Chapter-6

Polarization of Light

6.1. Introduction

Interference and diffraction of light can be explained by the wave nature of light. These effects occur for all types of waves but they do not predict the nature of the light waves—whether they are *longitudinal (vibrations are always parallel to the direction of propagation)* or *transverse (vibrations are always perpendicular to the direction of propagation)*. **The phenomenon of polarization of light indicates that the light waves are transverse in nature.**

6.2. Experiment of Polarization

S is a source of ordinary light. When light falls on transparent hexagonal shape of tourmaline crystal P_1, the light passes out from the crystal P_2, provided the optic axes (which behaves like narrow slits on a cardboard) of crystal P_1 and P_2 are parallel to each other. But, when the crystal P_2 is rotated, the intensity of emergent light decreases. This intensity **becomes zero when the optic axis of P_2 is at right angle to that of P_1**. So, *the light waves are transverse in nature, otherwise the intensity of emergent light remains unaffected due to rotation of P_2, as seen for any longitudinal wave.*

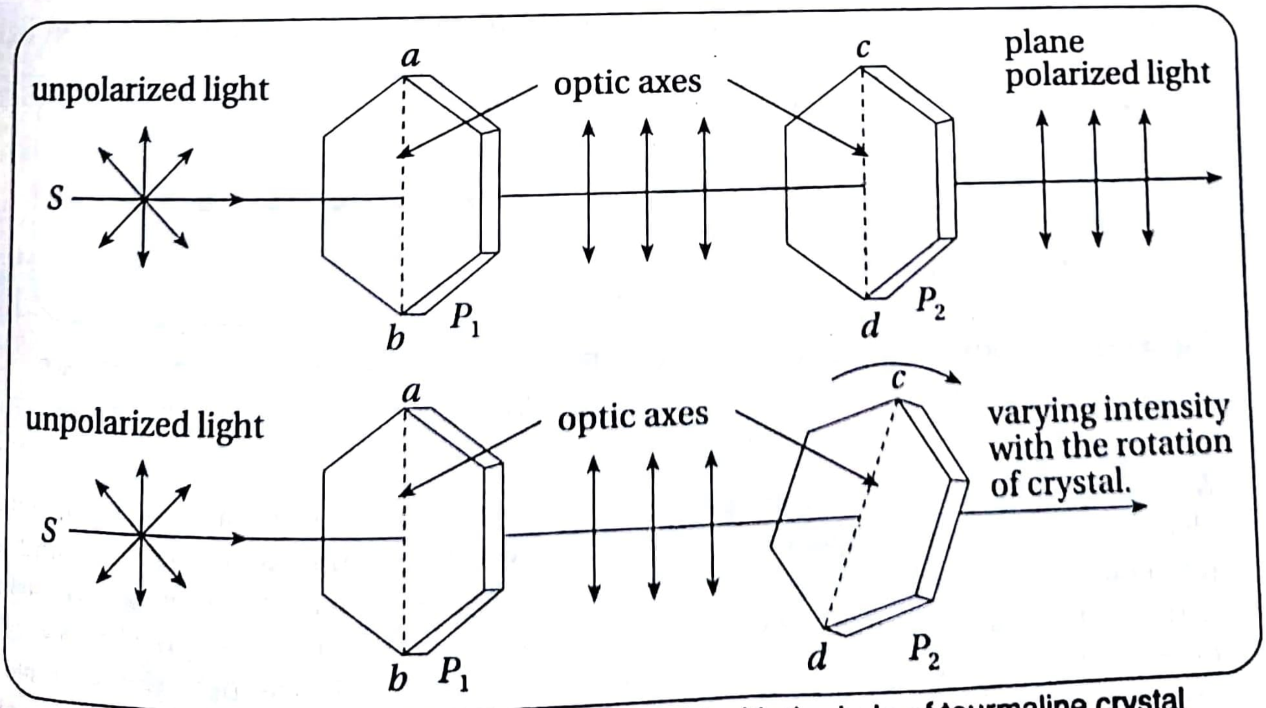

Fig. 1 ▷ An arrangement to get polarized light with the help of tourmaline crystal

This shows that the light coming from P_1 is not symmetrical about the direction of propagation. Now, the emergent light is confined to a particular plane. This phenomenon is called **polarization of light**.

Conclusions

① Light is propagated as transverse wave.

② The ordinary light waves which are incident on crystal P_1, are unpolarized.

The light vector (electric field vector)[1], that vibrates along all possible directions and perpendicular to the direction of propagation, is called an unpolarized light. Ordinary light or white light is an example of unpolarized light. It is represented by a star [Fig. 2].

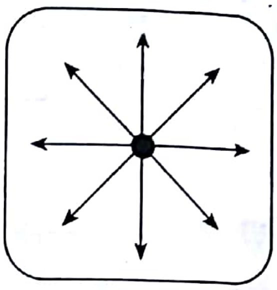

Fig. 2 ▷ Unpolarized light

③ When ordinary light passes through the optic axis (ab) of crystal P_1 [Fig. 1], the electric field vectors of ordinary light vibrate only in one direction *i.e.* light becomes polarized. Here, *the crystal P_1 is called polarizer as light is polarized after passing through it.* This polarized light can pass through P_2, only when the optic axis (cd) of P_2 is parallel to the optic axis (ab) of P_1. So, *the crystal P_2 is now acting as analyzer, as it indicates whether the beam is polarized or unpolarized.*

6.3. Polarized Light and Its Pictorial Representation

The phenomenon due to which the vibrations of light are restricted to a particular plane and its corresponding direction of vibration is perpendicular to the direction of propagation of light is called polarization of light.

Usually, a plane polarized light vibrates in the plane of the paper and they are represented by straight arrows (↕↕) [Fig. 3(a)]. When the vibrations of polarized light

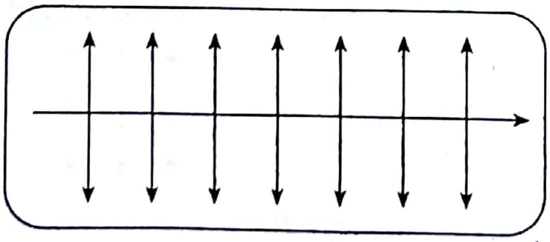

Fig. 3 (a) ▷ Polarized light (vibrations are in the plane of the paper)

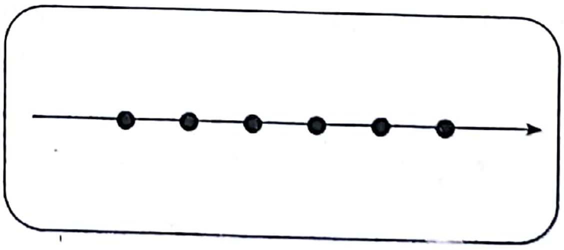

Fig. 3 (b) ▷ Polarized light (vibrations are perpendicular to the plane of the paper)

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[1] According to Maxwell's electromagnetic theory, light is an electromagnetic wave having electric field vector and magnetic field vector mutually perpendicular to each other and also perpendicular to the direction of propagation of the wave. Both the field vectors are varying in time. Since our eyes are sensitive to electric field only, we describe the optical phenomena only by electric field vector. But in an ordinary light, the electric field vectors have all possible orientations due to its random oscillations in the plane perpendicular to direction of propagation. So, the ordinary light is unpolarized light. Therefore, the electric field vector oscillation is restricted to a particular plane only, the light ray is called polarized light.

are perpendicular to the plane of the paper, they are represented by dots (•) [Fig. 3(b)]. ***When the vibrations are confined to a single plane*** (either in the direction along the plane of the paper or in the direction perpendicular to the plane of the paper, ***it is called plane polarized or linearly polarized light***. So, unpolarized light consists of infinite number of plane polarized light which have their own direction of vibrations [Fig. 4 and Fig. 5c].

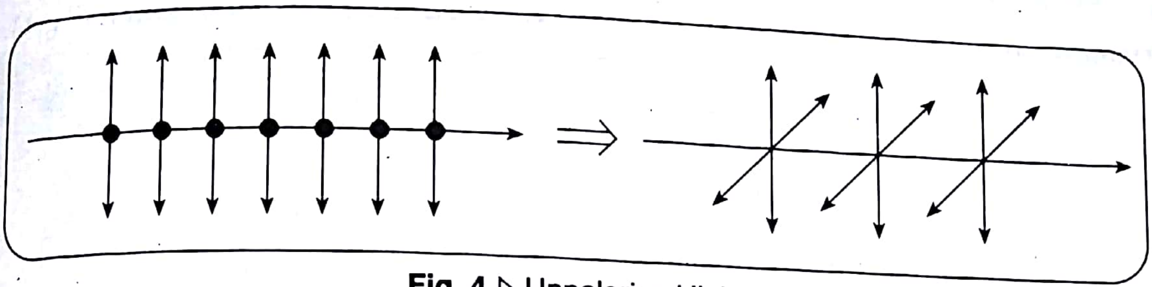

Fig. 4 ▷ Unpolarized light

Special Note :

Few examples of polarized lights confined to different planes are represented below [Fig. 5(a) and 5(b)]. Fig 5(c) represents unpolarized light.

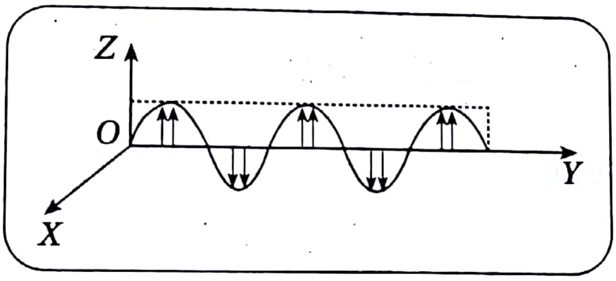

Fig. 5 (a) ▷ Vibrations are confined in YZ plane. Direction of propagation is along Y-axis

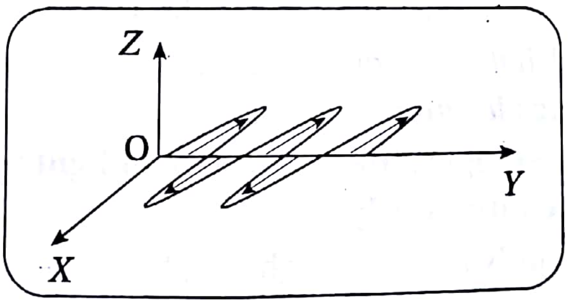

Fig. 5 (b) ▷ Vibrations are confined in XY plane (linearly polarized). Direction of propagation is along Y-axis

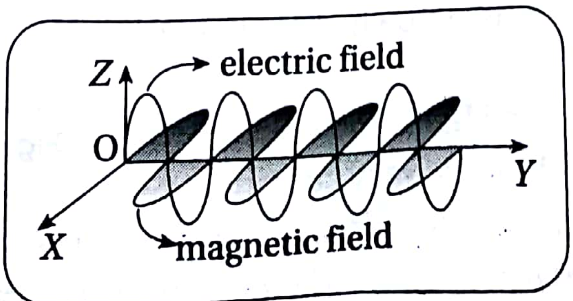

Fig. 5c ▷ Unpolarized light (*i.e.* electromagnetic wave)

6.4. Plane of Vibration and Plane of Polarization

Plane of vibration *The plane containing the direction of propagation of light and the direction of vibration of light, is known as plane of vibration.* [Fig. 6]

Plane of polarization A plane having *the direction of propagation of light and perpendicular to the direction of vibration, is known as plane of polarization* [Fig. 6].

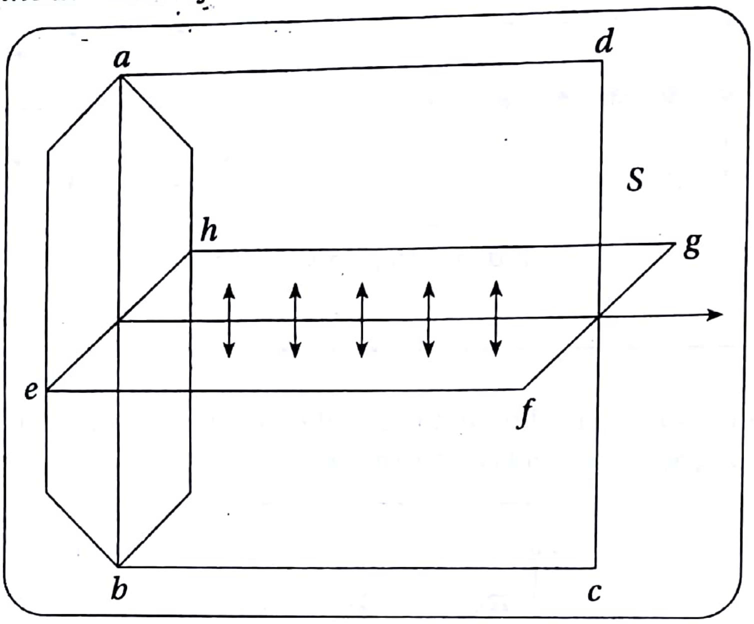

Fig. 6 ▷ Plane of polarization (*efgh*) and plane of vibration (*abcd*)

6.5. Classification of Polarized Light

There are two types of polarized light waves other than plane polarized light. They are—① circularly polarized light wave and ② elliptically polarized light wave.

In plane polarized light, the orientations of light vectors remain fixed but its magnitude changes during vibration.

In circularly polarized light, the magnitude of light vectors remains same but the orientations change continuously.

The concept of circularly polarized light can be given by a mechanical example. If one rotates the end of a string on the circumference of a circle, then each point on the string moves in a circular path and the corresponding wave is known as circularly polarized wave [Fig. 7].

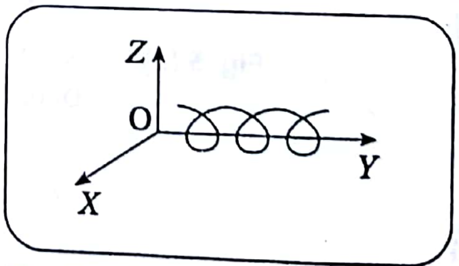

Fig. 7 ▷ Circularly polarized wave

In elliptically polarized light, the orientations and the magnitude of light wave vary continuously.

The circularly or elliptically polarized light can be formed by the superposition of two plane polarized light waves with a definite phase difference. The another way to get circularly or elliptically polarized light, we can use a thin slice of crystal called quarter-wave plate.

6.6. Methods of Producing Plane Polarized Light

The plane polarized light can be produced by different methods. The following few methods are usually used to produce plane polarized light.

① Polarization by reflection.
② Polarization by refraction.
③ Polarization by double refraction.

