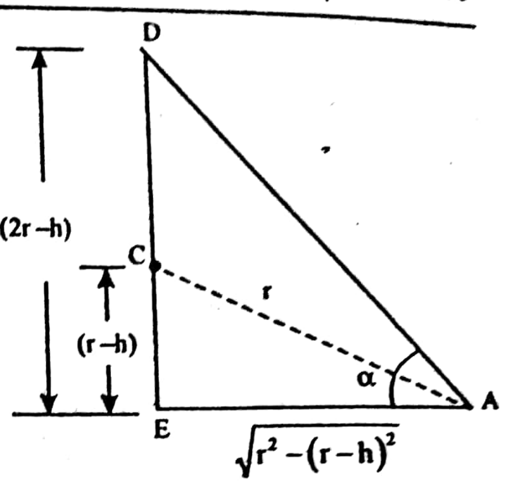

The line of actions of P & Q meet at D at which $R_a$ passes from A to D

From the $\Delta$ ACE

$AE = \sqrt{r^2 - (r-h)^2}$

Let $R_a$ makes an angle '$\alpha$' with the horizontal

From the $\Delta$ AED, $DE = 2r - h$




$$\therefore \tan\alpha = \frac{DE}{AE} = \frac{2r-h}{\sqrt{r^2-(r-h)^2}}$$

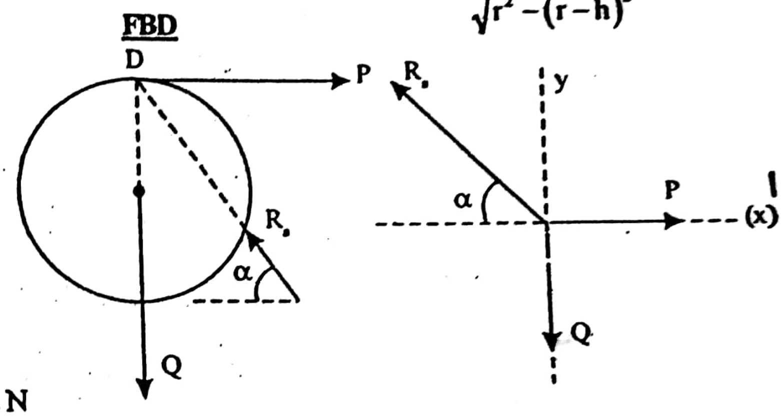
$$= \frac{(2\times 304.8) - 152.4}{\sqrt{(304.8)^2 - (304.8-152.4)}}$$

$$= \frac{457.2}{\sqrt{69677.28}} = \frac{457.2}{263.96}$$

or $\tan\alpha = 1.732$

$\Rightarrow \alpha = \tan^{-1}(1.732) = 60^\circ$

The FBD is shown in the figure




Resolving vertically $\Sigma y = 0$

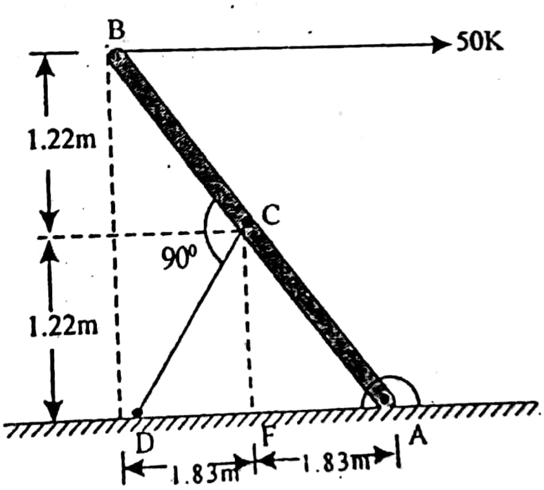
$R_a \sin\alpha = Q$

$$\Rightarrow R_a = \frac{2225}{\sin 60^\circ} \Rightarrow R_a = 2569.2\,N$$

Resolving horizontally $\Sigma x = 0$

$P = R_a \cos\alpha = 2569.2 \times \cos 60^\circ = 1284.6$ N (Ans.)

13. *A bar AB hinged to the foundation at A and supported by a strut CD is subjected to a horizontal 50 kN load at B, as shown in fig. Find the tensile force S in the strut and the reaction $R_a$ at A.*




**Soln.** $R_a = ?$

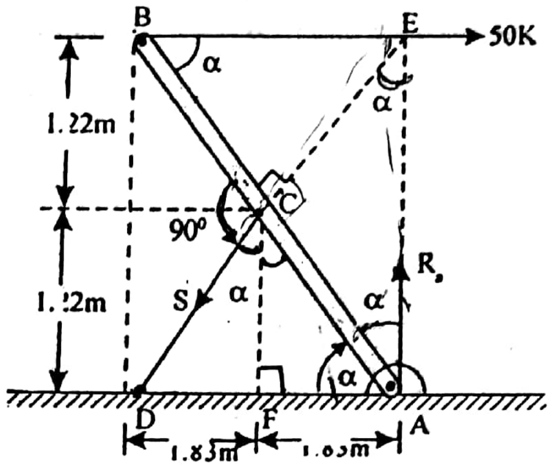
$S = ?$

The axial force S as the tension acts on the strut CD away from the joint C. The line of actions of 50 KN and S meet at E, at which $R_a$ passes from A to E

From the $\Delta$ACF

$$\tan\alpha = \frac{1.22}{1.83}$$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{1.22}{1.83}\right) \quad \Rightarrow \alpha = 33^\circ 41'$




Due to symmetry

$m\angle DCF = \alpha$

$m\angle ABE = \alpha$

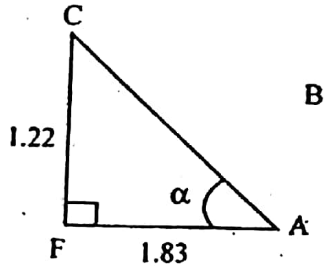
Since C is the midpoint of AB and $m\angle BCD = 90^\circ$,

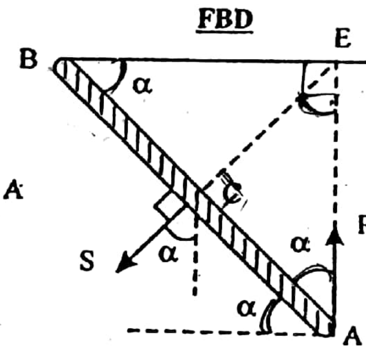
the $\Delta$ ABE is an isoscele.

Thus $m\angle EBA = \alpha = m\angle EAB$

In otherward $R_a$ makes $2\alpha$ with horizontal.

The FBD shown in the figure



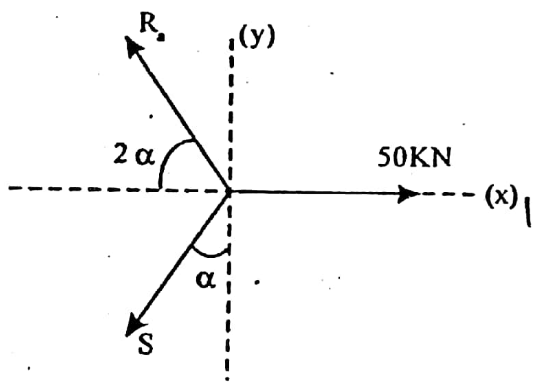

$R_a = S \cos\alpha$

Resolving vertically $\sum y = 0$

$R_a \sin 2\alpha = S \cos\alpha$

$$\Rightarrow R_a = S \frac{\cos 33^\circ 41'}{\sin(2 \times 33^\circ 41')}$$

$\Rightarrow R_a = 0.902\ S$ ...(i)




Resolving horizontally $\sum x = 0$

$R_a \cos 2\alpha + S \sin\alpha = 50$ ....(ii)

Substituting the value of '$R_a$' in equation (ii)

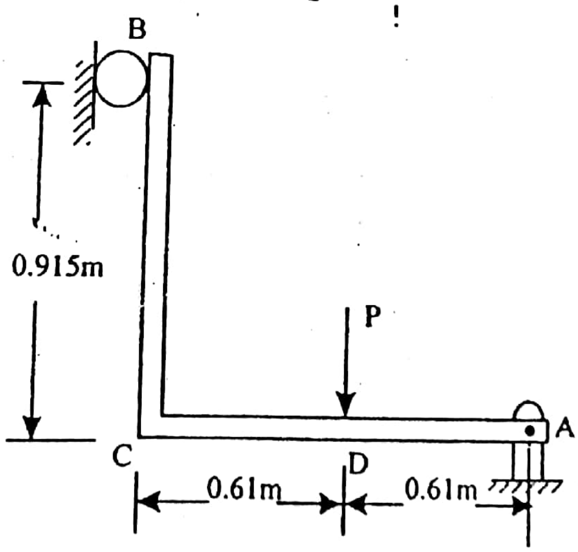
$0.902\ S \cos(2 \times 33^\circ 41') + S \sin 33^\circ 41' = 50$

$\Rightarrow S\ 0.902 = 50 \Rightarrow S = 55.44$ KN (Ans.)

$\therefore R_a = 0.902 \times 55.44$

or $R_a = 50.01$ KN (Ans.)

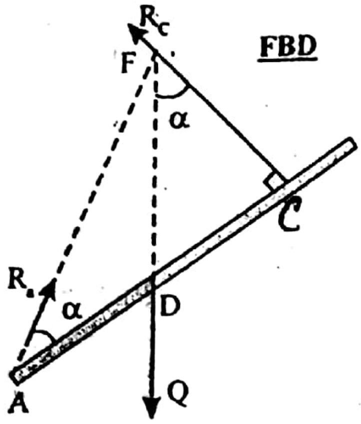
14. *Find the reactions $R_a$ and $R_b$ induced at the supports A and B of the right-angle bar ACB supported as shown in fig. and subjected to a vertical load P applied at the mid point of AC. ($R_a = 1.2P$; $R_b = 0.67\ P$).*




**Soln.** The line of actions of $R_b$ and P meet at E, at which $R_a$ passes from A to E making an angle $\alpha$ with horizontal.

Let $m\angle EAD = \alpha$

Let E is the center of the semicircle.




In the Δ ACF ,

$$AF = 2r$$

$$m\angle FAC = \alpha$$

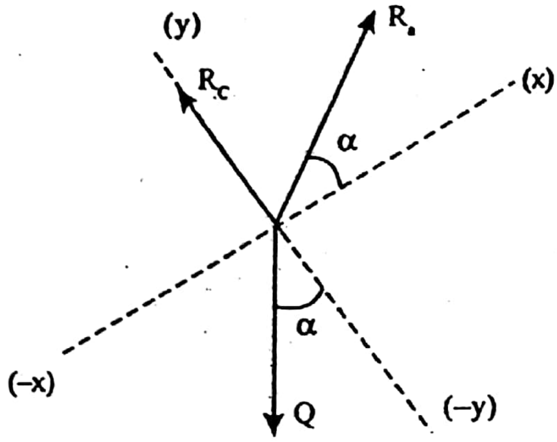
$$AC = 2\,r\cos\alpha$$

$$FC = 2\,r\sin\alpha$$

$$DC = AC - AD = 2\,r\cos\alpha - a$$

$$= 2 \times 0.866\,a\cos\alpha - a \;= a\,(2 \times 0.866\cos\alpha - 1)$$

From Δ CDF




$$\tan\alpha = \frac{DC}{CF} = \frac{a(2 \times 0.866\cos\alpha - 1)}{2r\sin\alpha}$$

$$\text{or}\quad \frac{\sin\alpha}{\cos\alpha} = \frac{a(1.732\cos\alpha - 1)}{2 \times 0.866\,a\sin\alpha}$$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{(1.732\cos\alpha - 1)}{1.732\sin\alpha}$$

$$\Rightarrow 1.732\sin^2\alpha = 1.732\cos^2\alpha - \cos\alpha$$

$$\Rightarrow 1.732\left(\cos^2\alpha - \sin^2\alpha\right) = \cos\alpha \qquad \Rightarrow 1.732 \times \left(2\cos^2\alpha - 1\right) = \cos\alpha$$

$$\Rightarrow 3.464\cos^2\alpha - \cos\alpha - 1.732 = 0$$

$$\Rightarrow \cos\alpha = \frac{1 \pm \sqrt{(1 + 4 \times 3.464 \times 1.732)}}{2 \times 3.464} = \frac{1 \pm \sqrt{24.998}}{6.928}$$

Taking + sign, $\cos\alpha = 0.866 \;\Rightarrow \alpha = 29°\,59' \approx 30°$ **(Ans.)**

Resolving horizontally $\sum x = 0$

$$R_a\cos\alpha = Q\sin\alpha$$

$$\Rightarrow R_a = Q\tan\alpha \qquad \Rightarrow R_a = Q\tan 30° \qquad \Rightarrow R_a = \frac{Q}{\sqrt{3}}$$ **(Ans.)**

Resolving vertically $\sum y = 0$

$$R_c + R_a\sin\alpha = Q\cos\alpha \qquad \Rightarrow R_c + Q\tan\alpha\,.\,\sin\alpha = Q\cos\alpha$$

$$\Rightarrow R_c = Q\left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}} \times \frac{1}{2}\right) \Rightarrow R_c = Q\left(\frac{3-1}{2\sqrt{3}}\right) \Rightarrow R_c = \frac{Q}{\sqrt{3}}$$ **(Ans.)**

***

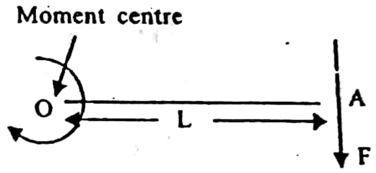
# Moments and Their Applications

## 2.6.1 INTRODUCTION

Every force in a system of forces has the tendency to produce rotation about any fixed point when the line of action does not meet that fixed point.

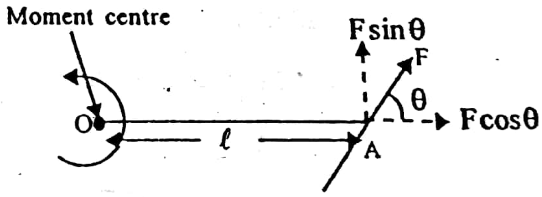
That turning effect of a force on the body is called moment. It the product of magnitude of the force and perpendicular distance of that point, about which moment is required. Mathematically

$M = F \times \ell$




Where L is the perpendicular distance from the moment centre 'O' to the line of action of the force F.

Moment of a force F inclined at an angle '$\theta$' with horizontal, about 'O' is




$M_O \quad = \quad F \sin\theta \times \ell + F \cos\theta \times 0$

$\quad\quad = \quad F \sin\theta \times \ell$ (anticlockwise)

Unit N – m in SI

Kgf – m in MKS

## 2.6.2 TYPES OF MOMENTS

(i) Clockwise moment, (ii) Anticlockwise moment

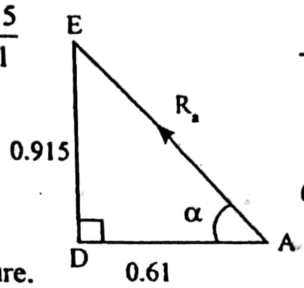
**Clockwise :** It is the moment of a force, whose effect is to turn a body in the direction in which a clock moves.

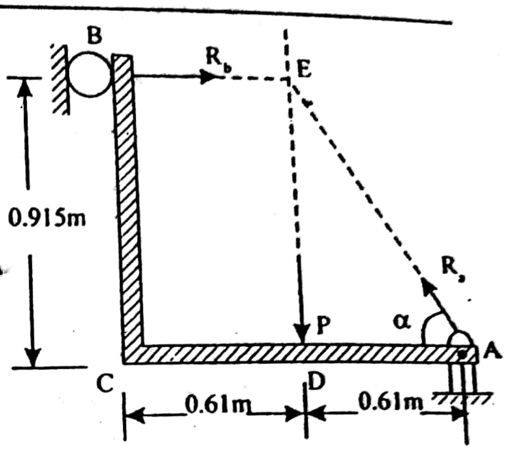
**Anticlockwise :** It is the moment of a force, whose effect is to turn a body in opposite direction in which a clock moves.

**Sign Convention**

Generally clockwise moment is taken as negative and anti-clockwise moment is taken as positive while solving problems in mechanics. But vice-versa in sign conventions may be considered.

From $\Delta$ ADE, $\tan\alpha = \dfrac{0.915}{0.61}$




$\Rightarrow \alpha = \tan^{-1}\left(\dfrac{0.915}{0.61}\right)$

$\Rightarrow \alpha = 56°\ 18'$

The FBD is shown in the figure.

Resolving vertically $\Sigma y = 0$

$R_a \sin\alpha = P$

$\Rightarrow R_a = P/\sin 56°\ 18'$

$\Rightarrow R_a = 1.201\ P$ **(Ans.)**

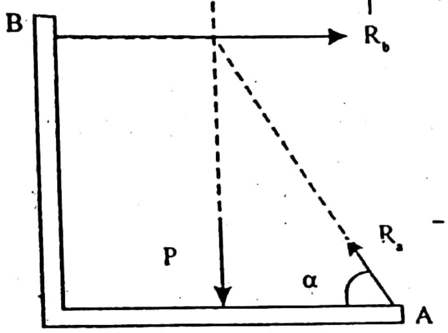
Resolving horizontally

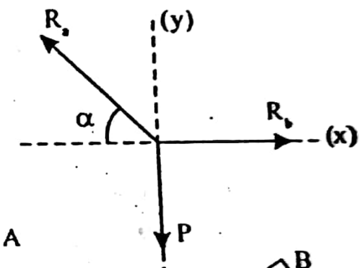
$R_b = R_a \cos\alpha$

$= 1.201\ P \times \cos 56°18'$

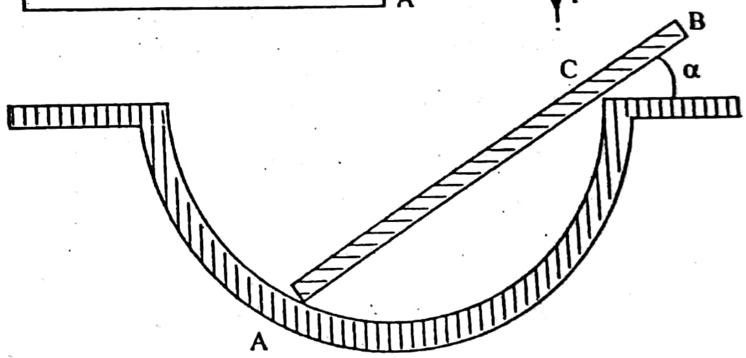
$= 0.666\ P$ **(Ans.)**

**BDF**




**15.** *The ends of a heavy prismatic bar AB of length 2a and weight Q supported by a hemispherical bowl of radius r as shown in fig. Assuming the smooth surfaces, determine the angle $\alpha$ corresponding to equilibrium of the bar, if r = 0.866a. Also find the reactions induced at the points of support A and C.*




**Soln.** Given data

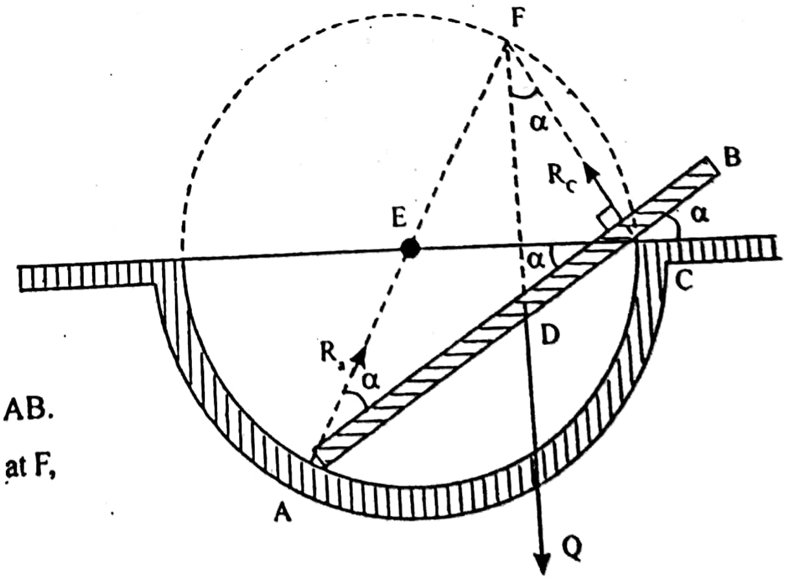
AB = 2a

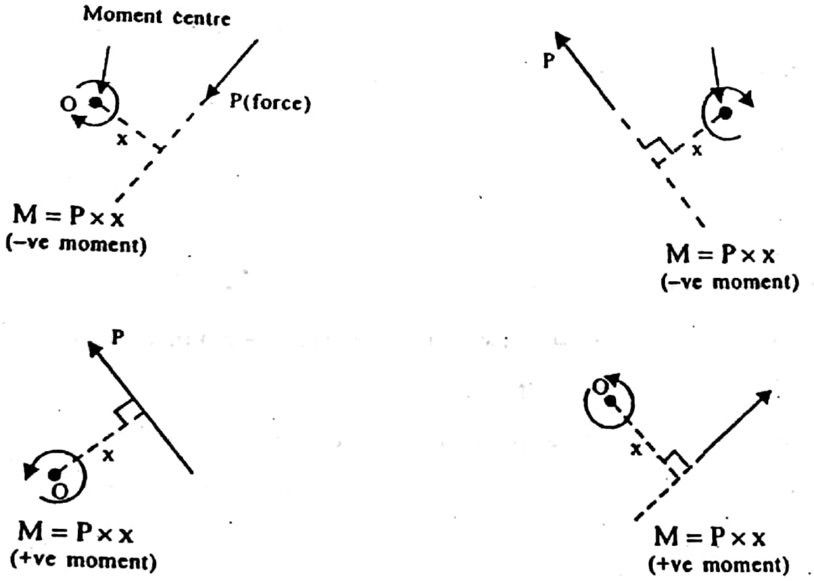
$\therefore$ AD = DB = a

r = 0.866 a

The reaction $R_c$ acts at C normal to AB.

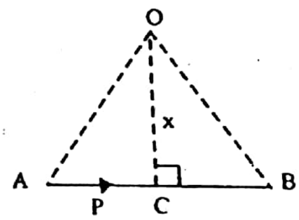
The line of actions of Q and $R_c$ meet at F, at which $R_a$ passes from A to F.






## 2.6.3 Graphical Representation of Moment

Let a force P be represented in magnitude and direction by the vector AB and let O be the moment centre. Now draw OC perpendicular to AB. Join OA & OB.




Moment of P about O

$$= P \times x \qquad = AB \times OC$$

$$= 2 \times \left[\frac{1}{2}\, AB \times OC\right]$$

$$= 2 \times [\text{Area of triangle OAB}]$$

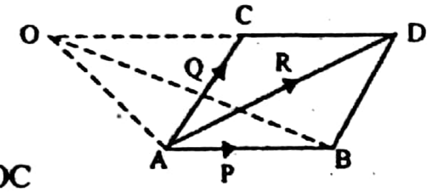
Hence the moment of a force about any moment centre is geometrically equal to twice the area of the triangle whose base represents the force and vertex represents the moment centre.

## 2.6.4 Varignon's Theorem or Law of Moments

The algebric sum of moments of all coplanner forces about any moment centre is equal to the moment of their resultant about that same moment centre.

**Proof:**

Let us consider two coplanner concurrent forces P & Q represents two sides AB and AC of a parallelogram ABCD. The diagonal AD represents the resultant. For simplicity extend the line CD to O. Assume 'O' is the moment centre. Join OA and OB.




Now the moment of P about O

$= 2 \times$ Area of $\Delta$ AOB

Similarly the moment of 'Q' about O $= 2 \times$ Area of $\Delta$ AOC

and the moment of the resultant R about O $= 2 \times$ Area of $\Delta$ AOD

But from the geometry of the figure

Area of $\Delta$ AOD = Area of $\Delta$ADC + Area of $\Delta$AOC

But area of $\Delta$ ADC = Area of $\Delta$ ABD = Area of $\Delta$ AOB

(Because the areas of all triangles between two parallel lines, having a common base, are equal)

$\therefore$ Area of $\Delta$ AOD = Area of $\Delta$ AOB + Area of $\Delta$ AOC

or $2 \times$ Area of $\Delta$ AOD = $2 \times$ Area of $\Delta$ AOB + $2 \times$ Area of $\Delta$ AOC

$\therefore$ Moment of R about O = moment of P about O + moment of Q about O

## 2.6.5 Principle of Moments

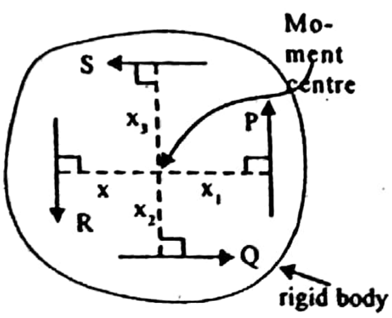
The algebric sum of moments of all coplanner forces acting on a rigid body in equilibrium, about any moment centre is zero.

Mathematically $\sum M = 0$

i.e., Clockwise moments = Anticlockwise moments

## 2.6.6 Position of Resultant Force

It is the perpendicular distance from the moment centre to the line of action of resultant force where the algebric sum of moments of all forces is equal to the moment of that resultant.

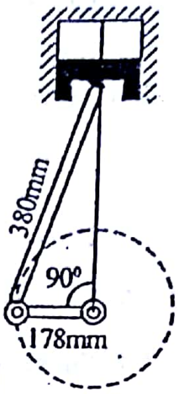



**Example**

$R \times x = P \times x_1 + Q \times x_2 + S \times x_3$

where x = Position of the resultant

1. *If the piston of the engine in the given fig. has a diameter of 101.6 mm and the gas pressure in the cylinder is 0.69 MPa, calculate the turning moment M exerted on the crank shaft for the particular configuration shown.*




**Soln.** Given data

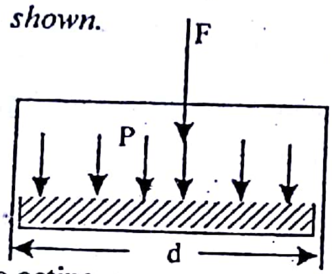
$d = 101.6$ mm

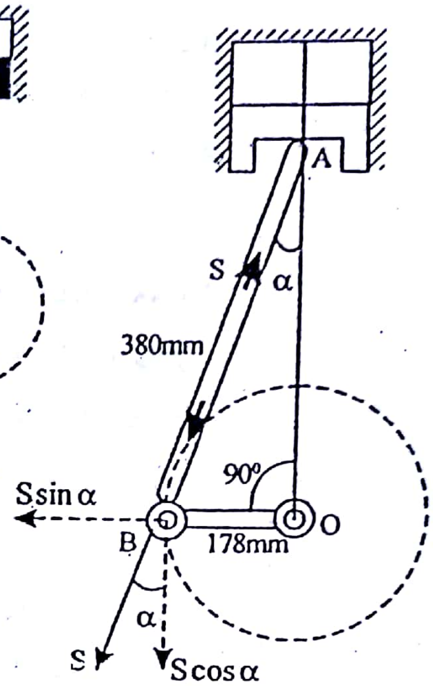
$P = 0.69$ MPa

$= 0.69$ N/mm²

Let $m\angle OAB = \alpha$

The normal pressure force acting on the piston




$$F = P \times \frac{\pi d^2}{4} = 0.69 \times \pi \times \frac{(101.6)^2}{4} = 5594.05 \text{ N}$$

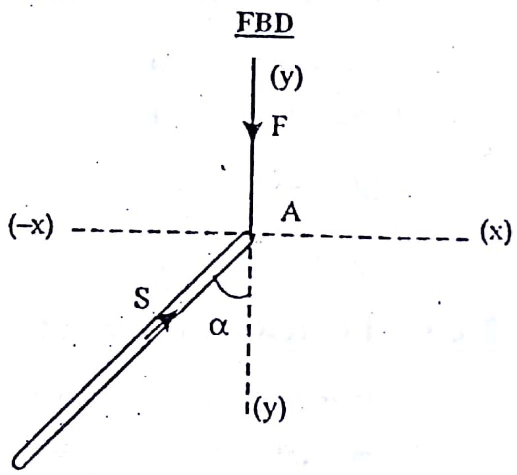
The FBD at A is shown in the figure.

The pressure force is transmitted along the connecting rod like the axial compressive force S.

i.e., Resolving vertically $\Sigma y = 0$

$S \cos\alpha = F$

$$\Rightarrow S = \frac{F}{\cos\alpha} \qquad \text{....(i)}$$



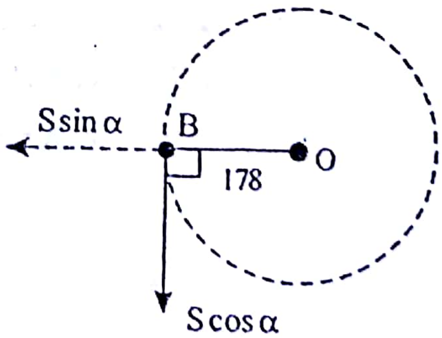

This compressive force S also acts towards B. It has two components S cos α acting vertically downward and S sin α acting horizontally left.

The moment, of S about crank shaft '0'.

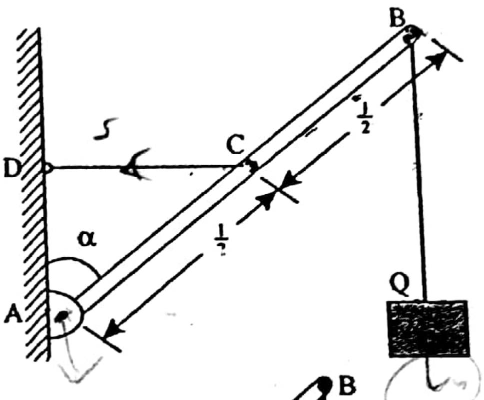
$$M_0 = S\cos\alpha \times 178 = \frac{F}{\cos\alpha} \times \cos\alpha \times 178 = 178\text{ F}$$

$= 178 \times 5594.05 = 995741.0013$ N – m

$= 0.995$ KN (Ans.)

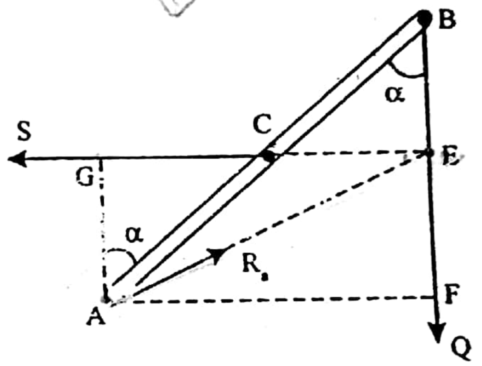


2. *A rigid bar AB is supported in a vertical plane and carries a load Q at its free end as shown in fig. Neglecting the weight of the bar itself, compute the magnitude of the tensile force S induced in the horizontal string CD.*




**Soln.** AB = $\ell$

The line of actions of S & Q meet at E at which $R_a$ passes from A to E.




The FBD is shown in the figure and using method of moment,

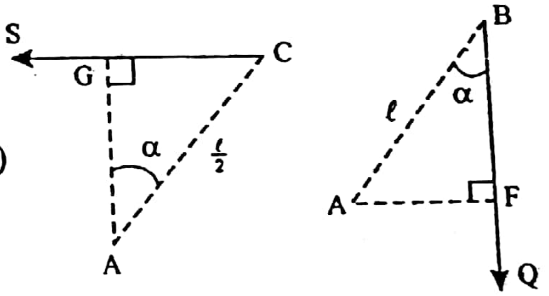
$\Sigma M_A = 0$

$Q \times AF = S \times AG$ ....(i)

From the triangles

$AF = \ell \sin\alpha$

$AG = \dfrac{\ell}{2}\cos\alpha$

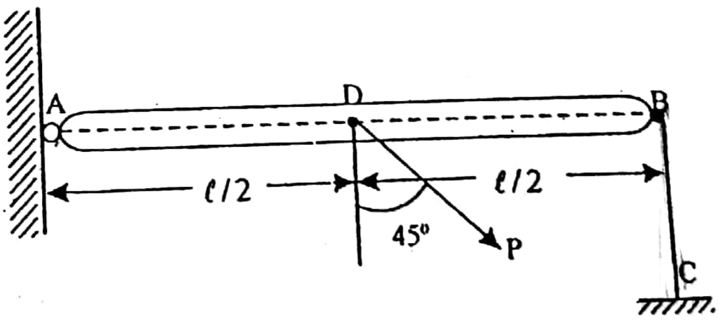



Substituting the values in equation (i)

$Q \times \ell \sin\alpha = S \times \dfrac{\ell}{2}\cos\alpha$

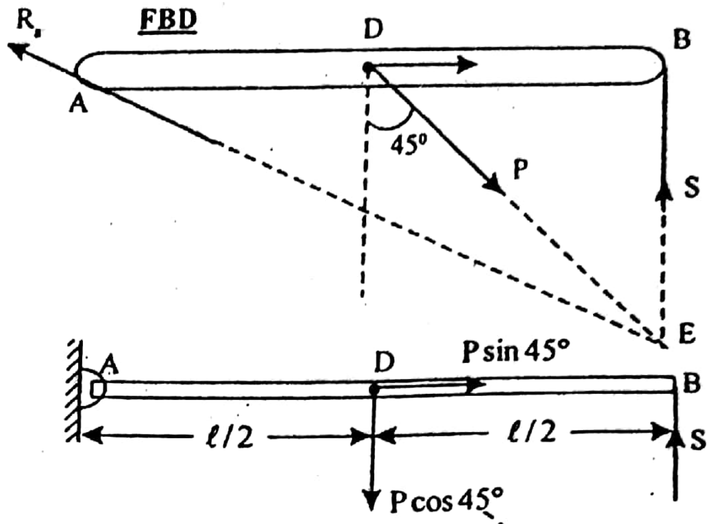
$\Rightarrow S = 2Q \tan\alpha$ **(Ans.)**

3. *A beam AB, hinged at A and supported at B by a vertical bar BC, is subjected to the action of a force P applied as shown in fig. Assuming ideal hinges at A, B and C, find the force S produced in the bar BC. Neglect the weight of the beam.*




**Soln.** The line of actions of P and compressive force S meet at E, at which $R_a$ passes from

E to A. The FBD is shown in the figure. Using method of moment

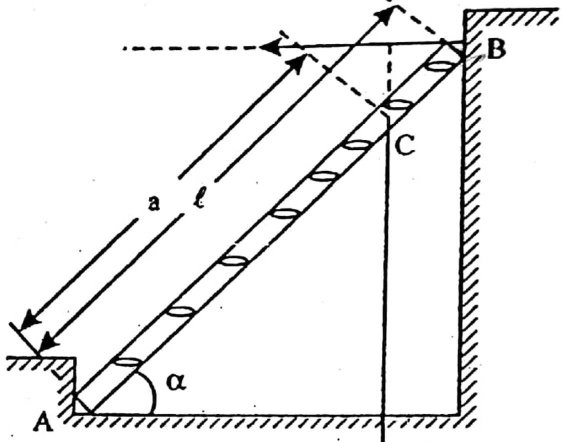



$\Sigma M_A = 0$

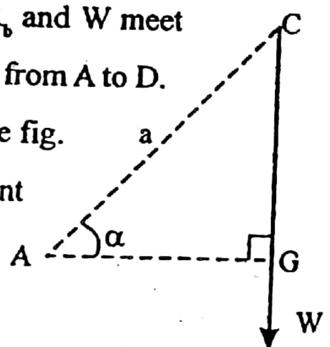
$$S \times \ell = P \cos 45^\circ \times \frac{\ell}{2}$$

$\Rightarrow S = 0.353\,P$ **(Ans.)**

**4.** *A long ladder supported at A and B, as shown in fig. a vertical load W can have any position as defined by the distance a from the bottom. Neglecting friction, determine the magnitude of the reaction $R_b$ at B. Neglect the weight of the ladder.*




**Soln.** The line of actions of $R_b$ and W meet at D, at which $R_a$ passes from A to D. The FBD is shown in the fig.



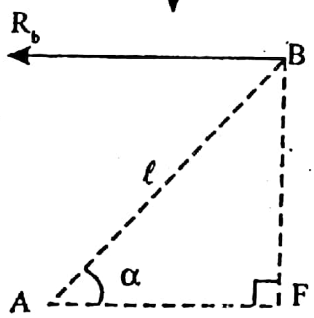

Using method of moment

$\Sigma M_A = 0$

But AE = BF

$R_b \times AE = W \times AG$

$\therefore\ R_b \times BF = W \times AG$



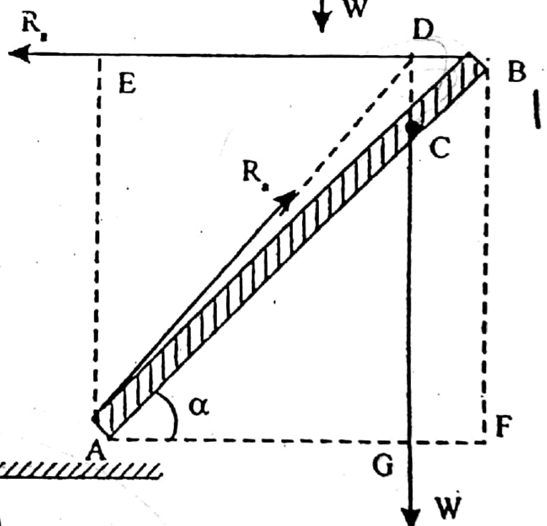

From the geometry of the fig.

$BF = \ell \sin \alpha$

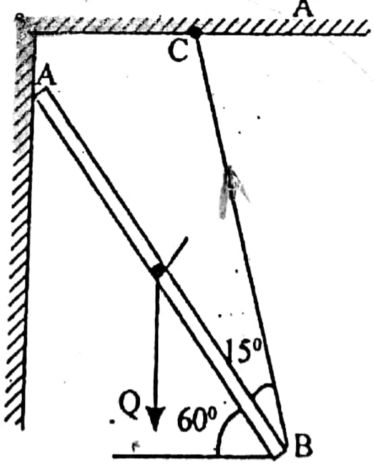
$AG = \cos \alpha$

$R_b \times \ell \sin \alpha = W \times a \cos \alpha$

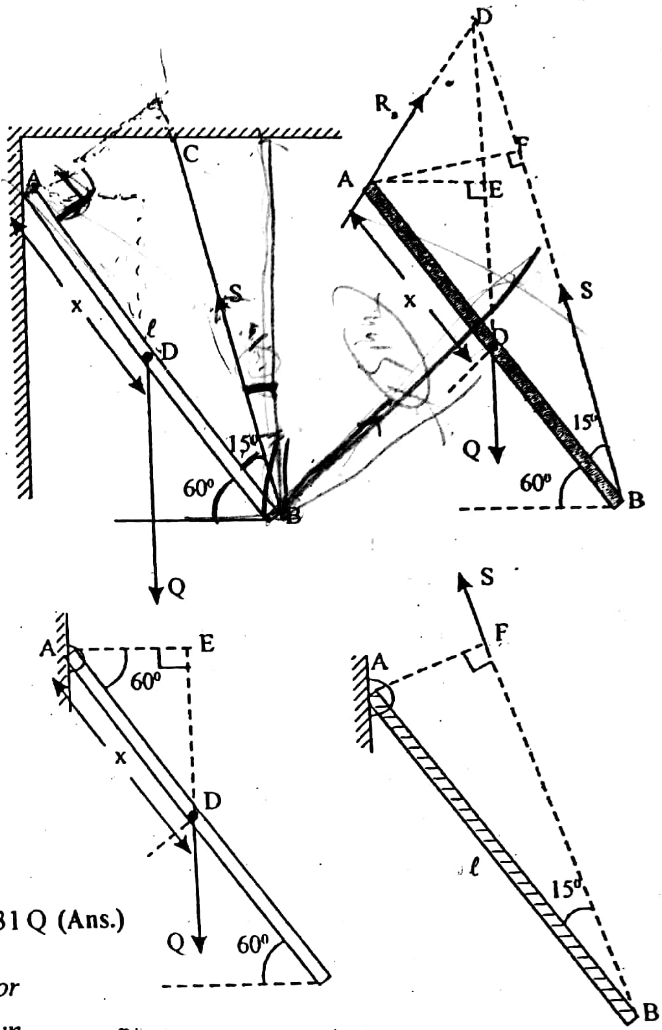
$$\Rightarrow R_b = \frac{W \times a}{\ell \tan \alpha} \quad \textbf{(Ans.)}$$




**5.** *A bar AB of length l is supported as shown in fig. At any point along its length a vertical load Q can be applied. Determine the position of this load for which the tensile force S in*



*the cable BC will be a maximum and evaluate same if the various angles are as shown in the figure. In calculation, neglect the weights of the bar and the cable.*



**Soln.** Given data

$AB = \ell$, $S_{Max} = ?$

The line of actions of S & Q meet at D at which $R_a$ passes from A to D.

Let the position of 'Q' from A along the bar AB is x. The FBD is shown in the figure.

$\Sigma M_A = 0$

$Q \times AE = S \times AF$

From the geometry of the triangles;

$AE = x \cos 60°$

$AF = \ell \sin 15°$

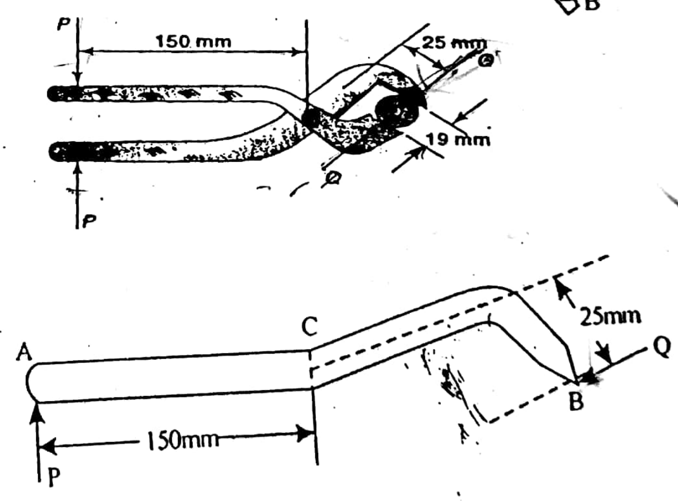
$\therefore\ S \times \ell \sin 15° = Q \times x \cos 60°$

$$\Rightarrow S = \frac{Q\ x \cos 60°}{\ell \sin 15°}$$

If $x = 0$, $S = 0$

If $x = \ell$, $S_{max} = \dfrac{Q \cos 60°}{\sin 15°} = 1.931\,Q$ **(Ans.)**

6. *A pair of adjustable plyers are used for turning a piece of 19 mm pipe as shown in fig. For the dimensions shown, what compressive forces Q are applied to the sides of the pipe when the hand grip is represented by applied collinear forces P as shown ?*

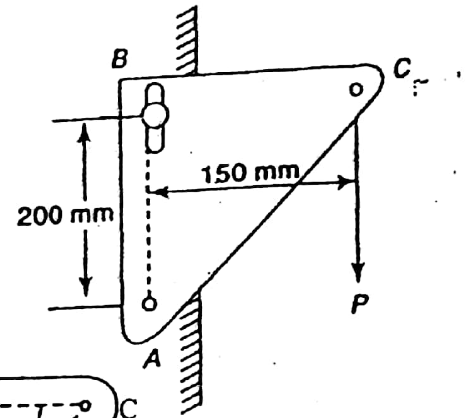


**Soln.** The plyer rotates at the fulcrum point C. The moment transmitted by P at C must be equal to the moment transmitted by 'θ' at C

Hence ; $P \times 150 = Q \times 25$

$Q = 6P$ **(Ans.)**

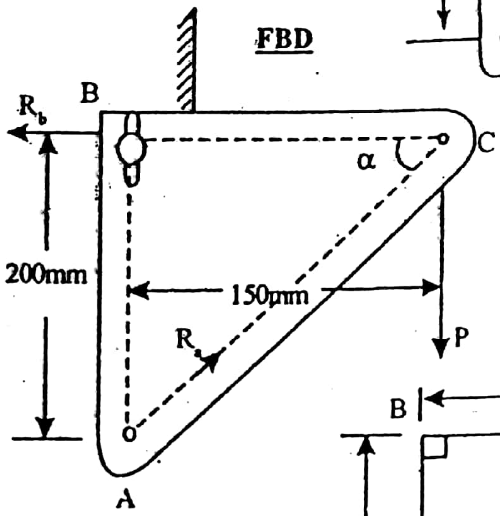
7. *A vertical load P is supported by a triangular bracket as shown in fig. Find the forces transmitted to the bolts A and B. Assume that the bolt B fits loosely in a vertical slot in the plate.*




**Soln.** $R_a = ?$

$R_b = ?$

The reaction $R_b$ at B acts from C to B as it is a guide. The reaction $R_a$ passes at the cuncurrent point 'C' from A to C.



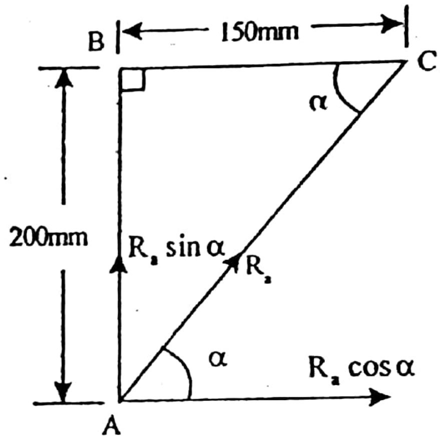

The FBD is shown in the fig.

$\Sigma M_A = 0$

$P \times 150 = R_b \times 200$

$\Rightarrow R_b = 0.75\ P$ **(Ans.)**

Let $m < BCA = \alpha$



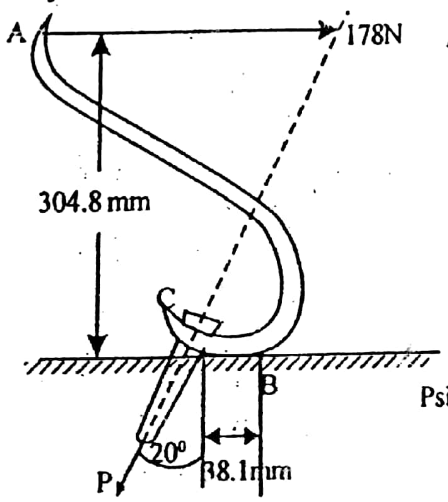

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{200}{150}\right) = 53^\circ\ 7'$$

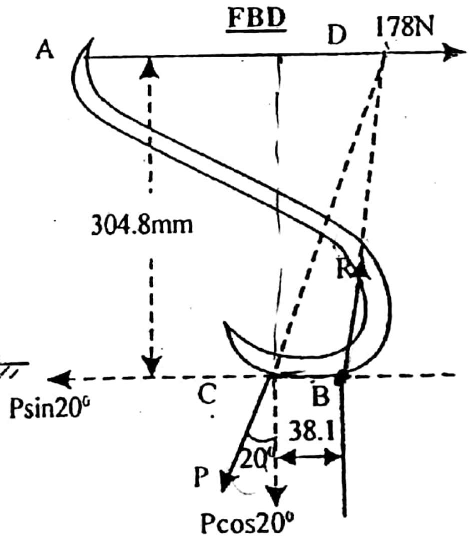
$\Sigma M_B = 0$

$R_a \cos\alpha \times 200 = P \times 150$

$$\Rightarrow R_a = \frac{P \times 150}{200 \times \cos 53^\circ\ 7'} \quad \text{or} \quad R_a = 1.249\ P$$ **(Ans.)**

8. *Find the magnitude of the pull P exerted on the nail C in fig. if a horizontal force of 178N is applied to handle of the wrecking bar as shown.*



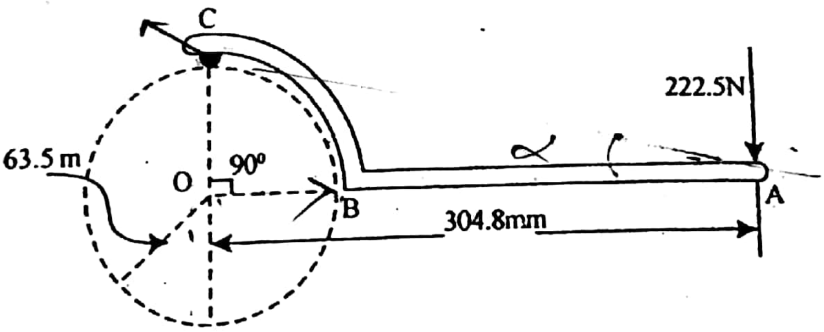

**Soln.** The line of actions of the axial pull 'P' on the nail and 178 N. Meet at D at which $R_b$ passes from B to D.

The FBD is shown in the figure

$\Sigma M_B = 0;\ P \cos 20^\circ \times 38.1 = 178 \times 304.8$

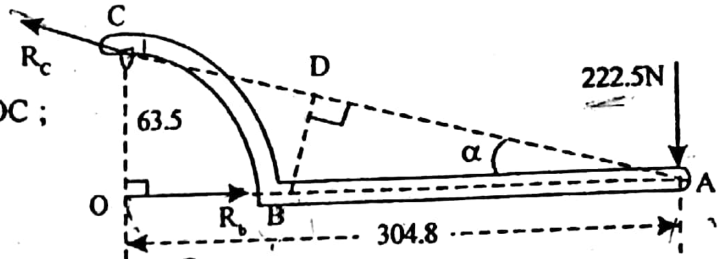
$$\Rightarrow P = \frac{178 \times 304.8}{38.1 \times \cos 20^\circ} \Rightarrow P = 1515.389\ N$$ **(Ans.)**

9. *Determine the forces exerted on the cylinder at B and C by the spanner wrench shown in fig. due to a vertical force of 222.5N applied to the handle as shown. Neglect friction at B.*



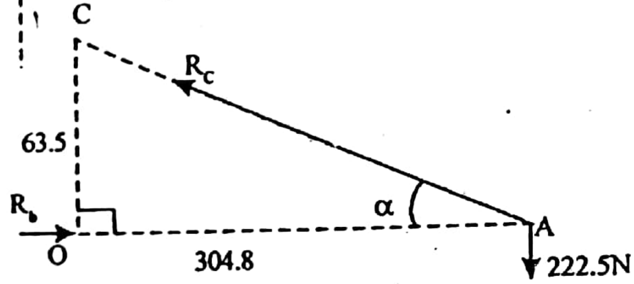

**Soln.** The line of actions of $R_b$ and 222.5N. meet at A at which $R_c$ passes from A to C.

The FBD is shown in the figure.




Let m $< CAO = \alpha$

From the geometry of the triangle AOC ;




$$\alpha = \tan^{-1}\left(\frac{63.5}{304.8}\right) = 11° 46'$$

$$\Sigma M_c = 0$$

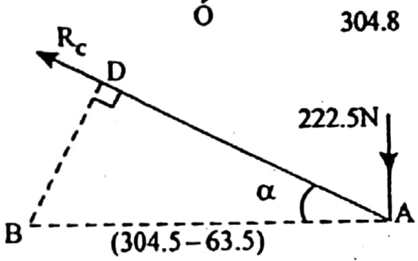
$$R_b \times 63.5 = 222.5 \times 304.8$$

$$\Rightarrow R_b = 1068 \text{ N} \qquad \textbf{(Ans.)}$$

$$\Sigma M_B = 0$$

$$R_c \times BD = 222.5 \times AB$$

Considering the triangle ABD;




$$\Rightarrow BD = (304.8 - 63.5) \sin \alpha$$

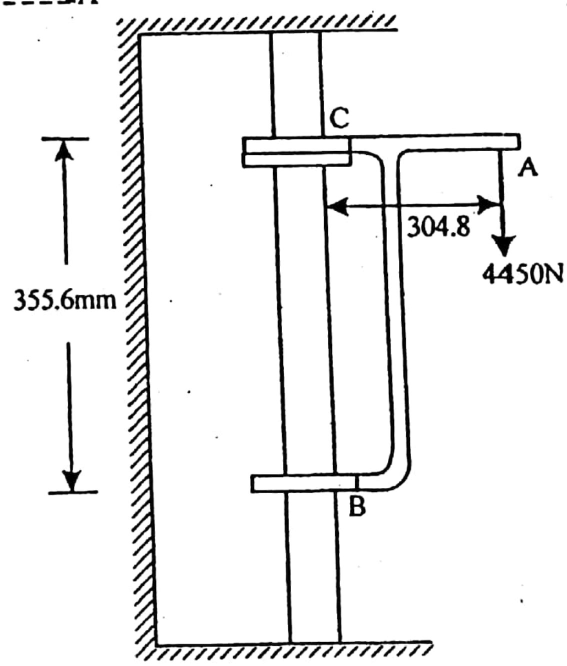
and; $AB = (304.8 - 63.5)$

Substituting the values;

$$R_c = \frac{222.5 \times (304.8 - 63.5)}{(304.8 - 63.8) \sin \alpha}$$

$$\Rightarrow R_c = \frac{222.5}{\sin 11° 46'} \Rightarrow R_c = 1091.079 \text{ N} \qquad \textbf{(Ans.)}$$

10. *A bracket ACB can slide freely on the vertical shaft BC but is held by a small collar attached to the shaft as shown in Fig. neglecting all friction. Find the reactions at B and C for the vertical load as shown.*




**Soln.** The line of actions of $R_b$ and 4450 N meet at D, at which $R_c$ passes from D to C

The FBD is shown in the figure

$$\Sigma M_C = 0$$

$$R_b \times 355.6 = 4450 \times 304.8$$

$$\Rightarrow R_b = \frac{4450 \times 304.8}{355.6}$$

$$\Rightarrow R_b = 3814.28 \quad \textbf{(Ans.)}$$

Let $m\angle CDA = \alpha$

From the $\Delta CDA$

$$\alpha = \tan^{-1}\left(\frac{355.6}{304.8}\right)$$

$$\Rightarrow \alpha = 49^\circ\, 23'$$

$$\Sigma M_B = 0$$

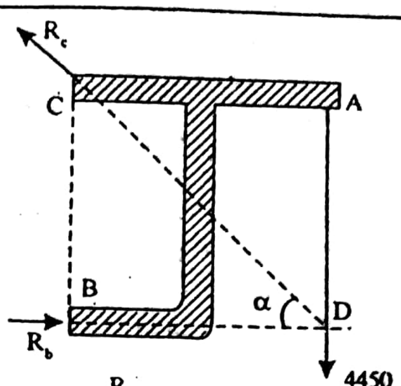
$$R_c \times BE = 4450 \times BD$$

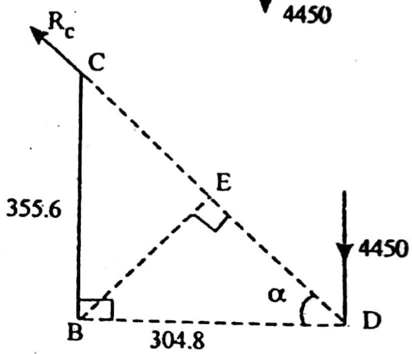
$$BE = 304.8 \sin\alpha$$

Substituting the value

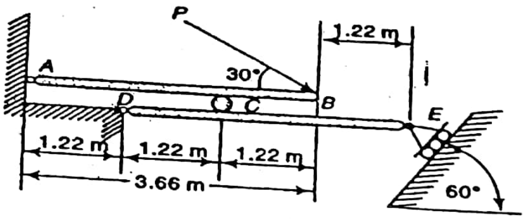
$$R_c \times 304.8 \sin\alpha = 4450 \times 304.8$$

$$\Rightarrow R_c = \frac{4450}{\sin 49^\circ\, 23'} \quad \Rightarrow R_c = 5862.34\text{ N} \quad \textbf{(Ans.)}$$




**11.** *Two beams AB and DE are arranged and supported as shown in fig. Find the magnitude of the reaction $R_c$ at E due to the force P = 890N applied at B as shown.*

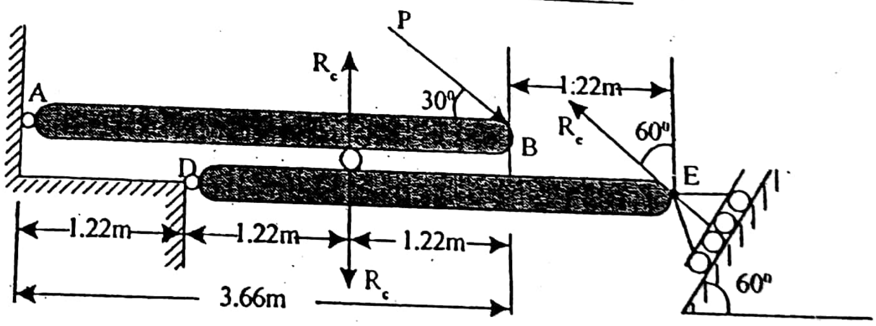



**Soln.** Given data

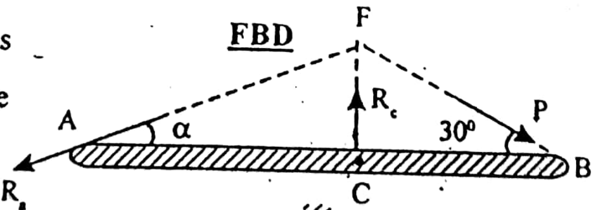
P = 890 N

**Consider the beam AB**

The line of actions of $R_c$ & P meet at 'F', at which $R_a$ passes from F to A. The FBD is shown in the figure



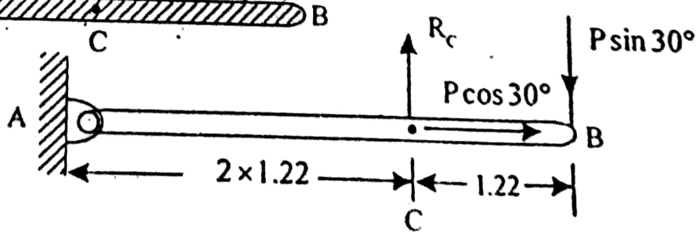

**FBD**




$$\Sigma M_A = 0$$

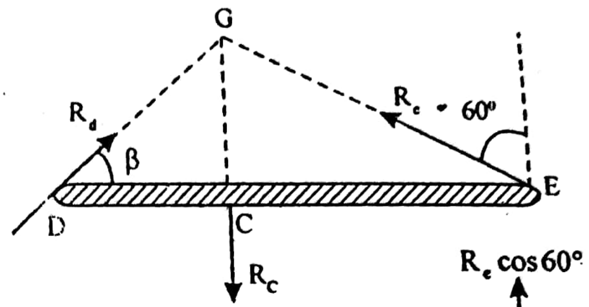
$$P \sin 30^\circ \times 3 \times 1.22 = R_c \times 2 \times 1.22$$

$$\Rightarrow R_c = 0.75\, P$$



**Consider the beam DE**

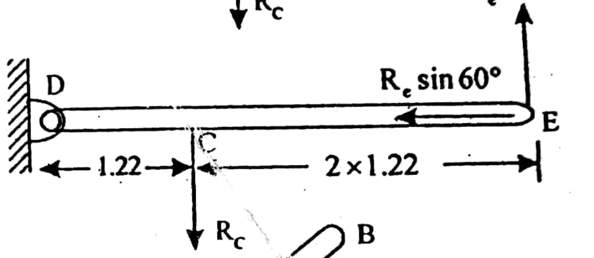
The line of actions of $R_C$ and $R_E$ meet at point G at which $R_d$ passes from D to G. Moreover $R_e$ makes 60° to the vertical.




The FBD is shown in the figure

$\Sigma M_D = 0$

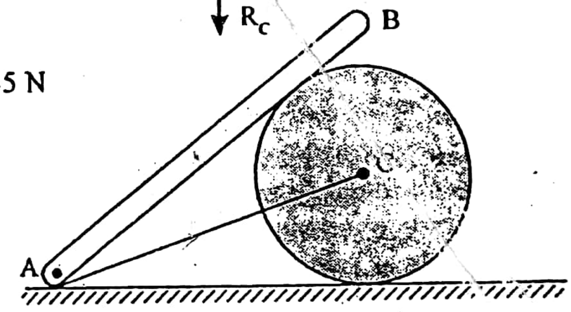
$R_e \cos 60° \times 3 \times 1.22 = R_c \times 1.22$




Substituting the value of '$R_C$'

$$R_e = \frac{0.75P}{3 \cos 60°}$$

$\Rightarrow R_e = 0.5$ P or $R_e = 0.5 \times 890 = 445$ N

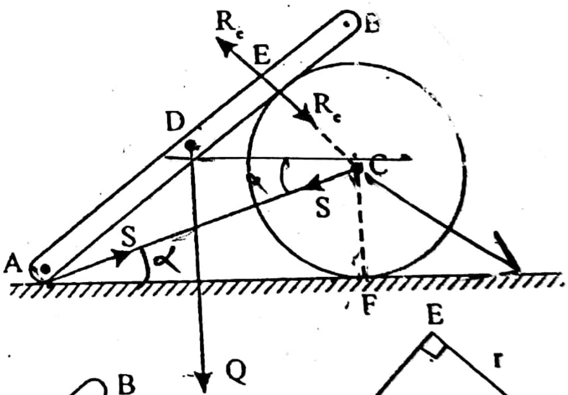



12. *A smooth right circular cylinder of radius r rests on a horizontal plane and is kept from rooling by an inclined string AC of length 2r as shown in the fig. A prismatic bar AB of length 3r and weight Q is hinged at point A and leans against the roller as shown. Find the tension S that will be induced in the string AC.*

**Soln.** Given data

AC = 2r

AB = 3r

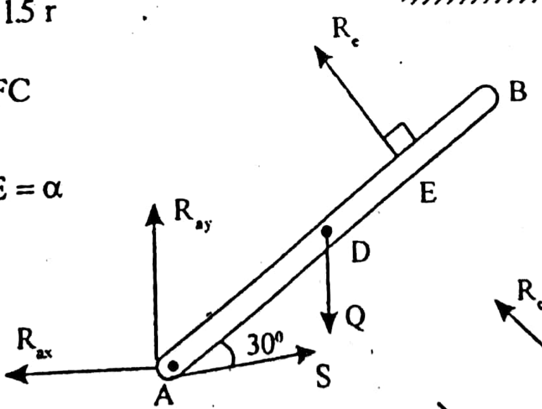


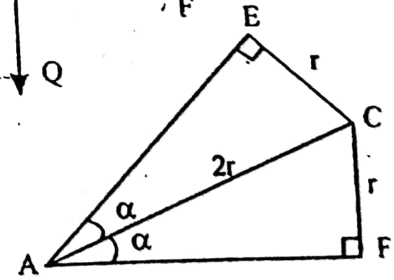

$$\therefore AD = DB = \frac{3r}{2} = 1.5 \text{ r}$$

The Δ AEC & Δ AFC are similar

$m < CAF = m < CAE = \alpha$

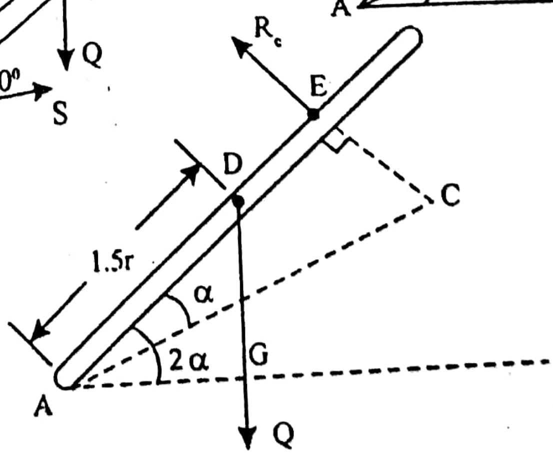
$$\therefore \ \alpha = \sin^{-1}\left(\frac{r}{2r}\right)$$




$\Rightarrow \alpha = 30°$

The FBD of the rod AB is shown in the figure.




$\Sigma M_A = 0$

$R_C \times AE = Q \times AG$ ............(i)

But from the geometry;

$AE = AC \cos\alpha$

$\Rightarrow AE = 2r \cos 30°$

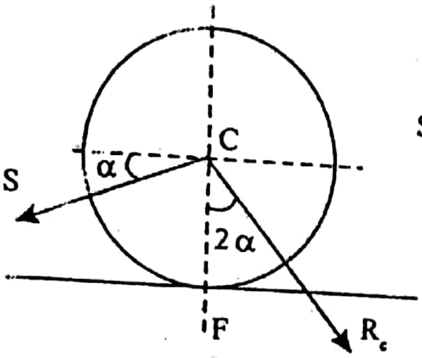
$AG = AD \cos 2\alpha$

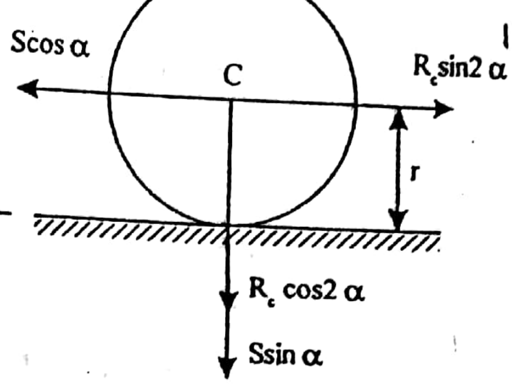
$\Rightarrow AG = 1.5r \cos 60°$

Substituting the values;

$$R_c = \frac{Q \times 1.5\, r \cos 60°}{2r \cos 30°}$$

$\Rightarrow R_c = 0.433Q$



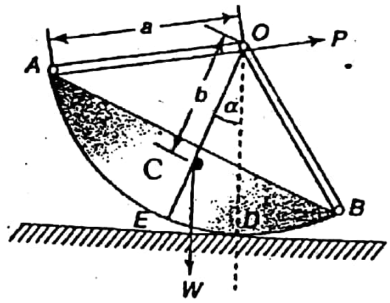

Considering the sphere, $\Sigma M_F = 0$

$S \cos\alpha \times r = R_c \sin 2\alpha \times r$

$$\Rightarrow S = \frac{0.433Q \sin 60°}{\cos 30°} \Rightarrow S = 0.433Q$$

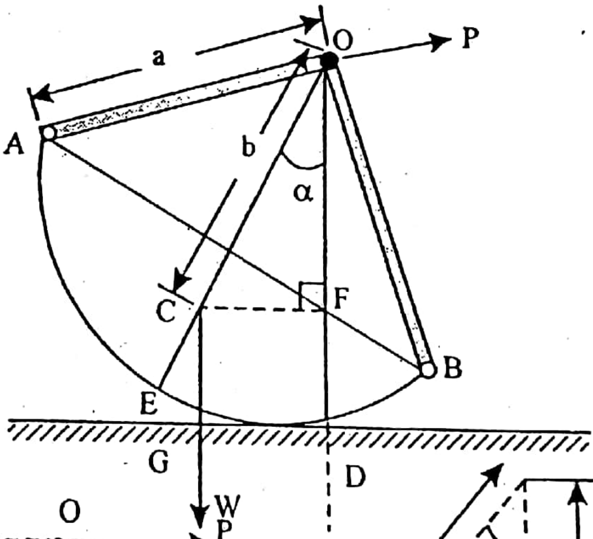
**(Ans.)**

**13.** *A rocker of weight W having a circular shoe AB of radius a and with center at O rests on a horizontal surface and is pulled by a horizontal force P applied at O, as shown in fig. Find the position of equilibrium, as defined by the angle α, which the rocker will assume if its center of gravity is at C, distance b from O along the bisecting radius OE.*




**Soln.** The line of actions of W and P meet at H, at which $R_d$ passes from D to H.

The FBD is shown in the figure.




$\Sigma M_D = 0$

$P \times OD = W \times GD$

But, $OD = OA = OB = a$

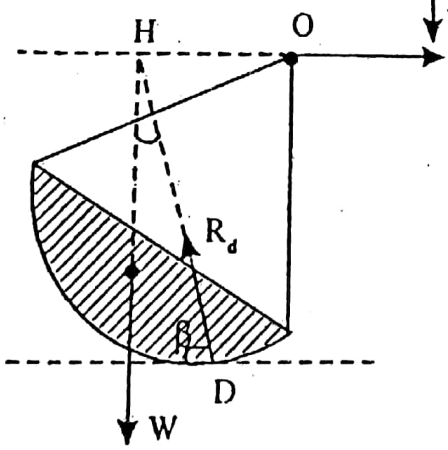
$GD = CF = b \sin\alpha$

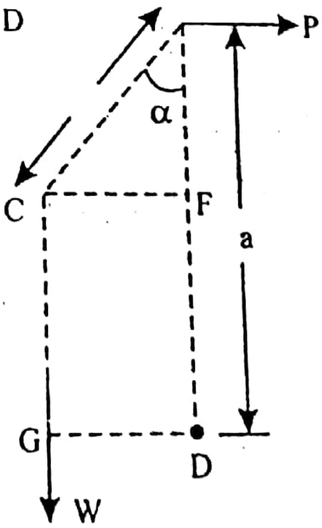
Sustituting the values

$\Rightarrow P \times a = W\, b \sin\alpha$

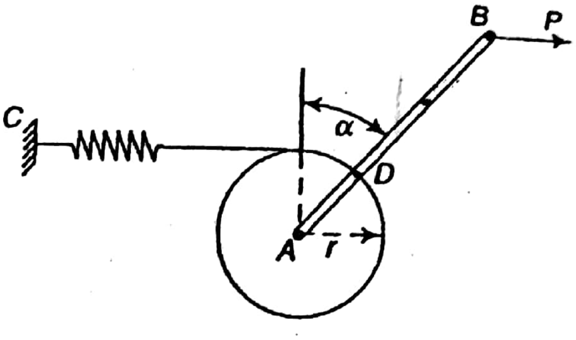
$$\Rightarrow \sin\alpha = \frac{P \times a}{W \times a}$$

or $\alpha = \sin^{-1}\left(\frac{P_a}{W_b}\right)$ **(Ans.)**



14. *A vertical prismatic bar AB of negligible weight and length, l, is hinged to a cylinder of radius r at A and supported at D by an elastic spring CD as shown in the fig. The stiffness of the spring is k and the spring is underformed when $\alpha = 0°$. The horizontal force P is applied to the bar AB at B as shown in figure. Find the position of equilibrium, as defined by the angle $\alpha$, in terms of P, k, l and r.*




**Soln.** AB = $\ell$

Let x is the displacement of the spring

$\therefore$ spring force S = k.x

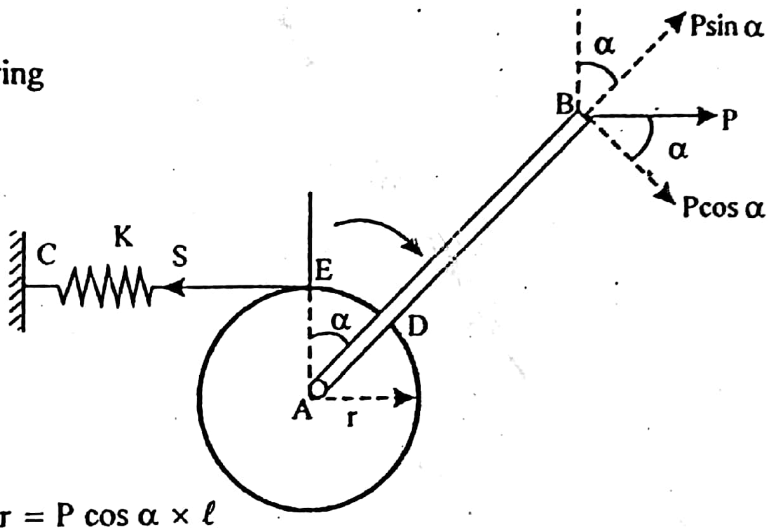
when $\alpha = 0, S = 0$

When $\alpha$ is more than 0°;

the spring deforms from E to D

$\therefore$ ED = r$\alpha$

$\therefore$ S = kr $\alpha$




Using method of moment;

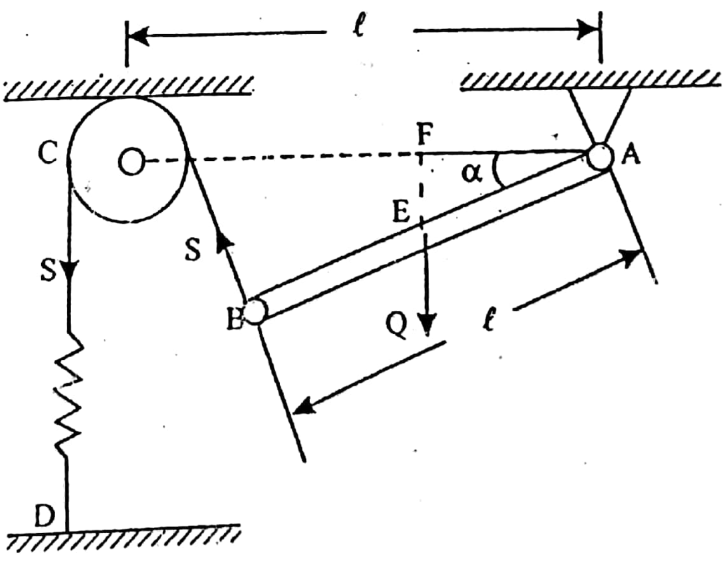
$\Sigma M_A = 0$

$S \times r = P\cos\alpha \times \ell \Rightarrow kr\,\alpha \times r = P\cos\alpha \times \ell$

$$\Rightarrow kr^2\alpha = P\cos\alpha \times \ell \Rightarrow \cos\alpha = \frac{kr^2\alpha}{P\ell}$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{kr^2\alpha}{P\ell}\right) \qquad \text{(Ans.)}$$

15. *A prismatic bar AB of weight Q and length l is hinged at A and supported at B by an elastic spring that passes over a pulley C. The spring is fixed at the other end D as shown in fig. The distance between the hinge A and the pulley C is equal to the length l of the bar AB. The stiffness of the spring is k and the spring is unstretched when the bar AB is horizontal. Find the configuration of equilibrium of the system as defined by the angle $\alpha$, which the bar makes with the horizontal as shown in figure in terms of Q, k, l.*



**Soln.** Let x is the displacement of the spring

$\therefore$ spring force $S = kx$

when $\alpha = 0$

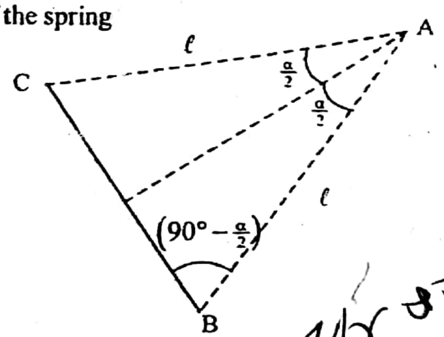
$S = 0$

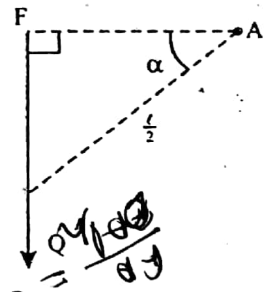
when $\alpha > 0$

$S = kx$

where $x = CB$

From the $\Delta$ ACB



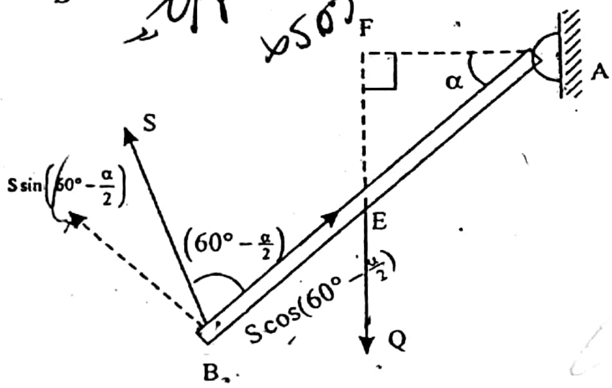

$$x = BC = 2\ell \sin\left(\frac{\ell}{2}\right)$$

$$\therefore S = K \times 2\,\ell \sin\frac{\alpha}{2}$$

$$\Sigma M_A = 0$$

$$S \sin\left(90° - \frac{\alpha}{2}\right) \times AB = Q \times AF$$

$$AF = \frac{\ell}{2}\cos\alpha, \quad AB = \ell$$



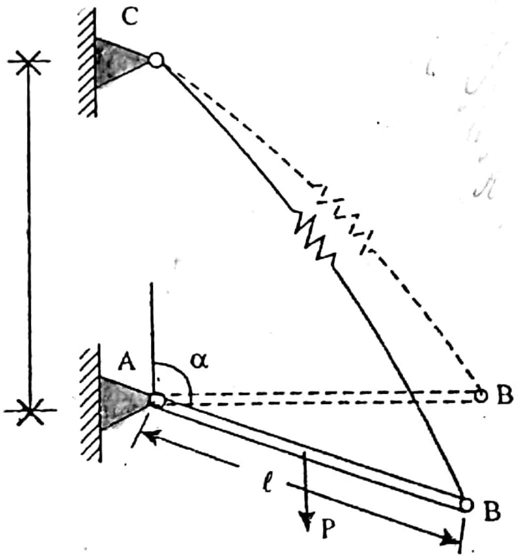

Substituting the values

$$2k\,\ell \sin\frac{\alpha}{2} \times \cos\frac{\alpha}{2} \times \ell = Q \times \frac{\ell}{2}\cos\alpha \quad \Rightarrow k\ell^2 \sin\alpha = \frac{Q\ell}{2}\cos\alpha$$

$$\Rightarrow \tan\alpha = \frac{Q}{2k\ell} \qquad \Rightarrow \alpha = \tan^{-1}\left(\frac{Q}{2k\ell}\right)$$

**(Ans.)**

*A Prismatic bar AB of weight P is hinged at point A and attached to a spring at point B and as shown in fig. The stiffnes of spring is k and the spring is undeformed when α = 90°. Find the α angle corresponding to equilibriumin terms of P, k and l as shown in the fig.*

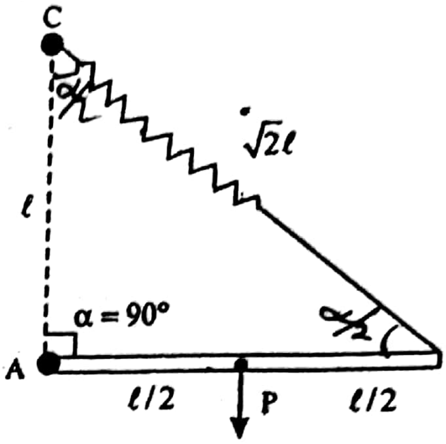



**Soln.** $T\sin\left(90 - \alpha\frac{1}{2}\right)\ell = P\frac{1}{2}\sin\alpha$

$k\ell\left\{2\sin\alpha\frac{1}{2} - \sqrt{2}\right\}\cos\alpha\frac{1}{2}$

$$k\ell^2 \cos\alpha\frac{1}{2}\left(2\sin\alpha\frac{1}{2}-\sqrt{2}\right)$$

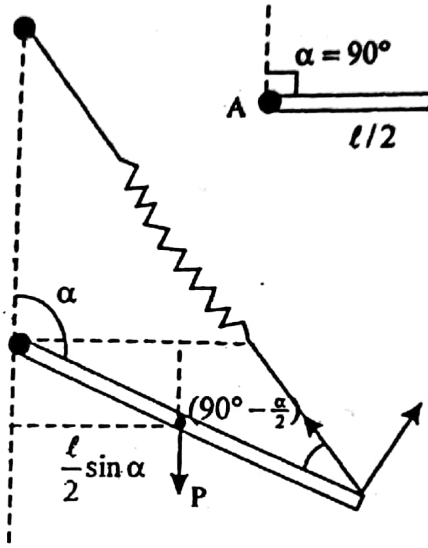
$$= P\ell\sin\alpha\frac{1}{2}\cos\alpha\frac{1}{2}$$




$$\ell' = \sqrt{\ell^2+\ell^2-2\ell^2\cos\alpha}$$

$$= 2\ell\sin\left(\frac{\alpha}{2}\right)$$

$$x = \left(2\sin(\alpha/2)-\sqrt{2}\right)\ell$$



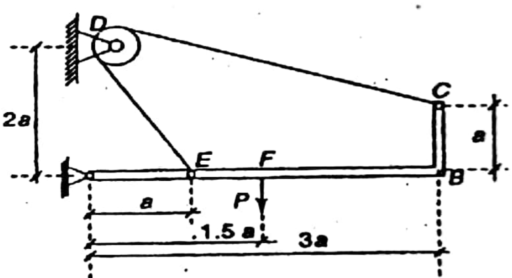

$$2k\ell^2\sin\left(\alpha\frac{1}{2}\right)-\sqrt{2}\,k l^2 = P l\sin\alpha\frac{1}{2}$$

$$\sin\alpha\frac{1}{2}\left[2kl^2 - Pl\right] = \sqrt{2}\,k l^2$$

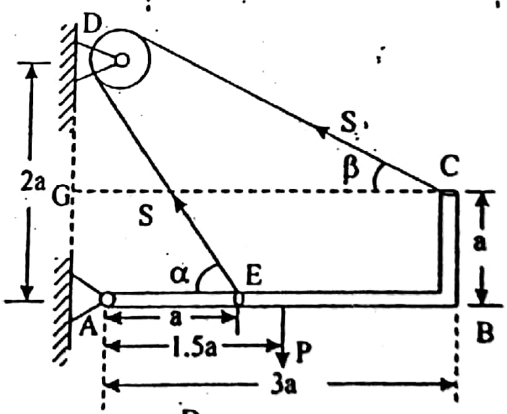
$$\sin\alpha\frac{1}{2} = \frac{\sqrt{2}\,kl}{2kl-P}$$

$$\left(\frac{\alpha}{2}\right) = \sin^{-1}\left(\frac{\sqrt{2}\,kl}{2kl-P}\right)$$

**17.** *Find the tension S induced in the string CDE attached at the points C and E of the right angle bar ABC of weight P supported as shown in fig. Assume a perfectly flexible string, frictionless pulley, and an ideal hinge at A.*

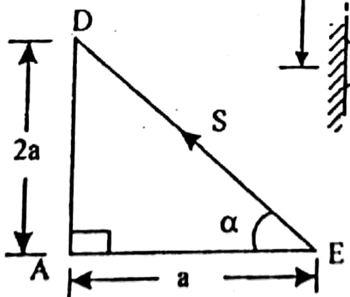



**Soln.** Let $m\angle DEA = \alpha$




From the triangle AED

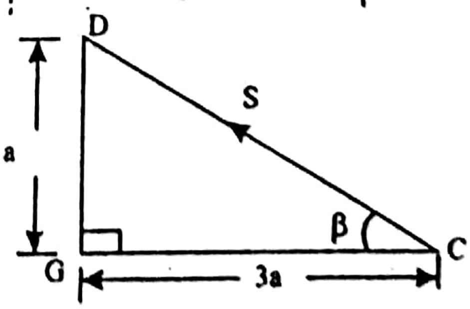
$$\alpha = \tan^{-1}\left(\frac{2a}{a}\right),$$



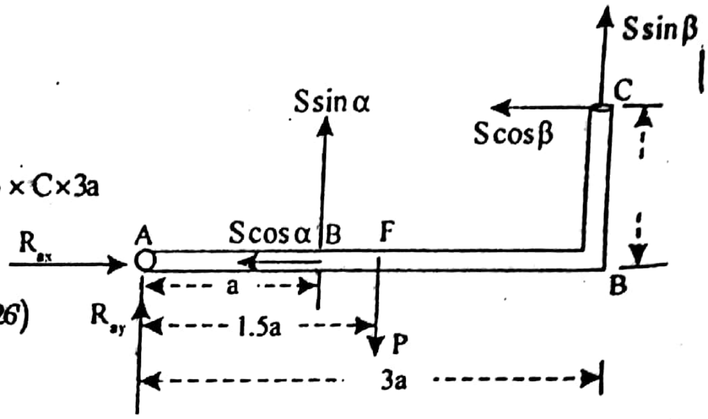

or $\alpha = \tan^{-1}(2) = 63°26'$

Let $m\angle DCG = \beta$

$$\therefore\ \beta = \tan^{-1}\left(\frac{a}{3a}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 18°\ 26'$$



The FBD of the angle bar is shown in the figure




$\Sigma M_A = 0$

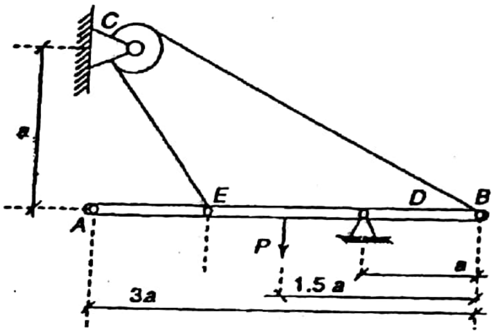
$P \times 1.5\,a = S \sin\alpha \times a + S\cos\beta \times a + S \sin\beta \times C \times 3a$

$\Rightarrow P \times 1.5 = S(\sin\alpha + \cos\beta + 3\sin\beta)$

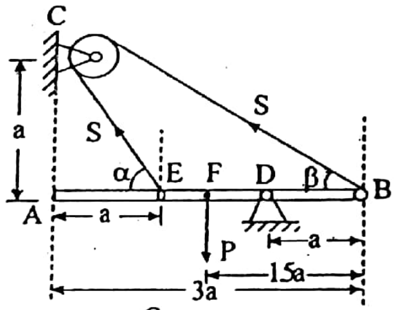
$\Rightarrow 1.5\,P = S(\sin 63^\circ 26' + \cos 18^\circ 26' + 3\sin 18^\circ 26')$

$\Rightarrow S = 0.537\ P$

**18.** *Find the tension S induced in the string ACB attached at the points A and B of a prismatic bar AB of weight P supported as shown in fig. Assume a perfectly flexible string, frictionless pulley with negligible weight and dimensions, and an ideal hinge at D.*




**Soln.** Let $m < CEA = \alpha$




From the geometry of the triangle

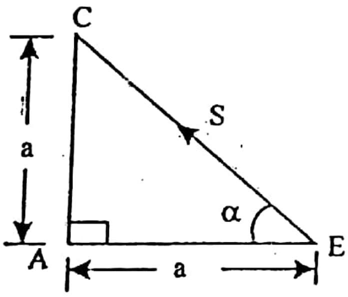
$$\alpha = \tan^{-1}\left(\frac{a}{a}\right)$$

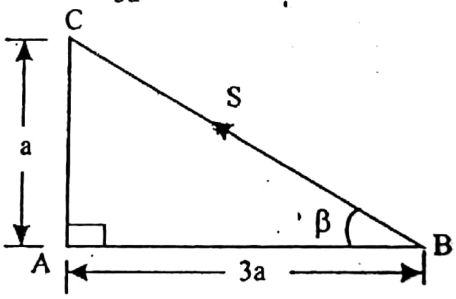
$$= \tan^{-1}(1) = 45^\circ$$

Now let $m < CBA = \beta$

$$\therefore \beta = \tan^{-1}\left(\frac{a}{2a}\right)$$

$$= 18^\circ 26'$$




The FBD is shown in the figure

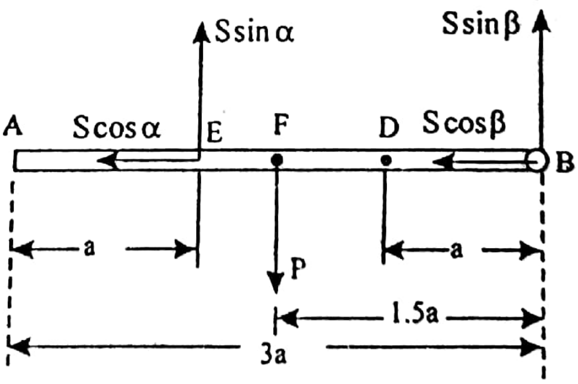
$\Sigma M_D = 0$

$P \times 0.5\,a + S\sin\beta \times a = S\sin\alpha \times a$

$$\Rightarrow P = \frac{S(\sin\alpha - \sin\beta)}{0.5} \Rightarrow S = \frac{0.5\,P}{(\sin\alpha - \sin\beta)}$$

$$\Rightarrow S = \frac{0.5\,P}{(\sin 45^\circ - \sin 18^\circ 26')}$$

$\Rightarrow S = 1.279\ P$ **(Ans.)**

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Chapter - 2.7

# Friction

## 2.7.1 Introduction

When a rigid body slides over another rigid body, a resisting force is exerted at the surface of contact called force of friction or simply friction. It always acts tangential to the surface of contact and in a direction, opposite to the tendency of motion or to the direction of motion. The major cause of such friction is believed to be the interlocking of microscopic protuberances (that is, minute projections on the surfaces) which oppose the relative motion. Such microscopic projections are always present howsoever smooth the surfaces may be. The forces of friction are present throughout in nature. Sometimes it is undesirable. But it can not be eliminated completely. It can be reduced to some extent by suitable means.

