Model Question Paper with effect from 2018-19
(CBCS Scheme)

USN | | | | | | | | | | |

17MAT41

Fourth Semester B.E.(CBCS) Examination
Engineering Mathematics - IV
(Common to all Branches)

Time: 3 Hrs

Max.Marks: 100

**Note: Answer any FIVE full questions, choosing at least ONE question from each module.
Use of statistical tables allowed.**

Module-I

(VK)

1. (a) Solve $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$ using Taylor's series method considering up to fourth degree terms and, find the $y(0.1)$ **(06 Marks)**

   (b) Use Runge - Kutta method of fourth order to solve $(x+y)\frac{dy}{dx} = 1, y(0.4) = 1$, to find $y(0.5)$. (Take $h = 0.1$). **(07 Marks)**

   (c) Given that $\frac{dy}{dx} = x - y^2$ and $y(0) = 1, y(0.1) = 0.9117, y(0.2) = 0.8494, \& \; y(0.3) = 0.8061$ find $y(0.4)$, using Adam-Bashforth predictor-corrector method. **(07 Marks)**

OR

2. (a) Solve the differential equation $\frac{dy}{dx} = x + y^2$ under the initial condition $y(0) = 1$ by using modified Euler's method at the point $x = 0.2$. Perform three iterations at each step, taking $h = 0.1$. **(06 Marks)**

   (b) Use fourth order Runge - Kutta method, to find $y(1.2)$, given $\frac{dy}{dx} = xy, y(1) = 2$. **(07 Marks)**

   (c) Apply Milne's predictor-corrector formulae to compute $y(0.3)$ given **(07 Marks)**
   $\frac{dy}{dx} = x^2 + y^2$ with

x	-0.1	0.0	0.1	0.2
y	0.9087	1.0000	1.1114	1.2525

Module-II

(KM)

3. (a) Using Runge - Kutta method, solve $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, for $x = 0.1$, correct to four decimal places, using initial conditions $y(0) = 1, y'(0) = 0$. **(06 Marks)**

   (b) If α and β are two distinct roots of $J_n(x) = 0$, then prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

   (c) Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(07 Marks)**

OR

4. (a) Apply Milne's predictor-corrector method to compute $y(0.4)$ given the differential equation $\frac{d^2y}{dx^2}=1+\frac{dy}{dx}$ and the following table of initial values: **(06 Marks)**

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y'	1	1.2103	1.4427	1.6990

(b) With usual notation, show that (i) $J_{1/2}(x)=\sqrt{(2/\pi x)}\sin x$ (ii) $J_{-1/2}(x)=\sqrt{(2/\pi x)}\cos x$. **(07 Marks)**

(c) With usual notation, derive the Rodrigues's formula viz., $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. **(07 Marks)**

Module-III

(SH)

5. (a) State and prove Cauchy's theorem. **(06 Marks)**

(b) Evaluate $\int_C \frac{2z^2+1}{(z+1)^2(z-2)}dz$ where C is the circle $|z|=3$, using Cauchy's residue theorem. **(07 Marks)**

(c) Discuss the transformation $w=e^z$. **(07 Marks)**

OR

6. (a) Find the analytic function $f(z)=u+iv$, given $v=[r-(1/r)]\sin\theta, r\neq 0$. **(06 Marks)**

(b) Derive Cauchy-Riemann equation in cartesian form. **(07 Marks)**

(c) Find the bilinear transformation which maps the points $z=i,1,-1$ into the points $w=1,0,\infty$. **(07 Marks)**

Module-IV

(TL)

7. (a) Derive mean and variance of the Binomial distribution. **(06 Marks)**

(b) A random variable X has the following probability function for various values of x:

$X(=x_i)$	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	k

Find (i) the value of k (ii) $P(x<1)$ (iii) $P(x\geq -1)$ **(07 Marks)**

(c) A fair coin is tossed thrice. The random variables X and Y are defined as follows : X=0 or 1 according as head or tail occurs on the first; Y= Number of heads. Determine (i) the distribution of X and Y (ii) joint distribution of X and Y. **(07 Marks)**

OR

8. (a) Two persons A and B play a game in which their chances of winning are in the ratio 3:2. If 6 games are played, find A's chance of winning at least three games. **(06 Marks)**

(b) In a normal distribution, 7% of the items are under 35 and 89% of the items are under 63. Find the mean and standard deviation of the distribution. **(07 Marks)**

(c) Let X be the random variable with the following distribution and Y is defined by X^2

$X(=x_i)$	-2	-1	1	2
$f(x_i)$	1/4	1/4	1/4	1/4

Determine (i) the distribution of g of Y (ii) joint distribution of X and Y (iii) $E(XY)$. **(07 Marks)**

Module-V

(B115)

9. (a) A manufacture claimed that at least 95%of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5%. **(06 Marks)**

(b) Explain (i) transient state (ii) absorbing state (iii) recurrent state of a Markov chain. **(07 Marks)**

(c) Show that probability matrix $P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ is regular stochastic matrix and find the associated unique fixed probability vector. **(07 Marks)**

OR

10. (a) Define the terms : (i)Null hypothesis (ii)Confidence intervals (iii)Type-I and Type-II errors **(06 Marks)**

(b) The following are the $I.Q.$'s of a randomly chosen sample of 10 boys: 70,120,110,101,88,83,95,98,107,100. Does this supports the hypothesis that the population mean of $I.Q.$'s is 100 at 5% level of significance? ($t_{0.05} = 2.262$ for 9 d.f.) **(07 Marks)**

(c) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball , find the probabilities that after three throws (i) A has the ball (ii) B has the ball and (iii) C has the ball. **(07 Marks)**

*****

Engineering Mathematics - IV (17 MAT 4)

1) (a) $\frac{dy}{dx} = 2y + 3e^{x}$ $y(0) = 0$, Taylor's Series

$y(0.1) = ?$

Sln $y(x) = y(x_0) + (x - x_0)\, y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \cdots$

$y' = 2y + 3e^{x}$ $y'(0) = 3$

$y'' = 2y' + 3e^{x}$ $y''(0) = 9$

$y''' = 2y'' + 3e^{x}$ $y'''(0) = 21$

$y(0.1) = 0 + (0.1)\, y'(0) + \frac{(0.1)^2}{2} y''(0) + \frac{(0.1)^3}{6} y'''(0)$