6.6.1. Polarization by Reflection

French scientist, E. L. Malus, in 1808, was able to show that, if a beam of ordinary light is incident on partially reflecting transparent medium (like glass), the reflected beam is partially plane polarized. The degree of polarization depends upon the angle of incidence.

In 1811, Sir Brewster performed a number of experiments to study the polarization of light by reflection from different media. It is to be found that, the reflected and refracted rays are perpendicular to each other and the reflected ray is completely plane polarized at a particular angle of incidence on the partially reflecting transparent medium.

The angle of incidence (of reflecting surface) at which the reflected and refracted rays become mutually perpendicular and the reflected ray is completely polarized is known as angle of polarization or Brewster's angle (i_p).

Brewster's law Consider a ray of unpolarized light incidents on a medium of refractive index μ [Fig. 8]. AB is the reflected polarized light with vibrations perpendicular to the plane of incidence and AC is the refracted light with vibrations parallel to the plane of incidence.

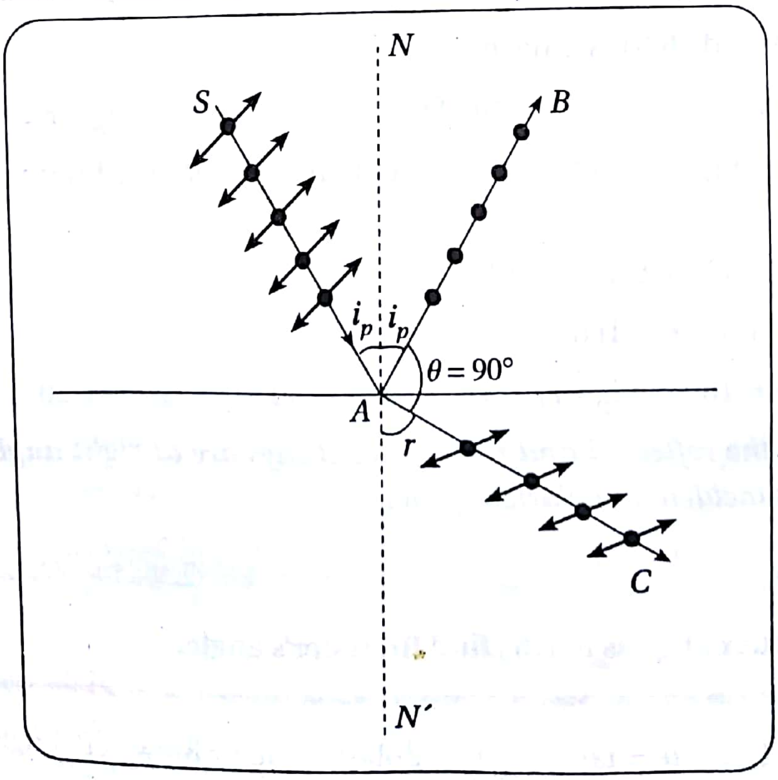

Fig. 8 ▷ Polarization in reflection

According to Snell's law,

$$\mu = \frac{\sin i}{\sin r}, \quad \mu = \text{refractive index of the medium}$$

when $i = i_p$; then $r = 90° - i_p$

$$[\because \text{ at } i = i_p,\ r = 180° - (90° - i_p)]$$

Hence, $\mu = \dfrac{\sin i_p}{\sin(90° - i_p)}$

or, $\boldsymbol{\mu = \tan i_p}$ $\cdots(6.1)$

This is known as **Brewster's law** and i_p = angle of polarization or Brewster's angle.

Brewster's law states that, **the tangent of the polarization angle is equal to refractive index of the medium, from the surface at which reflection of unpolarized light takes place.**

6.6.2. At the Polarization Angle, the Reflected and Refracted Rays are Perpendicular to Each Other

According to Brewster's law,

$$\mu = \tan i_p = \frac{\sin i_p}{\cos i_p} \quad \cdots(6.2)$$

But, by Snell's law, we have

$$\mu = \frac{\sin i_p}{\sin r} \quad \cdots(6.3)$$

From (6.2) and (6.3), we have

$$\cos i_p = \sin r \quad \text{or,} \quad \sin(90° - i_p) = \sin r \quad \text{or,} \quad \boldsymbol{i_p + r = 90°} \quad \cdots(6.4)$$

If 'θ' is the angle between the reflected and the refracted ray, we can write from figure 8,

$$\angle NAB + \angle BAC + \angle CAN' = 180°$$

or, $i_p + \theta + r = 180°$

or, $\theta = 180° - (i_p + r) = 180° - 90° = 90° \quad [\because i_p + r = 90°]$

Therefore, ***the reflected and the refracted rays are at right angle to each other, when the light is incident at polarizing angle.***

Problem 1

If refractive index of glass is 1.5, find Brewster's angle.

Solution We have, $\mu = \tan i_p$ [i_p = polarization or Brewster's angle]

or, $i_p = \tan^{-1}(\mu) = \tan^{-1}(1.5) = 56.30°$

Problem 2

A ray of light is incident on a glass of refractive index 1.732 at the polarization angle. Find (i) angle of polarization and (ii) the angle of refraction of the ray.
[W.B.U.T. 2004]

Solution We have from Brewster's law,

$$i_p = \tan^{-1}(\mu); \quad i_p = \text{polarization or Brewster's angle}$$
$$= \tan^{-1}(1.732) = 59.99 \simeq 60°$$

When the ray is incident at the polarization angle, the angle between the reflected ray and refracted ray is $90°$.

$\therefore \quad i_p + r = 90°$, r = angle of refraction,

or, $\quad r = 90° - i_p = 90° - 60° = 30°$

Problem 3

If a glass slab of refractive index 1.5 is immersed in water of refractive index $\frac{4}{3}$, find the angle of polarization.

Solution Refractive index of water *w.r.t.* air $= {}_a\mu_w = \frac{4}{3}$

Refractive index of glass slab *w.r.t.* air $= {}_a\mu_g = 1.5$

$\therefore$ Refractive index of glass slab *w.r.t.* water $= {}_w\mu_g = \dfrac{{}_a\mu_g}{{}_a\mu_w} = \dfrac{1.5}{\frac{4}{3}} = 1.125$

$\therefore$ The polarizing angle, $i_p = \tan^{-1}\mu = \tan^{-1}(1.125) = 48°22'$

Problem 4

The critical angle of glass with respect to air is $41.81°$. What is the angle of refraction for light, incident on glass at the polarization angle?

Solution If θ_c is the critical angle, the refractive index (μ) is given by,

$$\mu = \frac{1}{\sin\theta_c} = \frac{1}{\sin(41.81°)} = 1.5$$

From Brewster's law, the polarization angle,

$$i_p = \tan^{-1}(\mu) = \tan^{-1}(1.5) = 56.30°$$

Again, $\quad i_p + r = 90°, \; r = 90° - i_p = 90° - 56.30° = 33.7°$

So, the angle of refraction at the polarization angle = 33.7°

6.6.3. Polarization by Refraction

When unpolarized ordinary light is refracted through any transparent medium (example: glass slab), the refracted light is partially polarized. In order to get completely polarized light, it has to be refracted through a number of plates, which are placed parallel to each other by an air gap. The greater the number of plates, the purity of polarized light will be more.

For an instance, if an unpolarized light incidents at polarizing angle on a number of plates, then the reflected ray will be completely polarized. But the refracted light is partially polarized (vibrations parallel to the plane of incidence). *The degree of polarization can be increased by increasing the number of plates* [Fig. 9)].

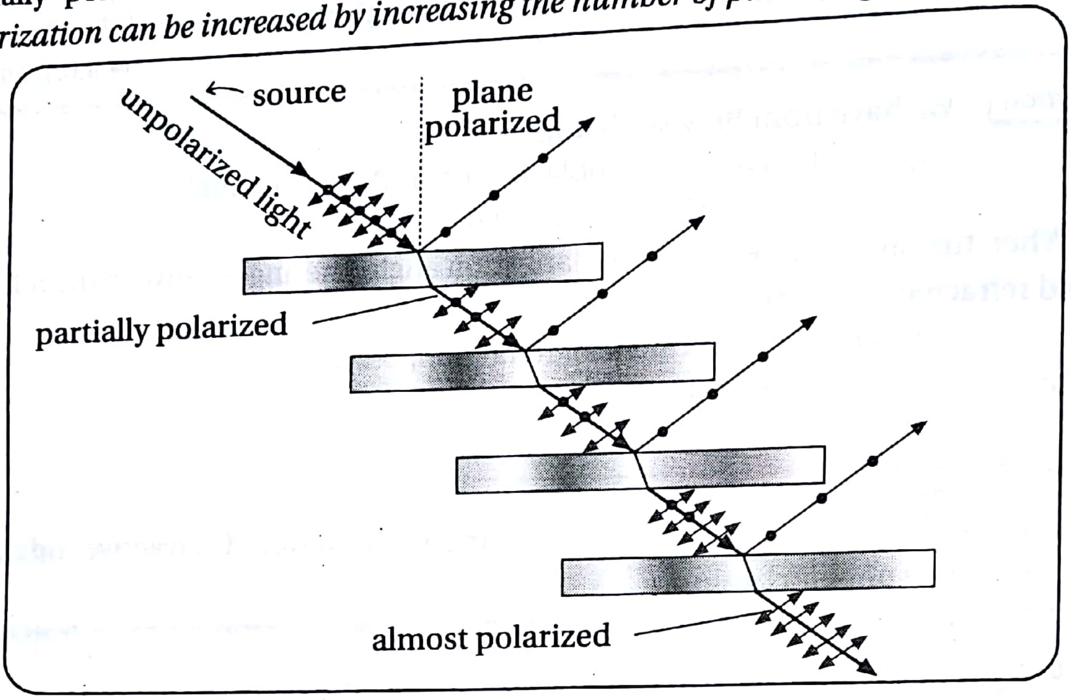

Fig. 9 ▷ Polarization by refraction

Law of Malus **It states that the intensity of plane polarized light transmitted through an analyzer is proportional to the square of cosine of the angle between the plane of transmission of the analyzer and the plane of polarizer.**

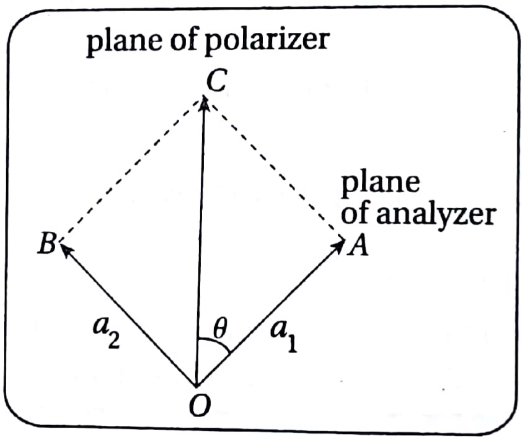

Fig. 10 ▷ Resolving of the amplitude 'a' of plane polarized light

Proof Let a be the amplitude of the plane polarized light and θ be the angle between the plane of transmission of the analyzer and the polarizer [Fig. 10] at any instant. The amplitude a of the plane polarized light emerging from the polarizer may be resolved into two components, along and perpendicular to the plane of transmission of the analyzer. Hence, the analyzer transmits the amplitude,

$a_1(= OA) = a\cos\theta$, which is along the plane of transmission.

Again, $a_2(= OB) = a\sin\theta$, which is perpendicular to the plane of transmission of the analyzer.

The component $a_2(= a\sin\theta)$ is reflected from the analyzer whereas *the component $a_1(= a\cos\theta)$ is transmitted through the analyzer.* So, the intensity of the transmitted ray,

$$I = a_1^2 \quad \text{or,} \quad \boxed{I = a^2\cos^2\theta} = \boxed{I_0\cos^2\theta} \qquad \cdots(6.5)$$

where $I_0 = a^2$ = intensity of the incident unpolarized light

This is *Malus' law.*

Discussion

① When $\theta = 90^\circ$, $I = 0$. So when the polarizer and analyzer are perpendicular to each other *the intensity of transmitted light from the analyzer is zero* (i.e., minimum).

② When $\theta = 0^\circ$ or 180°, $\cos\theta = \pm 1$ and $I = I_0$ = intensity of the incident unpolarized light.

So, when the polarizer and analyzer are parallel, *the intensity of light transmitted from the analyzer is same as that falls on it from polarizer.*

③ If the light incident on an analyzer is unpolarized, $I = \frac{I_0}{2}$.

This is because, *an unpolarized light vibrates randomly* in a plane perpendicular to the direction of propagation. So, *the intensity of transmitted light from the analyzer depends on the average value of* $\cos^2\theta$

i.e., $\cos^2\theta = \frac{1}{2}$.

Problem 1

The axes of a polarizer and an analyzer are oriented at 30° to each other. When unpolarized light of intensity I_0 is incident on the polarizer, what is the intensity of transmitted light?

Solution As unpolarized light of intensity I_0 is incident on the polarizer, the polarizer transmits light of intensity

$$I_1 = \frac{I_0}{2}.$$

∴ Light transmitted by the analyzer

$$I_2 = I_1\cos^2\theta$$

$$= \frac{I_0}{2}\cos^2 30^\circ$$

$$= 0.375 I_0$$

Problem 2

The axes of a polarizer and an analyzer are oriented at 30° to each other. A polarized light of intensity I_0 is incident on this polarizer–analyzer system. If the amplitude of light makes an angle 30° with the axis of the polarizer, what is the intensity of the transmitted light?

Solution When a polarized light is incident on polarizer, the polarizer transmits light of intensity

$$I_1 = I_0\cos^2 30^\circ = 0.75 I_0$$

This light is polarized at $30°$ to the axis of the system.

Again, as the 2nd polarizer (acting as an analyzer) axis is also making an angle $30°$, the light will be transmitted completely through it.

$\therefore$ The intensity of the transmitted light from the analyzer

$$I_2(=I_1) = 0.75I_0$$

Problem 3

A group of four polarizing sheets are lined up so that the characteristic direction of each is rotated $30°$ clockwise with respect to the proceeding sheet. When an unpolarized beam of light is incident on this group of polarizing sheets, what fraction of the incident intensity is transmitted?

Solution Due to the arrangement of four polarizer, the angle between successive polarizers, $\theta = 30°$.

Let I_0 be the intensity of the unpolarized incident light.