## 2.7.2 Classification

(i) Static friction, (ii) Dynamic friction

## 2.7.3 Static Friction

It is the force of friction exerted between two contact surfaces when a body tends to move over another body.

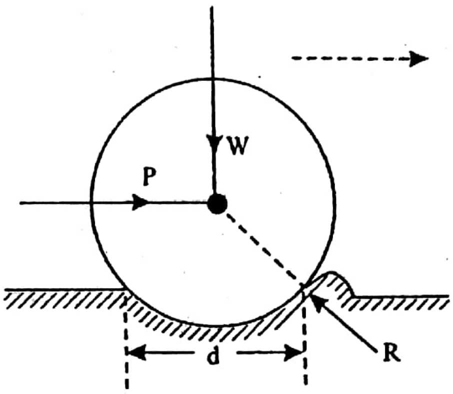
## 2.7.4 Dynamic Friction

It is the force of friction exerted between two contact surfaces when a body moves over another body. (Force of dynamic friction is approximately equal to the maximum static friction. It decreases slightly if speed increases). The dynamic friction is of two types (i) Sliding friction, (ii) Rolling friction.

**Sliding Friction:** It is the force of friction between two contact surfaces when a body slides over another body.

**Rolling Friction:** It is the force of friction between two contact surfaces when a body rolls over another body.

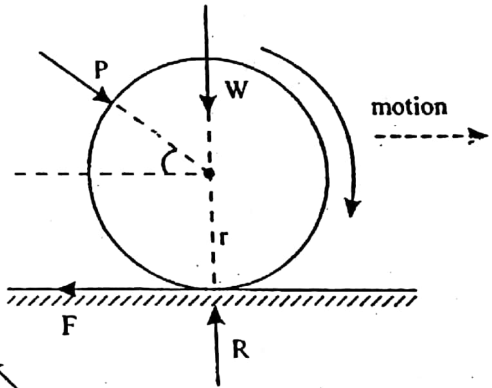
## 2.7.5 Rolling Resistance



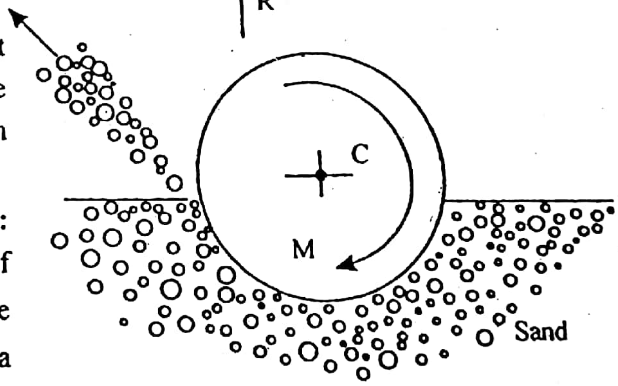

When a sphere or wheel rolls on the ground, the point of the sphere in contact with the ground at any instant, has no relative motion with respect to the ground. Thus a large amount of frictional force is eliminated. But in practice the resistance to the rolling motion exists. It is only due to the deformation of the surface upon which the sphere rolls. Thus, the contact between the roller and the ground is not limited to a single point but extends over an area. Hence the distance 'd' is refered as 'the forward length of deformation" and is defined as the coefficient of rolling resistance.

**Force of friction on a Roller**

**Case - I When a force is applied.**

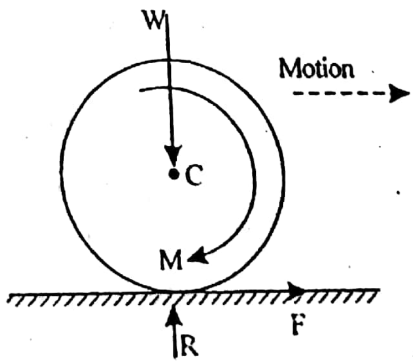



When a force P is applied to a roller or wheel, the basic tendency of applied force is to move the roller forward. The force of friction acts at the contact surface, which opposes the motion. Moreover frictional force 'F' will exert a turning moment ( $F \times r$ ) about the centre 'C', thus making the roller rolls along the direction of motion (clockwise in this case). Whether the roller rolls or slides or rolls and slides, it depends upon the amount of friction present. If the body rolls without sliding, then $F < \mu R$ . But when the body tends to slide or slip then $F = \mu R$ .




**Case - II When a torque is applied in the absence of friction:**

When a roller is applied with torque 'M' (in case of an automobile wheel - powered by the engine) in the absence of friction (for an example, the wheels of a car sunk in the sand) then the roller continues to spin throwing back sand, without moving forward.



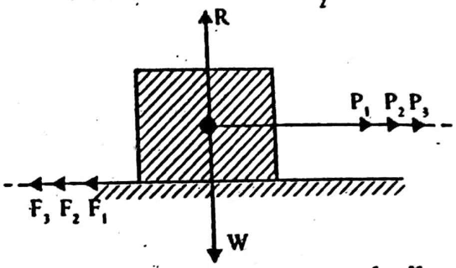

**Case - III When a torque is applied in the presence of friction:**

When sufficient friction is provided, the roller moves forward. When the roller is powered with a torque, the basic tendency is to rotate the roller and it is the friction that provides the necessary forward force to make it move. Hence the friction in this case acts in the same direction of motion of the roller.

## 2.7.6 LIMITING FRICTION (MAX. STATIC FRICTION)

Let a body of weight W rest on a horizontal surface. If a force of magnitude $P_1$ is applied and the body does not tend to move, then the force of friction $F = P_1$. If a slightly greater force $P_2$ is applied and the body does not tend to move, then the force of friction $F = P_2$.

Similarly if a slightly greater force $P_3$ is applied and the body just tends to move, then force of friction $F = F_3$ i.e. the maximum value of force of friction or called limiting friction.



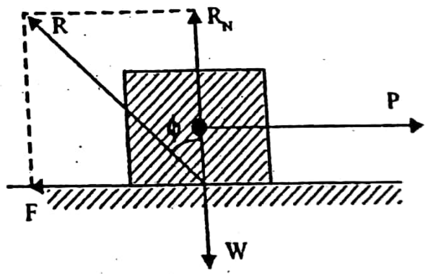

If we increase the applied forces to $P_4$, $P_5$, $P_6$......., then the body moves gradually with an accelerating speed but the force of friction remains same as $F = P_3$. But it slightly decreases at high speed.

Hence the limiting friction is the maximum force of friction between two contact surfaces when a body tends to move over another body.

[Note : If $P = 0$, then $F = 0$, i.e., force of friction varies from zero to limiting value]

## 2.7.7 ANGLE OF FRICTION ($\phi$)

Let $R_N$ be the normal reaction due to active force W and F be maximum force of friction due to applied force P. Applying parallelogram law of forces to $R_N$ and F, the resultant force




$$R = \sqrt{R_N^2 + F^2} \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{F}{R_N}\right)$$

$R = \sqrt{R_N^2 + f}$

Where $\phi$ is the direction of resultant R with the normal reaction $R_N$, called angle of friction.

Hence angle of friction ($\phi$) is the maximum angle made by resultant reaction and normal reaction at which the body just tends to move over another body.

## 2.7.8 COEFFICIENT OF FRICTION ($\mu$)

From the above figure, we have derived an equation

$$\tan\phi = \frac{F}{R_N} \quad \text{or} \quad F = \tan\phi \cdot R_N$$

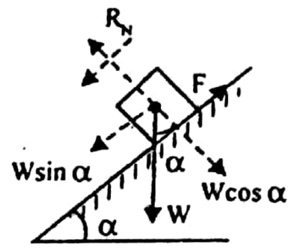
Since we know that $\phi$ is constant for the particular surfaces, $\tan\phi$ is also constant.

i.e., $\quad F \propto R_N$

or $F = \text{constant} \times R_N$

where constant of propertionallity is

$\tan\phi = \mu$, called coefficient of friction.



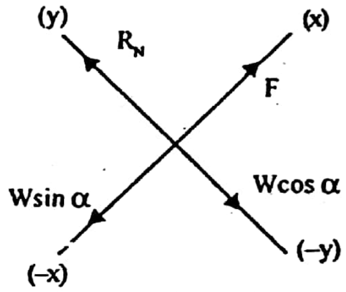

or $F = \mu R_N$

or $\mu = \dfrac{F}{R_N}$

Hence coefficient friction $(\mu)$ is the ratio between maximum force of friction and the normal reaction when a body just tends to move over another body.

## 2.7.9 ANGLE OF REPOSE ($\alpha$)

Consider a body of weight W is at rest on an inclined plane of angle $\alpha$. The normal reaction, force of friction and the components of W are shown in the figure below.




Let the angle of inclination ($\alpha$) be gradually increased, till the body just tends to slide down the plane. At these limiting conditions of equilibrium;

$\Sigma x = 0$

$\therefore\ F = W\sin\alpha$

or $\mu R_N = W\sin\alpha$ .........(i)

$\left(\text{Because } \dfrac{F}{R_N} = \mu\right)$

Similarly $\Sigma Y = 0$ $\quad R_N = W\cos\alpha$ .......(ii)

Solving equations (i) & (ii)

$\mu(W\cos\alpha) = W\sin\alpha$

or $\tan\alpha = \mu$

or $\tan\alpha = \tan\phi$ (Because $\mu = \tan\phi$)

or $\alpha = \phi$

i.e., maximum angle of inclination is equal to maximum angle of friction.

Hence angle of repose is the maximum angle of inclination at which the body just tends to move down the plane under the action of the component force of self weight. It is equal to the angle of friction.

### 2.7.10 Laws of Friction (Static & Dynamic)

(1) The force of friction always acts tangential to the surfaces of contact and in a direction opposite to the tendency of the body to move, or acts opposite to the direction of motion in case of dynamics.

(2) The force of friction varies from zero to its maximum value, depends upon the resultant force tending to cause the motion.

(3) The ratio between maximum force of friction and normal reaction remains constant for the particular contact surfaces i.e., $\mu$ remains constant.

(4) Force of friction is independent of the area and shape of contact surfaces but depends upon surface roughness.

(5) For average speed the limiting force of friction remains constant. But it slightly decreases at high speed. [Accordingly '$\mu$' decreases slightly at high speed]

### 2.7.11 Nature of Friction

There are generally two types of frictions according to the nature

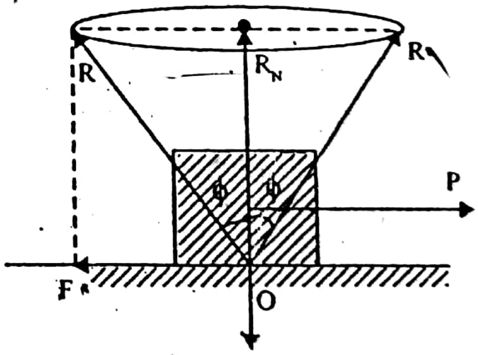
(i) Dry friction or solid friction, (ii) Fluid friction

(i) **Dry friction (coulomb friction) :** It is the force of friction exterted between two direct contact surfaces which are dry Ex- (i) Sliding friction, (ii) Rolling friction

(ii) **Fluid friction :** It is the force of friction exerted between two indirect contact surfaces when they are separated by some fluids or between two fluid layers.

Ex- greasy oil, lubricants, water etc. are when introduced between two surfaces

### 2.7.12 Cone of Friction




From the condition equilibrium, we know that the resultant $R = \sqrt{F^2 + R_N^2}$ makes an angle $\phi$ with the normal reaction : Since $\phi$ is maximum, there will be no any other resultant force beyond R. Keeping 'O' as vertex and the line of the action of the normal reaction as the axis, if the resultant reaction is rotated at angle $\phi$, a cone will

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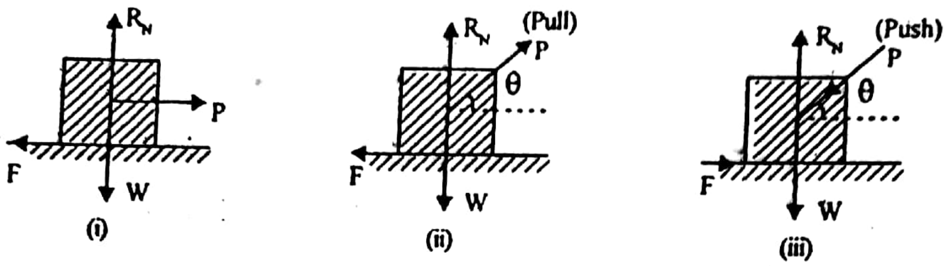
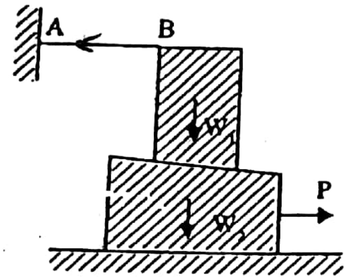
### 2.7.13 EQUILIBRIUM OF BODIES ON ROUGH HORIZONTAL PLANE

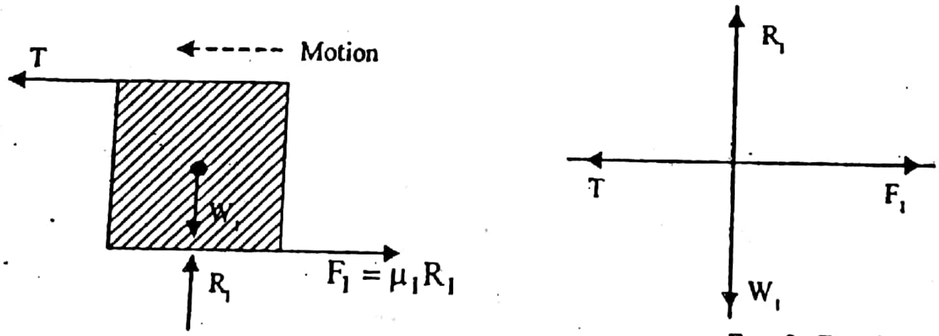
(a) For a single body on horizontal plane; $\sum x = 0, \sum y = 0$ and then solve the problem for unknown values. Everywhere $F = \mu R_N$.




(b) [AB is a string & $\mu_1$ & $\mu_2$ are the coefficient of frictions between $W_2$ and horizontal surface, and between $W_1$ & $W_2$ respectively]

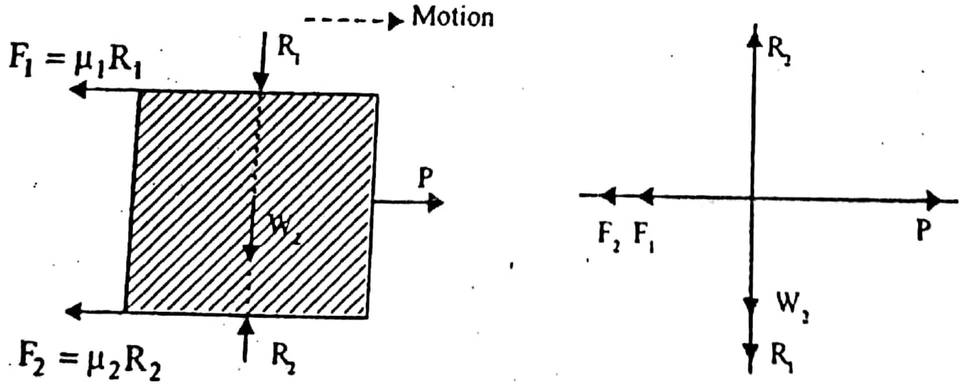



For two bodies on horizontal plane, draw free body diagrams of each and apply conditions of equilibrium for this said problem.




$\Sigma x = 0, \ \Sigma y = 0$

$R_1$ is the normal reaction, T be the tension in the string AB and $F_1$ be the force of friction acts rightward due to the tendency of the body -1 to move left.

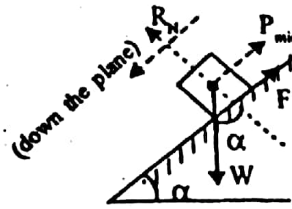



$\Sigma x = 0, \ \Sigma y = 0$

$R_2$ be the normal reaction at the horizontal surface, $R_1$ be the equal opposite normal reaction due to the body -1, $F_1$ and $F_2$ are force of frictions due to the tendency of body -2 to move rightward.

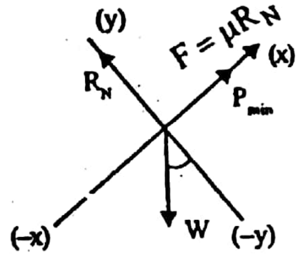
## 2.7.14 EQUILIBRIUM OF BODIES ON ROUGH INCLINED PLANE

(a) **When the tendency of the body is to move down the plane and it is subjected a force along the inclined plane.**




The applied force required is minimum and the force of friction will act up the plane.

∴ For equilibrium of the body;




$\Sigma y = 0$

∴ $R_N = w \cos\alpha$ (i)

$\Sigma x = 0$

$P_{min} + F = W \sin\alpha$

or $P_{min} + \mu R_N = W \sin\alpha$ (ii)

Solving equation (i) and (ii)

$$P_{min} + \mu\,(W\cos\alpha) = W\sin\alpha$$

or $$P_{min} = W(\sin\alpha - \mu\cos\alpha) \quad \text{(iii)}$$

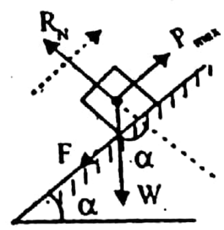
But we know that $\mu = \tan\phi = \dfrac{\sin\phi}{\cos\phi}$

∴ The equation (iii) becomes $P_{min} = w\left(\sin\alpha - \dfrac{\sin\phi}{\cos\phi}\cos\alpha\right)$ or $P_{min} = \dfrac{w\sin(\alpha - \phi)}{\cos\phi}$

(b) **When the tendency of the body is to move up the plane and applied force is along the inclined plane.**

The applied force will be maximum and the force of friction will act down along the plane.

For equilibrium of the body;



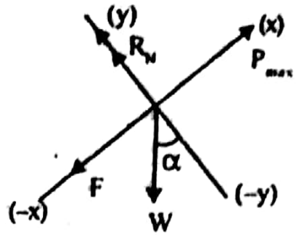

$\Sigma y = 0$

$R_N = W\cos\alpha$ .........(i)

$\Sigma x = 0$

$P_{max} = F + W\sin\alpha$

or $\quad P_{max} = \mu R_N + W \sin\alpha \quad$ ......(ii)




Solving equation (i) and (ii)

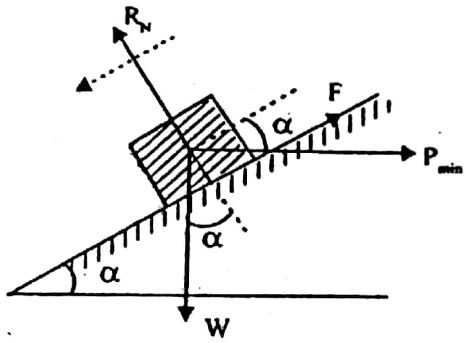
$$P_{max} = \mu W\cos\alpha + W\sin\alpha$$

$$= W(\mu\cos\alpha + \sin\alpha) \text{ ......(iii)}$$

But we know that $\mu = \tan\phi = \dfrac{\sin\phi}{\cos\phi}$

$$\therefore \quad P_{max} = W\left(\frac{\sin\phi}{\cos\phi}\cos\alpha + \sin\alpha\right) \quad \text{or} \quad P_{max} = \frac{W\sin(\alpha+\phi)}{\cos\phi}$$

(c) **When the tendency of the body is to move down the plane and force applied horizontally.**



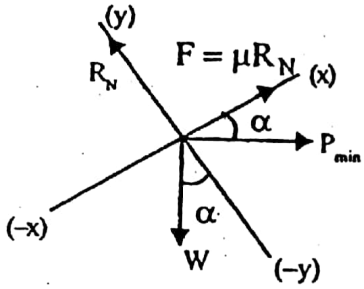

The applied force is minimum and the force of friction will act up the plane.

For equilibrium of the body ;

$\Sigma y = 0$

$R_N = W\cos\alpha + p_{min}\sin\alpha \quad$ .......(i)

$\Sigma x = 0$




$P_{min}\cos\alpha + F = W\sin\alpha$

or $P_{min}\cos\alpha + \mu R_N = W\sin\alpha \quad$ ......(ii)

Solving equation (i) and (ii)

$P_{min}\cos\alpha + \mu(W\cos\alpha + P_{min}\sin\alpha) = W\sin\alpha$

or $P_{min}(\cos\alpha + \mu\sin\alpha) = W(\sin\alpha - \mu\cos\alpha)$

But we know that $\mu = \tan\phi = \dfrac{\sin\phi}{\cos\phi}$

Substituting , $P_{min}\left(\cos\alpha + \dfrac{\sin\phi}{\sin\phi}\sin\alpha\right) = W\left(\sin\alpha - \dfrac{\sin\phi}{\cos\phi}\cos\alpha\right)$

or $\quad P_{min}\dfrac{\cos(\alpha-\phi)}{\cos\phi} = W\dfrac{\sin(\alpha-\phi)}{\cos\phi}$

or $\quad P_{min} = W\dfrac{\sin(\alpha-\phi)}{\cos(\alpha-\phi)}$

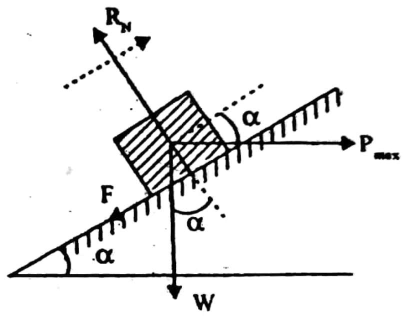
or $\quad P_{min} = W \tan(\alpha - \phi)$.............(when $\alpha > \phi$)

$= W \tan(\phi - \alpha)$.............(when $\phi > \alpha$)

**(d). When the tendency of the body is to move up the plane and the applied force is horizontal**

The applied force is maximum and the force of friction will act down the plane

For equilibrium of the body;




$\sum y = 0$

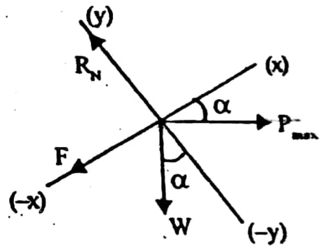
$\therefore R_N = W\cos\alpha + p_{max}\sin\alpha$ ....(i)

$\sum x = 0$

$p_{max}\cos\alpha = F + W\sin\alpha$

But $F = \mu R_N$

$\therefore P_{max}\cos\alpha = \mu R_N + W\sin\alpha$ .....(ii)




Solving equations (i) and (ii)

$P_{max}\cos\alpha = \mu(W\cos\alpha + P_{max}\sin\alpha) + W\sin\alpha$

or $P_{max}(\cos\alpha - \mu\sin\alpha) = W(\mu\cos\alpha + \sin\alpha)$

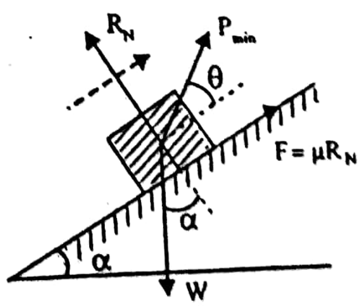
But we know that $\mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$

$$\therefore P_{max}\left(\cos\alpha - \frac{\sin\phi}{\cos\phi}\sin\alpha\right) = W\left(\frac{\sin\phi}{\cos\phi}\cos\alpha + \sin\alpha\right)$$

$$\text{or } P_{max}\frac{\cos(\alpha-\phi)}{\cos\phi} = \frac{W\sin(\alpha+\phi)}{\cos\phi}$$

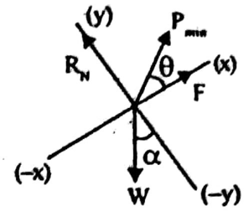
$$\text{or } P_{max} = \frac{W\sin(\alpha+\phi)}{\cos(\alpha+\phi)} \qquad \text{or } P_{max} = W\tan(\alpha+\phi)$$

**(e) When the tendency of the body is to move down the plane and the applied force is inclined to inclined plane.**




The applied force is minimum and the force of friction will act up the plane.

For equilibrium of the body;




$\Sigma y = 0$

$\therefore\ R_N + P_{min} \sin\theta = W\cos\alpha$ (i)

$\Sigma x = 0$

$\therefore\ P_{min}\cos\theta + F = W\sin\alpha$

or $P_{min}\cos\theta + \mu R_N = W\sin\alpha$ (ii)

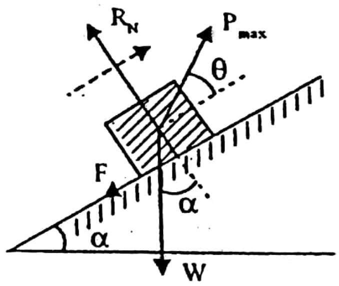
solving equations (i) and (ii) $P_{min}\cos\theta + \mu(w\cos\alpha - P_{min}\sin\theta) = W\sin\alpha$

or $P_{min}(\cos\theta - \mu\sin\theta) = W(\sin\alpha - \mu\cos\alpha)$ but $\mu = \tan\phi = \dfrac{\sin\phi}{\cos\phi}$ substituting,

$$\therefore\ P_{min}\left(\cos\theta - \frac{\sin\phi}{\cos\phi}\sin\theta\right) = W\left(\sin\alpha - \frac{\sin\phi}{\cos\phi}\cos\alpha\right)$$

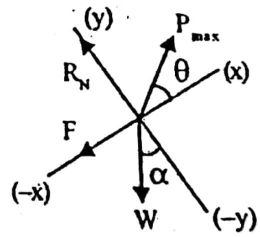
$$\text{or } P_{min}\frac{\cos(\theta+\phi)}{\cos\phi} = \frac{W\sin(\alpha-\phi)}{\cos\phi} \qquad \text{or } P_{min} = W\frac{\sin(\alpha-\phi)}{\cos(\theta+\phi)}$$

(f) **When the tendency of the body is to move up the plane and the applied force is inclined to inclined plane.**




The applied force is maximum and the force of friction will act down the plane.

For equilibrium of the body;




$\Sigma y = 0$

$\therefore\ R_N + P_{max}\sin\theta = W\cos\alpha$

$\Sigma x = 0$

$\therefore\ P_{max}\cos\theta = F + W\sin\alpha$

or $P_{max}\cos\theta = \mu R_N + W\sin\alpha$ ........... (ii)

Solving equation (i) and (ii)

$P_{max}\cos\theta = \mu(W\cos\alpha - p_{max}\sin\theta) + W\sin\alpha$

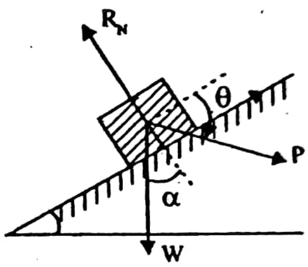
or $P_{max}(\cos\theta + \mu\sin\theta) = W(\sin\alpha + \mu\cos\alpha)$

But $\mu = \tan\phi = \dfrac{\sin\phi}{\cos\phi}$

$$\therefore\ P_{max}\left(\cos\theta + \frac{\sin\phi}{\cos\phi}\sin\theta\right) = W\left(\sin\alpha + \frac{\sin\phi}{\cos\phi}\cos\alpha\right)$$

$$\text{or } P_{max}\frac{\cos(\theta-\phi)}{\cos\phi} = \frac{W\sin(\alpha+\phi)}{\cos\phi} \quad \text{or } P_{max}\cos(\theta-\phi) = W\sin(\alpha+\phi)$$

$$\therefore P_{max} = \frac{W\sin(\alpha+\phi)}{\cos(\theta-\phi)}$$

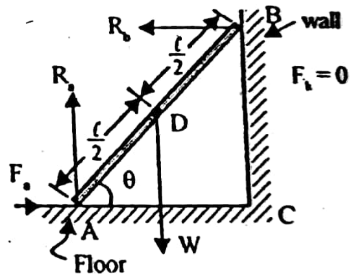



***Note :*** If the applied force is inclined clockwise to the inclined plane; then θ will be negative. The equations will be :

$$P_{min} = \frac{W\sin(\alpha-\phi)}{\cos(-\theta+\phi)} \qquad P_{max} = \frac{W\sin(\alpha+\phi)}{\cos(-\theta-\phi)}$$

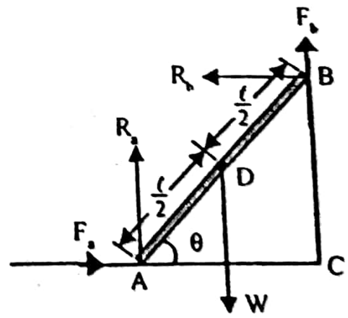
## 2.7.15 Equilibrium of Ladder

(a) **Wall is smooth :** Normal reaction $R_a$ and $R_b$ will act at the supports A and B. The force of friction at A will act right word along the floor. If $\mu_a$ is the coefficient of friction at A, then $F_a = \mu_a R_a$. The force of friction at B is zero due to smooth wall. Then apply condition of equilibrium;




$$\sum y = 0,\ \sum x = 0\ \ \&\ \sum M_A = 0 \text{ or } \sum M_B = 0$$

(b) **Wall is rough :** Let $\mu_a$ & $\mu_b$ are the coefficient of frictions at A & B




$\therefore\ F_a = \mu_a R_a$ and $F_b = \mu_b R_b$

For equilibrium

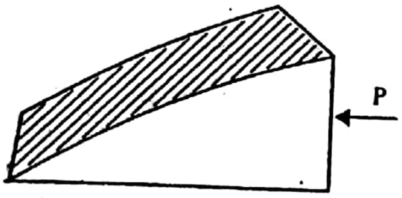
$$\sum y = 0,\ \sum x = 0\ \ \&\ \sum M_A = 0 \text{ or } \sum M_B = 0$$

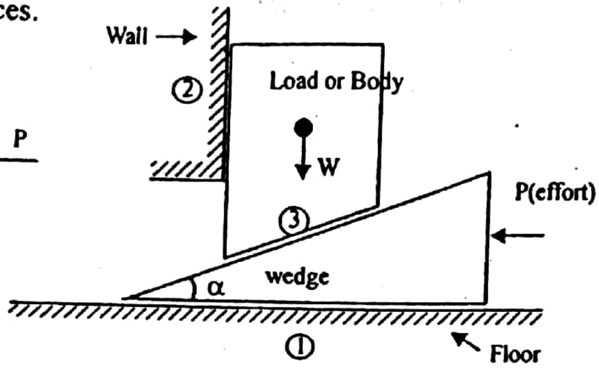
## 2.7.16 Equilibrium of Wedge

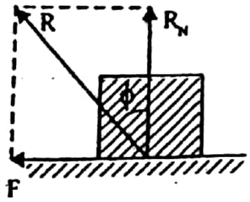
A wedge is a triangular or trapezoidal block which is used to lift heavy loads by the application of effort. It is also used for tightening fits or keys for shafts.

$\alpha$ = wedge angle, $\mu_1$, $\mu_2$ & $\mu_3$ are the coefficient of friction at surfaces 1, 2 and 3 as shown in the figure below.

$\therefore$ $\phi_1 = \tan^{-1}(\mu_1)$, $\phi_2 = \tan^{-1}(\mu_2)$ and $\phi_3 = \tan^{-1}(\mu_3)$ are the angles of friction respectively at corresponding surfaces.

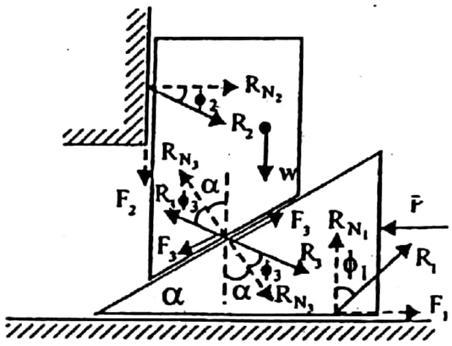


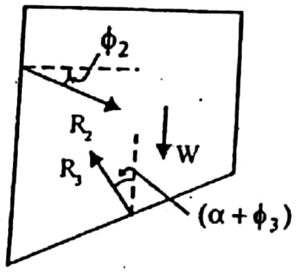

As we know that the normal reaction and force of friction can be reduced to a single force called resultant reaction which makes an angle $\phi$ with normal reaction.

Hence it is very easy to solve problems relating wedge by using resultant reactions.

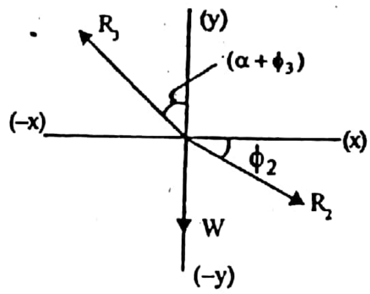



For example :

**FBD of Body**



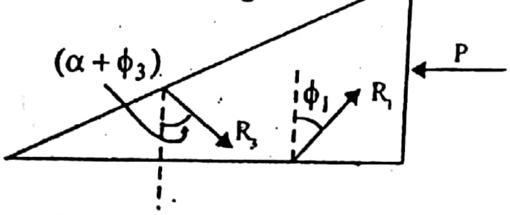

**Force diagram**



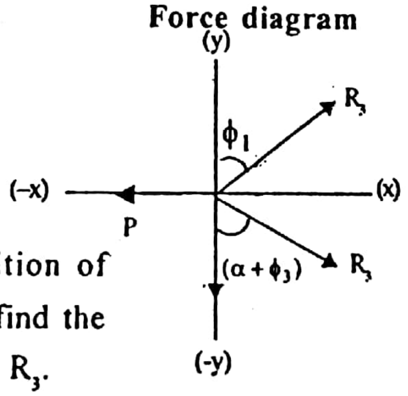

$\left.\begin{array}{l}\Sigma x = 0\\ \Sigma y = 0\end{array}\right\}$ find $R_3$ ?

From this force diagram, apply conditions of equilibrium, then find $R_3$ from a known value of W.

**FBD of wedge**



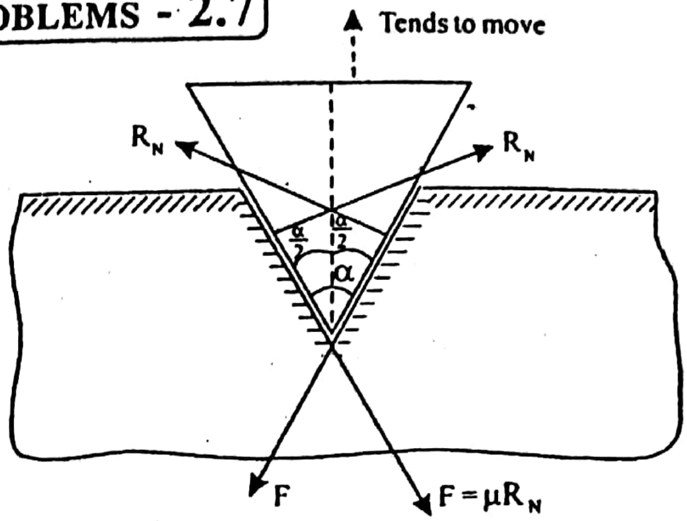

**Force diagram**




$\left.\begin{array}{l}\Sigma y = 0\\ \Sigma x = 0\end{array}\right\}$ find P ?

From this force diagram, apply condition of equiibrium, $\Sigma x = 0$ and $\Sigma y = 0$, then find the minimum effort P from a known value of $R_3$.

## SOLVED PROBLEMS - 2.7




**1.** *What must be the angle $\alpha$ between the plane faces of a steel wedge used for splitting logs if there is to be no danger of the wedge slipping out after each blow of the sledge ?*

**Soln.** The steel wedge has two faces, hence two normal reactions $R_N$ to two force of friction 'F'.

We know that $F = \mu R_N$

The FBD is shown in the figure

Resolving horizontally $\sum x = 0$

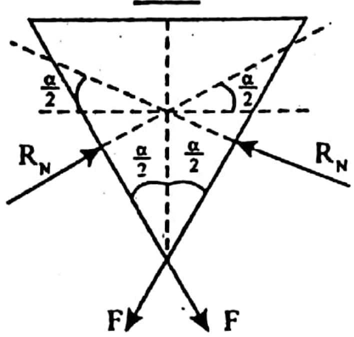
$$2R_N \cos\frac{\alpha}{2} = 2F \sin\frac{\alpha}{2}$$

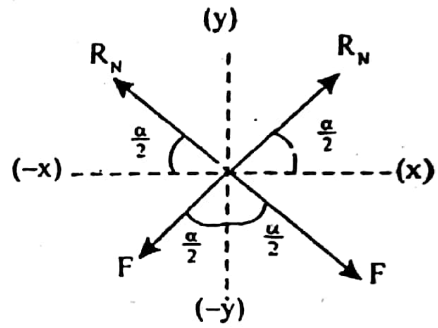
or $$2R_N \cos\frac{\alpha}{2} = 2\mu R_N \sin\frac{\alpha}{2}$$

$$\Rightarrow \tan\frac{\alpha}{2} = \mu \quad \Rightarrow \tan\frac{\alpha}{2} = \tan\phi$$

$$\Rightarrow \frac{\alpha}{2} = \phi \Rightarrow \alpha = 2\phi$$

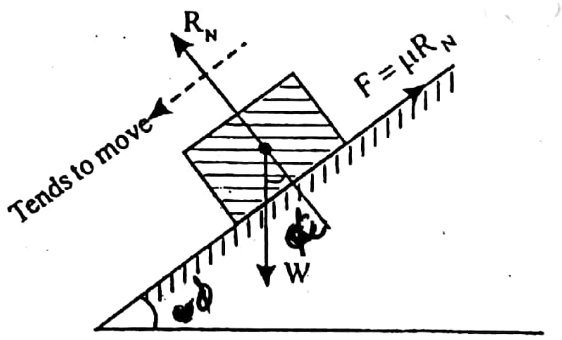
**FBD**




**(Ans.)**

**2.** *A flat stone slab rests on an inclined skidway that makes an angle $\alpha$ with the horizontal. What is the condition of equilibrium if the angle of friction is $\phi$ ?*




**Soln.** Let $\alpha$ is the angle of inclination with horizontal.

The FBD is shown in the figure

Resolving vertically $\sum y = 0$

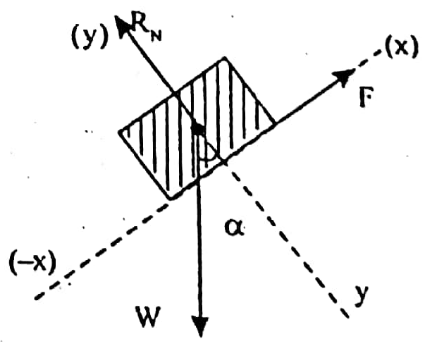
$$R_N = W \cos\alpha \quad \text{....(i)}$$

Resolving horizontally $F = W \sin\alpha$

or $$\mu R_N = W \sin\alpha \quad \text{...(ii)}$$

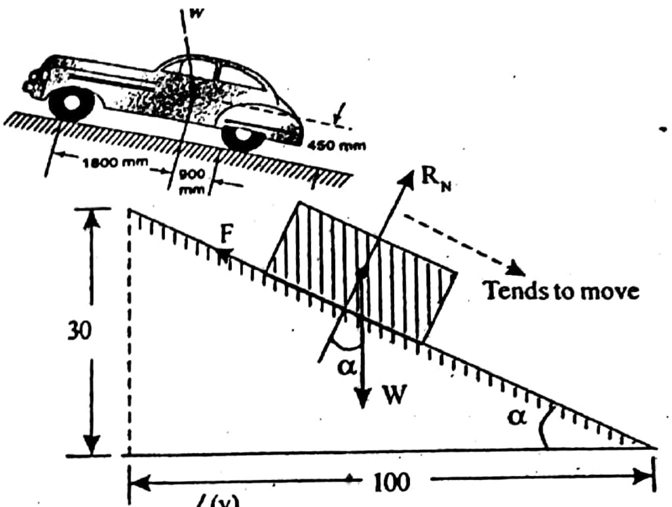
Substituting the value of $R_N$

$$\mu W \cos\alpha = W \sin\alpha$$




$$\Rightarrow \tan\alpha = \mu \quad \Rightarrow \tan\alpha = \tan\phi \quad \Rightarrow \alpha = \phi$$ **(Ans.)**

**3.** ***What is the necessary co-efficient of friction between tires and roadway to enable the four wheel drive automobile to climb a 30 percent grade ?***




**Soln.** Given data

30% Grade means 30 unit of height per hundred unit of horizontal plane.

The FBD is shown in the figure

Resolving vertically $\Sigma y = 0$

$$R_N = W \cos \alpha \qquad \text{....(i)}$$

Resolving horizontally $\Sigma x = 0$

$$F = W \sin \alpha$$

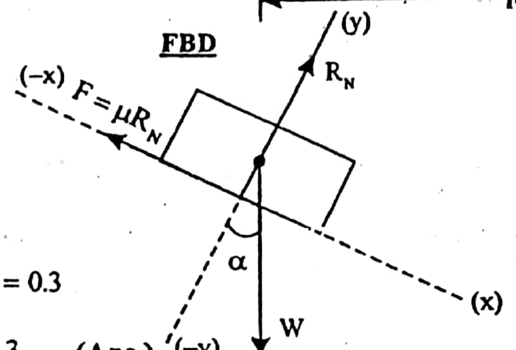
$$\Rightarrow \mu R_N = W \sin \alpha \qquad \text{....(ii)}$$

Substituting the value of "$R_N$"

$$\mu W \cos \alpha = W \sin \alpha$$

$$\Rightarrow \mu = \tan \alpha \qquad \text{....(iii)}$$

**FBD**

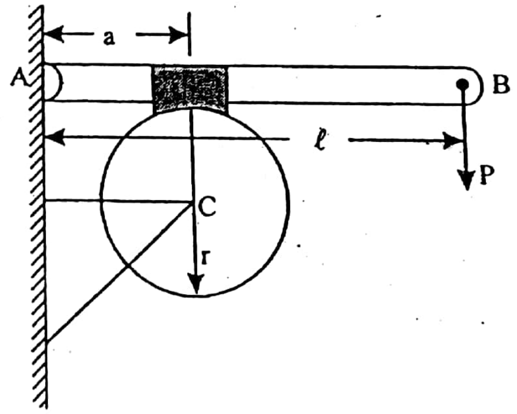



From the geometry, $\tan \alpha = \dfrac{30}{1000} = 0.3$

$\therefore$ co-efficient of friction $\mu = 0.3$ **(Ans.)**

21

**4.** ***A heavy rotating drum of radius r is supported in bearings at C and is braked by the device as shown in Fig- . Calculate the braking moment $M_c$ with respect to point C if the co-efficient of kinetic friction between drum and brake shoe is μ.***




**Soln.** The FBD of the lever is shown in the fig. Due to the application of the brake equal and opposite normal reactions act on the lever as well as on the drum. Similarly equal opposite forces of friction act at the contact surface. Considering the lever

$R_{ax}$

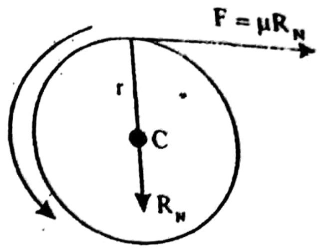

$\Sigma M = 0$

$\therefore$ Force of friction $F = \mu R_N$

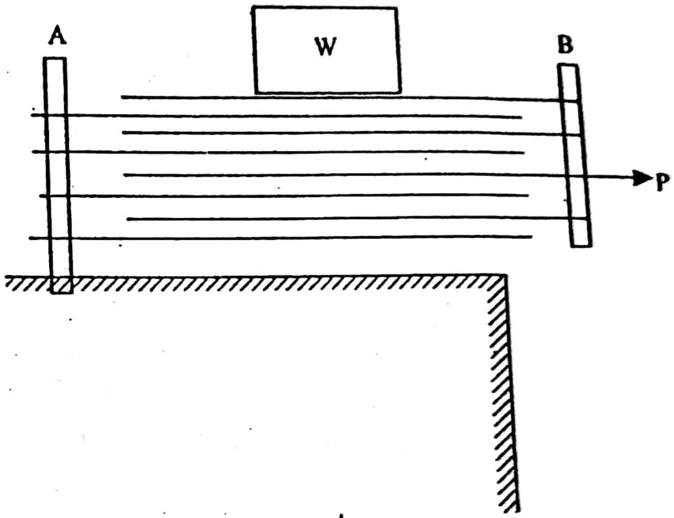
$$F = \mu\left(\frac{P\ell}{a}\right)$$

The turning moment at C,

$$M_c = F \times r \qquad = \mu\, r \frac{P\ell}{a} \qquad \textbf{(Ans.)}$$



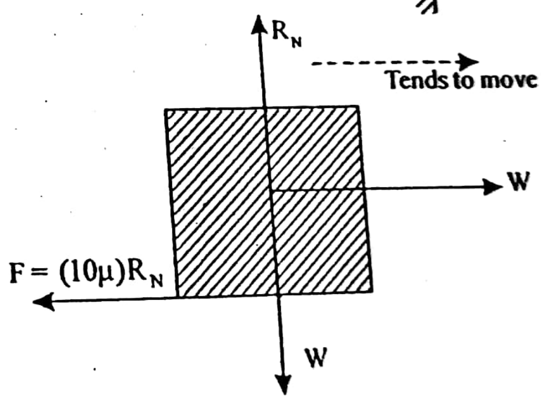

**5.** *To determine experimentally the coefficient of friction for steel on steel, flat plates, of negligible weight compared with the large top weight W, are stacked on a horizontal plane as shown in Fig. Alternate plates are held together by loose fitting vertical pins A and B. The pin A is anchored to a steel slab, and a horizontal pull applied to the pin B as shown. If there are five movable plates and slipping occurs when the horizontal pull has the magnitude P, what is the coefficient of friction μ?.*




**Soln.** We have 5 movable plates and 5 fixed plates, hence the no. of contact surfaces are ten.

$\therefore$ Total co-efficient of friction between the block W and the horizontal surface is $10\,\mu$

The FBD is shown in the fig.




Resolving vertically, $\Sigma y = 0$

$$R_N = W \qquad \text{....(i)}$$

Resolving horizontally $\Sigma x = 0$

$$F = P$$

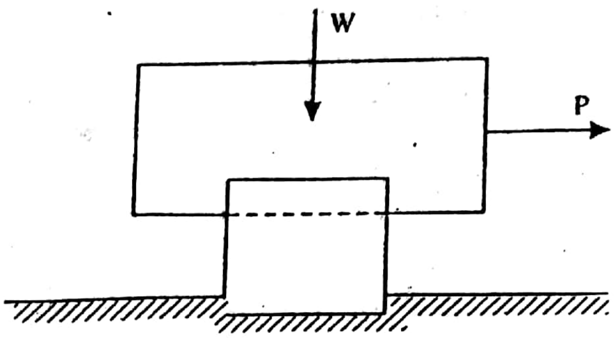
or $\quad 10\,\mu\, R_N = P$

Substituting the value of $R_N$

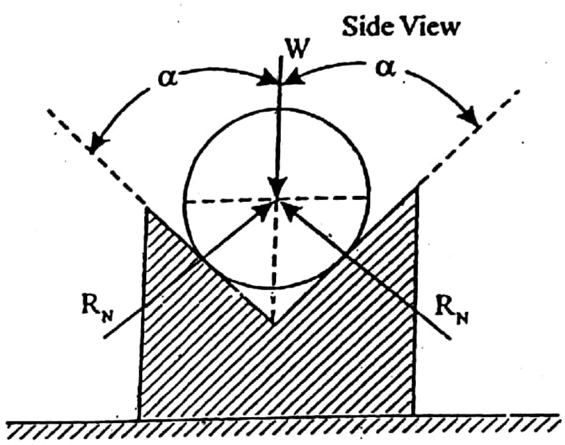
$$10\,\mu\, W = P$$

$$\Rightarrow \quad \mu = \frac{P}{10W} \qquad \textbf{(Ans.)}$$

6. *A short right circular cylinder of weight $W$ rests in a horizontal V notch having the angle $2\alpha$ as shown in fig. If the coefficient of friction is $\mu$, find the horizontal force $P$ necessary to cause slipping to impend.*




**Soln.** From the side view of the fig. cylinder rests on two supports, hence two normal reactions $R_N$ acting at an angle $\alpha$ with horizontal. The FBD of side view is shown on the fig.

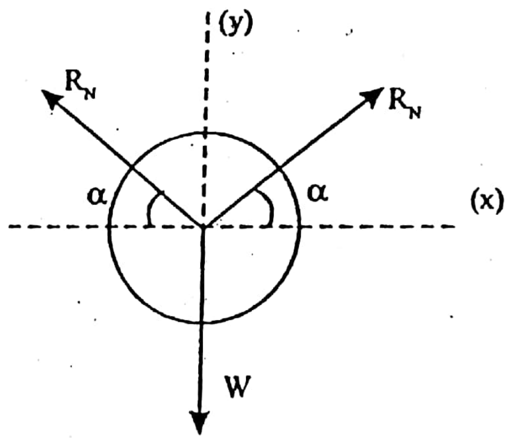



Resolving vertically,

$$\Sigma y = 0$$

$$2R_N \sin\alpha = W$$

or $$R_N = \frac{W}{2\sin\alpha} \qquad \text{....(i)}$$




From the front view, two forces of friction 'F' act at contact point horizontally

Resolving horizontally,

$$\Sigma x = 0$$

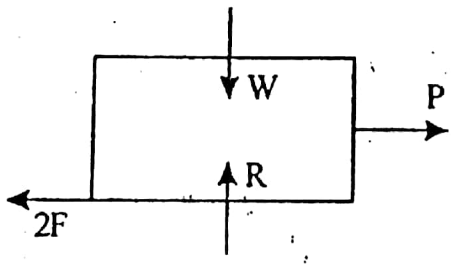
$$P = 2F$$

$$\Rightarrow P = 2 \times \mu R_N \qquad \text{....(ii)}$$

Substituting the value

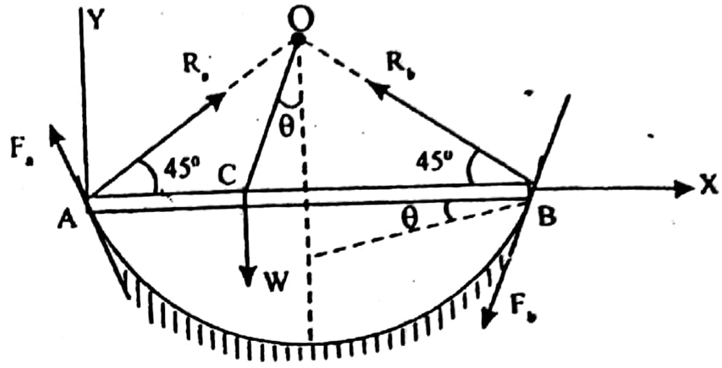
$$P = 2 \times \mu \frac{W}{2\sin\alpha}$$

$$\Rightarrow P = \frac{\mu W}{\sin\alpha} \qquad \text{(Ans.)}$$

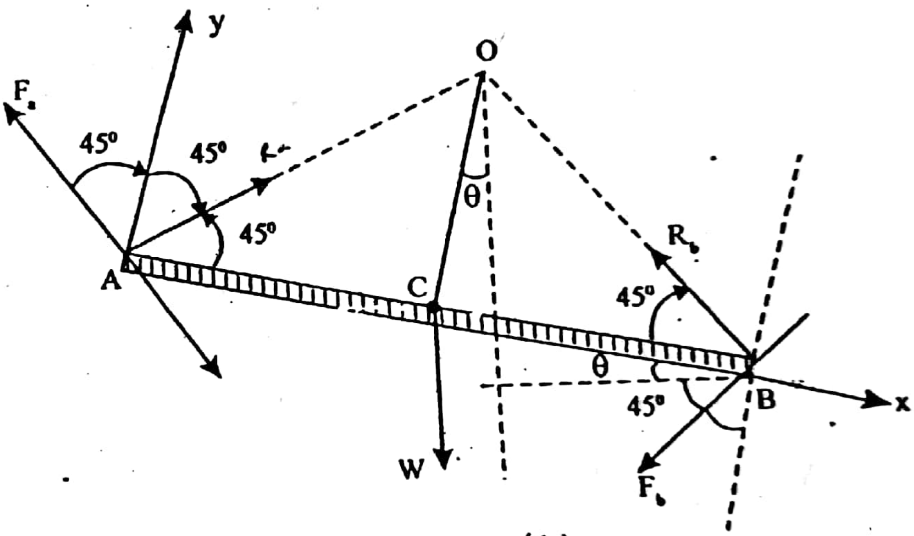



(Front view)

* **7.** *The ends of a heavy prismatic bar AB are supported by a circular ring in a vertical ring as shown in fig. If the length of the bar is such that it subtends an angle of 90° in the ring and the angles of friction at A and B are each φ, what is the greatest angle of inclination θ that the bar can make with the horizontal in a condition of equilibrium.*




**Soln.** The normal reactions $R_a$ & $R_b$ act at contact points A & B passing through the center of the circular ring and making an angle of 45° to the bar. The force of friction $F_a$ and $F_b$ act normal to $R_a$ and $R_b$, if the bar slides down. The FBD is shown in the fig. Resolving along x-axis $\sum x = 0$




$$W \sin\theta + R_a \cos 45° = R_b \cos 45° + \mu R_b \cos 45° + \mu R_a \cos 45°$$

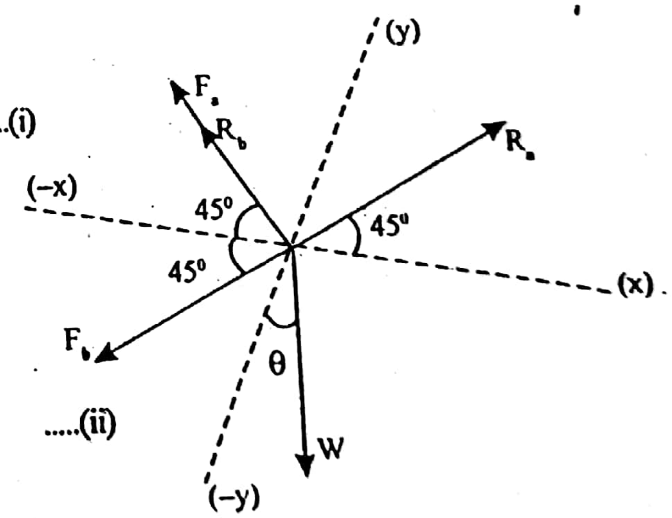
$$\Rightarrow W\sqrt{2}\sin\theta = (R_b - R_a) + \mu(R_a + R_b) \quad \text{.......(i)}$$

Resolving along Y-axis

$\sum y = 0$

$$R_b \sin 45° + R_a \sin 45° + \mu R_a \cos 45° = W\cos\theta + \mu R_b \sin 45°$$

$$\Rightarrow W\sqrt{2}\cos\theta = (R_b + R_a) + \mu(R_a - R_b) \quad \text{.....(ii)}$$




Taking moment about 'C'

$\sum M_c = 0$

$$R_b \sin 45° \times CB = \mu R_b \sin 45° \times CB + R_a \sin 45° \times AC + \mu R_a \sin 45° \times AC$$

Since AC = BC

$$R_b = \mu R_a + R_a + \mu R_b$$

or $\quad R_a = R_b(1-\mu)/(1+\mu)$ .....(iii)

Dividing equation (i) by (ii)

$$\tan\theta = \frac{(R_b - R_a) + \mu(R_a + R_b)}{(R_b + R_a) + \mu(R_a - R_b)} \qquad \text{.....(iv)}$$

Substituting the value of $R_a$

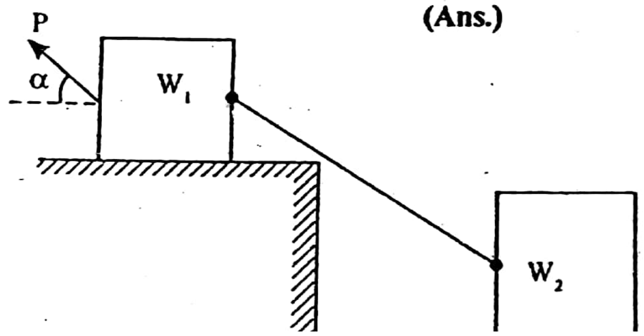
$$\tan\theta = \frac{\left[R_b - \frac{R_b(1-\mu)}{1+\mu}\right] + \mu\left[\frac{R_b(1-\mu)}{1+\mu} + R_b\right]}{\left[R_b + \frac{R_b(1-\mu)}{1+\mu}\right] + \mu\left[\frac{R_b(1-\mu)}{1+\mu} - R_b\right]}$$

or $\quad \tan\theta = \dfrac{(1+\mu-1+\mu) + \mu(1-\mu+1+\mu)}{(1+\mu+1-\mu) + \mu(1-\mu-1-\mu)}$ $\quad$ or $\quad \tan\theta = \dfrac{2\mu + 2\mu}{2 - 2\mu^2} = \dfrac{2\mu}{1-\mu^2}$

But we know that $\mu = \tan\phi$

Substituting the value of $\mu$, $\quad \tan\theta = \dfrac{2\tan\phi}{1-\tan^2\phi} = \tan 2\phi$

$\therefore\ \theta = 2\phi$ **(Ans.)**




8 *Two blocks having weights $W_1$ and $W_2$ are connected by a string and rest on horizontal planes as shown in Fig. If the angle of friction for each block*

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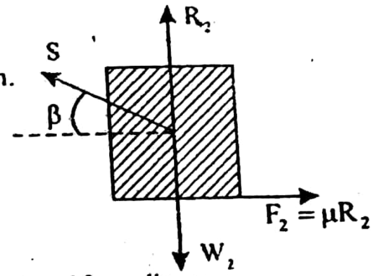
The two FBDS are shown in the figure.

In these two figures, unknowns are five ( $\alpha$ , $\beta$ , S, $R_1$ and $R_2$).

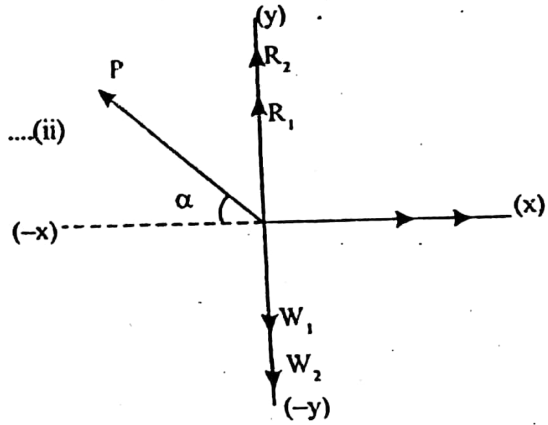
But we have four equations (For each, $\Sigma X = 0, \Sigma Y = 0, \Sigma Z = 0$ )

Hence it will be easy to solve, if we draw combined force diagram.

**FBD - II**




Combined force diagram




Resolving vertically $\sum y = 0$

$$R_1 + R_2 + P \sin\alpha = W_1 + W_2$$

or $\quad R_1 + R_2 = (W_1 + W_2) - P \sin\alpha \qquad$ ....(i)

Resolving horizontally

$$\sum x = 0$$

$$P \cos\alpha = \mu R_1 + \mu R_2$$

$\Rightarrow P \cos\alpha = \mu(R_1 + R_2) \qquad$ ....(ii)

Substituting the value of ($R_1 + R_2$) from equation (i) in equation (ii)

$$P \cos\alpha = \mu[(W_1 + W_2) - P \sin\alpha]$$

$$P(\cos\alpha + \mu \sin\alpha) = \mu(W_1 + W_2)$$

We known that $\mu = \tan\phi = \dfrac{\sin\phi}{\cos\phi}$

Substituting the value of $\mu$, the equation becomes;

$$P\cos(\phi - \alpha) = (W_1 + W_2)\sin\phi \quad \text{or} \quad P = \frac{(W_1 + W_2)\sin\phi}{\cos(\phi - \alpha)}$$

P will be minimum, when $\cos(\phi - \alpha)$ will be maximum.

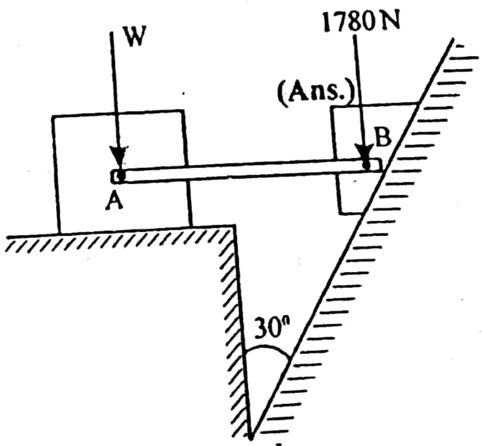
The maximum value of $\cos(\phi - \alpha) = 1 = \cos 0°$

$\therefore \phi - \alpha = 0 \quad$ or $\quad \phi = \alpha$

Substituting the value $P_{min} = (W_1 + W_2)\sin\phi$

or $\quad P_{min} = (W_1 + W_2)\sin\alpha$

Q. *Two blocks connected by a horizontal link AB are supported on two rough planes as shown in Fig. The coefficient of friction for block A on the horizontal plane is $\mu = 0.4$. The angle of friction for block B on the inclined plane is $\phi = 15°$. What is the smallest weight W of block A for which equilibrium of the system can exist.*

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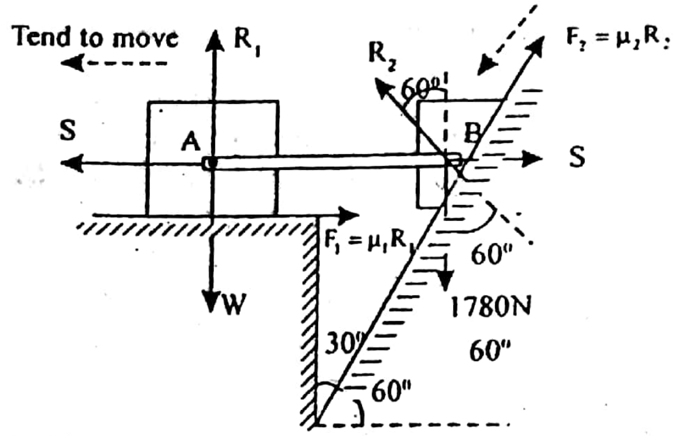
**Soln.** The normal reaction $R_1$ & $R_2$ act at A & B and force of frictions act along the contact surface, when the block 'B' tends to slide downward.

An equal opposite compressive force acts along the rod AB towards A & B.

The FBD's shown in the figure.

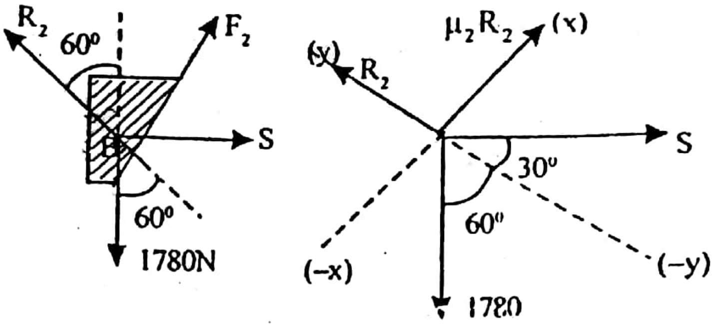



**Considering Block - B**

Resolving along y-axis.

$\sum y = 0$

**FBD - B**




$R_2 = 1780 \cos 60° + S \cos 30°$ .....(i)

Resolving along x-axis, $\sum x = 0$

$\mu_2 R_2 + S \sin 30° = 1780 \sin 60°$ .....(ii)

or $R_2 = \dfrac{170 \sin 60° - S \sin 30°}{0.268}$ .....(iii)

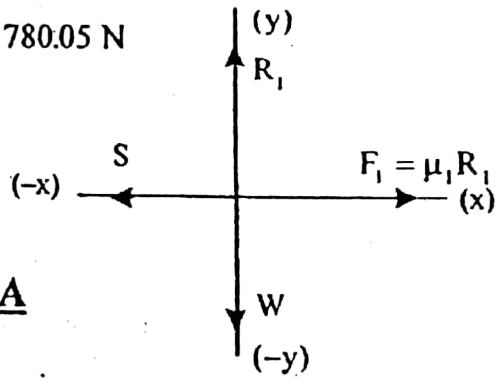
Equating (i) and (iii)

$$\frac{1780 \sin 60° - S \sin 30°}{0.268} = 1780 \cos 60° + S \cos 30°$$

$\Rightarrow 1780 \sin 60° - S \sin 30° = 238.52 + 0.232S$

$\Rightarrow 1541.52 - 238.52 = S(0.232 + 0.5) \quad \Rightarrow S = \dfrac{1303}{0.732} = 1780.05 \text{ N}$

**For 'A'**



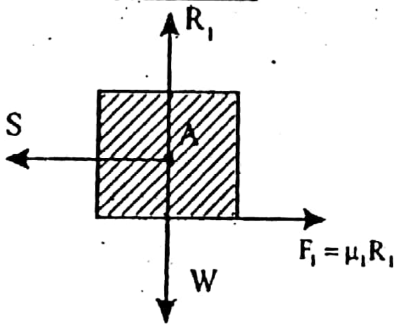

Substituting the value of 'S'

$$R_2 = \frac{1780 \sin 60° - 1780.05 \sin 30°}{0.268} = 2430.97 \text{ N}$$

**Considering Block - A**

Resolving vertically

**FBD - A**




$\sum y = 0$

$R_1 = W$ ....(i)

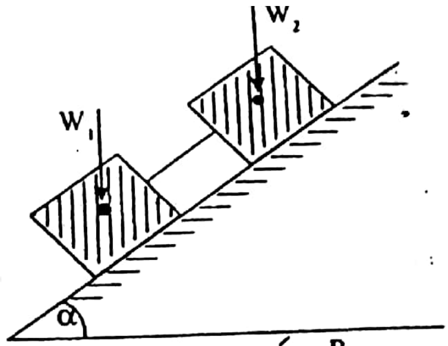
Resolving horizontally $\sum x = 0$

$\mu_1 R_1 = S$ .....(ii)

or $\mu_1 W = S$

$\therefore \quad W = \dfrac{S}{\mu_1} = \dfrac{1780.05}{0.4} = 4450 \text{ N}$ **(Ans.)**

10. *Two blocks of weights $W_1$ and $W_2$ rest on a rough inclined plane and are connected by a short piece of string as shown in Fig-. If the co-efficients of friction are $\mu_1 = 0.2$ and $\mu_2 = 0.3$ respectivley . Find the angle of inclination of the plane for which siding will impend. Assume $W_1 = W_2 = 22.25N$.*




**Soln.** Given data

$W_1 = W_2 = 22.25N$

The FBDs of two bodies are shown for fig. - I

**Considering block of wt. '$W_2$'**

Resolving along y-axis,

$\Sigma y = 0$

$$R_2 = W_2 \cos\alpha \qquad \text{....(i)}$$

Resolving along x-axis, $\Sigma x = 0$

$$\mu_2 R_2 = S + W_2 \sin\alpha \qquad \text{.....(ii)}$$

or $\quad \mu_2 W_2 \cos\alpha = S + W_2 \sin\alpha$

or $\quad S = W_2(\mu_2 \cos\alpha - \sin\alpha) \qquad \text{.....(iii)}$

**Considering block of wt. '$W_1$'**

Resolving along y-axis, $\Sigma y = 0$

$$R_1 = W_1 \cos\alpha \qquad \text{....(iv)}$$

Resolving along x-axis, $\Sigma x = 0$

$$S + \mu_1 R_1 = W_1 \sin\alpha \qquad \text{....(v)}$$

or $\quad S + \mu_1 W_1 \cos\alpha = W_1 \sin\alpha$

or $\quad S = W_1(\sin\alpha - \mu_1 \cos\alpha) \qquad \text{....(vi)}$

Equating (iii) and (iv)

$$W_2(\mu_2 \cos\alpha - \sin\alpha) = W_1(\sin\alpha - \mu_1 \cos)$$

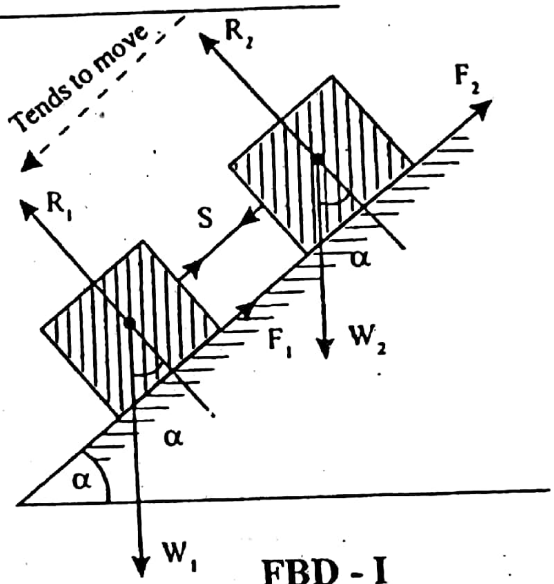
But $W_1 = W_2$ (given)

$$\mu_2 \cos\alpha - \sin\alpha = \sin\alpha - \mu_1 \cos\alpha$$

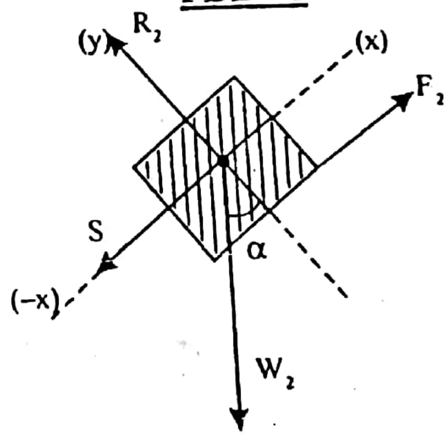
or $\quad 2\sin\alpha = \cos\alpha(\mu_2 + \mu_1)$

$$\tan\alpha = \frac{(\mu_2 + \mu_1)}{2} = \frac{(0.3 + 0.2)}{2}$$

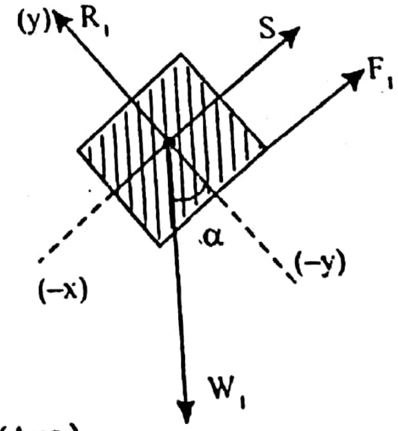
$$\alpha = \tan^{-1}\left(\frac{0.5}{2}\right) = 14° \, 2'$$




**FBD - I**

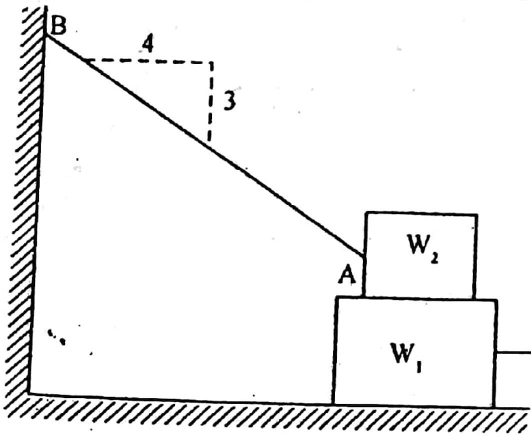



**FBD - II**




(Ans.)

*A block of weight $W_1$ = ~~890~~ 900 N rests on a horizontal surface and supports on top of it another block of weight $W_2$ = 225 ~~222.5~~ N. The block $W_2$ is attached to a vertical wall by the inclined string AB. Find the magnitude of the horizontal force P, applied to the lower block as shown in Fig. that will be necessary to cause slipping to impend. The co-efficeint of friction for all contact surfaces is = 0.3.*

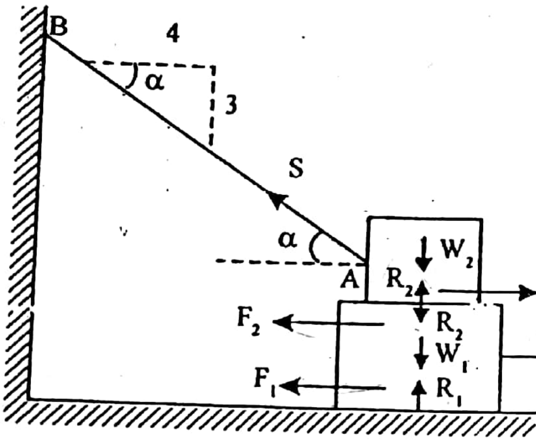



**Soln.** Given data

$W_1 = 890$ N, $W_2 = 222.5$ N

$\mu_1 = \mu_2 = \mu = 0.3$

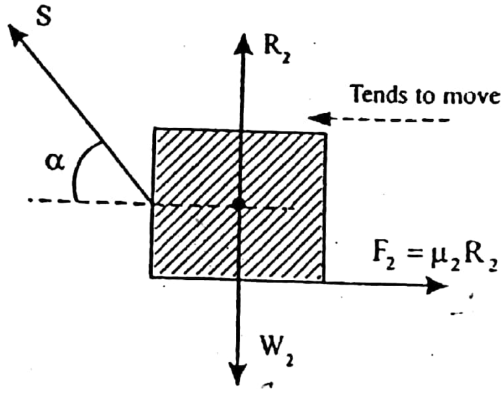
$$\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36°52'$$




The normal reaction $R_2$ is equal and opposite between two bodies hence two equal opposite forces of friction act at these contact surfaces.

The FBDs are shown

**For fig. - 1**




Resolving vertically, $\Sigma y = 0$

$$R_2 + S\sin\alpha = W_2 \quad \text{....(i)}$$

Resolving horizontally, $\Sigma x = 0$

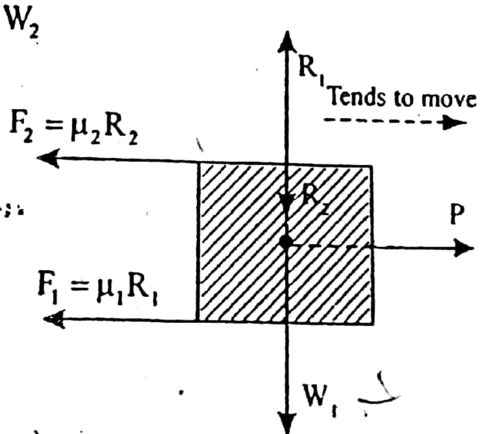
$$F_2 = \mu_2 R_2 = S\cos\alpha \quad \text{....(ii)}$$

Substituting the value of 'S'

$$R_2 + \frac{\mu_2 R_2}{\cos\alpha}\sin\alpha = W_2 \text{ or } R_2(1+\mu_2\tan\alpha) = W_2$$

$$\Rightarrow R_2 = \frac{\overset{225}{\cancel{222.5}}}{(1+0.3\tan 36.52)} = 181.636 \text{ N}$$

**For FBD - II**



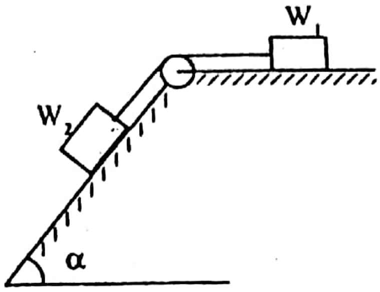

Resolving vertically, $\Sigma y = 0$

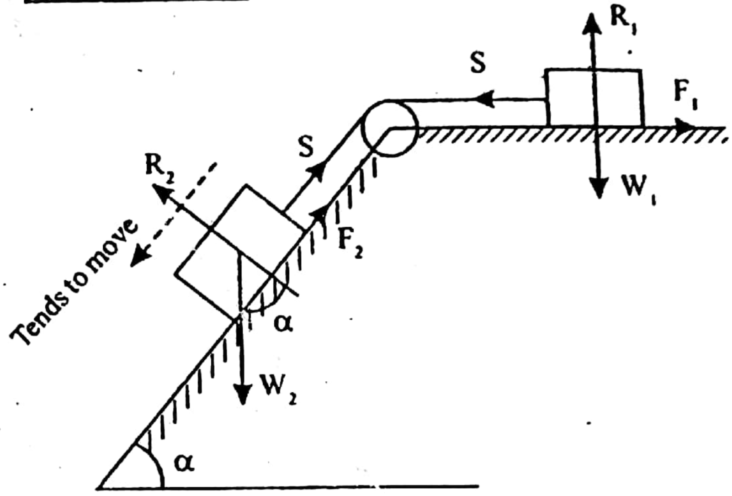
$R_1 = W_1 + R_2 = \overset{900}{\cancel{890}} + 181.636 = 1071.636$ N

Resolving horizontally, $\Sigma x = 0$

$P = F_1 + F_2 = \mu(R_1 + R_2) = 0.3\,(1071.636 + 181.636) = 375.98$ N **(Ans)**

12. *Two rectangular blocks of weights $W_1$ and $W_2$ are connected by a flexible cord and rest upon a horizontal and an inclined plane, respectively with the cord passing over a pulley as shown in Fig. In the particular case where $W_1 = W_2$ and the co-efficient of static friction is the same for all contact surfaces, find the angle of inclination of the inclined plane, at which motion of the system will impend. Neglect friction in the pulley.*




**Soln.** Given data

$W = W_1 = W_2$.

