B.Tech 2nd semester
Assignment -II

1. Evaluate the followings:
   a) $\int_0^{\frac{\pi}{4}} \int_0^{\sqrt{\cos 2\theta}} \frac{r}{(1+r^2)^2} dr d\theta$
   b) $\int \int r^2 \sin\theta \, dr \, d\theta$, over the cardoid $r = a\,(1 + cos\theta)$ above the initial line.
   c) $\int_0^1 \int_{\sqrt{x-x^2}}^{\sqrt{1-x^2}} \frac{xye^{-(x^2+y^2)}}{x^2+y^2} dxdy$
   d) $\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} \frac{x^2-y^2}{x^2+y^2} dxdy$
2. Using the transformation $u = x - y, v = x + y$, evaluate $\int \int cos\left(\frac{x-y}{x+y}\right) dxdy$ over the region bounded by the lines $x = 0, y = 0, x + y = 1$.
3. Using the transformation $u = x^2 - y^2, v = 2xy$, evaluate $\int \int (x^2 + y^2)\, dxdy$ over the region bounded by the hyperbola $x^2 - y^2 = 1, x^2 - y^2 = 9, xy = 2, xy = 4$.
4. Evaluate $\int \int \int x^2yz\, dx\, dy\, dz$ over the region bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$.
5. Evaluate $\int \int \int \frac{dxdydz}{(x^2+y^2+z^2)^{\frac{3}{2}}}$ over the region bounded by the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$,$a > b > 0$.
6. Evaluate $\int \int \int \frac{dxdydz}{(x^2+y^2+z^2)^{\frac{1}{2}}}$ over the region bounded by the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$,$a > b > 0$.
7. Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z}\, dz\, dy\, dx$

Vector Calculus

1. Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$.
2. Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$.
3. Find the directional derivative of $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$. Also calculate the magnitude of the maximum directional derivative.
4. Find the directional derivative of $\phi = 5x^2y - 5y^2z + 2.5z^2x$ at the point $P(1, 1, 1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = z$.
5. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.
6. A fluid motion is given by $\bar{v} = (ysin\, z - \sin x)i + (xsinz + 2yz)j + (xycos\, z + y^2)k$ is the motion irrational? If so, find the velocity potential.
7. Show that the vector field represented by $\vec{F} = (z^2 + 2x + 3y)i + (3x + 2y + z)j + (y + 2zx)k$ is irrotational but not solenoidal. Also obtain a scalar function ϕ such that $grad\ \phi = \vec{F}$.
8. If the directional derivative of the function $\emptyset = xyz$ at $(1, 1, 1)$ in the direction of $\alpha i + j + k$ is $\sqrt{3}$, find α.
9. Find the angle between the surface $xlog\, z = y^2 - 1$ and $x^2y = 2 - z$ at the points $(1, 1, 1)$.
10. Find the values of a and b so that the surfaces $ax^3 - by^2z = (a + 3)x^2$ and $4x^2y - z^3 = 11$ may cut orthogonally at $(2, -1, -3)$.
11. Find the values of a, b, c so that the vector $\bar{F} = (x + y + az)\hat{\imath} + (bx + 2y - z)\hat{\jmath} + (-x + cy + 2z)\hat{k}$ may be irrotational.
12. Prove that $\bar{A} = (z^2 + 2x + 3y)i + (3x + 2y + z)j + (y + 2zx)k$ is conservative.
13. Prove that, $\int_C \bar{F}.d\bar{r} = 3\pi$, where $\bar{F} = zi + xj + yk$ and C is the arc of the curve $\bar{r} = \cos t\, i + \sin t\, j + tk$ from $t = 0$ to $t = 2\pi$.
14. If $\bar{F} = (2x - y + 2z)i + (x + y - z)j + (3x - 2y - 5z)k$, calculate the circulation of $\bar{F}$ along the circle in the xy-plane of radius of 2 unit radius and centre at the origin.

15. If $\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ then,

(i) Prove that $\bar{F}$ is conservative

(ii) Find its scalar potential

(iii) Find the work done in moving a particle under this force field from $(1, 1, 0)$ to (2, 0, 1).

16. If $\bar{F} = 2xye^z i + x^2e^z j + x^2ye^z k$ is conservative then find

(i) The scalar potential φ

(ii) The work done in moving a particle under this force field from (0, 0, 0) to (1, 1, 1).

17. Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2 dy]$ where C is the region bounded by $y = x, y = x^2$.

18. Verify Green's theorem for $\oint_C (2xy\, dx - y^2\, dy)$, where C is the boundary of the region bounded by the ellipse $3x^2 + 4y^2 = 12$.

19. Evaluate the following integrals by using Green's theorem **:**

(a) $\oint_C [(x^2 + y^2)dx + (5x^2 - 3y)dy]$, where C is the region bounded by the parabola $x^2 = 4y$ and the line $y = 4$.

(b) $\oint_C [e^x(\sin y\, dx + \cos y\, dy)]$, where C is the boundary of the region bounded by the ellipse $4(x + 1)^2 + 9(y - 3)^2 = 36$.