$\therefore$ 1st polarizer transmits light of intensity $I_1 = \frac{I_0}{2}$

2nd polarizer (acts as analyzer) transmits light of intensity $I_2 = I_1\cos^2\theta$

3rd polarizer transmits light of intensity $I_3 = I_2\cos^2\theta$

4th polarizer transmits light of intensity $I_4 = I_3\cos^2\theta$

Now, we can write,

$$I_4 = I_3\cos^2\theta = (I_2\cos^2\theta)\cos^2\theta = (I_1\cos^2\theta)\cos^4\theta = \frac{I_0}{2}\cos^6\theta.$$

So, the fraction of incident light transmitted is

$$\frac{I_4}{I_0} = \frac{\left(\frac{I_0}{2}\right)\cos^6\theta}{I_0} = \frac{(\cos 30°)^6}{2} = \frac{1}{2}(0.866)^6 = 0.211.$$

6.6.4. Isotropic and Anisotropic Medium

An **isotropic medium (example: glass slab)** consists of a periodic regular and identical arrangement of atoms in all directions. So, *when a light wave passes through this medium, it will move with the same velocity in all directions. So, this medium refracts only one single ray.*

But there are many *crystalline substances,* where the interatomic distances along different directions are different. The atoms are arranged in an irregular manner. So, if a light wave passes through this medium, it will not maintain a constant velocity in all directions. Thus, the velocity of light through this medium is not constant in all directions due to different optical properties. This type of medium is known as **anisotropic medium**. So, more than one refracted rays are obtained from this medium.

6.6.5. Double Refraction

When an unpolarized light is incident on an anisotropic crystal (example: calcite), the incident light is splitted up into two refracted plane polarized rays (extraordinary *i.e.* E-ray and ordinary *i.e.* O-ray). This phenomena is known as double refraction. Dr. Dane Erasmus Bartholinus in 1669 discovered this phenomenon.

6.6.6. O-ray and E-ray

When an unpolarized light is incident on an anisotropic crystal (example: calcite), it produces two refracted rays by double refraction process. One of these two refracted rays obeys the ordinary Snell's law of refraction and having vibrations *perpendicular to the principal section*[2] *of the crystal.* This ray is known as *ordinary ray* or *O-ray.*

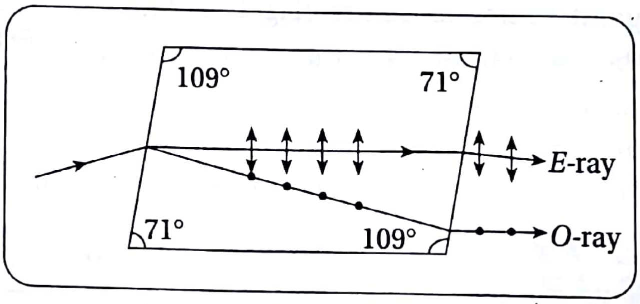

Fig. 11 ▷ Vibrations of E-ray and O-ray

The other refracted ray does not obey the ordinary Snell's law of refraction and having vibrations along the principal section of the crystal. This is called *extraordinary ray* or *E-ray.*

6.6.7. Uniaxial and Biaxial Crystals

The crystal in which there exists only one direction, along which the velocities of the two refracted rays (E-ray and O-ray) are same is known as uniaxial crystal.

- ***Examples :*** calcite ($CaCO_3$), quartz (SiO_2).

A doubly refracting crystal possessing one optic axis may be called a uniaxial crystal.

The crystal in which there exists two such directions (known as optic axes) along which the O-ray and E-ray travel with the same velocity is known as biaxial crystal.

- ***Examples :*** aragonite, copper sulphate etc.

6.6.8. Huygen's Theory of Double Refraction

According to Huygen's theory **each point on a wavefront acts as a source of secondary wavelet that spreads its disturbances in all directions from their centres with a speed of propagation of the wave. This new wavefront will appear as an envelope of the secondary wavelets, those will again spread their disturbances in all directions.**

[2] **Principal section :** Principal section of a crystal is defined as a plane passing through the optic axis and normal to a crystal surface.
In calcite crystal, principal section always cuts the surface of the crystal in a parallelogram with angles $71°$ and $109°$.

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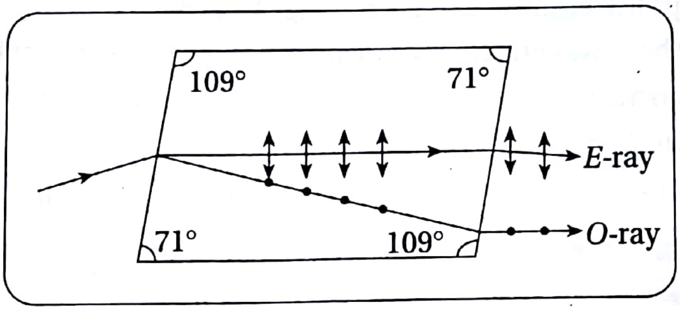

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- ***Examples :*** calcite ($CaCO_3$), quartz (SiO_2).

A doubly refracting crystal possessing one optic axis may be called a uniaxial crystal.

The crystal in which there exists two such directions (known as optic axes) along which the O-ray and E-ray travel with the same velocity is known as biaxial crystal.

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2 **Principal section :** Principal section of a crystal is defined as a plane passing through the optic axis and normal to a crystal surface.

In calcite crystal, principal section always cuts the surface of the crystal in a parallelogram with angles 71° and 109°.

If we imagine a point source of unpolarized light within a doubly refracting crystal, two wavefronts are produced. The wavefront at any time corresponding to the O-ray *will be a sphere* (spherical wave surface) as O*-ray propagates* with same velocity in all directions obeying Snell's law about the point source as centre [Fig 12]. Whereas the wavefront at the same instant corresponding to E*-ray will be an ellipsoid as* E*-ray propagates with different velocities* in different directions and this does not obey Snell's law in anisotropic doubly refracting crystals.

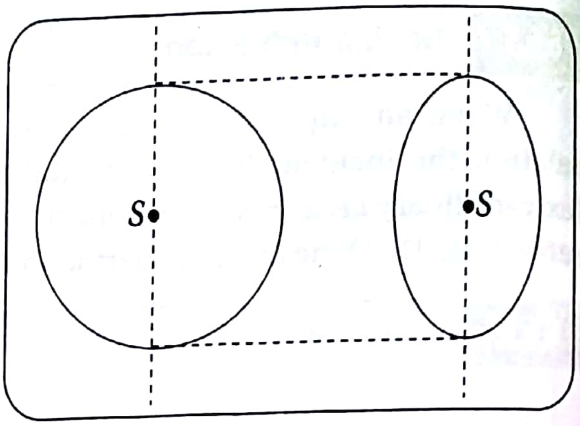

Fig. 12 ▷ Spherical wavefront and ellipsoidal wavefront

6.6.9. Positive and Negative Crystal

Depending on the velocities of O*-ray and* E*-ray* inside the crystal, the crystals (anisotropic) are classified into *two types*—

1. Positive crystal and **2. Negative crystal.**

① **Positive crystal:** *If the velocity of* O*-ray is greater than that of* E*-ray inside a crystal, then this type of crystal is known as positive crystal.*

Hence, the ellipsoidal wavefront due to E-ray lies inside the spherical wavefront due to O-ray [Fig. 13].

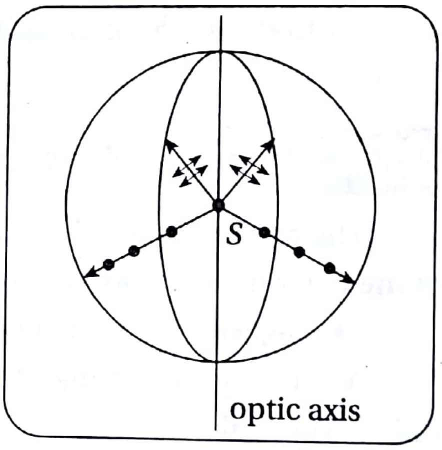

Fig. 13 ▷ Positive crystal

It is to be noted that—

i Since, refractive index $\mu = \frac{c}{v}$, the refractive index for E-ray (μ_e) will be greater than that of O-ray (μ_o) for the positive crystal.

ii The velocities of O-ray and E-ray are the same along the optic axis within the crystal.

• ***Examples:*** quartz, iron oxide etc.

② **Negative crystal:** *If the velocity of* E*-ray is greater than that of* O*-ray inside the crystal, the crystal is known as negative crystal.*

Hence, the spherical wavefront due to O-ray lies inside the ellipsoidal wavefront due to E-ray [Fig. 14].

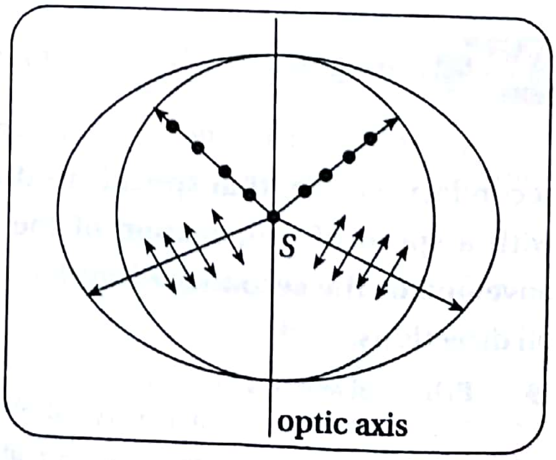

Fig. 14 ▷ Negative crystal

It is to be noted that—

i As $\mu = \frac{c}{v}$, the refractive index for O-ray (μ_o) will be greater than that of E-ray (μ_e) for the negative crystal.

ii The velocities of O-ray and E-ray are the same along the optic axis within the crystal.

• ***Examples :*** calcite, tourmaline etc.

For a positive crystal, the amount of double refraction or birefringence is measured by $\mu_e - \mu_o = \Delta\mu$.

For a negative crystal, the amount of double refraction is $-\Delta\mu$.

Problem 1

Calculate the velocities of ordinary and extraordinary rays in calcite crystal in a plane perpendicular to the optic axis. The refractive indices of calcite crystal for E-ray and O-ray are 1.485 and 1.659 respectively.

Solution If c_o and c_e are the velocities of O-ray and E-ray respectively then,

$$\mu_o = \frac{c}{c_o} \text{ and } \mu_e = \frac{c}{c_e} \quad [c = \text{velocity of light in vacuum}]$$

Therefore, $$c_e = \frac{c}{\mu_e} = \frac{3 \times 10^8}{1.485} = 2.02 \times 10^8 \text{ m}$$

and $$c_o = \frac{c}{\mu_o} = \frac{3 \times 10^8}{1.659} = 1.808 \times 10^8 \text{ m}$$

6.6.10. Calcite Crystal

Calcite crystals are transparent and colourless. It is formed from calcium carbonate ($CaCO_3$). It is a rhombohedral crystal containing six parallelogram faces. The two opposite angles of each parallelogram are at 102° and 78°. The two diametrically opposite corners of the rhombohedron, where three obtuse angles of 102° meet, are called the blunt corners of the crystal.

6.6.11. Optic Axis

Optic axis of a crystal is defined as a direction inside a double refracting crystal along which if a ray passes, there will be no double refraction of the incident ray due to the same velocity of E-ray and O-ray along this direction. So both the E-ray and O-ray behave alike along optic axis in all respects.

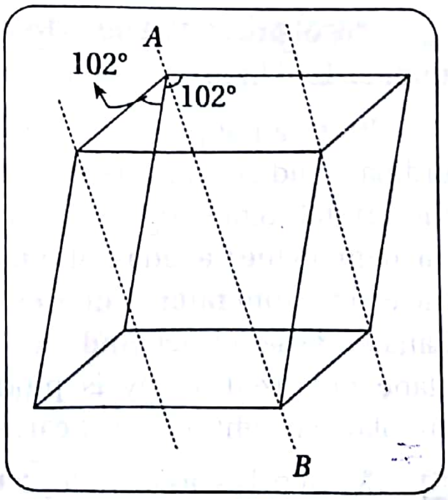

Fig. 15 ▷ Optic axis

A line drawn through one of the blunt corners and that bisects these blunt corners is known as optic axis (AB say in Fig. 15). One thing may be noted that optic axis cannot be obtained by joining the two blunt corners of the rhombohedron.

An unpolarized light moving along the optic axis or parallel to the optic axis cannot be divided into ordinary and extraordinary ray. So, the phenomena of double refraction cannot occur along the optic axis as the velocities of the two rays (O-ray and E-ray) are same along this direction.

6.6.12. Nicol Prism and its Use as Polarizer and Analyzer

Nicol Prism: It is an optical device, that can produce plane polarized E-ray from unpolarized light and also has an ability to analyze the plane polarized light. In the year of 1826, William Nicol invented this device.

Principle *The activity of Nicol Prism is based on the principle of double refraction in a uniaxial crystal.* So, when an unpolarized light is incident on a uniaxial crystal, it splits up into polarized E-ray and O-ray. *By the process of total internal reflection (using Canada balsam layer), the O-ray is somehow eliminated and the E-ray is transmitted through this crystal as a plane polarized light.*

Construction It is constructed by a rhombohedron of calcite crystal, whose length is three times its breadth. The angles of principal section are 112° and 68° [Fig. 16]. The crystal is cut into two pieces along the plane perpendicular to both the principal section and the two end faces (AB and CD) of the crystal. The two cut surfaces of the crystal are then cemented together by adhesive material 'Canada balsam', whose refractive index ($\mu_{CB} = 1.55$) is greater than the refractive index for extraordinary ray ($\mu_e = 1.486$), but less than the refractive index for ordinary ray ($\mu_o = 1.66$).

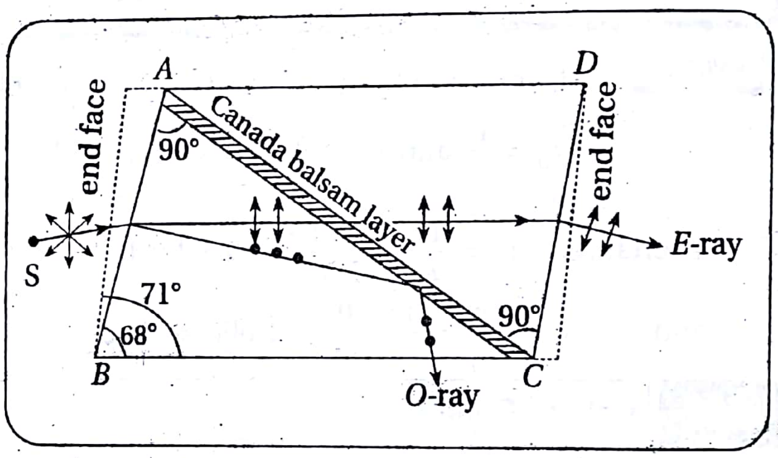

Fig. 16 ▷ Cross-section of a Nicol prism showing the elimintation of O-ray by total internal reflection

Action

Nicol prism as a polarizer **If an optical device can restrict the vibration of any unpolarized light in a single direction, then it is called polarizer.**

When an unpolarized light [Fig. 16] falls on the Nicol prism, the ray splits up into ordinary and extraordinary ray by the process of double refraction and travels through the crystal. Since $\mu_o > \mu_{CB}$, the O-ray suffers total internal reflection when it is incident on the Canada-Balsam layer. But the E-ray incidents on Canada-Balsam layer traversing from rarer to denser medium ($\mu_{CB} > \mu_e$) and it is transmitted through the Canada-Balsam layer. And finally it emerges through the Nicol prism. In this way, the plane polarized E-ray is produced by Nicol prism eliminating O-ray from the unpolarized light. This is the use of Nicol prism as a polarizer.