$\mu_1 = \mu_2 = \mu$

$\alpha = ?$

The FBDs are shown in the figure

**For fig. - I**

Resolving vertically, $\Sigma y = 0$

$R_1 = W_1 = W$ .....(i)

Resolving horizontally, $\Sigma x = 0$

$S = F_1 = \mu_1 R_1$ ....(ii)

$\therefore S = \mu W$ .....(iii)

**For fig. - II**

Resolving along y - axis $\Sigma y = 0$

$R_2 = W_2 \cos\alpha = W\cos\alpha$ ....(iv)

Resolving along x-axis, $\Sigma x = 0$

$$S + F_2 = W_2 \sin\alpha$$

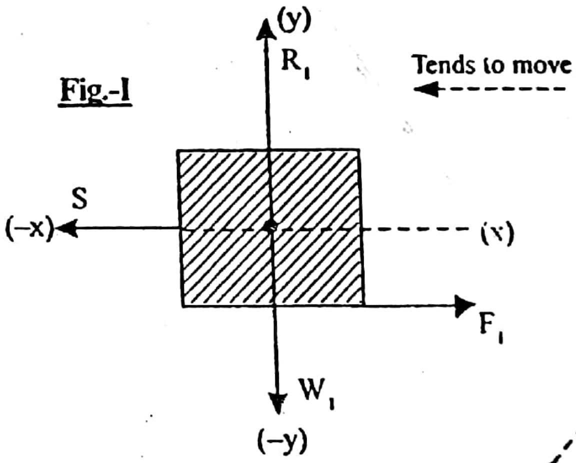
or $S = W\sin\alpha - \mu R_2$ .....(v)

$\therefore S = W\sin\alpha - \mu W\cos\alpha$ ....(vi)

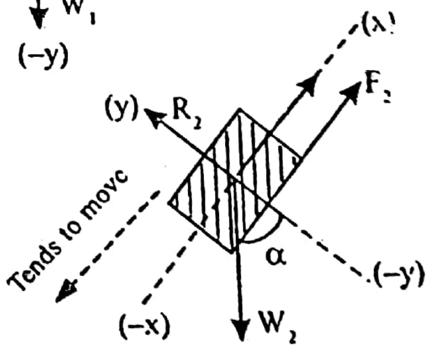
Equating equation (iii) and (vi)

$\mu W = W\sin\alpha - \mu W\cos\alpha$

<u>Fig.-I</u>




<u>Fig.-II</u>

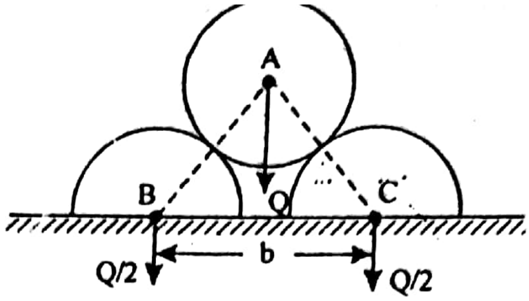



or $\mu W(1+\cos\alpha) = W\sin\alpha$ or $\mu \times 2\cos^2\frac{\alpha}{2} = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

or $\mu\cos\frac{\alpha}{2} = \sin\frac{\alpha}{2}$ or $\tan\frac{\alpha}{2} = \mu$

But we know that $\mu = \tan\phi$ $\therefore \tan\frac{\alpha}{2} = \tan\phi$ or $\alpha = 2\phi$ **(Ans.)**

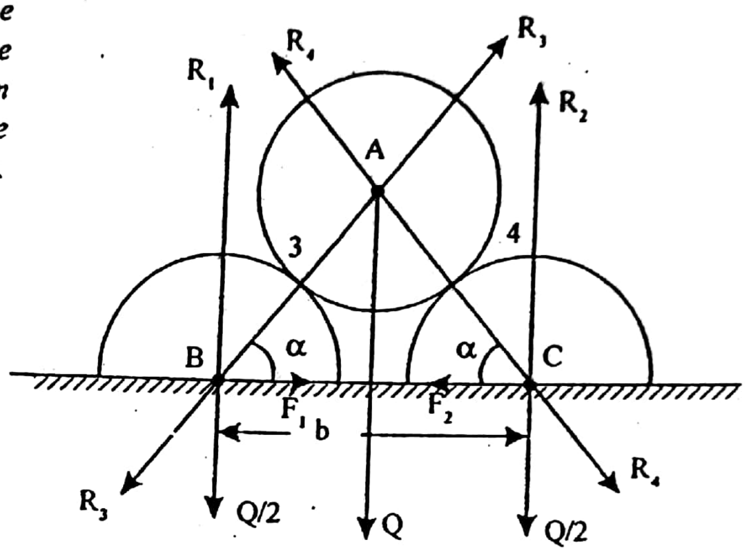
13. *A smooth circular cylinder of weight Q and radius r is supported by two semicircular cylinders each of the same radius r and weight Q/2, as shown in Fig. If the co-efficient of static friction between the flat faces of the semi circular cylinders and the horizontal plane on which they rest is μ = 1/2, and friction between the cylinders themselves is neglected, determine the maximum distance b between the centres B and C for which equilibrium will be possible without the middle cylinder touching the horizontal plane.*




**Soln.** Given data

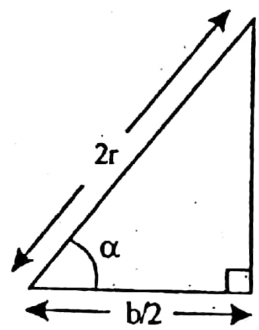
$$\mu = \frac{1}{2} = 0.5$$

$$b_{max} = ?$$




The normal reactions $R_3$ and $R_4$ act at the contact surfaces 3 and 4 towards each cylinders

From the geometry of the triangles



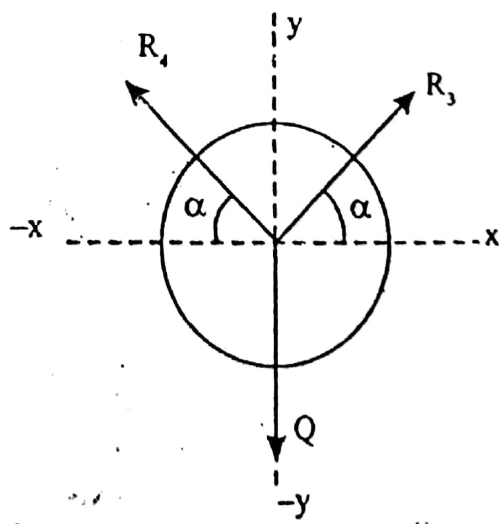

$$\cos\alpha = \frac{b/2}{2r} = \frac{b}{4r}$$

$$\therefore \sin\alpha = \sqrt{1-\cos^2\alpha}$$

$$= \sqrt{1-\frac{b^2}{(4r)^2}} = \frac{\sqrt{16r^2-b^2}}{4r}$$

The FBD of the cylinder A is shown.

**FBD - at 'A'**




Resolving horizontally, $\sum x = 0$

$$R_3 \cos\alpha = R_4 \cos\alpha$$

$$\therefore R_3 = R_4 \qquad \text{....(i)}$$

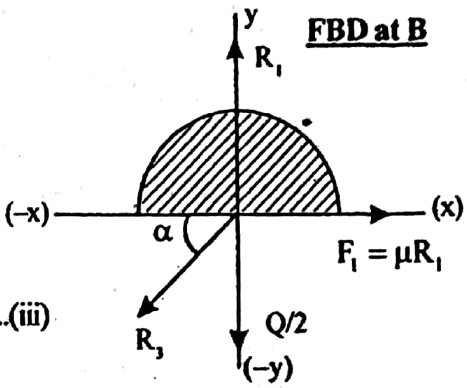
Resolving vertically, $\sum y = 0$

$$2R_4 \sin\alpha \text{ or } 2R_3 \sin\alpha = Q$$

$$\therefore R_4 = R_3 = \frac{Q}{2\sin\alpha} \qquad \text{....(ii)}$$

**Considering FBD at B**

Resolving horizontally, $\sum x = 0$




$$\mu R_1 = R_3 \cos\alpha \quad \text{or} \quad R_1 = \frac{Q}{2\mu \sin\alpha} \cdot \cos\alpha$$

$$= \frac{Q}{2 \cdot \mu \dfrac{\sqrt{16r^2 - b^2}}{4r}} \times b/4r = \frac{Qb}{2\mu\sqrt{16r^2 - b^2}} \quad \text{....(iii)}$$

Resolving vertically, $\sum x = 0$

$$R_1 = \frac{Q}{2} + R_3 \sin\alpha \quad \text{....(iv)}$$

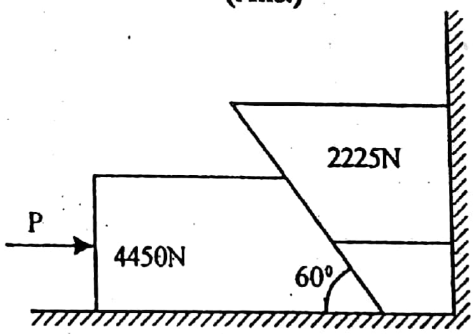
Substituting the values $\dfrac{Qb}{2\mu\sqrt{16r^2 - b^2}} = \dfrac{Q}{2} + \dfrac{Q}{2\sin\alpha} \cdot \sin\alpha$

$$\Rightarrow \frac{Qb}{2\mu\sqrt{16r^2 - b^2}} = Q \quad \text{or} \quad b = 2\mu\sqrt{16r^2 - b^2}$$

Squaring both sides $b^2 = 4\mu^2\left(16r^2 - b^2\right) = 4 \times \left(\frac{1}{2}\right)^2 \left(16r^2 - b^2\right) = 16r^2 - b^2$

or $2b^2 = 16r^2$ or $b = \sqrt{\dfrac{16}{2}r^2} = 2.83r$ **(Ans.)**

14. *Referring to the given Fig. the coefficient of frictions are as follows : 0.25 at the floor 0.30 at the wall and 0.2 between blocks. Find the minimum value of a horizontal force P applied to the lower block that will hold the system in equilibrium.*




**Soln.** $W_1 = 4450$ N

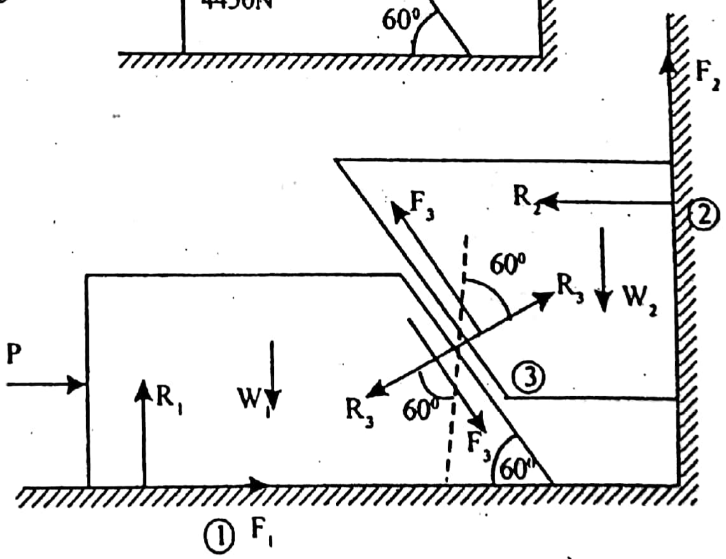
$W_2 = 2225$ N

$\mu_1 = 0.25$

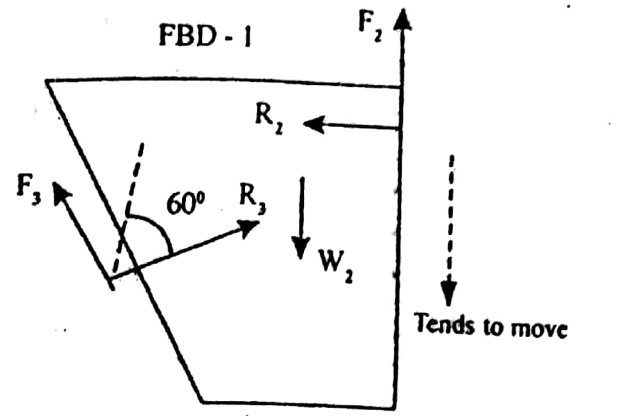
$\mu_2 = 0.3$

$\mu_3 = 0.2$

Since required 'P' is minimum to hold the upper block falling down; the block of wt.4450N tends to move left.



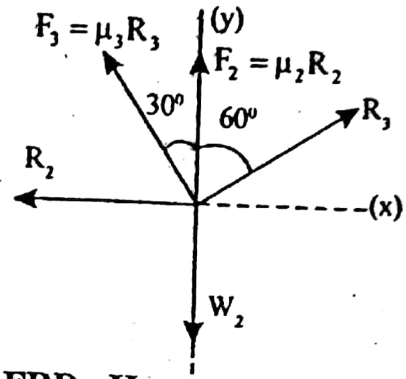
The normal reaction $R_3$ acts at the contact surface between two bodies towards each other, making 60° to the vertical. The force of friction $F_3$ also acts at the contact surface towards each other.




The FBD - 1 is shown in the figure.

Resolving horizontally $\sum x = 0$

$$R_3 \sin 60° = R_2 + \mu_3 R_3 \sin 30° \quad ....(i)$$




Resolving vertically $\quad \sum y = 0$

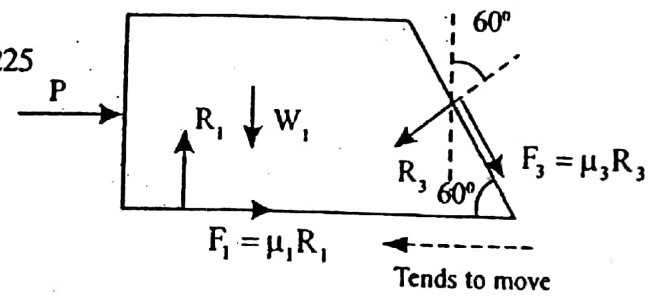
$$\mu_2 R_2 + \mu_3 R_3 \cos 30° + R_3 \cos 60° = 2225 \quad .....(ii)$$

From equation (i) we get;

$$R_2 = R_3 \left[\sin 60° - 0.2 \sin 30°\right]$$

$$\Rightarrow \; R_2 = 0.766\ R_3 \quad ....(iii)$$

**FBD - II**




Substituting the value in equation (ii)

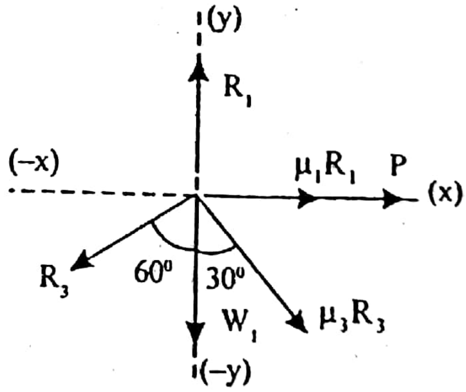
$$\mu_2 \times 0.766\ R_3 + \mu_3 R_3 \cos 30° + R_3 \cos 60° = 2225$$

$$\Rightarrow R_3 (0.2298 + 0.173 + 0.5) = 2225$$

$$\Rightarrow R_3 = 2464.55\ N$$

**Considering the lower block**

Resolving vertically $\quad \sum y = 0$




$$R_1 = R_3 \cos 60° + W_1 + \mu_3 R_3 \cos 30°$$

$$\Rightarrow R_1 = 2464.55 \cos 60° + 4450 + 0.2 \times 2464.55 \times \cos 30°$$

$$\Rightarrow R_1 = 6109.14\ N$$

Resolving horizontally $\sum x = 0$

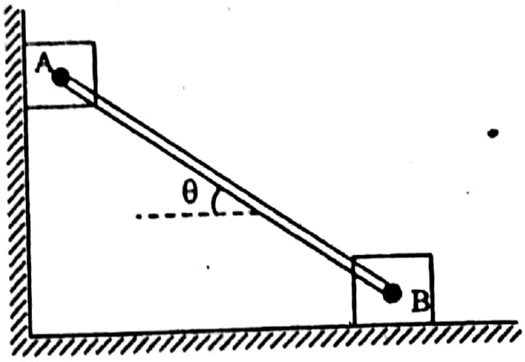
$$P + \mu_1 R_1 + \mu_3 R_3 \sin 30° = R_3 \sin 60°$$

$$\Rightarrow P = 2464.55 \times \sin 60° - (0.25 \times 6109.14) - (0.2 \times 2464.55 \times \sin 30°)$$

$$= 2134.36 - 1527.28 - 246.45$$

$P = 360.6\,N$ **(Ans.)**

15. ***Two identical blocks A and B are connected by a rods and rest against vertical and horizontal planes, respectively as shown in Fig-. If sliding impends when θ = 45°, determine the co-efficient of friction, assuming it to be same at both floor and wall.***



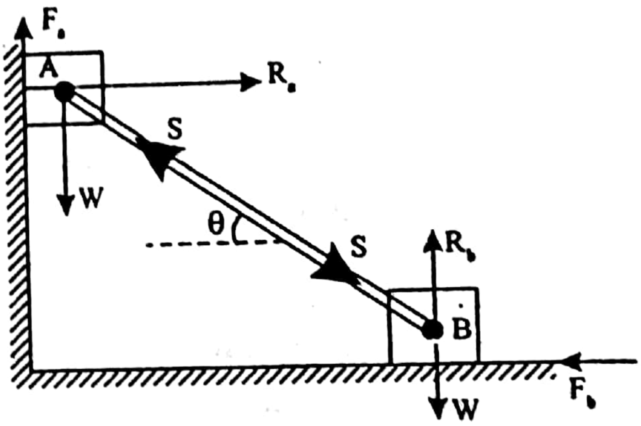

**Soln.** Given data

$\theta = 45°$

$W_1 = W_2 = W$

$\mu_1 = \mu_2 = \mu$

Since the two bodies are connected by a rod, equal opposite compressive forces 'S' act along AB, towards A & B.




The FBD for block A is shown in the figure

Resolving horizontally $\Sigma x = 0$

$$R_a = S\cos 45° = \frac{S}{\sqrt{2}} \qquad \text{...(i)}$$

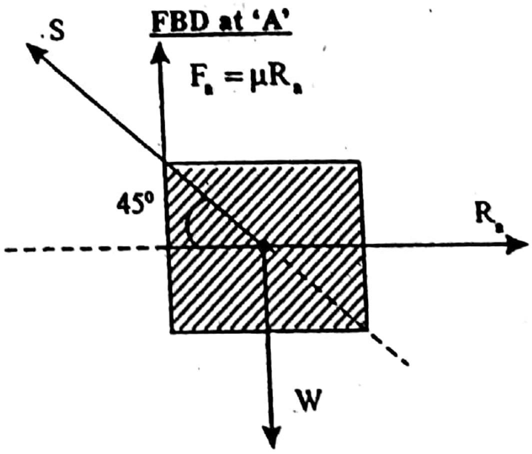
Resolving vertically $\Sigma y = 0$

$$\mu R_a + S\sin 45° = W \qquad \text{....(ii)}$$

Substituting the value of '$R_a$' $\mu\dfrac{S}{\sqrt{2}} + \dfrac{S}{\sqrt{2}} = W$

$$\therefore \frac{S}{\sqrt{2}}(\mu + 1) = W$$

$$\Rightarrow S = \frac{\sqrt{2}W}{\mu + 1} \qquad \text{.....(iii)}$$




The FBD for block B is shown in the figure

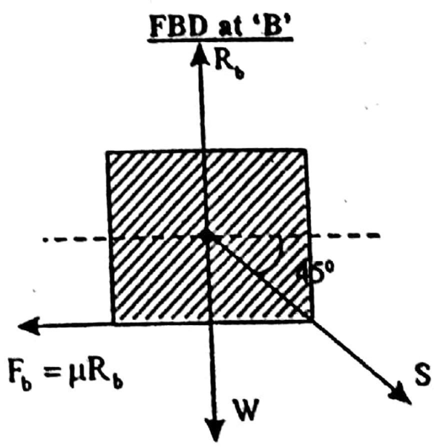
Resolving vertically $\Sigma y = 0$

$$R_b = W + S\sin 45° \qquad \text{......(iv)}$$

Resolving horizontally $\Sigma x = 0$

$$S\cos 45° = \mu R_b \qquad \text{.....(v)}$$

Substituting the value of '$R_b$' $\dfrac{S}{\sqrt{2}} = \mu\left(W + \dfrac{S}{\sqrt{2}}\right)$



$$\Rightarrow \frac{S}{\sqrt{2}} = (1-\mu) = \mu W \qquad \Rightarrow S = \frac{\sqrt{2}\,\mu W}{1-\mu} \qquad ...(vi)$$

Equating (iii) and (vi)

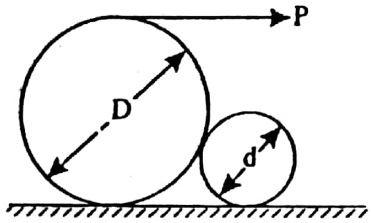
$$\frac{\sqrt{2}\,\mu W}{1-\mu} = \frac{\sqrt{2} W}{\mu+1} \qquad \Rightarrow \mu(\mu+1) = 1-\mu$$

$$\Rightarrow \mu^2 + \mu = 1-\mu \qquad \Rightarrow \mu^2 + 2\mu - 1 = 0 \quad \Rightarrow \mu = \frac{-2 \pm \sqrt{4+4}}{2}$$

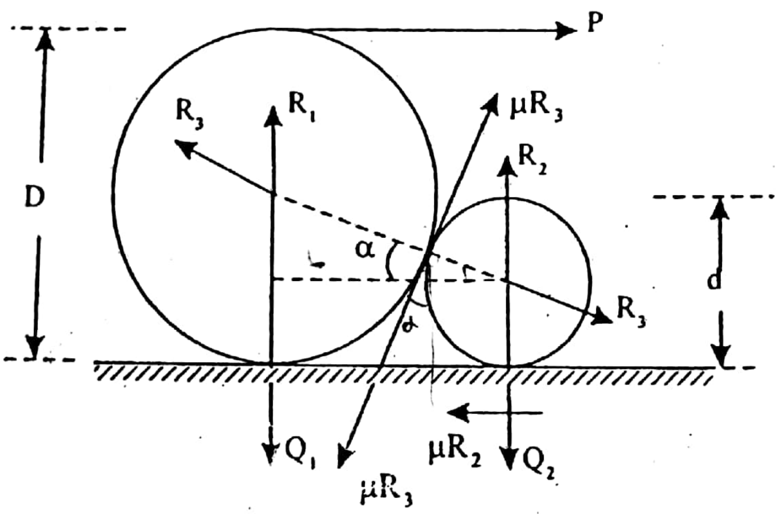
Taking + sign, $\frac{-2+\sqrt{8}}{2} = 0.414$

$\therefore \mu = 0.414$ **(Ans.)**

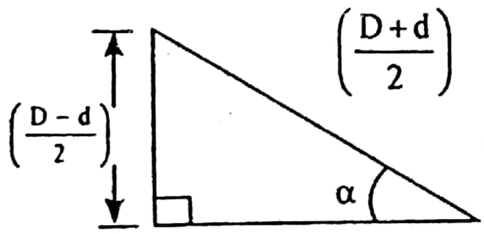
16. *Two heavy right circular rollers of diameters D and d, respectively, rest on a rough horizontal plane as shown in Fig. The larger roller has a string wound around it to which a horizontal force P can be applied as shown. Assuming that the co-efficient of friction μ has the same value for all surfaces of contact, determine the necessary condition under which the large roller can be pulled over the small one.*




**Soln.** Let $Q_1$ and $Q_2$ are the weight of two rollers. The normal reactions $R_1$ & $R_2$ acting normally towards the two cylinders. The reaction $R_3$ acts in between two cylinders at point of contact. From the geometry of the fig.




$$\sin\alpha = \frac{\left(\frac{D-d}{2}\right)}{\left(\frac{D+d}{2}\right)} \Rightarrow \sin\alpha = \frac{(D-d)}{(D+d)}$$




$$\therefore \cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{(D-d)^2}{(D+d)^2}} = \frac{2\sqrt{Dd}}{D+d}$$

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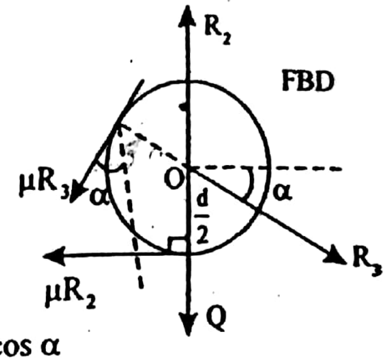
Considering the smaller roller




$$\Sigma M_0 = 0, \quad \mu R_2 \times \frac{d}{2} = \mu R_3 \times \frac{d}{2}$$

$$\Rightarrow R_2 = R_3 \qquad \text{.........(i)}$$

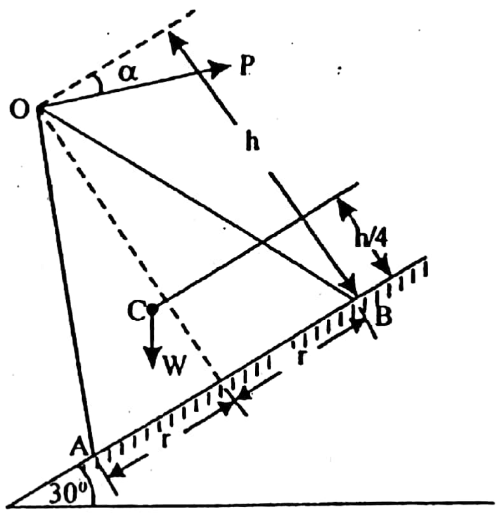
Resolving horizontally $\Sigma x = 0$

$$R_3 \cos\alpha = \mu R_2 + \mu R_3 \sin\alpha \qquad \text{.......(ii)}$$

or $\cos\alpha = \mu + \mu \sin\alpha \Rightarrow \cos\alpha = \mu(1 + \sin\alpha)$ or $\mu = \dfrac{\cos\alpha}{1+\sin\alpha}$

$$\therefore \quad \mu = \frac{2\frac{\sqrt{Dd}}{D+d}}{1+\frac{(D-d)}{(D+d)}} = \frac{2\frac{\sqrt{Dd}}{D+d}}{\frac{D+d+D-d}{D+d}} = \frac{2\sqrt{Dd}}{2D} = \frac{\sqrt{D}\sqrt{d}}{\sqrt{D}\sqrt{D}} \cdot \quad \mu = \sqrt{\frac{d}{D}} \qquad \textbf{(Ans.)}$$

11. *A solid right circular cone of radius r = 76.2mm and altitude h = 4.6mm has its centre of gravity C on its geometric axis at the distance h/4 = 76.2mm. above the base. This cone rests on an inclined plane AB, which makes an angle of $30^0$ with the horizontal and for which the coefficient of friction is μ= 0.5 . A horizontal force P is applied to the vertex O of the cone and about in the vertical plane of the Fig- as shown. Find the maximum and minimum values of P consistent with equilibrium of the cone if the weight W = 44.5 N.*

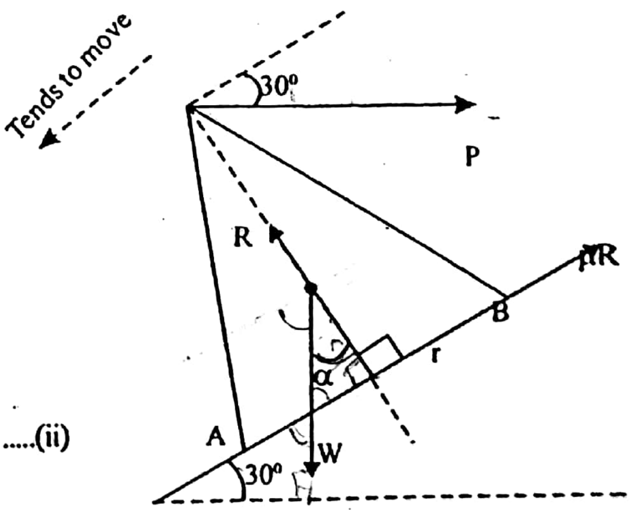



**Soln.** Given data

h = 304.6 mm, r = 76.2 mm

$$\mu = 0.5, \quad \frac{h}{4} = 76.2 \text{ mm}$$

$$\alpha = 30°, \; W = 44.5 \text{ N}$$




**For minimum P when cone tends to slides**

Resolving along 'y' $\Sigma y = 0$

$$R = W\cos 30° + P\sin 30° \qquad \text{...(i)}$$

Resolving along 'x', $\Sigma x = 0$

$$\mu R + P\cos 30° = W\sin 30° \qquad \text{.....(ii)}$$

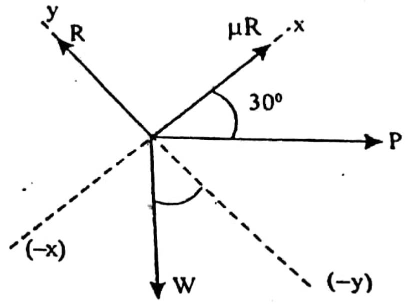
Substituting the value of R in equation (ii)

$$\mu(W\cos 30° + P\sin 30°) + P\cos 30° = W\sin 30°$$

$\Rightarrow \mu P \sin 30° + P \cos 30° = W \sin 30° - \mu W \cos 30°$

$\Rightarrow P(0.5 \sin 30° + \cos 30°) = 44.5\,(\sin 30° - 0.5\cos 30°)$

$\Rightarrow P = \dfrac{2.980}{1.116} = 2.67\text{ N}$ **(Ans.)**




**For maximum value of P**

When the cone tends to topple at B

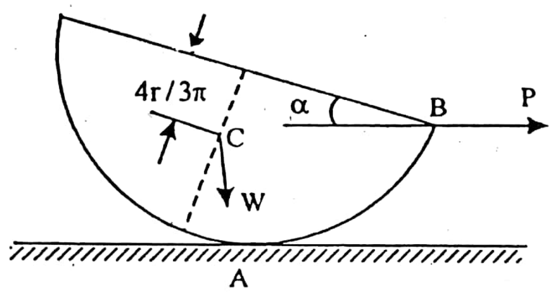
$\Sigma M_B = 0$

$$W \cos 30° \times r + W \sin 30° \times \frac{h}{4} + P \sin 30° \times r = P \cos 30° \times h$$

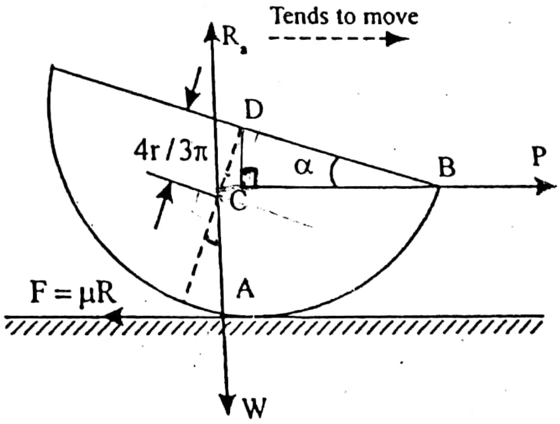
$$\Rightarrow (44.5 \cos 30° \times 76.2) + (44.5 \sin 30° \times 76.2) = P\left[(\cos 30° \times 304.6) - (\sin 30° \times 76.2)\right]$$

$$\Rightarrow 2936.60 + 1695.45 = P(263.79 - 38.1) \Rightarrow P = \frac{4632.05}{225.69} = 20.523\text{ N}$$ **(Ans.)**

**18.** *A short semicircular right cylinder of radius r and weight W rests on a horizontal surface and is pulled at right angles to it geometric axis by a horizontal force P applied at the middle B of the front edge. Find the angle α that the flat face will make with the horizontal plane just before sliding begins if the coefficient of friction at the line of contact A is μ. The gravity force W must be considered as acting at the center of gravity C as shown in the figure.*




**Soln.** The FBD is shown in the figure




Resolving vertically $\Sigma y = 0$

$R_a = W$ .....(i)

Resolving horizontally $\Sigma x = 0$

$P = \mu R_a = \mu W$ ....(ii)

$$\Sigma M_D = 0,\ P \times r \sin\alpha + W \sin\alpha \cdot \frac{4r}{3\pi} = \mu w \times r \Rightarrow \mu W \sin\alpha + W\frac{4}{3\pi}\sin\alpha = \mu W$$

$$\Rightarrow \sin\alpha\left(\mu + \frac{4}{3\pi}\right) = \mu \Rightarrow \sin\alpha = \frac{\mu 3\pi}{3\pi\mu + 4} \Rightarrow \alpha = \sin^{-1}\left(\frac{3\pi\mu}{4 + 3\pi\mu}\right)$$ **(Ans.)**

## 3.1 INTRODUCTION

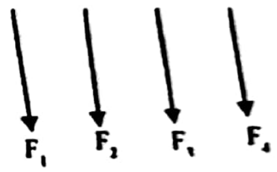
The coplanar forces whose lines of action are parallel to each other, are known as parallel forces.

## 3.2 CLASSIFICATION

(i) Like parallel forces, (ii) Unlike parallel forces

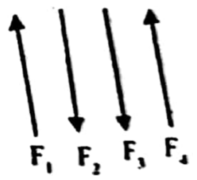
**(i) Like parallel forces**

The coplanar parallel forces when act in the same directions, are known as like parallel forces.




**(ii) Unlike parallel forces**

The coplanar parallel forces when act in different directions are known as unlike parallel forces.

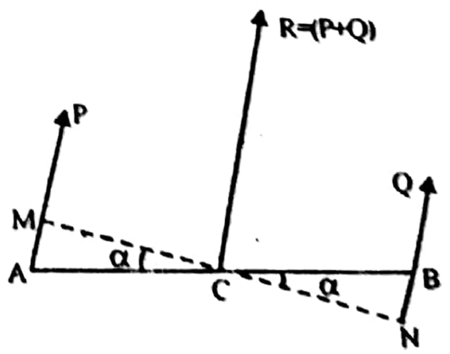



## 3.3 RESULTANT OF LIKE PARALLEL FORCES

Let P & Q are two like parallel forces act at the points A & B. Since the angle between them is 0°, the resultant,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = (P + Q)^2 = P + Q \qquad \text{....(i)}$$

Also the resultant should pass in-between them and let it acts at 'C'. Through 'C' draw MN perpendicular to P & Q. Hence;




$CM = AC\cos\alpha$ and $CN = BC\cos\alpha$

According to Varignon's principle, the moment of R about 'C' is equal to the algebric sum of moments of P & Q about C.

i.e., $R \times O = Q \times CN - P \times CM$

or $P \times AC \cos\alpha = Q \times BC \cos\alpha$

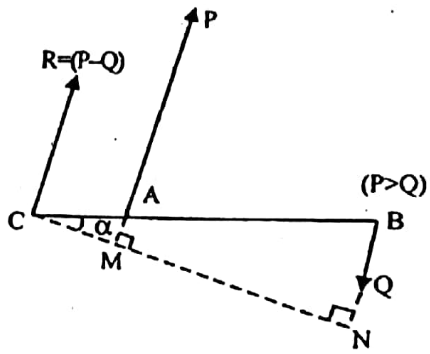
or $P \times AC = Q \times BC$ ....(i)

Also $AB = AC + BC$ ....(ii)

It is seen that the resultant of like parallel forces divide AB internally and acts parallel to P & Q.

## 3.4 Resultant of Unlike Parallel Forces

Let P & Q are two unlike parallel forces act at the points A & B. Since the angle between them is 180°, the resultant does not lie inbetween them. The magnitude is




$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180°} = P - Q \text{ ....(i)}$$

Assume it acts at C from A where larger force P acts. It may act at C from B if 'Q' is larger.

Through C, draw MN perpendicular to P & Q. Then $CM = AC\cos\alpha$ and $CN = BC\cos\alpha$

According to Varignon's theorem; the moment of R about C must be equal to the sum of moments of P & Q about C.

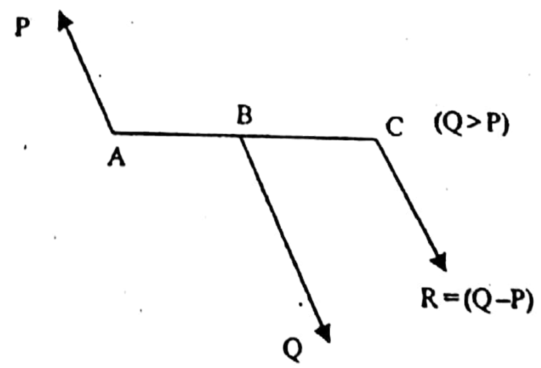
$\therefore$ $R \times 0 = P \times CM - Q \times CN$

or $P \times AC\cos\alpha = Q \times BC\cos\alpha$

or $P \times AC = Q \times BC$ ....(ii)

Also $BC = AB + AC$ ....(ii)

It is seen that the resultant divides the distance between P & Q externally and acts parallel to them.




***Note:*** If Q > P, then the figure shows the position of the resultant.

In same way;

$R = Q - P$ ....(i)

$P \times AC = Q \times BC$ ....(ii)

and $AC = AB + BC$

The surface area 'dA' generated by the rotation of 'dL' is ; $dA = dL \cdot \theta \cdot y$

Therefore the total area generated by the segment 'L' be

$$A_x = \int dL \cdot \theta \cdot y \qquad \text{....(i)}$$

But; $\int dL \cdot y = L\, y_c$

Hence the equation (i) can be written as; $A_x = L\,\theta\, y_c$

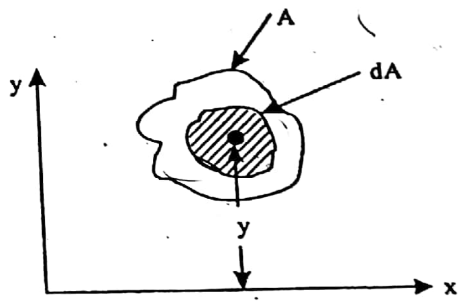
Similarly, area generated about y-axis; $A_y = L\,\theta\, x_c$

### 3.3.13 Papus Theorem - II

The volume of the solid generated by rotating any plane figure about a non intersecting axis is equal to the product of area of the figure, angle of rotation and centroidal distance of that figure from that non intersecting axis.

i.e., $V_x = A \times \theta \times y_c$

and $V_y = A \times \theta \times x_c$




**Proof**

Consider an infinite small area dA, is at a distance of 'y' from x-axis, rotates at an angle '$\theta$'.

The volume generated dV by the rotation of dA is;

$$dV = dA \cdot \theta \cdot y$$

Therefore the total volume of the solid generated;

$$V_x = \int dA \cdot \theta \cdot y \qquad \text{.......(i)}$$

But $\int dA \cdot y = A y_c$

Hence; $V_x = A \cdot \theta \cdot y_c$

Similarly the volume generated about y-axis;

$V_y = A \cdot \theta \cdot x_c$

*Note :-* For one complete revolution, $\theta = 2\pi$ ·

$V_x = A \times \theta \times y_c$

$V_y = A \times \theta \times x_c$

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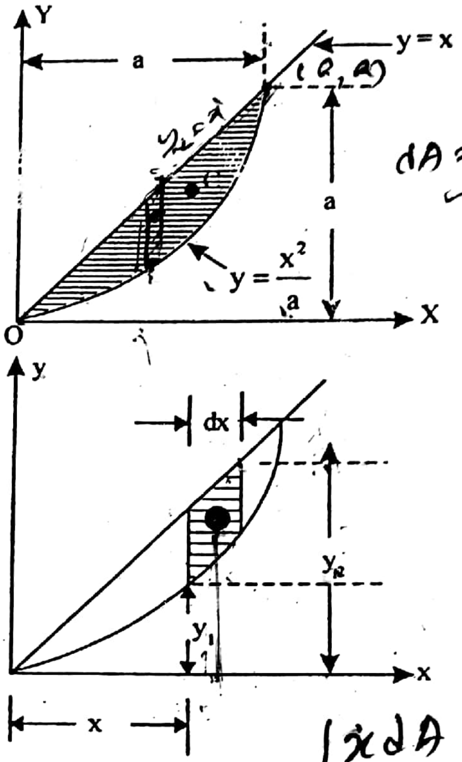
2. *Determine the coordinates $x_c$ and $y_c$ of the centroid C of the area between the parabola $y = x^2/a$ and the straight line $y = x$ as shown in the fig.*




$dA = (y_2 - y_1)dx$

$y = \frac{x^2}{a}$

**Soln.** Consider a small strip of thickness dx is at a distance of x from y-axis.

The height of the strip is $y_2 - y_1$, where $y_2 = x$ & $y_1 = x^2/a$

From the principle $x_c = \dfrac{\int x\,dA}{\int dA}$

$x_c = \frac{\int x\,dA}{\int dA}$

where $dA = (y_2 - y_1)\,dx$
Substituting the values

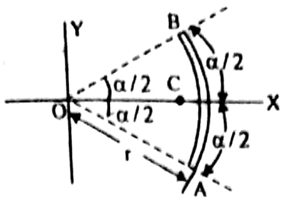
$$x_c = \frac{\int_0^a x.(y_2 - y_1)dx}{\int_0^a (y_2 - y_1)dx} = \frac{\int_0^a x\left[x - \frac{x^2}{a}\right]dx}{\int_0^a \left(x - \frac{x^2}{a}\right)dx}$$

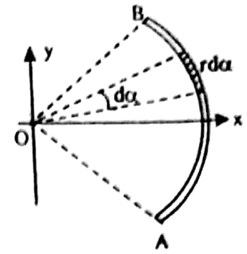
$$= \frac{\int_0^a \left(x^2 - \frac{x^3}{a}\right)dx}{\int_0^a \left(x - \frac{x^2}{a}\right)dx} = \frac{\left[\frac{x^3}{3} - \frac{x^4}{4a}\right]_0^a}{\left[\frac{x^2}{2} - \frac{x^3}{3a}\right]_0^a} = \frac{\frac{a^3}{3} - \frac{a^3}{4}}{\frac{a^2}{2} - \frac{a^2}{3}} = \frac{a}{2}$$

$$\text{Similarly} \quad y_c = \frac{\int y.dA}{\int dA} = \frac{\int_0^a \left(\frac{y_2 + y_1}{2}\right)(y_2 - y_1)dx}{(y_2 - y_1)dx} = \frac{\frac{1}{2}\int_0^a \left[(y_2)^2 - (y_1)^2\right]dx}{\int_0^a (y_2 - y_1)dx}$$

$$= \frac{\frac{1}{2}\int_0^a \left(x^2 - \frac{x^4}{a^2}\right)dx}{\int_0^a \left(x - \frac{x^2}{a}\right)dx} = \frac{\frac{1}{2}\left[\frac{x^3}{3} - \frac{x^5}{5a^2}\right]_0^a}{\left[\frac{x^2}{2} - \frac{x^3}{3a}\right]_0^a} = \frac{\frac{1}{2}\left[\frac{a^3}{3} - \frac{a^3}{5}\right]}{\left[\frac{a^2}{2} - \frac{a^2}{3}\right]} = \frac{\frac{1}{2}\left(\frac{2a^3}{15}\right)}{\frac{a^2}{6}} = \frac{a^3}{15} \times \frac{6}{a^2} = \frac{2}{5}a \quad \textbf{(Ans.)}$$

3. *Determine the coordinates $x_c$ and $y_c$ of the centriod C of the length of circular arc AB of radius r and central angle α as shown in Fig.*




**Soln.** Consider an elementary length $d\ell$ subtending angle $d\alpha$ at the center.

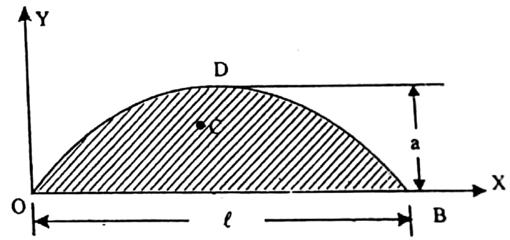
$\therefore d\ell = r d\alpha$

Let us assume $x = r \cos \alpha$

$$\therefore x_c = \frac{\int x\, d\ell}{\int d\ell} = \frac{\int_{-\alpha/2}^{\alpha/2} r.\cos\alpha\, .rd\alpha}{\int_{-\alpha/2}^{\alpha/2} rd\alpha} = \frac{r[\sin\alpha]_{-\alpha/2}^{\alpha/2}}{[\alpha]_{-\alpha/2}^{\alpha/2}} = \frac{r\left[\sin\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right]}{\frac{\alpha}{2} + \frac{\alpha}{2}} = \frac{2r}{\alpha}\sin\frac{\alpha}{2}$$

Similarly,
$$y_c = \frac{\int y\, .d\ell}{\int d\ell} = \frac{\int_{-\alpha/2}^{\alpha/2} r\, .\sin\alpha\, .\, r\, d\alpha}{\int_{-\alpha/2}^{\alpha/2} r.d\alpha} = \frac{-r\, .[\cos\alpha]_{-\alpha/2}^{\alpha/2}}{[\alpha]_{-\alpha/2}^{\alpha/2}} = \frac{-r\left[\cos\frac{\alpha}{2} - \cos\frac{\alpha}{2}\right]}{\left[\frac{\alpha}{2} + \frac{\alpha}{2}\right]} = 0$$

4. *Determine the coordinates $x_c$ and $y_c$ of the centriod C of the area between the x-axis and the half sine wave ODB as shown in fig..*

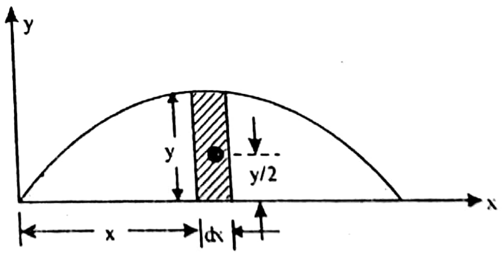



**Soln.** Consider a strip of thickness dx is at a distance of x from Y and axis.

Let y be its height

Since it is a sine curve

Assume $Y = a \sin \frac{\pi x}{\ell}$



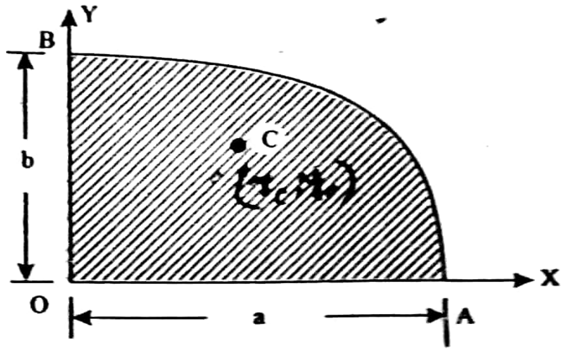

$$x_c = \frac{\int x\, dA}{\int dA}$$

$\therefore$ where $dA = ydx$

Substituting the value

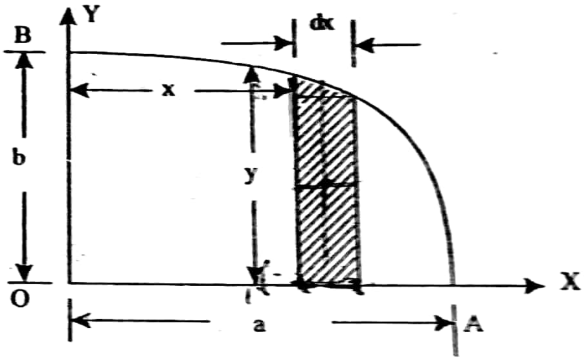
## SOLVED PROBLEMS - 3.3

1. *Determine the coordinates $x_c$ and $y_c$ of the centriod C of the area of one-quadrant of an ellipse OAB with major and minor semiaxes a and b, respectively.*




**Soln.** Consider a small strip of thickness dx is at a distance of X from Y axes.
Let y be the vertical length.

$$\therefore\ x_c = \frac{\int x\,dA}{\int dA}$$




where $dA = y\,dx$

We know that equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Assume $x = a\sin\alpha,\ y = b\cos\alpha$

$\therefore\quad dx = a\cos\alpha\,d\alpha,\ dy = b\sin\alpha\,d\alpha$

If $x = 0$, then $\sin\alpha = 0$ when $\alpha = 0$

Similarly if $x = a$, $\sin\alpha = 1$ when $\alpha = 90°$

$$\therefore\ x_c = \frac{\int_0^a x\,y\,dx}{\int_0^a y\,dx} = \frac{\int_0^{\pi/2} a\sin\alpha\ b\cos\alpha\ .\ a\cos\alpha\ .\ d\alpha}{\int_0^{\pi/2} b\cos\alpha\ .\ a\cos\alpha\ \ d\alpha} = \frac{a\int_0^{\pi/2}\cos^2\alpha\sin\alpha\ \ d\alpha}{\int_0^{\pi}\cos^2\alpha\,d\alpha}$$

$$= \frac{a\int_0^{\pi/2}\cos^2\alpha\ \sin\alpha\ \ d\alpha}{\frac{1}{2}\int_0^{\pi/2}(1+\cos 2\alpha)\ d\alpha} = \frac{a(-1)\left[\frac{\cos^3\alpha}{3}\right]_0^{\frac{\pi}{2}}}{\frac{1}{2}\left(\alpha + \frac{\sin 2\alpha}{2}\right)_0^{\frac{\pi}{2}}} = \frac{\frac{-2a}{3}(0-1)}{\frac{\pi}{2}} = \frac{4a}{3\pi}$$

Similarly $\quad y_c = \dfrac{\int_0^a \frac{y}{2}\,y\,dx}{\int_0^a y\,dx} = \dfrac{4b}{3\pi}$ **(Ans.)**

$$x_c = \frac{\int_0^\ell x.y.dx}{\int_0^\ell y.dx} = \frac{\int_0^\ell x.a\sin\frac{\pi x}{\ell}.dx}{\int_0^\ell a\sin\frac{\pi x}{\ell}.dx}$$

$$= \frac{\left|x\left[-\frac{a\ell}{\pi}\cos\frac{\pi x}{\ell}\right]-1\left[\frac{a\ell^2}{\pi^2}\sin\frac{\pi x}{\ell}\right]\right|_0^\ell}{\left(-\frac{a\ell}{\pi}\cos\frac{\pi x}{\ell}\right)_0^\ell} = \frac{\ell-\frac{a\ell}{\pi}(-1)}{-\frac{a\ell}{\pi}(-1)+\frac{a\ell}{\pi}} = \frac{a\ell^2/\pi}{2a\ell/\pi} = \frac{\ell}{2}$$

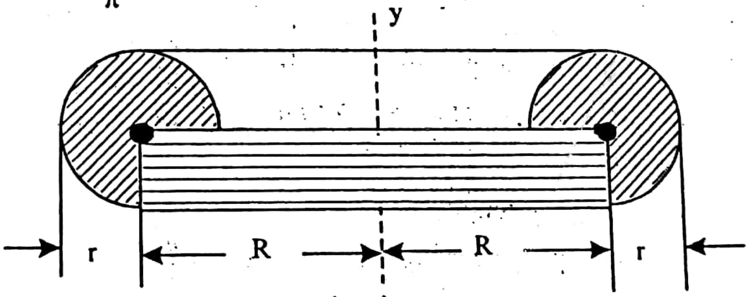
Similarly

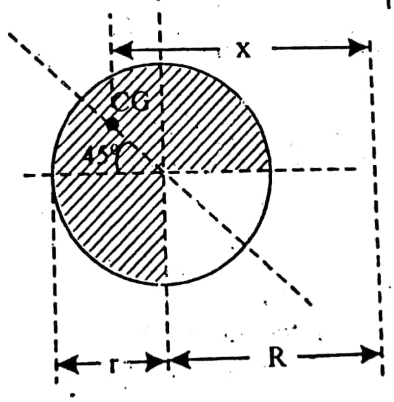
$$y_c = \frac{\int \frac{y}{2}dA}{\int dA} = \frac{\int_0^\ell \frac{y}{2}y.dx}{\int_0^\ell y.dx} = \frac{\frac{1}{2}\int_0^\ell y^2\,dx}{\int_0^\ell y.dx} = \frac{\frac{1}{2}\int_0^\ell\left(a\sin\frac{\pi x}{\ell}\right)^2dx}{\int_0^\ell a\sin\frac{\pi x}{\ell}dx}$$

$$= \frac{\frac{a^2}{4}\int_0^\ell\left(1-\cos\frac{2\pi x}{\ell}\right)dx}{\int_0^\ell a\sin\frac{\pi x}{\ell}dx}, \quad \text{since}\left[\sin^2\theta = \frac{(1-\cos 2\theta)}{2}\right]$$

$$= \frac{\frac{a^2}{4}\left[x-\frac{\ell}{2\pi}\cos\frac{2\pi x}{\ell}\right]_0^\ell}{\left[\frac{-a\ell}{\pi}\cos\frac{\pi x}{\ell}\right]_0^\ell} = \frac{\frac{a^2}{4}\left[\ell-\frac{\ell}{2\pi}(-1)\right]}{\frac{2a\ell}{\pi}} = \frac{a^2\ell}{4}\times\frac{\pi}{2a\ell} = \frac{\pi a}{8}$$ (Ans.)

5 *Using the second theorem of Pappus, calculate the volume of the ring as shown in Fig. If R = 250 mm, r = 100 mm.*




**Soln.** Given data

R = 250 mm

r = 100 mm

According to the 2nd theorem of papus, the volume generated

$V_x = A\theta \times x_c$

For full revolution $\theta = 2\pi$

Since $\frac{3}{4}$th of the circular area is rotated about y-axis,

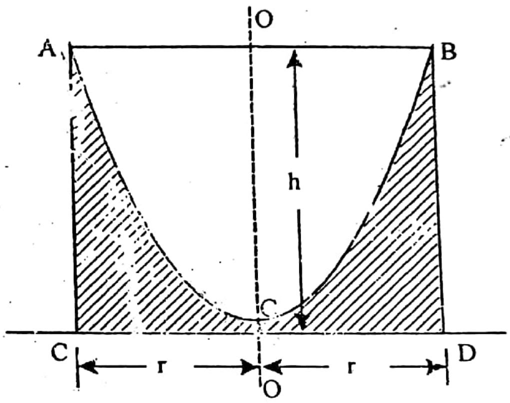
$$A = \frac{3}{4}\pi r^2 \quad \text{or} \quad A = \frac{3}{4}\pi(100)^2 = 7500\pi$$

$$x_c = R + \left(\frac{4r}{3\alpha}\sin\frac{\alpha}{2}\right)\sin 45° \qquad = 250 + \frac{4 \times 100}{3\left(\frac{3}{4} \times 2\pi\right)}\sin 135° \times \frac{1}{\sqrt{2}}$$

$$= 250 + \frac{400}{\frac{9}{2}\pi} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \qquad = 250 + \frac{400}{9\pi} = 264.147 \text{ mm}$$

$V = A\theta\, x_c = 7500\,\pi \times 2\pi \times 257.07 = 3.91 \times 10^7\ \text{mm}^3$ **(Ans.)**

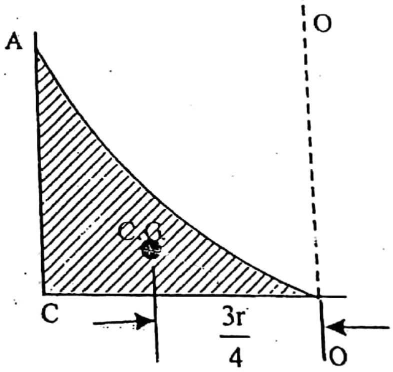
*A right circular cylindrical tank containing water, spins about its vertical geometric axis OO at such speed that the free water surface is a paraboloid ACB as shown in fig. What will be the depth of water in the tank when it comes to rest ?*




**Soln.** When the water spins about the geometrical axis '0-0', the volume generated $V = A\,\theta\, x_c$

Since the surface of water is a paraboloid, the area;

$$A = \frac{rh}{3} \quad \text{And} \quad x_c = \frac{3r}{4}$$



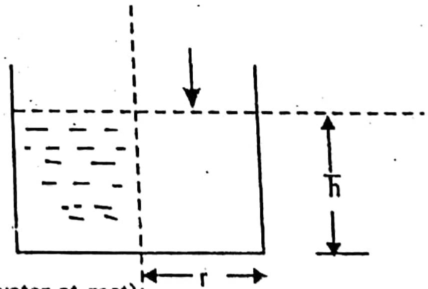

$\theta = 2\pi$

According to the 2nd theorem of Papus;

$$\therefore V = \frac{rh}{3} \times 2\pi \times \frac{3r}{4} = \frac{r^2 h\pi}{2} \qquad \text{....(i)}$$

When the water comes to rest in the tank, the volume remains same as before, generated by rotation.

Let $\bar{h}$ is the height of the water in the tank when it is at rest.



(water at rest)

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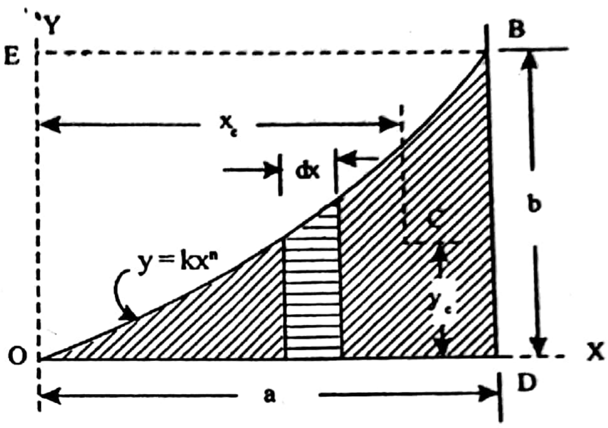
$\therefore$ the volume of the water $V = \pi r^2 \bar{h}$ ....(ii)

Equating (i) and (ii) $\dfrac{r^2 h \pi}{2} = \pi r^2 \bar{h} \Rightarrow \bar{h} = \dfrac{h}{2}$ **(Ans.)**

7. *Referring to the given Fig. prove that, if the equation of the curve OB, referred to the coordinate axes x and y taken along two adjacent sides of a rectangle OEBD, is $y = kx^n$, then the coordinates $x_c$ and $y_c$ of the centroid C of the area of the shaded spandrel OBD are given by the formulas*



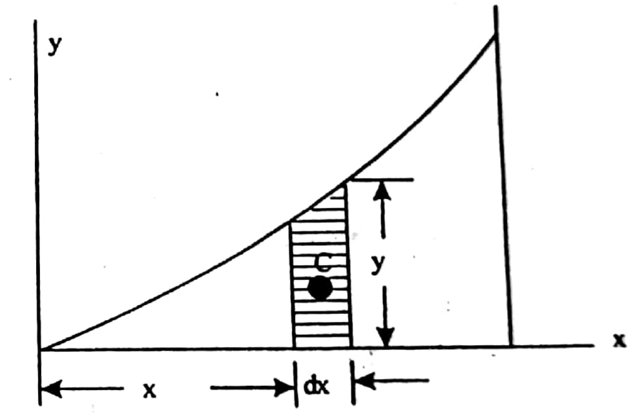

$$x_c = \frac{n+1}{n+2}a, \quad x_c = \frac{n+1}{4n+2}b$$

**Soln.** Considering a strip of thickness dx is at distance of x from Y axis.

Let y be its height.

The equation of parabola is given

$$y = Kx^n$$




$$\therefore x_c = \frac{\int x\, dA}{\int dA}$$

where $dA = ydx$

$$\text{Substituting; } x_c = \frac{\int_0^a x.y\, dx}{\int_0^a y.dx} = \frac{\int_0^a x\,.\,Kx^n\,.\,dx}{\int_0^a Kx^n\, dx} = \frac{\int_0^a x^{n+1} dx}{\int_0^a x^n dx}$$

$$= \frac{\left.\dfrac{x^{n+2}}{n+2}\right]_0^a}{\left.\dfrac{x^{n+1}}{n+1}\right]_0^a} = \frac{a^{n+2}}{n+2} \times \frac{n+1}{a^{n+1}} = \frac{n+1}{n+2}a$$

$$\text{Similarly } y_c = \frac{\int y\, dA}{\int dA} = \frac{\int_0^a \frac{y}{2}\, ydx}{\int_0^a y.dx} = \frac{\frac{1}{2}\int_0^a K^2 x^{2n} dx}{\int_0^a Kx^n dx} = \frac{\frac{1}{2}K\left[\dfrac{x^{2n+1}}{2n+1}\right]_0^a}{\left[\dfrac{x^{n+1}}{n+1}\right]_0^a}$$

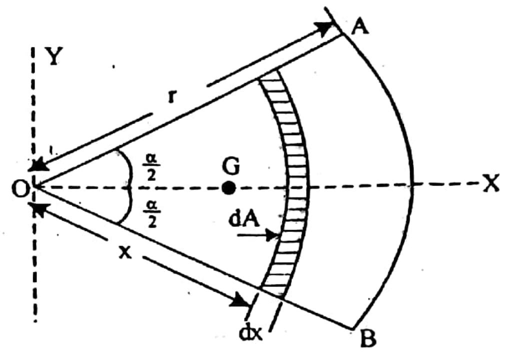
For the maximum range x = a, y = b; substituting the values in the equation $y = kx^n$ ;

$\Rightarrow b = Ka^n \therefore K = \dfrac{b}{a^n}$

Substituting the values of x, y and K;

$$y_c = \frac{1}{2}K\left[\frac{a^{2n+1}}{2n+1}\right] \times \frac{n+1}{a^{n+1}} = \frac{k}{2}\left[\frac{n+1}{2n+1}\right] \times a^n, \qquad y_c = \left[\frac{n+1}{2(2n+1)}\right]b$$

**8.** *Using result of Problem. 3 and an elemental area dA as shown in the Fig. locate the centroid G of the area of the circular sector OAB by integration.*




**Soln.** Consider a small circular arc of thickness dx at a distance of x from the center.

$\therefore dA = \alpha x \, dx$

We know that the centroid of that circular arc from the center is $\dfrac{2x}{\alpha} \sin\dfrac{\alpha}{2}$

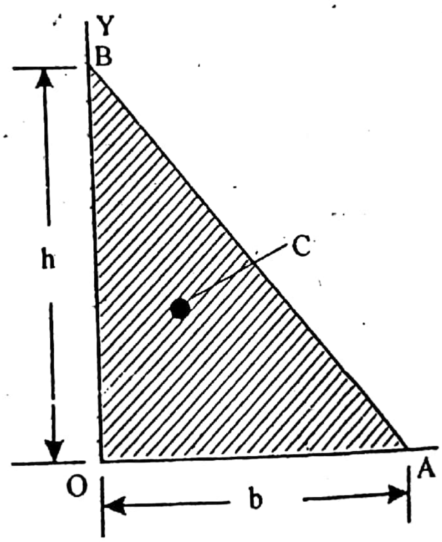
$$\therefore OG = x_c = \frac{\int x dA}{\int dA} = \frac{\int_0^r \frac{2x}{\alpha} \sin\frac{\alpha}{2} . \alpha x . dx}{\int_0^r \alpha \, x \, dx}$$

$$= \frac{2 \sin\frac{\alpha}{2}\left[\frac{x^3}{3}\right]_0^r}{\alpha\left[\frac{x^2}{2}\right]_0^r} = 2\left(\sin\frac{\alpha}{2}\right) \times \frac{r^3}{3} \times \frac{2}{\alpha r^2} = \frac{4r}{3\alpha} \sin\frac{\alpha}{2} \qquad \textbf{(Ans.)}$$

Since the sector is symmetrical about x-axis, the centroid lies on x-axis.

Hence $y_c = 0$ **(Ans.)**

**9.** *Determine the co-ordinates $x_c$ and $y_c$ of the C of the area of a right-angled triangle OAB with base and height b and h, respectively as shown in the fig.*

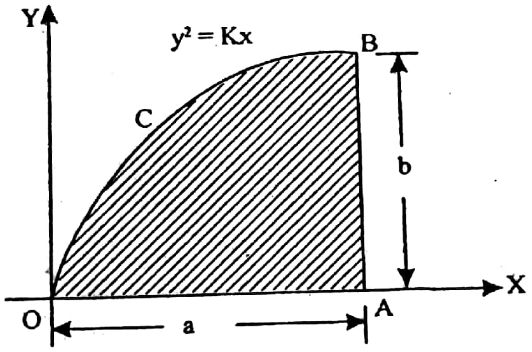


$\therefore x_c = OG \cos 45°$

or $\quad x_c = \dfrac{2\pi}{3 \times \pi} \sin 45° \times \cos 45° = \dfrac{4r}{3\pi}$ **(Ans.)**

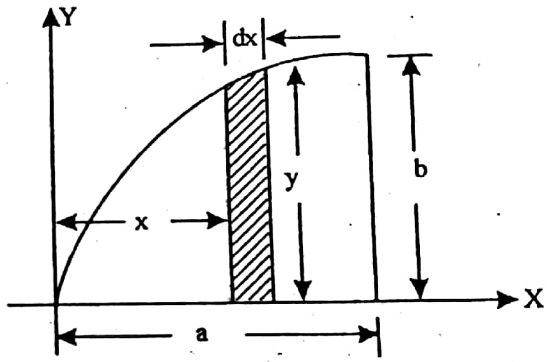
Similarly $\quad Y_c = OG \sin 45°, \qquad Y_c = \dfrac{4r}{3\pi}$ **(Ans.)**

**11.** *Determine the coordinates $x_c$ and $y_c$ of the centroid C of the area bounded by the parabloa $y^2 = kx$, the straight line $x = a$ and x-axis as shown in fig.*




**Soln.** Consider a vertical strip of thickness dx is at a distance of x from the Y-axis.

Let y be its height.




$$\therefore X_c = \frac{\int x dA}{\int dA}$$

where $dA = ydx$

From the equation $y^2 = kx$

$y = \sqrt{kx}$

$$\therefore X_c = \frac{\int_0^a x\, ydx}{\int_0^a ydx}$$

$$= \frac{\int_0^a x.\sqrt{kx}\,.dx}{\int_0^a \sqrt{kx.dx}} = \frac{\int_0^a x^{3/2}dx}{\int_0^a x^{1/2}dx} = \frac{\left[\dfrac{(x)^{5/2}}{\frac{5}{2}}\right]_0^a}{\left[\dfrac{(x)^{3/2}}{\frac{3}{2}}\right]_0^a} = \frac{\left[\dfrac{(a)^{5/2}}{\frac{5}{2}}\right]}{\left[\dfrac{(a)^{3/2}}{\frac{3}{2}}\right]} = \frac{(a)^{\frac{5}{2}}}{\frac{5}{2}} \times \frac{\frac{3}{2}}{(a)^{\frac{3}{2}}}$$

$$= \frac{(a)^{5/2}}{5} \times \frac{3}{(a)^{3/2}} = \frac{3a}{5}$$ **(Ans)**

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$$y_c = \frac{\int \frac{y}{2} dA}{\int dA} = \frac{\int \frac{y}{2} \times y dx}{\int y\, dx} = \frac{\frac{1}{2}\int y^2 dx}{\int y dx} = \frac{\frac{1}{2}\int_0^a kx\, dx}{\int_0^a \sqrt{kx} dx}$$

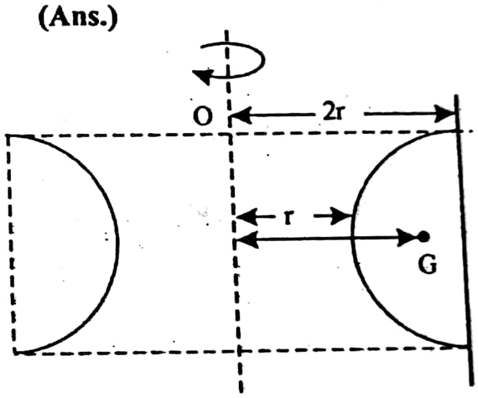
$$= \frac{\frac{k}{2}\left(\frac{x^2}{2}\right)_0^a}{\sqrt{k}\left(\frac{x^{3/2}}{3/2}\right)_0^a} = \frac{\frac{\sqrt{k}}{2} \times \frac{a^2}{2}}{\frac{(a)^{3/2}}{3/2}} = \frac{\frac{\sqrt{k}a^2}{4} \times 3}{2(a)^{3/2}} = \frac{3\sqrt{k}a^2}{8\, a^{3/2}} = \frac{3\sqrt{k}\,\sqrt{a}}{8}$$

We know that $y^2 = Kx$, when $x = a$, $y = b$, substituting;

$$b^2 = ka \quad \Rightarrow \quad K = \frac{b^2}{a}$$

Substituting the value, $y_c = \dfrac{3 \times \frac{b}{\sqrt{a}} \times \sqrt{a}}{8} = \dfrac{3b}{8}$ **(Ans.)**

**12.** *Using the first theorem of Pappus, calculate the area of the surface of revolution shown in Fig. which is obtained by rotating a semicircular arc about a vertical axis.*

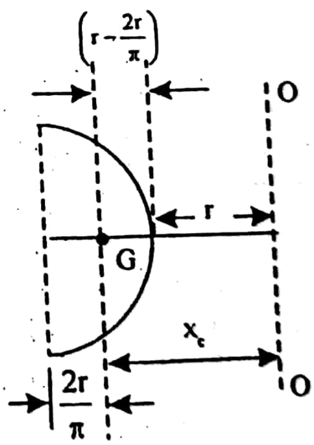



**Soln.** $L = \pi \times r$

$$x_c = r + \left(r - \frac{2r}{\pi}\right) = \left(2r - \frac{2r}{\pi}\right)$$

$\theta = 2\pi$

$\therefore$ According to 1st theorem of papus the area generated about the axis O–O;.



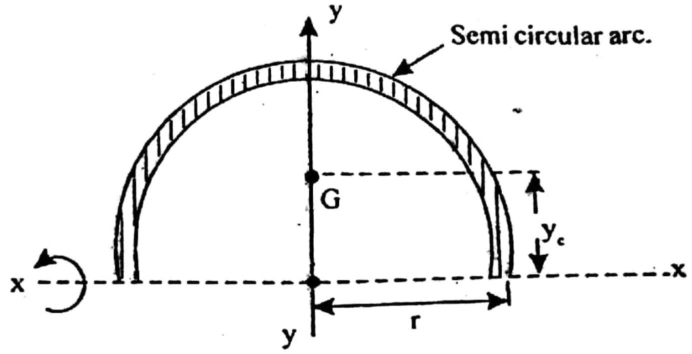

$$A_{0-0} = L \times \theta \times x_c$$

$$= \pi r \times 2\pi \times \left(2r - \frac{2r}{\pi}\right) = \pi r^2 \times 4 \times r^2\left(1 - \frac{1}{\pi}\right)$$

$$= \pi^2 \times 4 \times r^2 \frac{(\pi - 1)}{\pi} = 4\pi r^2 (\pi - 1)$$

**(Ans.)**

**13.** *Using the first theorem of Pappus, calculate the centroid of a semicircular arc of radius r, if the surface area of sphere is* $A = 4\pi r^2$




**Soln.** Given data : $A = 4\pi r^2$

According to the 1st theorem of papus

$A_{x-x} = L \times \theta \times y_c$

where $L = \pi r,\ \theta = 2\pi$

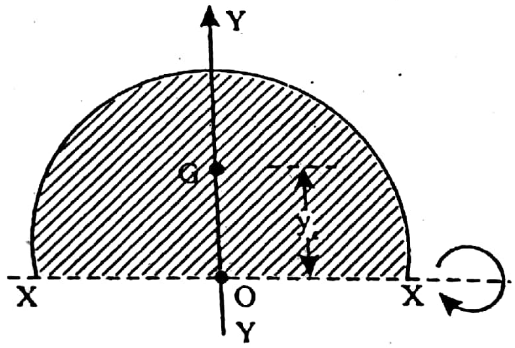
Substituting the values

$4\pi r^2 = \pi r \times 2\pi \times y_c$

$$\therefore y_c = \frac{4\pi r^2}{2\pi^2 r} = \frac{2r}{\pi} \qquad \text{(Ans.)}$$

Since the arc is symmetrical about y-axis, $x_c = 0$

**14.** *Using the second theorem of Pappus, determine the centroid of semicircular area of radius r, if the volume of the sphere* $V = \frac{4}{3}\pi r^3$ *? (Fig. P)*




**Soln.** Given data

$$V = \frac{4}{3}\pi r^3$$

According to the 2nd theorem of papus

$V_{x-x} = A \times \theta \times y_c$

where $A = \frac{\pi r^2}{2},\ \theta = 2\pi$

Substituting the values,

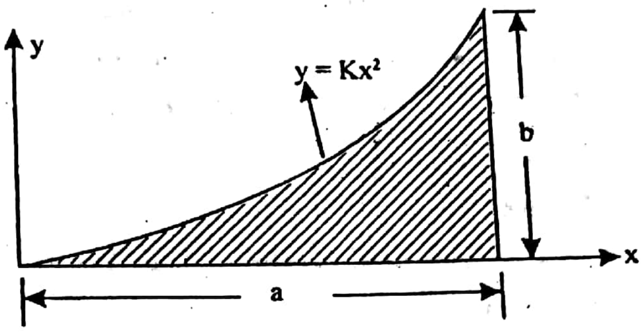
$$\frac{4}{3}\pi r^2 = \frac{\pi r^2}{2} \times 2\pi \times y_c$$

$$\therefore y_c = \frac{4r}{3\pi}$$

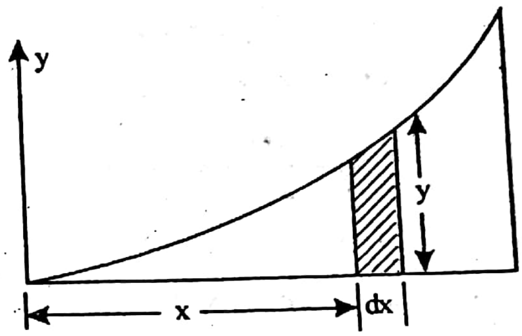
Since the figure is symmetrical about y-axis,

$x_c = 0$

**15.** *Locate the centroid of a parabola as shown in the figure.*




**Soln.** Consider a vertical strip of thickness dx, is at a distance of x from y-axis.




$$\therefore\ x_c = \frac{\int x\, dA}{\int dA} \qquad \text{where, } dA = y\,dx$$

$$\therefore\ x_c = \frac{\int x\, ydx}{\int ydx} = \frac{\int_0^a x.kx^2.dx}{\int_0^a kx^2.dx} = \frac{\int_0^a x^3 dx}{\int_0^a x^2 dx}$$

$$= \frac{\left[\frac{a^4}{4}\right]}{\left[\frac{a^3}{3}\right]} = \frac{3a}{4} \qquad \textbf{(Ans.)}$$

$$\text{Similarly } y_c = \frac{\int \frac{y}{2} dA}{\int dA} = \frac{\int_0^a \frac{y}{2}.ydx}{\int_0^a y\,dx} = \frac{\frac{1}{2}\int_0^a y^2 dx}{\int_0^a ydx}$$

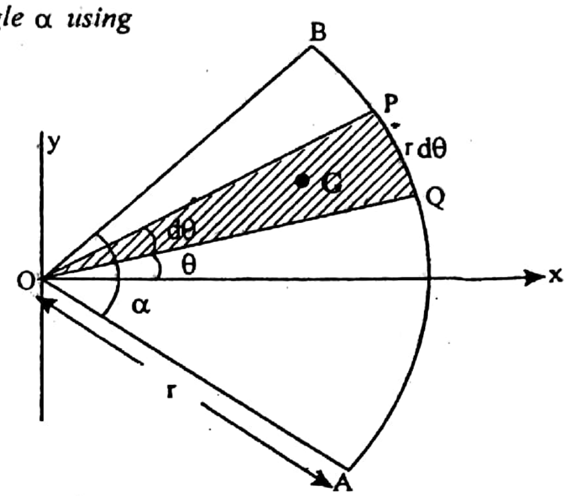
$$= \frac{\frac{1}{2}\int_0^a k^2x^4.dx}{\int_0^a kx^4 dx} = \frac{\frac{1}{2}\,k^2\left[\frac{x^5}{5}\right]_0^a}{k\left[\frac{x^3}{3}\right]_0^a} = \frac{3Ka^2}{10}$$

When $x = a$, $y = b$, then $b = Ka^2$

$$\therefore\ K = \frac{b}{a^2} \qquad \text{Substitute the value of K.}$$

$$\therefore\ y_c = \frac{3b}{10} \qquad \textbf{(Ans.)}$$

**16.** *Determine the centroid of a sector OAB at an angle α using Ist principle as shown in the figure.*

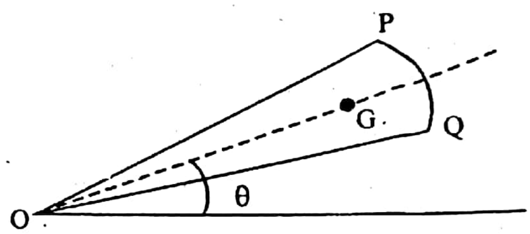



**Soln.** The elementary section of the sector OPQ is so small that it can be treated as a triangle.

$\therefore$ Area of the $\Delta OPQ = dA = \frac{1}{2} \times r \times rd\theta$

$= \frac{1}{2} r^2 \, d\theta$

The centroid of the $\Delta OPQ$ is $OG = \frac{2}{3} r$




$\therefore$ The x-coordinate, $x = OG \cos\theta = \frac{2}{3} r \cos\theta$

$\therefore$ The y-coordinate, $y = OG \sin\theta = \frac{2}{3} r \sin\theta$

$$X_c = \frac{\int x \, dA}{\int dA} = \frac{\int_{-\alpha/2}^{\alpha/2} \frac{2}{3} r \cos\theta \frac{1}{2} r^2 \, d\theta}{\int_{-\alpha/2}^{\alpha/2} \frac{1}{2} r^2 \, d\theta}$$

$$= \frac{\frac{2r}{3} \int_{-\alpha/2}^{\alpha/2} \cos\theta \, d\theta}{\int_{-\alpha/2}^{\alpha/2} d\theta} = \frac{\frac{2r}{3}\left[\sin\frac{\alpha}{2} - \sin\left(-\frac{\alpha}{2}\right)\right]}{\left[\frac{\alpha}{2} - \left(\frac{\alpha}{2}\right)\right]} = \frac{4r}{3\alpha} \sin\left(\frac{\alpha}{2}\right)$$

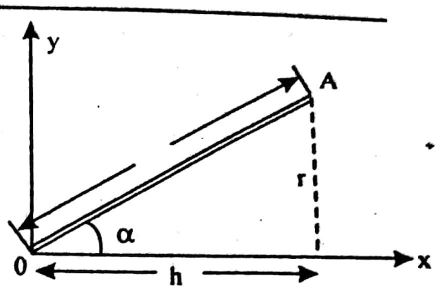
$\therefore X_c = \frac{4r}{3\alpha} \sin\left(\frac{\alpha}{2}\right)$ **(Ans.)**

Similarly; $$Y_c = \frac{\int y dA}{\int dA} = \frac{\int \frac{2}{3} r \sin\alpha \frac{1}{2} r^2 d\theta}{\int \frac{1}{2} r^2 \, d\theta}$$

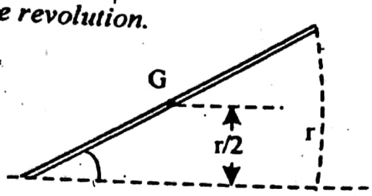
$$= \frac{\frac{2r}{3} \int_{-\alpha/2}^{\alpha/2} \sin\theta d\theta}{\int_{-\alpha/2}^{\alpha/2} d\theta} = \frac{-\frac{2r}{3} [\cos\theta]_{-\alpha/2}^{\alpha/2}}{[\theta]_{-\alpha/2}^{\alpha/2}} = 0$$

$y_c = 0$ **(Ans.)**

**17.** *Using papus theorem, find the surface area generated by a segment as shown in the fig. for one complete revolution.*




**Soln.** L = L

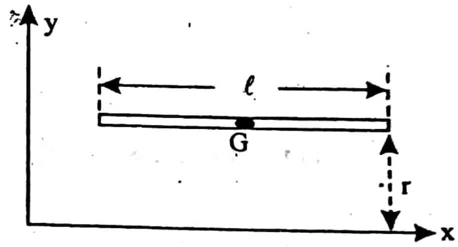



$\theta = 2\pi, \quad y_c = \frac{r}{2}$

$\therefore \quad L.\theta.y_c = L \times 2\pi \times \frac{r}{2} = \pi\, r\, L$ **(Ans.)**

(Surface area of hollow cone)

**18.** *Using papus theorem find the surface area generated by rotating a horizontal bar of length 'l' at a distance of 'r' from x-axis, for one complete revolution.*



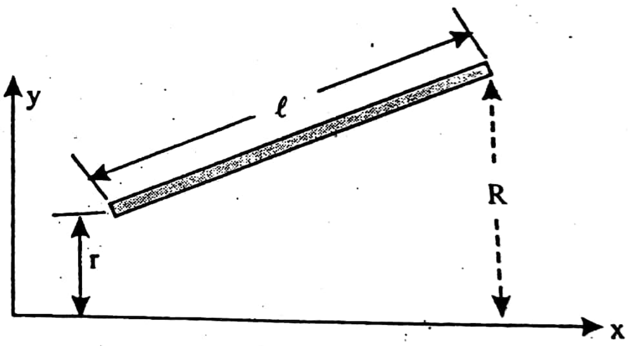

**Soln.** $L = \ell, \quad \theta = 2\pi$

$y_c = r$

$\therefore$ Surface area; $A = L.\theta.y_c = \ell \times 2\pi \times r = 2\pi\, r\, \ell$ **(Ans.)**

(Surface area of a hollow cylinder)

**19.** *Find the surface area generated by an inclined bar as shown in the fig.*



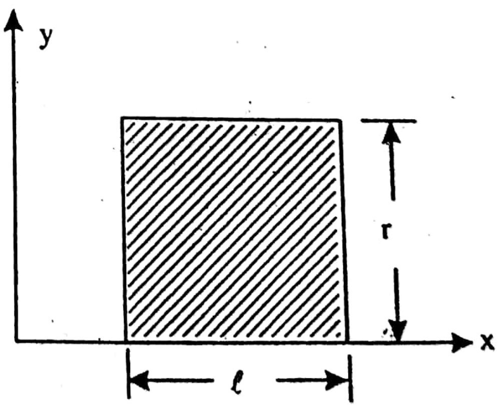

**Soln.** $L = \ell, \; y_c = \frac{R+r}{2}, \; \theta = 2\pi$

$\therefore$ Area generated, $A = L \times \theta \times y_c$

or $A = \ell \times 2\pi \times \left(\frac{R+r}{2}\right) = \pi\,(R+r)\,\ell$

(Surface area of hollow frustrum)

**20.** *Using papus theorem, find the volume of solid generated, if a rectangular area rotates about x-axis as shown in the fig.*



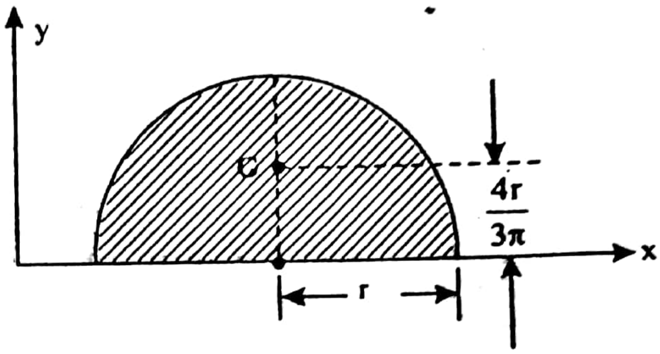

**Soln.** $A = \ell \times r, \quad \theta = 2\pi, \quad y_c = \frac{r}{2}$

$\therefore$ Volume of the solid generated;

$V = A \times \theta \times y_c$

or $V = \ell \times r \times 2\pi \times \frac{r}{2} = \pi r^2 \ell$ **(Ans.)**

(Volume of a solid cylinder)




**21.** *Using papus theorem, find the volume of the body generated, if a semicircle rotates full revolution about x-axis as shown in the fig.*

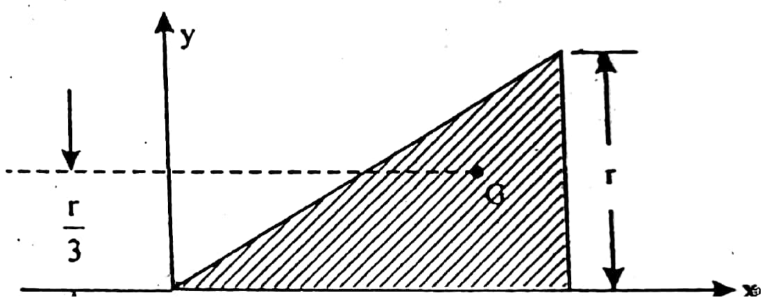
**Soln.** $A = \frac{\pi r^2}{2}$

$\theta = 2\pi,\ y_c = \frac{3r}{3\pi}$

$\therefore$ volume of solid generated;

$$V = A \times \theta \times y_c = \frac{\pi r^2}{2} \times 2\pi \times \frac{4r}{3\pi} = \frac{4}{3}\pi r^3 \text{ (Ans.)}$$

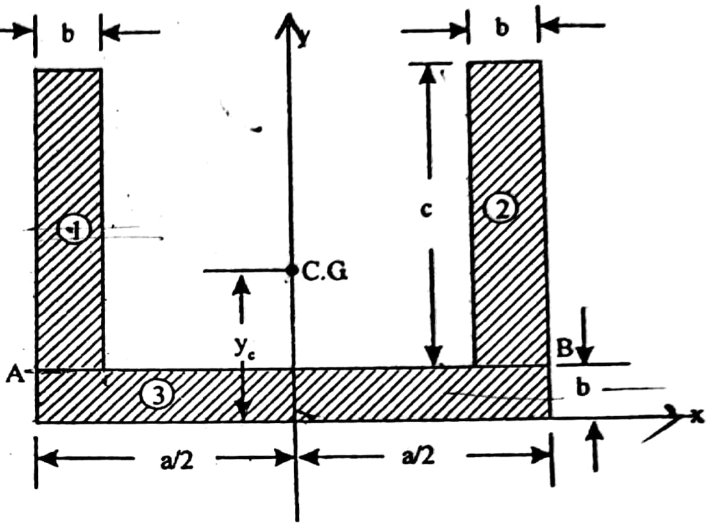
(Volume of a solid sphere)




**22.** *Find the volume of the solid cone generated, if a right angled triangle rotates for full revolution as shown in the fig.*

**Soln.** $A = \frac{1}{2} \times h \times r,\ \theta = 2\pi,\ y_c = \frac{r}{}$

*Determine the coordinate $y_c$ of the shaded area of the figure shown in the Fig. The following dimensions are given: $a = 150$ mm, $b = 25$ mm, $c = 50$ mm*




**Soln.** Given data

$a = 150$ mm, $b = 25$mm, $c = 50$ mm

Dividing the figure in to three rectangles, of areas

$A_1 = b \times c = A_2$, $\quad A_3 = a \times b$

$y_1 = \left(b + \frac{c}{2}\right) = y_2$, $\quad y_3 = \frac{b}{2}$

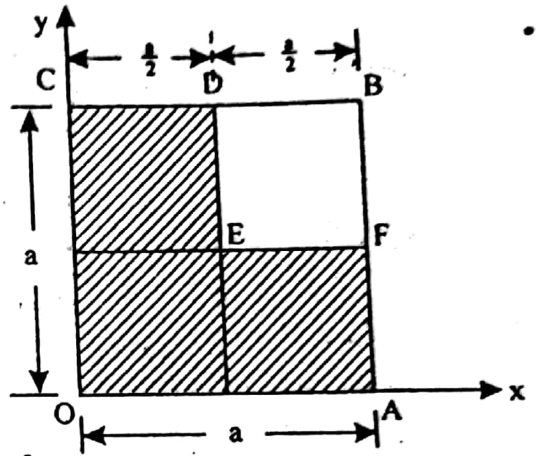
$$y_c = \frac{A_1y_1 + A_2y_2 + A_3y_3}{\quad} \quad 2\left[bc\left(\frac{2b+c}{2}\right)\right] + ab\left(\frac{b}{2}\right) \quad 2b^2c + bc^2 + \frac{ab^2}{2} = \frac{4b^2c + 2bc^2 + ab^2}{\quad}$$

Earlier we have found that

$$y_c = \frac{4b^2c + 2bc^2 + ab^2}{4bc + 2ab} \qquad \therefore\ b = \frac{4b^2c + 2bc^2 + ab^2}{4bc + 2ab}$$

$$\Rightarrow\ 4b^2c + 2bc^2 + ab^2 = 4b^2c + 2ab^2 \Rightarrow 2bc^2 = ab^2 \quad \text{or} \quad 2c^2 = ab \quad \Rightarrow c = \sqrt{\frac{ab}{2}}$$

3. *Locate the centroid of the shaded three-quarters of the area of a square of dimension a as shown in the Fig.*




**Soln.** Consider square OABC as (i) and square BDEF as (2)

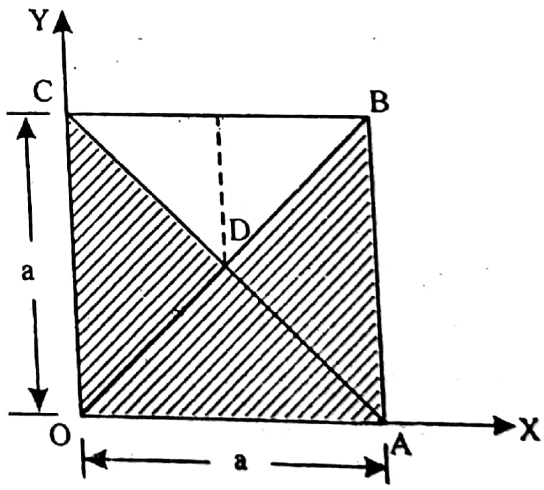
$\therefore\ A_1 = a^2 \qquad A_2 = a^2/4$

$$y_1 = x_1 = \frac{a}{2} \qquad y_2 = x_2 = \frac{a}{2} + \frac{a}{4} = \frac{3a}{4}$$

$$\therefore\ y_c = \frac{A_1y_1 - A_2y_2}{A_1 - A_2} = \frac{a^2\left(\frac{a}{2}\right) - \frac{a^2}{4} \times \frac{3a}{4}}{a^2 - \frac{a^2}{4}} = \frac{\frac{a^3}{2} - \frac{3a^3}{16}}{\frac{3a^2}{4}} = \frac{5a^3}{16} \times \frac{4}{3a^2} = \frac{5a}{12} \qquad \textbf{(Ans.)}$$

Similarly $X_c = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2} = \dfrac{5a}{12}$ (due to symmetry) **(Ans.)**

4. *Locate the centroid of the shaded three-quaters of the area of a square of dimesion a as shown in the Fig.*




**Soln.** Consider square OABC as (1), Δ BCD as (2)

$$A_1 = a^2 \qquad A_2 = \frac{1}{2} \times a \times \frac{a}{2} = \frac{a^2}{4}$$

$$y_1 = \frac{a}{2} \qquad y_2 = \frac{a}{2} + \frac{2}{3}\left(\frac{a}{2}\right) = \frac{5a}{6}$$

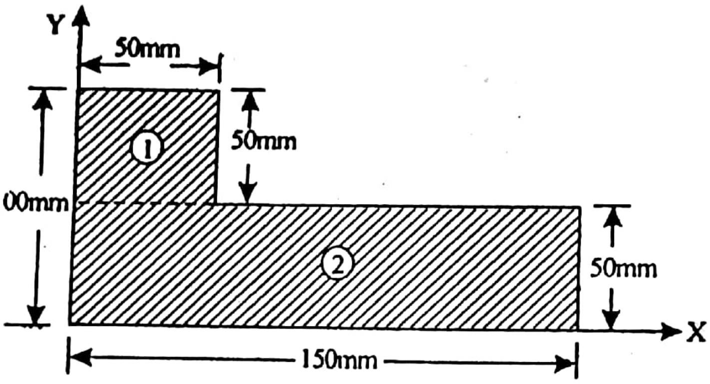
$$x_1 = x_2 = \frac{a}{2}$$

$$\therefore\ y_c = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{a^2\left(\frac{a}{2}\right) - \left(\frac{a^2}{4} \times \frac{5a}{6}\right)}{a^2 - \frac{a^2}{4}} = \frac{\frac{a^3}{2} - \frac{5a^3}{24}}{\frac{3a^2}{4}} = \frac{7a^3}{24} \times \frac{4}{3a^2} = \frac{7a}{18} \quad \textbf{(Ans.)}$$

Similarly; $$x_c = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{a^2\left(\frac{a}{2}\right) - \left(\frac{a^2}{4} \times \frac{a}{2}\right)}{a^2 - \frac{a^2}{4}} = \frac{\frac{a^3}{2} - \frac{a^3}{8}}{\frac{3a^2}{4}} = \frac{3a^3}{8} \times \frac{4}{3a^2} = \frac{a}{2}$$

$$x_c = \frac{a}{2} \quad \textbf{(Ans.)}$$

**5.** *Locate the centroid of the shaded area of the figure shown in the given Fig.*




**Soln.** $A_1 = 50 \times 50 = 2500\ mm^2$

$A_2 = 150 \times 50 = 7500\ mm^2$

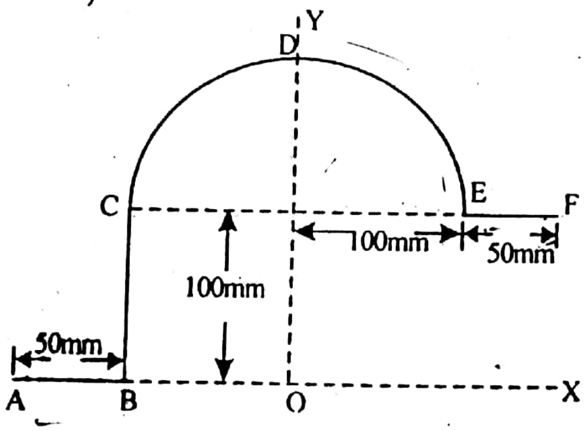
$x_1 = 25\ mm$ $\quad y_1 = 75\ mm$

$x_2 = 75\ mm$ $\quad y_2 = 25\ mm$

$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(2500 \times 75) + (7500 \times 25)}{(2500 + 7500)} = 37.5\ mm \quad \textbf{(Ans.)}$$

Similarly; $$x_c = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{(2500 \times 25) + (7500 \times 75)}{(2500 + 7500)} = 62.5\ mm \quad \textbf{(Ans.)}$$

**6.** *Referring to the given Fig. locate the centroid of the length of the mean center line with the dimensions shown.*




Previous

**Soln.** Consider AB is (1), BC – (2), CDE – (3)

EF – (4)

$L_1 = 50\ mm$ $\quad L_3 = 100\pi\ mm$

$L_2 = 100\ mm$ $\quad L_4 = 50\ mm$

$x_1 = -125\ mm$ $\quad x_2 = -100\ mm$

$x_3 = 0$ $\qquad x_4 = 125$ mm

$y_1 = 0$ $\qquad y_2 = 50$ mm

$$y_3 = 100 + \left[\frac{4(100)}{3\pi}\right] = 142.44 \text{ mm}, \qquad y_4 = 100 \text{ mm}$$

$$y_c = \frac{L_1y_1 + L_2y_2 + L_3y_3 + L_4y_4}{L_1 + L_2 + L_3 + L_4}$$

$$= \frac{(50 \times 0) + (100 \times 50) + (100\pi \times 142.44) + (50 \times 100)}{(50 + 100 + 100\pi + 50)} = 106.48 \text{ mm}$$

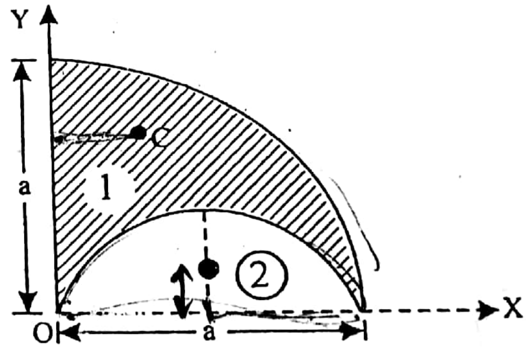
Similarly; $$x_c = \frac{L_1x_1 + L_2x_2 + L_3x_3 + L_4x_4}{L_1 + L_2 + L_3 + L_4}$$

$$= \frac{[50 \times (-125)] + [100 \times (-100)] + [100\pi \times 0] + (50 \times 125)}{(50 + 100 + 100\pi + 50)}$$

$$= \frac{(-6250) + (-10000) + 0 + 6250}{514.16} = -19.45 \text{ mm}$$ **(Ans.)**

Previous.