$y(0.1) = 0.3485$

b) $(x+y)\frac{dy}{dx} = 1$, $y(0.4) = 1$, $y(0.5) = ?$

$\frac{dy}{dx} = 1/_{x+y}$ $y_0 = 1$, $x_0 = 0.4$

$x_0 + h = 0.5$, $h = 0.1$

$$K_1 = h\, f(x_0, y_0) = (0.1)\, f(0.4, 1) = 0.0714$$

$$K_2 = h\, f\left(x_0 + h/2,\ y_0 + \frac{K_1}{2}\right) = 0.0673$$

$$K_3 = h\, f\left(x_0 + h/2,\ y_0 + K_2/2\right) = 0.0674$$

$$K_4 = h\, f(x_0 + h,\ y_0 + K_3) = 0.0638$$

$$y(x_0 + h) = y_0 + 1/6\,(K_1 + 2K_2 + 2K_3 + K_4)$$

$$\boxed{y(0.5) = 1.0674}$$

c) $\frac{dy}{dx} = x - y^2$ & $y(0) = 1$, $y(0.1) = 0.9117$, $y(0.2) = 0.8494$, $y(0.3) = 0.8061$, $y(0.4)$?

x	y	$y' = x - y^2$
0	1	$-1 = y_0'$
0.1	0.9117	$-0.7312 = y_1'$
0.2	0.8494	$-0.5214 = y_2'$
0.3	0.8061	$-0.3498 = y_3'$
0.4	?	

$$y_4^P = y_3 + h/24\,(55y_3' - 59y_2' + 37y_1' - 9y_0')$$

$$y_4^P = 0.7789$$

$$y_4^C = y_3 + h/24\,(9y_4' + 19y_3' - 5y_2' + y_1')$$

2) a) $\frac{dy}{dx} = x+y^2$; $y(0)=1$

$y(0.2) = ?$

$y_1^{(0)} = y_0 + h f(x_0, y_0) = 1+(0.1)(0+1^2) = 1.1$

$y_1^{(1)} = y_0 + h/2 \{ f(x_0, y_0) + f(x_1, y_1^{(0)}) \} = 1.1155$

$y_1^{(2)} = y_0 + h/2 \{ f(x_0, y_0) + f(x_1, y_1^{(1)}) \} = 1.1172$

New y_2 at 0.2 New $x_0 = 0.1$ & $y_0 = 1.1172$

$y_1^{(0.2)} = y_1^{(2)} + h f(x_1, y_1^{2}) = 1.252$

$y_2^{(1)} = y_1^{(2)} + h/2 \{ f(x_1, y_1^{(2)}) + f(x_2, y_2^{(0)}) \} = 1.27298$

$y_2^{(2)} = 1.2756$

$\therefore y(0.2) \sim 1.2756.$

2) b) $\frac{dy}{dx} = xy$; $y(1) = 2$ here $x_0 = 1$; $y_0 = 2$

$k_1 = h f(x_0, y_0) = (0.2)(1 \times 2) = 0.4$

$k_2 = h f(x_0 + h/2, y_0 + k_1/2) = 0.484$

$k_3 = h f(x_0 + h/2, y_0 + k_2/2) = 0.49324$

$k_4 = h f(x_0 + h, y_0 + k_3) = 0.59835$

$K = 1/6 (k_1 + 2k_2 + 2k_3 + k_4)$

$= 0.49214$

$y_1 = y_0 + k = 1 + 0.4921 = 2.4921$

—— x ——

c) $\frac{dy}{dx} = x^2 + y^2$

x	y	$y' = x^2 + y^2$
-0.1	0.9087	
0.0	1	1
0.1	1.1114	1.24532
0.2	1.2525	1.60883

$$y_4^{P} = y_0 + \frac{4h}{3}\left(2y_1' - y_2' + 2y_3'\right)$$

$$= 0.90878 + \frac{4\times 0.1}{3} \times (2 - 1.24522 + 3.21766)$$

$$= 1.43843$$

$$y_4' = f(x_4, y_4^{P}) = (0.3)^2 + (1.43843)^2 = 2.15908$$

$$y_4^{C} = y_2 + h/3\left(y_2' + 4y_3' + y_4'\right)$$

$$= 1.11145 + \frac{0.1}{3}\left(1.24522 + 6.43532 + 2.15908\right)$$

$$= 1.43944$$

New $y_4' = (0.3)^2 + (1.43944)^2 = 2.16199$

New $y_4^{(C)} = y_2 + h/3\left(y_2' + 4y_3' + y_4'\right)$

$$= 1.439538$$

Hence $y(0.3) = 1.439538$.

3b. If α and β are the distinct roots of $J_n(\alpha x)=0$ then

$$\int_0^a x J_n(\alpha x) J_n(\beta x)\, dx = 0$$

Proof Let $u = J_n(\alpha x)$

Put $t = \alpha x$ so that $u = J_n(t)$

$$\frac{du}{dx} = \frac{du}{dt}\frac{dt}{dx} = \alpha \frac{dJ_n}{dt}$$

$$\frac{d^2u}{dx^2} = \frac{d}{dt}\left(\frac{du}{dx}\right)\frac{dt}{dx} = \alpha^2 \frac{d^2J_n}{dt^2}$$

Since $J_n(t)$ satisfies Bessel's DE

$$t^2\frac{d^2J_n}{dt^2} + t\frac{dJ_n}{dt} + (t^2-n^2)J_n = 0$$

$$(\alpha x)^2 \frac{1}{\alpha^2}\frac{d^2u}{dx^2} + \alpha x \frac{1}{\alpha}\frac{du}{dx} + (\alpha^2x^2-n^2)u = 0$$

$$\frac{d^2u}{dx^2} + \frac{1}{x}\frac{du}{dx} + \left(\alpha^2 - \frac{n^2}{x^2}\right)u = 0 \quad (2)$$

III^{ly} if $v = J_n(\beta x)$

$$\frac{d^2v}{dx^2} + \frac{1}{x}\frac{dv}{dx} + \left(\beta^2 - \frac{n^2}{x^2}\right)v = 0 \quad (3)$$