Nicol prism as an analyzer **When an optical device can indicate whether light is polarized or unpolarized then it is called an analyzer.**

We know, when an unpolarized light incidents on a Nicol prism (used as polarizer), a plane polarized E-ray is transmitted through the prism. Now, another Nicol prism is placed on the path of this emerging E-ray. The vibrations of E-ray

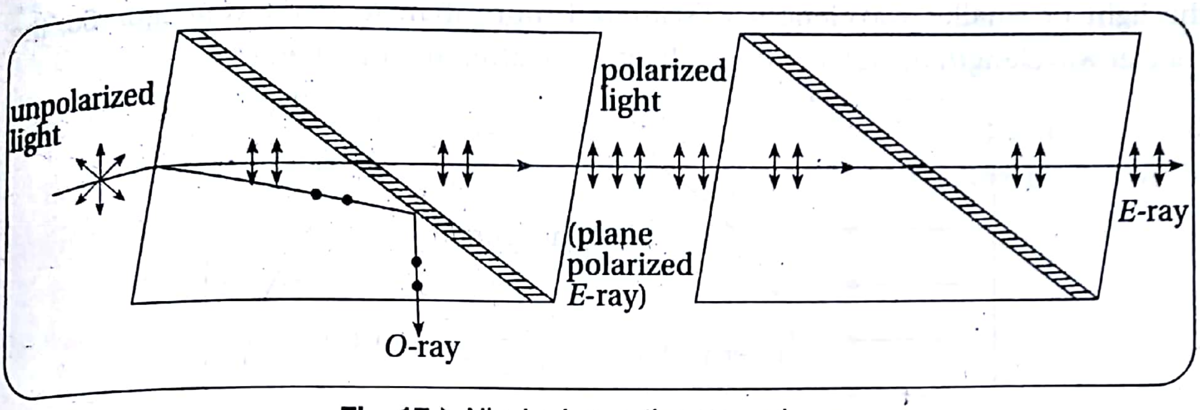

Fig. 17 ▷ Nicol prism acting as analyzer

always lie in the plane of principal section of the crystal. So, as long as the principal section of the second Nicol prism remains parallel to the principal section of the first Nicol prism, E-ray will be transmitted through the second Nicol prism [Fig. 17].

If the second Nicol prism is rotated gradually, the intensity of the emitted E-ray from the second Nicol prism also decreases. When the principal section of the second Nicol prism is perpendicular to that of the first, the intensity becomes zero [Fig. 18]. Therefore, the second Nicol prism analyzes the E-ray and acts as an analyzer.

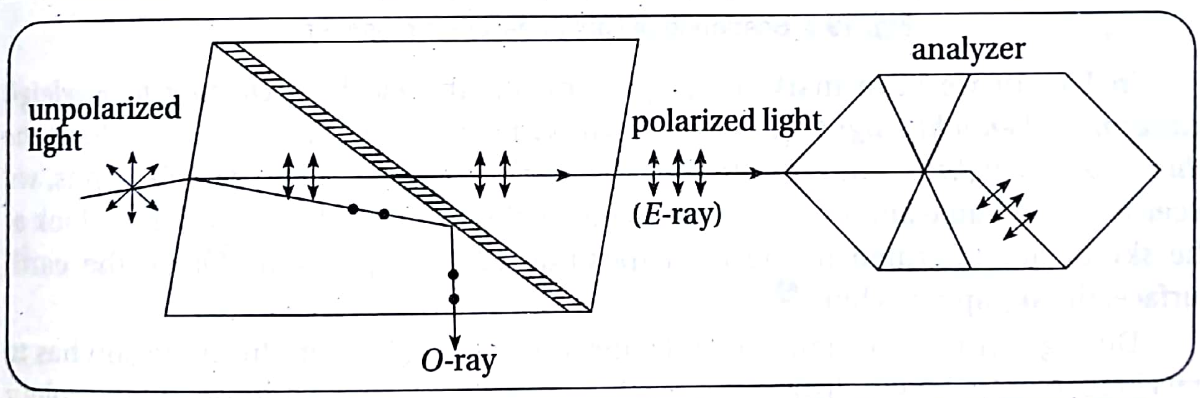

Fig. 18 ▷ Nicol prisms at crossed position for which the intensity of emerged E-ray is zero

6.6.13. Polarization by Scattering[3]

When an unpolarized light (e.g. sunlight) to made to is pass through a medium consisting of small ultramicroscopic particles, the light is scattered and the scattered light is found to be partially polarized. The degree of polarization is maximum for light scattered in a direction at right angles to the original beam.

Explanation of blue colour of the sky ; red colour during sunrise and sunset

Due to the scattering of light by small dust particles or molecules of atmosphere, the sky appears red in colour during sunrise and sunset and rest of the day it appears blue. The scattered light is found to be partially polarized. As the sizes of the dust particles or molecules of the atmosphere are comparable to the wavelength of light, these particles

[3] Polarization by scattering is not included in W.B.U.T. syllabus.

show Rayleigh scattering of light. The intensity of scattered light is inversely proportional to the fourth power of the wavelength (λ) *i.e.* $I \propto \frac{1}{\lambda^4}$. This implies that the light of smaller wavelength is scattered more than longer wavelength. So, the shorter wavelength of violet and blue light are scattered more than red.

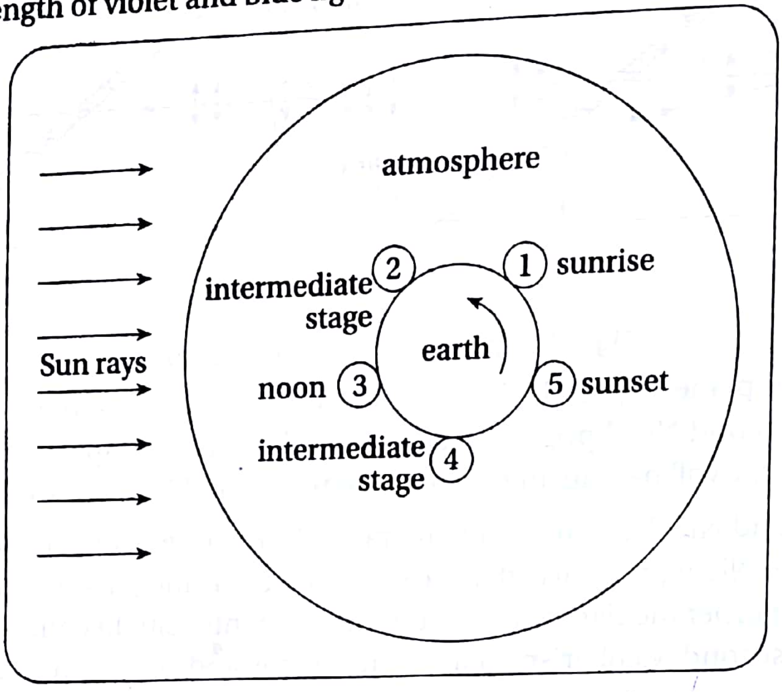

Fig. 19 ▷ Scattering of light at earth's atmosphere

In Fig. 19 we have marked our position on the earth. *According to Rayleigh scattering, when white light falls on the dust particles or molecules of atmosphere, the blue colour of light is scattered* 10 *times more than the red colour of light.* Thus, we receive considerable amount of scattered blue colour of light. Hence, when we look at the sky in our intermediate position [position ② and position ④] on the earth surface, the sky appears blue.[4]

During sunrise [position ①] and sunset [position ⑤], light from the sun has to travel much more longer distance to reach our eyes. So a larger portion of blue colour of light is scattered sidewise. As a result the blue colour is removed and the remaining colour is red. Hence, the sky appears red during sunrise and sunset.

At noon [position ③], when the sun is higher in the sky, the sunlight travels relatively small distance of atmosphere (*i.e.*, thin layer of atmosphere) to reach us. As a result only a small fraction of incident sunlight will be scattered. Basically we do not receive any scattered light. So, sky appears white during noon.

6.7. Retardation Plate

The optical device, which makes a finite path difference between O-ray and E-ray by retarding the motion of one of these rays is known as retardation plate. There are two types of retardation plates.

[4] Though shorter wavelength, violet colour is scattered more than blue, the sky is blue rather than violet. The reason is that the human eye is more sensitive to blue colour rather than violet colour.

① **Quarter-wave plate** *If the thickness of a doubly refracting crystal is so adjusted that it produces a path difference of $\frac{\lambda}{4}$ or phase difference of $\frac{\pi}{2}$ between the O-ray and E-ray, then the plate is called quarter-wave plate.* It is made up in such a way that the refracting faces are parallel to the optic axis.

For a **negative uniaxial crystal** $\mu_o > \mu_e$. Hence, the *path difference* between the ordinary and extraordinary rays in negative crystal for normal incidence will be $(\mu_o - \mu_e)t$, where t is the thickness of the plate. For **positive crystals** (*i.e.* $\mu_e > \mu_o$), the path difference between two rays (E-ray and O-ray) will be $(\mu_e - \mu_o)t$.

Thus, *for negative crystal,*

$$(\mu_o - \mu_e)t = \frac{\lambda}{4} \quad \text{or,} \quad \boxed{t = \frac{\lambda}{4(\mu_o - \mu_e)}} \qquad \cdots(6.6)$$

and *for positive crystal,*

$$(\mu_e - \mu_o)t = \frac{\lambda}{4} \quad \text{or,} \quad \boxed{t = \frac{\lambda}{4(\mu_e - \mu_o)}} \qquad \cdots(6.7)$$

- ***Uses:*** Quarter-wave plate is used to produce circularly and elliptically polarized light by placing them in the path of a plane polarized light.

② **Half-wave plate** *If the thickness of a doubly refracting crystal is so adjusted, that it produces a path difference of $\frac{\lambda}{2}$ or phase difference of π between the O-ray and E-ray, then the plate is called a half-wave plate.* The refracting faces are parallel to the optic axis.

For a **negative crystal** (*i.e.* $\mu_o > \mu_e$), the path difference $(\mu_o - \mu_e)t$ between the O-ray and E-ray will be equal to $\frac{\lambda}{2}$,

$$\therefore \quad (\mu_o - \mu_e)t = \frac{\lambda}{2}$$

$$\text{or,} \quad \boxed{t = \frac{\lambda}{2(\mu_o - \mu_e)}} \qquad \cdots(6.8)$$

For **positive crystals** (*i.e.* $\mu_e > \mu_o$),

and hence, $(\mu_e - \mu_o)t = \frac{\lambda}{2}$

$$\text{So, the thickness } \boxed{t = \frac{\lambda}{2(\mu_e - \mu_o)}} \qquad \cdots(6.9)$$

6.8. General Method for the Production of Plane, Circularly and Elliptically Polarized Light

Plane polarized light When an unpolarized light is passed through a Nicol prism, it splits up into O-ray and E-ray. The O-ray shows total internal reflection at the Canada-Balsam layer while the E-ray finally emerges through the prism [Fig. 16]. This emergent ray is plane polarized.

Circularly polarized light To produce circularly polarized light, two waves vibrating at right angle to each other with same amplitude and a phase difference of $\frac{\pi}{2}$ are to be superposed.

Again, when an unpolarized light is allowed to fall on a Nicol prism, it allows the emergent E-ray as plane polarized. Now, if this plane polarized light is allowed to fall on a quarter-wave plate so that, the angle between the optic axis and the plane of vibration is $45°$, then the emergent beam from the quarter-wave plate will be circularly polarized.

Elliptically polarized light To produce elliptically polarized light, two waves of unequal amplitude with a phase difference of $\frac{\pi}{2}$ are to be superposed.

When an unpolarized light is allowed to fall on a Nicol prism, it allows the emergent ray as plane polarized. The plane polarized light emerging from the Nicol prism is allowed to fall normally on the quarter-wave plate so that, the angle (θ) between the optic axis and the vibration of plane polarized light is other than $0°, 45°, 90°$. In this case the emergent beam from the quarter-wave plate becomes elliptically polarized.

It is to be noted that, if $\theta = 0°$ (or, $90°$) and $45°$, then the emergent beam from the quarter-wave plate will be linearly polarized and circularly polarized respectively.

6.8.1. Detection of Plane, Circularly and Elliptically Polarized Light

The light under test is allowed to pass through a rotating Nicol prism. Now the variation of intensities of the emergent ray from the Nicol prism is recorded. By observing the variation of intensities we can make the following conclusions —

① If there are *variations in intensities with minimum value zero*, then the light under test is *plane polarized.*

② If there are *variations in intensities with its minimum value not equal to zero*, then the light under test is *elliptically* or *partially polarized.*

③ If there is *no variation in intensity of the emergent ray* from the Nicol prism, then the light under test is *either circularly polarized* or *unpolarized.*

So, we have to distinguish between partially polarized light and elliptically polarized light or unpolarized light and circularly polarized light. For that we can take a quarter-wave plate which can produce circularly or elliptically polarized light.

To separate these pairs *i.e.*

[1] partially polarized and elliptically polarized or [2] circularly polarized and unpolarized.

The light under test is sent through quarter-wave plate and then with the help of another rotating Nicol prism, *the variation in intensities* of emergent rays from the quarter-wave plate is observed.

① **For the pair of partially polarized and elliptically polarized light** If the *intensity* is changing with its *minimum value zero*, then the light under test is *elliptically polarized* [Fig. 20(a)].