*Locate the centroid C of the shaded area obtained by cutting a semicircle of diameter a from the quadrant of a circle of radius a as shown in the Fig.*




**Soln.** Quadrant as (1)

Semicircle as (2)

$$A_1 = \frac{\pi a^2}{4} \qquad A_2 = \frac{1}{2}\pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{8}$$

$$x_1 = y_1 = \frac{4a}{3\pi} \qquad x_2 = \frac{a}{2},\ y_2 = \frac{4}{3\pi}\left(\frac{a}{2}\right) = \frac{4a}{6\pi}$$

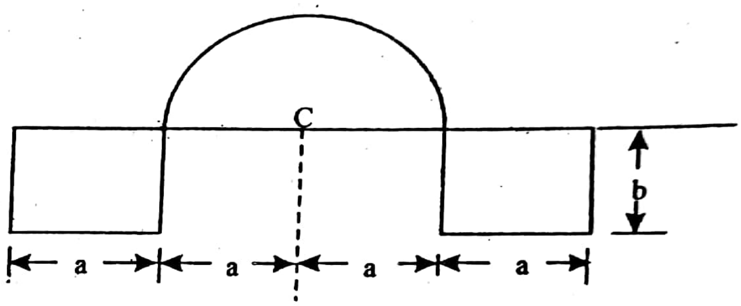
$$y_c = \frac{A_1y_1 - A_2y_2}{A_1 - A_2} = \frac{\left(\frac{\pi a^2}{4} \times \frac{4a}{3\pi}\right) - \left(\frac{\pi a^2}{8} \times \frac{4a}{6\pi}\right)}{\frac{\pi a^2}{4} - \frac{\pi a^2}{8}} = \frac{a\left(\frac{1}{3} - \frac{1}{12}\right)}{\pi\left(\frac{1}{4} - \frac{1}{8}\right)} = \frac{a\left(\frac{4-1}{12}\right)}{\pi\left(\frac{2-1}{8}\right)}$$

$$= \frac{a}{4} \times \frac{8}{\pi} = \frac{2a}{\pi} = 0.636\ a \qquad \textbf{(Ans.)}$$

Similarly; $x_c = \dfrac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$

$$= \frac{\left(\frac{\pi a^2}{4} \times \frac{4a}{3\pi}\right) - \left(\frac{\pi a^2}{8} \times \frac{a}{2}\right)}{\frac{\pi a^2}{4} - \frac{\pi a^2}{8}} = \frac{a\left(\frac{1}{3} - \frac{\pi}{16}\right)}{\pi\left(\frac{1}{4} - \frac{1}{8}\right)} = \frac{0.136\ a}{0.392} = 0.349\ a \qquad \textbf{(Ans.)}$$

**8.** *A slender homogeneous wire of uniform cross-sections is bent into the shape as shown in the fig. If the dimension a is fixed, find the dimension b so that the center of gravity of the wire will coincide with the center C of the semicircular portion.*

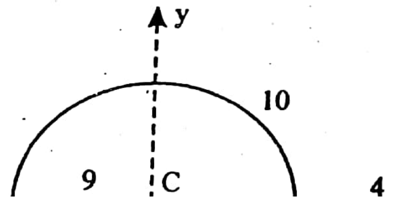



**Soln.** Since centroid lies at C, $y_c = 0$

$L_1 = a = L_2 = L_3 = L_4$ $\qquad$ $L_5 = b = L_6 = L_7 = L_8$

$L_9 = 2a$ $\qquad$ $L_{10} = \pi a$

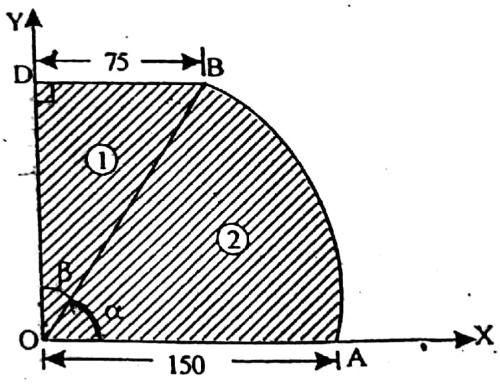
$y_1 = y_2 = -b$ $\qquad$ $y_3 = y_4 = 0$



$\Rightarrow ab + b^2 = a^2$ or $a^2 = ab + b^2 \Rightarrow b^2 + ab - a^2 = 0 \Rightarrow b = \dfrac{-a \pm \sqrt{a^2 + 4a^2}}{2}$

Taking (+ve) sign $= \dfrac{-a + a\sqrt{5}}{2} = \dfrac{a(-1+\sqrt{5})}{2} = 0.618\,a \quad \therefore\ b = 0.618\,a$ **(Ans.)**

2. *Locate the centroid C of the shaded area OABD as shown in the Fig.*




**Soln.** From geometry of the figure;

$$\beta = \sin^{-1}\left(\frac{75}{150}\right) = 30^\circ$$

$$\therefore\ \alpha = 90^\circ - 30^\circ = 60^\circ$$

$OA = OB = 150\,\text{mm}$

$$OD = OB\cos\beta = 150\cos 30^\circ = \frac{150\sqrt{3}}{2} = 75\sqrt{3}\ \text{mm}$$

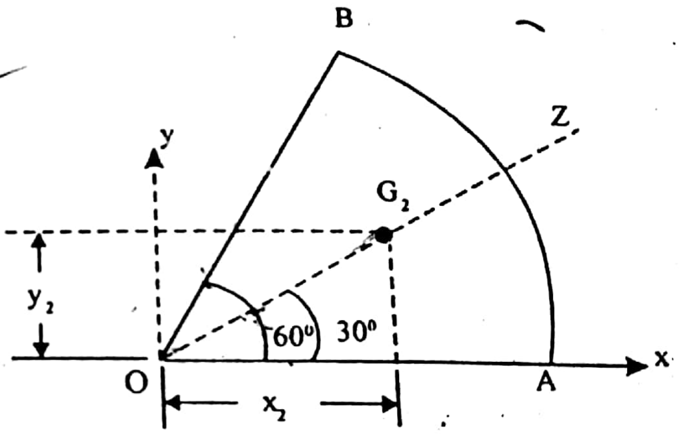
$$A_1 = \frac{1}{2} \times 75 \times 75\sqrt{3} = 4871.392\ \text{mm}^2$$

$$A_2 = \frac{1}{6} \times \pi \times (150)^2 = 11780.972\ \text{mm}^2$$

$$x_1 = \frac{75}{3} = 25\ \text{mm}$$

$$y_1 = \frac{2}{3} \times 75\sqrt{3} = 86.6\ \text{mm}$$

Here $\alpha = 60^\circ = \dfrac{\pi}{3}$




From the figure $\boxed{OG_2 = \dfrac{4r}{3\alpha}\sin\dfrac{\alpha}{2}} = \dfrac{4 \times 150}{3 \times \dfrac{\pi}{3}}\sin 30^\circ = 95.492\ \text{mm}$

$$\therefore\ x_2 = OG_2\cos 30^\circ = 95.492 \times \cos 30^\circ = 82.698\,\text{mm}$$

$$y_2 = OG_2\sin 30^\circ = 95.492 \times \sin 30^\circ = 47.746\,\text{mm}$$

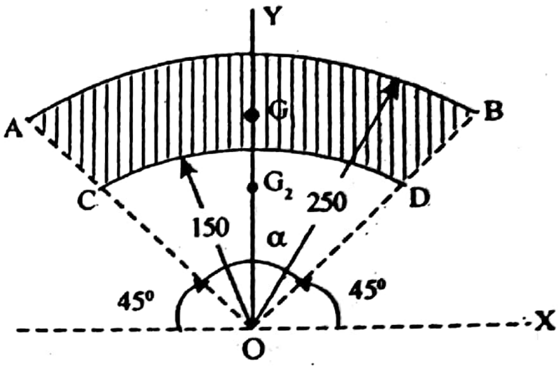
$$\therefore\ x_c = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{(4871.392 \times 25) + (11780.972 \times 82.698)}{(4871.392 + 11780.972)}$$

$$\therefore\ x_c = 66\ \text{mm}$$ **(Ans.)**

$$\therefore y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(4871.392 \times 86.6)+(11780.972 \times 47.746)}{(4871.392+11780.972)}$$

$\therefore y_c = 59.11$ mm **(Ans.)**

**10.** *Locate the centroid C of the shaded sector of a ring subtending 90° central angle and symmetrical about the y-axis, as shown in the fig.*




**Soln.** Here $\alpha = 90°$

$r_1 = 250$mm, $r_2 = 150$ mm

Sector OAB ......(1),

Sector OCD ...(2)

$$A_1 = \frac{1}{4} \times \pi \times (250)^2 = 49087.385 \text{ mm}^2$$

$$A_2 = \frac{1}{4} \times \pi \times (150)^2 = 17671.458 \text{ mm}^2$$

$$OG_1 = y_1 = \frac{4 \times 250}{3 \times \frac{\pi}{2}} \sin 45° = 150.052 \text{ mm}$$

$$OG_2 = y_2 = \frac{4 \times 150}{3 \times \frac{\pi}{2}} \sin 45° = 90.031 \text{ mm}$$

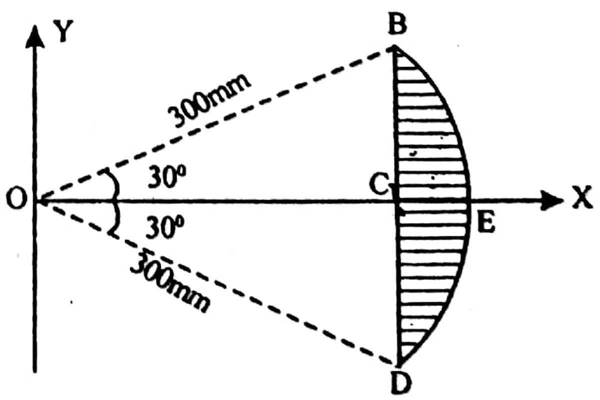
$x_1 = 0$ $\qquad$ $x_2 = 0$

$$y_c = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{(49087.385 \times 150.052) - (17671.458 \times 90.031)}{(49087.385 - 17671.458)}$$

$y_c = 183.8$ mm **(Ans.)**

Due to symmetry about Y-axis $x_c = 0$ **(Ans.)**

**11.** *Locate the centroid G of the shaded area of the circular segment BD shown in the Fig.*

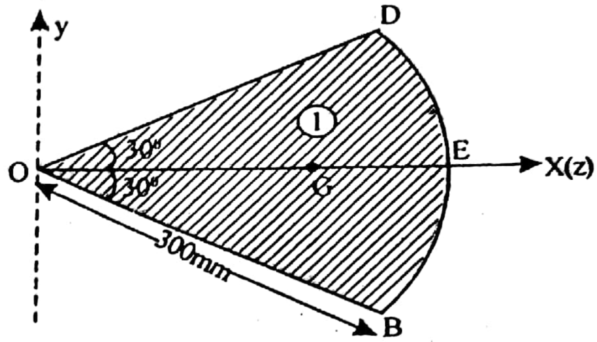



**Soln.** $OC = 150 \cos 30° = 150 \frac{\sqrt{3}}{2}$

$= 75\sqrt{3}$ mm

$BD = 2 \times BC$

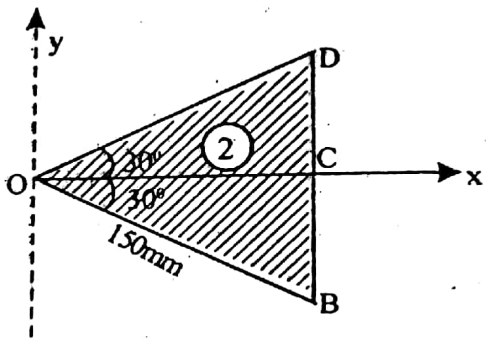
$= 2 \times 150 \sin 30° = 2 \times 150 \times \frac{1}{2} = 150$ mm




$A_1 = \frac{1}{6} \times \pi \times 300^2 = 47123.889 \text{ mm}^2$

$A_2 = \frac{1}{2} \times BD \times OC$

$= \frac{1}{2} \times 150 \times 75\sqrt{3} = 9742.785 \text{ mm}^2$




$x_1 = OG = \dfrac{4 \times 300}{3 \times \frac{\pi}{3}} \sin 30° = 190.985$ mm

$x_2 = \frac{2}{3} \times OC = \frac{2}{8} \times 75\sqrt{3} = 86.602$ mm

$y_1 = 0$

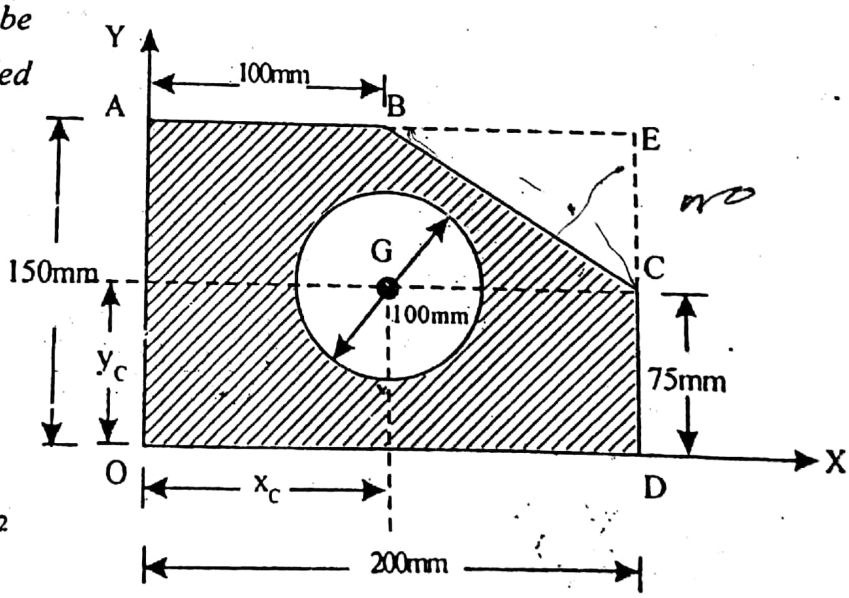
$y_2 = 0$

$$x_c = \frac{A_1x_1 - A_2x_2}{A_1 - A_2} = \frac{(47123.889 \times 190.985) - (9742.785 \times 89.602)}{(47123.889 - 9742.785)}$$

$x_c = 218.2$ mm (Ans.)

Due to symmetry about 'X' axis, $y_c = 0$ (Ans.)

12. *Referring to the given Fig. determine the coordinates $x_c$ and $y_c$ of the center of a 100 mm diameter, circular hole cut in a thin plate so that this point will be the centroid of the remaining shaded area.*




**Soln.**

AEDO - Rectangle .....(1)

Circle ....(2)

BEC - Triangle ....(3)

$\therefore \quad A_1 = 200 \times 150 = 30000 \text{ mm}^2$

$A_2 = \pi \times 50^2 = 7853.981 \text{ mm}^2$

$A_3 = \frac{1}{2} \times 100 \times 75 = 3750 \text{ mm}^2$

$x_1 = 100\text{ mm}$ $\qquad$ $y_1 = 75\text{ mm}$

$x_2 = x_c$ $\qquad$ $y_2 = y_c$

$$x_3 = 100 + \left(\frac{2}{3} \times 100\right) = 166.666\text{ mm}$$

$$y_3 = \left[75 + \left(\frac{2}{3} \times 75\right)\right] = 125\text{ mm}$$

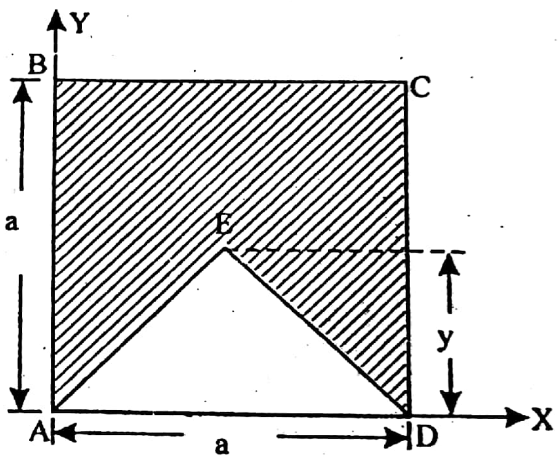
$$x_c = \frac{A_1x_1 - A_2x_2 - A_3x_3}{A_1 - A_2 - A_3} = \frac{(30000 \times 100) - (7853.981 \times x_c) - (3750 \times 166.666)}{(30000 - 7853.98 - 3750)}$$

$\therefore x_c = 90.476\text{ mm}$ **(Ans.)**

$$y_c = \frac{A_1y_1 - A_2y_2 - A_3y_3}{A_1 - A_2 - A_3} = \frac{(30000 \times 75) - (7853.981 \times y_c) - (3750 \times 125)}{(30000 - 7853.98 - 3750)}$$

$\therefore y_c = 67.857\text{ mm}$ **(Ans.)**

**13.** *An isoscele triangle ADE is to be cut from a square ABCD of dimension a as shown in the Fig. Find the altitude y of this triangle so that its vertex E will be the centroid of the remaining shaded area.*



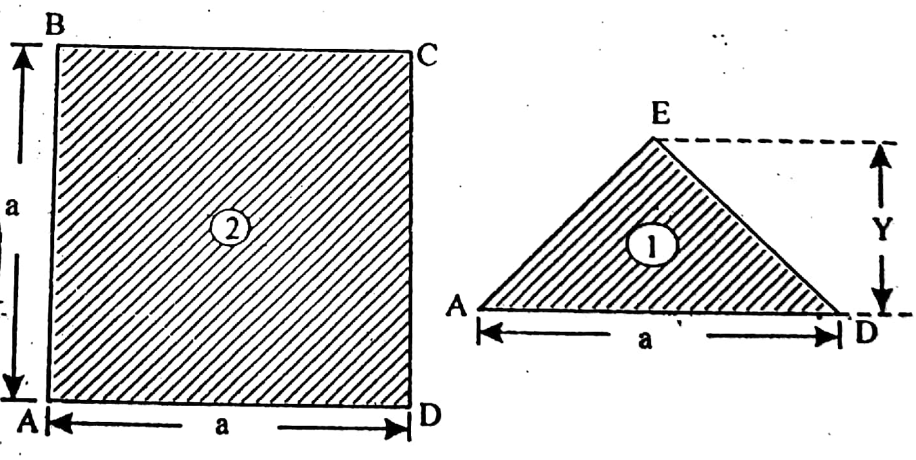

**Soln.** Since the centroid lies at 'E'

$y_c = y$

$A_1 = a^2 \qquad A_2 = \frac{1}{2} \times a \times y$

$y_1 = \frac{a}{2} \qquad y_2 = \frac{y}{3}$

$$y_c = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$




$$\Rightarrow y = \frac{\left(a^2 \times \frac{a}{2}\right) - \left(\frac{ay}{2} \times \frac{y}{3}\right)}{\left(a^2 - \frac{ay}{2}\right)}$$

$$\Rightarrow y = \frac{\frac{a^2}{2} - \frac{y^2}{6}}{\frac{2a - y}{2}}$$

$$\Rightarrow y = \frac{3a^2 - y^2}{6} \times \frac{2}{2a - y} \qquad \Rightarrow y = \frac{3a^2 - y^2}{6a - 3y}$$

$$\Rightarrow 3a^2 - y^2 = 6ay - 3y^2 \qquad \Rightarrow 2y^2 - 6ay + 3a^2 = 0$$

$$\Rightarrow y = \frac{6a \pm \sqrt{36a^2 - 24a^2}}{4}$$

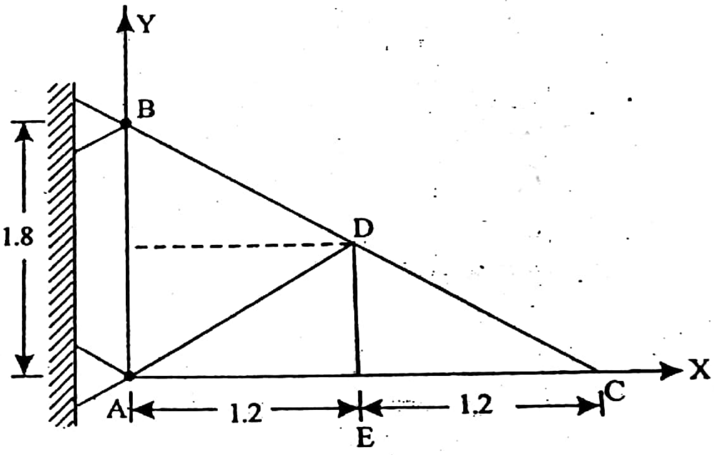
Taking (+ve) sign

$$\Rightarrow y = \frac{6a + a \times 3.464}{4} = 2.36a$$

(The ans. is not possible because the value exceeding a)
Hence taking (-ve) sign

$$\Rightarrow y = \frac{6a - (a \times 3.464)}{4} \Rightarrow y = 0.634\ a$$ **(Ans.)**

**14.** *Locate the center of gravity of the plane truss shown in the Fig. if all bars have the same weight per unit length.*




**Soln.** $AB - (1) \rightarrow L_1 = 1.8$ m

$AC - (2) \rightarrow L_2 = 2.4$ m

$BC - (3) \rightarrow L_3 = \sqrt{(1.8^2 + 2.4^2)} = 3$ m

$AD - (4) \rightarrow L_4 = \sqrt{(0.9^2 + 1.2^2)} = 1.5$ m

$DE - (5) \rightarrow L_5 = \frac{1.8}{2} = 0.9$ m

$x_1 = 0 \qquad y_1 = 0.9$ m

$x_2 = 1.2$ m $\qquad y_2 = 0$

$x_3 = 1.2$ m $\qquad y_3 = 0.9$ m

$x_4 = 0.6$ m $\qquad y_4 = 0.45$ m

$x_5 = 1.2$ m $\qquad y_5 = 0.45$ m

$$x_c = \frac{(L_1x_1 + L_2x_2 + L_3x_3 + L_4x_4 + L_5x_5)}{(L_1 + L_2 + L_3 + L_4 + L_5)}$$

$$= \frac{(1.8 \times 0) + (2.4 \times 1.2) + (3 \times 1.2) + (1.5 \times 0.6) + (0.9 \times 1.2)}{}$$

$x_c = 0.882$ m **(Ans.)**

$$y_c = \frac{(L_1y_1 + L_2y_2 + L_3y_3 + L_4y_4 + L_5y_5)}{(L_1 + L_2 + L_3 + L_4 + L_5)}$$

$$= \frac{(1.8 \times 0.9) + (2.4 \times 0) + (3 \times 0.9) + (1.5 \times 0.45) + (0.9 \times 0.45)}{(1.8 + 2.4 + 3 + 1.5 + 0.9)}$$

$y_c = 0.5625$ m **(Ans.)**

**15.** ***With respect to coordinate axes x and y, locate the centroid of the shaded area as shown in the Fig.***

**Soln.** $A_1 = 225 \times 25 = 5625 \text{ mm}^2$

$A_2 = 200 \times 25 = 5000 \text{ mm}^2$

$A_3 = 25 \times 25 = 625 \text{ mm}^2$

$x_1 = -12.5$ m

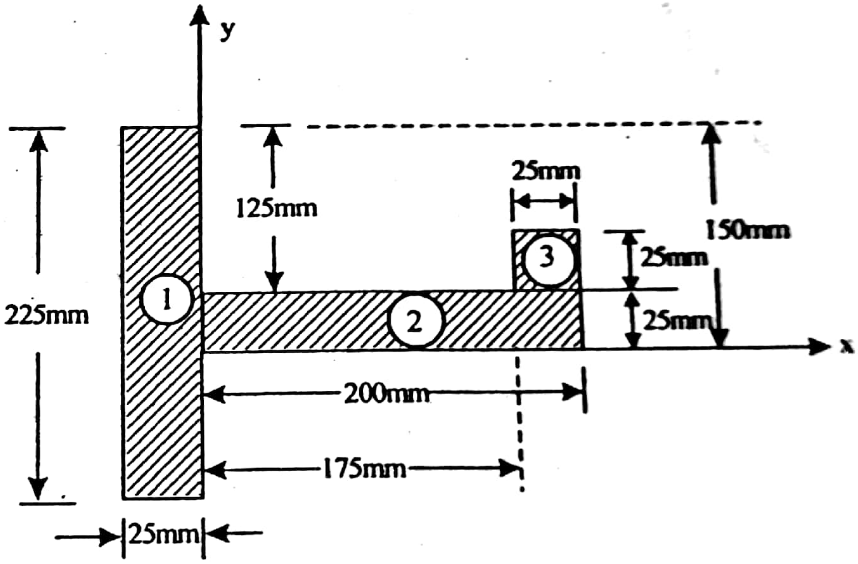
$$y_1 = \left(150 - \frac{225}{2}\right) = 37.5 \text{ mm}$$

$x_2 = 100$ mm

$y_2 = 12.5$ mm

$$x_3 = \left(175 + \frac{25}{2}\right) = 187.5 \text{ mm}$$

$$y_3 = \left(25 + \frac{25}{3}\right) = 37.5 \text{ mm}$$




$$x_c = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$= \frac{[5625 \times (-12.5)] + (5000 \times 100) + (625 \times 187.5)}{(5625 + 5000 + 625)}$$

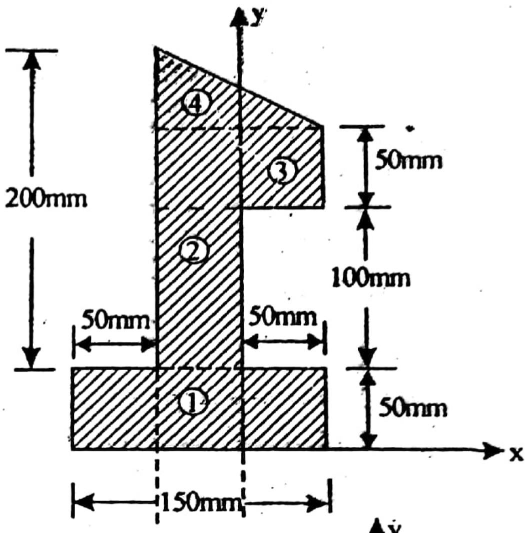
$= 48.61$ mm **(Ans.)**

$$y_c = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$= \frac{[5625 \times 37.5] + (5000 \times 12.5) + (625 \times 37.5)}{(5625 + 5000 + 625)}$$

$= 26.38$ mm **(Ans.)**

**16.** *With reference to the coordinate axes x and y, locate the centroid of the shaded area of the plane figure as shown in Fig.*




**Soln.** $A_1 = 150 \times 50 = 7500\ \text{mm}^2$

$A_2 = 100 \times 50 = 5000\ \text{mm}^2$

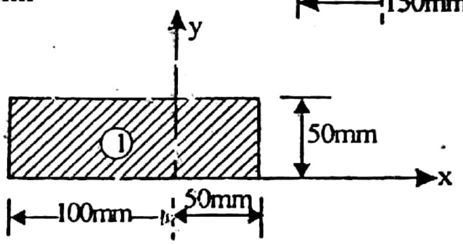
$A_3 = 100 \times 50 = 5000\ \text{mm}^2$

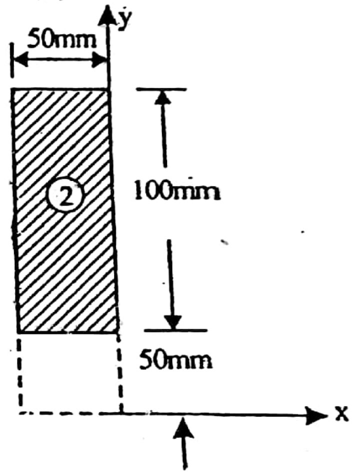
$A_4 = \dfrac{1}{2} \times 100 \times 50 = 2500\ \text{mm}^2$

$x_1 = -25\ \text{mm}, \qquad y_1 = 25\text{mm}$

$x_2 = -25\ \text{mm},$

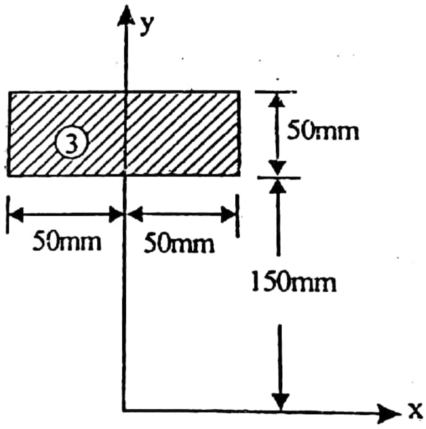
$y_2 = \left(50 + \dfrac{100}{2}\right) = 100\ \text{mm}$




$x_2 = 0$

$y_3 = 150 + \dfrac{50}{2} = 175\ \text{mm}$



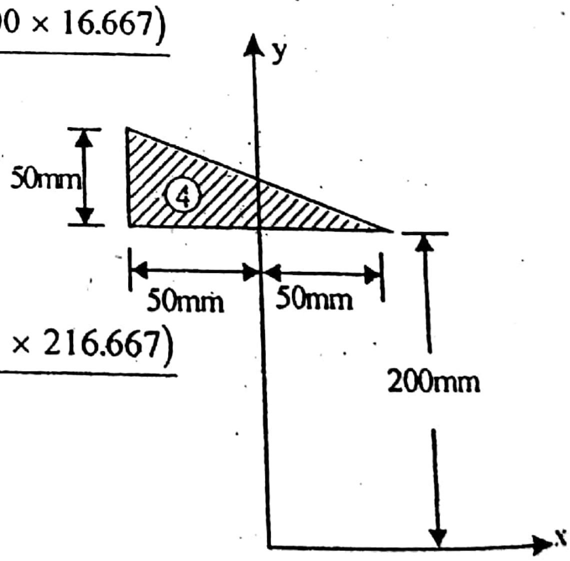

$x_4 = \left(\dfrac{2}{3} \times 100\right) - 50 = 16.667\ \text{mm}$

$y_4 = 200 + \dfrac{50}{3} = 216.667\ \text{mm}$

$$x_c = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{[7500 \times (-25)] + (5000 \times (-25)) + (5000 \times 0) + (2500 \times 16.667)}{(7500 + 5000 + 5000 + 2500)}$$

$= -13.54\ \text{mm}$ **(Ans.)**

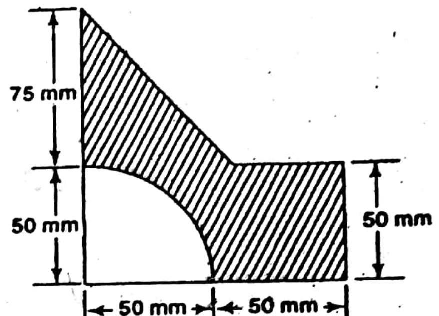



$$y_c = \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{[7500 \times 25] + (5000 \times 100) + (5000 \times 175) + (2500 \times 216.667)}{(7500 + 5000 + 5000 + 2500)}$$

$= 105.20\ \text{mm}$ **(Ans.)**

17. *With respect to coordinate axes x and y, locate the centroid of the shaded area as shown in the Fig.*




Previous

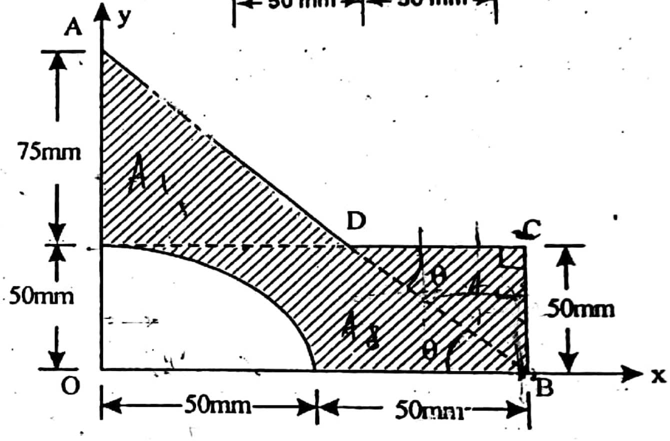
**Soln.** In the triangle AOB,

$$\theta = \tan^{-1}\left(\frac{125}{100}\right) = 51°20'$$

In the triangle BDC

$$\tan\theta = \frac{BC}{DC}$$

$$\therefore\ DC = \frac{BC}{\tan\theta} = \frac{50}{\tan 51°\ 20°} = 40 \text{ mm}$$




Δ AOB .....(1)

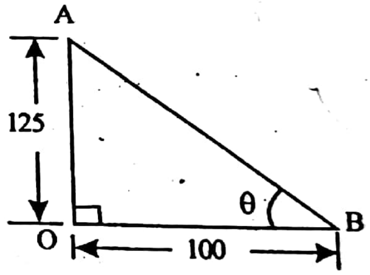
Δ DCB ....(2)

Quadrant ....(3)

$$A_1 = \frac{1}{2} \times 100 \times 125 = 6250 \text{ mm}^2$$

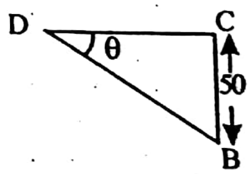
$$A_2 = \frac{1}{2} \times 50 \times 40 = 1000 \text{ mm}^2$$

$$A_3 = \frac{1}{4} \times \pi \times (50)^2 = 1963.495 \text{ mm}^2$$




$$x_1 = \frac{100}{3} = 33.333 \text{ mm} \qquad y_1 = \frac{125}{3} = 41.666 \text{ mm}$$

$$x_2 = \left(100 - \frac{40}{3}\right) = 86.666 \text{ mm} \qquad y_2 = \left(\frac{2}{3} \times 50\right) = 33.333 \text{ mm}$$



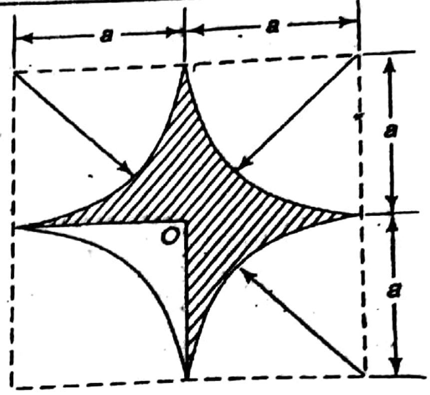

$$x_3 = 4 \times \frac{50}{3\pi} \qquad y_3 = 21.220 \text{ mm}$$

$$x_c = \frac{[(A_1x_1 + A_2x_2) - A_3x_3]}{[(A_1 + A_2) - A_3]} = \frac{[(6250 \times 33.333) + (1000 \times 86.666) - (1963.495 \times 21.220)]}{[(6250 + 1000) - 1963.495]}$$

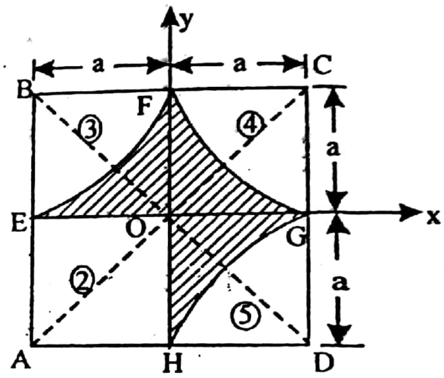
$x_c$ = 48 mm **(Ans.)**

$$y_c = \frac{[(A_1y_1 + A_2y_2) - A_3y_3]}{} = \frac{[(6250 \times 41.666) + (1000 \times 33.333) - (1963.495 \times 21.220)]}{}$$

**18.** *With respect to coordinate axes x and y, locate the centroid of the shaded area as shown in the Fig.*




**Soln.** Square ABCD ......(1)
Square EAHO .....(2)
Quadrant BEF ...(3)
Quadrant FCG ....(4)
Quadrant HDG .....(5)




$A_1 = 2a \times 2a = 4a^2, \quad A_2 = a^2$

$$A_3 = A_4 = A_5 = \frac{1}{4}(\pi a^2) = \frac{\pi a^2}{4}$$

$x_1 = 0 \qquad\qquad y_1 = 0$

$$x_3 = \left[-\left(a - \frac{4a}{3\pi}\right)\right] = -0.575a \qquad y_3 = \left(a - \frac{4a}{3\pi}\right) = 0.575\ a$$

$$x_2 = \left(-\frac{a}{2}\right) \qquad y_2 = \left(-\frac{a}{2}\right)$$

$$x_4 = \left(a - \frac{4a}{3\pi}\right) = 0.575a \qquad y_4 = \left(a - \frac{4a}{3\pi}\right) = 0.575a$$

$$x_5 = \left(a - \frac{4a}{3\pi}\right) = 0.575a \qquad y_5 = \left[-\left(a - \frac{4a}{3\pi}\right)\right] = -0.575a$$

$$x_c = \frac{(A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4 - A_5x_5)}{(A_1 - A_2 - A_3 - A_4 - A_5)}$$

$$= \frac{\left[(4a^2 \times 0) - \left[a^2 \times \left(-\frac{a}{2}\right)\right] - \left[\frac{\pi a^2}{4} \times (-0.575a)\right] - \left(\frac{\pi a^2}{4} \times 0.575a\right) - \left(\frac{\pi a^2}{4} \times 0.575a\right)\right]}{\left(4a^2 - a^2 - \frac{\pi a^2}{4} - \frac{\pi a^2}{4} - \frac{\pi a^2}{4}\right)}$$

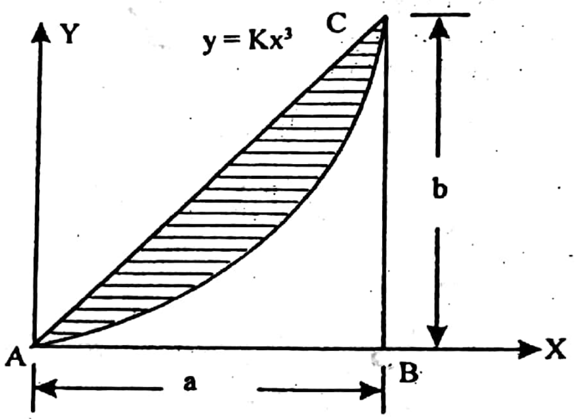
$$= \frac{(0.5a^3)[1 + 0.903 - 0.903 - 0.903]}{a^2\left[\frac{16 - 4 - 3\pi}{4}\right]} = \frac{2a \times 0.097}{2.575} = 0.074\ a \qquad \textbf{(Ans.)}$$

$$y_c = \frac{(A_1y_1 - A_2y_2 - A_3y_3 - A_4y_4 - A_5y_5)}{(A_1 - A_2 - A_3 - A_4 - A_5)}$$

$$= \frac{\left[(4a^2 \times 0) - \left[\left(a^2 \times \left(-\frac{a}{2}\right)\right)\right] - \left(\frac{\pi a^2}{4} \times 0.575a\right) - \left(\frac{\pi a^2}{4} \times 0.575a\right) - \left(\frac{\pi a^2}{4} \times 0.575a\right)\right]}{4a^2 - a^2 - \frac{\pi a^2}{4} - \frac{\pi a^2}{4} - \frac{\pi a^2}{4}}$$

$= 0.074\ a$ **(Ans.)**

**19.** *For the shaded area as shown in the Fig. find the ratio a/b for which the x and y coordinates of the centroid are equal.*

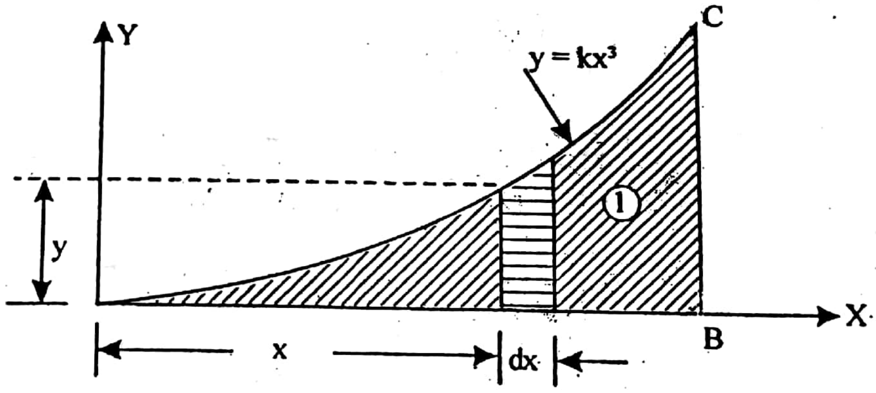



**Soln.** Parabola section (1)

$$x_c = \frac{\int x\, dA}{\int dA} \qquad \therefore dA = y dx$$

$$\therefore x_c = \frac{\int_0^a x.\, y dx}{\int_0^a y dx} = \frac{\int_0^a K x^4\, dx}{\int_0^a K x^3\, dx} = \frac{\left[\frac{x^5}{5}\right]_0^a}{\left[\frac{x^4}{4}\right]_0^a} = \frac{a^5}{5} \times \frac{4}{a^4} = \frac{4a}{5}$$

$$\therefore x_1 = \frac{4a}{5}$$

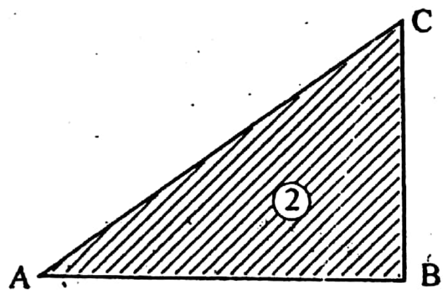



$$y_c = \frac{\int y/2\, dA}{\int dA} = \frac{\frac{1}{2}\int y^2\, dx}{\int y\, dx}$$

$$= \frac{\frac{1}{2}\int_0^a k^2 x^6\, dx}{\int_0^a kx^3\, dx} = \frac{\frac{k}{2}\left[\frac{x^7}{7}\right]_0^a}{\left[\frac{x^4}{4}\right]_0^a} = \frac{k}{2} \times \frac{a^7}{7} \times \frac{4}{a^4} = \frac{2Ka^3}{7}$$

When $x = a,\ y = b,\ \therefore \quad y = kx^3$

$\Rightarrow b = ka^3 \quad \Rightarrow \quad k = \frac{b}{a^3}$




Sustituting the value of 'K' :

$$y_c = \frac{2 \times \frac{b}{a^3} \times a^3}{7} = \frac{2b}{7} \text{ i.e. } y_1 = \frac{2b}{7}$$

∴ Area of the parabola,

$$A_1 = \int dA = \int_0^a y dx = \int_0^a kx^3 dx = k\left[\frac{x^4}{4}\right]_0^a = \frac{ka^4}{4}$$

Substituting the value of k : $A_1 = \frac{b}{a^3} \times \frac{a^4}{4} = \frac{ab}{4}$

Let triangular section (2)

∴ Area of the triangle $\qquad A_2 = \frac{1}{2} \times a \times b$

$$x_2 = \left(\frac{2}{3} \times a\right)$$

$$y_2 = \left(\frac{b}{3}\right)$$

$$\therefore x_c = \frac{A_2x_2 - A_1x_1}{A_2 - A_1} = \frac{\left(\frac{ab}{2} \times \frac{2a}{3}\right) - \left(\frac{ab}{4} \times \frac{4a}{5}\right)}{\left(\frac{ab}{2} - \frac{ab}{4}\right)}$$

$$= \frac{\frac{a^2b}{3} - \frac{a^2b}{5}}{\frac{2ab - ab}{4}} = \frac{5a^2b - 3a^2b}{15} \times \frac{4}{ab} = \frac{2a^2b}{15} \times \frac{4}{ab} = \frac{8a}{15}$$ **(Ans.)**

$$y_c = \frac{A_2y_2 - A_1y_1}{A_2 - A_1}$$

$$= \frac{\left(\frac{ab}{2} \times \frac{b}{3}\right) - \left(\frac{ab}{4} \times \frac{2b}{7}\right)}{\left(\frac{ab}{2} - \frac{ab}{4}\right)} = \frac{\frac{ab^2}{6} - \frac{ab^2}{14}}{\frac{ab}{4}} = \frac{\frac{7ab^2 - 3ab^2}{42}}{\frac{ab}{4}}$$

$$= \frac{4ab^2}{42} \times \frac{4}{ab} = \frac{8ab^2}{21ab} = \frac{8b}{21}$$ **(Ans.)**

When the coordinates $x_c = y_c$, then

$$\frac{8a}{15} = \frac{8b}{21} \quad \text{or} \quad \frac{a}{b} = \frac{15}{21} = 0.714$$ **(Ans.)**

## 4.3.4. Analysis of a Truss

A truss can be analysed by following methods :

(i) Method of joints

(ii) Method of sections

(iii) Graphical method.

## 4.3.5. Method of Joints

Each joint of the truss is equilibrium under the actions of axial forces in concurrent bars and external forces if acting on it.

**Procedure :**

(i) Select the joint as FBD where maximum unknown forces are two. (Because we have two equations i.e. $\Sigma X = 0, \Sigma Y = 0$ for concurrent system of forces)

**Note** : We can select a joint where maximum unknown forces are three provided out of three, two are collinear.

(ii) Apply the conditions of equilibrium i.e. $\Sigma X = 0, \Sigma Y = 0$ at that joints.

(iii) If required determine the support reactions at the supports of a truss considering it as FBD.

## 4.3.6. Method of Sections

Any cutting section of a truss is assumed to be equilibrium under the actions of cutting forces and the external forces acting on that side.

**Procedure :**

(i) Cut the section where maximum unknown cutting forces are three. (Because we have three equations of equilibrium, $\Sigma X = 0, \Sigma Y = 0$, $\Sigma M = 0$ for coplanner system of forces)

**Note :** We can cut a section where unknown cutting forces are four provided out of four two are collinear.

(ii) Assume any part of the cutting section (left part, or right part) is equilibrium under the actions of cutting forces and external forces on that sides.

(iii) Assume all the cutting forces as tensions or compressions or even both.

(iv) Apply the conditions of equilibrium $(\Sigma X = 0, \Sigma Y = 0, \Sigma M = 0)$ to the cutting forces and external forces, acting on that equilibrium part.

(v) If required determine the support reactions at the supports of the truss considering the whole truss as FBD.

## 4.3.7. Frame

A frame may be defined as a structure made up several bars, design to carry external loads on it (either at joints or at any other places of the bars).

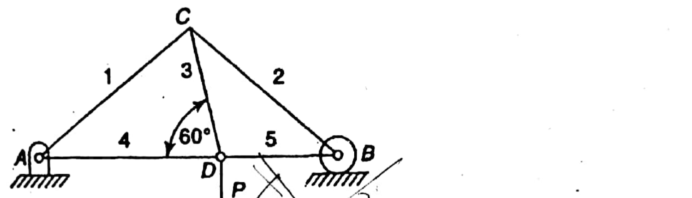
It the center lines of all the bars of a frame lie in one plane, the frame is called plane frame.

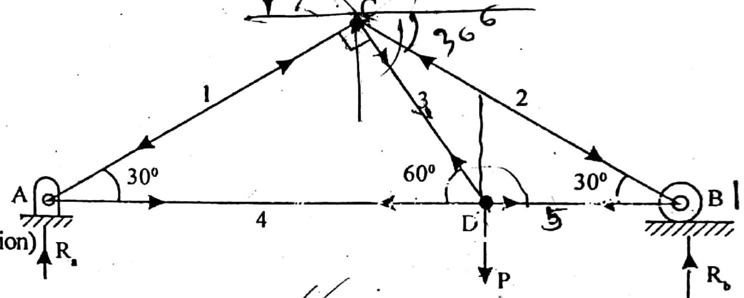
The bars are designed to carry axial load (tension, compression) along with may or may not have additional load due to bending.

## SOLVED PROBLEMS - 4.3

### (Method of Joint)

1. *Calculate the axial forces $S_i$ in each bar of the simple truss supported and loaded as shown in the Fig. The triangle ACB is isosceles with 30° angles at A and B and P = 5 kN.*




**Soln.** Given data :

P = 5 kN

**Select the joint 'D' as FBD**

Resolving vertically, $\Sigma Y = 0$ at D

$S_3 \sin 60° = 5 \Rightarrow S_3 = 5.773$ kN (Tension)

**Select the joint C as FBD**

Resolving vertically, $\Sigma Y = 0$

$S_3 = S_2 \cos 30° \Rightarrow S_2 = 6.67$ kN (Comp.)

Resolving horizontally, $\Sigma X = 0$

$S_1 = S_2 \sin 30° \Rightarrow S_1 = 3.335$ kN (Comp.)

**Select the joint B as FBD**

Resolving horizontally, $\Sigma X = 0$

$S_5 = S_2 \cos 30° \Rightarrow S_5 = 5.776$ kN (Tension)

Resolving vertically, $\Sigma Y = 0$

$R_b = S_2 \sin 30° \Rightarrow R_b = 3.335$ kN

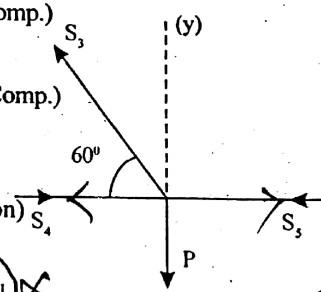
**Select the jont D as FBD**

Resolving horizontally, $\Sigma X = 0$

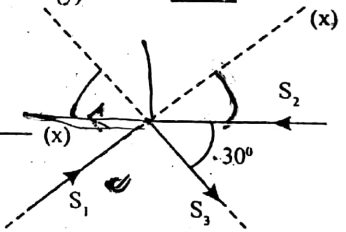
$S_4 + S_3 \cos 60° = S_5$

$\Rightarrow S_4 = 2.89$ kN (Tens)

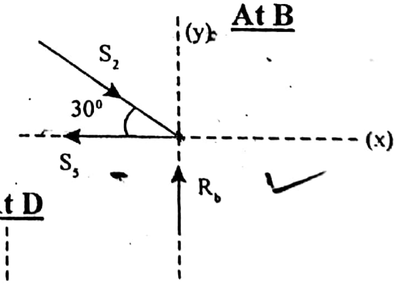
**At D**




**At C**

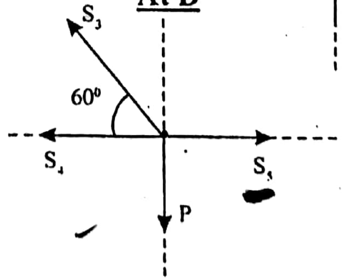



**At B**

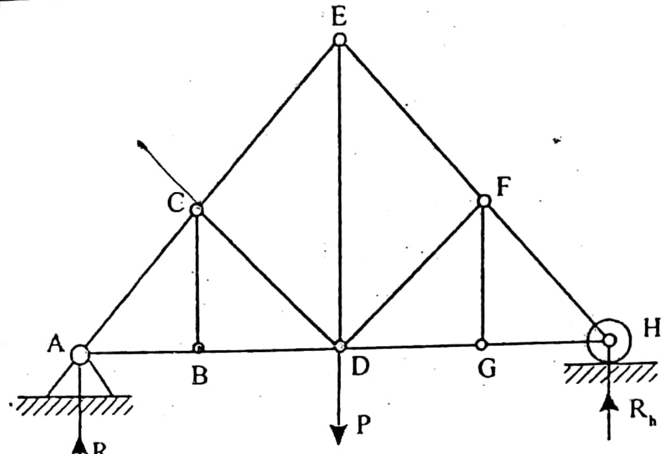



| Bars | Axial Force | Magnitude | Nature |
|---|---|---|---|
| AC | $S_1$ | 3.335kN | Comp. |
| BC | $S_2$ | 6.67kN | Comp. |
| CD | $S_3$ | 5.773kN | Tens. |
| AD | $S_4$ | 2.89kN | Tens. |
| BD | $S_5$ | 5.776kN | Tens. |

**At D**



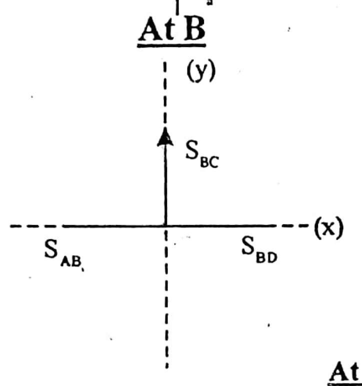
**2.** *Prove that a tensile force equal to the applied load P is produced in the bar DE of the truss shown in the Fig.*




**Soln.** **Select the joint B as FBD**

Resolving vertically,

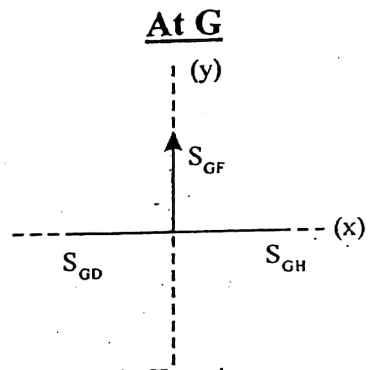
$\Sigma Y = 0,\ S_{BC} = 0$




**Select the joint G as FBD**

Resolving vertically,

$\Sigma Y = 0,\ S_{GF} = 0$




**Select the joint C as FBD**

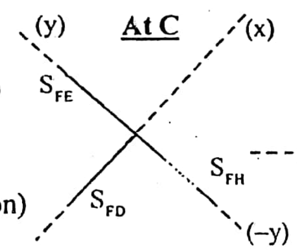
Resolving vertically,

$\Sigma Y = 0,\ S_{CD} = 0$

**Select the joint F as FBD**

Resolving along y-axis

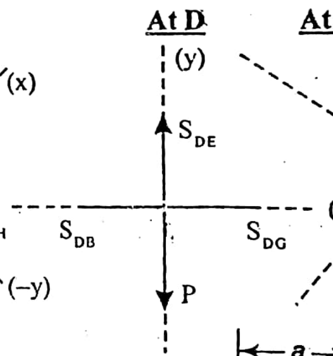
$\Sigma Y = 0,\ S_{FD} = 0$

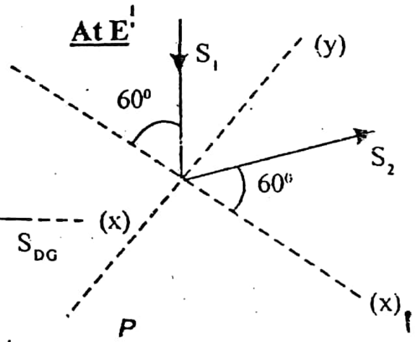



**Select the joint D as FBD**

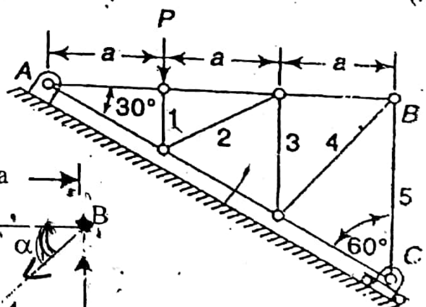
Resolving vertically,

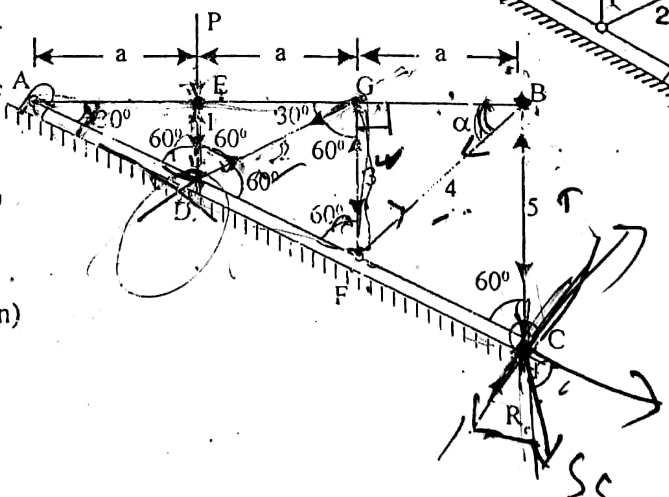
$\Sigma Y = 0,\ S_{DE} = P$ (Tension)




**3.** *Determine the axial forces in the bars 1, 2, 3, 4 and 5 of the plane truss supported and loaded as shown in the Fig.*




**Soln.** **Select the joint E as FBD**

Resolving vertically,

$\Sigma Y = 0,\ S_1 = P$ (Tension)

$S_5 \sin 60° = R_c$

Select the joint D as FBD

Resolving along y-axis

$\Sigma Y = 0,$

$S_2 \sin 60° = S_1 \sin 60°$

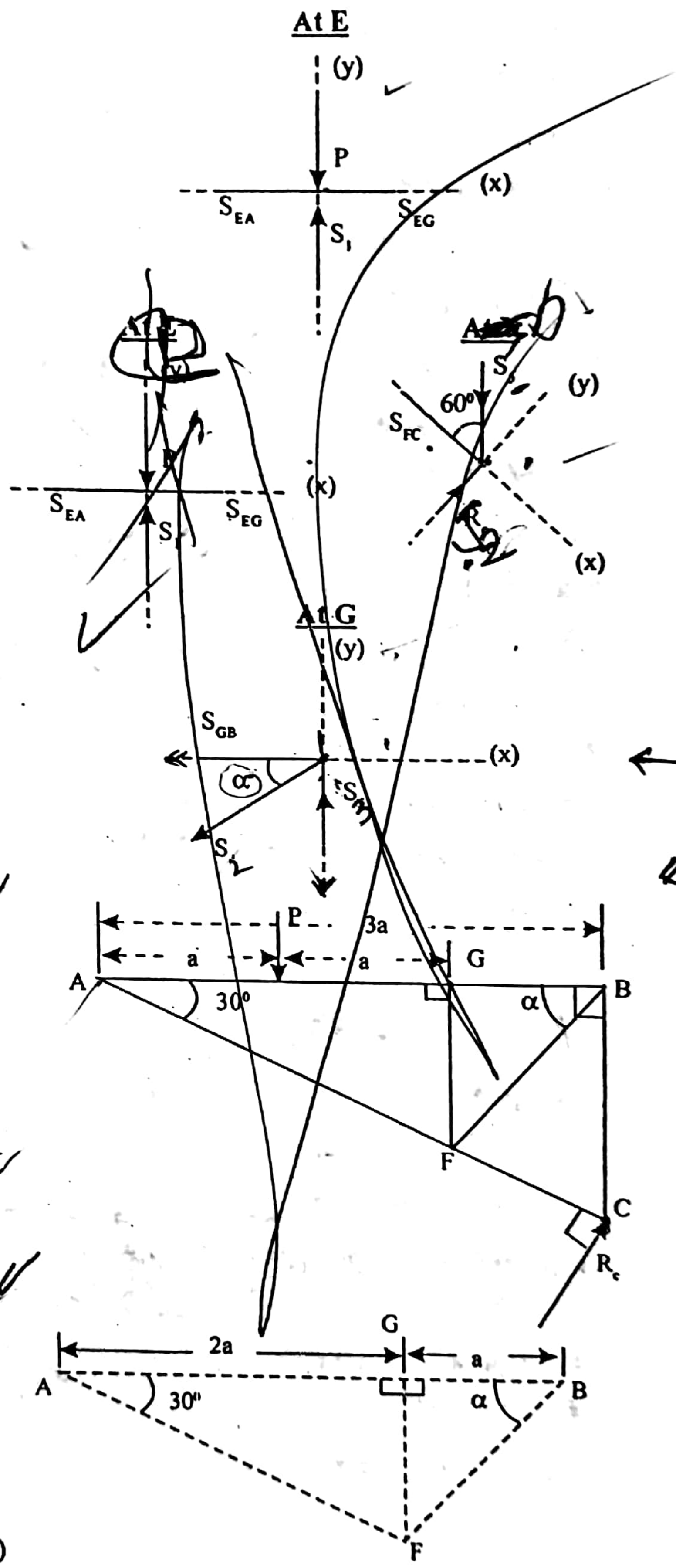
$\therefore S_2 = P$ (Comp.)

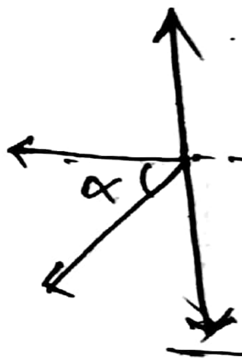
Select the joint G as FBD

Resolving vertically

$\Sigma Y = 0$

$S_3 = S_2 \sin 30° = \dfrac{P}{2}$ (Comp.)




Considering the triangle $\Delta ABC$,

$\cos 30° = \dfrac{3a}{AC}$

$\therefore AC = \dfrac{3a}{\cos 30°} = \dfrac{6a}{\sqrt{3}}$

Taking the moment about A,

$\Sigma M_A = 0$

$R_C \times AC = P \times a$

$\therefore R_c = \dfrac{P \times a}{6a/\sqrt{3}} = \dfrac{\sqrt{3}P}{6}$

Considering the triangle AFG

$\tan 30° = \dfrac{FG}{2a}$

$\therefore FG = 2a \times \dfrac{1}{\sqrt{3}} = \dfrac{2a}{\sqrt{3}}$

Considering the triangle FGB

$\tan\alpha = \dfrac{FG}{GB} = \dfrac{2a}{\sqrt{3}}/a$

$\therefore \alpha = \tan^{-1}\left(\dfrac{2}{\sqrt{3}}\right) = 49°6'$

Select the joint C as FBD

Resolving along y-axis, $\Sigma Y = 0$

$S_5 \sin 60° = R_c$

$\therefore S_5 = \dfrac{\sqrt{3}P}{6} \times \dfrac{2}{\sqrt{3}} = \dfrac{P}{3}$ (Comp.)

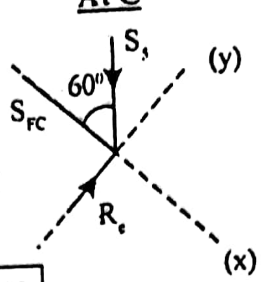
Select the joint B as FBD

Resolving vertically, $\Sigma Y = 0$

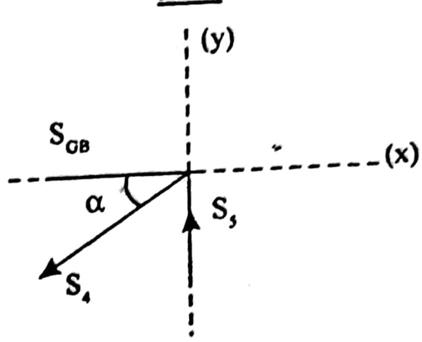
$S_4 \sin\alpha = S_5$

$\therefore S_4 = \frac{P}{3} \times \frac{1}{\sin 49°6'} = 0.441P$ (Tension)

At C

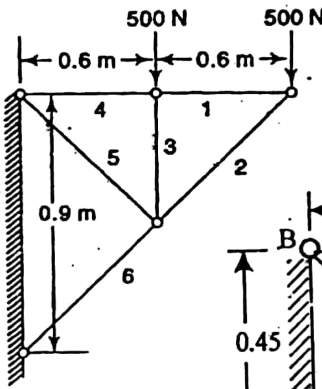



At B



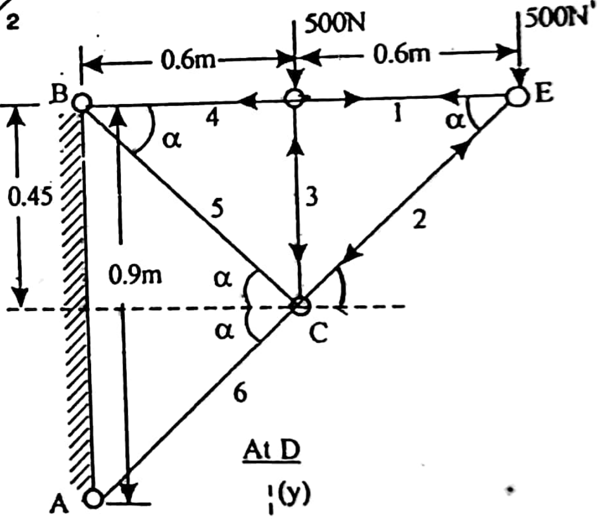

| Bars | Axial force | Magnitude | Nature |
|---|---|---|---|
| ED | $S_1$ | P | tens. |
| DG | $S_2$ | P | Comp. |
| GF | $S_3$ | P/2 | Comp. |
| FB | $S_4$ | 0.441P | tens. |
| BC | $S_5$ | P/3 | Comp. |

4. *Determine the axial force in each bar of the plane truss loaded as shown in the Fig.*




**Soln.** Select the joint E as FBD

$$\alpha = \tan^{-1}\left(\frac{0.45}{0.6}\right) = 36°\ 52'$$




Resolving vertically, $\Sigma Y = 0$

$S_2 \sin 36°\ 52' = 500$

$\Rightarrow S_2 = 833.4$ N. (Comp.)

Resolving horizontally, $\Sigma x = 0$

$S_1 = S_2 \cos 36°\ 52'$

$\Rightarrow S_1 = 666.748$ N (Tens.)

Select the joint D as FBD

Resolving vertically, $\Sigma Y = 0$

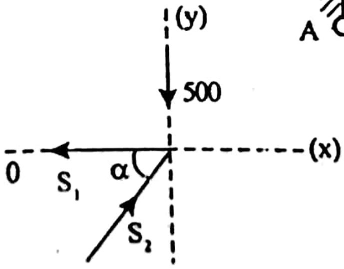
$S_3 = 500$ N. (comp.)

Resolving horizontally, $\Sigma x = 0$

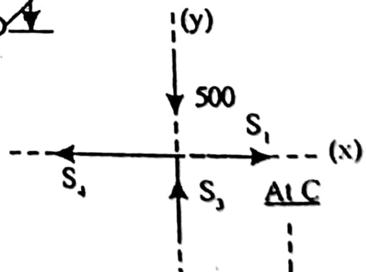
$S_4 = S_1$

$\Rightarrow S_4 = 666.748$ N. (tens)

At E




At D



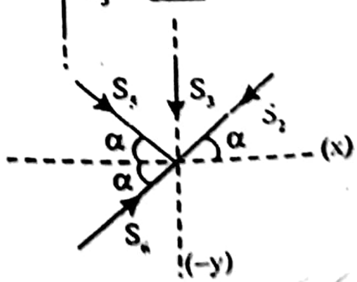

Select the joint C as FBD

Let us assume that $S_5$ and $S_6$ are compression

Resolving vertically, $\Sigma Y = 0$

$S_5 \sin 36°52' + S_2 \sin 36°52' + S_3 = S_6 \sin 36°52$

At C



Dividing throughout $\sin 36°52'$

$S_5 + S_2 + 1.667\, S_3 = S_6$

$\Rightarrow S_6 - S_5 = 833.4 + 833.4$

$\Rightarrow S_6 - S_5 = 1666.8$ ........(i)

Resolving horizontally, $\Sigma x = 0$

$S_5 \cos 36°52' + S_6 \cos 36°52' = S_2 \cos 36°52'$

$\Rightarrow S_6 + S_5 = S_2$

$\Rightarrow S_6 + S_5 = 833.4$ .......(ii)

Solving (i) and (ii)

$2S_6 = 1666.8 + 833.4 \quad \Rightarrow S_6 = 1250\text{ N.}$ (comp.)

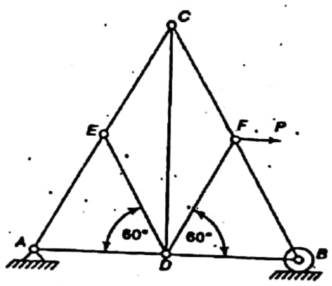
Substituting the value ($S_6$) in equation (1)

$S_5 = 1250 - 1666.8 = -416.8\text{ N}$

–ve sign indicates, $S_5$ is tension i.e., $S_5 = 416.8\text{ N}$ (tens.)

| BARS | Axial Forces | Magnitude | Nature |
|---|---|---|---|
| DE | $S_1$ | 666.748 N | tens. |
| CE | $S_2$ | 833.4 N | comp. |
| CD | $S_3$ | 500 N | comp. |
| BD | $S_4$ | 666.748 N | tens. |
| BC | $S_5$ | 416.8 N | tens. |
| AC | $S_6$ | 1250 N | comp. |

5. *Determine the force S in the bar CD of the simple truss supported and loaded as shown in the Fig. The triangle ABC is equilateral.*




**Soln.** **Select the joint 'E' as FBD**

Let $S_{ED}$ inclines at an angle $\alpha$ with y-axis.

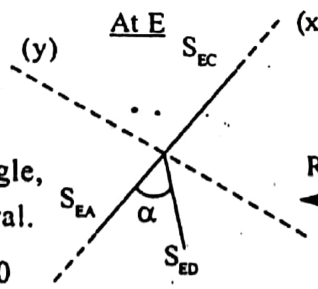
Resolving along x-axis, $\Sigma y = 0$

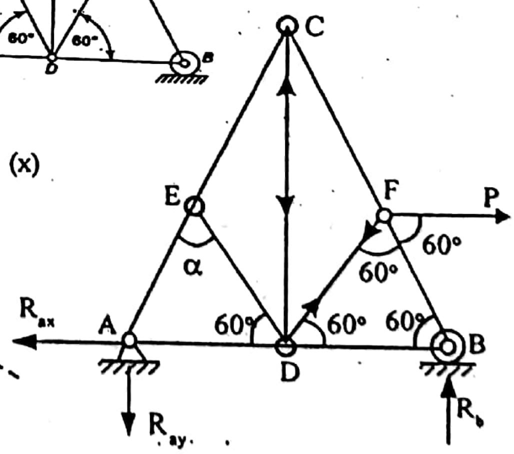
$S_{ED} = 0$

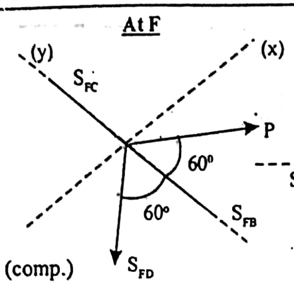
**Select the joint 'F' as FBD**

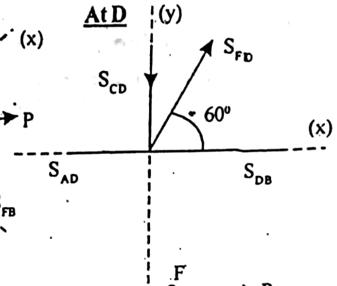
Since ABC is an equilateral triangle, the triangle DFB is also equilateral.

Resolving along X - axis, $\Sigma x = 0$






$S_{FD}$ sin 60° = P sin 60°

$\therefore S_{FD} = P$ (tens)

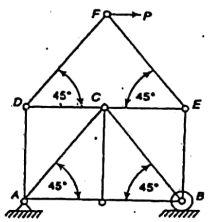
**Select the joint 'D' as FBD**

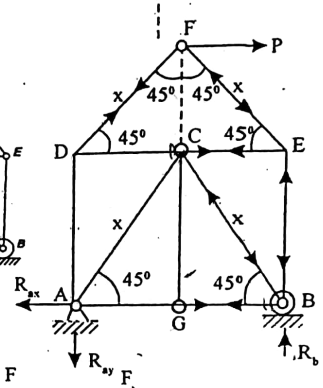
Resolving vertically, $\Sigma y = 0$

$S_{CD} = S_{FD} \sin 60°$

$\Rightarrow S_{CD} = P \times \frac{\sqrt{3}}{2} = 0.866\ P$ (comp.)

**6.** ***Determine the force S in the bar AB of the simple truss supported and loaded as shown in the Fig.***






**Soln.** Let length of the bars FD, FE, AC and BC are 'x'.

$\Sigma M_A = 0$

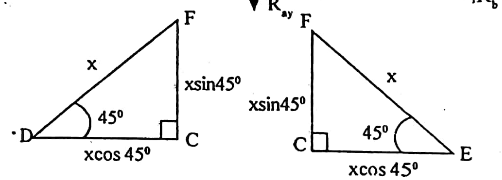
$R_b \times 2x \cos 45° = P \times 2x \sin 45°$

$\therefore R_b = P$

**Select the joint 'G' as FBD**

Resolving vertically, $\Sigma y = 0$

$S_{CG} = 0$




**Select the joint 'F' as FBD**

Resolving vertically $\Sigma y = 0$

$S_{FD} \cos 45° = S_{FE} \cos 45°$

or $S_{FD} = S_{FE}$

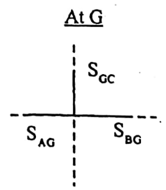
Resolving horizontally, $\Sigma x = 0$

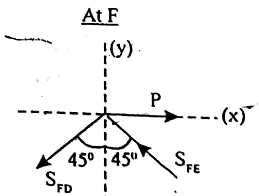
$S_{FD} \sin 45° + S_{FE} \sin 45° = P$

$\Rightarrow 2\ S_{FD} = \frac{P}{2 \sin 45°}$

$\Rightarrow S_{FD} = \frac{P}{\sqrt{2}}$ (tens)

$S_{FE} = \frac{P}{2 \sin 45°} = \frac{P}{\sqrt{2}}$ (comp.)



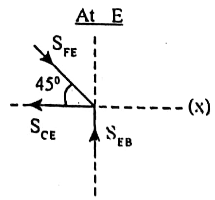



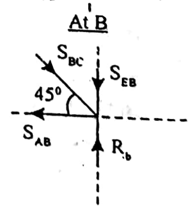

**Select the joint 'E' as FBD**

Resolving vertically, $\Sigma y = 0$

$S_{FE} \sin 45° = S_{EB}$

$\therefore S_{EB} = \frac{P}{2}$ (comp.)



**Select the Joint 'B' as FBD**

Resolving vertically, $\Sigma y = 0$

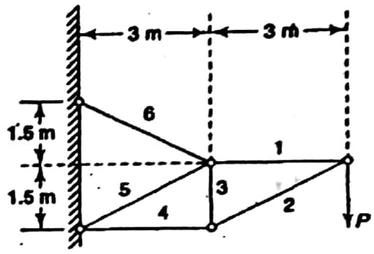
$$S_{BC} \sin 45° + S_{EB} = R_b$$

$$S_{BC} = (P - 0.5P)\sqrt{2} \quad \Rightarrow S_{BC} = \frac{P}{\sqrt{2}} \text{ (comp.)}$$

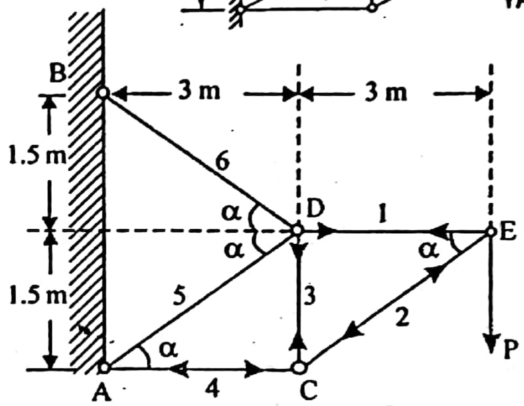
Resolving horizontally, $\Sigma x = 0$

$$S_{AB} = S_{BC} \cos 45° \quad \Rightarrow S_{AB} = \frac{P}{2} = 0.5\,P \text{ (tens.)} \textbf{ (Ans.)}$$

7. *Determine the axial force in each bar of the plane truss loaded as shown in the Fig.*




**Soln.** **Select the joint 'E' as FBD**




Resolving vertically, $\Sigma y = 0$

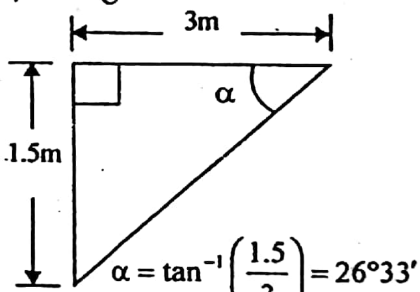
$$S_2 \sin\alpha = P$$

$$\Rightarrow S_2 = 2.237\ P \text{ (comp.)}$$

Resolving horizontally, $\Sigma x = 0$

$$S_1 = S_2 \cos\alpha = 2P \text{ (tens.)}$$

**Select the joint 'c' as FBD**




Resolving horizontally, $\Sigma x = 0$

$$S_4 = S_2 \cos\alpha = 2P \text{ (comp.)}$$

Resolving vertically, $\Sigma y = 0$

$$S_3 = S_2 \sin\alpha = 2.237 \times \sin 26° 33' = P \text{ (tens.)}$$

**Select the joint 'D' as FBD**

Let us assume that, $S_5$ & $S_6$ are tensile in nature.