$v(2) - u(3)$

$$\left(v\frac{d^2u}{dx^2} - u\frac{d^2v}{dx^2}\right) + \frac{1}{x}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)$$

$$+ (\alpha^2 - \beta^2)uv = 0$$

or $$\frac{d}{dx}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right) + \frac{1}{x}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)$$

$$= (\beta^2 - \alpha^2)uv$$

$$(\beta^2 - \alpha^2)x\,uv = x\frac{d}{dx}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right) + \left(v\frac{du}{dx} - u\frac{dv}{dx}\right)$$

$$= \frac{d}{dx}\left\{x\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)\right\}$$

$$(\beta^2 - \alpha^2)\int_0^a x\,uv\,dx = \left[x\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)\right]_0^a$$

$$= a\left[v(a)\frac{du}{dx}(a) - u(a)\frac{dv}{dx}(a)\right]$$

$$\frac{du}{dx} = \alpha\frac{dJ_n}{dt} = \alpha J_n'(t)$$

$$= \alpha J_n'(\alpha x)$$

$$\frac{du}{dx}(a) = \alpha J_n'(\alpha a)$$

$|||^{ly}$ $v = J_n(\beta x)$ $\quad \frac{dv}{dx}(a) = \beta J_n'(\beta a)$

Since $u = u(x) = J_n(\alpha x)$

$v = v(x) = J_n(\beta x)$

$u(a) = J_n(\alpha a)$ $\quad v(a) = J_n(\beta a)$

$$(\beta^2 - \alpha^2)\int_0^a x J_n(\alpha x) J_n(\beta x)\,dx$$

$$= \alpha\left[J_n(a\beta)\alpha J_n'(a\alpha) - \beta J_n(a\alpha)J_n'(a\beta)\right]$$

If α and β are the distinct roots

of $J_n(ax) = 0$ then $J_n(a\alpha) = 0$

$J_n(a\beta) = 0$

with $\alpha \neq \beta$

$$\int_0^a x J_n(\alpha x) J_n(\beta x)\,dx = 0$$

$$\int_0^1 x J_n(\alpha x) J_n(\beta x)\,dx = 0$$

$\underline{a = 1}$

3c) $P_0(x) = 1$ $\quad P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2 - 1)$

$P_3(x) = \frac{1}{2}(5x^3 - 3x)$

$x^2 = \frac{1}{3}\left[2P_2(x) + P_0(x)\right]$

$x^3 = \frac{1}{5}\left[2P_3(x) + 3P_1(x)\right]$

$\frac{1}{5}\left[2P_3(x) + 3P_1(x)\right] + 2\cdot\frac{1}{3}\left[2P_2(x) + P_0(x)\right]$

$- P_1(x) = 3P_0(x)$

$\frac{2}{5}P_3(x) + \frac{4}{3}P_2(x) - \frac{2}{5}P_1(x) = \left(3 - \frac{2}{3}\right)P_0(x)$

$6P_3(x) + 20P_2(x) - 6P_1(x) = 35P_0(x)$

3a) Set $t = \frac{dy}{dx}$

$\frac{dt}{dx} = 1 + 2xy + x^2t$ with given $y_0 = 1$ $t_0 = 0$ $x_0 = 0$

$f(x, y, t) = t$, $\phi(x, y, t) = 1 + 2xy + x^2t$

$$k_1 = hf(x_0\ y_0\ t_0) = h\,t_0 = (0.1)(0) = \underline{0}$$

$$l_1 = hg(x_0\ y_0\ t_0) = h(1 + 2x_0 y_0 + x_0^2 t)$$

$$= 0.1(1 + 0 + 0) = \underline{0.1}$$

$$k_2 = hf\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_1}{2},\ t_0 + \frac{l_1}{2}\right)$$

$$= 0.1\left(t_0 + \frac{l_1}{2}\right) = 0.1\left(0 + \frac{0.1}{2}\right)$$

$$= \underline{0.005}$$

$$l_2 = hg\left(x_0 + \frac{h}{2}\ \ y_0 + \frac{k_1}{2}\ \ t_0 + \frac{l_1}{2}\right)$$

$$= 0.1\left[1 + 2(0.05) + (0.05)^2(0.05)\right]$$

$$= 0.1\left[1 + 0.1 + 0.000125\right]$$

$$= \underline{0.11001}$$

$$k_3 = hf\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2},\ z_0 + \frac{l_2}{2}\right)$$

$$= 0.1\left(z_0 + \frac{l_2}{2}\right) = 0.1\left(0 + \frac{0.11001}{2}\right)$$

$$= \underline{0.0055}$$

$$l_3 = hg\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2},\ z_0 + \frac{l_2}{2}\right)$$

$$= 0.1\left[1 + 2(0.05)(1.0025) + (0.05)^2(.055)\right]$$

$l_3 = \underline{0.11004}$

$k_4 = h f(x_0 + h \quad y_0 + k_3 \quad z_0 + l_3)$

$= 0.1 \; (0.1104) = \underline{0.01104}$

$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 0.00534$

Soln $\quad y_1 = y_0 + k$

$y(0.1) = 1 + 0.00534 \sim \underline{1.00534}$

4 a) $y'' = 1 + y'$

Let $y' = \frac{dy}{dx} = z$ $\quad y'' = \frac{d^2y}{dx^2} = \frac{dz}{dx}$

$$\frac{dz}{dx} = 1 + z \quad \text{or } z' = 1 + z$$

$z'(0) = 1 + 1 = 2$

$z'(\cdot 1) = 1 + z(0.1) = 1 + 1.2103 = 2.2103$

$z'(0.2) = 1 + z(0.2) = 1 + 1.4427 = 2.4427$

$z'(0.3) = 1 + z(0.3) = 1 + 1.699 = 2.699$

$x_0 = 0 \quad y_0 = 1 \quad z_0 = 1 \quad z_0' = 2$

$x_1 = 0.1 \quad y_1 = 1.1103 \quad z_1 = 1.2427 \quad z_1' = 2.2103$

$x_2 = 0.2 \quad y_2 = 1.2427 \quad z_2 = 1.4427 \quad z_2' = 2.4427$

$x_3 = 0.3 \quad y_3 = 1.399 \quad z_3 = 1.699 \quad z_3' = 2.699$

Milne Predictor

$$y_4^P = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_3)$$

$$z_4'^P = z_0 + \frac{4h}{3}(2z_1' - z_2' + 2z_3')$$

$$y_4^P = 1 + \frac{4}{3}(0.1)\left(2(\cancel{*}.2103) - 1.4427 + 2(1.699)\right)$$

$$\simeq 1.5835$$

$$Z_4^P = 1 + \frac{4}{3}(0.1)\left\{2(2.2103) - 2.4427 + 2(2.699)\right\}$$

$$= 1.9835$$

Milne Corrector

$$y_4^C = y_2 + \frac{h}{3}\left(Z_2 + 4Z_3 + Z_4\right)$$

$$= 1.2427 + \frac{0.1}{3}\left(1.4427 + 4(1.1699) + 1.9835\right)$$

$$= 1.58344$$

$$Z_4^C = Z_2 + \frac{h}{3}\left(Z_2' + 4Z_3' + Z_4'\right)$$

$$= 1.4427 + \frac{0.1}{3}\left(2.4427 + 4(2.699) + 2.9835\right)$$

$$= 1.98344$$

$$y(0.4) = 1.5834$$

4.b) $$J_{\frac{1}{2}}(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\,\Gamma\left(\frac{1}{2}+r+1\right)} \left(\frac{x}{2}\right)^{\frac{1}{2}+2r}$$