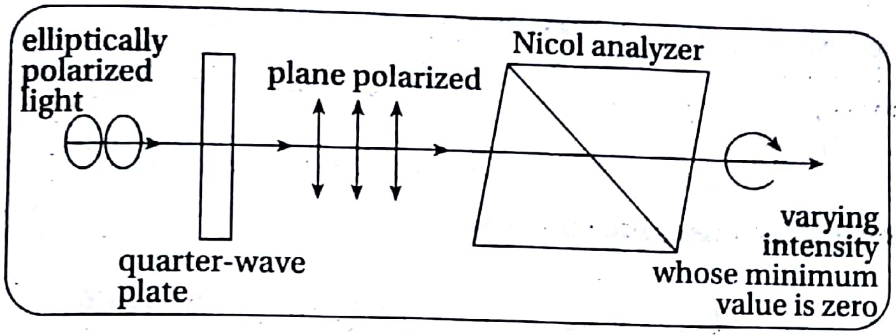

Fig. 20(a)

If the *intensity is changing with its minimum value not equal to zero*, then the light is *partially polarized* [Fig. 20(b)].

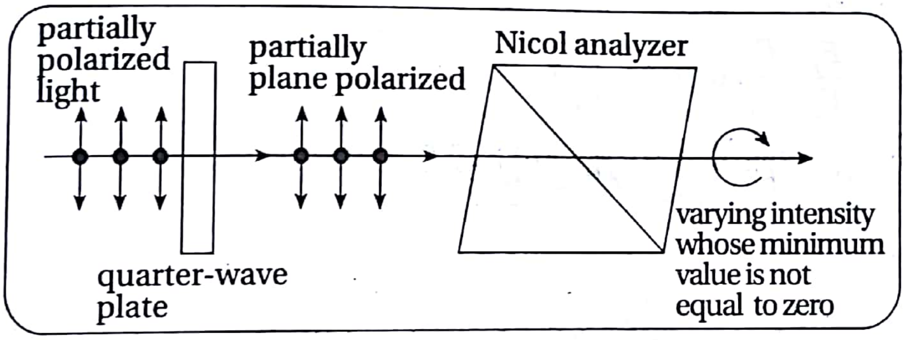

Fig. 20(b)

② **For the pair of circularly polarized and unpolarized light** If the *intensity is changing with its minimum value equal to zero, then it is circularly polarized* [Fig. 21(a)].

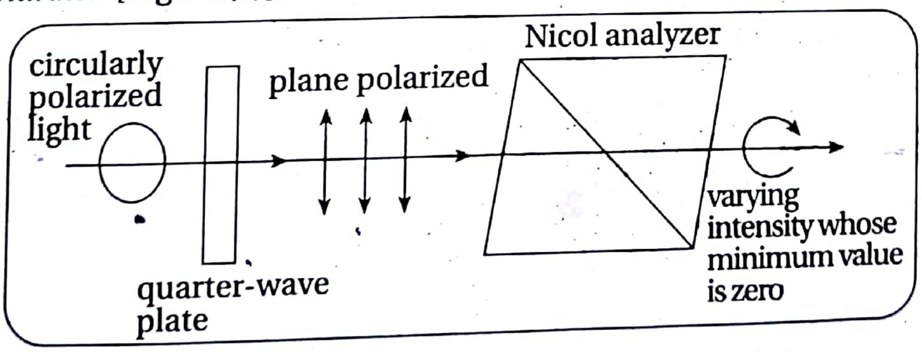

Fig. 21(a)

If there is no variation in intensity, then it is *unpolarized* [Fig. 21(b)].

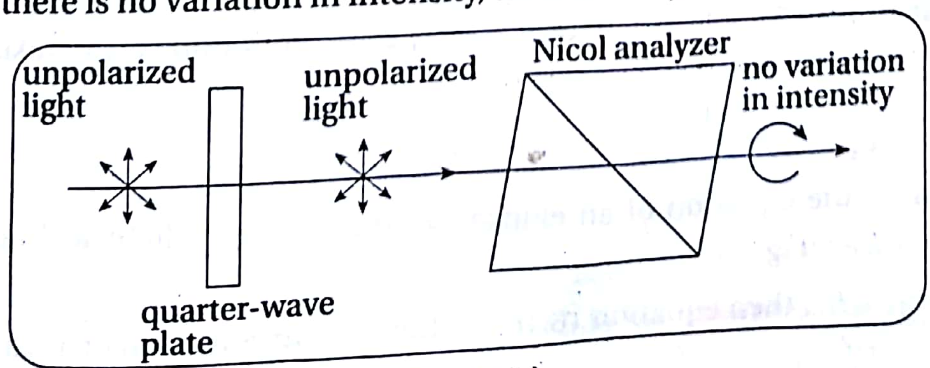

Fig. 21(b)

6.8.2. Theoretical Method (or Analytical Method) for Production of Plane, Circularly and Elliptically Polarized Light

Consider two orthogonal optical vectors (*i.e.* two electric vectors) of frequency as given below,

$$E_x = E_1 \sin\omega t \quad \cdots (6.10)$$

$$E_y = E_2 \sin(\omega t + \delta), \quad \text{where } \omega = 2\pi n \quad \cdots (6.11)$$

From equations (6.10) and (6.11) we have,

$$\frac{E_x}{E_1} = \sin\omega t \quad \cdots (6.12) \quad \text{and} \quad \frac{E_y}{E_2} = \sin(\omega t + \delta) \quad \cdots (6.13)$$

Now, $$\frac{E_y}{E_2} = \sin\omega t\cos\delta + \cos\omega t\sin\delta = \frac{E_x}{E_1}\cos\delta + \sqrt{1 - \frac{E_x^2}{E_1^2}}\sin\delta$$

or, $$\left(\frac{E_y}{E_2} - \frac{E_x}{E_1}\cos\delta\right)^2 = \left(\sqrt{1 - \frac{E_x^2}{E_1^2}}\sin\delta\right)^2$$

or, $$\frac{E_y^2}{E_2^2} + \frac{E_x^2}{E_1^2}\cos^2\delta - \frac{2E_xE_y\cos\delta}{E_1E_2} = \left(1 - \frac{E_x^2}{E_1^2}\right)\sin^2\delta$$

or, $$\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} - \frac{2E_xE_y}{E_1E_2}\cos\delta = \sin^2\delta \quad \cdots (6.14)$$

Equation (6.14) represents the *general equation for an ellipse.*

Special cases

① When the phase difference $\delta = 0$ or, $\delta = n\pi$, where $n = 0, 1, 2, \cdots$.

Then, equation (6.14) takes the form,

$$\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} - \frac{2E_xE_y}{E_1E_2} = 0 \quad \text{or,} \quad \left(\frac{E_y}{E_2} - \frac{E_x}{E_1}\right)^2 = 0$$

or, $$E_y = \frac{E_2}{E_1}E_x \quad \cdots (6.15)$$

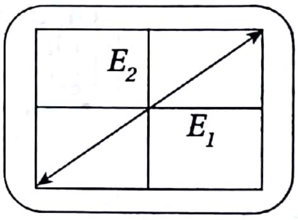

Fig. 22

Equation (6.15) represents a *straight line with slope* $\frac{E_2}{E_1}$. So, the *emergent* light will be *plane polarized* [Fig. 22].

② When $\delta = (2n+1)\frac{\pi}{2}$, where $n = 0, 1, 2, \cdots$, equation (6.14) takes the form,

$$\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} = 1 \quad \cdots (6.16)$$

This is the equation of an ellipse. So, the emergent light will be *elliptically polarized* [Fig. 23].

If $E_1 = E_2$, then equation (6.16) reduces to the equation of a circle,

$$E_x^2 + E_y^2 = E_1^2 \quad \cdots (6.17)$$

So, the emergent beam will be *circularly polarized* [Fig. 24].

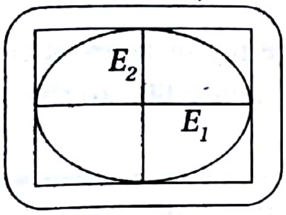

Fig. 23

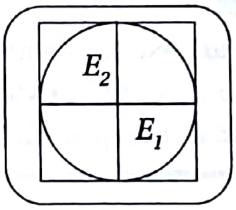

Fig. 24

Problem 1

Calculate the thickness of a calcite crystal which will convert the plane polarized light into circularly polarized light. The refractive indices for ordinary and extraordinary rays are 1.65 and 1.48 respectively. Given the wavelength of light is 5890 Å.

Solution A quarter-wave plate can convert the plane polarized light into circularly polarized light. Therefore, for a quarter-wave plate of negative crystal,

$$t = \frac{\lambda}{4(\mu_o - \mu_e)} = \frac{5890 \times 10^{-8}}{4(1.65 - 1.48)} = 8.66 \times 10^{-5}\text{cm}$$

Problem 2

For half-wave plate of quartz crystal, the refractive indices for ordinary and extraordinary rays are 1.544 and 1.553 respectively. The thickness of the half-wave plate is 2.65×10^{-3} cm. Find the wavelength of the light used.

Solution For half-wave plate of positive crystal, thickness $t = \dfrac{\lambda}{2(\mu_e - \mu_o)}$

or, $$2.65 \times 10^{-3} = \frac{\lambda}{2(1.553 - 1.544)}$$

or, $$\lambda = 2.65 \times 10^{-3} \times 2 \times (1.553 - 1.544) = 4770\text{ Å}$$

Problem 3

Find the state of polarization when the x and y components of the electric field are given by $E_x = E_0 \sin(\omega t + kz)$ and $E_y = E_0 \cos(\omega t + kz)$. **[W.B.U.T. 2013]**

Solution
$$E_x = E_0 \sin(\omega t + kz)$$
$$E_y = E_0 \cos(\omega t + kz)$$

where, E_0 is the amplitude of the wave.

$\therefore$ $$E_x^2 + E_y^2 = E_0^2 [\sin^2(\omega t + kz) + \cos^2(\omega t + kz)]$$

or, $$E_x^2 + E_y^2 = E_0^2$$

This represents a circular vibration.

At any position, say, $z = 0$, the electric vector

$$\vec{E} = \hat{i}E_x + \hat{j}E_y = \hat{i}E_0 \sin\omega t + \hat{j}E_0 \cos\omega t$$

Since, the electric field vector rotates in clockwise direction, the light is a right circularly polarized light.

Problem 4

A linearly polarized light wave of angular frequency ω is propagating along $+z$ direction with its plane of vibration making an angle $60°$ to the xz plane. Find the expression of this polarized light.

Solution The x component of the amplitude (E_0) of the wave,

$$E_{0x} = E_0 \cos 60° = \frac{E_0}{2}$$

The y component of the amplitude (E_0) of the wave,

$$E_{0y} = E_0 \sin 60° = \frac{\sqrt{3}}{2} E_0$$

$\therefore$ The expression of linearly polarized light wave is

$$\vec{E}(z, t) = \hat{i}\frac{E_0}{2}\cos(kz - \omega t) + \hat{j}\frac{\sqrt{3}}{2}E_0 \cos(kz - \omega t)$$

Problem 5

Find the state of polarization when the x and y components of the electric field are given by

$$E_x = E_0 \cos(\omega t + kz)$$

$$E_y = \frac{E_0}{\sqrt{2}} \cos(\omega t + kz + \pi)$$

[C.U. 1989]

Solution $E_y = \frac{E_0}{\sqrt{2}}\cos(\pi + \omega t + kz) = -\frac{E_0}{\sqrt{2}}\cos(\omega t + kz) = -\frac{E_x}{\sqrt{2}}$

So the light is linearly polarized which makes an angle $\tan^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ with the X axis.

Problem 6

When a plane polarized light passes through a quartz crystal cut parallel to its optic axis, it changes to a circularly polarized light. The wavelength of the incident plane polarized light is 700×10^{-9} m. Given the refractive indices for E-ray and O-ray are $\mu_e = 1.553$ and $\mu_o = 1.544$.

Solution To produce circularly polarized, the thickness (d) of quarter wave plate will be such that it will produce a phase difference of $\frac{\pi}{2}$ between E-ray and O-ray.

Thus, $$\frac{2\pi}{\lambda}(\mu_e d - \mu_o d) = \frac{\pi}{2}$$

or, $$d = \frac{\lambda}{4(\mu_e - \mu_o)}$$

$$= \frac{700 \times 10^{-9}}{4(1.553 - 1.544)} \text{ m} = \frac{700 \times 10^{-9}}{4 \times 0.009} \text{ m}$$

$$= 1.94 \times 10^{-5} \text{ m} = 1.94 \times 10^{-2} \text{ mm} = .019 \text{ mm}$$

Problem 7

If a left circularly polarized light is passing through a half wave plate, prove that the emergent light is a right handed circularly polarized light.

Solution Let us consider the incident left circularly polarized light is represented by

$$E_x = E_0 \cos(\omega t - kz),\ E-\text{wave}$$

$$E_y = E_0 \sin(\omega t - kz),\ O-\text{wave}$$

As the phase of x component will advance by π relative to y component while passing through half wave plate (made of negative crystal, say), the emergent ray will be

$$E_x = E_0 \cos(\omega t - kz + \pi) = -E_0 \cos(\omega t - kz)$$

and $$E_y = E_0 \sin(\omega t - kz)$$

This represents a right circularly polarized light.

6.9. Polaroids

Plane polarized beam of light of large cross-section cannot be produced by tourmaline crystal or Nicol prism (made by calcite crystal) due to non-availability of very large size of these crystals.

A polaroid is a thin and large transparent sheet or film of crystalline polarizing material (made artificially) capable of producing and analyzing plane polarized beams of light of large cross-section.

Initially, a paste of *organic synthetic crystalline material of iodoquinine, called herapathite, is prepared.* Herapathite is a small needle shaped crystal and possess the property of polarizing light. These crystals are not stable and cannot bear even a slight strain. So, the *commercial polaroid is prepared by suspending these small herapathite crystal in nitrocellulose.* In this way, a large fine sheet is produced, which contains millions of tiny herapathite crystals with their optic axes parallel. This is mounted between two thin sheets of glass to give it more stability. The construction of polaroid is based on **dichroism.**[5]

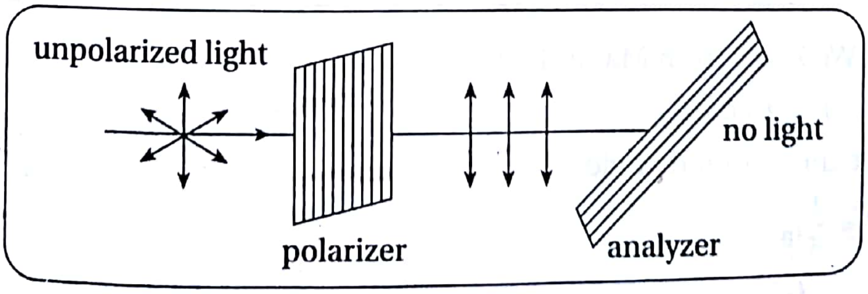

Fig. 25 ▷ Action of polaroid

[5] The *phenomenon by which a doubly refracting crystal* (example : tourmaline crystal, calcite crystal) *absorbs ordinary and extraordinary rays unequally is called dichroism.* The phenomenon of selective absorption is also called dichroism.