Resolving vertically, $\Sigma y = 0$

$$S_6 \sin\alpha = S_3 + S_5 \sin\alpha$$

$$\Rightarrow S_6 - S_5 = S_3 / \sin\alpha$$

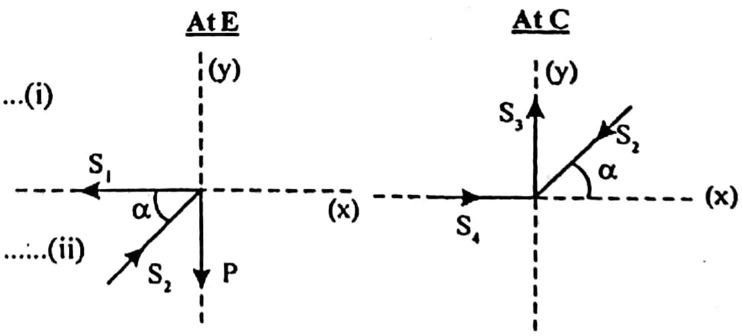
$$\Rightarrow S_6 - S_5 = 2.235\ P \quad \text{...(i)}$$

Resolving horizontally, $\Sigma x = 0$

$$S_6 \cos\alpha + S_5 \cos\alpha = S_1$$

$$\Rightarrow S_6 + S_5 = 2.235\ P \quad \text{......(ii)}$$

Solving (i) and (ii)



$$S_6 = \left(\frac{2.235 + 2.237}{2}\right)P$$

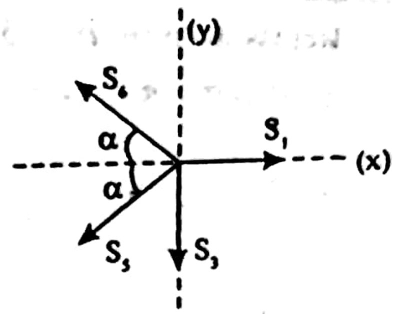
$\Rightarrow S_6 = 2.237\ P$ (tens.)

Substituting the $S_6$ value in equation (ii)

$S_5 = (2.235 - 2.235)\ P$

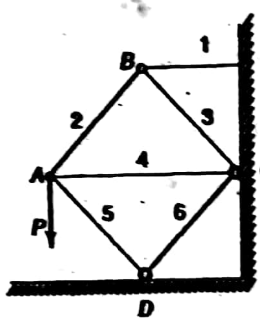
$\Rightarrow S_5 = 0$ **(Ans.)**

At D



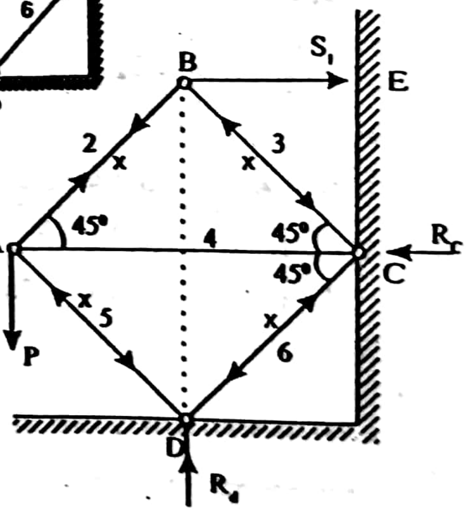
| Bars | Axial force | Magnitude | Nature |
|---|---|---|---|
| DE | $S_1$ | 2P | tens. |
| CE | $S_2$ | 2.237P | comp. |
| CD | $S_3$ | P | tens. |
| AC | $S_4$ | 2P | comp. |
| AD | $S_5$ | 0 | – |
| BD | $S_6$ | 2.237P | tens. |

**8.** ***Determine the axial force in each bar of the plane truss supported and loaded as shown in Fig. ABCD is a square : AC is horizontal.***



**Soln.** Let each bar of the square be 'x'

Consider whole frame as FBD



Resolving vertically $\Sigma y = 0$,

$R_d = P$ ----(i)

Resolving Horizontally $\Sigma x = 0$,

$R_c = S_1$ ----(ii)

Taking moment about 'D' $\Sigma M_D = 0$

$S_1 \times 2x \sin 45° = R_c \times x \sin 45° + P \times x \cos 45°$

$\Rightarrow 2S_1 = R_c + P$ ----(iii)

$\Rightarrow 2S_1 = S_1 + P$

or $S_1 = P$ (tension)

$\therefore R_c = S_1 = P$

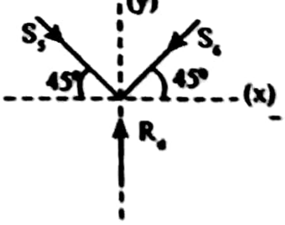
Select the joint 'D' as FBD

Resolving horizontally $\Sigma x = 0$

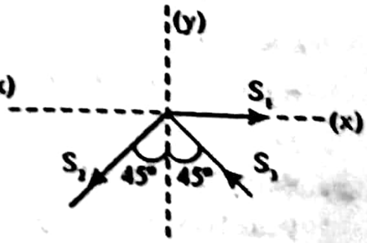
$S_6 \cos 45° = S_5 \cos 45°$

$\Rightarrow S_6 = S_5$

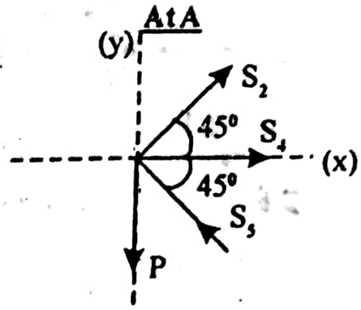
At D



At B



Resolving vertically $\Sigma y = 0$




$S_6 \sin 45° + S_5 \sin 45° = R_d$

$$\Rightarrow S_6 = \frac{P}{2 \sin 45°} = \frac{P}{\sqrt{2}} \text{ (comp.)}$$

$$\therefore S_5 = \frac{P}{\sqrt{2}} \text{ (comp.)}$$

**Select the Joint 'B' as FBD**

Resolving vertically $\Sigma y = 0$

$S_2 \cos 45° = S_3 \cos 45° \Rightarrow S_2 = S_3$

Resolving horizontally $\Sigma x = 0$

$S_2 \sin 45° + S_3 \sin 45° = S_1 \Rightarrow 2 S_2 \sin 45° = P$

$$\Rightarrow S_2 = \frac{P}{\sqrt{2}} \text{ (tens.)} \quad S_3 = \frac{P}{\sqrt{2}} \text{ (comp.)}$$

**Select the joint 'A' as FBD**

Let us assume, '$S_4$' as the tension

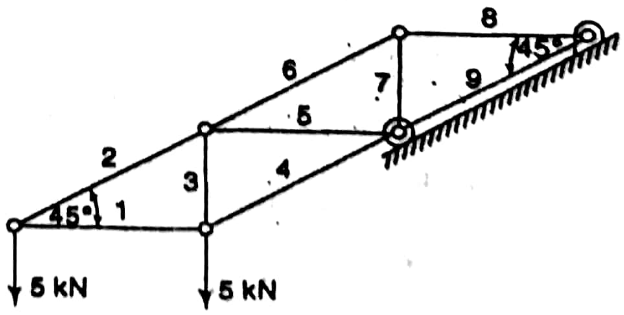
Resolving horizontally $\Sigma x = 0$

$S_2 \cos 45° + S_4 = S_5 \cos 45°$

$$\Rightarrow \frac{P}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + S_4 = \frac{P}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \Rightarrow S_4 = 0 \qquad \text{(Ans.)}$$

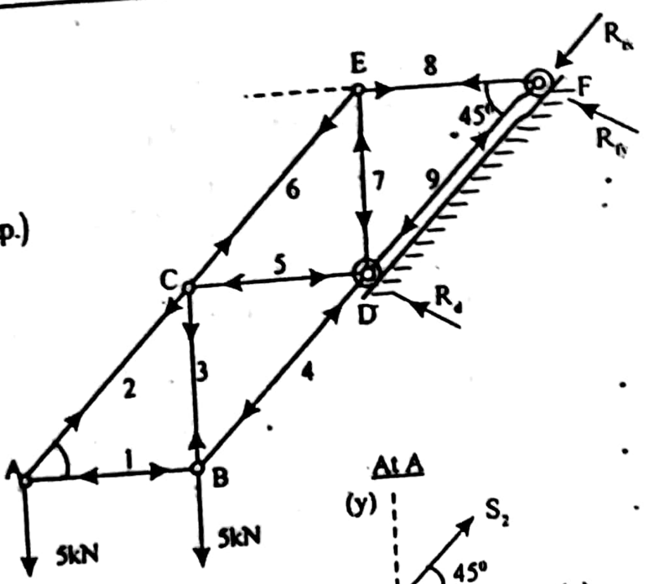
| Bar | Axial force | Magnitude | Nature |
|---|---|---|---|
| BE | $S_1$ | P | tens. |
| AB | $S_2$ | $P/\sqrt{2}$ | tens. |
| BC | $S_3$ | $P/\sqrt{2}$ | comp. |
| AC | $S_4$ | 0 | – |
| AD | $S_5$ | $P/\sqrt{2}$ | comp. |
| CD | $S_6$ | $P/\sqrt{2}$ | comp. |

9. *Determine the axial force $S_i$ in each bar of the plane truss supported and loaed as shown in Fig.*




**Soln. Select the joint 'A' as FBD**

Resolving vertically $\Sigma y = 0$



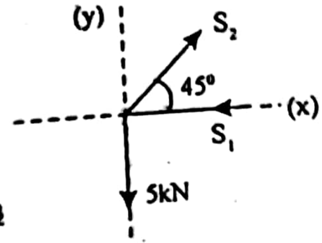

$S_2 \sin 45° = 5$

$\Rightarrow S_2 = 5\sqrt{2}$ KN (tens)

Resolving horizontally $\Sigma x = 0$

$$S_1 = S_2 \cos 45° = 5\sqrt{2} \times \frac{1}{\sqrt{2}} = 5 \text{ KN. (comp.)}$$

At A




Select the joint 'B' as FBD

Resolving horizontally, $\Sigma x = 0$

$S_4 \cos 45° = S_1 \Rightarrow S_4 = 5\sqrt{2}$ KN (comp)

Resolving vertically, $\Sigma y = 0$

$\cdot\, S_3 = 5 + S_4 \sin 45° \Rightarrow S_3 = 10$ KN (tens.)

Select the joint 'C' as FBD

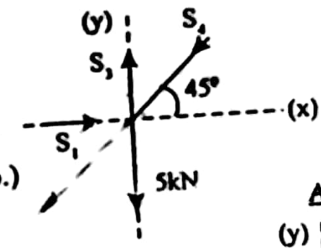
Let us assume $S_3$ and $S_6$ as tension

Resolving vertically $\Sigma y = 0$

$S_6 \sin 45° = S_3 + S_2 \sin 45°$

$\Rightarrow S_6 - (10 + 5)\sqrt{2} \Rightarrow S_6 = 15\sqrt{2}$ KN (tens)

At B




Resolving horizontally $\Sigma x = 0$

$S_5 + S_6 \cos 45° = S_2 \cos 45°$

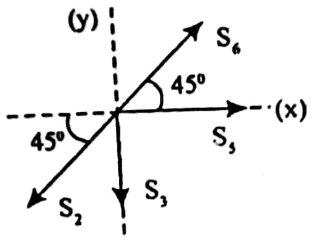
$\Rightarrow S_5 = 5 - 15 = -10$ KN $\quad \therefore S_5 = 10$ KN (comp.)

Select joint joint 'E' as FBD

Resolving horizontally, $\Sigma x = 0$

$$S_8 = S_6 \cos 45° = 15\sqrt{2} \times \frac{1}{\sqrt{2}} = 15 \text{ KN (Tension)}$$

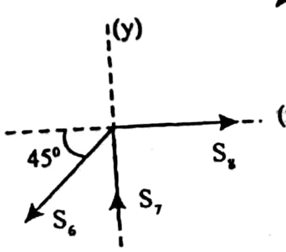
At C




Resolving vertically, $\Sigma y = 0$

$S_7 = S_6 \sin 45° = 15$ KN (comp)

At E




Select the joint D as FBD

Let us assume $S_9$ as compression.

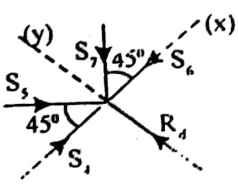
Resolving Horizontally $\Sigma x = 0$

$S_5 \cos 45° + S_4 = S_9 + S_7 \cos 45°$

$$\therefore S_9 = 10 \times \frac{1}{\sqrt{2}} + 5\sqrt{2} - 15 \times \frac{1}{\sqrt{2}} = \frac{10 + 10 - 15}{\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ KN}$$

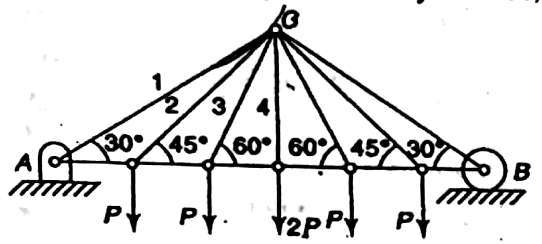
$\therefore S_9 = 3.535$ KN (comp.)

At D



| Arial forces | Bars | Magnitude | Nature |
|---|---|---|---|
| $S_1$ | AB | 5 KN | Comp. |
| $S_2$ | AC | $5\sqrt{2}$ KN | Tens. |
| $S_3$ | BC | 10 KN | Tens. |
| $S_4$ | BD | $5\sqrt{2}$ KN | Comp. |
| $S_5$ | CD | 10 KN | Comp. |
| $S_6$ | CE | $15\sqrt{2}$ KN | Tens. |
| $S_7$ | ED | 15 KN | Comp. |
| $S_8$ | EF | 15 KN | Tens. |
| $S_9$ | DF | 3.535 KN | Comp. |

**10.** *Using method of joints, calculate the axial force in each of the bars 1, 2, 3 and 4 of the plane truss shown in Fig.*

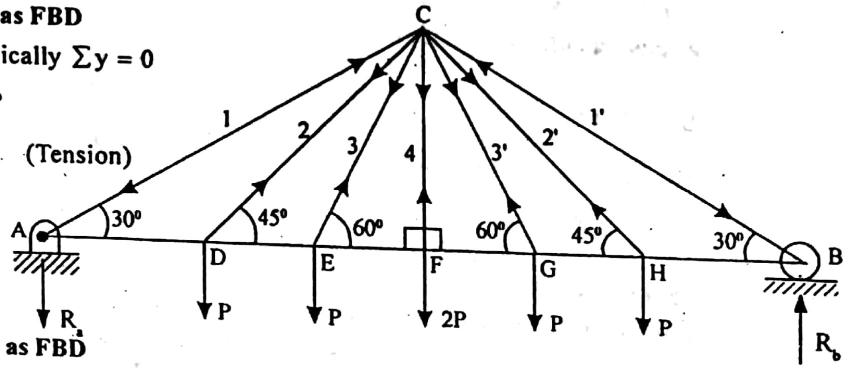



**Soln.** **Select joint D as FBD**

Resolving vertically $\Sigma y = 0$

$S_2 \sin 45° = P$

$\therefore S_2 = \sqrt{2}\ P$ (Tension)




**Select joint E as FBD**

Resolving Vertically $\Sigma y = 0$

$S_3 \sin 60° = P$

$\therefore S_3 = \dfrac{2P}{\sqrt{3}}$ (Tension)

**Select joint F as FBD**

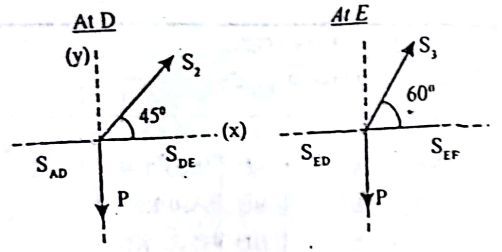
Resolving vertically, $\Sigma y = 0$

$S_4 = 2P$ (Tension)

Due to symmetry at joint H and G

$S'_2 = S_2 = \sqrt{2}\,P$ (Tension)

$S'_3 = S_3 = \dfrac{2P}{\sqrt{3}}$ (Tension)




And $S_1 = S'_1$

Select joint C as FBD

Let us assume $S_1$ as compression

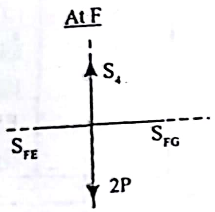
Resolving vertically, $\Sigma y = 0$

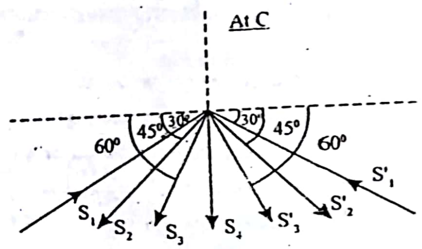
$2S_1 \sin 30° = 2S_2 \sin 45° + 2S_3 \sin 60° + S_4$

or $S_1 \times \dfrac{1}{2} = S_2 \times \dfrac{1}{\sqrt{2}} + S_3 \times \dfrac{\sqrt{3}}{2} + \dfrac{S_4}{2}$

or $\dfrac{S_1}{2} = \left(\sqrt{2}\,P\right) \times \dfrac{1}{\sqrt{2}} + \dfrac{2P}{\sqrt{3}} \times \dfrac{\sqrt{3}}{2} + \dfrac{2P}{2}$

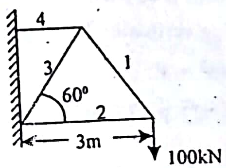
or $\dfrac{S_1}{2} = P + P + P$ $\quad \therefore S_1 = 6P$ (comp.)




| Bars | Axial forces | Magnitude | Nature |
|---|---|---|---|
| AC | $S_1$ | 6P | comp. |
| DC | $S_2$ | $\sqrt{2}P$ | tens. |
| EC | $S_3$ | $2P/\sqrt{3}$ | tens. |
| FC | $S_4$ | 2P | tens. |

11. *A cantilever truss of 3 m span is loaded as shown in fig. Find the forces in the various member of the framed truss, and tabulate the results.*




Soln. Select joint 'A' as FBD

Resolving vertically, $\Sigma Y = 0$

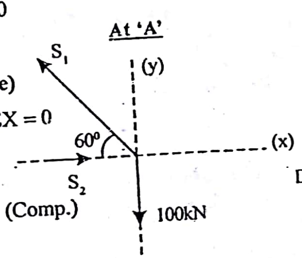
$S_1 \sin 60° = 100$

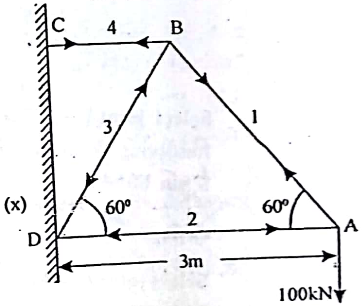
$\Rightarrow S_1 = 115.47$ kN (Tensile)

Resolving horizontallly, $\Sigma X = 0$

$S_2 = S_1 \cos 60°$

$= 11547 \times \dfrac{1}{2} = 57.735$ kN (Comp.)



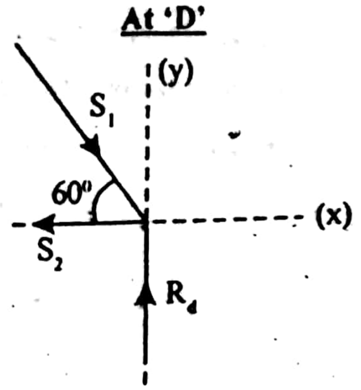



$\Rightarrow S_1 = 11.54$ kN (Comp.)

Resolving horizontally, $\Sigma X = 0$

$S_2 = S_1 \cos 60°$

$= 11.54 \times \cos 60° = 5.77$ kN (Tens.)



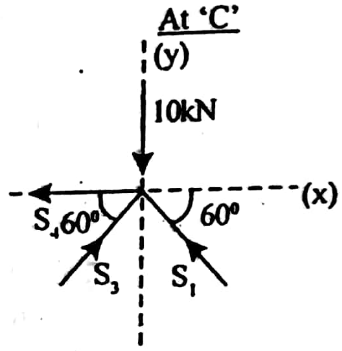

**Select 'C' as FBD**

Let, $S_3$ is compression and $S_4$ is tension.

Resolving vertically, $\Sigma Y = 0$

$S_1 \sin 60° + S_3 \sin 60° = 10$

$$\Rightarrow S_3 = \frac{10-(11.54\times\sin 60°)}{\sin 60°} \Rightarrow S_3 = 0$$



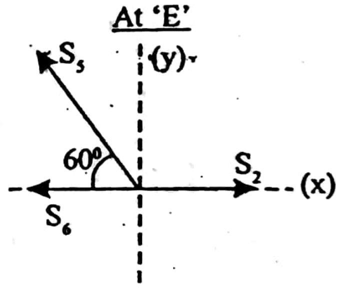

Resolving horizontally, $\Sigma X = 0$

$S_4 + S_1 \text{Cos} 60° = 0$

$$\Rightarrow S_4 = -S_1 \cos 60° \qquad \Rightarrow S_4 = -\left(11.54 \times \frac{1}{2}\right)$$

$\Rightarrow S_4 = -5.77$ kN $\qquad \Rightarrow S_4 = 5.77$ kN (Comp.)

**Select the joint 'E' as FBD,**



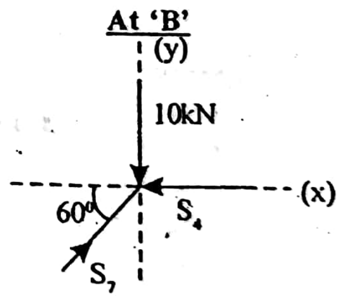

Resolving vertically, $\Sigma Y = 0$

$S_5 \sin 60° = 0$ or $S_5 = 0$

Resolving horizontally, $\Sigma X = 0$

$S_6 = S_2 = 5.77$kN (Tens.)

**Select the jont 'B' as FBD**



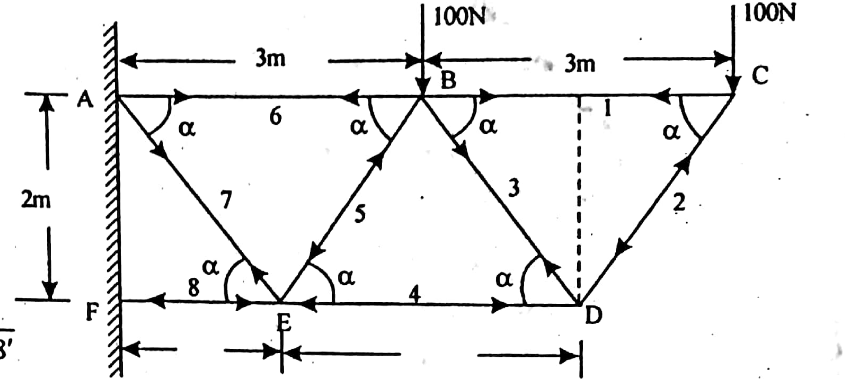

Resolving horizontally, $\Sigma X = 0$

$$S_7 \cos 60° = S_4 \qquad \Rightarrow S_7 = \frac{5.77}{(1/2)}$$

$\Rightarrow S_7 = 11.54$kN (Comp.)

| Members | Axial force | Magnitude | Nature |
|---|---|---|---|
| CD | $S_1$ | 11.54kN | Comp. |
| ED | $S_2$ | 5.77kN | Tens. |
| CE | $S_3$ | 0 | – |
| BC | $S_4$ | 5.77kN | Comp. |
| BE | $S_5$ | 0 | – |
| AC | $S_6$ | 577kN | Tens. |
| AB | $S_7$ | 11.54kN | Comp. |

**13.** *A cantilever truss is loaded is shown in the fig. Determine axial force in each members of the truss.*




**Soln.** **Select Joint C as FBD**

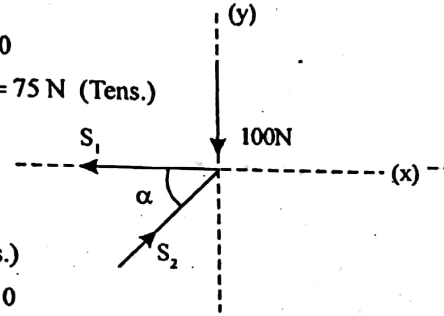
Resolving vertically, $\Sigma Y = 0$

$S_2 \sin\alpha = 100$ or $S_2 = \dfrac{100}{\sin 53°.8'}$

$\therefore\ S_2 = 125N$ (Comp.)

Resolving horizontally, $\Sigma X = 0$

$S_1 = S_2 \cos\alpha = 125 \cos 53°8' = 75\ N$ (Tens.)

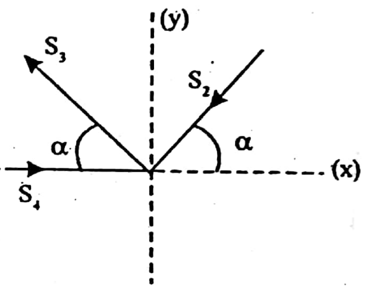



**Selet joint D as FBD**

Resolving vertically, $\Sigma Y = 0$

$S_3 \sin\alpha = S_2 \sin\alpha$

$\Rightarrow S_3 = S_2 = 125N$ (Tens.)




Resolving horizontally, $\Sigma X = 0$

$S_4 = S_3 \cos\alpha + S_2 \cos\alpha$

$= (125 \times \cos 53°8') + (125 \times \cos 53°8') = 150\ N$ (Comp.)

**Select the joint 'B' as FBD**

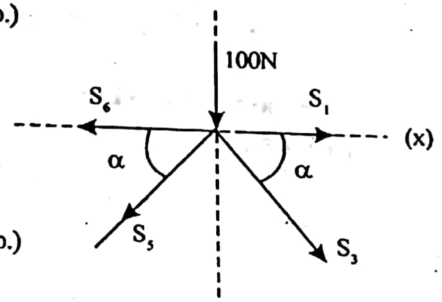
Let $S_5$ be the compression and $S_6$ as tension.

Resolving vertically, $\Sigma Y = 0$

$S_5 \sin\alpha = S_3 \sin\alpha + 100$

$\Rightarrow S_5 = \dfrac{(125 \times \sin 53°8') + 100}{\sin 53°8'} \Rightarrow S_5 = 250N$ (Comp.)

**At 'B'**




Resolving horizontally, $\Sigma X = 0$

$S_6 = S_1 + S_3 \cos\alpha + S_5 \cos\alpha$

$\therefore\ S_6 = 75 + (125 \times \cos 53°8') + (250 \times \cos 53°8')$

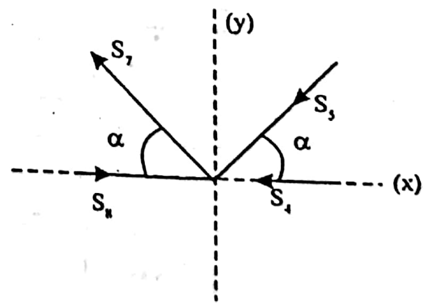
$= 300\ N$ (Tens)

**Select the Joint "E" as FBD**

Resolving vertically, $\Sigma Y = 0$

$S_7 = \sin\alpha = S_7 \quad \Rightarrow S_7 = S_5 = 250N$ (Tens.)

**At 'E'**



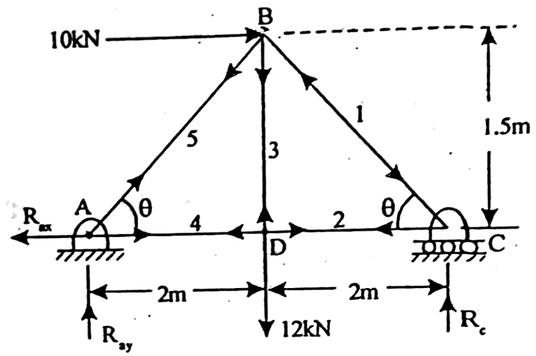

Resolving horizontally, $\Sigma X = 0$

$S_8 = S_4 + S_5 \cos\alpha + +S_7 \cos\alpha$

$S_8 = 150 + (250 \times \cos 53°8') + (250 \times \cos 53°8') = 450\ N$ (Comp.)

| Members | Axial force | Magnitude | Nature |
|---|---|---|---|
| BC | $S_1$ | 75N | Tens. |
| CD | $S_2$ | 125N | Comp. |
| BD | $S_3$ | 125N | Tens. |
| ED | $S_4$ | 150N | Comp. |
| BE | $S_5$ | 250N | Comp |
| AB | $S_6$ | 300N | Tens. |
| AE | $S_7$ | 250N | Tens. |
| EF | $S_8$ | 450N | Comp. |

**14.** *A framed structure of 4 m span and 1.5 m height subjected to two points loads at B and D. in Fig. Find graphically, or other wise, the forces in all the members of the structure?*




**Soln.** From the geometry of the fig,

$$\theta = \tan^{-1}\left(\frac{1.5}{2}\right) = 36°.52'$$

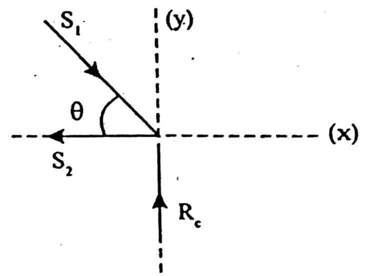
Considering whole truss as FBD, $\Sigma M_A = 0$

$R_C \times 4 = (12\times2)+(10\times1.5)$

$\Rightarrow R_c = 9.75$ kN

**Select joint 'C' as FBD**

**At 'C'**




Resolving vertically, $\Sigma Y = 0$

$S_1 \sin\theta = R_c$

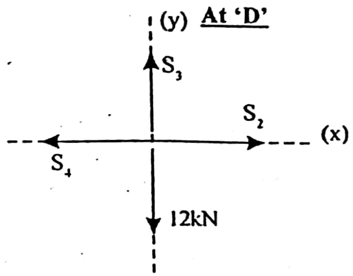
$$\Rightarrow S_1 = \frac{9.75}{\sin 36°52'} \Rightarrow S_1 = 16.25\text{kN (Comp.)}$$

Resolving horizontally, $\Sigma X = 0$

$S_2 = S_1\cos\theta = 16.25\times\cos 36°52' = 13$ kN (Tens.)

**Select joint 'D' as FBD**

**At 'D'**




Resolving Vertically, $\Sigma Y = 0$

$S_3 = 12$ kN (Tens.)

Resolving horizontally, $\Sigma X = 0$

$S_4 = S_2 = 13$kN (Tens.)

Select joint 'B' as FBD

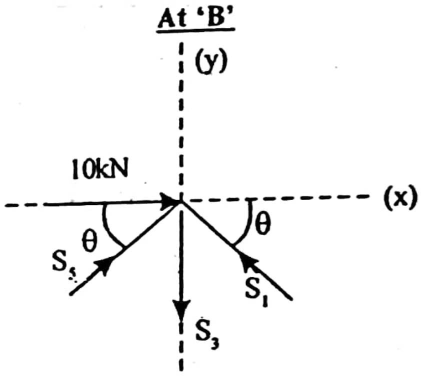
Let $S_5$ is compression,

Resolving horizontally, $\Sigma X = 0$

$$S_5 \cos\theta^\circ = (10 - S_1 \cos\theta)$$

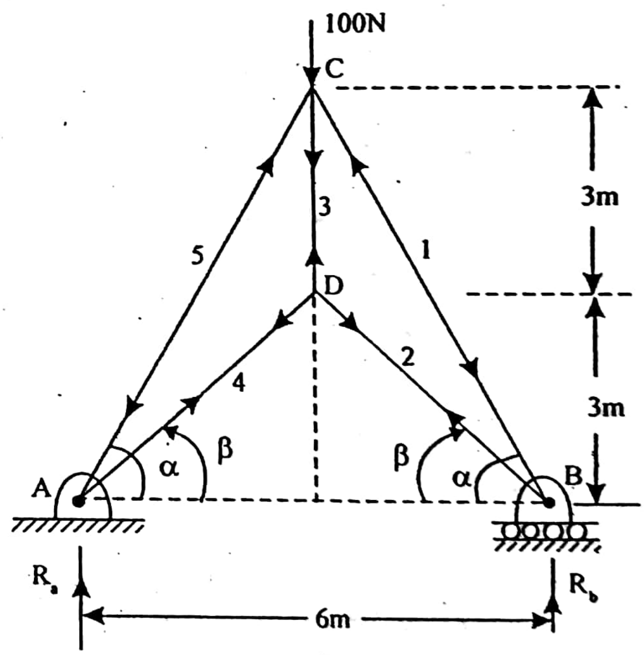
$$\Rightarrow S_5 = \frac{10 - (16.25 \times \cos 36°52')}{\cos 36°52'}$$

$$\Rightarrow S_5 = -3.75\text{kN} \Rightarrow S_5 = 3.75 \text{ kN} \qquad \text{(Tens.)}$$




| Members | Axial force | magnitude | Nature |
|---|---|---|---|
| BC | $S_1$ | 16.25kN | Comp. |
| CD | $S_2$ | 13kN | Tens |
| BD | $S_3$ | 12kN | Tens. |
| AD | $S_4$ | 13kN | Tens. |
| AB | $S_5$ | 3.75kN | Tens. |

**15.** *A framed structure of 6 m span is carrying a central load of 100 N as shown in fig. Find the magnitude and nature of forces in all members of the structure and tabulate the results?*




**Soln.** Let $m\angle BDA = m\angle DAB = \beta$

$$m\angle CBA = m\angle CAB = \alpha$$

From geometry of the figure

$$\alpha = \tan^{-1}\left(\frac{6}{3}\right) = 63°26'$$

$$\beta = \tan^{-1}\left(\frac{3}{3}\right) = 45°$$

Considering whole structure as FBD

$\Sigma Y = 0$

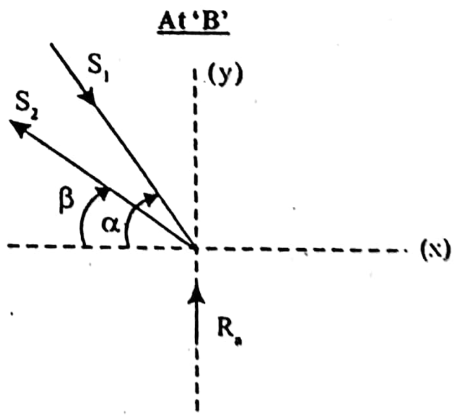
$R_a + R_b = 100\text{N}$ (i)

$\Sigma M_A = 0$

$R_a \times 6 = 100t \times 3$ (ii)

$\therefore\ R_b = 50\text{N}$

$\therefore R_a = 50\text{N}$



Select joint - B as FBD

Let $S_1$ is Compression and $S_2$ be the tension.

Resolving horizontally, $\Sigma X = 0$

$$S_1 \cos\alpha = S_2 \cos\beta$$

$$\Rightarrow S_1 = S_2 \times \frac{\cos\beta}{\cos\alpha} \qquad \Rightarrow S_1 = S_2 \times \frac{\cos 45°}{\cos 63°26'}$$

$$\Rightarrow S_1 = 1.581 S_2 \qquad ....(iii)$$

Resolving vertically, $\Sigma Y = 0$

$$S_1 \sin\alpha = R_a + S_2 \sin\beta \qquad .....(iv)$$

Solving (iii) and (iv);

$$1.581 S_2 \times \sin 63°26' = 50 + S_2 \times \sin 45°$$

$$\Rightarrow 1.414 S_2 = 50 + 0.707 S_2 \Rightarrow 0.707 S_2 = 50$$

$$\Rightarrow S_2 = 70.721\text{N} \quad \text{(Tens.)}$$

$$\therefore S_1 = 1.581 \times 70.721 \qquad \Rightarrow S_1 = 111.81\text{ N} \quad \text{(Comp.)}$$

Select joint 'A' as FBD

Due to symmetry,

$$S_4 = S_2 = 70.721\text{N} \qquad \text{(Tens.)}$$

$$S_5 = S_1 = 111.81\text{N} \qquad \text{(Comp.)}$$

Select joint 'C' as FBD

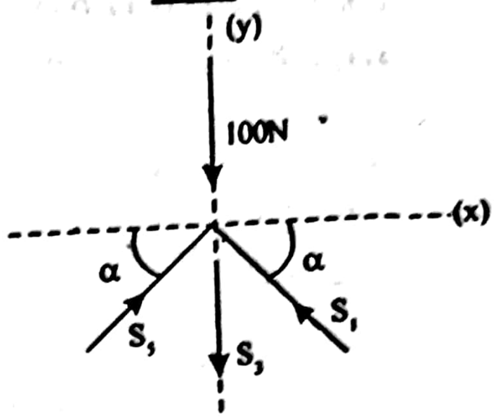
Assume $S_6$ as tension, resolving vertically, $\Sigma Y = 0$

$$S_3 + 100 = S_1 \sin\alpha + S_5 \sin\alpha$$

$$\Rightarrow S_3 = \left[(111.81 \times \sin 63°26') + (111.81 \times \sin 63°26')\right] - 100$$

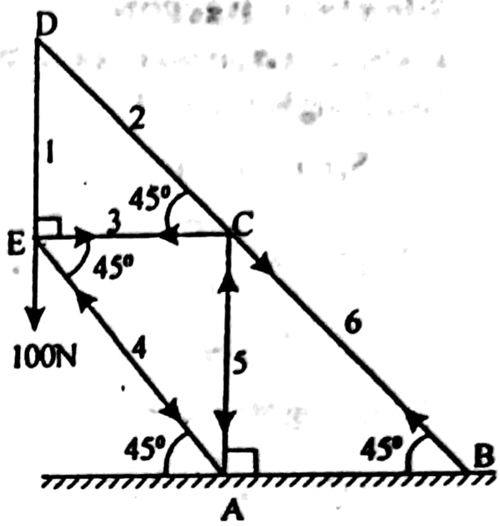
$$\Rightarrow S_3 = 100\text{N} \qquad \text{(Tens.)}$$

At 'C'




| Member | axial force | Magnitude | Nature |
|---|---|---|---|
| BC | $S_1$ | 111.81N | Comp. |
| BD | $S_2$ | 70.721N | Tens. |
| CD | $S_3$ | 100N | Tens. |
| AD | $S_4$ | 70.721N | Tens. |
| AC | $S_5$ | 111.81N | Comp. |

**16.** *Calculate the axial forces in each bar of a simple truss supported and loaed as shown in the fig?*




**Soln.** Select 'D' as FBD

Resolving horizontally, $\Sigma X = 0$

$S_2 \cos 45° = 0$

$\Rightarrow S_2 = 0$

Resolving vertically, $\Sigma Y = 0$

$S_1 = 0$

Select the joint 'E' as FBD

Resolving vertically, $\Sigma Y = 0$

$S_4 \sin 45° = 100$

$\Rightarrow S_4 = 141.42\text{N}$ (Comp.)

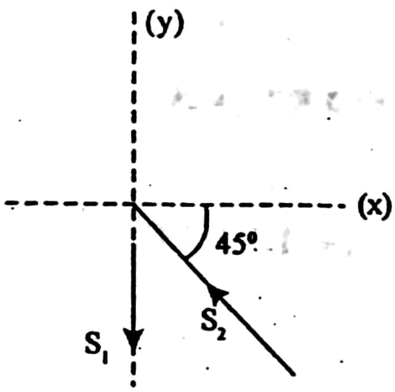
Resolving horizontally, $\Sigma X = 0$

$S_3 = S_4 \cos 45°$

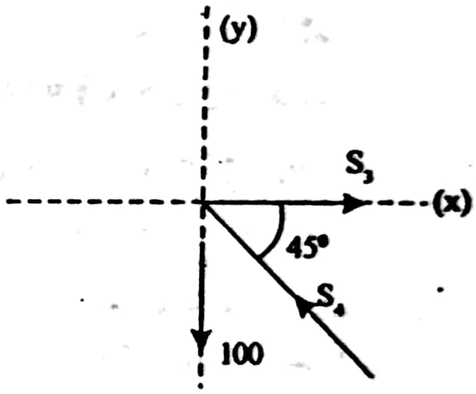
$\Rightarrow S_3 = 141.42 \times \cos 45°$

$\Rightarrow S_3 = 100\text{N}$ (Tens.)

At 'D'




At 'E




Select the joint 'C' as FBD

Resolving horizontally, $S_6 \cos 45° = S_3$

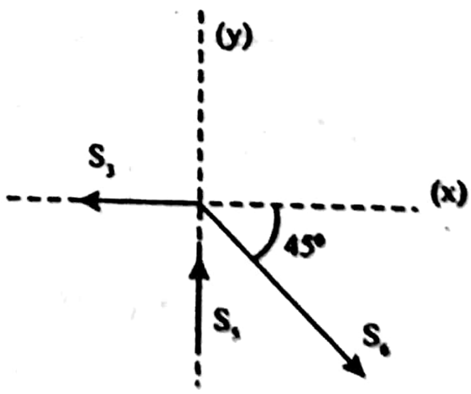
$\Rightarrow S_6 = \dfrac{100}{\cos 45°} \quad \Rightarrow S_6 = 141.42\text{N}$ (Tens.)

Resolving vertically, $\Sigma Y = 0$

$S_5 = S_6 \sin 45°$

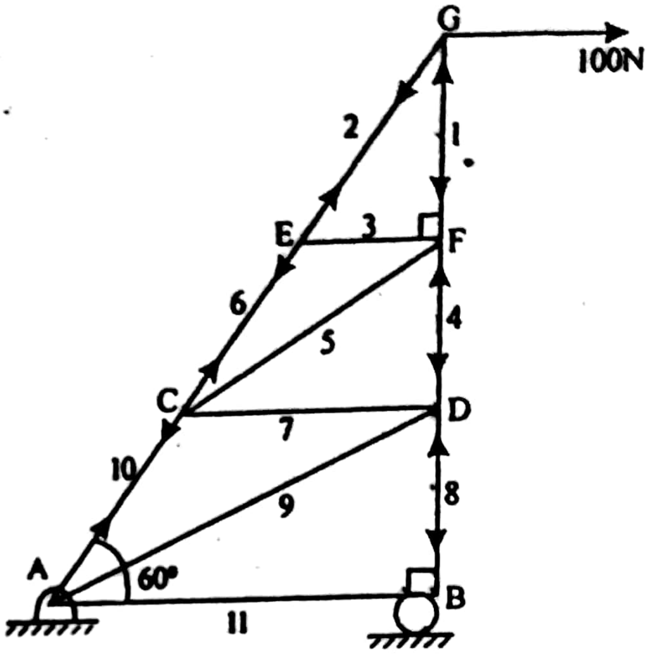
$\Rightarrow S_5 = 141.42 \times \sin 45° \Rightarrow S_5 = 100\text{N}$ (Comp.)

At 'C'




| Members | Axial force | Magnitude | Nature |
|---|---|---|---|
| ED | $S_1$ | 0 | – |
| CD | $S_2$ | 0 | – |
| CE | $S_3$ | 100N | Tens. |
| AB | $S_4$ | 141.42N | Comp. |
| AC | $S_5$ | 100N | Comp. |
| BC | $S_6$ | 141.42N | Tens. |

17. ***Determine the forces in all members of the plane truss loaded and supported as shown in the figure?***




**Soln.** Select 'G' as FBD

Resolving horizontally, $\Sigma X = 0$

$S_2 \cos 60° = 100$

$\Rightarrow S_2 = \dfrac{100}{\cos 60°} = 200N$ (Tens.)

Resolving vertically, $\Sigma Y = 0$

$S_1 = S_2 \sin 60°$

$\Rightarrow S_1 = 200 \times \sin 60° = 200N$ (Comp.)

Select joint 'E' as FBD

REsolving along y-axis, $\Sigma Y = 0$

$S_3 \sin 60° = 0$

$\Rightarrow S_3 = 0$

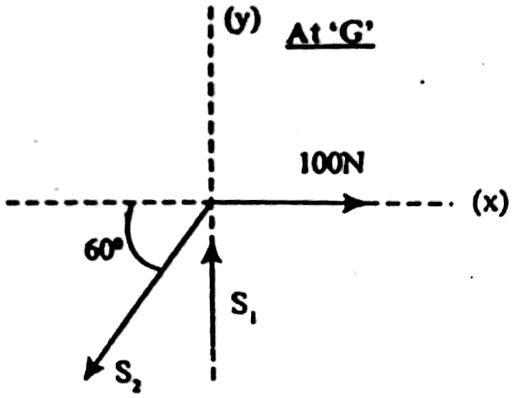
Resolving along x-axis, $\Sigma X = 0$

$S_6 = S_2 = 200N$ (Tens.)

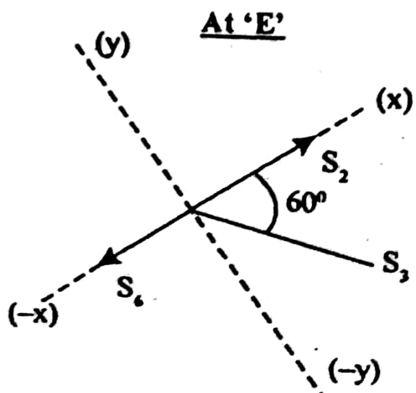
Similarly at joint F, C and D

$S_5 = 0,\ S_7 = 0, S_9 = 0$

$S_{10} = S_6 = S_2 = 200N$ (Tens.)




| Member | Axial force | Magnitude | Nature |
|---|---|---|---|
| GF | $S_1$ | 200N | Comp. |
| GE | $S_2$ | 200N | Tens. |
| EF | $S_3$ | 0 | – |
| DF | $S_4$ | 200N | Comp. |
| CF | $S_5$ | 0 | – |
| CE | $S_6$ | 200N | Tens. |
| CD | $S_7$ | 0 | – |
| BD | $S_8$ | 200N | Comp. |
| AD | $S_9$ | 0 | – |
| AC | $S_{10}$ | 200N | Tens. |



Chapter - 7.7

# D'Alembert's Principle

## 7.7.1 INTRODUCTION

The differential equation of plane motion of a body is

$$\left.\begin{aligned}\Sigma F_x &= m\ddot{x}\\ \Sigma F_y &= m\ddot{y}\end{aligned}\right\} \qquad \text{......(i)}$$

In general $\Sigma F = ma$, can be written in the form

$$\Sigma F - ma = 0 \qquad \text{....(ii)}$$

From the equation (ii), we see that the sum of resultant of external forces $(\Sigma F)$ and the force (-ma) is zero i.e., this equation of motion of a body is of the same form as an equation of static equilibrium. Hence equation (ii) is called as equation of dynamic equilibrium.

The force (-ma) is called inertial force.

i.e., $$F_i = -ma = \frac{-W}{g} \times a$$

F = ma

In otherword, it is the product of mass and acceleration and acts oposite to the direction of acceleration due to negative sign and supposed to act at C.G of the body.

The equation (ii) can be written as $\Sigma F + F_i = 0$

In the rectangular component form, the equation are $\Sigma F_x + F_{ix} = 0$

where $F_{ix} = -ma_x$ and $\Sigma F_y + F_{iy} = 0$ where $F_{iy} = -ma_y$

## 7.7.2 D'ALEMBERT'S PRINCIPLE

To write the equation of dynamic equilibrium, introduce an imaginary force equal to the inertia force to the external resultant force acting on the body and equate the sum to zero.

This concept is known as D'Alemberts principle and is very useful in the solution of engineering problems of dynamics.

**Advantages :** Using D'Alembert's principle, the problems under dynamic equilibrium can virtually be converted into static equilibrium problems.

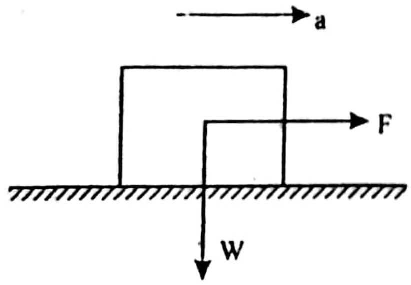
Hence all conditions of static equilibrium such as

$\Sigma x = 0,\ \Sigma y = 0,\ \Sigma M = 0$

can be readily used while solving problems.

**Examples**

(i) Let a body of 'W' is moving along a horizontal plane with a constant acceleration 'a' and external force 'F'.




Introduce inertia force opposite to the direction of acceleration.

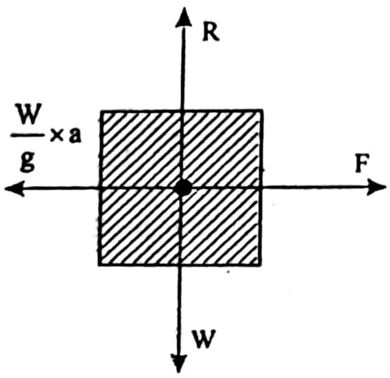
Then applying condition of static equilibrium as

$\Sigma y = 0$

$\therefore\ R = w$

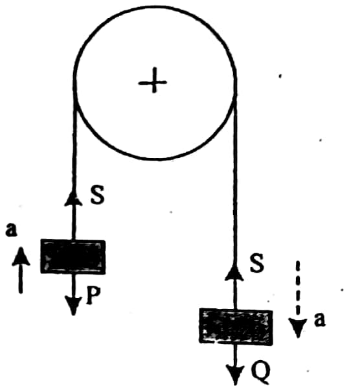
$\Sigma x = 0$

$$\therefore\ F = \frac{W}{g} \times a$$




(ii) Let a pulley system is loaded as shown in the figure. Assume Q > P and 'a' be the acceleration of Q.

Since 'Q' is moving down, its inertia force acts upward. Now using equation of statics,




$\Sigma y = 0$

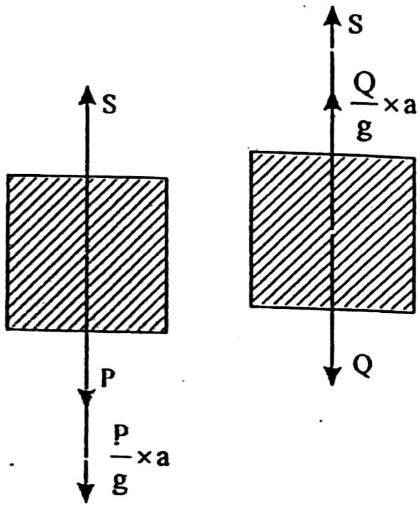
$$S + \frac{Q}{g} \times a = Q \qquad \text{.....(i)}$$

Now considering 'P' introduce inertia force

$\left(\frac{P}{g} \times a\right)$ acting down.

$$\text{Then } \Sigma y = 0,\ S = \left(P + \frac{P}{g} \times a\right) \qquad \text{.....(ii)}$$

Solving (i) and (ii) S & a can be found out if P & Q are known to us.



(iii) Let a body is deaccelerating due to friction on a horizontal track.

Now we can say the body is accelerating in opposite direction.

Hence inertia force will be introduced in forward direction i.e., towards deacceleration.

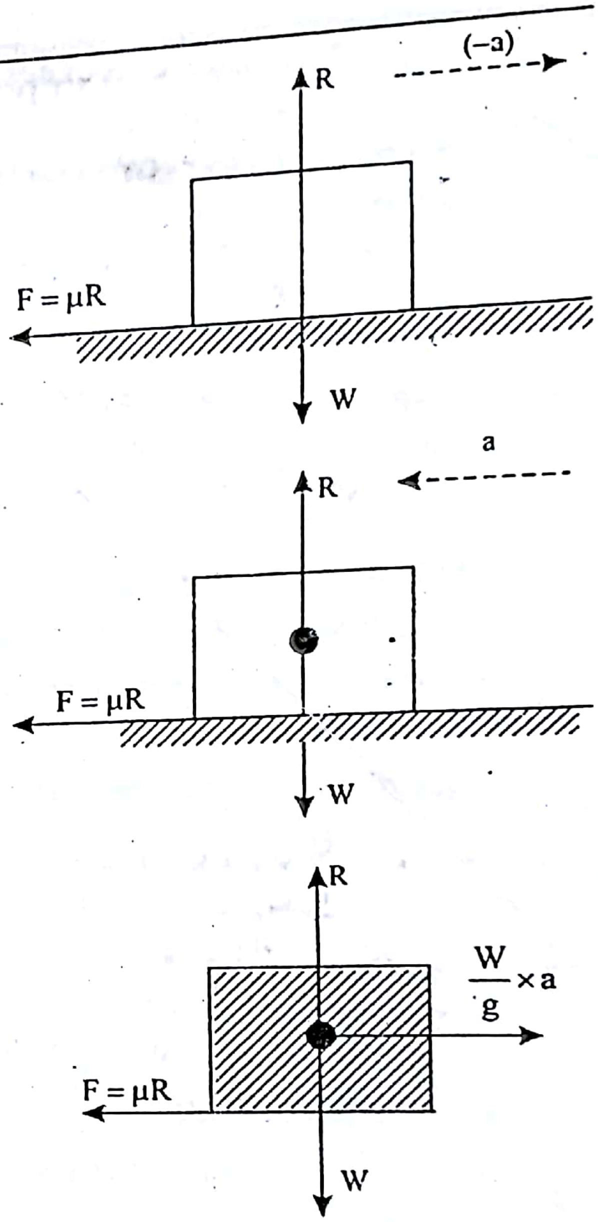
Then using equation of statics.

$$\sum y = 0, \quad R = W$$

$$\sum x = 0, \quad \mu R = \frac{W}{g} \times a$$

or $$\mu W = \frac{W}{g} \times a$$

or $$a = \mu g$$




*Note :* The equation of motion and the equation of dynamic equilibrium of a body are two methods of expression which differ only in the concept used and in the manner of writing the equations. Otherwise both are same.

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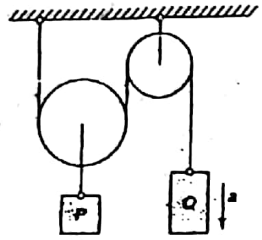
## SOLVED PROBLEMS - 7.7

inertia $F = -mg$

1. *Two weights P and Q are connected by the arrangement as shown in the Fig. Neglecting friction and the inertia of the pulleys and cord, find the acceleration a of weight Q. Assume that $p = 178N$. $Q = 133.5N$*




**Soln.** Given data

$P = 178\text{ N}$

$Q = 133.5\text{ N}$

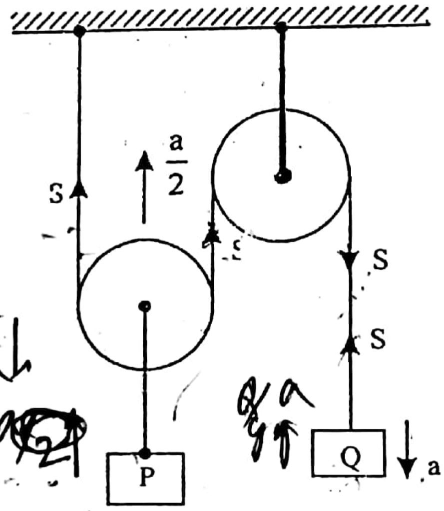
Let 'S' be the tension in the rope

Since Q is moving downward with an acceleration 'a', the inertia force will act in opposite direction.

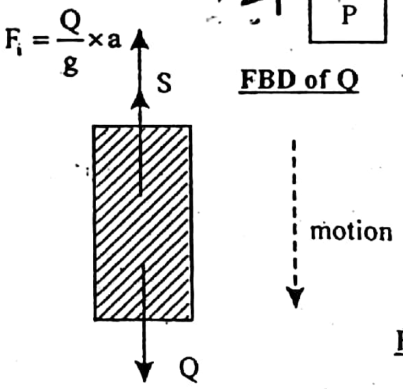
Applying 'D' Alembert's principle

$$\Sigma y = 0, \quad \frac{Q}{g} \times a + S = Q$$

$$\Rightarrow S = 133.5\left(1 - \frac{a}{9.81}\right) \quad \text{.....(i)}$$




$\frac{P}{g} \cdot \frac{a}{2} \downarrow$ $\frac{Q}{g} a \uparrow$




**FBD of Q**

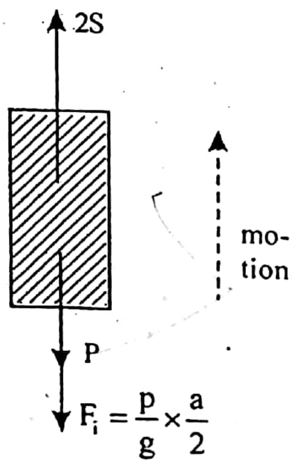
Since P is acted upon by 2S; the acceleration will be a/2.

Applying D'Alembert's principle

$$\Sigma y = 0$$

$$2S = P + \frac{P}{g} \times \frac{a}{2} \quad \text{........(ii)}$$

**FBD of P**




Substituting the values of S & P in equation (ii)

$$\Rightarrow 2\left[133.5\left(1 - \frac{a}{9.81}\right)\right] = 178\left(1 + \frac{a}{2 \times 9.81}\right)$$

$$\Rightarrow 267 - 27.21a = 178 + 9.072a$$

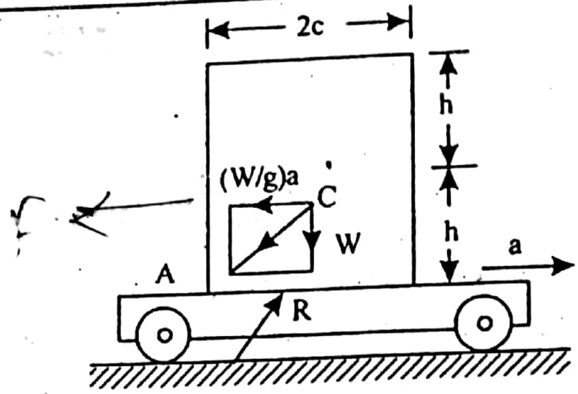
$$\Rightarrow 267 - 178 = a\,(9.072 + 27.21)$$

$$\Rightarrow a = 2.45 \text{ m/sec}^2 \qquad \textbf{(Ans.)}$$

2. *A block of weight W, height 2h and width 2C rests on a flat car which moves horizontally with constant acceleration a. Determine the*

(a) *the value of the acceleration a at which slipping of the block on the car will impend if the coefficient of friction is 'μ'.*

(b) *the value of the acceleration at which tipping of the block about the edge A will impend, assuming sufficient friction to prevent slipping.*




**Soln. Case - I**

Let $a_1$ be the acceleration of the block on the car

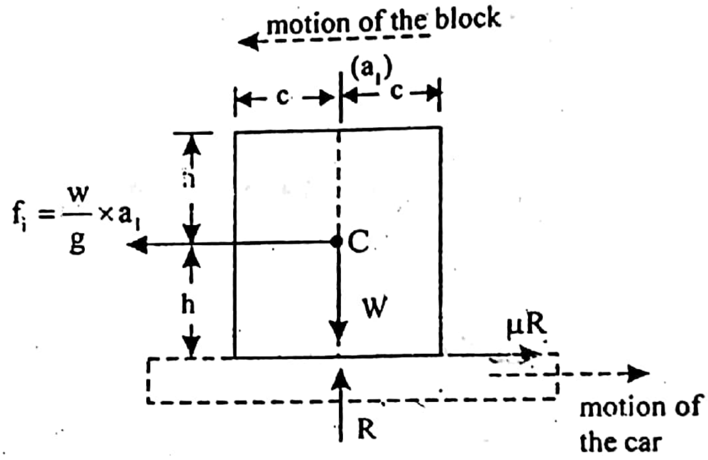
When the block tends to slip on the car. Then the conditions of equillibrium can be applied. i.e.,

$\Sigma y = 0$ , $R = W$

Similarly; $\Sigma x = 0$

$$\frac{W}{g} \times a_1 = \mu R \quad \text{or} \quad \frac{W}{g} \times a_1 = \mu W$$

$$\Rightarrow \quad a_1 = \mu g \quad \text{........(i)} \qquad \textbf{(Ans.)}$$



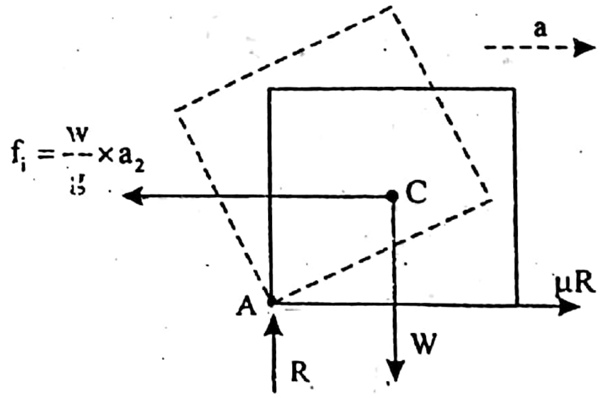

**Case - II**

When the block tends to tip at the point 'A'. The normal reaction will act at point 'A' only. Let $a_2$ be the acceleration of the block due to tipping. Using - D'Alembert's principle and taking moment about A.

$\Sigma M_A = 0$

$$\frac{W}{g} \times a_2 \times h = W \times c$$

$$\Rightarrow a_2 = \frac{cg}{h} \qquad \textbf{(Ans.)}$$



3. *Assuming the car as shown in the earlier Fig. has a velocity of 6 m/sec. find the shortest distance S in which it can be stopped with constant deceleration without disturbing the block. The following data are given:* $c = 0.6$m $h = 0.9$m, $\mu = 0.5$

**Soln.** Given data $V = 6$m/sec $c = 0.6$m $h = 0.9$m, $\mu = 0.5$ $u = 0$

**Case - 1**

When the block tends to slip on the car, the acceleration of the block

$$a_1 = \mu g = 0.5 \times 9.81 = 4.905 \text{ m/sec}^2$$

**Case - 2**

When the block tends to tip at 'A', then the acceleration of the block,

$$a_2 = \frac{Cg}{h} = \frac{0.6 \times 9.81}{0.9} = 6.54 \text{ m/sec}^2$$

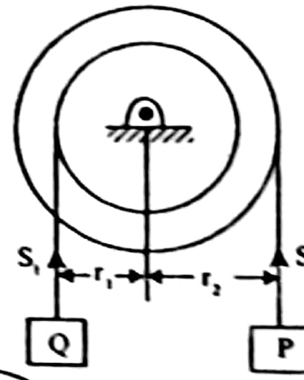
Since $a_1$ is less than $a_2$ the safest acceleration for the block is $a_1 = 4.905$ m/sec$^2$

$\therefore$ The shortest and safest distance covered by the car as follows : $V^2 - u^2 = 2a_1 S$

Where, u = initial velocity = 0

$$\Rightarrow S = \frac{V^2}{2a_1} = \frac{(6)^2}{2 \times 4.905} \qquad \Rightarrow S = 3.67 \text{ m} \qquad \text{(Ans.)}$$

4. *Neglecting friction and the inertia of the two - step pulley shown, find the acceleration 'a' of the falling weight P. Assume P = 35.6 N, Q = 53.4 N and $r_2 = 2r_1$*




**Soln.** Given data

$P = 35.6$ N, $Q = 53.4$ N, $r_2 = 2r_1$

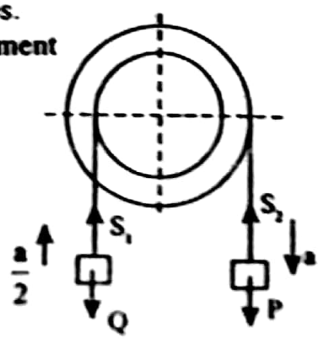
Assume $S_1$ and $S_2$ be the tensions in the left and right ropes.

Let the angular displacement of pulley block is $d\theta$

$AA' = r_2 d\theta$

$BB' = r_1 d\theta$

$$\therefore d\theta = \frac{AA'}{r_2} = \frac{BB'}{r_1}$$



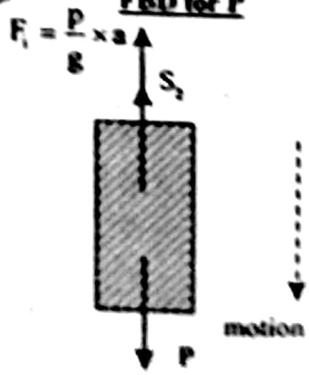


or $$BB' = \frac{AA'}{2}$$

$\therefore$ If the load 'P' is displaced to a distance 'x' then the load 'Q' will be displaced at a distance of (x/2).

Therefore, if 'a' = acceleration of 'P',

**FBD for P**



then '(a/2)' is the acceleration of 'Q'.

Considering the equillibrium of 'P'.

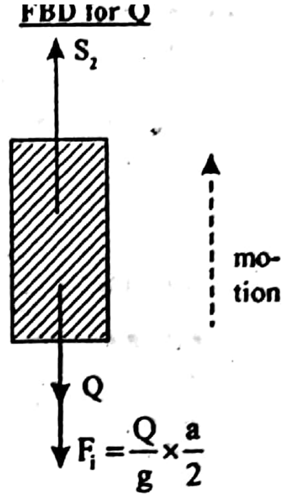
$\Sigma y = 0$

$$\frac{P}{g} \times a + S_2 = P \Rightarrow S_2 = P\left(1 - \frac{a}{g}\right) \quad \text{.....(i)}$$

Considering equillibrium of 'Q'

$\Sigma y = 0$

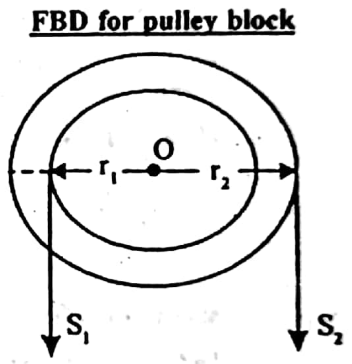
$$S_1 = Q + \frac{Q}{g} \times \frac{a}{2} \quad \Rightarrow S_1 = Q\left(1 + \frac{a}{2g}\right) \quad \text{......(ii)}$$




Considering equillibrium of pulley block,

$$\Sigma M_0 = 0 \qquad S_1 \times r_1 = S_2 \times r_2 \quad \text{......(iii)}$$

$$\text{or} \quad S_1 \times r_1 = S_2 \times 2r_1 \Rightarrow S_1 = 2S_2 \quad \text{......(iv)}$$




Substituting the values of $S_1$ and $S_2$

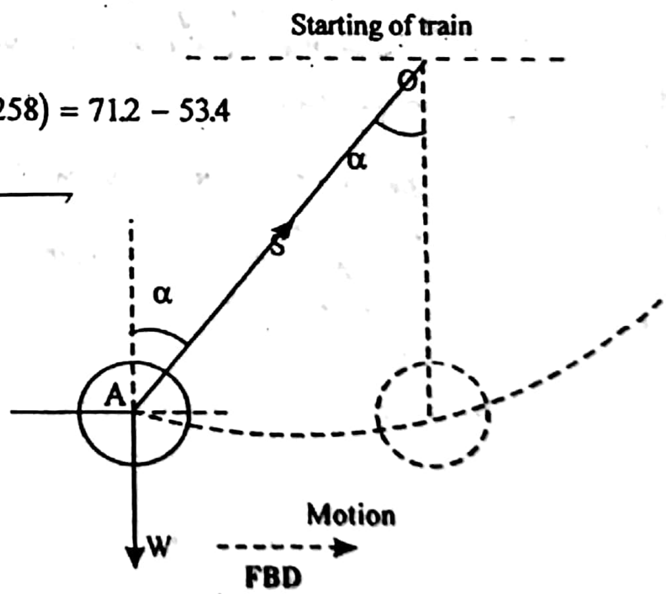
$$\Rightarrow Q\left(1 + \frac{a}{2g}\right) = 2\left[P\left(1 - \frac{a}{g}\right)\right]$$

$$\Rightarrow 53.4\left(1 + \frac{a}{2 \times 9.81}\right) = 2\left[35.6\left(1 - \frac{a}{9.81}\right)\right]$$

$$\Rightarrow 53.4 + 2.721\,a = 71.2 - 7.258a \Rightarrow a(2.721 + 7.258) = 71.2 - 53.4$$

$$\Rightarrow 9.979a = 17.8 \Rightarrow a = 1.78 \text{ m/sec}^2$$

5. *A mathematical pendulum hanging from, the ceiling of a railway car inclines to the vertical by an angle α during starting of the train ? What is the corresponding acceleration of the train ?*




**Soln.** S = tension in the rope 'OA'

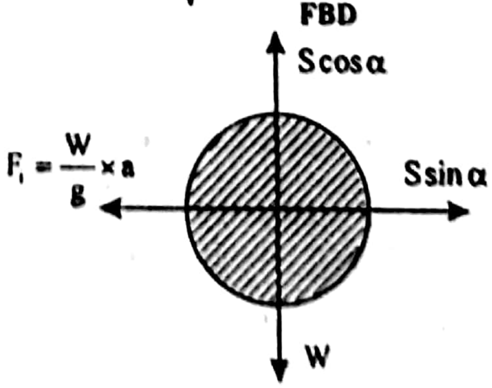
Using D'Alembert's principle and applying conditions of equillibrium

$$\Sigma y = 0 \qquad S\cos\alpha = W \quad \text{.....(i)}$$

$$\Sigma x = 0 \qquad S\sin\alpha = \frac{W}{g} \times a \quad \text{.....(ii)}$$

dividing (ii) ÷ (i)

$$\tan\alpha = \frac{a}{g} \Rightarrow a = g\tan\alpha \qquad \textbf{(Ans.)}$$

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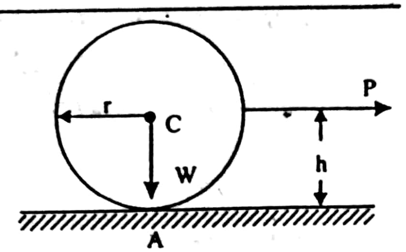
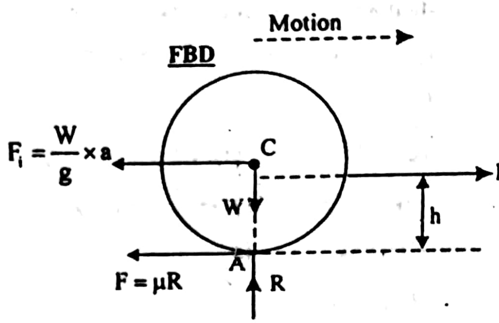
7. *A homogeneous sphere of radius r and weight W slides along the floor under the action of a constant horizontal force p applied to a string, as shown. Determine the height h during this motion if the coefficient of friction between sphere and floor is μ.*




**Soln.** The body is equillibrium under the actions of 'W', normal reaction 'R' at 'A', force of friction μR at 'A' inertia force at 'c' and the applied force 'P'.

**FBD**




Using D'Alembert's principle and applying conditions of equillibrium

$\Sigma y = 0;\ R = W$ .....(i)

$\Sigma x = 0;\ \frac{W}{g} \times a + \mu R = P$

or $\frac{W}{g} \times a + \mu W = P$ .....(ii)

Taking moment about 'A'

$\Sigma M_A = 0;\quad \frac{W}{g} \times a \times r = P \times h$ .....(iii)

Substituting the value of P in equation (iii)

$$\frac{W}{g} \times a \times r = \left(\frac{w}{g} \times a + \mu W\right) h \quad \text{or} \quad h = \frac{ar}{g} \times \frac{g}{a + \mu g} = \frac{ar}{(a + \mu g)}$$

**Alternatively**

From equation (iii)

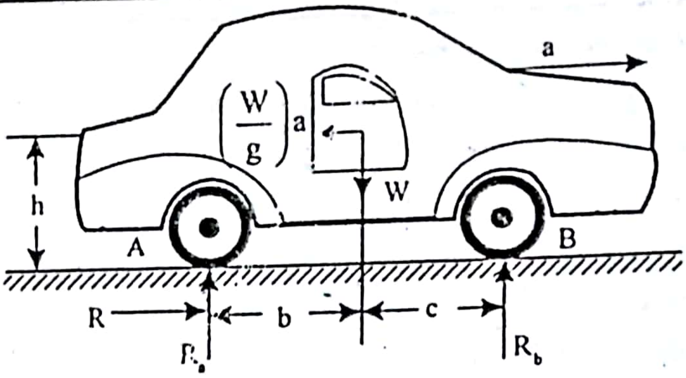
$a = \frac{Phg}{Wr}$ ....(iv)

Substituting the value of a in equation (ii)

$$\frac{W}{g} \times \frac{Phg}{Wr} + \mu W = P \ \Rightarrow \ \frac{Ph}{r} + \mu W = P \ \Rightarrow \ \frac{Ph}{r} = P - \mu W$$

$$\Rightarrow h = \frac{r(P - \mu W)}{P} = r\left[1 - \frac{\mu W}{P}\right]$$ (Ans.)

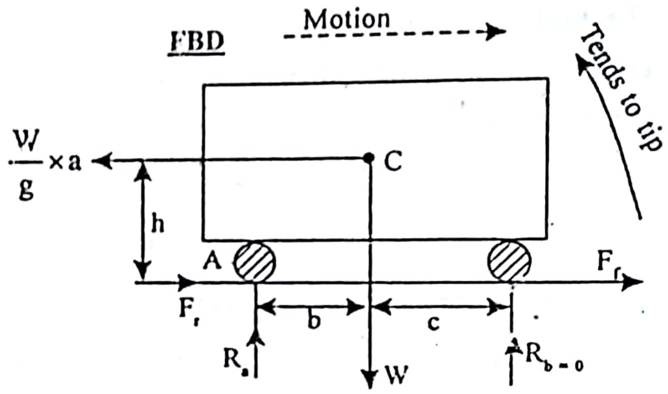
8. *Assuming that a car of dimensions shown, has sufficient power and that there is sufficient friction, find the maximum accn that it would be able to develop without tipping over backward.*




**Soln.** When the car is about to tip backward at point 'A'.

$\Sigma M_A = 0$

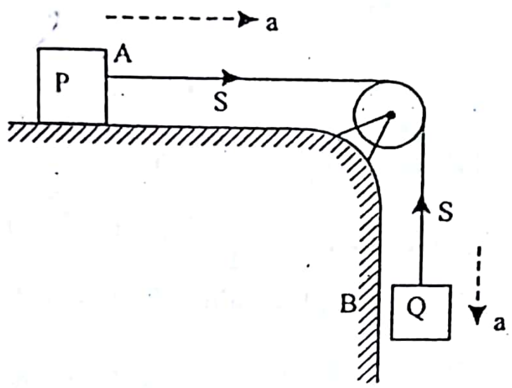
Using D'Alembert's principle and applying conditions of equilibrium;




$\Sigma M_A = 0$

$\frac{W}{g} \times a \times h = W \times b \quad \Rightarrow a = \frac{bg}{h}$

9. *Two blocks of weights P and Q are connect by a flexible but inextensible cord and supported as shown. If the coefficient of friction between the block P and the horizontal surface is 1/3 and all other frictions are negligible, find (a) the accn of the system and (b) the tension force S in the cord. The following numerical data are given : P = 53.4 N, Q = 26.7 N.*



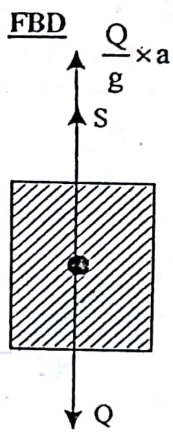

**Soln.** Given data

P = 53.4 N, Q = 26.7 N, μ = 1/3

Let 'S' be the tension in the cord 'AB'

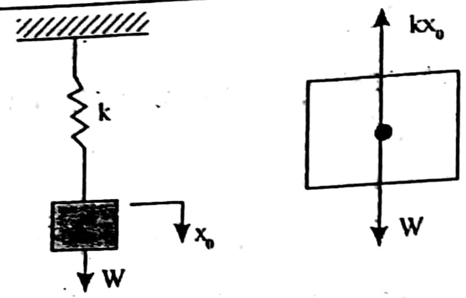
Using D'Alembert's principle and applying conditions of equillibrium to 'Q'

$\Sigma y = 0$




$$\frac{Q}{g} \times a + S = Q \qquad \text{....(i)}$$

6. *A spring-suspended mass hangs from the ceiling of an elevator cage. How will its natural period of free vertical vibration be affected by acceleration of the cage ?*

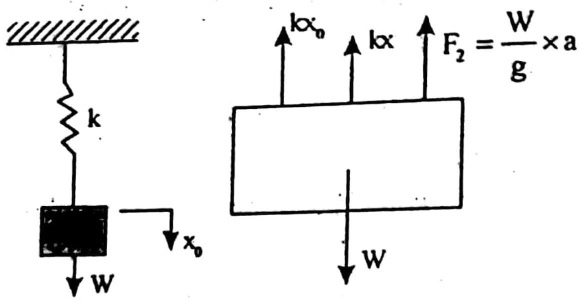



**Soln.** **Case - 1**

When the body is with initial displacement $x_0$ then the body remain equillibrium under the actions of spring force and weight of the body.

i.e., $\Sigma y = 0$ $\quad kx_0 = W$ ......(i)

where k = spring constant in N/m.




Case - 2

When body is displaced to a distance of 'x' from its initial position with an acceleration 'a'. Then using D'Alembert's principle and applying conditions of equilibrium, the equation becomes $\Sigma y = 0$

$$kx_0 + kx + \frac{W}{g} \times a = W \quad \text{....(ii)}$$

The equation (ii) now reduces to

$$\frac{W}{g} \times a + kx = 0 \quad \text{or} \quad \frac{W}{kg} \times a + x = 0 \quad \Rightarrow \quad \frac{W}{kg}\ddot{x} + x = 0$$

This represents the equation of vibration in the spring mass system.

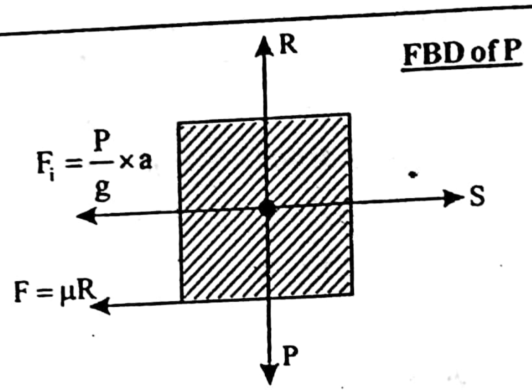
The above equation also can be written in the form of $(\omega_n)^2\ \ddot{x} + x = 0$

where $\omega_n = \sqrt{\frac{W}{kg}}$ is the natural circular frequency of oscillation

Time period $t = 2\pi\omega_n$ or $t = 2\pi\sqrt{\frac{W}{Kg}}$

Thus the time period will never be affected by the acceleration of the spring mass system (cage), rather depends upon weight and spring constant.

Applying conditions of equilibrium for 'P'




$\Sigma y = 0$

$R = P$ ......(ii)

$\Sigma x = 0 \quad \frac{P}{g} \times a + \mu R = S$

or $\frac{P}{g} a + \mu P = S$ ....(iii)

Solving (i) and (iii) $\quad \frac{P}{g} \times a + \mu P = Q - \frac{Q}{g} \times a$

or $\frac{53.4}{9.81} \times a + \frac{1}{3} \times 53.4 = 26.7 - \frac{26.7}{9.81} \times a$

$\Rightarrow 5.44\, a + 17.8 = 26.7 - 2.73\, a \quad \Rightarrow (5.44 + 2.7)a = 26.7 - 17.8$

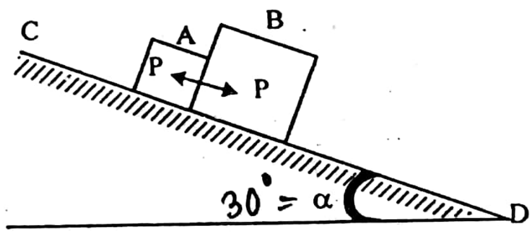
or $a = 1.089\ m/sec^2$ **(Ans.)**

Substituting the value of 'a' in equation (i)

$\frac{Q}{g} \times a + S = Q \quad \Rightarrow S = Q - \frac{Q}{g} \times a$

$\Rightarrow S = 26.7 - \frac{26.7}{9.81} \times 1.089 \quad \Rightarrow S = 23.73\ N$ **(Ans.)**

**10.** *Two blocks A and B under the action of gravity slide down the inclined plane CD that makes with the horizontal the angle $\alpha = 30°$. If the weights of the blocks are $W_a = 44.5N$ and $W_b = 89\ N$ and the coefficients of friction between them and the inclined plane are $\mu_a = 0.15$ and $\mu_b = 0.30$ find the pressure P existing between the blocks during the motion.*

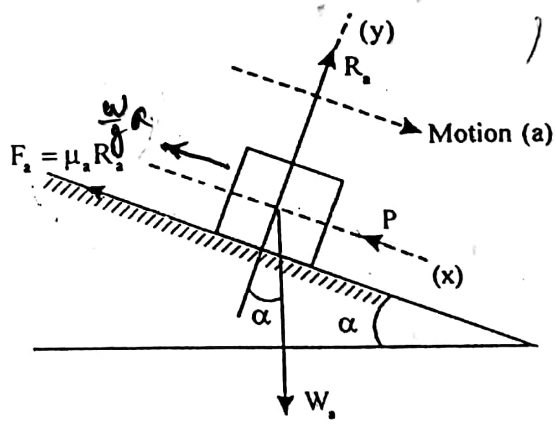



**Soln.** Given data

$W_a = 44.5\ N, \quad W_b = 89\ N, \quad \mu_a = 0.15,$

$\mu_b = 0.30, \quad \alpha = 30°$

**FBD for A**




Considering the block 'A' is equillibrium under actions of $W_a$, $F_a$, $F_i$, $R_a$ and P.

Where 'P' is the common normal forces between the two bodies.

