15CS36

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define Tautology. Verify the following compound proposition is a tautology or not : $\{(p \vee q) \to r\} \leftrightarrow \{\sim r \to \sim (p \vee q)\}$. **(04 Marks)**

b. Check whether the following argument is valid or not :
If I study, then I will not fail in exam.
If I do not watch TV in the evenings, then I will study.
I failed in exam.
$\therefore$ I must have watched TV in the evenings. **(04 Marks)**

c. Define : i) open sentence ii) quantifiers. Write the following proposition in symbolic form and find its negation : "All integers are rational numbers and some rational numbers are integers". **(04 Marks)**

d. Give a direct proof of the statement, "For all integers K and l, if K and l are both even then $K + l$ is even and Kl is even". **(04 Marks)**

OR

2 a. Define converse, inverse and contra positive of an implication. Hence find converse, inverse and contra positive for "$\forall x, (x > 3) \to (x^2 > 9)$" where universal set is the set of real numbers R. **(04 Marks)**

b. Using the laws of logic, prove the following logical equivalence : $[(\sim p \vee \sim q) \wedge (F_0 \vee p) \wedge P] \Leftrightarrow p \wedge \sim q$. **(04 Marks)**

c. What are bound variables and free variables. Identify the same in each of the following expressions :
i) $\forall y, \exists z \{\cos(x + y) = \sin(z - x)\}$
ii) $\exists x, \exists y \{(x^2 - y^2) = z\}$. **(04 Marks)**

d. Verify the validity of the following argument : If a triangle has two equal sides, then it is isosceles. If a triangle is isosceles, then it has two equal angles. The triangle ΔABC does not have two equal angles. $\therefore \Delta ABC$ does not have two equal sides. **(04 Marks)**

Module-2

3 a. Prove by mathematical induction $1.3 + 2.4 + 3.5 + \cdots + n(n+2) = \dfrac{n(n+1)(2n+7)}{6}$. **(04 Marks)**

b. Give a recursive definition for each of the following integer sequence :
i) $a_n = 7n$ ii) $a_n = 2 - (-1)^n$ for $n \in z^+$. **(04 Marks)**

c. How many positive integers can be formed by using the digits 3, 4, 4, 5, 5, 6, 7 to exceed 5,000,000? **(04 Marks)**

d. In how many ways can we distribute seven apples and six oranges among four children so that each child receives at least one apple? **(04 Marks)**

OR

4 a. If $F_0, F_1, F_2, ----$ are Fibonacci numbers, then prove by induction $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$. **(04 Marks)**

b. A sequence $\{a_n\}$ is defined recursively as $a_1 = 7$ and $a_n = 2a_{n-1} + 1$ for $n \geq 2$. Find a_n in explicit form. **(04 Marks)**

c. Find the number of arrangements of all the letters in the word "TALLAHASSEE". How many of these arrangements have no adjacent A's? **(04 Marks)**

d. Find the coefficient of $w^3x^2yz^2$ in the expansion of $(2w - x + 3y - 2z)^8$. **(04 Marks)**

Module-3

5 a. Define Cartesian product of two sets. For any three non-empty sets A, B and C. Prove that $A \times (B - C) = (A \times B) - (A \times C)$. **(04 Marks)**

b. Let f and g be two functions form R to R defined by $f(x) = 2x + 5$ and $g(x) = \frac{x-5}{2}$. Show that f and g are invertible to each other. **(04 Marks)**

c. Define partition of a set. If R is a relation defined on $A = \{1, 2, 3, 4\}$ by $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$, determine the partition induced by R. **(04 Marks)**

d. Let $A = \{a, b, c\}$, $B = P(A)$ where $P(A)$ is the power set of A. Let R be a subset relation on A. Show that (B, R) is a POSET and draw its Hasse diagram. **(04 Marks**

OR

6 a. Let R be an equivalence relation on set A and $a, b \in A$. Then prove the following are equivalent :

i) $a \in [a]$

ii) a R b iff $[a] = [b]$

iii) if $[a] \cap [b] \neq \phi$ then $[a] = [b]$. **(04 Marks)**

b. Prove that a function $f : A \to B$ is invertible iff it is one – one and onto. **(04 Marks)**

c. State Pigeonhole principle. Show that if any seven numbers from 1 to 12 are chosen, then two of them will add to 13. **(04 Marks)**

d. Show that the set of positive divisors of 36 is a POSET and draw its Hasse diagram. Hence find its i) least element ii) greatest element. **(04 Marks**

Module-4

7 a. Out of 30 students in a hostel, 15 study history, 8 study economics and 6 study geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(04 Marks)**

b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all these derangements. **(04 Marks)**

c. Find the rook polynomial for the following board [refer Fig.Q7(c)] :

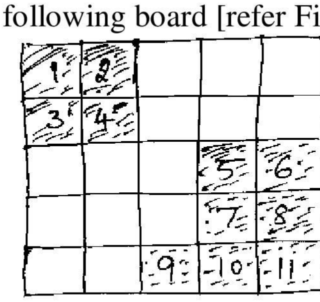

Fig. Q7(c) **(04 Marks)**

d. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(04 Marks)**

OR

8 a. Determine the number of integers between 1 and 300 (inclusive) which are
i) divisible by exactly two of 5, 6, 8 ii) divisible by at least two of 5, 6, 8. **(04 Marks)**

b. In how many ways can be integers 1, 2, - - - -, 10 be arranged in a line so that no even integer is in its natural place. **(04 Marks)**

c. An apple, a banana, a mango and an ornage are to be distributed to four boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have apple, the boy B_3 does not want banana or mango, B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(04 Marks)**

d. Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$ given that $F_0 = 0$, $F_1 = 1$. **(04 Marks)**

Module-5

9 a. Define the following with an example for each :
i) Complete graph ii) regular graph iii) bipartite graph iv) complete bipartite graph. **(04 Marks)**

b. Define isomorphism of two graphs. Verify the following graphs are isomorphic or not :
[Refer Fig.Q9(b)] **(04 Marks)**

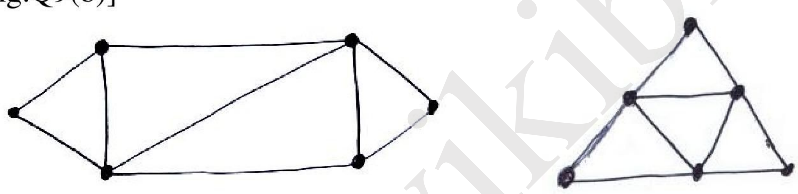

Fig.Q9(b)

c. Show that a tree with n vertices has n – 1 edges. **(04 Marks)**

d. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(04 Marks)**

OR

10 a. Explain Konigsberg bridge problem. **(04 Marks)**

b. Define the following with an example :
i) subgraph ii) spanning subgraph
iii) induced subgraph iv) edge–disjoint and vertex – disjoint subgraphs. **(04 Marks)**

c. If a tree T has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4 and one vertex of degree 5, find the number of leaves in T. **(04 Marks)**

d. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. **(04 Marks)**