$$= \left(\frac{x}{2}\right)^{\frac{1}{2}} \sum_{r=0}^{\infty} \frac{(-1)^r x^{2r}}{2^{2r}\, r!\, \Gamma\left(\frac{1}{2}+r+1\right)}$$

$$= \left(\frac{x}{2}\right)^{\frac{1}{2}} \sum_{r=0}^{\infty} \frac{(-1)^r x^{2r}}{2^{2r}\, r!\, \frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right) \cdots \left(\frac{1}{2}+r\right) \Gamma\left(\frac{1}{2}\right)}$$

$$= \sqrt{\frac{x}{2}}\, \frac{1}{\Gamma\left(\frac{1}{2}\right)} \left\{ \frac{1}{\left(\frac{1}{2}\right)} - \frac{x^2}{2^2\, \frac{1}{2}\left(\frac{1}{2}+1\right)} + \frac{x^4}{2^4\, 2!\left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)} - \cdots \right\}$$

$$= \sqrt{\frac{x}{2}}\, \frac{1}{\frac{1}{2}\,\Gamma\left(\frac{1}{2}\right)} \left\{ 1 - \frac{x^2}{2\cdot 3} + \frac{x^4}{2\cdot 3\cdot 4\cdot 5} - \cdots \right\}$$

$$= \sqrt{2x}\, \frac{1}{\sqrt{\pi}} \left\{ 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots \right\}$$

$$= \sqrt{\frac{2}{\pi x}} \left\{ x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \right\}$$

$$= \sqrt{\frac{2}{\pi x}}\, \sin x$$

4

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x.$$

$$J_{-\frac{1}{2}}(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\,\Gamma(r+\frac{1}{2})} \left(\frac{x}{2}\right)^{-\frac{1}{2}+2r}$$

$$= \left(\frac{x}{2}\right)^{-\frac{1}{2}} \sum_{r=0}^{\infty} \frac{(-1)^r x^{2r}}{2^{2r} r!\, \Gamma(r+\frac{1}{2})}$$

$$= \left(\frac{x}{2}\right)^{-\frac{1}{2}} \sum_{r=0}^{\infty} \frac{(-1)^r x^{2r}}{2^{2r} r!\, (-\frac{1}{2})(-\frac{1}{2}+1)(-\frac{1}{2}+2) \cdots (-\frac{1}{2}+r)\,\Gamma(-\frac{1}{2})}$$

$$= \sqrt{\frac{2}{x}} \frac{1}{(-\frac{1}{2})\Gamma(-\frac{1}{2})} \left\{ 1 - \frac{x^2}{2(-\frac{1}{2}+1)} + \frac{x^4}{2^4\, 2!\,(-\frac{1}{2}+1)(-\frac{1}{2}+2)} - \cdots \right\}$$

$$= \sqrt{\frac{2}{x}} \frac{1}{(-\frac{1}{2})\Gamma(-\frac{1}{2})} \left\{ 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \right\}$$

$$= \sqrt{\frac{2}{x}} \frac{1}{\sqrt{\pi}} \left\{ 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \right\}$$

$$= \sqrt{\frac{2}{\pi x}} \cos x$$

7 c) Let $v = (x^2-1)^n$

$$v_1 \equiv \frac{dv}{dx} = 2nx(x^2-1)^{n-1}$$

$$(x^2-1)v_1 = 2nx(x^2-1)^{n-1}$$

$$\cancel{= 2nx(x^2-1)^n}$$

$$\cancel{x^2}$$

$$= 2nx(x^2-1)^{n-1}(x^2-1)$$

$$= 2nx(x^2-1)^n$$

$$= 2nxv$$

$$(x^2-1)v_1 = 2nxv$$

$$(x^2-1)v_1 - 2nxv = 0$$

Differentiating $(x^2-1)v_2 + 2xv_1 - 2nxv_1 - 2nv = 0$

$$(x^2-1)v_2 + 2(1-n)xv_1 - 2nv = 0$$

Using Leibniz

$$\left\{(x^2-1)v_{n+2} + nC_1(2x)v_{n+1} + nC_2\, 2v_n\right\}$$

$$+ \left\{2(1-n)xv_{n+1} + nC_1 v_n\right\} - 2nv_n = 0$$

$$(x^2-1)v_{n+2} + \{2nx + 2(1-n)x\} v_{n+1} - n(n+1)v_n = 0$$

$$(1-x^2)v_{n+2} - 2x v_{n+1} + n(n+1)v_n = 0$$

This shows that v_n satisfies Legendre differential equation

$$P_n(x) = k v_n = k \frac{d^n v}{dx^n}$$

$$= k \frac{d^n}{dx^n}(x^2-1)^n$$

$$= k \frac{d^n}{dx^n}\{(x+1)^n (x-1)^n\}$$

$$= k\Big[\{(x+1)^n n! + nC_1\, n(x+1)^{n-1} n!(x-1) + nC_2\, n(n-1)(x+1)^{n-2} \frac{n!}{2}(x-1)^2\} + \dots + nC_{n-1}\{n!(x+1)n(x-1)^{n-1}\} + n!(x-1)^n\Big]$$

at $x=1$

$$P_n(1) = k\, n!\, 2^n$$

$P_n(1) = 1$

$$k = \frac{1}{n!\, 2^n}$$

$$\therefore P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$$

Module - III

5. (a)

Statement : If $f(z)$ is analytic at all points inside and on a simple closed curve C then $\int_C f(z)\,dz = 0$.

Pf :- Let $f(z) = u + iv$

Then $\int_C f(z)\,dz = \int_C (u+iv)(dx+idy)$

$$\Rightarrow \quad = \int_C (u\,dx - v\,dy) + i\int_C (v\,dx + u\,dy) \quad \text{(1)}$$

We have Green's theorem in a plane stating that if $M(x,y)$ and $N(x,y)$ are two real valued fn. having continuous first order partial derivatives in a region R b'ded by the curve C then

$$\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

Thus, RHS of (1) becomes

$$\int_C f(z)\,dz = \iint_R \left(-\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,dy + i\iint_R \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) dx\,dy$$

Since $f(z)$ is analytic, we have Cauchy-Riemann equations:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

We have

$$\int_C f(z)\,dz = \iint_R \left(\frac{\partial u}{\partial y} - \frac{\partial u}{\partial y}\right) dx\,dy + i \iint_R \left(\frac{\partial v}{\partial y} - \frac{\partial v}{\partial y}\right) dx\,dy$$

$$\Rightarrow \int_C f(z)\,dz = 0.$$

Hence the proof.