Applying D'Alembert's principle at A

$\Sigma y = 0$

$R_a = W_a \cos\alpha = 44.5 \times \cos 30°$

or $R_a = 38538$ N ...(i)

$\Sigma x = 0$

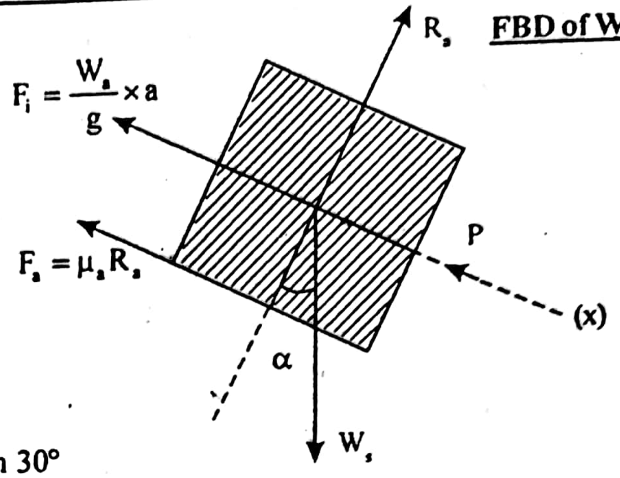
$\frac{W_a}{g} \times a + \mu_a R_a + P = W_a \sin\alpha$

or $\frac{44.5}{9.81} \times a + 0.15 \times 38.538 + P = 44.5 \times \sin 30°$

$\Rightarrow 4.536\,a + P = 22.25 - 5.78$

$\Rightarrow 4.536a + p = 16.469$ .....(ii)

FBD of $W_a$




Considering equillibrium of 'B' and applying D'Alembert's principle

$\Sigma y = 0$

$R_b = W_b \cos\alpha$ .....(iii)

$= 89 \times \cos 30° \quad = 77.076$ N

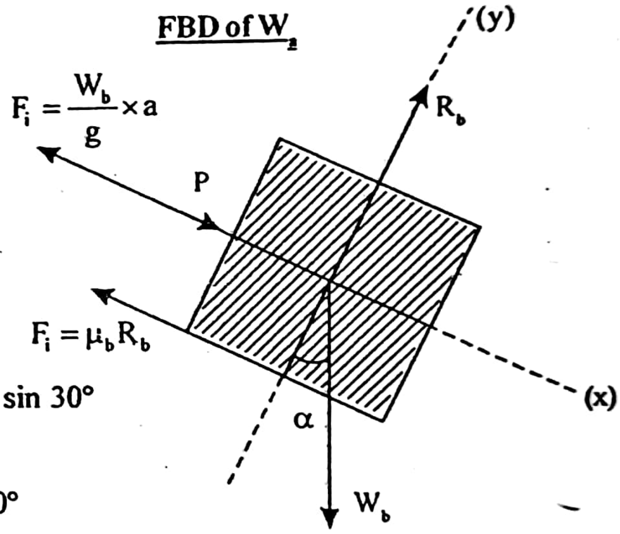
$\Sigma x = 0 = \frac{W_b}{g} \times a + (0.30 \times 77.076) = P + 89 \sin 30°$

$\Rightarrow \frac{89}{9.81} \times a + (0.30 \times 77.076) = P + 89 \sin 30°$

$\Rightarrow 9.072\,a + 23.122 = P + 44.5$

$\Rightarrow 9.072a - p = 21.378$ .....(iv)

FBD of $W_b$

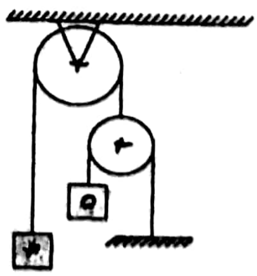



Solving (ii) and (iv)

$\frac{16.469 - P}{4.536} = \frac{21.378 + P}{9.072} \Rightarrow 149.4 - 9.072\,P = 96.97 + 4.536\,P$

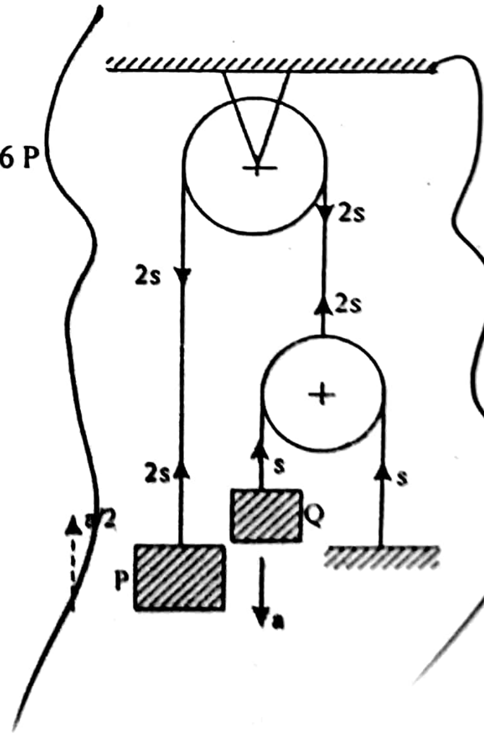
$\Rightarrow P(13.608) = 52.43$ or $P = 3.85$ N (Ans.)

1. *Neglecting friction & the inertia of the two pulleys as shown in Fig- find the acceleration of the weight Q, assuming that P=Q.*




**Soln.** Given data, $P = Q$

Let S be he tension in the rope attached to 'Q'.

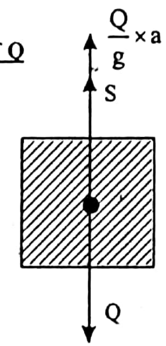


Considering 'Q' as equillibrium body and by using D'Alembert's principle,

$\Sigma y = 0$

$$\frac{Q}{g} \times a + S = Q \Rightarrow S = Q\left(a - \frac{a}{g}\right) \quad \text{....(i)}$$

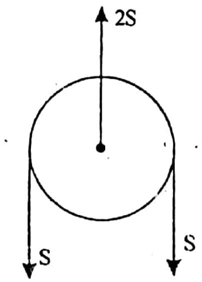
**FBD of Q**




Since the load P is supported by '2s' then the acceleration of 'P' will be (a/2)

Applying conditions of equillibrium and D'Alembert's principle, $\Sigma y = 0$

**FBD for pulley**




$$\frac{P}{g} \times \frac{a}{2} + P = 2S \Rightarrow 2S = P\left(1 + \frac{a}{2g}\right)$$

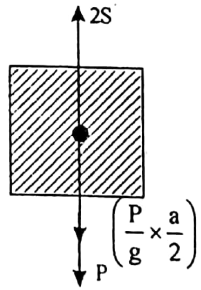
$$\text{or} \quad S = \frac{P}{2}\left(1 + \frac{a}{2g}\right) \quad \text{.....(ii)}$$

Equating (i) and (ii)

$$Q\left(1 - \frac{a}{g}\right) = \frac{P}{2}\left(1 + \frac{a}{2g}\right)$$

But given that P = Q

**FBD of P**

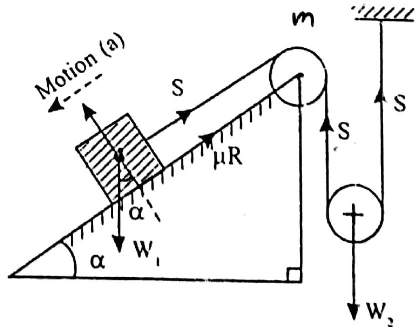



$$\therefore \left(1 - \frac{a}{g}\right)2 = \left(1 + \frac{a}{2g}\right) \Rightarrow \left(\frac{g-a}{g}\right)2 = \left(\frac{2g+a}{2g}\right)$$

$$\Rightarrow 4g - 4a = 2g + a \Rightarrow 2g = 5a \Rightarrow a = \frac{2}{5}g \quad \text{(Ans.)}$$

$$\Rightarrow a = 3.924 \text{ m/sec}^2$$

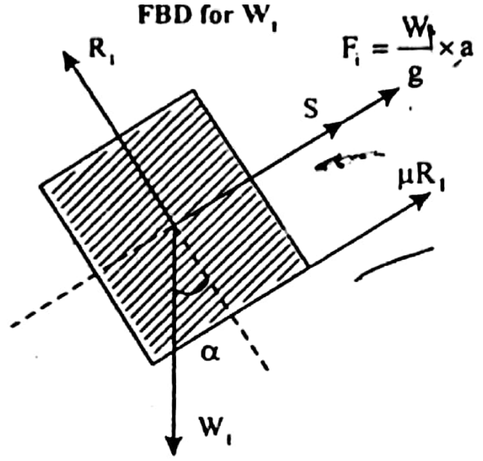
12. *Find the tension S in the string during motion of the system as shown in Fig. if $W_1 = 890$ N, $W_2 = 445$ N. The system is in a vertical plane, and the coefficient of friction between the inclined plane and the block $W_1$ is $\mu = 0.2$. Assume the pulleys to be without mass and $\alpha = 45°$.*



**Soln.** Given data

$W_1 = 890$ N, $W_2 = 445$ N, $\mu = 0.2$, $\alpha = 45°$

Considering equillibrium of the body $W_1$ and applying D'Alembert's principle ;




$\Sigma y = 0;\quad R_1 = W_1 \cos\alpha$ ....(i)

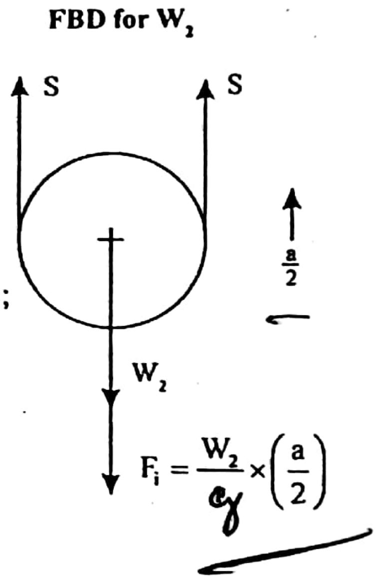
$= 890 \times \cos 45° = 629.325$ N

$\Sigma x = 0;\quad S + \frac{W_1}{g} \times a + \mu R_1 = W_1 \sin\alpha$

$\Rightarrow S + \frac{890}{9.81} \times a + 0.2\,(629.325) = 890 \sin 45°$

$\Rightarrow S + 90.72\ a + 125.865 = 629.325$

$\Rightarrow S + 90.72a = 503.46$ ....(ii)




$\Rightarrow$ Considering the equilibrium and applying D'Almbert's principle;

$\Sigma y = 0$

$2S = W_2 + \frac{W_2}{g} \times \frac{a}{2} \Rightarrow 2S = 445 + \frac{445}{9.81} \times \frac{a}{2}$

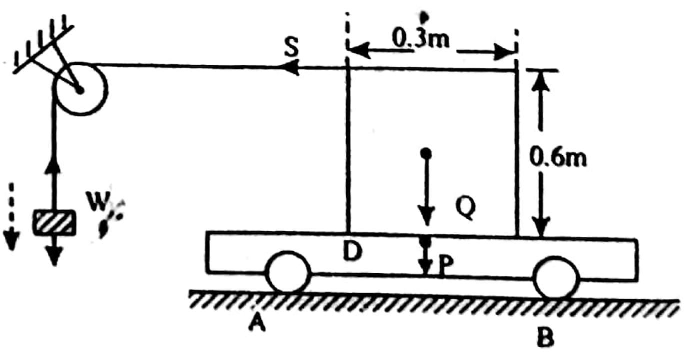
$\Rightarrow 2S = 445 + 22.68\ a$ ....(iii)

Solving (ii) and (iii)

$\frac{503.46 - S}{90.72} = \frac{2S - 445}{22.68} \Rightarrow 11418.47 - 22.68S = 181.44S - 40370.4$

$\Rightarrow 51788.87 = 204.12S \Rightarrow S = 253.71$ N (Ans.)

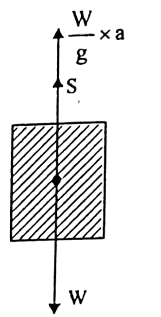
**13.** *A rectangular block of weight $Q = 890$ N rests on a flat car of weight $P = 445$ N which may roll along the horizontal plane AB without friction. The car and block together are to be accelerated by the weight W arranged as shown. Assuming that there is sufficient friction between the block and the car to prevent sliding find the maximum value of weight W by which the car can be accelerated. What will this acceleration be?*



**Soln.** **Given data** $Q = 890\ N$, $P = 445\ N$

Considering equillibrium of the body 'W' and applying D'Alembert's principle,

**FBD of W**



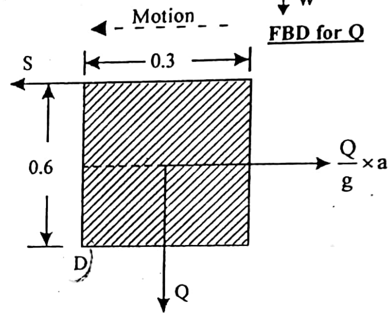

$$\Sigma y = 0 \quad S + \frac{W}{g} \times a = W$$

$$\Rightarrow S = W\left(1 - \frac{a}{g}\right) \qquad .....(i)$$

Since there is sufficient friction between the block and car; the block never slides; but tends to tip at 'D'.

i.e. $\Sigma M_D = 0$

**FBD for Q**

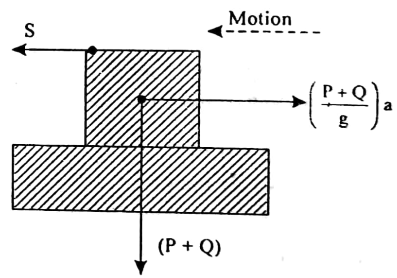



$$S \times 0.6 = Q \times \frac{0.3}{2} + \frac{Q}{g} \times a \times 0.3$$

$$\Rightarrow S = \frac{Q(0.15 + 0.030a)}{0.6} \qquad ....(ii)$$

Considering P & Q as single body and applying conditions of equillibrium

$\Sigma x = 0$




$$\Rightarrow S = \left(\frac{P + Q}{g}\right)a \qquad \Rightarrow S = \left(\frac{445 + 890}{9.81}\right)a$$

$$\Rightarrow S = 136.08\ a \qquad .....(iii)$$

Substituting the value of 'S' in equation (ii)

$$136.08\ a = \frac{890(0.15 + 0.03a)}{0.6}$$

$$\Rightarrow 81.648\ a = 133.5 + 26.7a$$

$$\Rightarrow a = \frac{133.5}{54.948} = 2.42\ m/sec^2 \qquad \textbf{(Ans.)}$$

Substituting ; $\therefore S = 136.08 \times 2.42 \Rightarrow S = 329.327$

Substituting the 'a' and 'S' in equation (i)

$$S = W\left(1 - \frac{a}{g}\right) \quad \text{or} \quad 329.327 = W\left(1 - \frac{2.42}{9.81}\right)$$

$$\Rightarrow W = \frac{329.327}{0.753} = 437.17, \quad W_{max} = 437.17\ N \qquad \textbf{(Ans.)}$$

Chapter - 7.8

# Momentum and Impulse

## 7.8.1 INTRODUCTION

Out of various methods, the principle of impulse and momentum is another method for solving problems relating dynamics and is derived from Newton's second law.

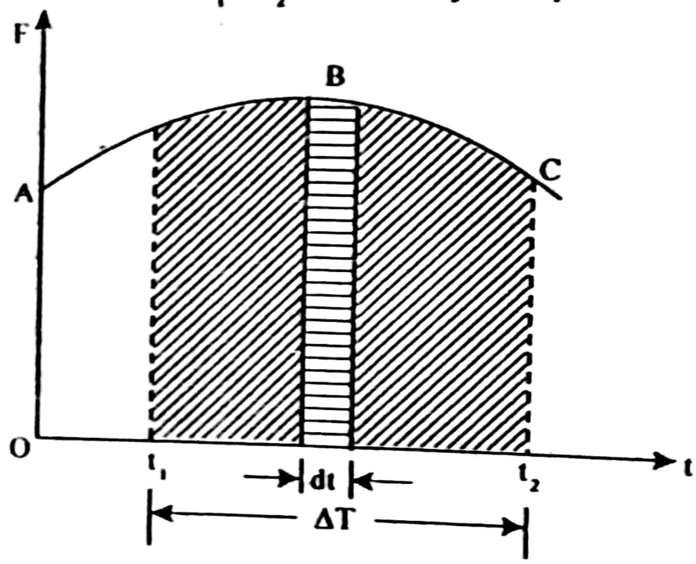
This principle relates force, mass, velocity and time and is particularly used when large forces act for a very small time.

**Impulse of a force**

When a large force acts over a short period of time, that force is called impulsive force or simply impulse.

. The impulse of a force 'F' acting over a time interval for $t_1$ to $t_2$ is defined by the equation;

$$I = \int_{t_1}^{t_2} F\,dt \quad \text{...(i)}$$




The impulse of a force can be shown in force time diagram.

It is the area under F - t diagram as shown in the figure.

When the variation of the force with respect to the time is unknown, the impulse can be measured as

$$I = F_{average} \times \Delta T$$

Unit : Newton - second (Ns)

*Note :* Impulse of a force is a vector quantity and its direction is the direction of applied force.

## 7.8.2 MOMENTUM

When a body of mass 'm' moves with velocity 'V', the total motion possessed by that body is called momentum. Mathematically, momentum = m × v

Let a body of mass m is accelerated rectilinearly under the action of a force F.

. The equation of motion in that direction is, $F = ma$

$$\text{or} \quad F = m\,.\frac{dv}{dt} \quad \text{or} \quad F = \frac{d}{dt}(mv) \quad \text{...(i)}$$

Which states that the force "F" acting on the body is equal to the rate of change of momentum.

### 7.8.3 Principle of Impulse and Momentum

The total change in the momentum of a body during a time interval is equal to the impulse of the acting force during the same interval of time.

From the equation (i) we can write

$$F.dt = d(mv) \qquad \text{...... (ii)}$$

Where Fdt is the impulse of the acting force 'F' and d(mv) be the differential change in the momentum of the body in the direction of motion.

Integrating equation (ii), we get

$$\int_{t_1}^{t_2} Fdt = \int_{v_1}^{v_2} d(mv) \quad \text{or} \quad mv_2 - mv_1 = \int_{t_1}^{t_2} F.dt \qquad \text{.....(iii)}$$

The equation (iii) represents a relation between vector quantities for a single particle or body.

### 7.8.4 System of Particles or Bodies

The principle of impulse and momentum is also useful when a problem involves the motion of several particles i.e., each particle is to be considered separately and then add vectorially the moments and impulses.

Mathematically, $\Sigma mv_2 - \Sigma mv_1 = \Sigma \int_{t_1}^{t_2} Fdt$

If $t_1 = 0$, $t_2 = t$, then the above equation can be written as

$$\Sigma mv_2 - \Sigma mv_1 = \Sigma \int_0^t Fdt$$

### 7.8.5 Conservation of Momentum

When sum of the impulses due to external forces is zero, the momentum of the system remains conserved.

i.e., when $\Sigma \int_{t_1}^{t_2} Fdt = 0$ then $\Sigma mv_2 = \Sigma mv_1$

$\therefore$ Final momentum of the system = Initial momentum of the system.

***Example -1***

A car of weight W rolls without resistance along a horizontal track. Initially the car together with a man of wiehgt w, is moving to right with a speed V as shown in the figure. Calculate the increase in the velocity of the car if the man runs with a speed of v relative to the floor and jumps off at the left.

**Soln.** Weight of the car = W

Weight of the man = w

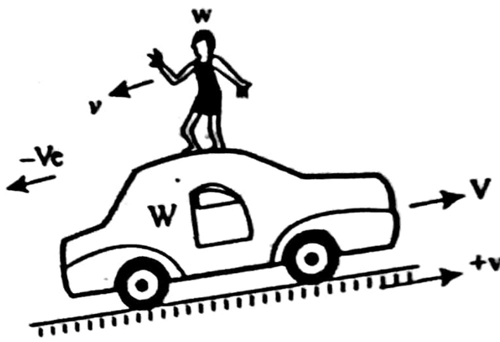
Initial velocity of car together with man to the right = V

Velocity of man to the left = u

Now, the initial momentum of the system (car + man) = $\frac{(W+w)}{g} \times V$

When the man runs with a velocity 'v'

Let $\Delta V$ be the increase in the velocity of the car when the man runs and then jumps off.




$\therefore$ Final velocity of the car $= V + \Delta V$

$\therefore$ Final momentum of the car $= \frac{W}{g}(V + \Delta V)$

Now, absolute velocity of man with respect to the car

= (Velocity of man + velocity of car) $= -v + (V + \Delta V)$

$\therefore$ Final momentum of the man $= \frac{w}{g}[-v + (V + \Delta V)]$

According to the conservation of momentum

$$\left(\frac{W+w}{g}\right) \times V = \frac{W}{g}(V + \Delta V) + \frac{w}{g}[-v + (V + \Delta V)]$$

or $$WV + wV = WV + W.\Delta V - wv + wV + w\Delta V$$

or $$(W + w)\,\Delta V = wv \qquad \text{or} \qquad \Delta V = \frac{wv}{(W+w)}$$

In terms of mass $$\Delta V = \frac{mv}{(M+m)}$$

From the above example it is clearly understood that the increased velocity $\Delta V$ of the car is independent of the initial velocity of the car V.

***Example -2***

Assuming that the car is initially at rest and holds 'n' men, each of weight 'w'

(i) If the men together run and jump off simultaneously, then increase in velocity of the car will be,

$$\Delta V = \frac{nwv}{(W + nw)}$$

(ii) If the car has already an initial velocity V, then total velocity of the car will be

$$V + \Delta V = V + \frac{nwv}{(W + nw)}$$

(iii) If each man runs and jumps in succession (one after one) then increase in velocity of the car will be,

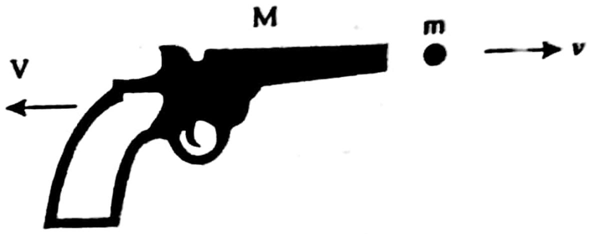
$$\Delta V = \sum_{i=n}^{i=1} \frac{wv}{(W+iw)} = \frac{wv}{(W+nw)} + \ldots + \frac{wv}{(W+3w)} + \frac{wv}{(W+2w)} + \frac{wv}{(W+w)}$$

(iv) If the weight of men are different, run and jump in succession, then increase in velocity of the said car will be

$$\Delta V = \frac{w_1 v}{[W+w_1+w_2+w_3+\ldots\ldots]} + \frac{w_2 v}{[W+w_2+w_3+w_4+\ldots\ldots]} + \frac{w_3 v}{[W+w_3+w_4+w_5+\ldots\ldots]} + \ldots\ldots$$

### 7.8.6 Recoil of Gun

Let a bullet of mass 'm' is fired from a gun of mass M, with a velocity 'v'. According to Newton's third law, the gun recoils back with a velocity, let 'v'. Then according to the conservation of momentum,

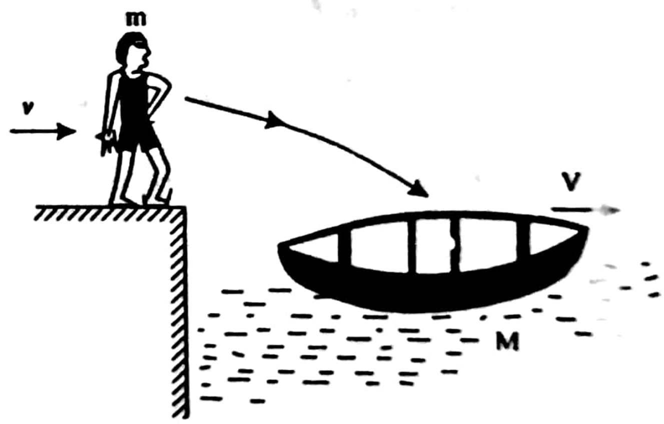



$$mv = MV \qquad \text{or} \qquad V = \frac{mv}{M}$$

### 7.8.7 Motion of a Boat

Consider a man of mass m runs and jumps with a velocity 'v' into the boat of mass M, which is at rest. Then the boat together with the man moves to the right with a velocity 'V'.

Hence accoring to the conservation of momentum,




$$mv = (m+M)V$$

or $$V = \frac{mv}{(m+M)}$$

According to conservation of momentum :

$$W_1 V_0 = (W_1 + W_2)V \quad \Rightarrow V = \frac{W_1 V_0}{(W_1 + W_2)} \qquad .....(ii)$$

∴ The distance covered by the boat in t sec.

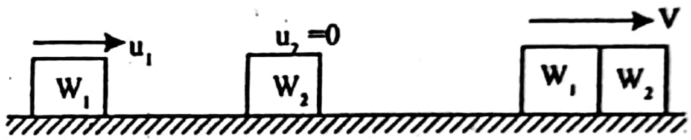
$$S = V.t \Rightarrow \qquad S = \frac{W_1\ V_0}{(W_1 + W_2)} \times t$$

Substituting the value of $V_0$ and other values $S = \frac{712 \times 2.4}{(712 + 890)} = 1.067$ m

∴ The position of the man from the pier (platform)

$= 4.5 + S - x = 4.5 + 1.067 - 2.4 = 3.167$ m **(Ans.)**

**3.** *A locomotive weighing 534 kN has a velocity of 16 kmph and backs into a freight car weighting 86 kN that is at rest on a level track. After coupling is made, with what velocity v will the entire system continue to move? Neglect all friction.*




**Soln.** Given data

$$W_1 = 534 \text{ KN}, \quad u_1 = 16 \text{ KMPh} = \frac{16 \times 1000}{3600} = 4.45 \text{ m/sec.}$$

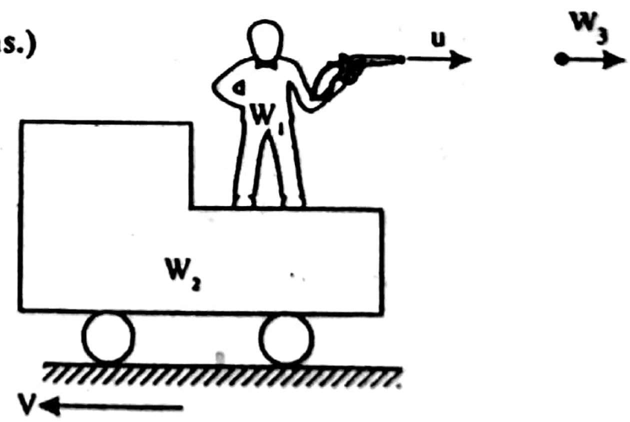
$W_2 = 86$ KN., $u_2 = 0$

Let V be the velocity of $W_1$ & $W_2$ after coupling is made.

According to conservation of momentum $W_1 u_1 + W_2 u_2 = (W_1 + W_2)v$

$$\Rightarrow \quad V = \frac{534 \times 4.45}{(534 + 86)} = 3.83 \text{ m/sec.} \qquad \textbf{(Ans.)}$$

**4.** *A 667.5 N man sits in a 333.75N canoe and fires a 0.277N rifle bullet horizontally directly over the bow of the canoe. Neglecting friction of the water, find the velocity V with which the canoe will move after the shot if the rifle has a muzzle velocity of 660 m/sec.*




**Soln.** Given data

Weight of the man $W_1 = 667.5$ N, Weight of the canoe $W_2 = 333.75$ N

u = velocity of nozzle = 660 m/sec., Weight of the bullet $W_3 = 0.277$ N

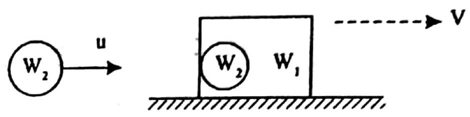
V = ?

According to the conservation of momentum $W_3 \times u = (W_1 + W_2)V$

where V is the velocity of recoil i.e., $V = \frac{W_3 \times u}{(W_1 + W_2)} = \frac{0.277 \times 660}{(667.5 + 333.75)}$

$\Rightarrow V = 0.182 \text{ m/sec.}$ **(Ans.)**

5. *A wood block weighing 22.25 kg rests on a smooth horizontal surface. A revolver bullet weighing 0.14N is shot horizontally into the side of the block. If the block attains a velocity of 3m/sec. what was the nozzle velocity V of the bullet ?*




**Soln.** Given data

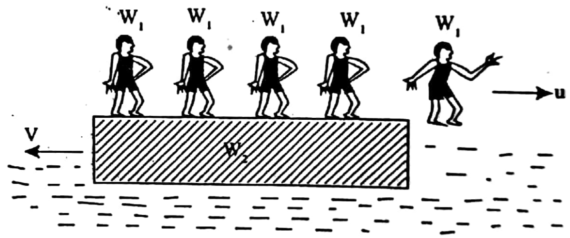
Weight of the wodden block = $W_1$ = 22.25 N, Weight of the bullet $W_2$ = 0.14 N

Velocity after impact, V = 3m/sec., Velocity of nozzle, u = ?

According to conservation of momentum $W_2 \times u = (W_1 + W_2)\, V$

$$\Rightarrow u = \frac{(W_1 + W_2)V}{W_2} \Rightarrow u = \frac{(22.25 + 0.14) \times 3}{0.14} \Rightarrow u = 479.78 \text{ m/sec.} \quad \textbf{(Ans.)}$$

6. *Five men lined up at one end of a floating raft, initially at rest, run in succession with velocity u = 3m/sec, relative to the raft and dive off at the far end. Neglecting resistance of the water to horizontal motion of the raft find its velocity after the last man dives.*




*Each man weighs 756.5 kg and the raft weighs 4450 N.*

**Soln.** Given data

Weight of each man = $W_1$ = 756.5 N, Weight of the raft = $W_2$ = 4450 N

Velocity of each man u = 3 m/sec., No. of men = 5

According to conservation of momentum

$$V = \frac{W_1 u}{W_2 + 5w_1} + \frac{W_1 u}{W_2 + 4w_1} + \frac{W_1 u}{W_2 + 3w_1} + \frac{W_1 u}{W_2 + 2w} + \frac{W_1 u}{W_2 + w_1}$$

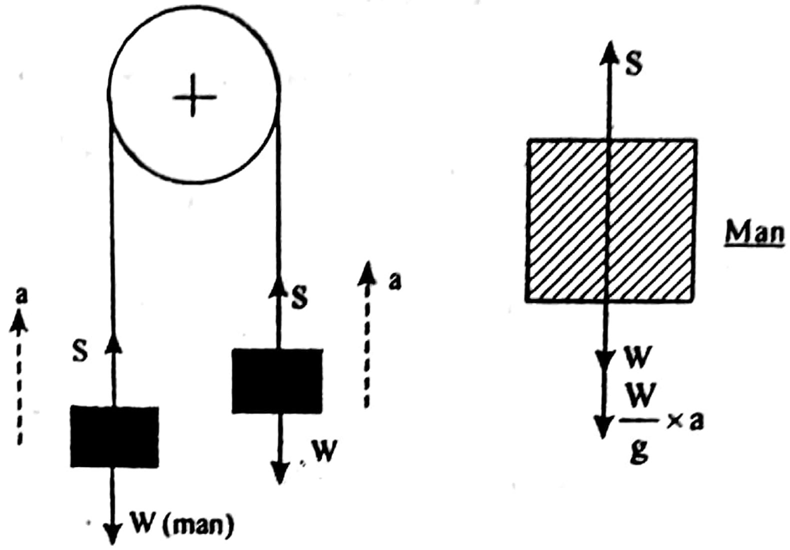
where V = Velocity of recoil of the raft.

$$V = W_1 u\left(\frac{1}{W_2 + 5w_1} + \frac{1}{W_2 + 4w_1} + \frac{1}{W_2 + 3w_1} + \frac{1}{W_2 + 2w_1} + \frac{1}{W_2 + w_1}\right)$$

$$= 756.5 \times 3\left[\frac{1}{4450 + 5(756.5)} + \frac{1}{4450 + 4(756.5)} + \frac{1}{4450 + 3(746.5)} + \frac{1}{4450 + 2(756.5)} + \frac{1}{4450 + (756.5)}\right]$$

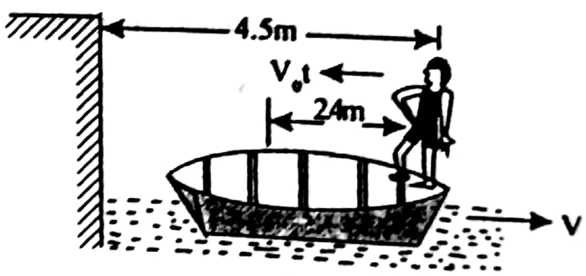
$= 1.733$ m/sec **(Ans.)**

1. *A man of weight W holds one end of a rope that passes over a frictionless pulley above his head and carries an inert weight W at its other end. Thus when the man puts his full weight on the rope, he will be just balanced by the weight at the other end. Discuss. what will happen if the man tries to climb upward along the rope.*




**Soln.** When a man climbs up with an acceleration 'a' then the dead weight 'W' hanged freely at the other end will move up with the same acceleration. Because the weight of the man is balanced by dead weight and tension in the rope in both side remain same.Hence the equations of dynamic equillibrium will hold good for both.

2. *A man weighing 712 N stands in a boat so that he is 4.5m from a pier on the shore. He walks 2.4m in the boat toward the pier and then stops. How far from the pier will he be at the end of this time? The boat weights 890N, and there is assumed to be no friction between it and the water.*




**Soln** Given data

Weight of the man $W_1 = 712$ N

Weight of the boat $W_2 = 890$ N

Let the man moves with an uniform velocity $V_0$ towards left in 't' sec. covering a distance 'x'.

$\therefore$ Distance travelled by the man on the boat

$$x = V_0 \times t \qquad .....(i)$$

or $\quad 2.4 = V_0 \times t \quad \Rightarrow V_0 = \dfrac{2.4}{t}$ m/sec.

Let V is the velocity of the boat along with man when it moves towards right.

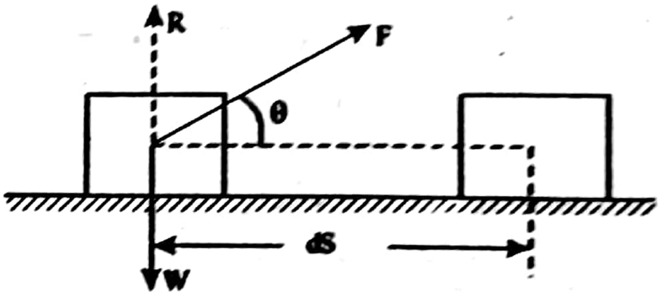
### 7.9.1 INTRODUCTION

Out of many methods principle of work and energy is also one of the method for solving problems involving the motion of a particle or motion of system of particles. It relates force, mass, velocity and displacement, but not acceleration of motion. Both work and energy are scalar quantities.

**Advantages**

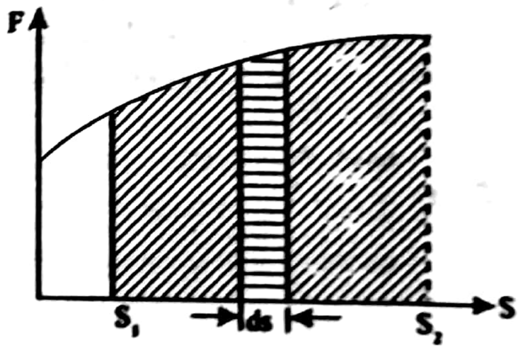
The advantages of this method is that we can directly determine the velocity of the particle without requiring to determine its acceleration.

### 7.9.2 WORK OF A FORCE




If a particle is displaced by an infinitesimal amount 'ds' under the action of a Force 'F' acting at angle 'θ' to the direction of displacement, then work is said to be done on it.

i.e., $\delta u = F \cos\theta . ds$




Hence it is the product of the displacement and the component of the force in the direction of the displacement.

The work done by a force during a finite displacement can be found out by integration.

$$U_{1-2} = \int_{1}^{2} \delta U = \int_{s_1}^{s_2} F \cos\theta . ds$$

**Unit :** Newton metre (Nm) or Joule.

**Note :**(i) Gravity force, W & normal reaction R have no work along the direction of displacement, which are perpendiculars.

(ii) If the displacement is zero, then work done by a force is also zero.

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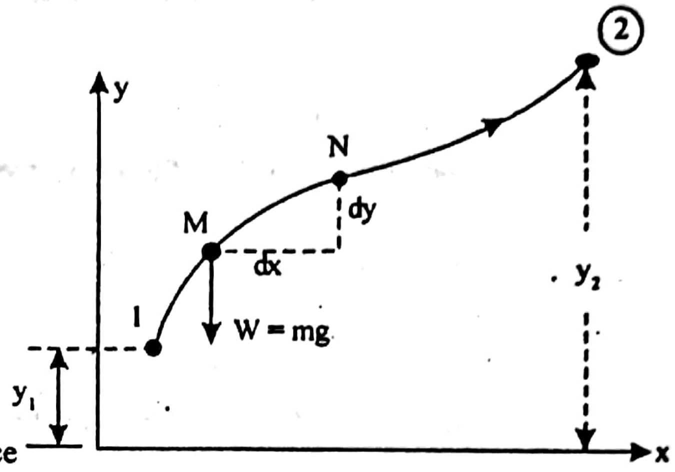
(iii) Work done by a force is positive if the direction of the force and the direction of displacement are the same. Otherwise negative.

(iv) Work done by a force depends on the path over which the force moves except conservative forces, like gravity force, spring force, elastic force etc.

**Work of various forces (conservatives)**

(i) **Work of Gravity force :** Let a particle of weight w moving along a path in x-y plane as shown in the figure.




Let this particle at any instant, at M be displaced vertically by dy to a new position N. $\therefore$ workdone $\delta u = -w\ dy$

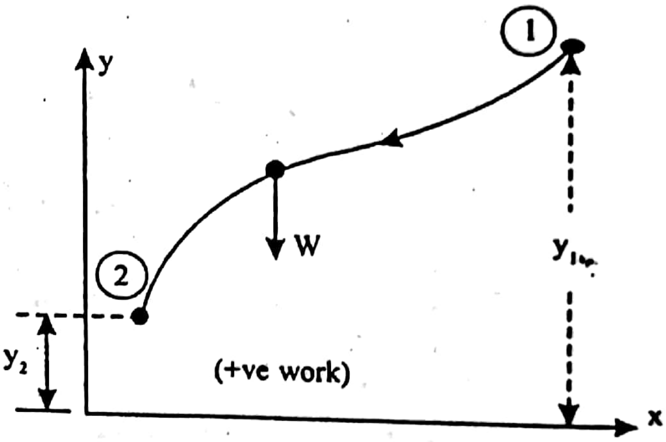
-ve sign, is due to work against the gravity force

Total work done from position (1) to (2)

$$U_{1-2} = \int_{y_1}^{y_2} -w\ dy = -mg\,(y_2 - y_1)$$

or Simply, $U_{1-2} = -mgy$

where $y = (y_2 - y_1)$




Work done by gravity force only depends elevation (vertical distance),never depends upon the path that the particle travels.

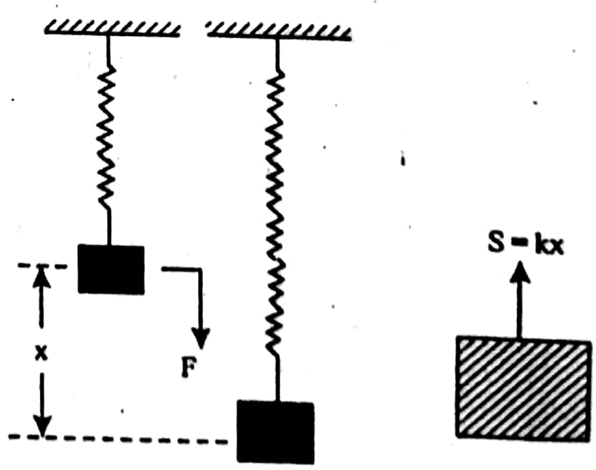
Work done by that gravity force is positive when the particle moves from a higher elevation to a lower elevation.

But it has no work in horizontal direction.

### 7.9.3 Spring Force

Let a spring of stiffness 'k' is streched to an amount 'x' from its initial undeformed position as shown in the figure.

Hence the force exerted by the spring (spring force as the tension) $F = S = kx$



Assume an infinite small displacement dx from any initial position (1) to final position (2)

$\therefore$ work done on the spring when streched;

$$\delta u = -F.dx \qquad .....(i)$$

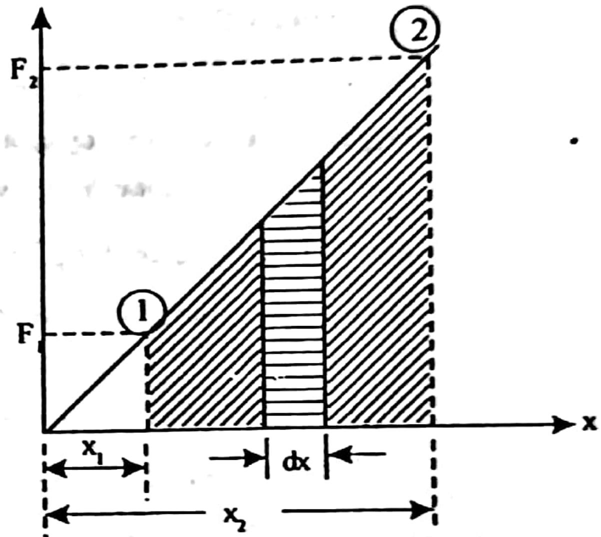
The force and displacement are opposite and hence work done is negative Hence total work done

$$U_{1-2} = -\int_{x_1}^{x_2} F dx \qquad ....(ii)$$

Substituting the value of F from equation (1) in equation (ii),we get

$$U_{1-2} = -\int_{x_1}^{x_2} kx.dx$$

$$= -\frac{1}{2}k\left(x_2^2 - x_1^2\right) \qquad ....(iii)$$




*Graphical representation of work done by a spring forces*

The equation (iii) can be rewritten as follows :

Let $F_1 = kx_1$ , $F_2 = kx_2$ .....(iv)

$$\therefore U_{1-2} = -\frac{1}{2}k(x_2 + x_1)(x_2 - x_1)$$

$$= -\frac{(F_1 + F_2)}{2} \times (x_2 - x_1) \quad ....(v)$$

$$\therefore U_{1-2} = -\frac{1}{2} \text{(average force) (displacement)}$$

## 7.9.4 ENERGY

It can be defined as the capacity to do work. The unit of work and energy are same (joule). There are various forms of energy, like mechanical energy, electrical energy, thermal energy, solar energy etc. Out of these potential energy and kinetic energy are one of the forms of mechanical energy.

**Kinetic Energy :** It is the energy possessed by a paritlce by virtue of its motion.

i.e., $$K.E. = \frac{1}{2}mv^2 = \frac{W}{2g}v^2$$

## 7.9.5 PRINCIPLE OF WORK AND ENERGY

It states that the differential change in kinetic energy of the moving particle is equal to the work done by the acting force on the corresponding infinitesimal displacement.

**Proof**

We know that equation of motion of a particle under the action of a constant force

$$F = \frac{w}{g} \times a = \frac{w}{g}\frac{dv}{dt}$$

$$= \frac{w}{g} \cdot \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{w}{g} v . \frac{dv}{dx} \quad \text{.....(i)}$$

When the particle moves an infinitesimal displacement 'dx' from an initial velocity '$V_1$' to final velocity '$V_2$' then the work done = F.dx

or $$F.dx = \frac{w}{g} v \frac{dv}{dx} .dx = \frac{W}{g} V\,dV \quad \text{.....(ii)}$$

Integrating equation (ii)

$$\int_{x_1}^{x_2} F.dx = \frac{w}{g} \int_{V_1}^{V_2} Vdv \quad \text{...(iv)}$$

or $$U_{1-2} = \frac{w}{g}\frac{v_2^2}{2} - \frac{w}{g}\frac{v_1^2}{2}$$

$$= \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2 \quad \text{...(v)}$$

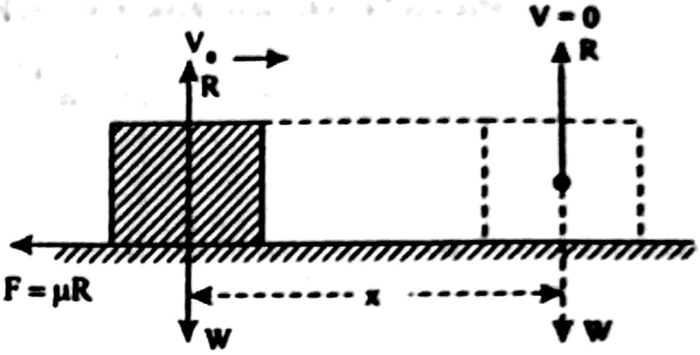
$$= (KE)_2 - (KE)_1$$

i.e., work done by a force acting on a particle is equal to change in K.E.

**Note:** When asystem of bodies are in motion, then add up the change in kinetic energy of all bodies and equate it to the work of all the forces involved during the displacement of the system of bodies.

i.e., $\sum U_{1-2} = \sum (KE_2 - KE_1)$

1. *A block of weight W is given an initial velocity $V_0$ along a rough horizontal plane and is brought to rest by friction in a distance x. Determine the coefficient of friction, assuming that it is independent of velocity.*




**Soln.** It is given that the body is moving with an initial uniform velocity '$V_0$' and comes to rest after travelling a distance 'x'.

∴ Final velocity of the body V = 0

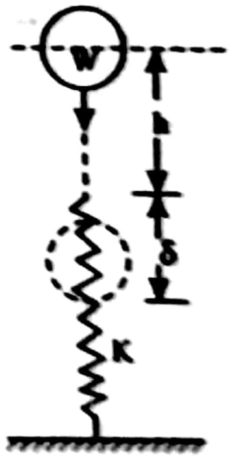
Since the body is equillibrium statically in vertical direction; R = W

Along the direction of x, the sum of energies = Σ Work Done
i.e., change in Kinetic energy = W. D.

$$\Rightarrow \frac{1}{2}\frac{W}{g}\left(V^2 - V_0^2\right) = (F \times x); \quad \text{where } F = \text{force of friction} = \mu R = \mu W$$

$$\Rightarrow -\left(\frac{1}{2} \times \frac{w}{g}\right) V_0^2 = -\mu W x \Rightarrow \mu = \frac{V_0^2}{2gx} \qquad \textbf{(Ans.)}$$

2. *When a ball of weight W rests on a spring of constant k, it produces a static deflection of 2.5mm. How much will the same ball compress the spring if it is dropped from a height h = 0.3 m? Neglect the mass of the spring.*




**Soln.** Given data

$\delta_0$ = static deflection = 0.025 m
h = 0.3 m
We know that spring constant

$$k = \frac{W}{\delta_0}$$

When the ball is allowed to fall from a distance of h and compresses the spring to δ, then the potential energy of the ball = strain energy of the spring i.e., $\frac{W}{g} \times g \times (h+\delta) = \frac{1}{2} k(\delta)^2$

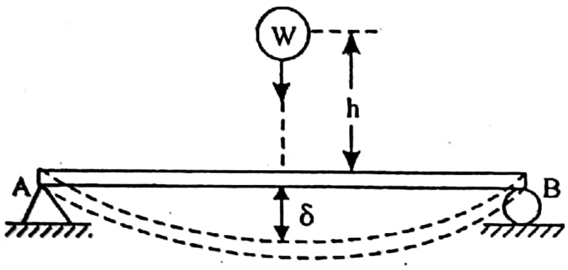
$\Rightarrow W(h+\delta) = \frac{1}{2}\frac{W}{\delta_0} \times \delta^2$ $\quad\Rightarrow (h+\delta) = \frac{\delta^2}{2\delta_0}$

or $2h\delta_0 + 2\delta_0 = \delta^2$ $\quad$ or $2 \times 0.3 \times 0.025 + 2 \times 0.025 \times \delta = \delta^2$

$\Rightarrow \delta^2 - 0.05\,\delta - 0.015 = 0$ $\quad$ or $\delta = \frac{+0.05 \pm \sqrt{(0.05)^2 + 4 \times 0.015}}{2}$

Taking (+ve) sign $\quad \delta = 0.15$ m $\quad$ **(Ans.)**

3. ***Determine the dynamical deflection '$\delta$' that will be produced at the centre of a simply supported beam by allowing a 17.8 kN weight to drop onto it from a height of 100mm. When gradually applied, the same load produces a static deflection of 2.5 mm. Neglect the mass of the beam.***




**Soln.** Weight of the ball, W = 17.8 KN, h = 0.1m

Static deflection, $\delta_0 = 0.0025$m

Assuming it as spring mass system,

the spring constant $k = \frac{W}{\delta_0}$

$\therefore k = \frac{17.8}{0.0025} = 7120$ KN/m

When the ball is allowed to fall from a distance of 'h' it tends to deflect the beam to a distance of '$\delta$'.

Hence, strain energy of the system = Potential energy of the ball

$\therefore \frac{1}{2} K\delta^2 = \frac{W}{g} \times g \times (\delta + h)$ $\quad$ or $\quad K\delta^2 = 2W\delta + 2Wh$

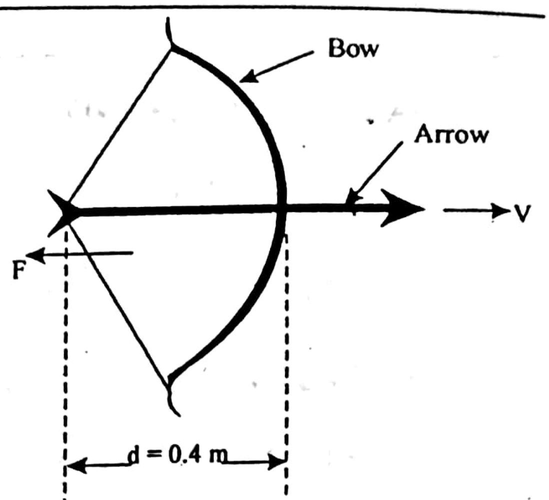
$7120\delta^2 - 35.6\delta - 3.56 = 0$

$\therefore \delta = \frac{35.6 \pm \sqrt{(35.6)^2 + (4 \times 7120 \times 3.56)}}{2 \times 7120}$

Taking +ve sign

$\delta = 0.025$ m = 25mm $\quad$ **(Ans.)**

4. *An arrow weighing 0.1433 N is shot from a 155.75N draw bow at full draw d = 400mm. Assuming a linear relation between draw and force, calculate the velocity V with which the arrow leaves the bow.*




**Soln.** Given

F = 155.75 N (Force applied to the arrow)

W = Weight of the arrow = 0.1433N

d = static deflection of the bow =0.4m .

Let V = Velocity of the arrow after leaving the bow.

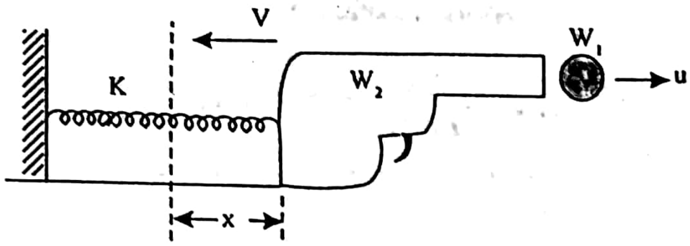
Since arrow and bow is equivalent to the spring mass sysem, then spring constant, $k = \frac{F}{d}$

Applying work energy principle, strain energy of the spring = change in K.E. of the arrow

$$\frac{1}{2}K.d^2 = \frac{1}{2}\frac{W}{g}V^2 \quad \text{or} \quad \frac{1}{2} \times \frac{F}{d} \times d^2 = \frac{W \times V^2}{2g}$$

$$\text{or} \quad V = \sqrt{\frac{F \times d \times g}{W}} = \sqrt{\frac{155.75 \times 0.4 \times 9.81}{0.1433}} = 65.30 \text{ m/s} \qquad \textbf{(Ans.)}$$

5. *A gun weighing 667.5 N fires a 4.45 kN projectile with a nozzle velocity of 1080 m/sec. The gun is nested in springs having a total spring constant k = 26,700 kN/cm. Assuming that the explosion is over before the gun has a chance to move perceptiably how far will it recoil after the explosion ?*




**Soln.** Weight of the projectile (bullet) $W_1$ = 4.45 KN

Weight of the gun, $W_2$ = 667.5 KN

Velocity of bullet = 1080 m/s

Spring constant , k = 26700 KN/m

Let 'x' be the distance that the gun travels towards left after recoil

V = Velocity of recoil

Acording to the conservation of momentum $W_1 u = W_2 V$

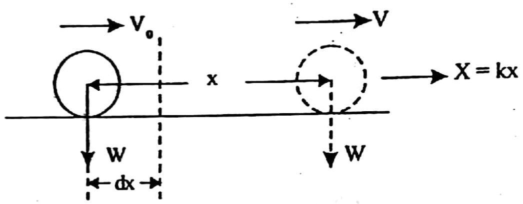
$$\therefore V = \frac{4.45 \times 1080}{6675} = 7.2 \text{ m/s}$$

According to work – energy principle

$$\frac{1}{2}kx^2 = \left(\frac{1}{2}\frac{W_2}{g}V^2 - 0\right) \quad \text{or} \quad x = \sqrt{\frac{W_2 V^2}{kg}} = \sqrt{\frac{6675 \times 7.2^2}{26700 \times 9.81}} = 0.364 \text{ m}$$

(Ans.)

6. *A particle of mass m moves rectilinearly along the x axis under the action of a force X = kx, where k is a constant. Find the velocity V as a function of displacement x if the initial conditions of motion are $x_0 = 0$ and vinitial $\dot{x}_0 = v_0$*




**Soln. Given**

$x_0$ = Initial displacement = 0

$\dot{x}_0 = V_0$, X = Force = kx

where k = constant

Let the particle moves a distance 'dx' by the action of the force 'X'

According to work - energy principle

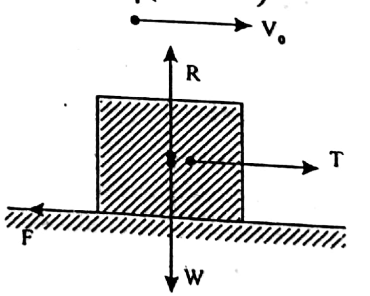
m.Vdv = X dx

Integrating both side

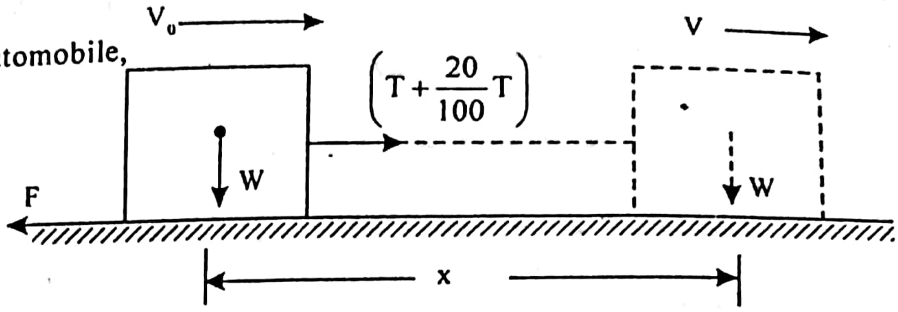
$$m\int_{V_0}^{V} V\,dV = \int_{x_0=0}^{x} X\,dx \quad \text{or} \quad m\int_{V_0}^{V} V\,dV = \int_{x_0}^{x} kx.dx$$

$$\text{or} \quad \frac{1}{2}m\left(V^2 - V_0^2\right) = \frac{kx^2}{2} \quad \text{or} \quad V^2 - V_0^2 = \frac{kx^2}{m} \quad \text{or} \quad V = \sqrt{\left(V_0^2 + \frac{kx^2}{m}\right)} \quad \textbf{(Ans.)}$$

7. *The driver of an automobile moving with constant speed $V_0$ = 64 kmph along a straight level road steps on the accelerator so as to increase the power by 20%. How far will the car travel before attaining a speed V = 80 kmph? Assuming that the resistance to motion remains constant and equal to 5% of the weight of the car?*



**Soln.** **Given data**




Uniform speed of the Automobile,

$V_0 = 64$ Kmph

$$= \frac{64 \times 1000}{3600} = 17.778 \text{ m/s}$$

Let T = Tractive effort on the automobile

When the power is increased by 20%, then tractive effort will also increased by 20% i.e.,

$$\left(T + \frac{20}{100}T\right)$$

After increasing, of tractive effort, the velocity of the automobile is

$$V = 80 \text{ kmph} = \frac{80 \times 1000}{3600} = 22.223 \text{ m/s.}$$

Frictional resistance to the motion,

$$F = \frac{5}{100} \times W, \quad \text{where } W = \text{wt of the Automobile}$$

**When the automobile is moving with uniform speed $V_0$**

The net acceleration is zero. i.e., $F - T = 0$

$$\text{or} \quad F = T = \frac{5}{100} \times W$$

Where F = Friction force due to resistance.

**When the automobile is accelerated by increasing 20% power**

Let 'x' be distance travelled by the automobile when velocity increases from $V_0$ to V.

According to work-energy principle;

$$\frac{W}{2g}\left(V^2 - V_0^2\right) = \left[T + \frac{20}{100}T - F\right] \times x$$

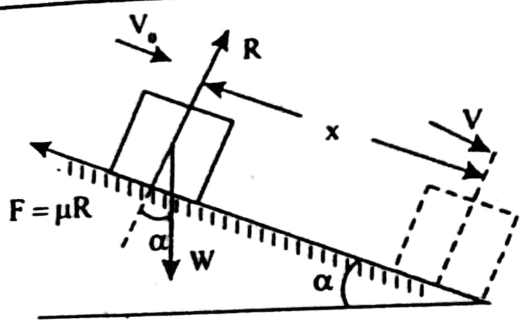
$$\text{or } \frac{W}{2g}\left(V^2 - V_0^2\right) = \frac{20\,T}{100} \times x = \frac{20}{100} \times \left(\frac{5}{100} \times W\right) \times x \text{ or } \frac{V^2 - V_0^2}{2g} = \frac{x}{100}$$

$$\text{or} \quad x = \frac{100\left(V^2 - V_0^2\right)}{2 \times g} = \frac{100\left(22.223^2 - 17.778^2\right)}{2 \times 9.81} = 906.24 \text{ m}$$

$$\text{or} \quad x = 0.90624 \text{ Km}$$

**(Ans.)**

8. *A small block of weight $W = 44.5N$ is given an initial velocity $v_0 = 3$ m/sec. down the inclined plane. If the coefficient of friction between the plane and the block is $\mu = 0.3$, find the velocity V of the block at B after it has travelled a distance $x = 15m$.*




**Soln.** Given data

$W = 44.5$ N

$V_0 = 3$ m/s

$\alpha = 30°$

$\mu = 0.3$

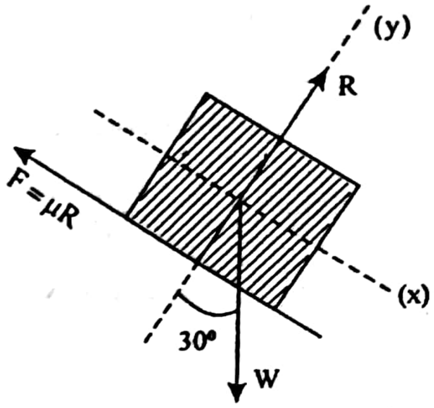
$x = 15$m

Applying condition of static equilibrium along y-axis

$R = W \cos 30°$

$\therefore$ Force of friction along -x-axis.

$F = \mu R = \mu W \cos 30°$




Applying work energy principle along -x- direction

$$\frac{W}{2g}\left(V^2 - V_0^2\right) = (W \sin \alpha - F) \times x$$

$$= (W \sin 30° - \mu W \cos 30°) \times x$$

or $\quad V^2 - V_0^2 = 2g\left[\sin 30° - \mu \cos 30°\right] \times x$

$$V^2 = 3^2 + 2 \times 9.81 \times 15\left(\sin 30° - 0.3 \cos 30°\right) = 79.688$$

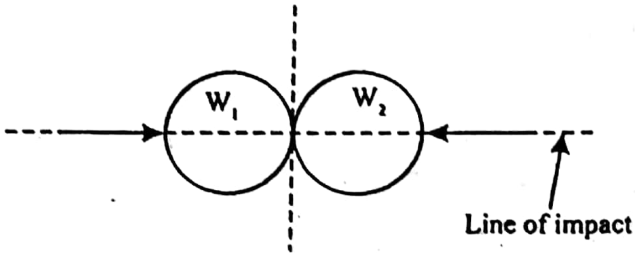
$$V = \sqrt{79.688} = 8.927 \text{ m/s}$$ **(Ans.)**

## 7.11.1 INTRODUCTION

**Impact:** The phenomenon of collision of two moving bodies where we have active and reactive forces of very large magnitude during a very short interval of time is called impact.

The magnitude of the forces and the duration of impact depend on
(i) the shapes of the bodies, (ii) velocities, (iii) elastic properties

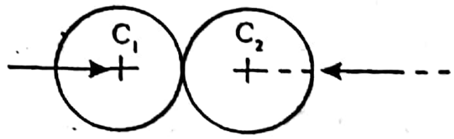
During impact, the body which strikes another body, completely loses its initial velocity. This indicates that there must takes place a very large deacceleration and forces produce at the point of contact are enormous in comparison with the weight of the bodies. Hence impulse during the impact be $\int_0^t F\,dt$




**Line of Impact:** The common normal to the surfaces of two bodies in contact during impact, is called line of impact.

## 7.11.2 TYPES OF IMPACT

**(i) Central/Non-central impact**

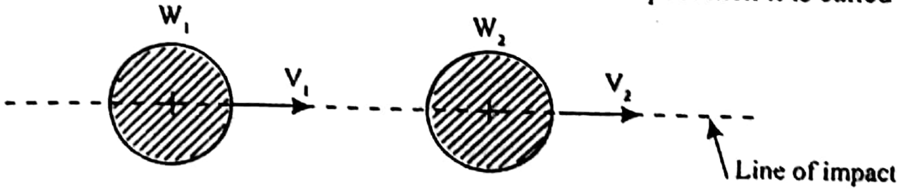



If the mass centres ($C_1$ and $C_2$) of the colliding bodies lie on the line of impact, it is called central impact.

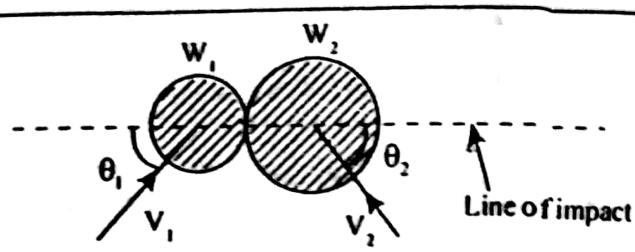
If the centre of mass of two bodies never lie on the line of impact, then it is called non-central impact.

**Direct impact/oblique impact**

If the velocities of the two bodies before collision lie in line of impact then it is called as direct impact.

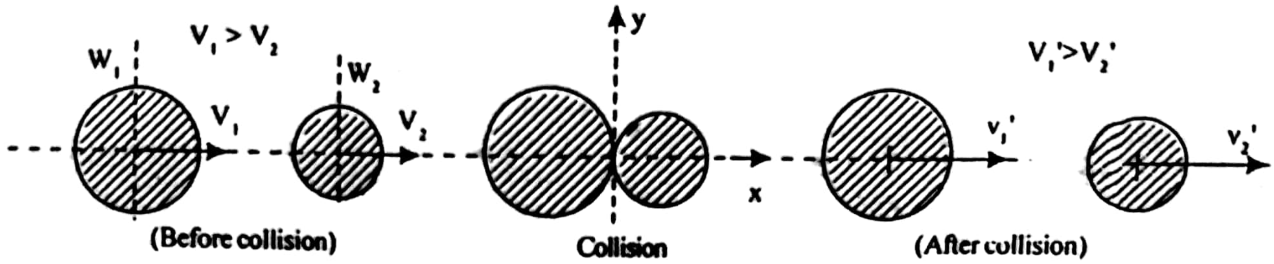


If the velocities do not coincide with the line of impact,then the collision is said to be indirect or oblique impact.




**Direct Impact**

Let two bodies of masses $m_1$ and $m_2$ moving in the same direction and along the same straight line with velocities $V_1$ and $V_2$ respectively.




When $V_1 > V_2$, the body -1 strikes the body -2.

Since no external force are involved, the total momentum of the system remain conserved.

i.e., $$\frac{w_1}{g} v_1 + \frac{w_2}{g} v_2 = \frac{w_1}{g} v_1' + \frac{w_2}{g} v_2'$$

Where $V_1'$ and $V_2'$ are the velocities of respective bodies after collision.

Now $(V_1 - V_2)$ is called as velocity of approach. $(V_2' - V_1')$ = velocity of separation

According to Newton's law of collision, velocity of separation is directly proportional to the velocity of approach.

i.e., $$(V_2' - V_1') \propto (V_1 - V_2) \quad \text{or} \quad e = \frac{(V_2' - V_1')}{(V_1 - V_2)}$$

Where e = constant of proportionality, called co-efficient of restitution(recovery)

### 7.11.3 Coefficient of Restitution (e)

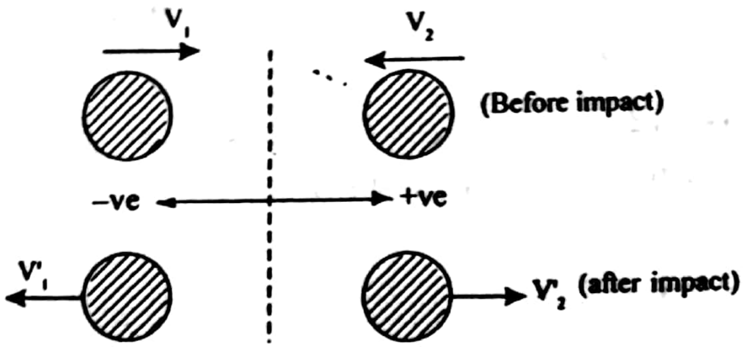
It is the ratio between velocity of separation and velocity of approach when two bodies collide. The coefficient of restitution depends upon the nature of impact.

*Note :* (i) $V_1$, $V_2$, $V_1'$, $V_2'$ are positive if they are directed towards +ve x-axis,otherwise -ve.

Scanned by CamScanner

*For example*

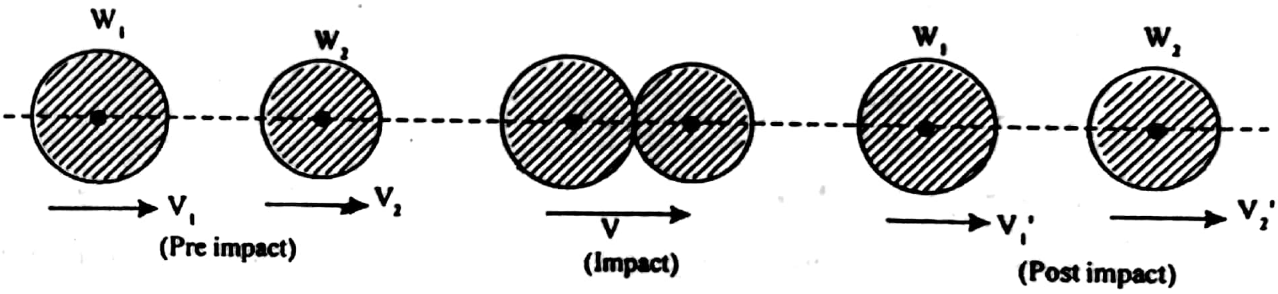
Figure below shows, $V_1$ and $V_2'$ are +ve, But $V_2$ & $V_1'$ are negative.




### 7.11.4 PHENOMENON OF IMPACT

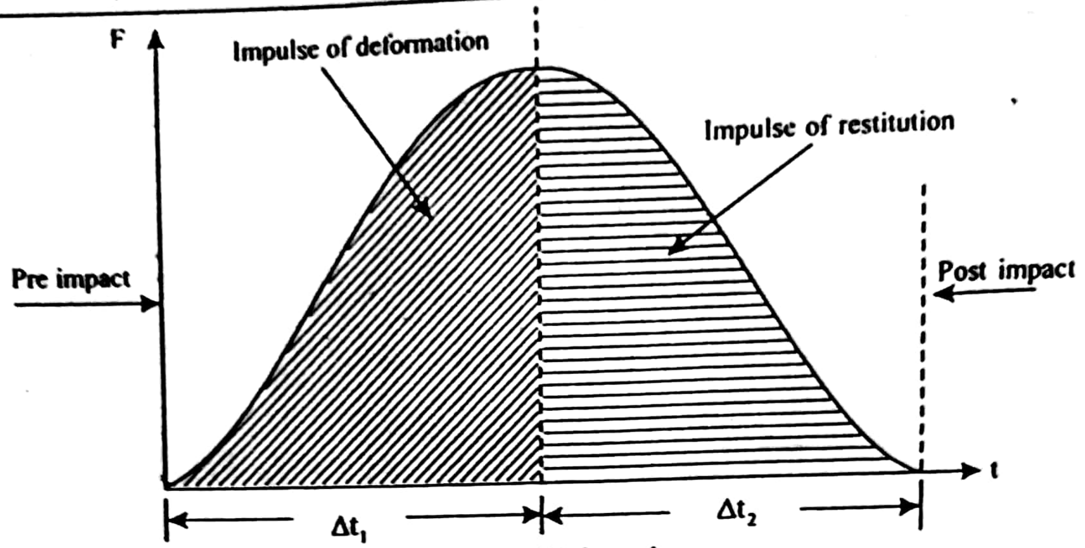
The phenomenon of impact consists of two phases.

i. **Period of deformation :** The bodies after collision, come momentarily to rest and then deform. The time interval from the first contact to the maximum deformation is called the period of deformation. At the end of this period of deformation, both bodies move with the same and common velocity 'v'.




ii. **Period of Restitution :** The process of regaining the original shape is called restitution. The period of deformation is followed by the period of restitution. At the end of which the two bodies either regain their original shapes fully or partially, or remain permanently deformed. This depend upon the magnitude of impact forces acting on them and the properties of the materials involved.

At the end the restitution the two bodies get separated and move with different velocities (when impact is elastic) or move with same velocities (when impact is plastic).




$\Delta t_1$ = Period of deformation

$\Delta t_2$ = Period of restitution

### 7.11.5 Elastic, Plastic and Semi-elastic impacts

1. **Elastic impact :** The impact is said to be perfectly elastic, if velocity of separation is equal to the velocity of approach. i.e., $(V_2' - V_1') = (V_1 - V_2)$

   Hence coefficient of restitution, e = 1 and $\Delta t_1 = \Delta t_2$

2. **Plastic impact:** The impact is said to be perfectly plastic if velocity of separation of two bodies is zero (permanent deformation) i.e., $V_2' - V_1' = 0$ or $V_2' = V_1' = V'$

   The both bodies after impact move with same velocity. Hence coefficient of restitution; e = 0

   $\therefore$ The period of restitution is also zero i.e., $\Delta t_2 = 0$

3. **Semi-elastic impact :** The impact is said to be semielastic, if velocity of separation is less than velocity of approach (partially deformed). i.e., $(v_2' - v_1') < (v_1 - v_2)$

   Hence coefficient of restitution; $0 < e < 1$

### 7.11.6 Application of conservation of Energy

(i) **Elastic impact :** According to conservation of momentum;

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \quad \text{or} \quad m_1(v_1 - v_1') = m_2(v_2' - v_2) \quad \text{.....(i)}$$

The coefficient of restitution for perfectly elastic impact

$$e = 1 \qquad \therefore \frac{v_2' - v_1'}{v_1 - v_2} = 1$$

$$\text{or} \quad v_2' - v_1' = v_1 - v_2 \quad \text{or} \quad v_1 + v_1' = v_2 + v_2' \quad \text{.....(ii)}$$

Multiplying the above two equation;

$$m_1(v_1 - v_1')(v_1 + v_1') = m_2(v_2' - v_2)(v_2 + v_2')$$

or $$m_1\left[V_1^2 - (V_1')^2\right] = m_2\left[(V_2')^2 - V_2^2\right]$$

$$m_1 v_1^2 + m_2 v_2^2 = m_1(v_1')^2 + m_2(v_2')^2$$

Dividing throughout by $\frac{1}{2}$

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1(v_1')^2 + \frac{1}{2}m_2(v_2')^2$$

$\therefore$ K.E. of the two bodies before impact = K.E. of the two bodies after impact.

We concluded that in case of perfectly elastic impact, both the momentum and energy are conserved.

**(ii) Plastic impact**

In case of plastic impact $e = 0$ $\quad \therefore V_2' = V_1' = V'$

$\therefore$ According to conservation of momentum $m_1 v_1 + m_2 v_2 = (m_1 + m_2)v'$

or $\quad m_1(v_1 - v') = m_2(v' - v_2)$ ....(i)

$e=0 \quad \therefore v_2' - v_1' = (v_1' - v') = 0$ ....(ii)

If we solve equation (i) and (ii), no finite solution for energy will be obtained.

Hence the energy of the system is not conserved

***Note:*** In any general case of impact wahen $e \neq 1$, the energy of the system never remains constant.

### 7.11.7 Loss of Kinetic Energy during Impact

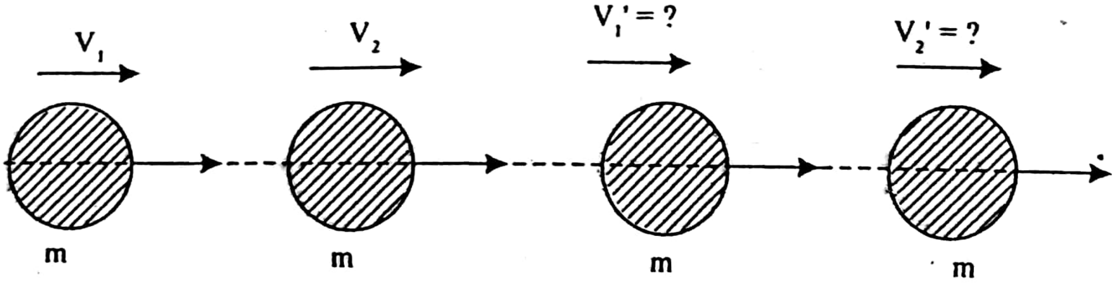
When the conservation of energy is not applicable (when $e \neq 1$) to a system, then there must be some losses. The sum of K.E. before impact must be more than sum of energy after impact.

i.e., $\quad (K.E)_1 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2, \quad (K.E)_2 = \frac{1}{2}m_1(v_1')^2 + \frac{1}{2}m_2(v_2')^2$

$\therefore$ Loss of energy ; $\Delta E = (K.E)_1 - (K.E)_2$

**Examples**

**Case - 1 Elastic impact of two equal masses**




(Pre impact) (Post impact)

According to conservation of momentum, $mv_1 + mv_2 = mv_1' + m_2 v_2'$

or $\qquad v_1 + v_2 = v_1' + v_2'$ ...(i)

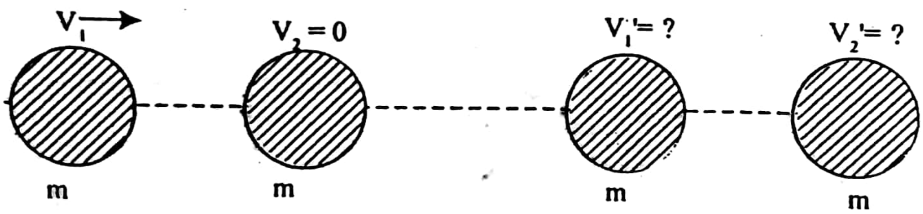
The coefficient of restitution, $\qquad e = \dfrac{v_2' - v_1'}{v_1 - v_2}$, since e = 1

$v_1 - v_2 = v_2' - v_1'$ .....(ii)

Solving equation (i) and (ii), we get $v_1' = v_2$, $v_2' = v_1$

i.e., after an elastic impact, two masses exchange their velocities.

**Case - 2 A body strikes Another body of equal mass at rest. (Elastic impact)**




We know that $mv_1 + 0 = mv_1' + mv_2'$

or $\qquad v_1 = v_1' + v_2'$ ....(i)

The coefficient of restitution, $e = 1 = \dfrac{v_2' - v_1'}{v_1 - 0}$

or $\qquad v_1 = v_2' - v_1'$ ....(ii)

Solving equation (i) and (ii), we get

$$v_1' = 0, \qquad v_2' = v_1$$

i.e., after an elastic collision, the first body comes to rest and the second body moves with the velocity of first body.

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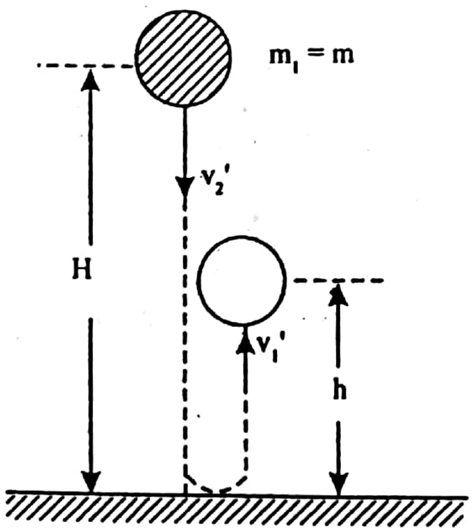
Case - 3 Semi-elastic impact of body on a fixed surface of infinite mass :

Let H - Height of the body from the fixed surface before falling




$$\left(m_2 = \infty,\ V_2 = 0,\ V_2' = 0\right)$$

h = height of the body after impact

$v_1$ = velocity at pre-impact

$v_1'$ = velocity of the body at post-impact

**Note:** The law of conservation of momentum and law of conservation of energy cannot be applied here, because of infinite mass of the second body.

$\therefore$ According work-energy principle W.D = KE

$$mgH = \frac{1}{2}mv_1^2$$

$$v_1 = \sqrt{2gH} \qquad ...(i)$$

After impact or rebound $mgh = \frac{1}{2}m\left(v_1'\right)^2$ or $v_1' = \sqrt{2gh}$

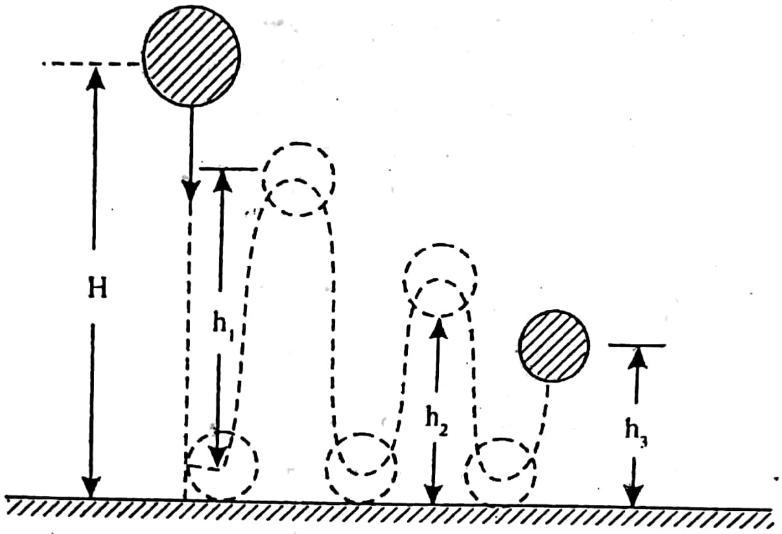
$\therefore$ Coefficient of restitution $e = \dfrac{\text{velocity after impact}}{\text{velocity before impact}} = \dfrac{v_1'}{v_1} = \dfrac{\sqrt{2gh}}{\sqrt{2gH}}$

$$\therefore e = \sqrt{\frac{h}{H}}$$

***Note :***(i) If the impact is an elastic one (e = 1), then h = H or $v_1' = v_1$ i.e., the body would rebound with which it strikes the immovable body.

(ii) **If the body rebounds, number of times after impact then**

$$e = \sqrt{\frac{h_1}{H}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{h_3}{h_2}} \ldots\ldots$$




**Case - 4**

**A body strikes the immovable body ($m = \infty$) at angle $\alpha$ and rebounds at an another angle '$\beta$' to the horizontal respectively.**

Then according conservation of momentum,

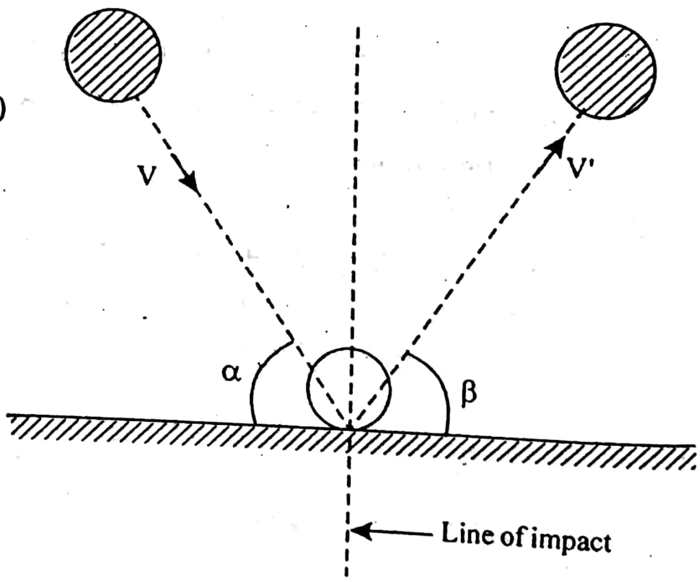
$$mu_1 \cos\alpha = mv \cos\beta$$

or $$u \cos\alpha = V \cos\beta \qquad \ldots\ldots(i)$$

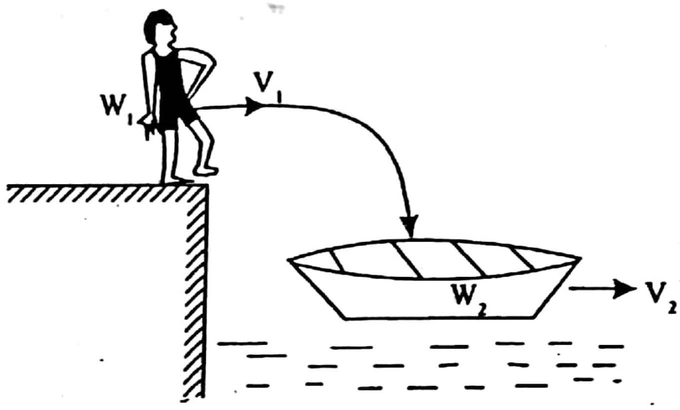
$\therefore$ The coefficient of restitution,

$$e = \frac{V \sin\beta}{u \sin\alpha} \qquad \ldots(ii)$$

Solving equations (i) and (ii), any unknown values can be found out.



1. *A man weighing 667.5N runs and jumps from a pier into a boat with a horizontal velocity $v_1$ = 3m/sec. Assuming that the impact is entirely plastic, find the velocity with which the man and boat will move away from the pier if the boat weights 890 N.*




**Soln.** Given data

$W_1$ = weight of the man = 667.5 N

$W_2$ = weight of the boat = 890 N

Velocity of man $V_1$ = 3 m/sec.

Velocity of (man + boat), $V_2$ = ?

According to the conservation of momentum; $W_1V_1 = (W_1 + W_2)V_2$

$$\Rightarrow V_2 = \frac{W_1V_1}{(W_1+W_2)} \quad \Rightarrow V_2 = \frac{667.5\times 3}{(667.5+890)} \quad \Rightarrow V_2 = 1.285\ \text{m/sec.} \qquad \textbf{(Ans.)}$$

2. *Using the data of prob.-1, compare the final K.E. of the boat and man together with the initial K.E. of the man and note that there is no conservation of energy in the case of plastic impact.*

**Soln.** Given data

$W_1$ = 667.5 N, $W_2$ = 890 N, $V_1$ = 3 m/sec

From the previous problem we have found out $V_2$ = 1.285 m/sec.

Initial kinetic energy of the system

$$(K.E.)_1 = \frac{1}{2}\times\frac{W_1}{g}\times V_1^2 = \frac{1}{2}\times\frac{667.5}{9.81}\times(3)^2 = 306.192\ \text{J}$$

Final K.E. of the system,

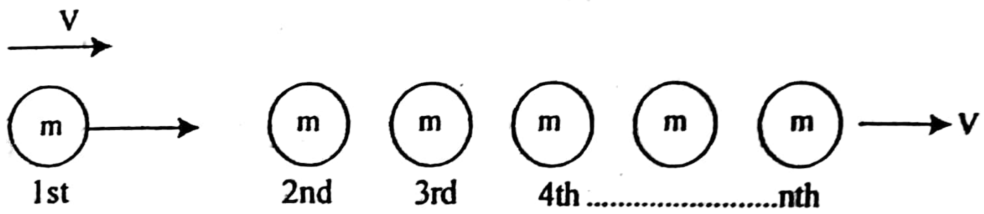
$$(K.E.)_2 = \frac{1}{2}\times\frac{(W_1+W_2)}{g}\times V_2^2$$

$$= \frac{1}{2}\times\frac{(667.5+890)}{9.81}\times(1.285)^2 = 131.079\ \text{J}$$

$(K.E.)_1$ is greater than $(K.E.)_2$

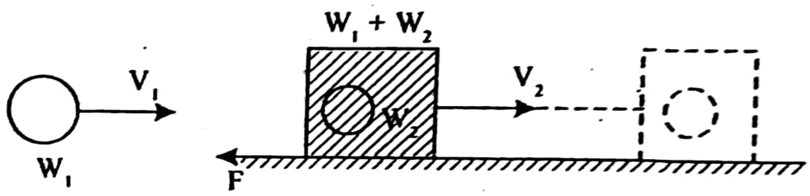
$\therefore$ Loss of energy $(K.E.)_1 - (K.E.)_2 = (306.192 - 131.079) = 175.112$ J

3. *Several identical blocks, each of mass m, rest in a row on a perfectly smooth horizontal plane so that their centers of gravity lie on a straight line. Another block, also of mass m, is moving along this line with velocity v and squarely strikes one end of the row. Discuss what will happen if the blocks are all perfectly elastic.*




**Soln.** When the 1st body strikes the second one, it suddenly comes to rest and 2nd body moves with the velocity of 1st one. Similarly when 2nd body strikes 3rd body, it comes to rest and the 3rd one moves with the velocity of 3rd one. In the way it continues.