To know whether the beam is plane polarized or not, we first allow a beam of unpolarized light through a polaroid sheet. Then we take another polaroid sheet to check whether the emergent beam from the first polaroid sheet is polarized or not. The second polaroid sheet is rotated slowly about the direction of propagation of light. When the second polaroid is rotated through 90° with respect to each other, the intensity of emergent light reduces to zero due to complete absorption of light. Thus the light under test is now plane polarized.

In recent developments, a better method is found in which individual molecules, rather than microcrystals are employed to construct a polaroid.

There are two types of such polaroids. They are **(i)** H-polaroid and **(ii)** K-polaroid.

For H-polaroid, a sheet of polyvinyl alcohol is subjected to a large strain due to increase in its original length by 3 to 8 times by heating. As a result the molecules get oriented in the direction of the applied strain. When the sheet is impregnated with iodine, the material becomes dichroic. This polaroid sheet is called ***H*-polaroid.** This polaroid is colourless and transmits more light than herapathite polaroid.

If the stretched sheet of polyvinyl alcohol is heated in presence of a dehydrating agent such as HCl, it becomes strongly dichroic as well as very stable. This polaroid sheet is called ***K*-polaroid.**

Uses

1. **In sunglasses** It is used in sunglasses to reduce the glare of light reflected from horizontal surfaces. Such goggles are more efficient in protecting the eye than those prepared from coloured glass.
2. **In wind shield of car** K-polaroids are not bleached by strong sunlight. They are used as headlight and wind screen of cars to cut off the dazzling light of the approaching vehicles and protect the eyes of the automobile drivers.
3. **In window panes** The polaroids are used in window panes of an aeroplane to control the light entering through the windows.

Problem 1

Two polaroids are adjusted in such a way that they give a maximam intensity. Through what angle should one polaroid be rotated to decrease the intensity of one fourth of its maximum intensity.

Solution We know from Malus' law

$$I = I_0 \cos^2\theta$$

Let θ be the required angle

Here, $I = \frac{1}{4} I_0$

Therefore, $\frac{I_0}{4} = I_0 \cos^2\theta$

or, $\cos^2\theta = \frac{1}{4}$ or, $\cos^2\theta = \pm\frac{1}{2}$ or, $\theta = \pm 60°$

6.10. Optical Activity

It is observed that certain substances like quartz, aqueous solution of sugar (optically active liquid) etc. rotate the plane of vibration of a plane polarized light that passes through them.

The property of a substance by virtue of which the plane of vibration of polarizing light passing through them is rotated through certain angle without bringing about any change in its type of polarization is called the optical activity of the substance. This phenomena is also called **rotatory polarization.**

For an example when a quartz crystal is placed between polarizing and analyzing Nicol prism having its face perpendicular to the face of the first Nicol prism, the field of view will not be completely dark. This experiment leads to the conclusion that the quartz crystal rotates the plane of polarization through some angle, for which a fraction of light comes out from the analyzing Nicol prism.

The amount of optical rotation which occurs inside the body of the crystal depends upon the following factors—

① temperature of the crystal.
② thickness of the crystal.
③ density of the material or concentration in case of solutions.
④ wavelength of light used.

The substances which can rotate the direction of vibration of the incident polarized light are called *optically active substances.*

There are two types of optically active substances—

① **Dextro-rotatory substances** *The substances which rotate the plane of vibration of incident polarizing light in clockwise direction i.e., towards the right of the direction of propagation of light are called right handed or dextro-rotatory substances.*

- ***Examples :*** quartz.

② **Laevo-rotatory substances** *The substances which rotate the plane of vibration in an anti-clockwise direction i.e. towards the left of the direction of propagation of light (provided the observer faces) are called left handed or laevo-rotatory substances.*

- ***Examples :*** sugar (fruit sugar).

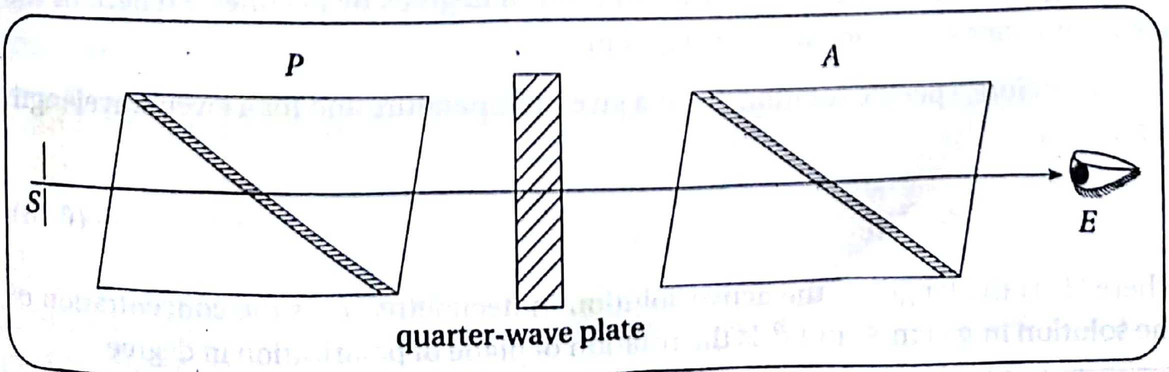

Fig. 26 ▷ Study of Rotation of plane of polarization

Biot's laws of optical activity

Scientist *Biot* studied the phenomenon of rotatory polarization with various optically active substances and different wavelengths of polarized light. Based on these studies, he formulated the following laws—

① *The angle of rotation θ of the plane of polarization* for a given wavelength and temperature is directly proportional to the length 'l' of the optically active substance *i.e.* $\theta \propto l$.

② The angle of rotation (θ) is inversely proportional to the square of the wave-length (λ) of light for a given length of the optically active substance *i.e.* $\theta \propto \frac{1}{\lambda^2}$.

③ The angle of rotation (θ) is directly proportional to the concentration (c) of the solution *i.e.* $\theta \propto c$.

④ The rotation (θ) produced by a mixture of optically active substances is equal to the algebraic sum of individual rotations, *i.e.* $\theta = \theta_1 + \theta_2 + \theta_3 + \cdots$.

Here, the clockwise rotation is considered as negative and anti-clockwise rotation as positive and the amount also depends on the nature and temperature of the substance.

6.11. Specific Rotation[6]

The phenomenon of rotating the plane polarized light about its direction of propagation in a medium is known as ***optical rotation.***

Liquids containing an optically active substance (example : sugar solution) rotate the plane of linearly polarized light. The angle through which the plane of polarized light is rotated depends upon—

① the length (l) of the optically active substance.

② the concentration of the solution (in $g \cdot cm^{-3}$) or density of the active substance in the solvent.

③ temperature and ④ wavelength of light.

The specific rotation of a solution at a given temperature and for a given wavelength of light is defined as the rotation in degrees by decimetre length of the active substance of concentration 1 $g \cdot cm^{-3}$.

Therefore, specific rotation S_λ at a given temperature and for a given wavelength is given by,

$$S_\lambda = \frac{\theta}{lc} \quad \cdots (6.18)$$

where 'l' is the length of the active solution in decimetre, 'c' is the concentration of the solution in $g \cdot cm^{-3}$ and θ is the rotation of plane of polarization in degree.

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[6] Not included in W.B.U.T. syllabus

6.12. Polarimeter[7]

It is an optical instrument, which measures the optical rotation of the plane of polarization of a plane polarized light produced by an optically active substance.[8]

There are two types of polarimeter—

① Laurent's half shade polarimeter

② Bi-quartz polarimeter.

They consist of two Nicol prisms capable of rotating about a common axis of a hollow tube for filling the solution of optically active substances.

6.12.1. Laurent's Half Shade Polarimeter[9]

Description of the apparatus S is a source of monochromatic light. The rays of light are made parallel by a convex lens L and they incident on the Nicol polarizer P which converts unpolarized light to polarized light. This polarized light incidents normally on half shade plate H [Fig. 27]. Half shade plate is a round plate, one half of

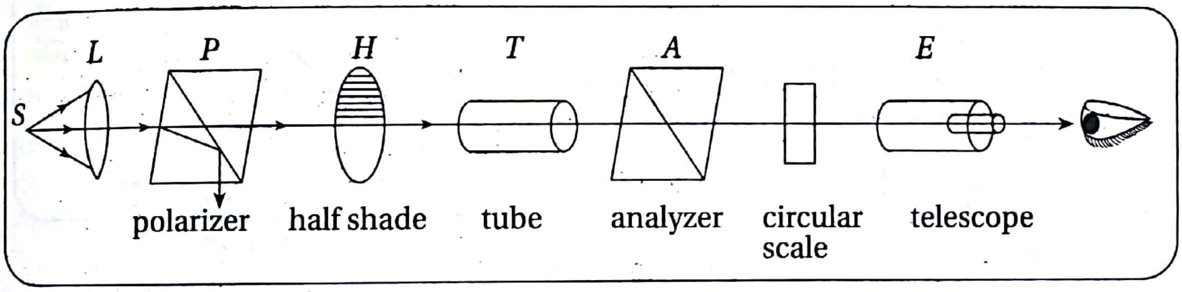

Fig. 27 ▷ Action of Laurent's half shade polarimeter

which is made up by quartz (ACB) [Fig. 28] and the other half (ADB) is a glass plate. The thickness of the quartz plate is such that, it introduces a path difference of $\frac{\lambda}{2}$ between O-ray and E-ray. The thickness of the glass plate is such that it absorbs some amount of light as the quartz plate does. The plane polarized light after passing from H, enters the tube T containing the active solution whose specific rotation is to be determined. The transmitted light from T passes through the Nicol analyzer 'A' which can be rotated about the direction of propagation of light as axis and its rotations can be read on a circular scale graduated in degrees with the help of a vernier. The light emerging from Nicol analyzer A is viewed through the telescope 'E'. The Nicol analyzer A and the telescope E are enclosed in a tube which can rotate about the axis of whole instrument.

Procedure to find specific rotation of sugar solution

① The polarimeter tube T is first filled with water. The analyzer A is slowly rotated till the two halves (ACB and ADB) of half shade device are equally

[7] Not included in W.B.U.T. syllabus

[8] When a polarimeter is used to measure the percentage of cane sugar in a solution, it is named as **saccharimeter**.

[9] Not included in W.B.U.T. syllabus

dark or bright. The position of the analyzer is recorded from the circular scale and the vernier scale.

② The tube T is now filled with the sugar, a solution of known concentration, replacing water (without any air gap). As the sugar solution rotates the plane of vibration in clockwise direction, the analyzer A is rotated in clockwise direction to obtain equally dark or bright field of view. This position of analyzer is recorded from scale. The difference between the two positions of the analyzer gives the angle of rotation (θ) for that concentration (c).

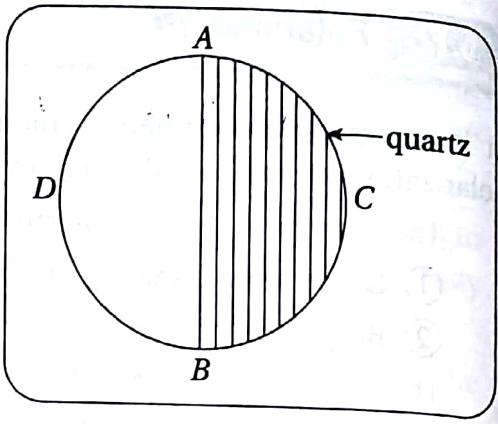

Fig. 28 ▷ Half shade plate

③ The experiment is repeated for various concentrations of solutions. A graph is plotted between θ and 'c'. The graph is a straight line, [Fig. 29]. The slope of the curve gives the value of $\frac{\theta}{c}$. Hence, the specific rotation (S_λ) of sugar solution is given by,

$$S_\lambda = \frac{10\theta}{lc} \quad \cdots (6.19)$$

where l is the length of the tube in cm.

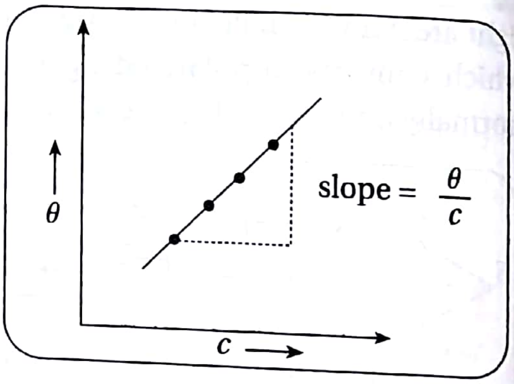

Fig. 29

6.12.2. Bi-Quartz Polarimeter[10]

Half shade plate used in Laurent's polarimeter can be used for a particular wavelength of light for which it is proposed. So, it cannot be used for any other wavelength of light. This disadvantage can be overcome by using a bi-quartz plate (in place of half shade plate), so that it can be used even with white light.

A bi-quartz plate consists of two semi-circular plates with one of left-handed quartz (laevo-rotatory) and the other of right-handed quartz (dextro-rotatory) [Fig. 30]. They are cut with their optic axis perpendicular to their refracting surfaces. The two plates are then joined and cemented to form a circular plate.

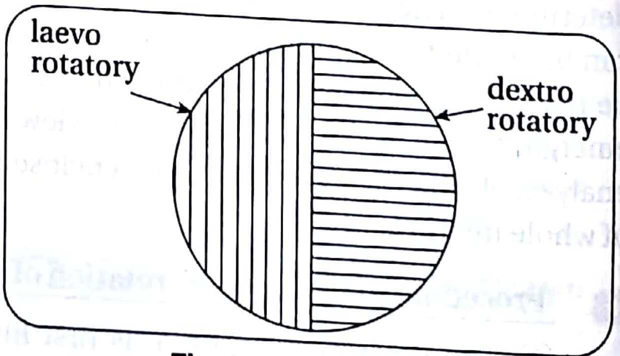

Fig. 30 ▷ Bi-quartz plate

Thus, the left handed plate rotates the plane of polarization of the incident beam in an anti-clockwise direction, while the right handed plate rotates the plane of

[10] Not included in W.B.U.T. syllabus

polarization in clockwise direction. The thickness of both the plates are same and they are so adjusted that only the wavelength corresponding to the yellow light will have $90°$ rotation of the plane of polarization.