(b) Let $f(z) = \dfrac{2z^2+1}{(z+1)^2(z-2)}$; $C : |z| = 3$.

$z=-1$ is a pole of order 2 and $z=2$ is a pole of order 1.

Both of them lie within the circle $|z| = 3$.

Residue at $z=-1$ be denoted by R_1 and we have

$$R_1 = \lim_{z \to -1} \frac{1}{(2-1)!} \frac{d}{dz} \left\{ (z+1)^2 \frac{2z^2+1}{(z+1)^2 (z-2)} \right\}$$

$$= \lim_{z \to -1} \frac{d}{dz} \left\{ \frac{2z^2+1}{z-2} \right\}.$$

$$= \lim_{z \to -1} \left\{ \frac{(z-2)(4z)-(2z^2+1)}{(z-2)^2} \right\}$$

$$= \lim_{z \to -1} \left\{ \frac{4z^2-8z-2z^2-2}{z^2-2z+4} \right\}$$

$$= \lim_{z \to -1} \left\{ \frac{2z^2-8z-2}{z^2-2z+4} \right\}.$$

$$= \frac{8}{7}.$$

Now, residue at $z=2$, R_2

$$= \lim_{z \to 2} \left\{ (z-2) \frac{2z^2+1}{(z+1)^2 (z-2)} \right\}$$

$$= \lim_{z \to 2} \left\{ \frac{2z^2+1}{z^2+2z+1} \right\}$$

$$= \frac{5}{9}$$

Hence by Cauchy's residue theorem,

$$R = 2\pi i \left(R_1 + R_2 \right) = 2\pi i \left(\frac{8}{7} + \frac{5}{9} \right)$$

$$= \frac{182 i \pi}{63}$$

(c) Discussion of $w = e^z$.

Consider $w = e^z$

i.e., $u + iv = e^{x+iy} = e^x . e^{iy}$

$$= e^x (\cos y + i \sin y)$$

$\therefore u = e^x \cos y, \quad v = e^x \sin y$ —— (1)

We shall find the image in the w-plane corresponding to the straight lines parallel to the co-ordinate axes in the z-plane. That is $x =$ constant and $y =$ constant.

Let us eliminate x and y seperately from (1).

Squaring and adding we get

$$u^2 + v^2 = e^{2x} \quad \text{—— (2)}$$

Also by dividing we get

$$\frac{u}{v} = \frac{e^x \sin y}{e^x \cos y}$$

i.e, $\frac{u}{v} = \tan y$ —— (3)

Case (1) :- Let $x = c_1$ where c_1 is a constant.

Eq (2) becomes $u^2 + v^2 = e^{2c_1} =$ constant $= r^2$ (Say).

This represents a circle with centre origin and radius 'r' in the w-plane.

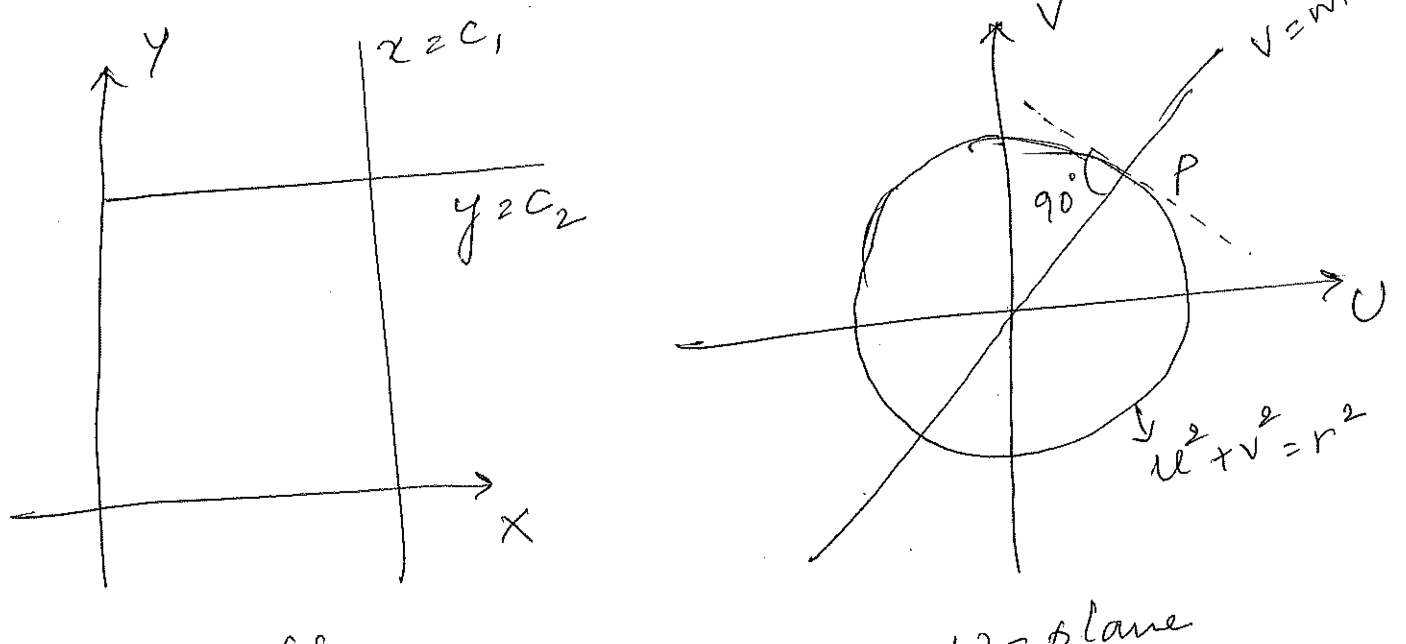
Case (2):- Let $y = C_2$ where C_2 is a constant.

Eq (3) becomes $\frac{v}{u} = \tan C_2 = m$ (say).

$$\therefore v = mu.$$

This represents a straight line passing thro' the origin in the w-plane.

Conclusion: The straight line parallel to



z-plane   w-plane

the x-axis ($y = C_2$) in the z-plane maps onto a straight line passing thro' the origin in the w-plane. The straight line parallel to the y-axis ($x = C_1$) in the z-plane maps onto a circle with centre origin and radius 'r' where $r = e^{C_1}$ in the w-plane.