4. *A wood block weighing 44.3N rests on a rough horizontal plane, the coefficient of friction between the two being $\mu = 0.4$. If a bullet weighing 0.23N is fired horizontally into the block with nuzzle velocity V = 600 m/sec, how far will the block be displaced from its initial position? Assume that the bullet remains inside the block*




**Soln.** $W_1$ = Weight of the bullet = 0.23 N

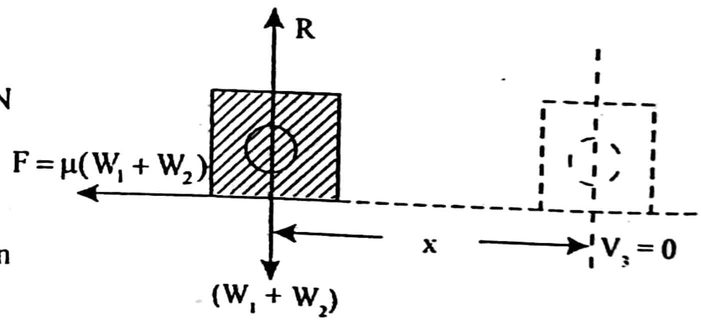
$W_2$ = Weight of the wodden block = 44.3 N

$V_1$ = velocity of bullet = 600 m/sec.

$\mu = 0.4$

Let x be the distance moved by wodden block from its initial position horizontally.

$V_2$ = Velocity of the wooden block and bullet.



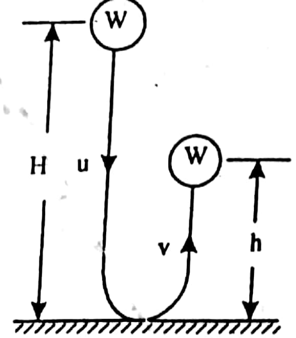

According to conservation of momentum $W_1V_1 = (W_1 + W_2)V_2$

$$\Rightarrow V_2 = \frac{(0.23 \times 600)}{(0.23 + 44.3)} = 3.099 \text{ m/sec.}$$

According to work energy principle; Net work done = change in K.E.

i.e., $\mu(W_1 + W_2)x = \dfrac{(W_1 + W_2)}{2g} \times V_2^2 \quad \Rightarrow 0.4x = \dfrac{(3.099)^2}{2 \times 9.81} \quad \Rightarrow x = 1.22 \text{ m}$ **(Ans.)**

5. *A golf ball dropped from rest onto a cement sidewalk rebounds eight tenths of the height through which it fell. Neglecting air resistance determine the coefficient of restitution.*




**Soln.** Given data :

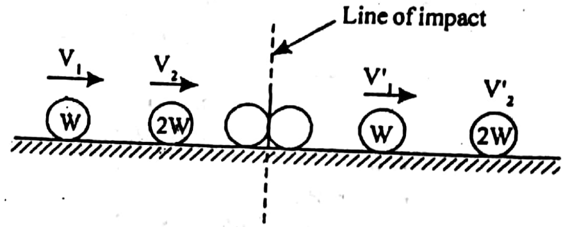
$$h = \frac{8}{10} H$$

We know that co-efficient of restitution;

$$\text{'e'} = \frac{\text{Velocity of separation}}{\text{Velocity of approach}} = \frac{V}{u} = \sqrt{\frac{h}{H}} = \sqrt{\frac{\frac{8}{10}H}{H}}$$

'e' = 0.894 **(Ans.)**

6. *For the two balls, find the velocities after impact if $v_1 = v$, $v_2 = 0$ and $W_2 = 2W_1$*




**Soln.** Weight of the 1st body $W_1 = W$

Weight of the 2nd body $W_2 = 2W_1 = 2W$

$V_1 = V$

$V_2 = 0$

$V_1^{\prime} = ?$

$V_2^{\prime} = ?$

Since impact is perfectly elastic, the co-efficient of restitution 'e' = 1

or $e = \dfrac{V_2^{\prime} - V_1^{\prime}}{V_1 - V_2} \Rightarrow 1 = \dfrac{V_2^{\prime} - V_1^{\prime}}{V}$

$\Rightarrow V_2^{\prime} - V_1^{\prime} = V$ ....(i)

According to the conservation of momentum

$W_1V_1 + W_2V_2 = W_1V_1^{\prime} + W_2V_2^{\prime} \Rightarrow WV = WV_1^{\prime} + 2WV_2^{\prime}$

$\Rightarrow 2V_2^{\prime} + V_1^{\prime} = V$ .....(ii)

Add (i) and (ii)

$V_2^{\prime} - V_1^{\prime} = V$

$2V_2^{\prime} + V_1^{\prime} = V$

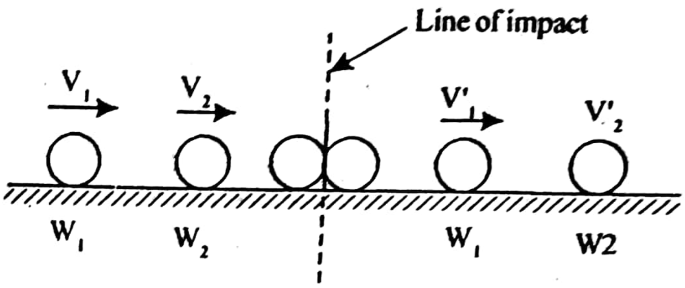
$$3V_2^{\prime} = 2V$$

$$V_2^{\prime} = \frac{2}{3}V \qquad \textbf{(Ans.)}$$

Substituting $V_2^{\prime}$ in equation - (ii)

$$2 \times \frac{2}{3}V + V_1^{\prime} = V \quad \Rightarrow V_1^{\prime} = V - \frac{4}{3}V \quad \Rightarrow V_1^{\prime} = V\left(1 - \frac{4}{3}\right) = -\left(\frac{V}{3}\right) \qquad \textbf{(Ans.)}$$

7. *For the two balls, in find the velocities $v_1$ and $v_2$ after an elastic impact if, before impact $v_1 = v$, $v_2 = 0$ and $W_2 = 2W_1$. and the coefficient of restitution $e = 0.5$.*




**Soln.** Given data

Weight of the 1st body $W_1 = W$

Weight of the 2nd body $W_2 = 3W$

$V_1 = V$

$V_2 = 0$

$e = 0.5$

We know that 'e' = coefficient of restitution;

$$= \frac{V_2^{\prime} - V_1^{\prime}}{V_1 - V_2} \quad \Rightarrow 0.5\,V = V_2^{\prime} - V_1^{\prime} \qquad \text{....(i)}$$

According to the conservation of momentum

$$W_1V_1 + W_2V_2 = W_1V_1^{\prime} + W_2V_2^{\prime}$$

$$\Rightarrow WV = WV_1^{\prime} + 3WV_2^{\prime} \quad \Rightarrow 3V_2^{\prime} + V_1^{\prime} = V \qquad \text{...(ii)}$$

Adding (i) and (ii)

$$V_2^{\prime} - V_1^{\prime} = 0.5\,V$$

$$3V_2^{\prime} + V_1^{\prime} = V$$

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$$4V_2^{\prime} = 1.5\,V$$

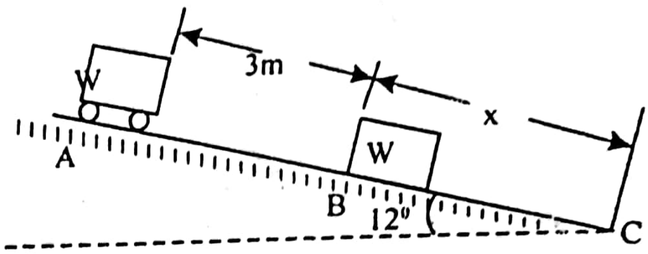
$$\text{or } V_2^{\prime} = \frac{1.5}{4}V = \frac{3}{8}V \qquad \textbf{(Ans.)}$$

Substituting the value in equation (i)

$$\frac{3}{8}V - V_1^{\prime} = 0.5\,V \Rightarrow V_1^{\prime} = -\left(\frac{V}{8}\right) \qquad \textbf{(Ans.)}$$

8. *A small car of weight W starts from rest at A and rolls without friction along an inclined plane to B where it strikes a block also of weight W and initially at rest. Assuming a plastic impact at B, the car and block will move from B to C as one particle. If the coefficient of friction between the block and plane is μ = 0.5, calculate the distance x to point C where the bodies come to rest.*

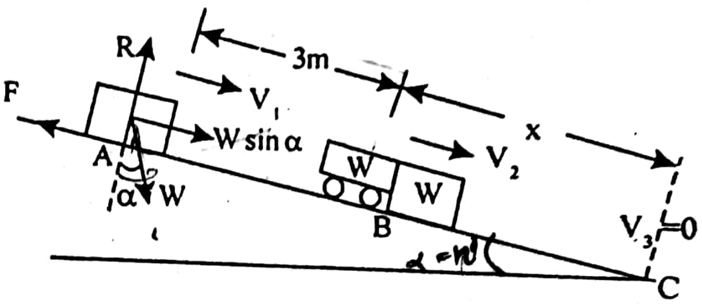
Previous




**Soln.** Given data

$\mu = 0.5, \quad \alpha = 12°$

**When the 1st body moves and strikes the 2nd body without friction.**




According to work energy Principle

$$W \sin\alpha \times 3 = \frac{1}{2} \times \frac{W}{g} V_1^2$$

where $V_1$ = Velocity of striking body

i.e, $V_1^2 = 6g \sin\alpha$

$$\Rightarrow V_1 = \sqrt{6 \times 9.81 \times \sin 12°}$$

$$\Rightarrow V_1 = 3.498 \text{ m/sec.}$$

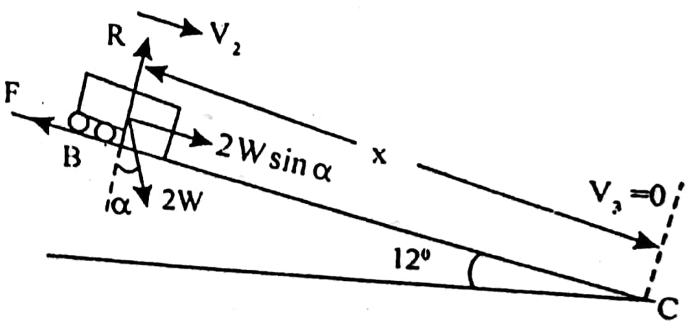
Let $V_2$ = Velocity of two bodies $(W + W)$ after plastic impact.

According to the conservation of momentum

$WV_1 = (W + W)\, V_2$

$\Rightarrow 3.498 = 2V_2$

$\Rightarrow V_2 = 1.749$ m/sec.




**When the two bodies travells a distance 'x' from 'B' to 'c' then $V_3 = 0$**

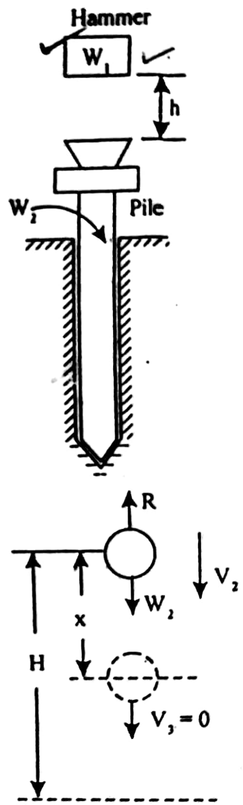
Applying work energy principle,

$$(2W \sin 12° - \mu W \cos 12°) \times x = \frac{2W}{2g} \times \left(V_3^2 - V_2^2\right)$$

or $\quad x[\mu \cos 12° - 2 \sin 12°] = \dfrac{V_2^2}{g}$

$$x = \frac{V_2^2}{g[\mu \cos 12° - 2 \sin 12°]} = \frac{1.749^2}{9.81\,[0.5 \cos 12° - 2 \sin 2°]} = 4.257 \text{ m} \qquad \text{(Ans.)}$$

**9.** ***For the pile and pile driver shown, the following numerical data are given : $W_1$ = 8900 N, $W_2$ = 4450 N, and the coefficient of restitution e = 0.25. If the resistance to penetration is constant and equal to 267,000 N, how many blows of the hammer will be required to drive the pile 0.3m?***




**Soln.** **Given data**

$W_1$ = 8900 N, $\quad W_2$ = 4450 N $\quad$ h = 1.5m

Co-efficient of restitution 'e' = 0.25

Resistance to the penetration R = 267000 N

Total depth of penetration, $\quad$ H = 0.3 m

Let $u_1$ be the initial velocity of hammer

similarly $u_2$ is the initial velocity of pile = 0

$V_1$ = velocity of hammer after blow.

$V_2$ = velocity of pile after blow

According to the conservation of energy,

$$W_1 \times h = \frac{W_1}{2g} \times u_1^2 \quad \Rightarrow u_1 = \sqrt{2gh},$$

$$\Rightarrow u_1 = \sqrt{2 \times 9.81 \times 1.5} \Rightarrow u_1 = 5.425 \text{ m/sec.}$$

According to the conservation of momentum

$$W_1u_1 + W_2u_2 = W_1V_1 + W_2V_2$$

$$\Rightarrow 8900 \times 5.425 = 8900 \times V_1 + 4450 \times V_2$$

$$\Rightarrow 48282.5 = 8900V_1 + 4450\, V_2 \qquad \text{...(i)}$$

We know that $e = \dfrac{V_2 - V_1}{u_1 - u_2} \quad \Rightarrow 0.25 = \dfrac{V_2 - V_1}{5.425}$

$$\Rightarrow V_2 - V_1 = 1.35625 \quad \Rightarrow V_2 = (V_1 + 1.35625) \qquad .....(ii)$$

Substituting the value of $V_2$ in equation (i)

$$8900\ V_1 + 4450(V_1 + 1.35625) = 48282.5$$

$$\Rightarrow 8900V_1 + 4450V_1 + 6023.3125 = 48282.5 \Rightarrow 13350V_1 = 42247.187$$

$\Rightarrow V_1 = 3.16$ m/sec. Substituting the value '$V_1$' in equation (ii)

$$V_2 = (3.16 + 1.35625) = 4.516 \text{ m/sec.}$$

Let x be the vertical penetration of pile for each blow.

$\therefore$ The net force acting on the pile along the direction of motion = $(W_2 - R)$

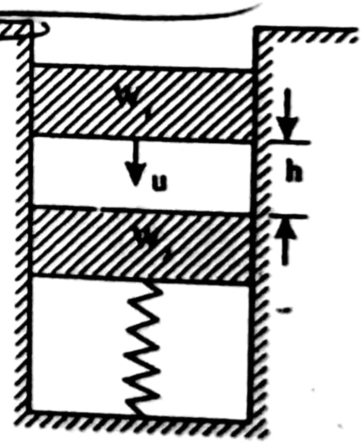
According to work energy principle, $\Sigma$ Work done = change in K.E.

i.e., $(W_2 - R)x = \frac{1}{2} \times \frac{W_2}{g}\left[(V_3)^2 - (V_2)^2\right]$ (where $V_3$ = Velocity of the after being blown up = 0)

$$\Rightarrow x(4450 - 267000) = -\left[\frac{1}{2} \times \frac{4450}{9.81}(4.516)^2\right] \Rightarrow 262550x = 4625.6 \Rightarrow x = 0.017 \text{ m}$$

Let n is the no. of blows for total pentration, $\therefore n = \frac{H}{x} = \frac{0.3}{0.017} = 17$ **(Ans.)**

**10.** *A weight $W_1$ falls through a height h onto a block of weight $W_2$ which is supported by a spring having a spring constant k. Assuming plastic impact, determine the maximum compression of the spring over and above that due to the static action of $W_2$. The following numerical datas are given :* $W_1 = W_2 = 44.5$N, $k = 178$ N/m, $h = 75$ mm

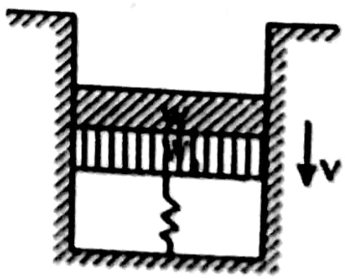



**Soln.** Given data

$W_1 = W_2 = 44.5$N

$k = 178$ N/m

$h = 75$ mm $= 0.075$m




Let $\delta_0$ be the static deflection of the spring due to the weight $W_2$

$$\therefore \delta_0 = \frac{W_2}{k} = \frac{44.5}{178} = 0.25 \text{ m}$$

When $W_1$ is allowed to fall through a height 'h'

Then $(W_1 \times h) = \frac{W_1}{2g} \times u^2$ ......(i)

Where u = initial velocity of $W_1$

Substituting the value in equation (i)

$\Rightarrow u = \sqrt{2 \times 9.81 \times 0.075} \quad \Rightarrow u = 1.213 \text{ m/sec.}$

After coupling is made,

let v = velocity of $(W_1 + W_2)$

According to the conservation of momentum

$$W_1 u = (W_1 + W_2)v \qquad \Rightarrow v = \frac{44.5 \times 1.213}{(44.5 + 44.5)} \Rightarrow v = 0.6065 \text{ m/sec.}$$

After coupling, let $\delta$ be the deflection of spring.

According to the conservation of energy, the total strain energy of the compressed spring = Total energy possessed by the system of $W_1$ & $W_2$

∴ Total strain energy of compressed spring

$$E_1 = \frac{1}{2}k(\delta_0 + \delta)^2 - \frac{1}{2}K\delta_0^2 \qquad \text{....(i)}$$

Total energy possessed by the entire system

$$E_2 = (W_1 + W_2)\delta + \frac{1}{2g}(W_1 + W_2)v^2 \qquad \text{...(ii)}$$

Equating (i) and (ii)

$$\Rightarrow E_1 = E_2 \Rightarrow \frac{1}{2}K(\delta_0 + \delta)^2 - \frac{1}{2}K\delta_0^2 = (W_1 + W_2)\delta + \frac{1}{2g}(W_1 + W_2)V^2$$

$$\Rightarrow \frac{1}{2}K\left(\delta_0^2 + \delta^2 + 2\delta_0\delta\right) - \frac{1}{2}K\delta_0^2 = (W_1 + W_2)\delta + \frac{1}{2g}(W_1 + W_2)$$

$$\Rightarrow \frac{1}{2}\left(K\delta_0^2 + K\delta^2 + 2K\delta_0\delta - K\delta_0^2\right) = (W_1 + W_2)\left(\delta + \frac{V^2}{2g}\right)$$

$$\Rightarrow K\delta^2 + 2K\delta_0\delta = 2(89)(\delta + 0.018) = 0$$

$$\Rightarrow 178\,\delta^2 + 89\delta = 178\delta + 3.3286$$

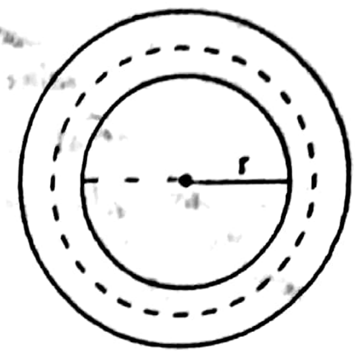
$$\Rightarrow 178\,\delta^2 - 89\delta - 3.3286 = 0$$

$$\delta = \frac{89 \pm \sqrt{(89)^2 + 4(178)(3.328)}}{2 \times 178}$$

Taking (+ve) sign $\qquad \delta = 0.535 \text{ m}$ **(Ans.)**

## SOLVED PROBLEMS – 8.4

1. *A circular ring has a mean radius r = 500 mm and is made of steel for which W = 77.12 kN/m³ and for which the ultimate strength in tension is 413.85 MPa. Find the uniform speed of rotation about its geometric axis perpendicular to the plane of the ring at which it will brust.*

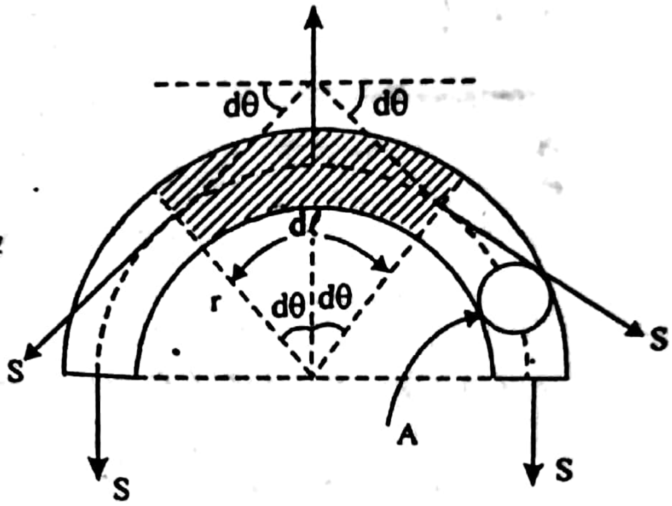



**Soln.** Given data $r = 500\text{ mm} = 0.5\text{ m}$

$w = (\text{weight density}) = 77.12\text{ kN/m}^3$

$\sigma_t = 413.85 \times 10^3\ \text{KN/m}^2$

Consider a small elementary ring, subtending an angle $2d\theta$




$$dF_c = \frac{dW}{g}\ \frac{V^2}{r}$$

Let S be the tension on the ring

A = Area of cross section

dW = Weight of the element = Weight density × volume = $w \times A \times d\ell$

where $d\ell = r \times 2d\theta$

$\therefore$ Centrifual force $\dfrac{w}{g} A d\ell \,.\, \dfrac{V^2}{r} = \dfrac{w}{g} A \,.\, r \,.\, 2d\theta \,.\, \dfrac{V^2}{r} = \dfrac{2\,w\,A\,d\theta\,.\,V^2}{g}$

Balancing forces along the radius $2S \sin d\theta = \dfrac{2w\,A\,d\theta\,V^2}{g}$

Since $d\theta$ is very small, $\sin d\theta \simeq d\theta$

$\therefore$ The above equation can be written as $2S\,d\theta = \dfrac{2w\,A\,d\theta\,V^2}{g}$ or $S = \dfrac{w\,A\,V^2}{g}$

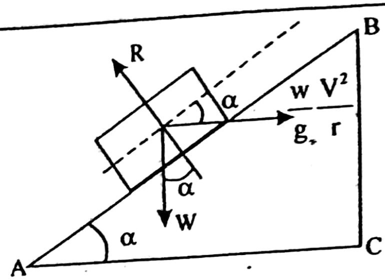
$\therefore$ Tensile stress on the ring $\sigma_t = \dfrac{S}{A} = \dfrac{wV^2}{g}$

Substituting the value, $413.85 \times 10^3 = \dfrac{77.12 \times V^2}{9.81} \Rightarrow V = 229.44\ \text{m/sec.}$

$\therefore$ We know that $V = \dfrac{\pi DN}{60}$ where $D = 2r$

$\Rightarrow 229.44 = \dfrac{\pi \times 2 \times 0.5 \times N}{60} \Rightarrow N = 4381\ \text{rpm}$ **(Ans)**

2. *Find the proper super-elevation 'e' for a 7.2 m highway curve of radius r = 600 m in order that a car travelling with a speed of 80 kph will have no tendency to skid side wise.*



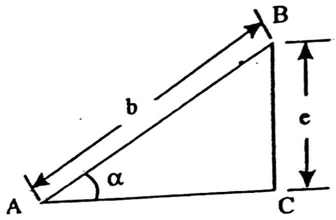

**Soln.** Given data

$b = 7.2$ m, $r = 600$ m

$V = 80$ kmph, $= \frac{80 \times 1000}{3600} = 22.223$ m/s

Resolving along X axis

$$W \sin\alpha = \frac{W}{g} \cdot \frac{V^2}{r} \cdot \cos\alpha$$




or $\tan\alpha = \frac{V^2}{rg}$ .........(i)

Let b = length of the highway, e = elevation

From the triangle ABC,

$\sin\alpha = \frac{e}{b}$ ..........(ii)

Since '$\alpha$' is very small, $\sin\alpha \cong \tan\alpha$

Substituting $\frac{V^2}{rg} = \frac{e}{b}$

or elevation $e = \frac{bv^2}{rg}$ ..........(iii)

Now substituting the values in equation (iii)

$$e = \frac{7.2 \times 22.223^2}{600 \times 9.81} = 0.6 \text{ m}$$ **(Ans.)**

3. *Racing car travels around a circular track of 300m radius with a speed of 384 kph. What angle 'α' should the floor of the track make with the horizontal in order to safeguard against skidding?*

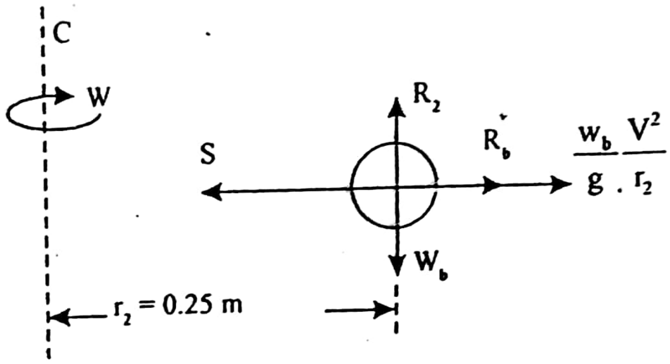
**Soln.** Given data, $r = 300$ m

$V = 384$ kmph $= \frac{384 \times 1000}{3600} = 106.67$ m / sec.

In the previous problem

$$\tan\alpha = \frac{V^2}{rg} \Rightarrow \alpha = \tan^{-1}\left(\frac{106.67^2}{300 \times 9.81}\right) \Rightarrow \alpha = 75^\circ\ 29'$$ **(Ans.)**

Let $R_a$ be the force exerted by $W_a$ on the stop
Considering equilibrium of forces along x-axis.




$$R_a + \frac{W_a}{g} \cdot r_1 \ \omega^2 = S$$

$$\Rightarrow R_a = 222.5 - \frac{44.5}{9.81} \times 0.25 \times (2\pi)^2$$

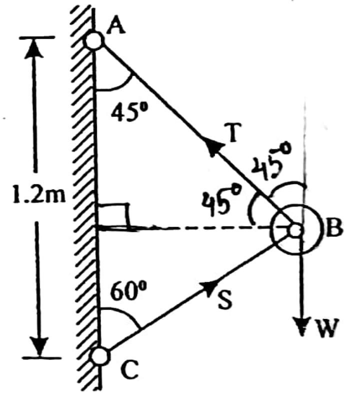
$$\Rightarrow R_a = 117.73 \text{ N}$$

**For Ball $W_b$**
Let $R_b$ be the force extered by $W_b$ on the stop.
Considering equilibrium of forces along x-axis

$$R_b + \frac{w_b}{g} \times r_2 \times \omega^2 = S \qquad \Rightarrow R_b = S - \frac{W_b}{g} r^2\omega^2$$

$$\Rightarrow R_b = 222.5 - \frac{66.75}{9.81} \times 0.25 \times (2\pi)^2 \qquad \Rightarrow R_b = 155.34 \text{ N}$$

6. *A ball of weight W is supported in a vertical plane as shown . Find the compressive force S in the bar BC (a) just before the string AB is cut and (b) just after the string AB is cut. Neglect the weight of the bar BC.*

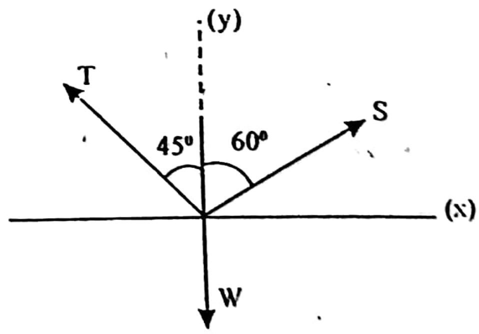



**Soln.** Let 'S' be the compressive force in the bar BC and 'T' is the tension in the string 'AB'.

For equillibrium, $\Sigma x = 0$

$S \sin 60° = T \sin 45°$

$$\Rightarrow T = \frac{S \sin 60°}{\sin 45°} \qquad \text{......(i)}$$




Resolving vertically,

$\Sigma y = 0$

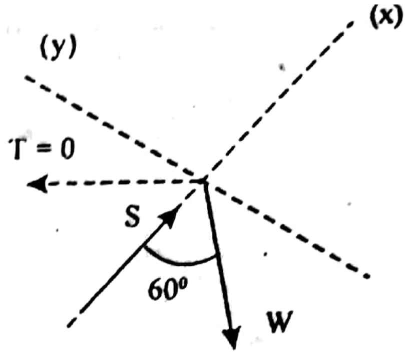
$T \cos 45° + S \cos 60° = w \qquad$ .......(ii)

Substituting the value of T in equation (ii)

$$S\left(\frac{\sin 60°}{\sin 45°} \times \cos 45° + \theta \cos 60°\right) = W$$

centrifugal

$\Rightarrow S = 0.732\ W$ (Ans.)




When AB is cut, $T = 0$

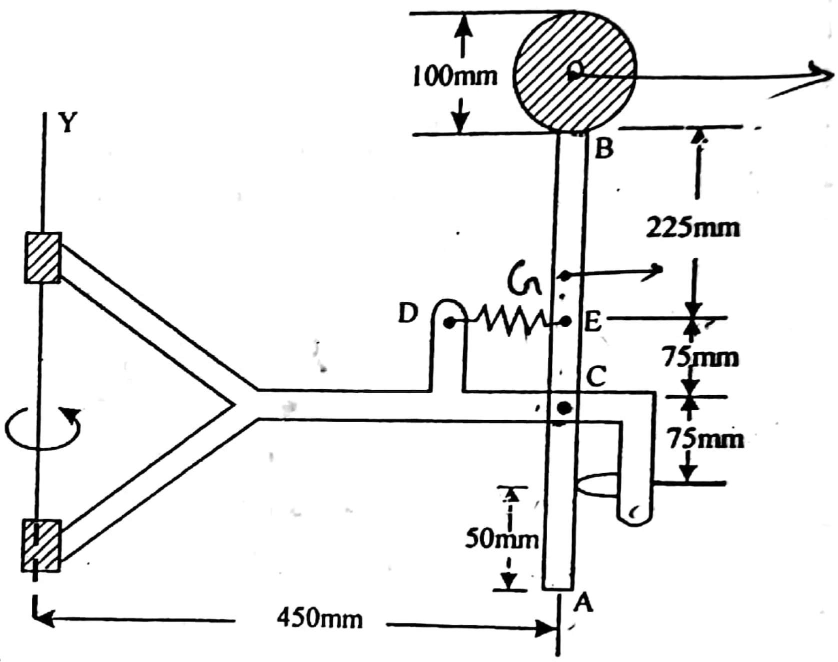
For equillibrium, resolving along x-axis.

$S = W \cos 60°$

$\Rightarrow S = 0.5\ W$ (Ans.)

7. *The arrangement shown in Fig. rotates about the vertical axis yy at constant rpm. The weight of the vertical bar AB, hinged at C, is 15N and the weight of the ball at the top is 30N. When the system is at rest the initial tension in the spring DE is 100 N. At what rpm will contact at A be broken ? Assume the frame and bar AB to be absolutely rigid.*

Previous




**Soln.** $W_1$ is the weight of the ball = 30N

$W_2$ = weight of the bar = 15N

r = radius of rotation = 450mm = 0.45m

Spring force in the portion

'DE' = S = 100N

The total vertical length of the bar AB = 425mm = 0.425m

∴ Weight of the bar acts at it midpoint G.

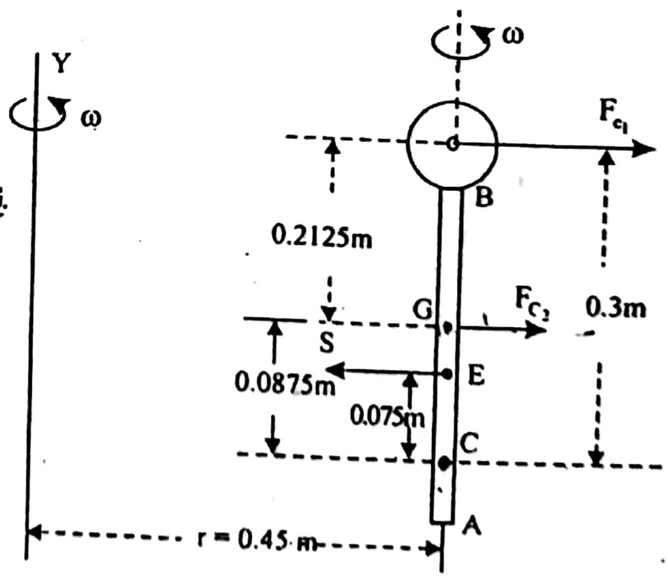
So that BG = 212.5 = 0.2125m.

Due to rotation the centrifugal force on the ball

$$F_{c_1} = \frac{W_1}{g} r\omega^2$$

Centrifugal force in the bar AB

$$F_{c_2} = \frac{W_2}{g} r\omega^2$$



The bar 'AB' remain equillibrium

only when $\Sigma M_c = 0$

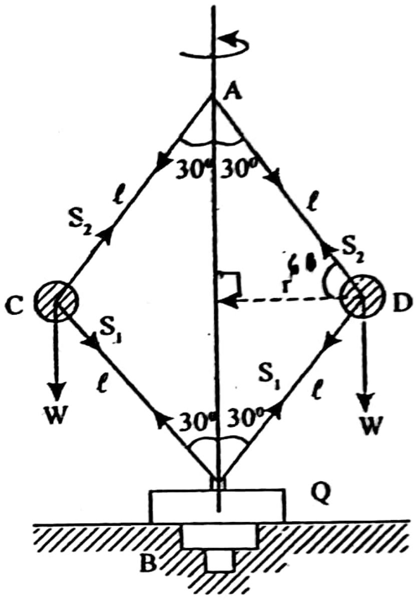
$F_{C_1} \times 0.3 + F_{C_2} \times 0.0875 = S \times 0.075$

$$\Rightarrow \frac{30}{9.81} \times 0.45\,\omega^2 \times 0.3 + \frac{15}{9.81} \times 0.45\,\omega^2 \times 0.0875 = 100 \times 0.075$$

$$\Rightarrow \omega^2 = \frac{7.5}{0.473} = 15.854 \qquad \Rightarrow \omega = 3.981 \text{ rad/sec.}$$

We know that $\omega = \dfrac{2\pi N}{60}$ $\Rightarrow 3.981 = \dfrac{2\pi N}{60} \Rightarrow N = 38.01$ rpm **(Ans.)**

*At what uniform speed of rotation around the vertical axis AB will be balls C and D of equal weights W begin to lift the weight Q of the device, shown. The following numerical data are given; W = 44.5 N, Q = 89N, l = 25 0mm,. Neglect all friction and the weights of the four hinged bars of length 'l'. The weight Q can slide freely along the shaft AB.*




**Soln.** Weight of the fly ball $W = 44.5$ N

Weight of the sleeve $Q = 89$ N

Length of each arm $\ell = 250$ mm $= 0.25$ m

From geometry of the fig.

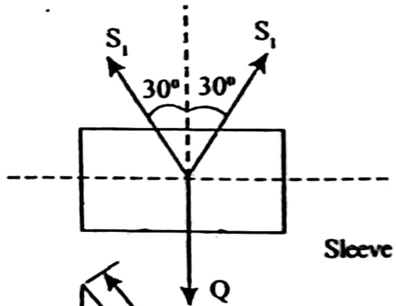
The radius of rotation $r = \ell \sin 30°$

$\Rightarrow r = \ell/2$

Let $S_1$ are the tensile forces in the bar BC and BD

$S_2$ are the tensile forces in the bar AC and AD

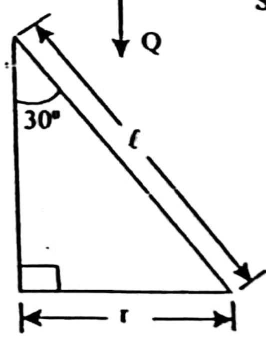
Considering equillibrium of sleeve



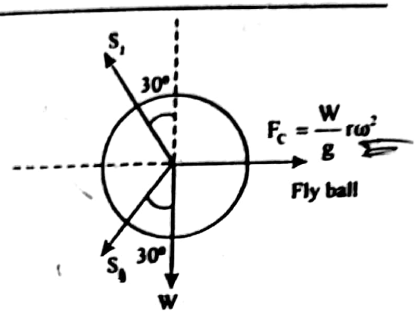

$\Sigma y = 0$

$2S_1 \cos 30° = Q$

$$\Rightarrow S_1 = \frac{89}{2 \times \cos 30°} = 51.284 \text{ N}$$



**Considering equilibrium of the fly ball**




$\Sigma y = 0$

$S_2 \cos 30° = W + S_1 \cos 30°$

$\Rightarrow S_2 = \frac{44.5 + 51.384 \times \cos 30°}{\cos 30°} \Rightarrow S_2 = 102.768 \text{ N}$

Resolving horizontally, $\Sigma x = 0$

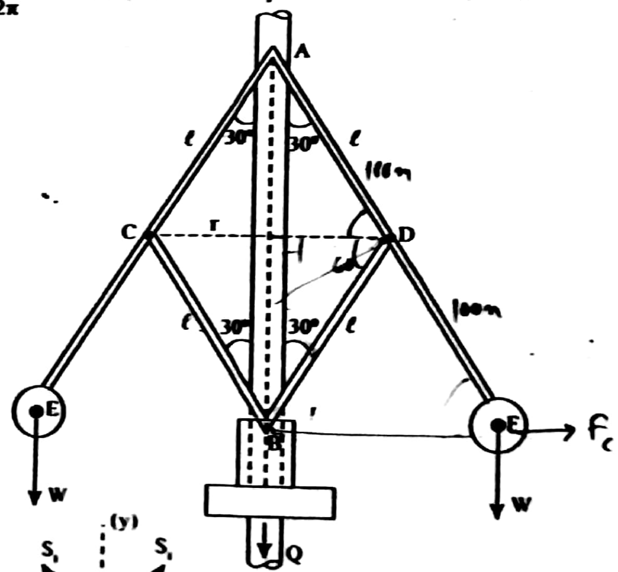
$S_1 \sin 30° + S_2 \sin 30° = F_C$

$\Rightarrow \sin 30° (S_1 + S_2) = \frac{W}{g} r\omega^2 \Rightarrow (51.384 + 102.768) \sin 30° = \frac{44.5}{9.81} \times 0.125\, \omega^2$

$\Rightarrow \omega = 11.65 \text{ rad/sec.}$

We know that $\omega = \frac{2\pi N}{60} \Rightarrow \frac{11.65 \times 60}{2\pi} = N \Rightarrow N = 111.33$ rpm. **(Ans.)**

9. ***The 8.9 N weight of the governor in Fig. Can slide freely on the vertical shaft AB. At what rpm about the axis AB will the 4.45N fly ball lift the sliding weight free of its supports? Neglect friction and weight of the bars.***



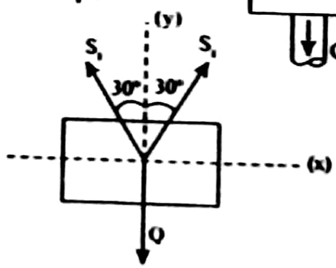

**Soln. Given data**

**$\ell$ = 1000 mm = 1m**

**BC = BD = $\ell$ = 1m**

**AF = AE = 2$\ell$ = 2m**

**Weight of the fly ball**
**W = 4.45 N**
**Weight of the sleeve (slidding weight)**
**Q = 8.9N = 2W**




**Let $S_1$ be the tension in the bars BC and BD. $S_2$ be the tension in the bars AE & AF.**

**Considering equilibrium of**

the slidding sleeve,

$\Sigma y = 0$

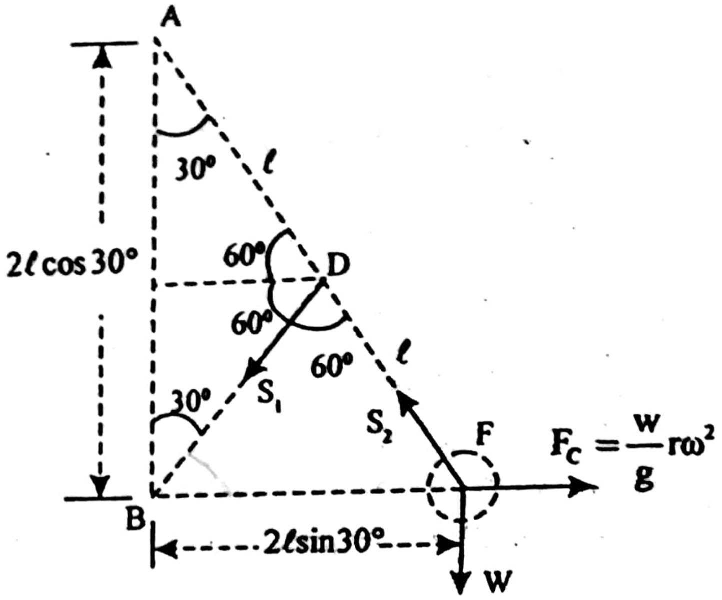
$2S_1 \cos 30° = Q$

$$\Rightarrow S_1 = \frac{8.9}{2 \times \cos 30°} = 5.138 \text{ N}$$

Radius of rotation of flyball from the axis of rotation $r = 2\ell \sin 30° = 1\text{m}$

The centrifugal force acting on the fly ball

$$F_c = \frac{W}{g} r\omega^2$$




**Considering equillibrium of the bar AF, $\Sigma M_A = 0$**

$W \times r + S_1 \sin 60° \times \ell = F_c \times 2\ell \cos 30°$

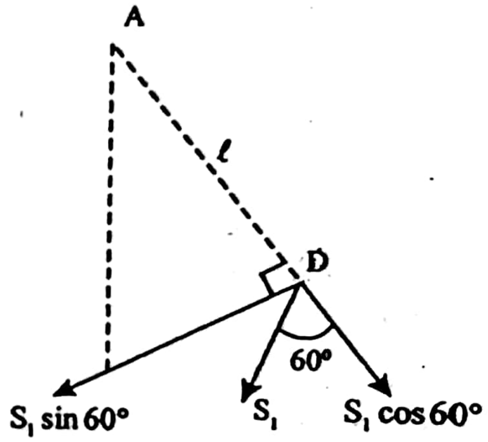
$\Rightarrow 4.45 \times 1 + 5.138 \sin 60° \times 1$

$$= \frac{4.45}{9.81} \times 1 \times \omega^2 \times 2 \times 1 \cos 30°$$

$\omega = 3.36$

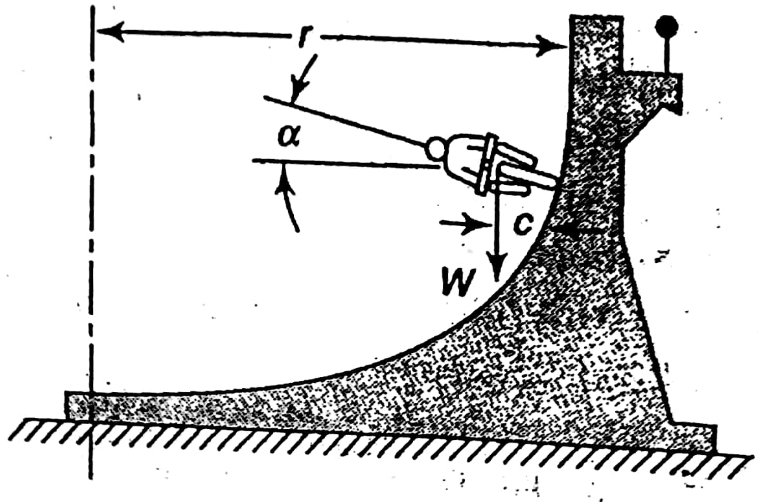
$$\omega = \frac{2\pi N}{60}$$

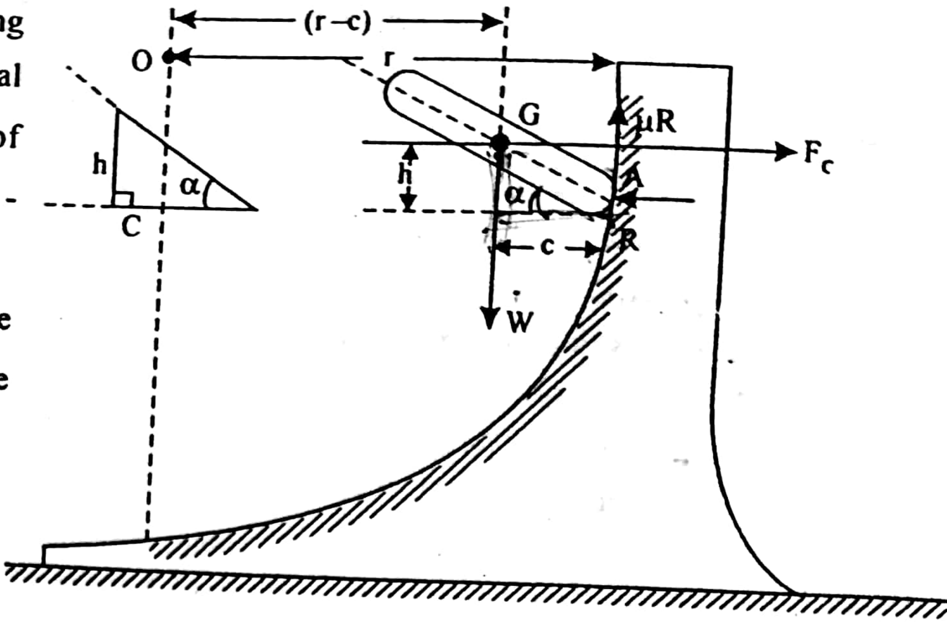
$$\Rightarrow \frac{3.36 \times 60}{2\pi} = N \Rightarrow N = 32.13 \text{ rpm.}$$




**(Ans.)**

10. *What is the minimum uniform speed that a man and motor cycle of weight W can have in going around the inside of a vertical circular drum of radius r in order to prevent slipping down the wall if the co-efficient of friction between the tires and the wall is μ When the motorcycle is running at this speed, what angle must its middle plane make with the horizontal in order to prevent tipping down ?*






**Soln.** When the motorcycle is going around the inside of a vertical circular drum, the radius of rotation is at a distance of (r - c) from the center of rotation. Since the wall is vertical the normal reaction 'R' to the cycle is horizontal.

For equillibrium $\Sigma x = 0$

$$R = F_c = \frac{W}{g}(r-c)\omega^2$$

$$= \frac{W}{g} \times \frac{BV^2}{(r-c)} \quad ....(i)$$

Resolving vertically, $\Sigma y = 0$

$$W = \mu R$$

$$\Rightarrow \quad W = \mu \times \frac{W}{g} \times \frac{v^2}{(r-c)}$$

or $\quad v^2 = \frac{g(r-c)}{\mu} \Rightarrow v = \sqrt{\frac{g(r-c)}{\mu}}$

**(Ans.)**

Let $\alpha$ be the angle of inclination of motorcycle with the horizontal,
h = elevation of center of gravity of motorcycle from horizontal.
From geometry of the figure

$\tan\alpha = h/c$ ....(ii)

For equillibrium of cycle, $\Sigma M_A = 0$

$$W \times c = F_c \times h \quad \Rightarrow W \times c = \frac{W}{g} \times \frac{v^2}{(r-c)} \times c\tan\alpha$$

Substituting the value of $v^2$

$$\Rightarrow c = \frac{1}{g} \times \frac{g(r-c)}{\mu(r-c)} \times c\tan\alpha \qquad \Rightarrow \frac{\tan\alpha}{\mu} = 1$$

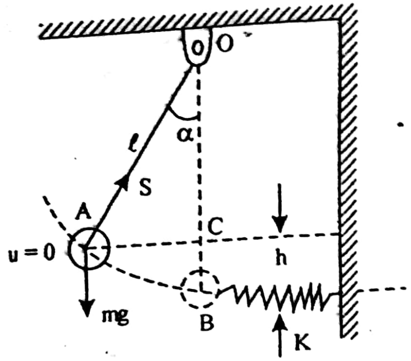
$$\tan\alpha = \mu = \tan\phi$$

$$\Rightarrow \alpha = \phi$$

**(Ans.)**

## SOLVED PROBLEMS - 8.6

1. *A simple pendulum of weight W and length l as shown. A is released from rest at A ($\alpha$ = 60°), swings downward under the influence of gravity, and strikes a spring of stiffness k at B. Neglecting the mass of the spring, determine the compression that it will suffer.*




**Soln.** Given data $\alpha = 60°$, $OA = OB = \ell$

Let $\delta$ be the compression of the spring when the ball of the simple pendulum strikes it.

Let 'h' be the vertical distance between A and B

From the geometry of the triangle OAC, $OC = OA \cos\alpha$

$= \ell \cos 60° = \ell/2$

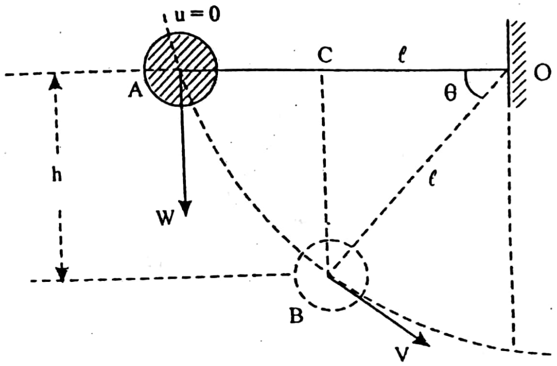
$\therefore \ h = OB - OC = \ell - \frac{\ell}{2} = \frac{\ell}{2}$

$\therefore$ Work done against gravity; $= mgh = W \times \frac{\ell}{2}$ ......(ii)

According to work-energy principle W.D = K.E

$W.\frac{\ell}{2} = \frac{1}{2}K.\delta^2$ or $\delta = \sqrt{\frac{W\ell}{k}}$ **(Ans.)**

2. *The simple pendulum as shown in Fig is released from rest at A with the string horizontal and swings downward under the influence of gravity. Express the velocity V of the bob as a function of the angle 'θ'.*




**Soln.** Given data $OA = OB = \ell$

Let 'h' be the vertical height of the pendulum from the Position A to B.

Hence; $h = \ell \sin\theta$

Work done against gravity

i.e., W.D = w.h ......(i)

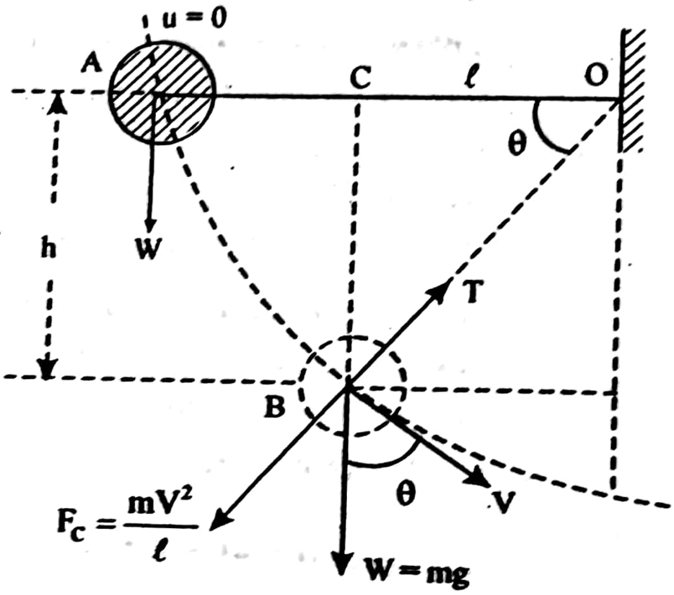
Let V be the tangential velocity at 'B'

$$\therefore \text{ Change in K.E} = \frac{1}{2}\frac{W}{g}V^2 - 0 \quad ....(ii)$$

According to the principle; W.D = change in K.E.

$$W \cdot \ell \sin\theta = \frac{1}{2}\frac{W}{g} \times V^2 \quad \text{or} \quad V = \sqrt{2g\,\ell \sin\theta}$$ **(Ans.)**

3. *If the simple pendulum of weight 'W' as shown in Fig is released form rest in the position A, find the tension T in the string OB as a function of the angle θ.*




**Soln.** Let T be the tension in the string
V = velocity of the ball at 'B'
From the previous problem, we have found $V = \sqrt{2g\,\ell \sin\theta}$
At B the ball is radially equilibrium under the action of

(i) Centrifugal force $F_c = \frac{mv^2}{e}$

(ii) Component of weight, $W \sin\theta$

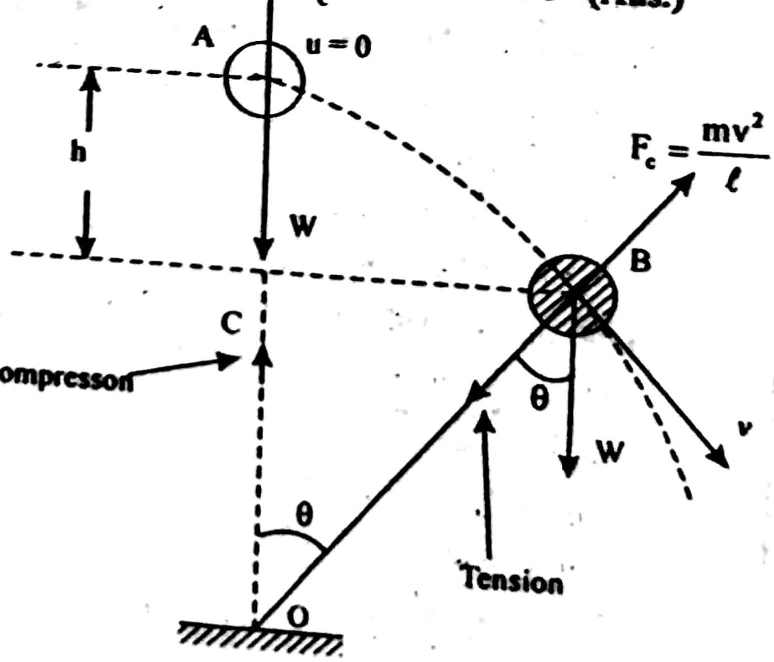
(iii) Tension in the spring

i.e., Resolving forces along OB; $T = \frac{mv^2}{\ell} + mg\sin\theta$

or $T = \frac{m}{\ell}\left(\sqrt{2g\ell\sin\theta}\right)^2 + mg\sin\theta = 3mg\sin\theta$

or $T = 3\,W\sin\theta$ **(Ans.)**

4. *If the pendulum as shown in Fig is released form rest in its position of unstable equilibrium, find the value of the angle θ defining the position in its downward fall at which the axial force in the rod changes from compression to tension.*




**Soln.** Given data
Let V be the velocity of the ball at B
h = vertical height from A to B
From the geometry of the figure
$h = OA - OC = \ell - \ell\cos\theta$

$\therefore h = \ell(1-\cos\theta)$

According to the principle; W.D = change in K.E

$\therefore$ W.h = $\left(\frac{1}{2}\frac{W}{g}V^2 - 0\right)$ or $\ell(1-\cos\theta) = \frac{V^2}{2g}$

or $V = \sqrt{2g\ell(1-\cos\theta)}$ ......(i)

At position A, the weight W compresses the rod OA, thus axial force in the rod is compression.

But the centrifugal force is zero.

Because u = 0

When the position of the ball changes to 'B', it is acted upon by the centrifugal force '$F_c$' along the radius '$\ell$'. Thus the axial force along the rod is tension i.e., the axial force in the rod changes from compression to tension when it changes the position from A to B.

At B, resolving forces along the radius OB; we get

$F_c = W\cos\theta$ or $\frac{W}{g}\frac{V^2}{\ell} = W\cos\theta$

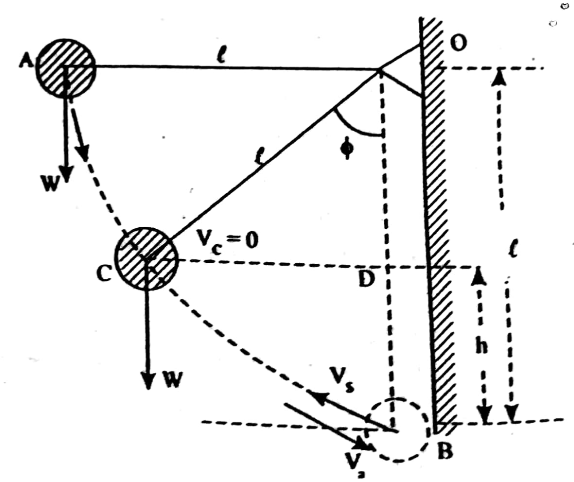
or $V = \sqrt{g\ell\cos 0}$ (ii)

Equating equation (i) and (ii) $\sqrt{2g\ell(1-\cos\theta)} = \sqrt{g\ell\cos\theta}$

or $2g\ell(1-\cos\theta) = g\ell\cos\theta$ or $3\cos\theta = 2$

$\theta = \cos^{-1}\left(\frac{2}{3}\right) = 48° \, 11'$ **(Ans.)**

**5.** ***In Fig a simple pendulum is released from rest in the horizontal position OA and falls in a vertical plane under the influence of gravity. If it strikes a vertical wall at B and the co-efficient of restitution is e = 1/2, find the angle $\psi$' defining its total rebound.***




**Soln.** Given data

$e = \frac{1}{2}$

OA = OB = OC = $\ell$

Let $V_a$ = velocity of approach before the collision with wall.

From A to B

According to work - energy Principle;

W.D = Change in K.E

or $\quad W \times \ell = \left(\frac{1}{2}\frac{W}{g} \times V_a^2 - 0\right) \quad$ or $\quad V_a = \sqrt{2g\ell} \quad$ ....(i)

**From B to C (Rebound)**

Let $V_s$ = velocity of separation after being struck on the wall

h = maximum elevation from B to C

According to the principle; $-(W \times h) = \left(0 - \frac{1}{2}\frac{W}{g} V_s^2\right) \quad$ or $\quad V_s^2 = 2gh$

or $\quad V_s = \sqrt{2gh} \quad$ ....(ii)

We know that coefficient of restitution, $e = \frac{V_s}{V_a} \quad$ or $\quad V_a = 2V_s$

or $\quad \sqrt{2g\ell} = 2\sqrt{2gh}$

Squaring both sides, $\ell = 4h$

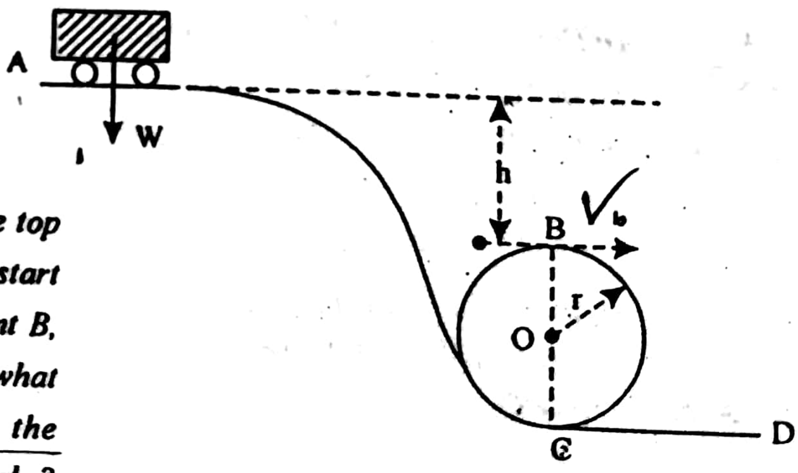
or $\quad h = \frac{\ell}{4} \quad$ ....(iii)

From the geometry of the triangle OCD $\cos\phi = \frac{\ell - h}{\ell} = \left(\ell - \frac{\ell}{4}\right)/\ell$

or $\quad \cos\phi = \frac{3}{4} \quad$ or $\quad \phi = \cos^{-1}\left(\frac{3}{4}\right) = 41° 25' \quad$ **(Ans.)**

*A small car of weight W starts from rest at A and rolls without friction along the loop ACBD as shown in the fig.*

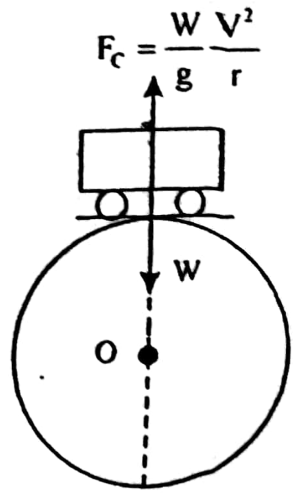
*What is the least height h above the top of the loop at which the car can start without falling off the track at point B, and for such a starting position, what velocity will the car have along the horizontal position CD of the track ? Neglect friction.*




**Soln.** Let $V_b$ be the velocity at the heighest point B.

At point 'B' the car is acted upon by centrifugal force, which is counter balanced by its own weight.

i.e., $\quad F_c = W$




or $$\frac{W}{g} \times \frac{V_b^2}{r} = W$$

or $$V_b = \sqrt{rg} \quad \text{......(i)}$$

**From A to B**

According to the work energy principle,

W.D. = Change in K.E

$$W \times h = \left(\frac{1}{2}\frac{W}{g}V_b^2 - 0\right) \quad \text{or} \quad V_b = \sqrt{2gh} \quad \text{....(ii)}$$

Equating (i) and (ii) $\sqrt{rg} = \sqrt{2gh}$

Squaring both sides, we get $h = \frac{r}{2}$ **(Ans.)**

**From A to C**

The total elevation from A to C be $(h + 2r)$

Let $V_c$ = velocity of the car at 'C'

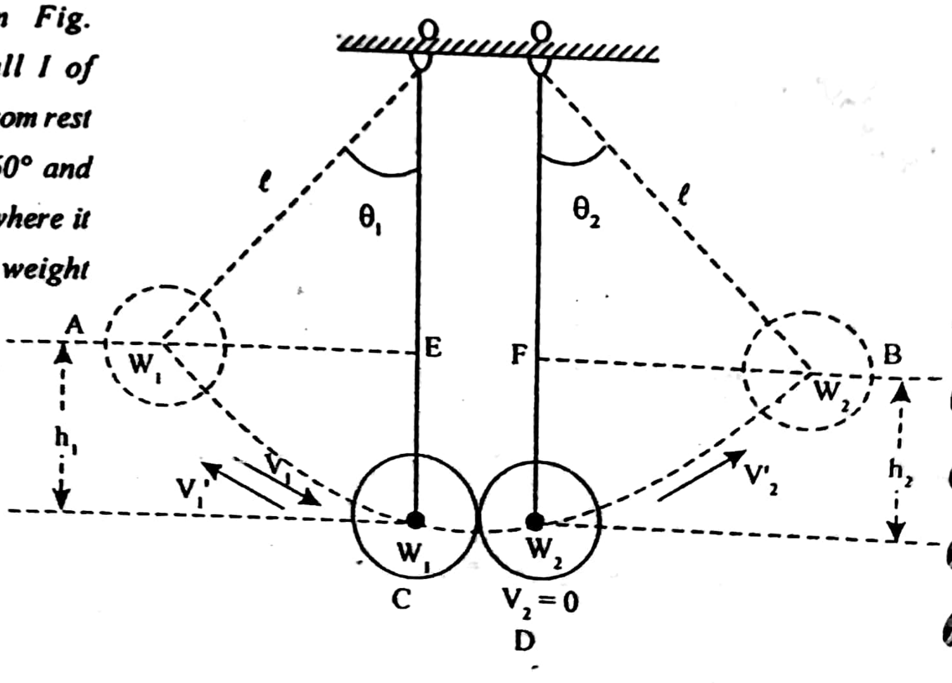
According to the work energy principle; $W(h + 2r) = \frac{1}{2}\frac{W}{g} \times V_c^2$

or $$V_c = \sqrt{2g(h + 2r)} \quad \text{....(iii)}$$

Now substituting the value of 'h'.

$$V_c = \sqrt{2g\left(\frac{r}{2} + 2r\right)} \quad \text{or} \quad V_c = \sqrt{5gr}$$ **(Ans.)**

7. *Referring to given Fig. assume that the ball I of weight W is released from rest in the position $\theta_1 = 60°$ and swings downward to where it strikes the ball II of weight 3W, which is at rest. Assuming an elastic impact, Calculate the angle $\theta_2$ through which the larger pendulum will swing after the impact.*



**Soln.** Given data

$W_1 = W, \quad W_2 = 3W, \quad \theta_1 = 60°$

$e = 1$ (elastic impact), $\quad V_2 = 0$ (Rest)

Let $V_1$ and $V_1'$ be the velocities of $W_1$ before and after impact.

$V_2$ & $V_2'$ are the corresponding velocities of $W_2$.

From the geometry of the figure, $h_1 = (\ell - \ell\cos\theta_1) = \ell - \ell\cos 60° = \frac{\ell}{2}$

Applying work energy principle to $W_1$ and we obtained

$$W_1 \times h_1 = \frac{W_1}{2g} \times V_1^2$$

or $\quad V_1 = \sqrt{2gh_1} = \sqrt{g\ell}$ .....(i)

**Considering elastic impact**

Velocity of approach before impact, $V_a = V_1 - V_2 = V_1 - 0$

or $\quad V_a = V_1 = \sqrt{g\ell}$ .....(ii)

Velocity of separation after impact, $V_s = V_2' - V_1'$ .....(iii)

$\therefore$ Coefficient of Restitution; $e = \frac{V_s}{V_a}$ or $V_2' - V_1' = V_1$ .....(iv)

Applying conservation of momentum; $\frac{W_1}{g}V_1 + \frac{W_2}{g}V_2 = \frac{W_1}{g}V_1' + \frac{W_2}{g}V_2'$

$\Rightarrow \quad WV_1 + 0 = WV_1' + 3WV_2'$

$\Rightarrow \quad 3V_2' + V_1' = V_1$ .....(v)

Adding equation (iv) and (v), we get $4V_2' = 2V_1$

or $\quad V_2' = \frac{V_1}{2} = \frac{\sqrt{g\ell}}{2}$ .....(vi)

Let $h_2$ be the elevation of $W_2$ after impact. From the geometry of the figure

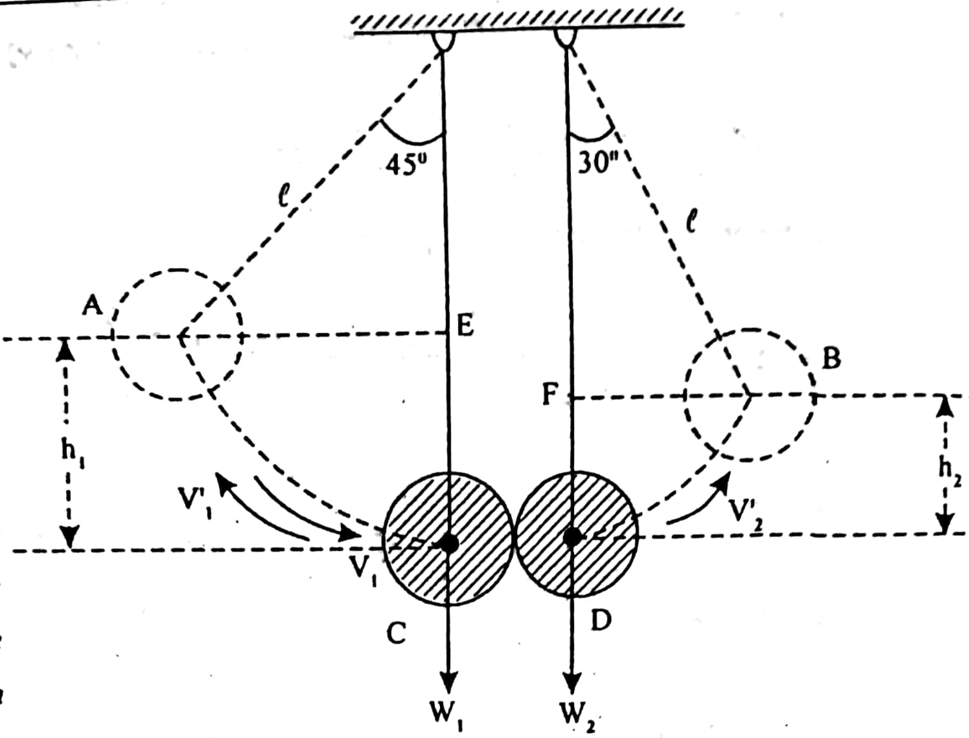
$h_2 = \ell - \ell\cos\theta_2 = \ell(1-\cos\theta_2)$

Applying work - energy principle; $\quad W_2 \times h_2 = \frac{W_2}{2g} \times (V_2')^2$

or $\quad (V_2')^2 = 2gh_2 \quad$ or $\quad \frac{g\ell}{4} = 2g\ell\,(1-\cos\theta_2)$

or $\quad (1-\cos\theta_2) = \frac{1}{8}$ or $\cos\theta_2 = \left(1-\frac{1}{8}\right) = \frac{7}{8}$ or $\theta_2 = \cos^{-1}\left(\frac{7}{8}\right) = 28°57'$ **(Ans.)**

8. *In the system shown in above Fig. the ball I is allowed to swing downward form rest in the position defined by the angle $\theta_1 = 45°$ and to strike the ball II, which after impact, swings upward to the position defined by the angle $\theta_2 = 30°$. If the weights of the balls are equal, find the coefficient of restitution 'e' for the materials.*




**Soln.** Given data $W_1 = W_2 = W$, $\theta_1 = 45°$, $\theta_2 = 30°$, $V_2 = 0$

From the geometry of the triangle; $h_1 = \ell - \ell\cos 45° = \ell(1-\cos 45°) = 0.293\ \ell$

$h_2 = \ell - \ell\cos 30° = \ell(1-\cos 30°) = 0.134\ \ell$

Let $V_1$ & $V_1'$ are the tantential velocities of $W_1$ before and after impact.

$V_2$ & $V_2'$ are corresponding velocities of $W_2$

**From A to C (Before impact)**

**Applying work - energy principle**

$$W_1 \times h_1 = \frac{W_1}{2g} \times V_1^2 \quad \text{or} \quad V_1 = \sqrt{2gh_1} \qquad \text{....(i)}$$

**From D to B (after impact)**

$$W_2 \times h_2 = \frac{W_2}{2g} \times \left(V_2'\right)^2 \quad \text{or} \quad V_2' = \sqrt{2gh_2} \qquad \text{....(ii)}$$

Applying conservation of momentum to the impact;

$$\frac{W_1}{g}V_1 + \frac{W_2}{g}V_2 = \frac{W_1V_1'}{g} + \frac{W_2V_2'}{g} \quad \Rightarrow WV_1 + 0 = WV_1' + WV_2'$$

$$\text{or} \quad V_2' + V_1' = V_1 \qquad \text{or} \quad V_1' = V_1 - V_2' \qquad \text{....(iii)}$$

Now the coefficient of restitution; $e = \dfrac{\text{velocity of separation}}{\text{velocity of approach}}$

$$= \frac{V_2' - V_1'}{V_1 - V_2} = \frac{V_2' - V_1'}{V_1} \qquad \text{.....(iv)}$$

Substituting the value of $V_1'$ from equation .......(iii) in (iv),

we get; $e = \dfrac{V_2' - (V_1 - V_2')}{V_1} = \dfrac{2V_2' - V_1}{V_1}$

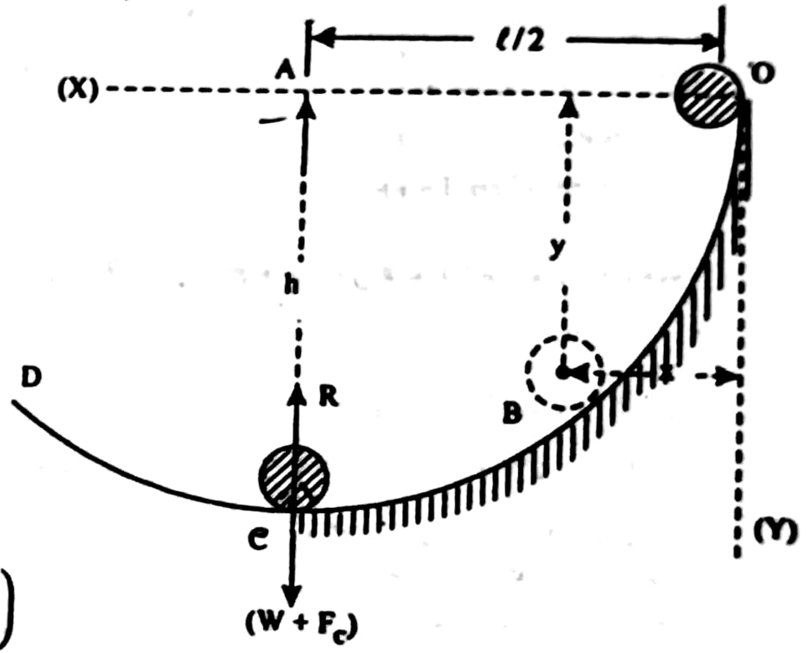
Now putting the value of $V_1$ & $V_2'$

We get, $e = \dfrac{2\sqrt{2gh_2} - \sqrt{2gh_1}}{\sqrt{2gh_1}}$ or $e = \dfrac{(2\sqrt{h_2} - \sqrt{h_1})}{\sqrt{h_1}} = \dfrac{(2\sqrt{0.134\ell} - \sqrt{0.293\ell})}{\sqrt{0.293\ell}}$

$\therefore e = 0.35$ **(Ans.)**

Previous

*In Fig a small ball of weight W = 22.25 N starts from rest at O and rolls down the smooth track OCD under the influence of gravity Find the reaction R exerted on the ball at C if the curve OCD is defined by the equation.*




$y = h\sin\left(\dfrac{\pi x}{\ell}\right)$ *and* $h = \dfrac{\ell}{3} = 0.9m$

**Soln.** Given data

$W = 22.25$ N, $y = h\sin\left(\dfrac{\pi x}{\ell}\right)$

$h = \dfrac{\ell}{3} = 0.9$ m

Let 'R' be the normal reaction at 'C'

Now the x and y cordinates at C be $\left(\dfrac{\ell}{2}, h\right)$

Given that $h = \dfrac{\ell}{3} = 0.9$ m $\qquad \therefore \ell = 0.9 \times 3 = 2.7$ m

From A to C

Let V be the velocity of the body at 'C'

According to work - energy principle, $W \times h = \dfrac{W}{2g} \times V^2$

or $\qquad V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.9} = 4.202$ m/s

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Now given that $y = h \sin\left(\frac{\pi x}{\ell}\right)$

$$\therefore \frac{dy}{dx} = \frac{\pi h}{\ell} \cos\left(\frac{\pi x}{\ell}\right)$$

At $x = \frac{\ell}{2}$ at 'C' $\frac{dy}{dx} = \frac{\pi h}{\ell} \cos\left(\frac{\pi \times \ell}{2\ell}\right) = 0$

Now $\frac{d^2y}{dx^2} = \frac{-\pi^2 h}{\ell^2} \sin\left(\frac{\pi x}{\ell}\right)$

At $x = \frac{\ell}{2}$ [at C]

$$\frac{d^2y}{dx^2} = \frac{-\pi^2 h}{\ell^2} \sin\left(\frac{\pi \times \ell}{2\ell}\right) = \frac{-\pi^2 h}{\ell^2}$$

Since y = f(x), the radius of curvature 'ρ' at any instance,

$$\rho = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right|$$

At C, $\rho = \left| \frac{\left[(1+0)^{3/2}\right]}{\left(\frac{-\pi^2 h}{\ell^2}\right)} \right| = \frac{\ell^2}{\pi^2 h}$ or $\rho = \left(\frac{2.7^2}{\pi^2 \times 0.9^2}\right) = 0.912$ m

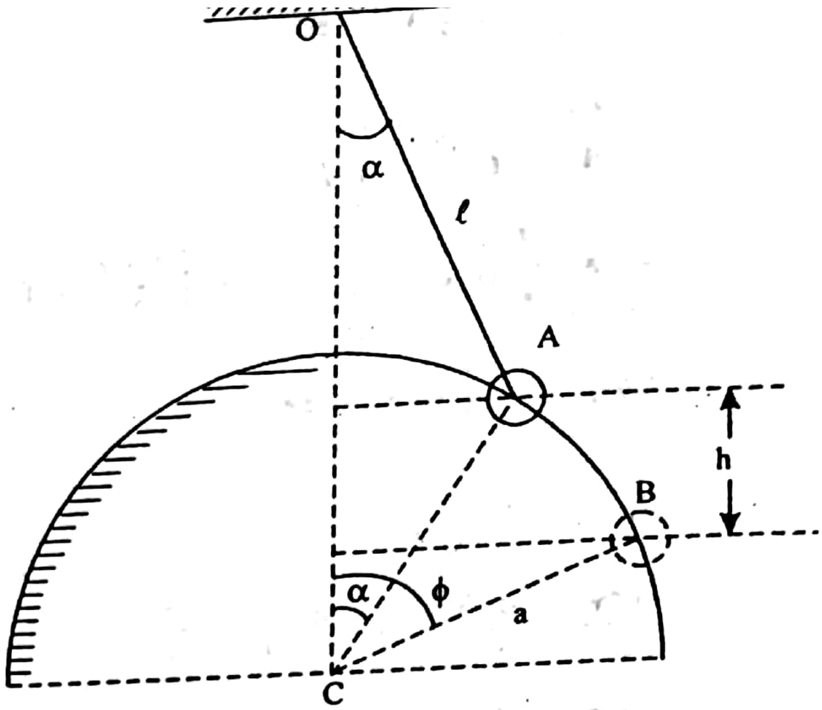
The body at 'C' is equilibrium under the actions of normal reaction $(\uparrow)$, its own weight $(\downarrow)$ and centrifugal force $F_c = \left(\frac{W}{g} \frac{V^2}{\rho}\right) (\downarrow)$

Now; $\sum y = 0$ $\quad R = W + F_c \quad = W + \frac{W}{g} \times \frac{V^2}{\rho}$

$$\therefore R = 22.25 + \frac{22.25}{9.81} \times \frac{(4.202)^2}{0.912}$$

$\therefore R = 66.16$ N **(Ans.)**

10. *Referring to given Fig find the value of the angle "$\phi$" defining the position of the point B where the particle will jump clear of the cylindrical surface after the string OA has been cut. Neglect friction.*




**Soln.** Given data

Radius of the cylinder = a

$\mu = 0$

Let h be the vertical elevation of the body from A to B

From the geometry of the figure

$$h = a\cos\alpha - a\cos\phi$$

or $h = a(\cos\alpha - \cos\phi)$

Let V be the velocity of the body at 'B'

Applying work - energy principle

From A to B,

$$W \times h = \frac{W}{2g} \times V^2 \text{ or } \quad V = \sqrt{2gh} \quad \text{....(i)}$$

**At 'B'**

When the string OA has been cut off, the body would jump clear the surface means, the normal reaction 'R' at the contact surface is zero. Resolving the forces along the radius CB, we get

$$F_c = W\cos\phi \quad \text{or} \quad \frac{W}{g}\frac{V^2}{a} = W\cos\phi$$

or $\quad V^2 = ga\cos\phi \quad$ ...(ii)

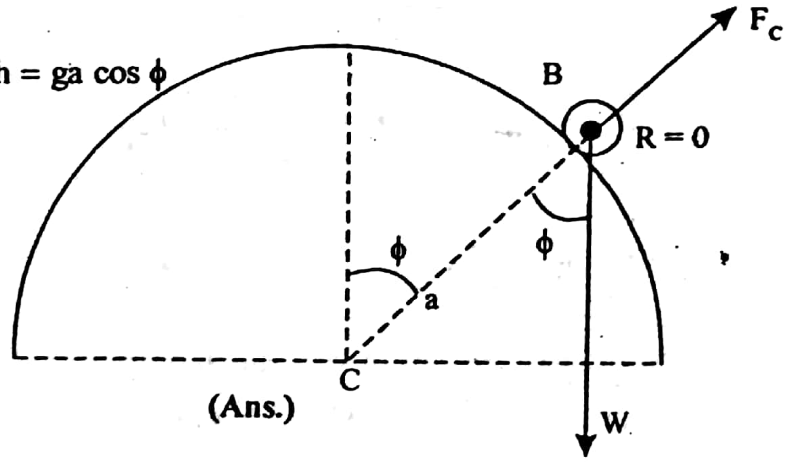
Substituting the value of 'V' from equation (i) in equation (ii), we get $2gh = ga\cos\phi$

$\Rightarrow \quad 2a(\cos\alpha - \cos\phi) = a\cos\phi$

$\Rightarrow \quad 3\cos\phi = 2\cos\phi$

$\Rightarrow \quad \cos\phi = \frac{2}{3}\cos\alpha$

$$\phi = \cos^{-1}\left[\frac{2}{3}\cos\alpha\right]$$




(Ans.)

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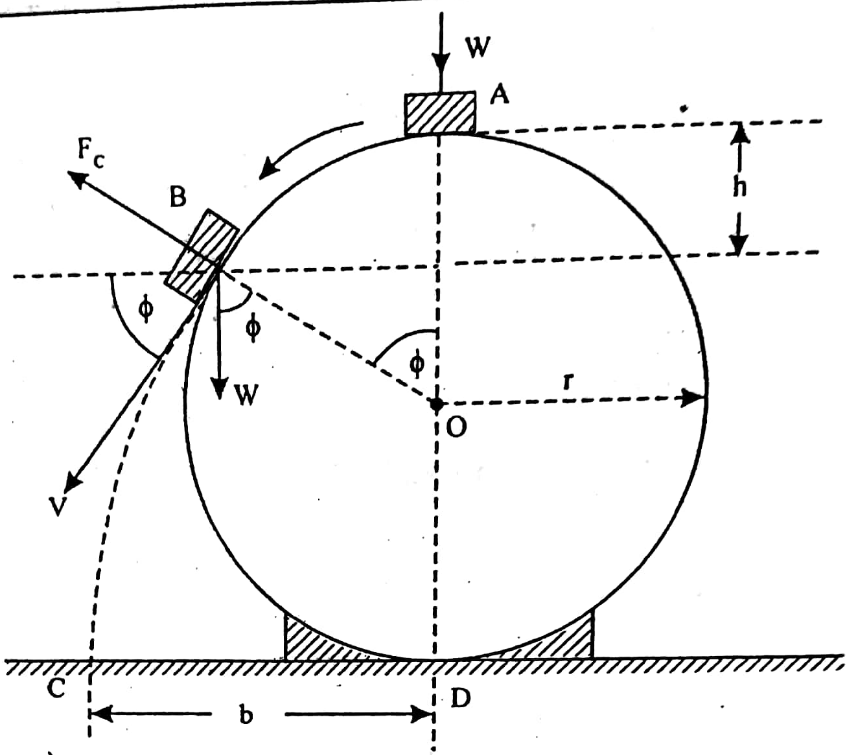
**11.** *A block of weight W starts from rest at A and slides in a vertical plane along the arc AB of a smooth circular cylinder of radius r. Neglecting friction, determine the distance b defining the position of the point C at which the block strikes the horizontal plane CD.*




**Soln.** From the geometry of the figure, the downward elevation from A to B;

$$h = r - r\cos\phi = r(1 - \cos\phi)$$

Let V be the velocity of the block at 'B'
Applying work - energy Principle;

$$Wh = \frac{W}{2g}V^2$$

or $$V = \sqrt{2gh} = \sqrt{2gh\, r(1-\cos\phi)} \qquad \text{....(i)}$$

The block is about to leave at B and hence the surface normal reaction at that point is zero (R = 0)
Resolving forces along its radius OB at 'B'

$$F_c = W\cos\phi \qquad \text{or} \qquad \frac{W}{g}\frac{V^2}{r} = W\cos\phi$$

or $$V = \sqrt{rg\cos\phi} \qquad \text{....(ii)}$$

Equating equations (i) and (ii)

We get; $$\sqrt{2gr(1-\cos\phi)} = \sqrt{gr\cos\phi}$$

or $$2(1-\cos\phi) = \cos\phi \qquad \text{or} \qquad 3\cos\phi = 2$$

$$\phi = \cos^{-1}\left(\frac{2}{3}\right) = 48^\circ\ 11'$$

**Consider the projectile motion from B to C :**

**Now consider vertical motion from B to C $(\downarrow)$**

$S = BE = r\cos\phi + r = r(\cos\phi + 1)$

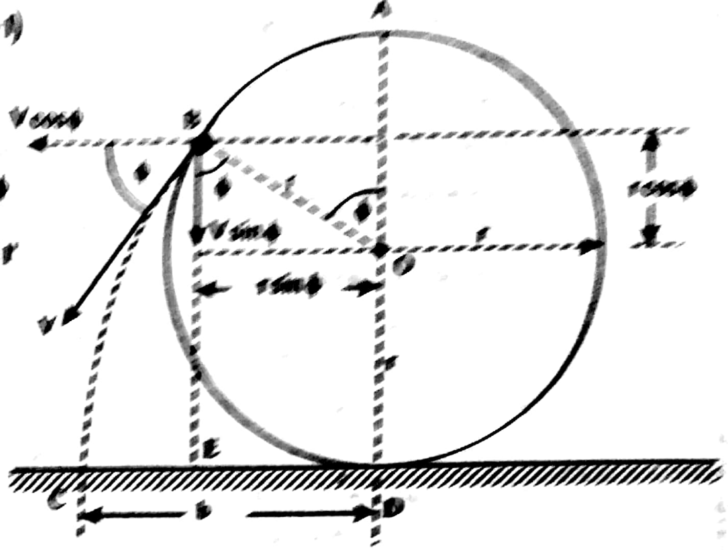
$= r(\cos 48^\circ 11' + 1) = 1.667\,r$

$a = +g$

$u = V\sin\phi = \left(\sqrt{rg\cos\phi}\right)\sin\phi$

$= \left(\sqrt{r}\ \sqrt{g\cos 48^\circ 11'}\right)\sin 48^\circ 11'$

$= 1.906\sqrt{r}$




t = time taken to reach at 'C'

Applying equation of plane rectilinear motion;

$S = ut + \frac{1}{2}at^2$

or $\quad 1.667\,r = 1.906\sqrt{r}\,.\,t + \frac{1}{2}\times 9.81\times t^2$

or $\quad 4.905\,t^2 + \left(1.906\sqrt{r}\right)t - 1.667r = 0$

or $\quad t = \dfrac{-1.906\sqrt{r} \pm \sqrt{\left(1.906\sqrt{r}\right)^2 - 4\times 4.905\times(-1.667r)}}{2\times 4.905}$

or $\quad t = 0.42\sqrt{r}$ sec.

