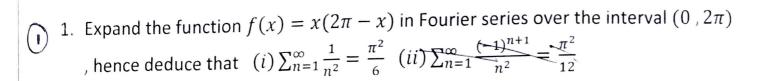
## MODULE-1(15MAT31) FOURIER SERIES



2. Expand the function  $f(x) = x2\pi - x^2$  in Fourier series over the interval  $(0, 2\pi)$ , hence deduce that  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ . Sketch the graph of f(x).



3. Find the Fourier series for the function  $f(x) = \frac{\pi - x}{2}$  in  $0 < x < 2\pi$ Hence deduce that  $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots$ 

4. Expand  $f(x) = 2x - x^2$  as a Fourier series in  $0 \le x \le 2$ .

5. Expand  $f(x) = e^{-x}$  as a Fourier series in the interval (-l, l)

6. Express  $e^x$  as Fourier series in [-1, 1].

7. Find a Fourier series to represent  $f(x) = x - x^2$  from  $-\pi$  to  $\pi$  and deduce that  $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots + \cdots$ 

9. Expand the function  $f(x) = x \sin x$  as a Fourier series in the interval  $-\pi \le x \le \pi$ . Hence deduce the following

(i)  $\frac{\pi}{2} = 1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7} \leftarrow (ii) \frac{\pi^{-2}}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - + \cdots$ 

11. Find a Fourier series for the function f(x) = |x| in  $-\pi \le x \le \pi$ Hence deduce that  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ 

3) 12.If  $f(x) = |\cos x|$ , expand f(x) as a fourier series in the interval  $(-\pi, \pi)$ 

Expand  $f(x) = \sqrt{1-\cos x}$ ,  $0 < x < 2\pi$  in a Fourier series. Hence evaluate  $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots$ 

1. Obtain the Fourier series for the function 
$$f(x) = \begin{cases} -\pi & for & -\pi < x < 0 \\ x & for & 0 < x < \pi \end{cases}$$
 and hence deduce that 
$$\frac{\pi^2}{\pi} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

2. Obtain the Fourier series for the function 
$$f(x) = \begin{cases} \pi x & for & 0 \le x \le 1 \\ \pi(2-x) & for & 1 \le x \le 2 \end{cases}$$
 and hence deduce that 
$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

3. Find a Fourier series to represent 
$$f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ x^2, & 0 \le x \le \pi \end{cases}$$

4. Obtain the Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0\\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$$

and hence deduce that  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \dots \dots \dots$ 

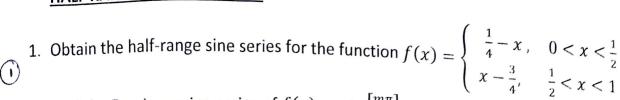
5. Find the Fourier series of the function 
$$f(x) = \begin{cases} -\cos x & (-\pi, 0) \\ \cos x & (0, \pi) \end{cases}$$

6. An alternating current after passing through a half wave rectifier has the form  $i = \left\{ \begin{array}{ll} I_0 \sin\theta & for \ 0 \leq x \leq \pi \\ 0 & for \ \pi \leq x \leq 2\pi \end{array} \right. \text{ (where $I_0$ in the maximum current). Express $i$ in a Fourier series.}$ 

(7.) Obtain the Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1 + \frac{4x}{3}, & \frac{-3}{2} \le x \le 0\\ 1 - \frac{4x}{3}, & 0 \le x \le \frac{3}{2} \end{cases}$$
 and hence deduce that 
$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \dots \dots \dots \dots$$

## HALF RANGE FOURIER SERIES



2. Find the Fourier cosine series of  $f(x) = sin\left[\frac{m\pi}{l}\right]x$ , where m is positive integer.

3. If 
$$f(x) = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \end{cases}$$
  
Show that i)  $f(x) = \frac{4}{\pi} \left[ sinx - \frac{1}{3^2} sin3x + \frac{1}{5^2} sin5x - \dots \right]$ 

*ii*) 
$$f(x) = \frac{\pi}{4} - \frac{2}{\pi} \left[ \frac{1}{1^2} \cos 2x + \frac{1}{3^2} \cos 6x + \frac{1}{5^2} \cos 10x - \dots \right]$$