Arrangement and description of the apparatus It has almost the same arrangement as that of Laurent's half shade polarimeter except a bi-quartz plate is used in place of half shade plate and white light is used in place of monochromatic light used in Laurent's half shade polarimeter.

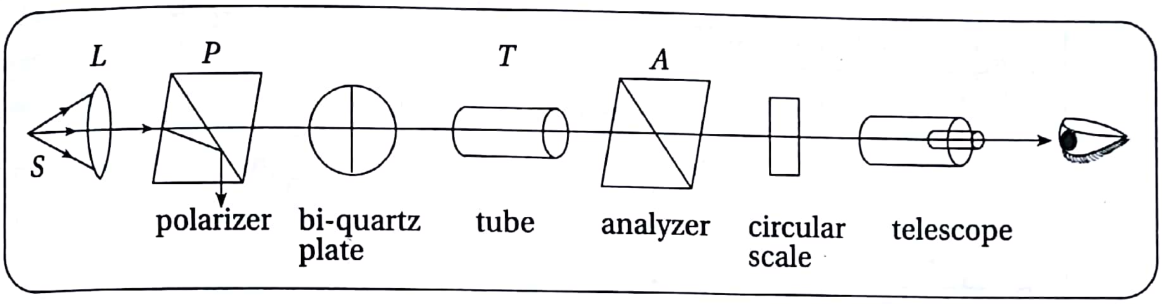

Fig. 31 ▷ Experimental arrangement of bi-quartz polarimeter

Action If white light is used, different wavelengths of incident light will have different rotations. Only the wavelength corresponding to yellow light will have a $90°$ rotation and will be totally obstructed by the analyzer. The emergent light in which yellow light will be missing will produce a dim-grey violet tint in both the halves which is known as tint of passage. By adjusting the particular position of the analyzer, field of view appears equally bright with the tint of passage. When the analyzer is rotated to one side of this position, one-half of the field of view appears blue while the other half appears red. If the analyzer is rotated in the opposite directions, the first half appears red and the second half appears blue. However in order to measure the specific rotation, the position of the analyzer A should be so adjusted that both the halves of the field of view get the tint of passage when the optically active solution is taken in the tube as well as the water are taken in the tube. The difference between the two positions of the analyzer gives the optical rotation θ of the optically active solution. Hence the specific rotation S_λ of the solution can be found out by using the equation,

$$S_\lambda = \frac{10\theta}{lc}$$, where l is the length of the tube taken in cm.

6.13. Babinet's Compensator[11]

One of the disadvantages of quarter-wave plate or a half-wave plate is that it can produce only a fixed path difference between the O-ray and E-ray for light of a particular wavelength. This disadvantage can be overcome by using Babinet's compensator.

Babinet's compensator is a device by means of which a desired path difference can be introduced between the O-ray and the E-ray for light of any wavelength.

[11] Not included in W.B.U.T. syllabus

Construction It consists of two small wedge shaped quartz prisms mounted in holder with their hypotenuse faces adjacent so that their optic axes are mutually perpendicular and both are perpendicular to the incident light. As a result, the speed of two rays (E-ray and O-ray) are interchanged at the interface. The wedge w_2 is fixed in the holder and other w_1 can be moved by a micrometer screw so that its hypotenuse face slides over that of the adjusting fixed wedge [Fig. 32]. Hence, the faces of the wedges are cut parallel to the respective optic axis.

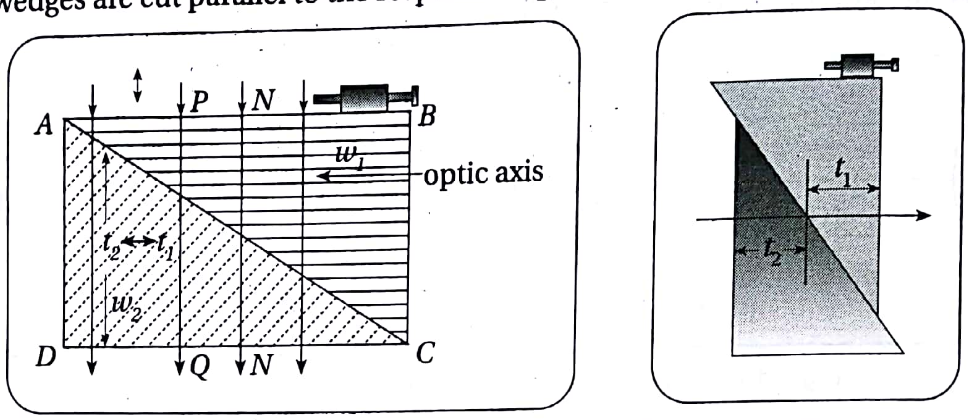

Fig. 32 (a) and (b) ▷ Construction of Babinet's compensator

Theory *Quartz is a positive crystal.* The optic axis of the quartz w_1 is parallel to the crystal surface while that of quartz w_2 is perpendicular to the crystal surface. Therefore, the plates have their optic axes mutually perpendicular. As a result, the E-ray in w_1 behaves as ordinary ray in w_2 while the O-ray in w_1 will behave as extra-ordinary ray in w_2 due to their interchange of speed at the interface.

If a plane polarized light from the polarizer is allowed to fall normally on the face AB, it will be split up into E-ray and O-ray in the crystal and they will travel along the same direction with different velocities (velocity of O-ray is more than that of E-ray). On transmission through the interface between the wedges, the O-ray in wedge w_1 becomes the E-ray in wedge w_2 and E-ray in wedge w_1 becomes the O-ray in wedge w_2 due to the interchange of their speed at the interface.

Consider ray PQ traverses a thickness t_1 in the first wedge w_1 and thickness t_2 in the second wedge w_2. If μ_e and μ_o be the refractive indices of quartz for the E-ray and O-ray respectively, then the path difference introduced between the two rays by the first wedge is $\Delta_1 = (\mu_e - \mu_o)t_1$ and that introduced by the second wedge is given by $\Delta_2 = (\mu_o - \mu_e)t_2$. Therefore, *the total path difference between the two rays will be,*

$$\Delta = \Delta_1 + \Delta_2 = (\mu_e - \mu_o)t_1 + (\mu_o - \mu_e)t_2$$

$$= (\mu_e - \mu_o)t_1 - (\mu_e - \mu_o)t_2 = (\mu_e - \mu_o)(t_1 - t_2) \qquad \cdots (6.20)$$

Therefore, the corresponding phase difference between the two rays,

$$\delta = \frac{2\pi}{\lambda} \times \text{path difference}$$

or,

$$\delta = \frac{2\pi}{\lambda} \times (\mu_e - \mu_o)(t_1 - t_2) \qquad \cdots (6.21)$$

So, by sliding the wedge w_1, the difference $(t_1 - t_2)$ can be given any desired value. As a result we may introduce any desired path difference or phase difference between the E-ray or O-ray. At the centre of the compensator (*i.e.* $t_1 = t_2$) the resultant phase difference is zero. So, the emergent light is plane polarized. On either side of this point the path difference gradually changes and the emergent light is polarized in various ways.

Problem 1

A sugar solution is prepared by adding 80g of cane sugar into 1L of water. It gives an optical rotation of 9.9° when it is filled in a 20cm tube. If specific rotation of the pure sugar is $60°\text{cm}^2\cdot\text{g}^{-1}$, find the percentage of purity of the sugar.

Solution S_λ = specific rotation = $60°\text{cm}^2\cdot\text{g}^{-1}$

θ = optical rotation = 9.9°, l = length of the tube = 20cm

c = concentration of the solution

$$\therefore \quad S_\lambda = \frac{10\theta}{lc} = \frac{10\times 9.9}{20\times c}$$

$$\text{or,} \quad c = \frac{9.9\times 10}{20\times 60} = 0.0825\,\text{g}\cdot\text{cm}^{-3}$$

$\therefore$ The concentration of sugar in the solution is $82.5\,\text{g}\cdot\text{L}^{-1}$.

Hence, the percentage of purity of sugar sample = $\frac{80}{82.5}\times 100\% = 96.6\%$

Problem 2

A tube of 20 cm length filled with a solution of 15 g of cane sugar in 100 cc of water is placed in the path of a polarized light. Find the angle of rotation of the plane of polarization if the specific rotation of cane sugar is 65°.

Solution We know, $S_\lambda = \frac{10\theta}{lc}$, where l is the length of the tube in cm.

Here, $S_\lambda = 65°$, $l = 20$ cm and $c = \frac{M}{V} = \frac{15}{100} = \frac{3}{20}\,\text{g}\cdot\text{cm}^{-3}$ = strength of the sugar solution.

$$\therefore \quad 65° = \frac{10\theta}{20\times\frac{3}{20}}$$

$$\text{or,} \quad \theta = 19.5°$$

Problem 3

A certain length (l_1) of 6% solution rotates the plane of polarization by 22°. How much length of 15% solution of the same substance will cause a rotation of 30°?

Solution Since, the same solution is used, so the specific rotation S_λ remains constant.

$$\text{Hence,} \quad S_\lambda = \frac{10\theta_1}{l_1c_1} = \frac{10\theta_2}{l_2c_2}$$

$$\text{or,} \quad \frac{\theta_1}{l_1c_1} = \frac{\theta_2}{l_2c_2}$$

$$\text{or,} \quad l_2 = \frac{\theta_2 l_1 c_1}{\theta_1 c_2} = \frac{30\times l_1\times 6}{22\times 15} = 0.545\,l_1 \text{ (nearly)}$$

Problem 4

The refractive indices of quartz for left handed and right handed circularly polarized sodium lights propagating along the optic axis are 1.54420 and 1.54427 respectively. Calculate the specific rotation of quartz. Take wavelength of sodium light as 6×10^{-5} cm.

Solution If μ_l and μ_r be the refractive indices of anti-clockwise and clockwise circularly polarized light, then the rotation θ may be expressed as,

$$\theta = \frac{\pi d}{\lambda}(\mu_l \sim \mu_r)$$ [12]

Here, $\mu_r = 1.54427$, $\mu_l = 1.54420$ and $\lambda = 6 \times 10^{-5}$ cm

Therefore, specific rotation $\frac{\theta}{d} = \frac{\pi}{\lambda}(\mu_r \sim \mu_l)$

$$= \frac{3.14}{6 \times 10^{-5}}(1.54427 - 1.54420)$$

$$= \frac{3.14 \times 0.00007}{6 \times 10^{-5}} = 3.66 \text{ rad} \cdot \text{cm}^{-1} = 210° \text{ cm}^{-1}$$

Exercise

Multiple Choice Questions

1. Which of the following phenomena proves the transverse characteristics of light?
   Ⓐ interference Ⓑ dispersion Ⓒ polarization **Ans.** Ⓒ

   [W.B.U.T. 2004]
2. Plane polarized light can be produced by—
   Ⓐ Nicol prism Ⓑ pile of plane Ⓒ all of them **Ans.** Ⓒ
3. An unpolarized light consists of—
   Ⓐ infinite number of plane polarized light
   Ⓑ finite number of plane polarized light
   Ⓒ none of them **Ans.** Ⓐ
4. In plane polarized light—
   Ⓐ the magnitudes of light vectors remain same but orientations change
   Ⓑ the magnitudes of light vectors change but orientations remain same
   Ⓒ the orientations and magnitudes both vary continuously **Ans.** Ⓑ
5. In elliptically polarized light—
   Ⓐ the magnitudes of light vectors remain same but orientations change
   Ⓑ the magnitudes of light vectors change but orientations remain same
   Ⓒ the orientations and magnitudes both vary continuously **Ans.** Ⓒ

[12] The derivation of this formula is beyond the scope of this book.

6. In circularly polarized light—
   Ⓐ the magnitudes of light vectors remain same but orientations change
   Ⓑ the magnitudes of light vectors change but orientations remain same
   Ⓒ the orientations and magnitudes both vary continuously **Ans.** Ⓐ
7. At polarization angle (or Brewster's angle) the reflected ray from a glass is—
   Ⓐ partially polarized light
   Ⓑ completely polarized light
   Ⓒ none of above **Ans.** Ⓑ
8. At polarization angle the angle between reflected ray and refracted ray is—
   Ⓐ 90° Ⓑ 45° Ⓒ 180° **Ans.** Ⓐ
9. A tangent of the polarization angle of a partially reflecting transparent medium is—
   Ⓐ refractive index of the medium
   Ⓑ conductivity of the medium
   Ⓒ resistivity of the medium **Ans.** Ⓐ
10. If refractive index of a medium is 1.732, then angle of polarization of the medium is approximately equal to—
   Ⓐ 57° Ⓑ 60° Ⓒ none of them **Ans.** Ⓑ
11. If polarization angle of a medium is $60°$, the angle of refraction is—
   Ⓐ 40° Ⓑ 150° Ⓒ 30° **Ans.** Ⓒ
12. The optic axis of a calcite crystal is—
   Ⓐ a line joining the two blunt corner of it
   Ⓑ a direction parallel to the line joining the blunt corners
   Ⓒ a plane containing the line joining the blunt corners **Ans.** Ⓑ
13. Which of the following phenomena is responsible for polarization of light?
   Ⓐ interference Ⓑ double reflection Ⓒ double refraction **Ans.** Ⓒ
14. The velocities of O-ray and E-ray in a crystal are same along the direction of—
   Ⓐ optic axis Ⓑ geometrical axis Ⓒ none **Ans.** Ⓐ
15. The E-ray inside of a calcite crystal does not obey the law of —
   Ⓐ reflection Ⓑ refraction Ⓒ both **Ans.** Ⓑ
16. If the velocities of E-ray and O-ray are v_e and v_o respectively, then, it has been seen that inside the positive crystal—
   Ⓐ $v_e > v_o$ Ⓑ $v_o > v_e$ Ⓒ $v_o = v_e$ **Ans.** Ⓑ
17. If the velocities of E-ray and O-ray are v_e and v_o respectively, then, it has been seen that inside the negative crystal—
   Ⓐ $v_e > v_o$ Ⓑ $v_e = v_o$ Ⓒ $v_o > v_e$ **Ans.** Ⓐ
18. If the refractive indices of E-ray and O-ray are μ_e and μ_o respectively, then it has been seen that inside the positive crystal—
   Ⓐ $\mu_e < \mu_o$ Ⓑ $\mu_e > \mu_o$ Ⓒ $\mu_e = \mu_o$ **Ans.** Ⓑ

19. If the refractive indices of E-ray and O-ray are μ_e and μ_o respectively, then it is found that inside the negative crystal—

Ⓐ $\mu_e < \mu_o$ Ⓑ $\mu_e > \mu_o$ Ⓒ $\mu_e = \mu_o$ **Ans.** Ⓐ

20. The action of Nicol prism is based on—

Ⓐ scattering Ⓑ double refraction Ⓒ reflection **Ans.** Ⓑ

21. Nicol prism is formed by—

Ⓐ calcite crystal Ⓑ quartz crystal Ⓒ none of them **Ans.** Ⓐ

22. At the interface of calcite crystal and Canada-Balsam layer, if the refractive indices of O-ray, Canada-Balsam and E-ray are μ_o, μ_{CB} and μ_e respectively, then which of the following is incorrect?