Considering horizontal motion from B to C (→)

$S = EC,\ a = 0,\ u = V\cos\phi$

or $\quad u = \left(\sqrt{rg\cos\phi}\right)\cos\phi = \left(\sqrt{r}\ \sqrt{9.81\times\cos 48^\circ 11'}\right)\cos 48^\circ 11' = 1.705\sqrt{r}$

$t = 0.42\sqrt{r}$

$\therefore S = ut + \frac{1}{2}at^2$

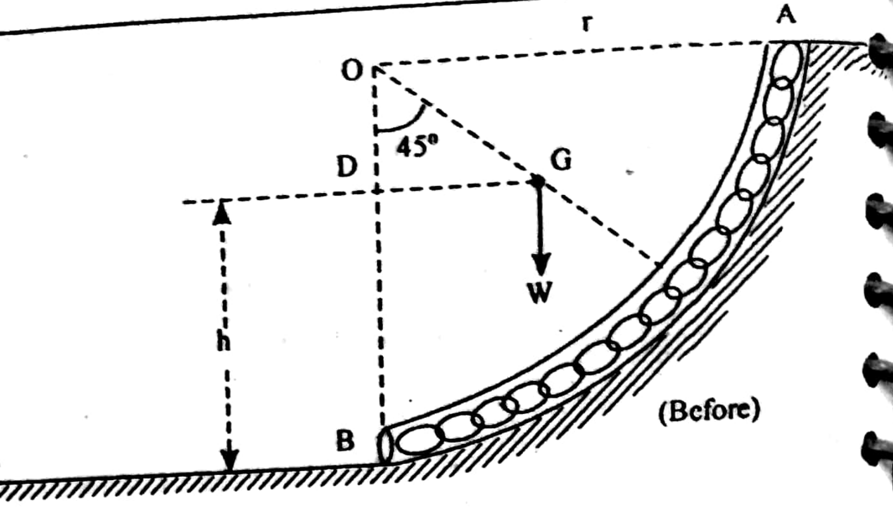
or $\quad EC = \left(1.705\sqrt{r}\times 0.42\sqrt{r}\right) + 0 = 0.716r$

$\therefore \quad b = DC = DE + EC$

$\quad = r\sin\phi + 0.716\,r = r\sin 48^\circ 11' + 0.716\,r$

$\therefore b = 1.461\,r$ (Ans)

**12.** *A smooth tube AB in the form of a quarter circle of mean radius r is fixed in a vertical plane and contains a flexible chain of length $\frac{\pi r}{2}$ and weight $\frac{\omega \pi r}{2}$ as shown if Fig. If released from rest in the configuration shown, find the velocity V with which the chain will move along the smooth horizontal plane BC after it emerges from the tube.*




**Soln.** Given data

Length of the chain, $\ell = \frac{\pi r}{2}$ Weight of the chain, $W = \frac{\omega \pi r}{2}$

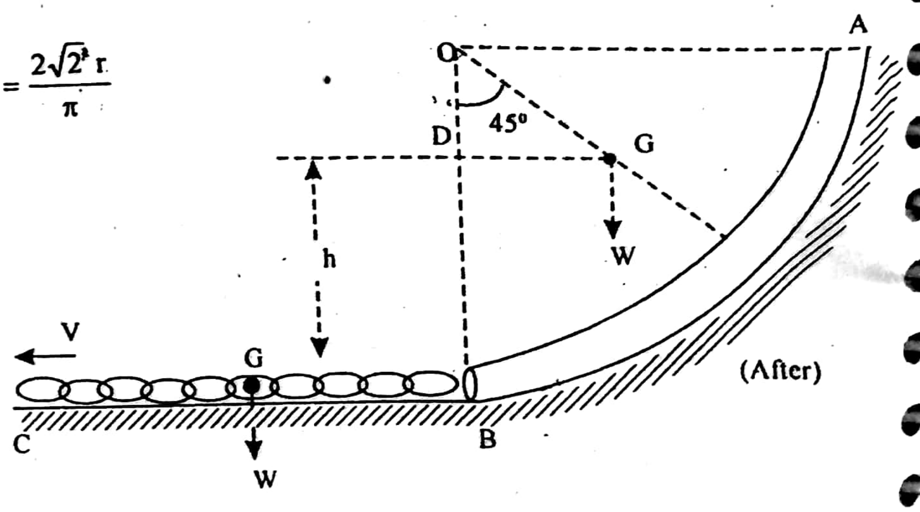
Before released, the chain takes the position of a quadrant curve. Hence the weight of the chain acts at C.G. of the curve.

From geometry of the figure,

$$OG = \frac{2r}{\alpha} \sin\frac{\alpha}{2}$$

Hence $\alpha = 90^\circ$ or $\frac{\pi}{2}$

$$\therefore\ OG = \frac{2r}{\frac{\pi}{2}} \sin 45^\circ = \frac{2\sqrt{2}\,r}{\pi}$$




Now when the chain is released from rest, all the links of the chain move with the same velocity 'v' and takes a new position in horizontal floor i.e., the C.G. will come down by DB.

From the geometry of the figure.

$$h = DB = OB - OD = OB - OG \cos 45^\circ$$

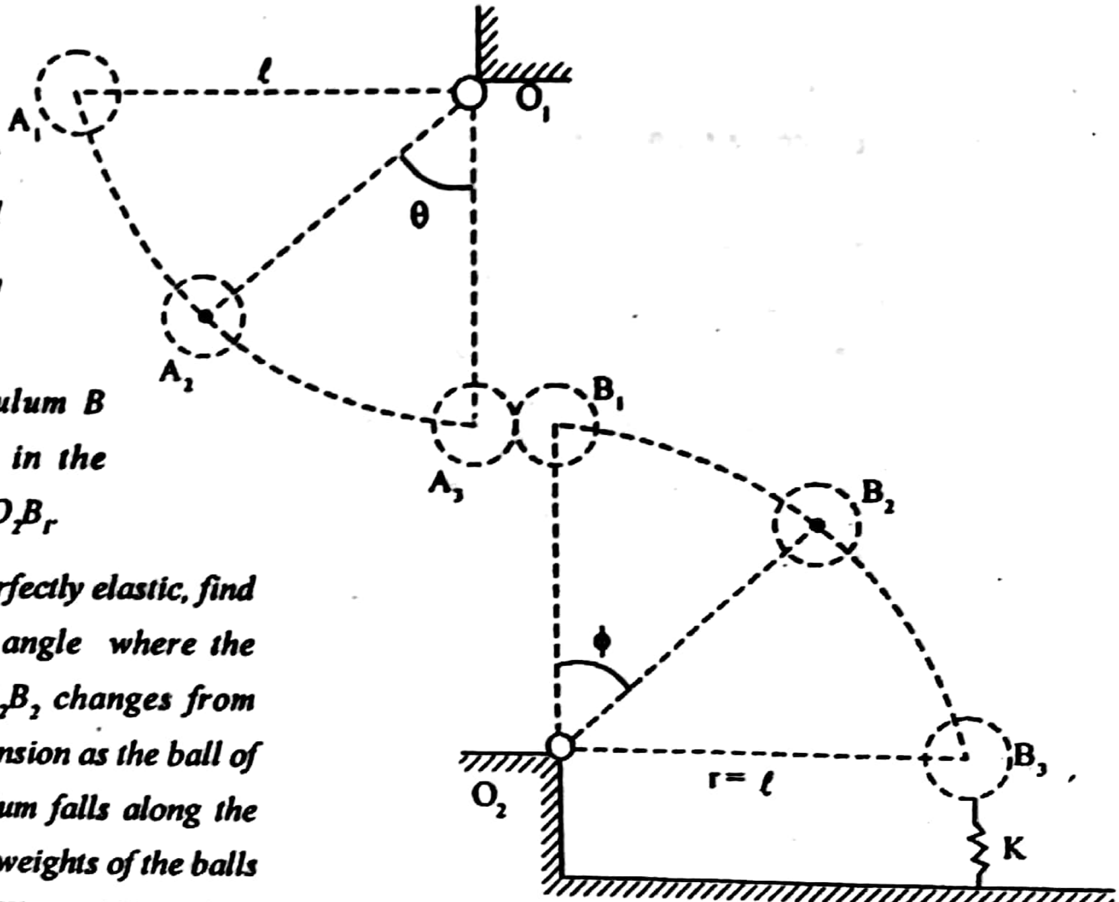
$$= r - \left(\frac{2\sqrt{2}.r}{\pi}\right) \times \frac{1}{\sqrt{2}} = r\left(1 - \frac{2}{\pi}\right) = 0.363\ r$$

Applying work - energy principle

$$W.h = \frac{W}{2g} \times V^2 \text{ or } \qquad V = \sqrt{2gh}$$

$$= \sqrt{2 \times g \times 0.363\ r} \qquad = \sqrt{0.728\ gr} \qquad \textbf{(Ans.)}$$

*In the side Fig- the pendulum A released from rest in the horizontal position $O_1A_1$ swings down and strikes the pendulum B initially at rest in the vertical position $O_2B_1$.*




*If the impact is perfectly elastic, find the value of the angle where the axial force in $O_2B_2$ changes from compression to tension as the ball of the lower pendulum falls along the path $B_1B_2B_3$. The weights of the balls are $W_a = 4.45N$, $W_b = 8.9N$, and the lengths $\ell = 300$ mm.*

**Soln.** Given data

$W_a = 4.45$ N $\qquad W_b = 8.9$ N

$\ell = 300$ mm $\qquad e = 1$ (elastic impact)

From the given data, $\qquad W_b = 2W_a$

i.e., If $W_a = W$, $\quad W_b = 2W$

Let $V_1$ & $V_1'$ are velocities of $W_a$ before and after impact.

$V_2$ & $V_2'$ are the corresponding velocities of $W_b$

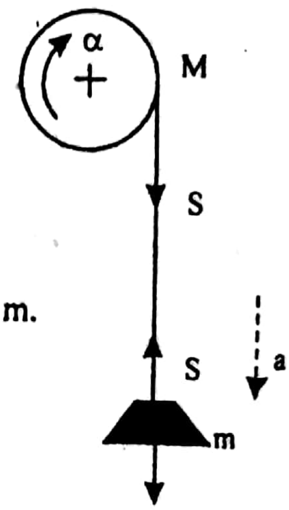
### 9.1.5 Torque and Angular Acceleration

According to 2nd law : $\Sigma T = I\alpha$

Where $\Sigma T$ = Total torque acting on a rotating body,

$I$ = mass moment of inertia, $\alpha$ = angular acceleration.

### 9.1.6 Motion of a body Tied to a string and passing over a pulley :




Let $m$ = mass of the rigid body. $M$ = Mass of the pulley

$r$ = radius of the pulley. $k$ = radius of gyration.

$I$ = mass moment of inertia.

$\alpha$ = Angular acceleration of pulley

$a$ = linear acceleration of a body.

$S$ = tension in the string.

∴ The net force acting along the direction of acceleration of mass m.

i.e. $\Sigma F = ma$ $\quad mg - S = ma \qquad$ (I)

The net torque acting along the direction of rotation of pulley.

$T = I\alpha \qquad$ (II)

where $T = S \times r$

Substituting the value of T. $\quad Sr = I\alpha$

Multiplying r by both sides $\quad Sr^2 = I\alpha r$

or $\quad Ia = Sr^2 \qquad [\because \alpha r = a] \qquad$ or $\quad S = \dfrac{Ia}{r^2}$

Substituting the 'S' value in equation (I) i.e. $mg - \dfrac{Ia}{r^2} = ma$ .

or $\quad a\left(m + \dfrac{1}{r^2}\right) = mg \qquad a = \dfrac{mg}{\left(m + \dfrac{I}{r^2}\right)}$

**Case - I :** when $I = \dfrac{Mr^2}{2}$ (mass moment of intertia of solid disc)

$$a = \frac{mg}{\left[m + \dfrac{\left(\dfrac{Mr^2}{2}\right)}{r^2}\right]} = \frac{2mg}{2m + M}$$

**Case - II** When $I = Mk^2$

$$\therefore a = \frac{mg}{\left(m + \dfrac{Mk^2}{r^2}\right)}$$

## 9.1.7 Motion of Two Bodies Connected by a String and Passing Over a Pulley

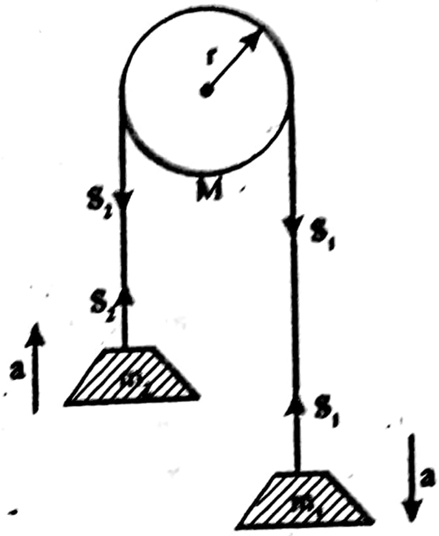
Let $m_1$ and $m_2$ = the masses of two hanging bodies.
$S_1$ and $S_2$ = tension in the right and left strings
The net force along the direction of motion of $m_1$,

$$m_1g - S_1 = m_1a \qquad \Rightarrow S_1 = m_1(g-a) \qquad \text{(i)}$$

Considering the motion of $m_2$,

$$S_2 - m_2g = m_2a \qquad \Rightarrow S_2 = m_2(a+g) \qquad \text{(ii)}$$

Considering the rotation of pulley, $T = I\alpha$ (iii)




Where Torque, $T = (S_1 - S_2)r$

Substituting the value in equation (iii) $(S_1 - S_2)r = I\alpha$

Multiplying both sides by 'r'

$$\Rightarrow (S_1 - S_2)r^2 = Ir\alpha$$

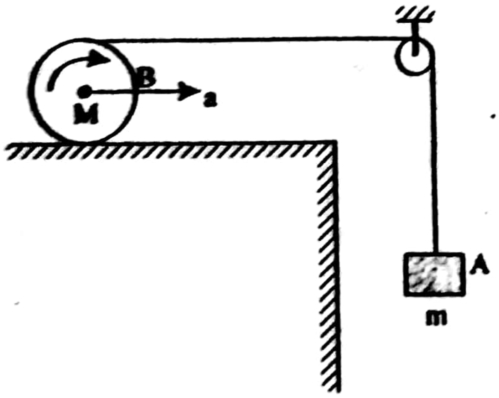
$$\Rightarrow (S_1 - S_2) = \frac{Ia}{r^2} \qquad [\because r\alpha = a] \qquad \text{(iv)}$$

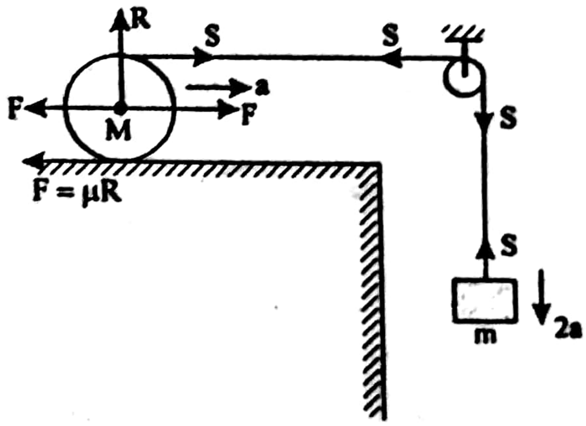
Substituting the value of $S_1$ and $S_2$ in equation (iv)

$$[m_1(g-a)] - [m_2(a+g)] = \frac{Ia}{r^2} \Rightarrow m_1g - m_1a - m_2a - m_2g = \frac{Ia}{r^2}$$

$$\Rightarrow g(m_1 - m_2) = \frac{Ia}{r^2} + m_1a + m_2a \quad \Rightarrow g(m_1 - m_2) = a\left[\frac{I}{r^2} + m_1 + m_2\right] \Rightarrow a = \frac{g(m_1 - m_2)}{\left[\frac{I}{r^2} + m_1 + m_2\right]}$$

## 9.1.8 Motion of a Body Rolling on a Rough Horizontal Plane Without Slipping




Let m = mass of the hanging body
M = mass of the roller.
$I = MK^2$

where I = mass moment of inertia

K = radius of gyration

r = radius of roller.

$\mu$ = co-efficient of friction.

$\alpha$ = angular acceleration of the roller.

a = linear acceleration of roller.

S = tension in the rope

Since the roller rolls with an acceleration 'a', the acceleration of hanging body will be 2a.

The net force along the direction of motion m,

$mg - S = 2ma \quad \Rightarrow S = m(g-2a)$ (i)

Similarly the net force acting along the direction of acceleration of roller M,

$S - F = Ma \quad$ or $S = Ma + \mu R$ (ii)

The net torque acting due to rotation of roller, $T = I\alpha$

or $F \times r = I\alpha \quad \Rightarrow Fr^2 = I\alpha r$

$\Rightarrow Fr^2 = Ia \quad [\because \alpha r = a] \quad$ or $\quad F = \frac{Ia}{r^2}$

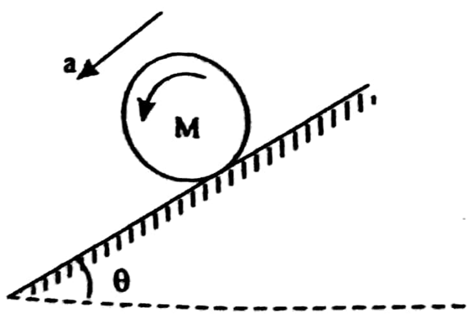
or $\quad F = \frac{Mk^2a}{r^2}$ (iii)

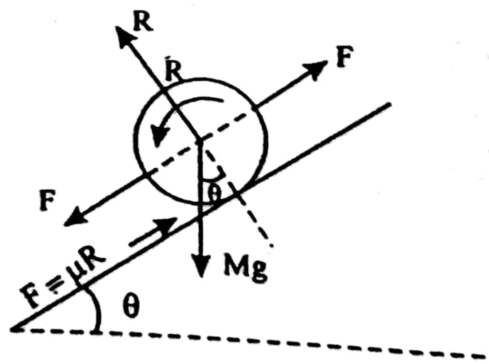
equating (i), (ii) and (iii) $m(g-2a) = Ma + F$

$$\Rightarrow m(g-2a) = Ma + \frac{MK^2a}{r^2} \quad \Rightarrow m(g-2a) = Ma\left(1+\frac{K^2}{r^2}\right) \quad \Rightarrow mg - 2ma = Ma + \frac{MK^2a}{r^2}$$

$$\Rightarrow mg = 2ma + Ma + \frac{MK^2a}{r^2} \quad \Rightarrow mg = a\left(2m + M + \frac{MK^2}{r^2}\right) \quad \Rightarrow a = \left(\frac{mg}{2m + M + \frac{MK^2}{r^2}}\right)$$

### 9.1.9 Motion of a body rolling down a rough inclined plane without slipping



= mass of the roller., $\theta$ = angle of inclination.

orce of friction, $F = \mu R$ or $F = \mu Mg\cos\theta$

he net force along the direction of acceleration $Mg\sin\theta - F = Ma$

$F = M(g\sin\theta - a)$ (i)

The net torque due to rotation of the roller $T = I\alpha$

or $F \times r = I\alpha$ $\Rightarrow Fr^2 = Ir\alpha$ $\Rightarrow Fr^2 = Ia$ $[\because r\alpha = a]$

$$\Rightarrow F = \frac{Ia}{r^2} = \frac{MK^2a}{r^2} \quad \text{(ii)}$$

Equating (i) and (ii)

$$M(g\sin\theta - a) = \frac{MK^2a}{r^2} \qquad \Rightarrow Mg\sin\theta - Ma = \frac{MK^2a}{r^2}$$

$$\Rightarrow Mg\sin\theta = a\left(M + \frac{MK^2}{r^2}\right)$$

$$\Rightarrow a = \frac{Mg\sin\theta}{M\left(1 + \frac{K^2}{r^2}\right)} \qquad \Rightarrow a = \frac{g\sin\theta}{\frac{r^2 + K^2}{r^2}}$$

From the above expression it was seens that acceleration is independent of the mass of the rolling body.

Substituting the value of 'a' in equation (ii).

$$F = \frac{MK^2\left(\frac{g\sin\theta}{\frac{r^2+K^2}{r^2}}\right)}{r^2}, \quad F = \frac{MK^2 g\sin\theta}{K^2 + r^2}$$

If the body rolls without slip, $F \le \mu Mg\cos\theta$

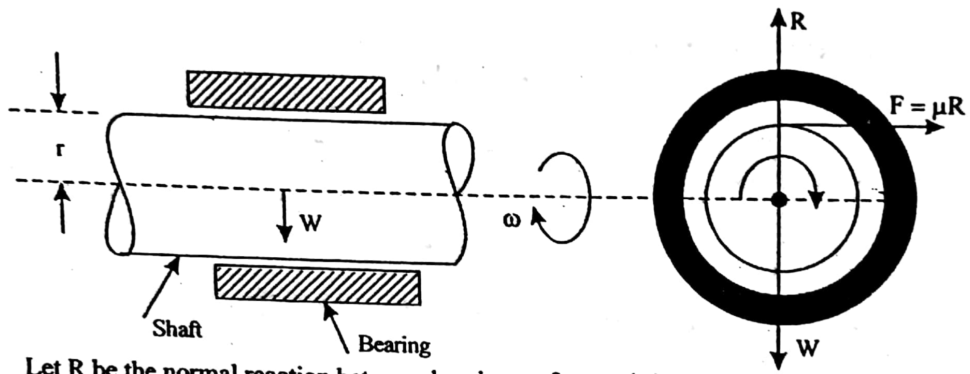
$$\text{or} \quad \frac{\mu K^2 g\sin\theta}{K^2 + r^2} \le \mu Mg\cos\theta$$

$$\Rightarrow \mu \ge \frac{\tan\theta}{\left(\frac{K^2 + r^2}{K^2}\right)}$$

Hence the minimum value of co-efficient of friction, $\mu = \dfrac{K^2(\tan\theta)}{K^2 + r^2}$

## SOLVED PROBLEMS - 9.3

1. *A shaft of radius r rotates with constant angular speed $\omega$ in bearings for which the coefficient of friction is $\mu$. Through what angle $\theta$ will it rotate after the driving torque is removed?*




**Soln.** Let R be the normal reaction between bearing surface and shaft.

$\therefore\ R = W$

$\therefore$ Force of friction $F = \mu R = \mu w$

Torque applied to the shaft, $T = F \times r = \mu wr$

Applying equation of rotation, $T = I\alpha$

Where I = mass moment of inertia $= \dfrac{mr^2}{2} = \dfrac{w}{2g} r^2$

$$\therefore \alpha = \frac{T}{I} = \frac{\mu Wr}{\left(\dfrac{W}{2g} r^2\right)} = \frac{2\mu g}{r}$$

When driving torque is removed, the angular velocity $\omega$ diminishes to zero and '$\alpha$' being the angular retardation.

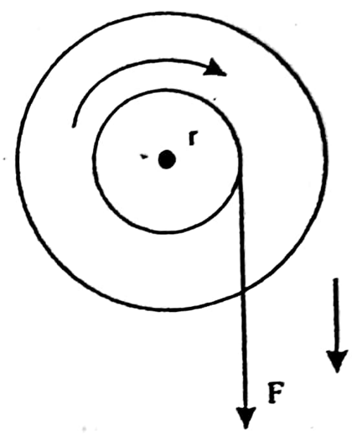
Let '$\theta$' be the total angular displacement.

Applying equation of Kinamatics.

$$\omega^2 - \omega_0^2 = 2\,\alpha\,\theta \quad \text{or} \quad 0 - \omega^2 = 2\left(-\frac{2\mu g}{r}\right)\theta$$

$$\theta = \frac{\omega^2 r}{4\mu g} \qquad \textbf{(Ans.)}$$

**2.** *The wheel of a small gyroscope is set spinning by pulling on a string wound around the shaft. Its moment of inertia is 5562.5 kg-mm² and the diameter of the shaft on which the string is wound is 12.5 mm. If 750 mm of string is pulled off with a constant force of 53.4N, what angular velocity will be imparted to the wheel ?*




**Soln.** Given data

$I = 5562.5\ \text{kg mm}^2,\quad d = 12.5\ \text{mm},\quad \ell = 750\ \text{mm}$

$F = 53.4\ \text{N} \qquad \omega = ?$

Net torque acting on the shaft $T = F \times r$

According to the Newton's 2nd law $T = I\alpha$ or $\alpha = \dfrac{T}{I}$

$$\therefore\ \alpha = \frac{F \times r}{I} = \frac{53.4}{5562.5} \times \frac{12.5}{2} = 0.06\ \text{rad/sec}^2$$

For one revolution, the angular displacement is $2\pi$.

When the whole length of the string is pulled off, the total angular distance;

$$\theta = \frac{\ell}{2\pi r} \times 2\pi = 120\ \text{rad}$$

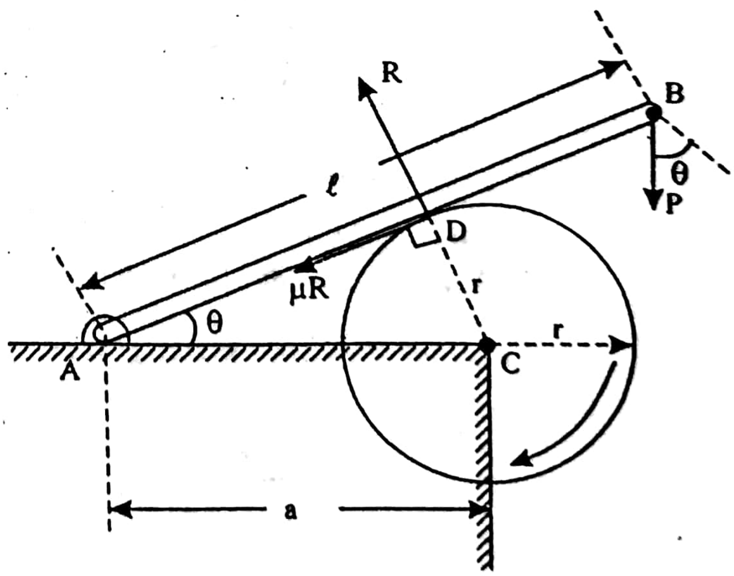
$\therefore$ Applying equation of kinematics,

$$\omega^2 - \omega_0^2 = 2\alpha\theta,\quad \omega^2 - 0 = 2 \times 0.06 \times 120$$

or $\quad \omega = \sqrt{(2 \times 0.06 \times 120)} = 3.795\ \text{rad/s}$ **(Ans.)**

**3.** *A right circular drum of radius r and weight W rotating at 600 rpm is braked by the device shown. Develop a formula for the time t required to bring the drum to rest if the coefficient of friction between the drum and breaking bar is μ. The following data are given :*

| | |
|---|---|
| *$l$ = 1.2m,* | *a = 0.75m,* |
| *r = 375mm,* | *μ = 0.25,* |
| *W = 1780N,* | *P = 445N* |



**Soln.** Given data

Initial rotation $N_0 = 600$ rpm

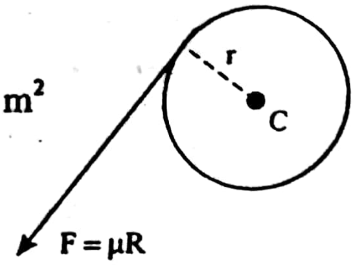
$$\therefore \omega_0 = \frac{2\pi N_0}{60} = 62.832 \text{ rad/sec.}$$

Final speed $N = 0$, $\ell = 1.2$ m,

$a = 0.75$m, $r = 375$ mm $= 0.375$ m, $\mu = 0.25$, $W = 1780$ N, $P = 445$N

We know that mass moment of inertia of circular drum

$$I = \frac{mr^2}{2} = \frac{W}{g} \times \frac{r^2}{2} = \frac{1780}{9.81} \times \frac{0.375^2}{2} = 12.758 \text{ kg} - \text{m}^2$$




Let 'R' be the reaction between plank AB and drum

Considering equilibrium of the plank 'AB',

$$\sum M_A = 0$$

$$R \times a \cos\theta = P \cos\theta \times \ell \text{ or } R = \frac{P \times \ell}{a} = \frac{445 \times 1.2}{0.75} = 712 \text{ N}$$

Considering rotation of drum

$$T = I\alpha$$

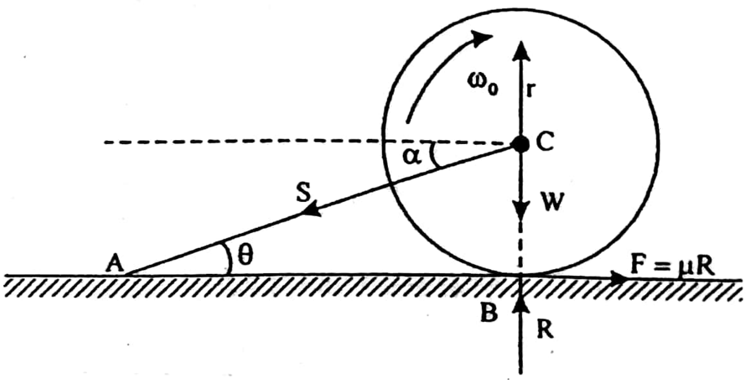
or $\quad F \times r = I\alpha \quad$ or $\quad \mu R \times r = I\alpha$

or $$\alpha = \frac{\mu R \times r}{I} = \frac{0.25 \times 712 \times 0.375}{12.758} = 5.232 \text{ rad/s}^2$$ (retardation)

Applying equation of kinamatics, $\omega - \omega_0 = \alpha t$

or $\quad 0 - \omega = (-\alpha)t \quad$ or $\quad t = \dfrac{62.832}{5.232} = 12$ sec. **(Ans.)**

4. *A solid right circular rotor of radius r and weight W tied to a horizontal plane by a rod AC has initial angular velocity $w_0$ as shown. If the rotor is suddenly allowed to rest its full weight on the plane, what time t will elapse before it comes to rest? The coefficient of friction at B is $\mu$. Numerical data are given as follows : $w_0 = 20$ rad/sec, $r = 0.3$m, $\mu = 1/4$, $\theta = 15°$.*



**Soln.** Given data

$$\omega_0 = 20\pi \text{ rad/s}, \quad r = 0.3\text{m}, \quad \mu = \frac{1}{4} = 0.2$$

$$\theta = 15^\circ \qquad I = \frac{mr^2}{2} = \frac{w}{g}\frac{r^2}{2}$$

Let 'S' be the tension in the string AC
**Considering linear equilibrium of the rotor.**

$$\sum y = 0$$

$$R = W + S \sin\theta \qquad \text{....(i)}$$

$$\sum x = 0$$

$$S\cos\theta = \mu R \qquad \text{....(ii)}$$

Solving equation (i) and (ii)

$$R = W + \frac{\mu R}{\cos\theta}\sin\theta \quad \text{or} \quad R(1-\mu\tan\theta) = W$$

or $$R = \frac{W}{(1-\mu\tan\theta)} \qquad \text{....(iii)}$$

**Considering rotation of the rotor**

$$T = I\alpha \quad \text{or} \quad F \times r = I\alpha$$

or $$\mu R \times r = I\alpha \quad \text{or} \quad \alpha = \frac{\mu W r}{(1-\mu\tan\theta)} \times \frac{2g}{Wr^2}$$

$$= \frac{2\mu g}{r(1-\mu\tan\theta)} \qquad \text{(retardation)}$$

**Applying equation of kinematics**

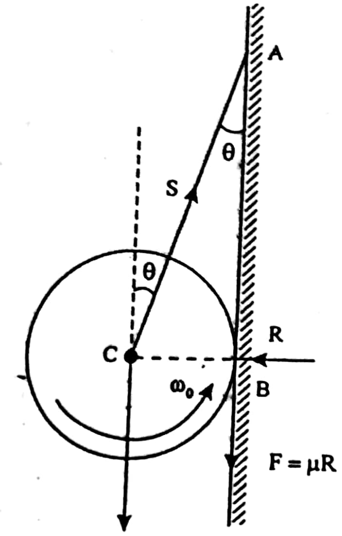
$$\omega - \omega_0 = \alpha t \quad \text{or} \quad 0 - \omega_0 = (-\alpha)t$$

or $$t = \frac{\omega_0}{\alpha} = \frac{\omega_0(1-\mu\tan\theta)}{2\mu g}$$

Substituting the values;

$$t = 20\pi \times 0.3\,\frac{(1-0.25\times\tan 15^\circ)}{2\times 0.25\times 9.81} = 3.6\text{ sec.} \qquad \textbf{(Ans.)}$$

**5.** ***Solve problem 4 for the case where AB is a vertical wall instead of a horizontal floor. Use numerical data as above.***




**Soln.** Given data

$\omega_0 = 20\pi$ rad / s

r = 0.3m

$\mu = 0.25,\ \theta = 15°$

$$I = \frac{mr^2}{2} = \frac{w}{g} \times \frac{r^2}{2}$$

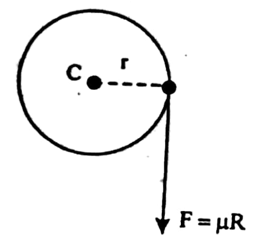
**Considering linear equation of the rotor**

$\sum y = 0 \qquad S\cos\theta = W + \mu R \quad$ (i)

$\sum x = 0 \qquad S\sin\theta = R \quad$ (ii)

**Dividing equation (i) and (ii)**

$$\cot\theta = \frac{W + \mu R}{R} \quad \text{or} \quad R\cot\theta = W + \mu R$$




$$\text{or } R(\cot\theta - \mu) = W \text{ or } R = \frac{W}{(\cot\theta - \mu)} \quad \text{(iii)}$$

**Considering rotation of the rotor**

$T = I\alpha \qquad$ or $\qquad \mu R \times r = I\alpha$

$$\text{or } \mu \times \frac{W}{(\cot\theta - \mu)} \times r = \frac{w}{g}\frac{r^2}{2} \times \alpha$$

$$\text{or } \alpha = \frac{2\mu g}{r(\cot\theta - \mu)} \quad \text{(retardation)} \quad \text{(iv)}$$

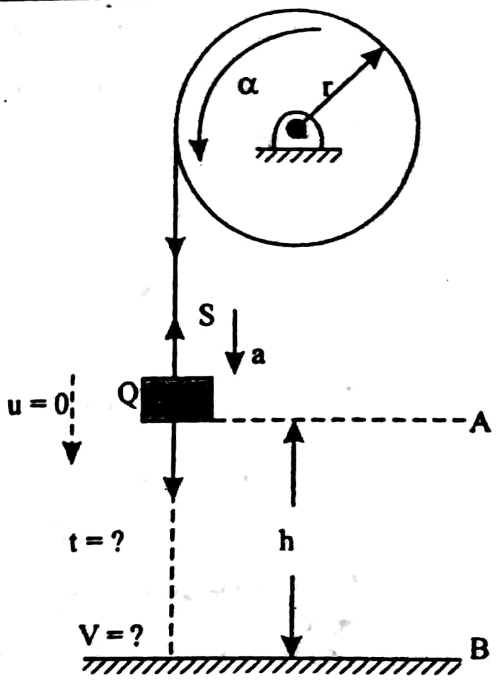
Applying equation of Kinamatics

$$\omega - \omega_0 = (-\alpha)t \quad \text{or } 0 - \omega_0 = -\alpha t \text{ or } t = \frac{\omega_0}{\alpha} = \frac{\omega_0 r(\cot\theta - \mu)}{2\mu g} \quad \text{(Ans.)}$$

Substituting the values

$$t = \frac{20\pi \times 0.3(\cot 15° - 0.25)}{2 \times 0.25 \times 9.81} = 13.4 \text{ sec.} \quad \text{(Ans.)}$$

6. *A solid right circular drum of radius r = 0.3m. and weight W = 143.3N is free to rotate about its geometric axis as shown in the fig. wound around the circumference of the drum is a flexible cord carrying at its free end a weight Q = 44.5N. If the weight Q is released from rest, (a) Find the time t required for it to fall through the height h = 3m. (b) With what velocity V will it strike the floor ?*




**Soln.** Given data r=0.3m, W=143.3N Q=44.5N

h=3m, $I = \dfrac{mr^2}{2} = \dfrac{w}{g}\dfrac{r^2}{2}$

**Considering linear motion of 'Q'**

$$Q - S = \frac{Q}{g} \times a \quad \text{or} \quad S = Q\left(1 - \frac{a}{g}\right) \qquad \text{....(i)}$$

**Considering rotation of drum**

$$T = I\alpha \quad \text{or} \quad S \times r = I \,.\, \alpha$$

But we know that $a = r\alpha$ or $\alpha = \dfrac{a}{r}$

Substituting the value of '$\alpha$' $S \times r = I \times \dfrac{a}{r} = \dfrac{W}{g}\dfrac{r^2}{2} \times \dfrac{a}{r}$

or $$S = \frac{Wa}{2g} \qquad \text{....(ii)}$$

Equating equations (i) and (ii)

$$Q\left(1 - \frac{a}{g}\right) = \frac{Wa}{2g} \quad \text{or } Qg - Qa = \frac{Wa}{2} \quad \text{or} \quad a\left(\frac{W}{2} + Q\right) = Qg$$

or $$a = \frac{2Qg}{(W + 2Q)} \qquad \text{.....(iii)}$$

Substituting the values; $a = \dfrac{2 \times 44.5 \times 9.81}{(143.3 + 2 \times 44.5)} = 3.758 \text{ m/s}^2$

**Considering the vertical motion of Q from A to B**
S = h, u = 0

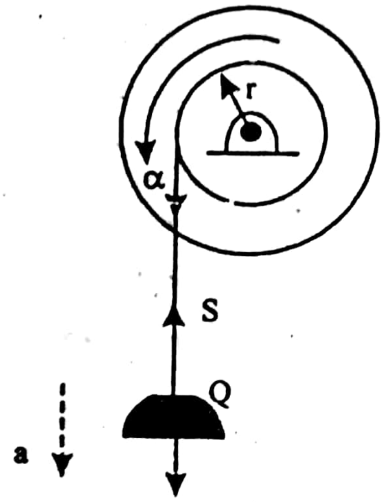
We know that $S = ut + \dfrac{1}{2} at^2$

or $3 = 0 + \dfrac{1}{2} \times 3.758 \times t^2$ or $t = 1.27$ sec. **(Ans.)**

Now, let V be the final velocity

$\therefore$ $V = u + at$ or $V = 0 + 3.758 \times 1.27 = 4.76 \text{ m/s}$ **(Ans.)**

7. *The rotor and shaft in fig. weigh 2225N, and the radius of gyration with respect to the axis of rotation is 250mm. Calculate the acceleration '$\ddot{x}$' of the falling weight W = 445N if the shaft radius r = 5 cm.*




Soln. **Given data**
W = 2225 N (wt of the rotor and shaft)
Radius of gyration, K = 250 mm = 0.25m
$\ddot{x} = a = ?$
Weight of the falling wt, Q = 445N, Shaft radius, r = 125mm = 0.125m

$\therefore I = mk^2 = \dfrac{W}{g} \times k^2$ Also $\alpha = \dfrac{a}{r}$, Given that W = 5Q

**Considering the motion of falling weight Q**

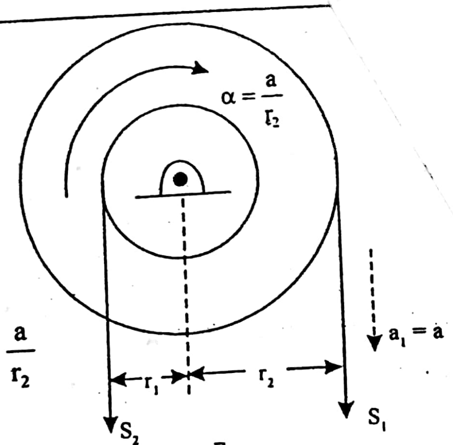
$$Q - S = \frac{Q}{g} \times a \quad \text{....(i)}$$

Where a = constant linear acceleration.
**Considering motion of the rotor and shaft.**

$T = I\alpha$ or $S \times r = \dfrac{W}{g} k^2 \times \dfrac{a}{r}$

or $$S = \frac{W}{g}\left(\frac{K}{r}\right)^2 .a \quad \text{.....(ii)}$$

**Motion of two steped pulley**




$$\sum T = I\alpha$$

$$S_1 \times r_2 - S_2 \times r_1 = I \cdot \alpha \quad \text{.....(iii)}$$

where $\alpha = \frac{a_1}{r_2} = \frac{a_2}{r_1}$, $\quad I = \frac{W}{g} k^2$

Substituting the values in equation (iii)

$$P\left(1-\frac{a}{g}\right) \times r_2 - P\left(1+\frac{2a}{3g}\right) r_1 = \frac{w}{g} \times k^2 \times \frac{a}{r_2}$$

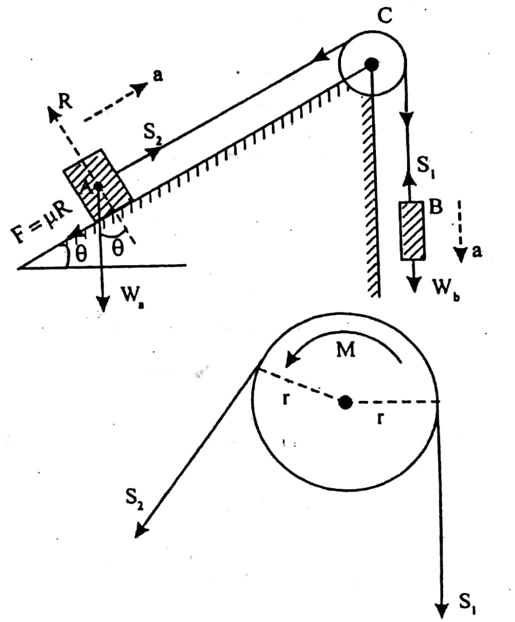
But, W = 8p; substituting;

$$\therefore \; r_2 - \frac{ar_2}{g} - r_1 - \frac{2ar_1}{3g} = \frac{8}{g} k^2 \times \frac{a}{r_2} \quad \text{or} \quad a\left[\frac{8k^2}{gr_2} + \frac{2r_1}{3g} + \frac{r_2}{g}\right] = r_2 - r_1$$

$$\frac{a}{g}\left[\frac{8 \times 0.17675^2}{0.375} + \frac{2 \times 0.25}{3} + 0.375\right] = 0.375 - 0.25 \quad \text{or} \quad a \times 1.208 = g(0.125)$$

$$\therefore \; a = 1.01 \text{ m/s}^2 \qquad \textbf{(Ans.)}$$

10. *Fig.. represents system consisting of a block A of weight $W_a$ = 3853.7 N, connected to a block B of weight $W_b$ = 2865.8N by a flexible cord which runs over a pulley of radius r = 300mm and weight $W_c$ = 1432.9N. The block A slides along a plane inclined to the horizontal by the angle θ= 30° and for which the coefficient of friction is μ = 0.1. Friction on the axle of the pulley is assumed to supply a constant resisting torque of 13.35N-m. Assuming that no slipping occurs between the cord and the pulley, determine (a) the acceleration of the blocks and (b) the maximum tensile force in the cord.*



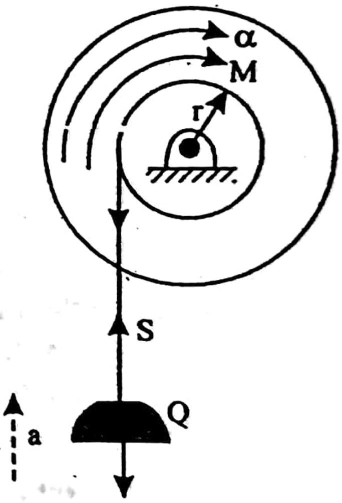
solving equation (i) and (ii)

$$Q - \frac{W}{g}\left(\frac{K}{r}\right)^2 . a = \frac{Q}{g} \times a \quad \text{or} \quad \frac{445}{g} \times a + \frac{2225}{g}\left(\frac{k}{r}\right)^2 a = 445$$

$$\text{or} \quad \frac{a}{g} + \frac{5}{g}\left(\frac{k}{r}\right)^2 a = 1 \quad \text{or} \quad \frac{a}{g}\left[1 + 5\left(\frac{K}{r}\right)^2\right] = 1$$

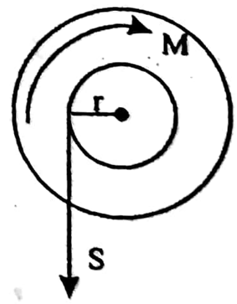
$$\text{or} \quad a = \frac{g}{\left[1+5\left(\frac{K}{r}\right)^2\right]} = \frac{g}{1+5\times\left(\frac{0.25}{0.125}\right)^2} = \frac{g}{(1+5\times 4)}$$

$$\text{or} \quad a = \frac{g}{21} \qquad \text{(Ans.)}$$




**8.** ***Using the same numerical data as in problem-7. Calculate the torque M that must be applied to the shaft of the rotor to produce an upward acceleration (g/3) of the attached weight W.***

**Soln.** Let M be the torque applied in opposite direction to lift the load 'Q'




Given that $a = \frac{g}{3}$

Upward motion of $S - Q = \frac{Q}{g} \times a$

$$\text{or} \quad S = Q\left(1+\frac{a}{g}\right) = Q\left(1+\frac{1}{3}\right) = \frac{4}{3}Q$$

**Rotation of rotor and shaft**

Net torque $\sum T = (M - Sr)$

$\therefore$ According to Newton's 2nd law

$$\sum T = I\alpha$$

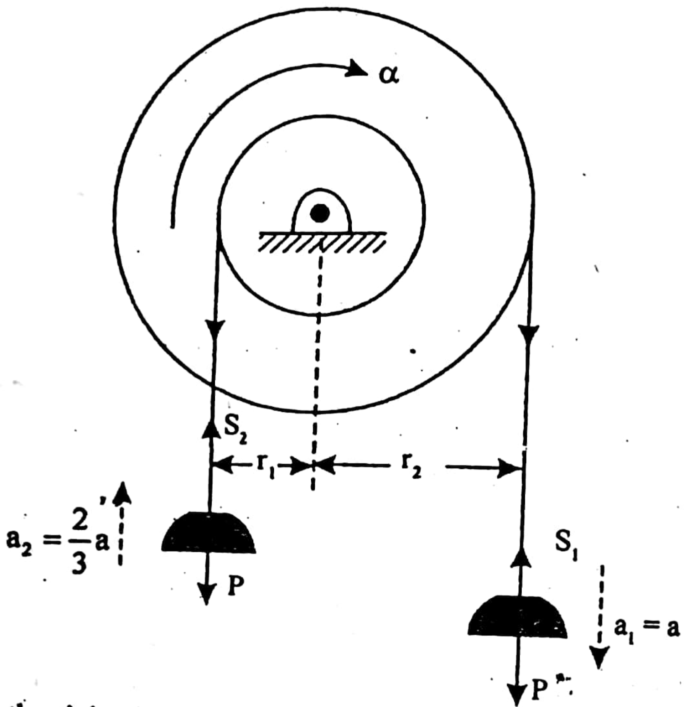
$$\text{or} \quad (M - Sr) = I\alpha$$

or $$M = \left(\frac{4}{3}Q\right) \times r + \left(\frac{W}{g}k^2\right) \times \frac{a}{r}$$

$$= \left(\frac{4}{3} \times 445 \times 0.125\right) + \left(\frac{2225}{9.81} \times 0.25^2 \times \frac{9.81}{3}\right) \times \frac{1}{0.125}$$

$\therefore M = 445\ N - m$ **(Ans.)**

9. *The two-step pulley as shown in fig. has weight W = 1780N and radius of gyration K = 176.75mm. Develop a formula for the downward acceleration of the falling weight p on the right if p = 222.5N, $r_1$ = 250mm and $r_2$ = 375mm.*




**Soln.** **Given data**

Weight of two step pulley, W = 1780 N, Radius of gyration, $i_0 = K = 176.75$ mm, P = 222.5N. $r_1$ = 250 mm, $r_2$ = 375 mm

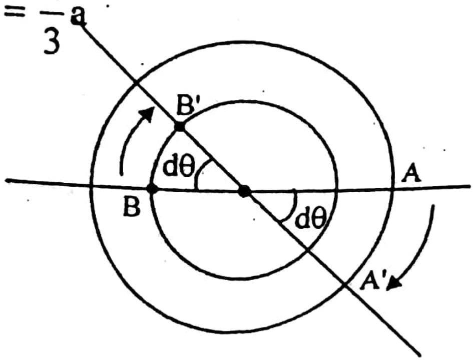
Given that $W = 8 \times p$

Let 'a' be the acceleration of falling weight 'P' on the right side

From the geometry of the figure

$$d\theta = \frac{AA'}{r_2} = \frac{BB'}{r_1} \text{ or } BB' = \frac{r_1}{r_2} \times AA'$$

$\therefore$ Accelerating of 'P' on the left (moving up) $= \frac{250}{375} \times a = \frac{2}{3}a$




Motion of P on the right

$$P - S_1 = \frac{P}{g} \times a \quad \text{or} \quad S_1 = P\left(1 - \frac{a}{g}\right) \quad \text{...(i)}$$

Motion of P on the left

$$S_2 - P = \frac{P}{g} \times \left(\frac{2}{3}a\right) \quad \text{or} \quad S_2 = P\left(1 + \frac{2a}{3g}\right) \quad \text{.....(ii)}$$

**Soln.** **Given data**

$W_a = 3853.7N$, $W_b = 2865.8\,N$, $r = 300mm = 0.3m$

Weight of the pulley, $W_c = 1432.9N$

$\theta = 30^\circ$ $\mu = 0.1$

Resisting torque, $M = 13.35$ N-m

**Motion of the block - $W_b$**

$$W_b - S_1 = \frac{W_b}{g} \times a \quad \text{or} \quad S_1 = W_b\left(1 - \frac{a}{g}\right) \qquad \text{...(i)}$$

**Motion of the block $W_a$**

$$S_2 - W_a \sin\theta - \mu R = \frac{W_a}{g} \times a$$

**But,** $R = w_a \cos\theta$

$$\therefore S_2 = W_a\left(\sin\theta + \mu\cos\theta + \frac{a}{g}\right) \qquad \text{.....(ii)}$$

**Motion of the pulley**

$$\sum T = I\alpha$$

$$\therefore S_1 \times r - S_2 \times r - M = I\alpha \qquad \text{.....(iii)}$$

But $\alpha = \dfrac{a}{r}$

and $I = \dfrac{mr^2}{2} = \dfrac{W_c}{g} \times \dfrac{r^2}{2}$

$$W_b\left(1 - \frac{a}{g}\right) - W_a\left(\sin\theta + \mu\cos\theta + \frac{a}{g}\right) - \frac{M}{r} = \frac{W_c}{g} \times \frac{r^2}{2} \times \frac{a}{r} \times \frac{1}{r}$$

Substituting the numerical values

$\Rightarrow$ $2865.8 - 292.13a - 1926.85 - 333.74 - 392.834a - 44.5 = 73.037\,a$

$\Rightarrow$ $758.001a = 560.71$

$\therefore a = 0.73\ m/s^2$ **(Ans.)**

**Maximum Tension '$S_1$'**

$$S_1 = W_b\left(1 - \frac{a}{g}\right) = 2865.8\left(1 - \frac{0.73}{9.81}\right) = 2649.7\,N.$$ **(Ans.)**

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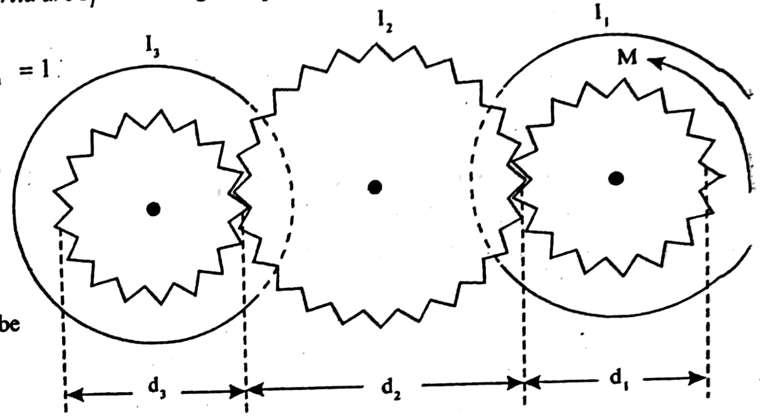
**11.** *Fig.. represents a system of two rotors of moments of inertia $I_1$ and $I_3$ which can rotate about their parallel geometric axes and are connected by an idling gear of moment of inertia $I_2$ as shown. If a constant driving torque $M = 133.5$ N-m is applied to the first-rotor $I_1$, what angular acc$^n$, of the rotor $I_3$ will be produced? Assume that the mechanical efficiency of the system is 1 and that the pitch diameters of the three gears are in the ratios $d_1 : d_2 : d_3 = 1 : 4 : 2$. The moments of inertia are $I_1 = 22.25 kg\text{-}m^2$, $I_2 = 8.9$ kg-m$^2$, $I_3 = 33.38 kg\text{-}m^2$.*

**Soln.** Given data




$M = 133.5$ N-m, $\eta_{mech} = 1$

$d_1 : d_2 : d_3 = 1 : 4 : 2$

$$I_1 = 22.25 \text{ kg} - \text{m}^2,$$

$$I_2 = 8.9 \text{ kg} - \text{m}^2$$

$$I_3 = 33.38 \text{ kg} - \text{m}^2$$

Ratio of their radie will be

$r_1 : r_2 : r_3 = 1 : 4 : 2$

Let $r_1 = r$

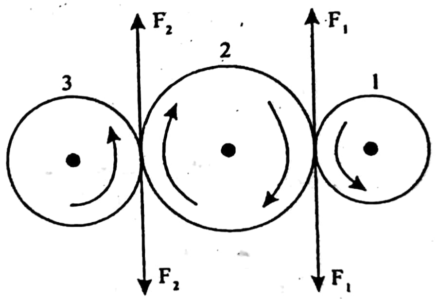
$$\therefore\ r_2 = 4r,\ r_3 = 2r.$$

Tangential acceleration at pitch point (contact point) between two gears remain same

i.e., $a_1 = a_2$

or $\quad r_1\alpha_1 = r_2\alpha_2$

or $\quad \alpha_1 = 4\alpha_2 \qquad$ ....(i)




Similarly between '2' and '3'

$a_2 = a_3$

or $\quad r_2\alpha_2 = r_3\alpha_3$

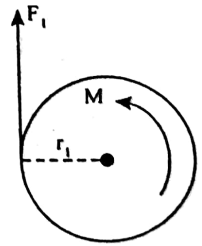
or $\quad 4r\alpha_2 = 2r\alpha_3$

$$\therefore\quad \alpha_2 = \frac{\alpha_3}{2} \qquad \text{....(ii)}$$

Equating (i) and (ii)

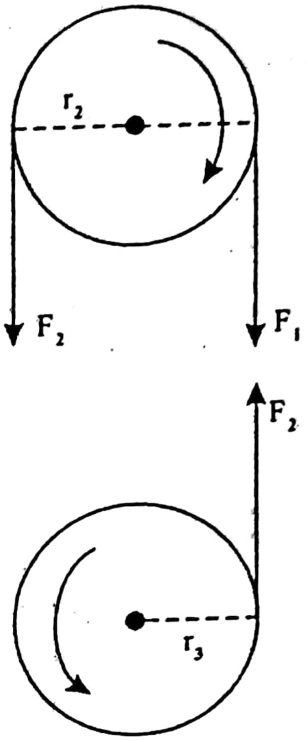
$$\alpha_1 = 4\alpha_2 = 4 \times \frac{\alpha_3}{2}$$

$$\therefore\ \alpha_1 = 2\alpha_3 \qquad \text{.....(iii)}$$




Let $F_1$ be the common tangential force between gear -1 and gear-2.
$F_2$ be the tangential force between gear -2 and gear -3
**Motion of Gear -1**

$$\sum T = I\alpha$$




or $\quad M - F_1 \times r = I_1 \cdot \alpha_1$

or $\quad M - F_1 r = 22.25 \times 2\alpha_3 = 44.5\,\alpha_3 \quad$ ....(iv)

**Motion of Gear - 2**

$$\sum T = I_2\alpha_2 = F_1 \times r_2 - F_2 \times r_2 = I_2\alpha_2$$

or $\; 4r\left[F_1 - F_2\right] \cancel{= 8.9 \times} \dfrac{\alpha_3}{2}$

or $\left(F_1 - F_2\right)r = 1.1125\,\alpha_3 \quad$ .....(v)

**Motion of Gear - 3**

$$\sum T = I\alpha, \quad F_2 \times r_3 = I_3\alpha_3$$

or $\quad F_2 \times 2r = 33.38 \times \alpha_3$

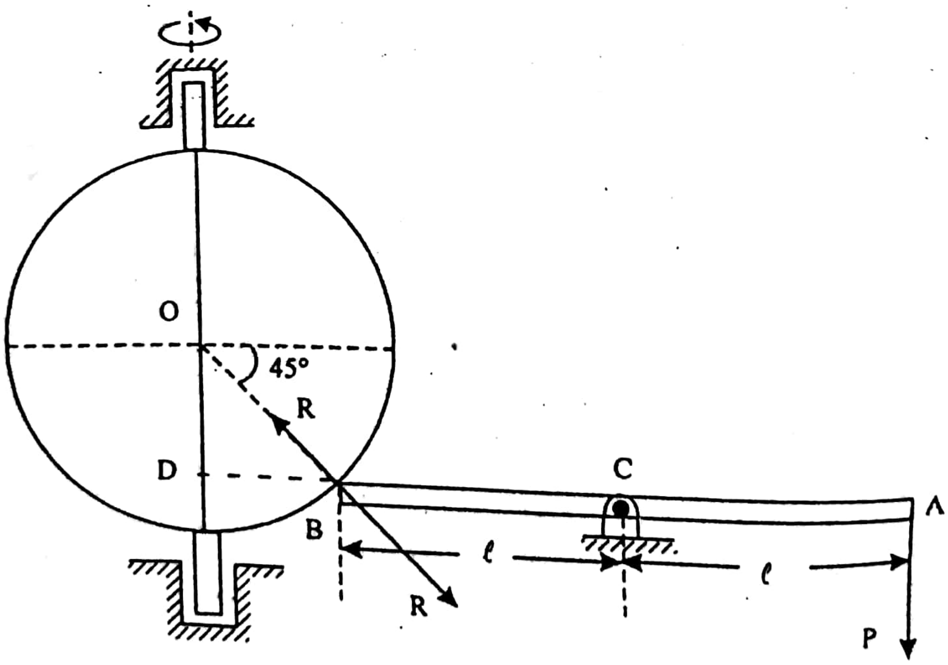
or $\quad F_2 r = 16.69\,\alpha_3 \quad$ .....(vi)

Now adding equation (iv), (v) and (vi)

We get $\left(M - F_1 r\right) + \left(F_1 r - F_2 r\right) + F_2 r = 44.5\,\alpha_3 + 1.1125\,\alpha_3 + 16.69\,\alpha_3$

$$\Rightarrow M = 62.3025\,\alpha_3 \quad \Rightarrow \alpha_3 = \frac{133.5}{62.3025} = 2.143 \text{ rad/s}^2 \qquad \text{(Ans.)}$$

12. *A homogeneous sphere of radius $r = 0.3m$ and weight $W = 178N$ rotates about a vertical diameter with initial angular velocity $\omega = 20\pi$ rad/sec and is braked by the device shown. Neglecting friction in the bearings, determine the time required for the brake to bring the sphere to rest if the coefficient of friction at B is $\mu = 1/3$ and $P = 44N$.*



**Soln.** Given data

$r = 0.3m$, $W = 178N$, $\omega_0 = 20\pi \text{ rad/s}$

$\mu = \frac{1}{3}$, $P = 44N$, $I = \frac{2}{5}\frac{W}{g}r^2$ (for solid sphere)

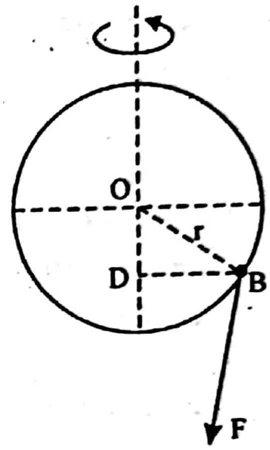

Let 'R' be the normal reaction at 'B'

Consider the equilibrium of 'AB'

$$\sum M_C = 0$$

$$P \times \ell = R\cos 45° \times \ell$$

or $R = \sqrt{2}P$ ....(i)




$\therefore$ Force of friction $F = \mu R = \mu \times \sqrt{2}\,P$

$\therefore$ Net resisting torque,

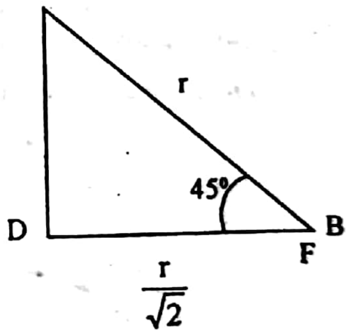
$T = F \times BD$

or $T = F \times r\cos 45°$

$$= \mu\sqrt{2} \times P \times \frac{1}{\sqrt{2}} \times r$$

$$= \mu pr$$

$\therefore$ According to Newton's law




$$\sum T = I\alpha$$

or $$\mu Pr = \left(\frac{2}{5}\frac{W}{g} \times r^2\right) \times \alpha$$

or $$\alpha = \frac{5\mu Pg}{2Wr} = \frac{5 \times 44 \times 9.81}{3 \times 2 \times 178 \times 0.3}$$

$$= 6.736 \text{ rad/s}^2 \text{ (retardation)}$$

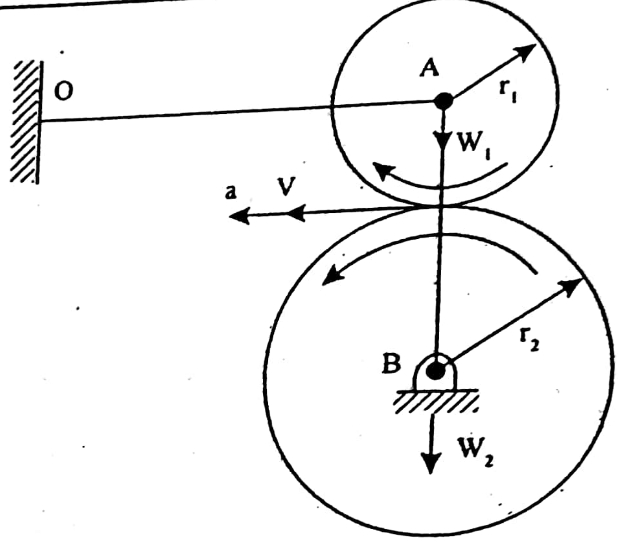
$\therefore$ Applying equation of Kinematics

$$\omega - \omega_0 = (-\alpha)t$$

or $0 - \omega_0 = -\alpha t$

or time required; $t = \frac{20\pi}{6.736} = 9.33$ sec. **(Ans.)**

**13.** *A circular rotor of weight $W_1$ = 133.5N and radius $r_1$ = 250 mm rotating with an initial angular velocity $\omega_0$ = 100 rad/sec is suddenly allowed to rest its full weight against another rotor of weight $W_2$ = 267N and radius $r_2$ = 375mm. which is free to rotate about its geometric axis parallel to that of the first but is initially at rest. At frist there will be slipping at the point of contact, but gradually the first rotor will be decelerated and the 2nd accelerated until no further slipping occurs. Find the time t that elapses before this condition prevails. Neglect friction in the bearings, and assume that the coefficient of friction between the surfaces of rotors is = 0.25.*



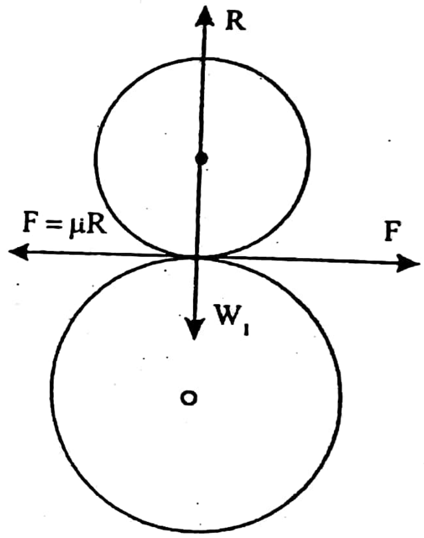

**Soln.** Given data

$W_1 = 133.5N \qquad r_1 = 250 \text{ mm} = 0.25\text{m}$

$\omega_0 = 100 \text{ rad/s}$

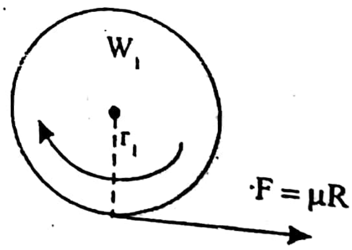
$W_2 = 267 \text{ N}, \; r_2 = 375 \text{ mm} = 0.375 \text{ m}$

$\mu = 0.25, \; I = \dfrac{m_1 r_1^2}{2}, \quad I_2 = \dfrac{m_2 r_2^2}{2}$




Since the roter $W_2$ is at rest, normal reaction between first rotor and 2nd rotor is the weight first rotor,

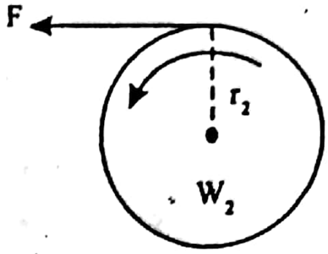
i.e., $\quad R = W_1 = 133.5N$




$\therefore$ Force of friction at the point of

contact $F = \mu R = \mu W_1$

$\therefore$ Rotation of 1st rotor




$$\sum T = I_1\alpha_1$$

or $$\mu W_1 \times r_1 = \frac{W_1}{g}\frac{r_1^2}{2} \times \alpha_1$$

or $$\alpha_1 = \frac{2\mu g}{r_1} = \frac{2 \times 0.25 \times 9.81}{0.25} = 19.62 \text{ rad/s}^2 \text{ (retardation)}$$

Using equation of kinamatics, $\omega_1 - \omega_0 = \alpha_1 t$

$\therefore\ \omega_1 = (100 - 19.62\,t)$

$\therefore$ Tangential velocity at the point of contact, $V = r_1\omega_1 = r_2\omega_2$

$\therefore$ $$V_1 = 0.25\,(100 - 19.62t) \qquad \text{....(i)}$$

**Rotation of 2nd rotor in same time period**

$$\sum T = I_2\alpha_2 \quad \text{or} \quad F \times r_2 = I_2\alpha_2$$

or $$\mu W_1 r_2 = \frac{W_2}{g} \times \frac{r_2^2}{2} \times \alpha_2 \quad \text{or} \quad \alpha_2 = \frac{\mu W_1 \times 2g}{r_2 W_2}$$

$$= \frac{0.25 \times 133.5 \times 2 \times 9.81}{0.375 \times 267} = 6.54 \text{ rad/s}^2$$

$\therefore$ Using equation of kinamatics

$$\omega_2 - \omega_0 = \alpha_2 t$$

or $$\omega_2 - 0 = \alpha_2 t$$

or $$\omega_2 = 6.54 \times t$$

$\therefore$ Tangential velocity at point of contact;

$$V_2 = r_2\omega_2 = 0.375 \times 6.54\,t = 2.4525\,t \qquad \text{......(ii)}$$

Since the tangential velocity at the point of contact is same

$$V_1 = V_2$$

or $$0.25\,(100 - 19.62t) = 2.4525t$$

or $$t = 3.4 \text{ sec.} \qquad \text{(Ans.)}$$

14. *A flywheel of mass 10 tonnes starts from rest and gets up a speed of 200 rpm in 2 minutes. Find the average torque exerted on it, if the radius of gyration of the flywheel is 60 cm.*

**Soln.** Given that

Mass of the flywheel (M) = 10t = 10000 kg

Initial angular speed ($\omega_0$) = 0 (because it starts from rest)

Final angular speed ($\omega$) = 200 rpm = $\frac{200 \times 2\pi}{60} = 6.67\ \pi$ rad / sec.

Time (t) = 2 min = 120 s

Radius of the gyration of the flywheel (k) = 60cm = 0.6m

Let $\alpha$ = constant angular acceleration of the flywheel

We know that the mass moment of inertia of the flywheel

$$I = Mk^2 = 10000 \times (0.6)^2 = 2600 \text{ kg m}^2$$

We know that $\omega = \omega_0 + \alpha t$ or $6.67\ \pi = \omega_0 + \alpha t = 0 + \alpha \times 120$

$$\therefore \alpha = \frac{6.67\pi}{120} = 0.174 \text{ rad/s}^2$$

Average torque exerted by the flywheel $T = I\alpha = 3600 \times 0.174 = 628.64$ N – m **(Ans.)**

15. *A flywheel is made up of steel ring 30 mm thick and 180 mm wide plate with mean diameter of 3 meters. If initially the flywheel is rotating at 250 rpm. Find the time taken by the wheel in coming to rest due to frictional couple of 100 Nm. Take mass density of the steel as 7500 Kg/m³. Neglect the effect of the spokes.*

**Soln.** Given data

Thickness of flywheel = 30 = 0.03m, Width of flywheel = 180mm = 0.18m

Mean dia. of flywheel = 3m, mean radius (r) = 1.5m

Initial speed ($N_0$) = 250 rpm

$$\omega_0 = \frac{2\pi N_0}{60} = \frac{2 \times \pi \times 250}{60} = 8.33\ \pi$$

Friction $\omega$ couple = 100N - m, density of steel = 7500 kg/m³

Let $\alpha$ = angular acceleration of flywheel

t = time taken by the flywheel in coming to rest.

We know that volume of flywheel; $v = \pi \times 0.03 \times 0.18 \times 3 = 0.05 \text{ m}^3$

Mass of flywheel, $M = 0.5 \times 7500 = 381.7$ kg

Mass moment of inertia $I = Mr^2 = 381.7 \times (1.5)^2 = 858.8 \text{ kg.m}^2$

We know that frictional couple $100 = I\alpha$

$$\Rightarrow \alpha = \frac{100}{858.8} = 0.116 \text{ rad/s}^2$$

We know that; $\omega = \omega_0 - \alpha t$ (retardation)

$$\Rightarrow 0 = 8.33\pi - 0.116 \times t \Rightarrow t = \frac{8.33\pi}{0.253} = 124.1 \text{ s}$$ **(Ans.)**

16. *A homogeneous solid cylinder of mass 150 kg and 2m. diameter, whose axis is horizontal, rotates about its axis, in frictionless bearings under the action of a falling block of mass 15 kg, which is carried by a thin rope wrapped around the cylinder. What will be the angular velocity of the cylinder, 3 seconds after the motion ? Neglect the weight of the rope ?*

**Soln.** Mass of cylinder (M) = 150 kg

Diameter of the cylinder (D) = 2m

radius = r = 1m

Mass of the block (m) = 15kg

time (t) = 3 sec.

Let a = linear acceleration of the solid cylinder

$\alpha$ = angular acceleration

We know that,

$$a = \frac{2mg}{2m+M} = \frac{2 \times 15 \times 9.81}{(2 \times 15) + 150} = 1.635 \text{ m/s}^2$$

$$\therefore \alpha = \frac{a}{r} = \frac{1.635}{1} = 1.635 \text{ rad/s}^2$$

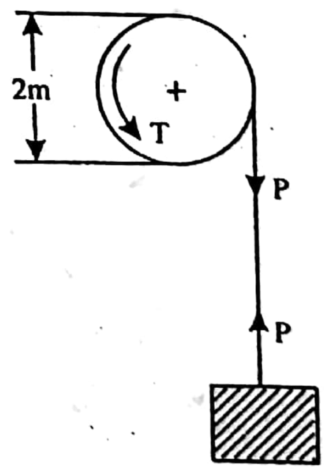
$\therefore$ Angular velocity of the cylinder 3 seconds after the motion :

$$\omega = \omega_0 + \alpha t = 0 + (1.635 \times 3) = 4.905 \text{ rad/s.}$$

17. *A solid cylinderical pulley of mass 1000 kg, having 0.8m, radius of gyration and 2m. diameter, is rotated by an electric motor, which exert a uniform torque of 50 KN/m. A body of mass 3 tonne is to lifted by a wire wrapped round the pulley.*

*Find (i) Acceleration of the body*

*(ii) Tension in the rope.*



**Soln.** Given data

Mass of pulley $M = 100$ kg

Radius of gyration $= k = 0.8$ m

Diameter of pulley $d = 2$m

radius $= r = 1$m

Torque $T = 50$ KN-m $= 50000$ N-m

Mass of the body to be lifted $m = 3t = 3000$ kg

$\therefore$ weight of the falling body

$$W = m \times g = 300 \times 9.81 = 29400 \text{ N}$$

(i) Let $a$ = acceleration of the body

$\alpha$ = angular acceleration

$P$ = tension in rope

Consider the motion of the hanging body, which is going upwards due to torque

$$P - W = \frac{W}{g} \times a$$

or $P - 29400 = 3000\,a$ ....(i)

We know that, $I = MK^2 = 1000 \times (0.8)^2 = 640 \text{ kg/m}^2$

Accelerating torque $T_1 = I\,\alpha = 640\,\alpha = 640\,a$

$\because \alpha = a$

Torque due to tension in the rope

$$T_2 = P \times r = P \times 1 = P$$

$\therefore$ Total torque $= (T_1 + T_2) = 640\,a + P$ ....(ii)

Total torque = torque exerted by the electric motor

$\Rightarrow 50000 = 640\,a + p$

$\Rightarrow P = 50000 - 640\,a$

$\Rightarrow$ Substituting the value of 'P' in equation (i)

$\therefore 50000 - 640\,a - 29400 = 3000\,a$

$\Rightarrow 20600 = 3640\,a \quad \Rightarrow a = 5.66 \text{ m/s}^2$

(ii) Tension of the rope

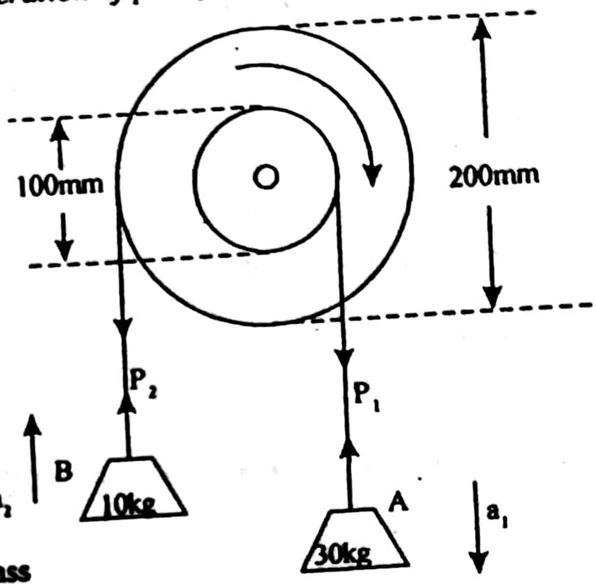
Substituting the value a in equation (iii)

$P - 29400 = 3000 \times 5.66 \quad \Rightarrow \quad P = 29400 + 16980$

$\Rightarrow \quad P = 46380 = 46.38$ KN

**(Ans.)**

**18.** *Two bodies A & B of masses 30 kg and 10 kg are tied to the two ends of a light string passing over a composite pulley of radius of gyration as 60mm and mass 40 kg as shown in fig. Find the pull in the two parts of the string and angular acceleration of pulley ?*




**Soln.** Given data

Mass of body $A(m_1) = 30$ kg

Mass of body $B(m_2) = 10$ kg

radius of gyration $K = 60mm = 0.06m$

mass of pully $(M) = 40$ kg

Internal dia of pulley $d_1 = 100$ mm $= 0.1$ m

$\therefore$ radius $r_1 = 0.05m$

external dia $d_2 = 200$ mm $= 0.2m$

$\therefore$ radius $r_2 = 0.1m$

Let $P_1$ = Pull in string carrying 30 kg mass

$P_2$ = Pull in string carry 10 kg mass

$\alpha$ = angular acceleration

$a_1, a_2$ are the acceleration of mass $m_1$ and $m_2$

Considering $m_1$ .......(i)

Net force along the direction of acceleration, $m_1 g - P_1 = m_1 a_1$

Considering $m_2$ .....(ii)

Net force along the direction of acceleration, $P_2 - m_2 g = m_2 a_2$

Considering pulley block

The net torque along the direction of motion,

$$\Sigma T = I\alpha \quad \text{or} \quad P_1 r_1 - P_2 r_2 = I\alpha \qquad \text{......(iii)}$$

But we know that,

$$a_1 = r_1\alpha$$
$$a_2 = r_2\alpha \qquad \text{.....(iv)}$$

$$\therefore I = M(k)^2 = 4 \times (0.06)^2 = 0.0144$$

From equation (i)

$$P_1 = m_1 (g - a_1)$$

From equation (ii)

$$P_2 = m_2 (a_2 + g)$$

Substituting the values of $P_1$, $P_2$, $a_1$ & $a_2$ in equation (iii)

values $P_1 r_1 - P_2 r_2 = I\alpha$

$$\Rightarrow [m_1(g - a_1)]r_1 - [m_2(a_2 + g)]r_2 = 0.0144\ \alpha$$

$$\Rightarrow \{m_1(g - r_1\alpha)\}\ r_1 - \{m_2(r_2\alpha + g)\}\ r_2 = 0.0144\ \alpha$$

$$\Rightarrow m_1 g r_1 - m_1 r_1^2\alpha - m_2 r_2^2 \alpha - m_2 g r_2 = 0.0144\ \alpha$$

$$\Rightarrow (30 \times 9.81 \times 0.05) - (30 \times (0.05)^2\alpha) - (10 \times (0.1)^2\alpha) - (10 \times 9.81 \times 0.1) = 0.0144\ \alpha$$

$$\Rightarrow 14.715 - 0.075\alpha - 0.1\alpha - 9.81 = 0.018\alpha$$

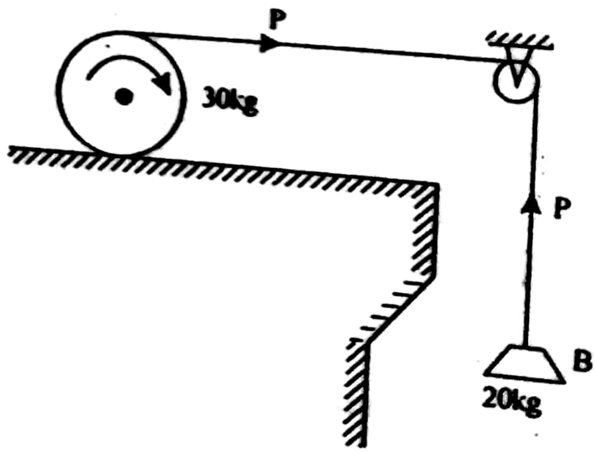
$$\Rightarrow 4.905 = 0.193\ \alpha$$

$$\Rightarrow \alpha = 25.11\ \text{rad/s}^2$$ (Ans.)

$$\therefore P_1 = m_1(g - a_1) = 30[9.81 - (0.05 \times 25.41)] = 256.17\text{N}$$ (Ans.)

$$\therefore P_2 = m_2(a_2 + g) = 10[(0.1 \times 25.41) + 9.81] = 123.51\text{ N}$$ (Ans.)

19. *Find the acceleration (a) on a solid right circular roller A of mass 30 kg when it is being pulled by another body B of mass 20kg along a horizontal plane as shown in fig. The mass B is attached to the end of a string wound round the circumference of t he roller. Assume that there is no slipping of the roller and the string is inextensible.*




**Soln.** Mass of roller (M) = 30Kg = A

mass of hanging body B(m) = 20kg

We know that for a solid right circular roller of radius r,

$$I = \frac{Mr^2}{2} = MK^2$$

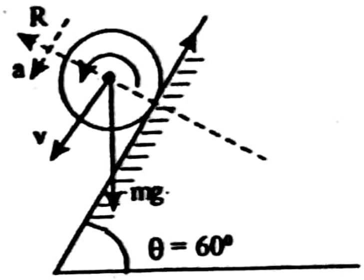
or $k^2 = 0.5r^2$

$$\therefore \quad a = \frac{mg}{M + 2m + \frac{Mk^2}{r^2}} = \frac{20 \times 9.81}{30 + (2 \times 20) + \frac{30 \times 0.5r^2}{r^2}}$$

$= 2.3\ m/s^2$ **(Ans.)**

**20.** *A solid uniformly thick wheel of radius 1m and mass 40 kg released with no initial velocity at the top of an inclined plane, which makes an angle of 60° with the horizontal. It rolls down without slipping. Determine (i) the minimum value of coefficient of friction (ii) the velocity of the center of the wheel after it has travelled a distance 5m down the inclined plane.*

**Soln.** Given data




r = radius of wheel = 1m

Mass of wheel (m) = 40 kg

inclination of plane $\theta = 60°$ ; $S = 5m$

(i) We know that for a uniformly thick wheel; $I = MK^2 = \frac{Mr^2}{2}$

$$\therefore \quad k^2 = 0.5\,r^2$$

Minimum value of co-efficient of friction,

$$\mu = \frac{\tan\theta}{\frac{k^2 + r^2}{k^2}} = \frac{\tan 60°}{\frac{0.5r^2 + r^2}{0.5r^2}} = \frac{\tan 60°}{\frac{r^2(0.5 + 1)}{0.5\,r^2}} = 0.577$$ **(Ans.)**

(ii) Let v = velocity at the centre of the wheel

We know that ;

$$a = \frac{g \sin\theta}{\frac{K^2 + r^2}{r^2}} = \frac{9.81 \times \sin 60°}{\frac{0.5r^2 + r^2}{r^2}} = 5.66\ m/s^2$$

Also, $V^2 = u^2 + 2as$

$= 0 + 2 \times 5.66 \times 4$

$V^2 = 45.31\ m/s$

$\Rightarrow V = 6.73\ m/s$