Suppose we draw a tangent at the point of intersection of these two curves in the w-plane the angle sublended is equal to 90°. Hence these two curves can be regarded as Orthogonal trajectories of each other.

6. a) $v = \left(r - \frac{1}{r}\right)\sin\theta, \quad r \neq 0.$

$$\Rightarrow v_r = \left(1 + \frac{1}{r^2}\right)\sin\theta$$

$$v_\theta = \left(r - \frac{1}{r}\right)\cos\theta$$

Consider $f'(z) = e^{-i\theta}(u_r + iv_r)$.

But $u_r = \frac{1}{r}v_\theta$ (CR eq^n)

$$\therefore f'(z) = e^{-i\theta}\left(\frac{1}{r}v_\theta + iv_r\right).$$

$$= e^{-i\theta}\left[\left(1 - \frac{1}{r^2}\right)\cos\theta + i\left(1 + \frac{1}{r^2}\right)\sin\theta\right] \quad (1)$$

$$= e^{-i\theta}\left[(\cos\theta + i\sin\theta) - \frac{1}{r^2}(\cos\theta - i\sin\theta)\right]$$

$$= e^{-i\theta}\left[e^{i\theta} - \frac{1}{r^2}e^{-i\theta}\right] = 1 - \frac{1}{(re^{i\theta})^2}$$

$$= 1 - \frac{1}{z^2}.$$

i.e. $f'(z) = 1 - \frac{1}{z^2}$

$$\therefore f(z) = \left(z + \frac{1}{z}\right) + c$$

$$u + iv = re^{i\theta} + \frac{1}{re^{i\theta}}$$

$$= r(\cos\theta + i\sin\theta) + \frac{1}{r}(\cos\theta - i\sin\theta)$$

$$u + iv = \left(r + \frac{1}{r}\right)\cos\theta + i\left(r - \frac{1}{r}\right)\sin\theta$$

$$\therefore u = \left(r + \frac{1}{r}\right)\cos\theta$$

$$\therefore f(z) = \left(r + \frac{1}{r}\right)\cos\theta + i\left(r - \frac{1}{r}\right)\sin\theta, \quad r \neq 0.$$

(b) The necessary condition that the fn. $w = f(z) = u(x,y) + iv(x,y)$ may be analytic at any point $z = x + iy$ is that, there exists four continuous first order partial derivatives $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ and satisfy the eqns: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$. These are known as Cauchy Riemann equations.

Pf :- Let $f(z)$ be analytic at $z = x+iy$.

$$\Rightarrow f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z) - f(z)}{\delta z}$$ exists and

is unique.

In the cartesian form $f(z) = u(x,y) + iv(x,y)$ and let δz be the increment in z correspon-ding to the increments δx, δy in x, y.

$$f'(z) = \lim_{\delta z \to 0} \frac{[u(x+\delta x, y+\delta y) + iv(x+\delta x, y+\delta y)] - [u(x,y) + iv(x,y)]}{\delta z}$$

$$= \lim_{\delta z \to 0} \frac{[u(x+\delta x, y+\delta y) - u(x,y)]}{\delta z}$$

$$+ i \lim_{\delta z \to 0} \frac{[v(x+\delta x, y+\delta y) - v(x,y)]}{\delta z} \quad \text{--- (1)}$$

Now $\delta z = (z+\delta z) - z$

$= [(x+\delta x) + i(y+\delta y)] - (x+iy)$

$= \delta x + i\delta y$

Since $\delta z \to 0$, we have

Case (i) :- Let $\delta y = 0$ so that $\delta z = \delta x$.

$\Rightarrow \delta x \to 0$.

Now (1) becomes,

$$f'(z) = \lim_{\delta x \to 0} \frac{u(x+\delta x, y) - u(x,y)}{\delta x} +$$

$$i \lim_{\delta x \to 0} \frac{v(x+\delta x, y) - v(x,y)}{\delta x}.$$

$$= \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}. \quad \text{——} (2)$$

Case (ii) :- Let $\delta x = 0$ so that $\delta z = i\delta y$
and $\delta z \to 0 \Rightarrow i\delta y \to 0$
$\Rightarrow \delta y \to 0.$

Now (1) becomes,

$$f'(z) = \lim_{\delta y \to 0} \frac{u(x, y+\delta y) - u(x,y)}{i\delta y} +$$

$$i \lim_{\delta y \to 0} \frac{v(x, y+\delta y) - v(x,y)}{i\delta y}$$

$$= -i \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}.$$

$$= \frac{\partial v}{\partial y} - i \frac{\partial u}{\partial y}. \quad \text{——} (3)$$

From (2) and (3)

$$\frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i \frac{\partial u}{\partial y}.$$

Equating real and imaginary parts,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}.$$

(c). Let $w = \dfrac{az+b}{cz+d}$ be the required bilinear transformation.

$z = i, 1, -1$
$w = 1, 0, \infty$

$z = i,\ w = 1$

i.e, $1 = \dfrac{ai+b}{ci+d}$

$\Rightarrow ai + b - ci - d = 0$ —— (1)

$z = 1,\ w = 0$

$\Rightarrow a + b = 0$ —— (2)

$z = -1,\ w = \infty.$

Consider $\dfrac{1}{w} = \dfrac{cz+d}{az+b}$

$\Rightarrow 0 = \dfrac{-c+d}{-a+b}$

$\Rightarrow -c + d = 0$ —— (3)

(1) + (3) gives

$ai + b - (1+i)c = 0$ —— (4)