4. Find the half range Cosine series for f(x) defined by,

$$f(x) = \begin{cases} kx & for \quad 0 \le x \le \frac{l}{2} \\ k(l-x) & for \quad \frac{l}{2} \le x \le l \end{cases}$$

And hence obtain the value of  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ 

5. Expand 
$$f(x)=(x-1)^2$$
 as half-range cosine series over  $0 \le x \le 1$ . Hence show that  $\pi^2=8\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\dots\right)$ 

6. Obtain the half range cosine series for the function f(x) = $sin x in 0 \le x \le \pi$ 

7. Find half range cosine series of 
$$f(x) = 1 - \frac{x}{l}$$
 in  $(0, l)$   
8. Find half range Fourier cosine series of  $f(x) = x(l-x)$  in  $(0, l)$ 

9. Find the half range sine series of  $f(x) = e^x$  in (0, 1)

10. Obtain the Fourier expansion of xsinx as a cosine series on  $(0, \pi)$ . Hence show that  $\frac{1}{13} + \frac{1}{35} + \frac{1}{57} + \dots = \frac{\pi - 2}{4}$ 

HARMONIC ANALYSIS  $a_0$  ,  $a_1$  ,  $a_2$  ,  $b_1$  ,  $b_2$   $for \, f(x)$  tabulated below :

Comput	e the r	ourier	COCTITION	7	3	4	5
	$\chi$	0	1	2.1	20	26	30
	6(2)	q	18	24	20	20	

 $f(x) \mid 9 \mid 18 \mid 24 \mid 28 \mid 26$ 2. Express y as a Fourier series up to first harmonic given

ace v as	a Fourie	er series u	p to first	Harmon	2400	3000	$360^{0}$
Less L	0	$60^{0}$	$120^{\circ}$	$180^{\circ}$	240	300	7.0
X	U	00	2.6	0.5	0.9	6.8	7.9
V	7.9	7.2	3.6	two har	monics i	n the Fo	urier serie

3. Obtain the constant term and the first two harmonics in the Fourier series

a the cor	istant te	rm and t	$2\pi$	VO Harring	$4\pi$	$5\pi$	$2\pi$
Tille	0	π	$2\pi$	π	$\frac{4\pi}{2}$	-	
Х	O	3	3		3	3	1.0
	1.0	1 4	1.9	1.7	1.5	1.2	1.0
F(x)	1.0	1.7		مانہ ا	current	over a pe	eriod

F(x) 1.0 1.4 1.9 1.7 1.5 1.2 1.0 The following table gives the Variating of periodic current over a period

lowing table	e gives th	e Variatir	T	$T_{I}$	$2T/_{2}$	$5T/_{6}$	T
t(sec)	0	$T_{6}$	1/3	./2	/ 3	0.25	1 98
A(Amp)	1.98	1.30	1.05	1.30	-0.88 he variab	-0.25 le curren	1.50

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic.

 $\stackrel{f{\circ}}{}$  Obtain the constant term and the coefficients of sin heta and sin2 heta in the Fourier expansion of y given the following data

ton of s	given the follow	wing data	3		200	360
ISION OF		120	180	240	300	300
$\theta^0$	) 60	120	17.0	17 3	11.7	0
Y	9.2	14.4	17.0	17.5		
				: 1110	n	

6. Express y in a Fourier series up to second Harmonics given,

				: -	· un to	SACON	d Harr	nonics	given,					1
re	<u>ن</u> د د	⁄in a f	ourie	rseries	up to	300011			210	240	270	300	330	
					00	120	150	LIXU	210	240	270	300		-
	Χ	0	30	60	90	120				4 20	1 [2	1 76	2.00	
-				0.20	0.16	0.50	1.30	2.16	1.25	1.30	1.52	1.70	2.00	
	Υ	1.80	1.10	0.30	0.10	0.50	1.00			1.30	((.)	war th	0	
									an cari	$ac \cap t t$	I(x)	ver un		

7. Obtain the first three coefficients in the Fourier cosine series of f(x) over the interval (0, 6). Given

	). Give				_	F
Χ	0	1	2	3	4	5
f(x)	4	8	15	7	6	2
. (/./						