Ⓐ $\mu_o > \mu_{CB}$ Ⓑ $\mu_{CB} > \mu_e$ Ⓒ $\mu_o < \mu_{CB}$ **Ans.** Ⓒ

23. In a Nicol prism, the O-ray is totally internally reflected but E-ray is transmitted. The statement is—

Ⓐ true Ⓑ false Ⓒ partially true **Ans.** Ⓐ

24. When an unpolarized ray falls on a Nicol prism, the emergent ray from it is—

Ⓐ plane polarized E-ray
Ⓑ plane polarized O-ray
Ⓒ circularly polarized E-ray **Ans.** Ⓐ

25. If the intensity of the light under test changes and vanishes, due to passage through the rotating Nicol prism, the light is—

Ⓐ plane polarized Ⓑ circularly polarized Ⓒ unpolarized **Ans.** Ⓐ

26. Light produced by a Nicol prism is—

Ⓐ plane polarized
Ⓑ elliptically polarized
Ⓒ circularly polarized **Ans.** Ⓐ

27. In quarter-wave plate the phase difference between E-ray and O-ray is—

Ⓐ π Ⓑ $\frac{\pi}{2}$ Ⓒ $\frac{\pi}{4}$ **Ans.** Ⓑ

28. In half-wave plate the path difference between E-ray and O-ray is—

Ⓐ λ Ⓑ $\frac{\lambda}{2}$ Ⓒ $\frac{\lambda}{4}$ **Ans.** Ⓑ

29. Light produced by polaroid is—

Ⓐ plane polarized
Ⓑ circularly polarized
Ⓒ elliptically polarized **Ans.** Ⓐ

30. The dichroism is—

Ⓐ the selective absorption of O-ray by the crystal
Ⓑ the selective absorption of E-ray and O-ray by the crystal
Ⓒ none **Ans.** Ⓑ

31. If one of the refracting ray is absorbed, the emergent ray is—
Ⓐ linearly polarized
Ⓑ circularly polarized
Ⓒ elliptically polarized
Ans. Ⓐ

32. When a plane polarized light is allowed to fall on a quarter-wave plate so that the angle between the optic axis and the plane of vibration is $45°$, then the emergent beam from the quarter-wave plate is—
Ⓐ elliptically polarized
Ⓑ circularly polarized
Ⓒ plane polarized
Ans. Ⓑ

33. If the angle between the optic axis and the vibration of plane polarized light is other than $0°$, $45°$ and $90°$, then the emergent beam from quarter-wave plate is —
Ⓐ elliptically polarized
Ⓑ plane polarized
Ⓒ circularly polarized
Ans. Ⓐ

34. If there is no variation of intensity of light under test, due to its passage through a rotating Nicol prism, then the light is—
Ⓐ circularly polarized
Ⓑ unpolarized
Ⓒ plane polarized
Ans. Ⓑ

35. The phenomena by which the plane of vibration of a polarizing light passing through quartz is rotated through a certain angle but the types of polarization remain the same, is called—
Ⓐ optical resistivity
Ⓑ photometry
Ⓒ optical activity (or optical rotation)
Ans. Ⓒ

36. If θ be the angle of rotation, l be the length of the active solution in cm and c be the concentration in $g \cdot cm^{-3}$, then the specific rotation is expressed as—
Ⓐ $S_\lambda = \frac{l\theta}{10c}$ Ⓑ $S_\lambda = \frac{10\theta}{lc}$ Ⓒ $S_\lambda = \frac{\theta c}{l}$
Ans. Ⓑ

37. The optical rotation produced by an optically active substance can be measured by using—
Ⓐ polaroid Ⓑ Nicol prism Ⓒ polarimeter
Ans. Ⓒ

38. The disadvantages of a quarter-wave plate or half-wave plate is that—
Ⓐ they produce variable path difference between O-ray and E-ray
Ⓑ they produce constant path difference between O-ray and E-ray
Ⓒ they produce variable phase difference between O-ray and E-ray
Ans. Ⓑ

39. The substance which rotates the plane of vibration in clockwise direction is known as—
(A) dextro-rotatory substance
(B) laevo-rotatory substance
(C) none of these
Ans. (A)

40. The substance which rotates the plane of vibration in anti-clockwise direction is known as—
(A) dextro-rotatory substance
(B) laevo-rotatory substance
(C) none of these
Ans. (B)

41. In a Laurent's polarimeter—
(A) half shade plate is used
(B) quarter-wave plate is used
(C) half-wave plate is used
Ans. (A)

42. In bi-quartz polarimeter—
(A) half shade plate is used
(B) bi-quartz plate is used
(C) half-wave plate is used
Ans. (B)

43. The plane of vibration and the plane of polarization of a beam of plane polarized light—
(A) are identical to each other
(B) are orthogonal to each other
(C) make an angle, which depends on the colour of the light
Ans. (B)
[W.B.U.T. 2006]

44. Which one of the following waves cannot be polarized?
(A) radio wave (B) X-ray (C) sound wave
Ans. (C)

45. Which one of the following is biaxial crystal?
(A) calcite (B) quartz
(C) argonite (D) none of these
Ans. (C)
[W.B.U.T. 2013]

46. The angle between the planes of vibration and polarization of a beam of polarized light is—
(A) 90° (B) 45°
(C) 0° (D) 180°
Ans. (A)
[W.B.U.T. 2013]

47. Polarization conclusively prove that light waves are—
(A) longitudinal (B) progressive
(C) stationary (D) transverse
Ans. (D)
[W.B.U.T. 2012]

Short Answer Type Questions

1. [a] Define polarization of light. [See Article 6.3]
   [b] Prove that the tangent of the polarization angle is equal to the refractive index of the medium. [See Article 6.6.1]
2. The critical angle of glass with respect to air is $41.81°$. Find the refractive index of the medium and the angle for refraction of the light incident on glass at the polarization angle.
3. State Malus' law and prove it. [See Article 6.6.3]
4. [a] Define double refraction [See Article 6.6.5]
   [b] When does double refraction occur in anisotropic crystal but not in isotropic crystal? [See Article 6.6.4]
5. [a] What are the differences between O-ray and E-ray? [See Article 6.6.6]
   [b] What are the differences between uniaxial and biaxial crystal? [See Article 6.6.7]
   [c] Why double refraction cannot occur along the optic axis of a crystal? [See Article 6.6.11]
6. [a] What is Nicol prism? [See Article 6.6.12]
   [b] Discuss its principle. [See Article 6.6.12]
7. Discuss Nicol prism as polarizer and analyzer. [See Article 6.6.12] **[W.B.U.T. 2004, 2005]**
8. What is retardation plate? How can you distinguish between circularly polarized light and unpolarized light with the help of quarter-wave plate and rotating Nicol prism? [See Article 6.7]
9. What are polaroids? State its construction and uses. [See Article 6.9]
10. [a] Define optical activity. [See Article 6.10]
    [b] On which factors of the crystal does the amount of optical rotation depend? [See Article 6.10]
11. [a] What is specific rotation of a solution? [See Article 6.11]
    [b] A 20 cm long tube contains sugar solution of 16 g in 100 cc of water. When it is placed in a saccharometer (a type of hydrometer that measures the concentration of sugar) optical rotation of 20° is observed. Find the specific rotation of sugar.

    [**Hint**: $S = \dfrac{10\theta}{lc}$, $\theta = 20°$, $l = 20$ cm, $c = \dfrac{16}{100}$ g · cm^{-3}]

Long Answer Type Questions

1. [a] What is polarization of light? (See Article 6.2) **[W.B.U.T. 2004, 2005]**
   [b] Define plane of vibration and plane of polarization. (See Article 6.4) **[W.B.U.T. B.OPT 2004]**

[c] State Brewster's law and hence prove that the angle between the reflected and refracted ray is $90°$.(See Article 6.6.2) **[W.B.U.T. 2005, W.B.U.T. B.OPT 2006]**

[d] The refractive index of a glass plate is 1.6. Calculate the angle of polarization and the corresponding angle of refracted ray. **[W.B.U.T. 2005, W.B.U.T. B.OPT 2006]**

2. [a] Explain how polarization can be obtained by reflection. (See Article 6.6.1)

   [b] State and explain Malus' law. [See Article 6.6.3] **[C.U.(Hons) 2001]**

   [c] What are the differences between (i) isotropic and anisotropic medium, (ii) uniaxial and biaxial crystal, (iii) positive and negative crystal? [See Article 6.6.4, 6.6.7, 6.6.9]

3. [a] Discuss the phenomenon of double refraction. [See Article 6.6.5] **[W.B.U.T. 2005]**

   [b] What are the differences between O-ray and E-ray? [See Article 6.6.6]

   [c] Describe the construction of Nicol prism. [See Article 6.6.12] **[C.U.(Hons) 1999]**

   [d] Describe how polarized light can be obtained with the help of Nicol prism. [See Article 6.8] **[C.U.(Hons) 2002, B.U.(Hons) 2000]**

   or,

   Write short notes on Nicol prism and its uses as polarizer and analyzer. **[W.B.U.T. 2004]**

4. [a] What is retardation plate? Distinguish between quarter-wave plate and half-wave plate. [See Article 6.7]

   [b] How can we distinguish between (i) an unpolarized light and circularly polarized light, (ii) an elliptically polarized and a mixture of plane polarized light and unpolarized light? **[W.B.U.T. B.OPT Optics II 2004]**

5. [a] Describe the construction of a quarter-wave plate and explain how you can produce circularly and elliptically polarized light in such a plate. [See Article 6.7, 6.8] **[C.U. (Hons) 1995]**

   [b] How are unpolarized, plane polarized, circularly polarized and elliptically polarized light distinguished?

   [c] What is specific rotation?

   [d] What are polaroids? Mention their uses. **[B.U.(Hons) 1999]**

   [e] What are the advantages of a polaroid over a Nicol prism? [See Article 6.9]

6. [a] Discuss optical activity and its application in any one polarimeter.

   Describe with necessary theorem, how to determine the concentration of sugar solution by using optical rotation method. [See Article 6.10] **[C.U. (Hons) 1996]**

[b] What is Babinet's compensator? What are the advantages of it over a half-wave or quarter-wave plate?

[c] Discuss the construction and necessary theory of thin device to get a desired path difference introducing between O-ray and E-ray for light of any wavelength. [See Article 6.13]

7. [a] Write a short note on Nicol prism and its use as polarizer and analyser. (See Article 6.6.12) **[W.B.U.T. 2012]**

[b] What are positive and negative crystals? Describe the construction of Nicol prism. (See Article 6.6.9, 6.6.12) **[W.B.U.T. 2013]**

[c] What is Brewster's angle? (See Article 6.6.1) **[W.B.U.T. 2010]**

Numerical Problems

1. Calculate the polarizing angle for diamond surface, if the angle of refraction of a beam of light through it is $12°$ when incident beam makes an angle of $60°$.

$\left[\textbf{Hint:}\ i_p = \tan^{-1}\left(\frac{\sin 60°}{\sin 12°}\right)\right]$ [Ans : $76.5°$] **[CU (Hons) 1966]**

2. The critical angle in certain substance is given to be $42°$. Calculate the polarizing angle. [Ans : $55°$]

3. Two polarizers are placed at cross position (angle between the polarizing planes is $90°$). A third polarizer with angle θ with the first one is placed between them. An unpolarized light of intensity I is incident on the first one and passes through all three polarizers. Find the intensity of light that comes out. **[WBUT 2008]** [Ans : $\frac{I}{8}\sin^2 2\theta$]

[**Hint:** Light transmitted from 1st polarizer $\frac{I}{2}$

Light transmitted from 2nd polarizer $= \frac{I}{2}\cos^2\theta$

Light transmitted from 3rd polarizer $= \left[\frac{I}{2}\cos^2\theta\right]\cos^2(90° - \theta)$

$= \frac{I}{8}\sin^2 2\theta$

4. Two polaroids are crossed to each other. Now one of them is rotated $60°$. What percentage of incident unpolarized light will pass through the system? [Ans : 37.5%]

[**Hint:** Light transmitted by 2nd polarizer $= \left(\frac{I_0}{2}\right)\cos^2(90° - 60°)$

Here, I_0 = intensity of incident unpolarized light. The angle between the transmission planes of the two polaroids on rotating through $60°$ becomes $(90° - 60°) = 30°$]

5. A ray of light is incident on a liquid of refractive index 1.40. Find the angle of refraction of the beam when the reflected ray is completely polarized. [Ans : $35.54°$]

6. The refractive indices of double refracting crystal for ordinary and extra ordinary rays are 1.584 and 1.592 respectively for wavelength $\lambda = 5600\text{Å}$. Determine the thickness of the crystal to produce half wave plate.

[Ans : 3.5×10^{-3}cm]

7. Unpolarized light is incident normally on a piece of quartz plate cut parallel to its principal axis. After emergence the path difference between ordinary and extra ordinary ray is $\frac{\lambda}{2}$.

Find the thickness of the plate. Given $\mu_o = 1.54442$ and $\mu_e = 1.5533$ and $\lambda = 5000\text{Å}$.

[C.U. 1993] [Ans : 2.74×10^{-3}cm]

8. In a Saccharimeter, the tube of 30cm length containing 60cm^3 of sugar solution produces an optical rotation of $15°$. If the specific rotation of sugar solutions is $65°$, find the quantity of sugar contained in the tube in the form of a solution.

[Ans : 4.56g]

[**Hint :** $c = \frac{10\theta}{l s_\lambda} = 0.076\text{g} \cdot \text{cm}^{-3}$

$\therefore$ Mass of sugar solution, $M = cV = 0.076 \times 60 = 4.56$ g]

9. The refractive index of E-ray and O-ray are respectively 1.65 and 1.45. Then find the thickness of the material required for a quarter-wave plate of light of wavelength 5000 Å.

[W.B.U.T. 2006]

$\left[\textbf{Hint :}\ t = \frac{\lambda}{4(\mu_e - \mu_o)}\right]$