$\therefore \quad 1a + 1b + 0c = 0$

$ia + 1b - (1+i)c = 0.$

Applying rule of cross multiplication,

$$\frac{a}{\begin{vmatrix} 1 & 0 \\ 1 & -(1+i) \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 1 & 0 \\ i & -(1+i) \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 1 \\ i & 1 \end{vmatrix}}$$

$$\Rightarrow \frac{a}{-(1+i)} = \frac{b}{1+i} = \frac{c}{1-i}$$

$\Rightarrow a = (-1+i),$

$b = 1+i$

$c = 1-i = d.$

Thus,

$$w = \frac{-(1+i)z + (1+i)}{(1-i)z + (1-i)}$$

$$= \frac{(1+i)}{(1-i)}\left(\frac{1-z}{1+z}\right).$$

$$= \frac{(1+i)^2}{1-i^2}\left(\frac{1-z}{1+z}\right)$$

$$= \frac{1+i^2+2i}{2}\left(\frac{1-z}{1+z}\right) = i\left(\frac{1-z}{1+z}\right).$$

module - IV

7.a. Derive mean and Variance of Binomial distribution

if p is the probability of success and q is the probability of failure, the probability of x success out of n trials is given by

$$\boxed{P(x) = nC_x\, p^x q^{n-x}}$$

Mean and standard deviation of the Binomial Distribution

$$\text{mean}(\mu) = \sum_{x=0}^{n} x p(x) = \sum_{x=0}^{n} x.\, nC_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x.\frac{n!}{x!(n-x)!} p^x q^{n-x} = \sum_{x=0}^{n} \frac{x.\, n!}{x(x-1)!\,(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n(n-1)!}{x-1!(n-x)!} p.p^{x-1} q^{n-x}$$

$$= np\sum_{x=1}^{n} \frac{(n-1)!}{(x-1)![(n-1)-(x-1)]} p^{x-1} q^{[(n-1)-(x-1)]}$$

$$\mu = np\sum_{x=1}^{n} {}^{n-1}C_{x-1}\, p^{x-1} q^{(n-1)-(x-1)}$$

$$\mu = np(p+q)^{n-1} = np$$

$$\boxed{\mu = np}$$

$$\text{variance}(v) = \sum_{x=0}^{n} x^2 p(x) - \mu^2 \quad \text{--- (1)}$$

$$\sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} [x(x-1)+x][p(x)]$$

$$= \sum_{x=0}^{n} x(x-1)p(x) + \sum_{x=0}^{n} xp(x)$$

$$= \sum_{x=0}^{n} x(x-1)nC_x p^x q^{n-x} + np$$

$$\sum_{x=0}^{n} x(x-1)\frac{n!}{x!(n-x)!} p^x q^{n-x} + np$$ 1

$$\sum_{x=2}^{n} \frac{n(n-1)(n-2)!}{(x-2)!(n-x)!} p^2 q^{n-x} p^{x-2} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{n-2!}{(x-2)![(n-2)-(x-2)]} p^{x-2} q^{(n-2)-(x-2)} + np.$$

$$= n(n-1)p^2(q+p)^{n-2} + np$$

Sub in ①

Variance $x = n(n-1)p^2 + np - n^2p^2 = np(1-p) = npq.$

Variance $(V) = npq.$

$S.D(\sigma) = \sqrt{V} = \sqrt{npq}.$

Thus we have for the Binomial distribution

6. A random variable X has the following probability function for various values of x

$X=(x_i)$	-2	-1	0	1	2	3
$P(x)$	0.1	K	0.2	2K	0.3	K

Find (i) the value of K (ii) $P(x<1)$ (iii) $P(x \geq -1)$

Sol: $P(x_i) \geq 0$ and $\sum p(x_i) = 1$ for a probability distribution

$\sum p(x_i) = 1$ $\quad 0.1 + K + 0.2 + 2K + 0.3 + K = 4K + 0.6 = 1 \Rightarrow \boxed{K = 0.1}$

$X = x_i$	-2	-1	0	1	2	3
$P(x)$	0.1	0.1	0.2	0.2	0.3	0.1

$P(x<1) = p(x=-2) + p(x=-1) + p(x=0)$

$= 0.1 + 0.1 + 0.2 = 0.4.$

$P(x \geq -1) = P(x=-1) + P(x=0) + P(x=1) + P(x=2) + P(x=3) = 0.1 + 0.2 + 0.2 + 0.3 + 0.1 = 0.9$

(c) A fair coin is tossed thrice. The random variables X and Y are defined as follows; X = 0 or 1 according as head or tail occurs on the first; Y = Number of heads. Determine (i) the distribution of X and Y (ii) Joint distribution of X and Y.

<u>Soln</u>: The sample space S and the association of random variables X and Y is given by the following Table.

S	HHH	HHT	HTH	HTT	THH	HTH	TTH	TTT
X	0	0	0	0	1	1	1	1
Y	3	2	2	1	2	1	1	0

(a) The probability distribution of X and Y is found as follows $X = \{x_i\} = \{0, 1\}$ and $Y = \{y_j\} = \{0, 1, 2, 3\}$

$P(X=0) = 4/8 = 1/2$; $P(X=1)$ is $1/2$

$P(Y=0) = 1/8$ $\quad P(Y=1) = 3/8$

$P(Y=2) = 3/8$ $\quad P(Y=3) = 1/8$

Thus we have following probability distribution of X & Y

x_i	0	1
$p(x_i)$	$1/2$	$1/2$

y_j	0	1	2	3
$g(y_j)$	$1/8$	$3/8$	$3/8$	$1/8$

(b) The Joint distribution of X and Y is found by computing $J_{ij} = P(X = x_i, Y = y_j)$ where we have $x_1 = 0$, $x_2 = 1$ and $y_1 = 0$, $y_2 = 1$, $y_3 = 2$, $y_4 = 3$

$J_{11} = P(X=0, Y=0) = 0$

($\because X = 0$ implies there is a head turn out & Y is the total number heads 0 is imposible)

$J_{12} = P(X=0, Y=1) = 1/8$ corresponding to the out come HTT

$J_{13} = P(X=0, Y=2) = 2/8 = 1/4$

$J_{14} = P(X=0, Y=3) = 1/8$

$J_{21} = 1/8$

$J_{22} = 1/4$

$J_{23} = 1/8$

$J_{24} = 0$

The Required joint probability distribution of x and y are as follows.

$x \backslash y$	0	1	2	3	Sum
0	0	1/8	1/4	1/8	1/2
1	1/8	1/4	1/8	0	1/2
Sum	1/8	3/8	3/8	1/8	1

8. a) Two persons A and B play a game in which their chances of wining are in the ratio 3:2 if 6 games are played, find A's chance of wining at least three games

Soln From what is given, the probability that A wins a game is $p = 3/(3+2) = 3/5$ Therefore, the probability function giving x success for A in 6 games is given by the binomial probability function

$$b\left(6, \frac{3}{5}, x\right) = 6C_x \left(\frac{3}{5}\right)^x \left(1 - 3/5\right)^{6-x}$$

$$= 6C_x \left(\frac{3}{5}\right)^x \left(\frac{2}{5}\right)^{6-x} = b(x) \text{ say}$$

hence, the probability of A wining at least three of six games is

$$b(x \geq 3) = 1 - b(x < 3) = 1 - \{b(x=0) + b(x=1) + b(x=2)\}$$

$$= 1 - \left\{\left(\frac{2}{5}\right)^6 + 6\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^5 + 15\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^4\right\}$$

$$1 - \frac{26}{5^6} \times 125 = 1 - 0.1792 = 0.8208$$

Thus, A has about 82% chance of ~~wining~~ winning at least three of the six games.

