Probability lesson plan 3

Random Variable

1. Two dice are rolled. Let X denote the random variable which counts the total number of points on the upturned faces, Construct a table giving the non-zero values of the probability mass function and draw the probability chart. Also find the distribution function of X.
2. The following is the distribution function of a discrete random variable X:

x	0	1	2	3	4	5	6	7
F(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) Find k, (ii) Evaluate $P(X < 6)$, $P(X \geq 6)$, and $P(0 < X < 5)$ (iii) Determine the distribution function of X.

3. An experiment consists of three independent tosses of fair coin. Let:
   X=the number of heads, Y=the number of head runs, Z=the length of head runs, a head run being defined as consecutive occurrence of at least two heads, its length then being the number of heads occurring together in three tosses of the coin.
   Find the probability function of (i) X, (ii) Y, (iii) Z, (iv) X+Y, (v) XY.
4. Suppose that the life in hours of a certain part of radio tube is a continuous random variable X with p.d.f. given by:

$$f(x) = \begin{cases} \frac{100}{x^2}, whenx \geq 100 \\ 0, \quad elsewhere \end{cases}$$

(i) What is the probability that all of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation?

(ii) What is the probability that none of three of the original tubes will have to be replaced during that first 150 hours of operation?

(iii) What is the probability that a tube will last less than 200 hours if it is known that the tube is still functioning after 150 hours of service?

5. A petrol pump is supplied with petrol once a day. If its daily volume of sales (X) in thousands of liters is distributed by: $f(x) = 5(1-x)^4, 0 \leq x \leq 1$, what must be the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01?
6. In a continuous distribution whose relative frequency density is given by: $f(x) = y_0.x(2-x), 0 \leq x \leq 2$, find the mean, variance, median, mode of the distribution and also show that for the distribution $\mu_{2n+1} = 0$.
7. The diameter, say X, of an electric cable, is assumed to be a continuous random variable with p.d.f. $f(x) = 6x(1-x), 0 \leq x \leq 1$
   (i) Check that f(x) is a p.d.f.
   (iii) Compute the number k such that $P(x < k) = P(x > k)$.
8. A random variable X has the probability low :

$$dF(x) = \frac{x}{b^2} e^{-\frac{x^2}{2b^2}}\, dx, \qquad 0 \leq x < \infty$$

Find the distance between quartiles and show that the ratio of this distance to the standard deviation of X is independent of the parameter ‘b’.

9. Calculate the standard deviation and mean deviation from mean if the frequency of $f(x)$ has the form :

$$f(x) = \begin{cases} \frac{3+2x}{18}, & for\ 2 \leq x \leq 4 \\ 0, & otherwise \end{cases}$$