8.b. In a normal distribution 7% of the items are under 35 and 89% of the items under 63. Find mean and Standard deviation of the distribution

Sol: Let μ and σ be the mean and S.D of the normal distribution

By data $P(x<35)=0.07$, $P(x<60)=0.89$

we have S.n.v $z=\frac{x-\mu}{\sigma}$

when $x=35$, $z=\frac{35-\mu}{\sigma}=z_1$ (say)

$x=60$ $z=\frac{60-\mu}{\sigma}=z_2$ (say)

Hence we have $P(z<z_1)=0.07$ & $P(z<z_1)=0.89$

$0.5+\phi(z_1)=0.07$ $\quad$ $0.5+\phi(z_2)=0.89$

$\phi(z_1)=-0.43$ $\quad$ $\phi(z_2)=0.39$

$\therefore \phi(z_1)=-\phi(1.4750)$ $\quad$ $\phi(z_2)=\phi(1.2263)$

$z_1=-1.475$ $\quad$ $z_2=1.2263$

ie $\frac{35-\mu}{\sigma}=-1.475$ $\quad$ $\frac{60-\mu}{\sigma}=1.2263$

$\mu-1.4759\sigma=35$ $\quad$ $\mu+1.2263\sigma=60$

$\mu=48.65$ $\quad$ $\sigma=9.25$

c. Let X be the random variable with the following distribution and Y is defined by X^2

$X=(x_i)$	-2	-1	1	2
$f(x_i)$	1/4	1/4	1/4	1/4

Determine (i) the distribution of g of y (ii) joint distribution of X and Y is defined by (iii) E(XY)

$Y \backslash X$	-2	-1	1	2	$g(y_j)$
4	$1/4$	0	0	$1/4$	$1/2$
1	0	$1/4$	$1/4$	0	$1/2$
$f(x_i)$	$1/4$	$1/4$	$1/4$	$1/4$	1

Sample Space $(-2,4), (2,4), (1,1), (-1,1)$ Each has prob $1/4$

OR $\Bigg[$ $P(-2,4) = P(-2/4)\,P(4)$ $\quad (P(A \cap B) = P(A/B)\,P(B))$

$= 1/2 \times 1/2 = 1/4$ $\quad$ or $\left(P(4/-2)\,P(-2) = 1 \times 1/4 = 1/4 \right)$

$P(2,4) = P(2/4)\,P(4) = 1/2 \times 1/2 = 1/4$

Module - 5 (Model Q.P. - 17MAT41)

9.(a) Let p be the prob. of success which being the prob. of the equipment supplied to the factory conformal to the specifications

$p = 0.95$, $q = 0.05$

$H_0 : p = 0.95$, claim is correct.

$H_1 : p < 0.95$ " " false.

$\mu = np = 200 \times .95 = 190$

$\sigma = \sqrt{npq} = \sqrt{200 \times .95 \times 0.05} = 3.082$

Expected no. of equipments = 192

Actual nos. = 182 (18 were faulty)

$\therefore$ difference = $190 - 182 = 8$

Now $z = \dfrac{x - np}{\sqrt{npq}} = 2.6 > 1.65$ at 5% level of significance

> 2.33 " 1% " "

$\therefore$ Claim is rejected at 5% as well as 1% level of significance.

9(b)(i) Transient State - A state (at least one) "j" to which we can go from state "i" but can't return to "i". Then state "i" is called transient state.

(iii) Recurrent State - A state "i" is called recurrent, if we go from that state to any other state "j", then there is at least one path to return back to "i"

(iv) Absorbing state -

9(c) $P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$

$P^2 = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$P^3 = \begin{bmatrix} 4 & 4 & 4 \\ 3 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

$\because$ All entries in P^3 are positive.

$\therefore$ P is regular stochastic matrix

Now we've to find out $v = (a, b, c)$ where $(a+b+c = 1)$

s.t. $vP = v$

$$[a \quad b \quad c]\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} = [a \quad b \quad c]$$

$\frac{a}{2} + \frac{b}{2} = a$; $\frac{a}{4} + c = b$; $\frac{a}{4} + \frac{b}{2} = c$

$\Downarrow$

$\frac{b}{2} = \frac{a}{2}$; $b - c = c - \frac{b}{2} \Rightarrow \frac{3b}{2} = 2c$

$\downarrow$

$a = b$, $b = \frac{4c}{3}$

we've $a+b+c = 1$

$\frac{4c}{3} + \frac{4c}{3} + c = 1 \Rightarrow \frac{11c}{3} = 1$

$\Rightarrow c = \frac{3}{11}$

$\therefore b = \frac{4}{11}$ & $a = \frac{4}{11}$

$\therefore \ \left(\frac{4}{11}, \frac{4}{11}, \frac{3}{11}\right)$ is the fixed prob. vector.

10(a) (i) Null Hypothesis – The hypothesis formulated for the purpose of its rejection under the assumption that it is true is called Null hypothesis denoted by H_0.

(ii) Confidence Intervals – A confidence interval is an interval of values instead of a single point estimate. The level of confidence corresponds to the expected proportion of intervals that will contain the parameter if many confidence intervals are constructed of the same sample size from the same population.

(iii) Type-I & Type-II error :– If the hypothesis is true but we reject it, then it's type-I error.

If the hypothesis is false but we accept it, then it's type-II error.

(b) $\mu = 100$, $n = 10$

$$\bar{x} = \frac{\Sigma x}{n} = 97.2$$

$$s^2 = \frac{1}{(n-1)} \Sigma (x - \bar{x})^2$$

$$s^2 = \frac{1}{9}\Big[(70-97.2)^2 + (120-97.2)^2 + (110-97.2)^2 + (101-97.2)^2 + (88-97.2)^2 + (83-97.2)^2 + (95-97.2)^2 + (98-97.2)^2 + (107-97.2)^2 + (100-97.2)^2\Big]$$

$$s^2 = \frac{1}{9}\big[1833.6\big] = 203.73 \Rightarrow s = 14.27$$

we have $t = \frac{\bar{x} - \mu}{s}\sqrt{n} = \frac{97.2 - 100}{14.27}\sqrt{10} = .6204 < 2.262$

$\therefore$ Hyp. is accepted.

(c) Sample space = $\{A, B, C\}$

$$\text{t.p.m } P = \begin{array}{c|c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

Initially if C has the ball, the associated initial prob. vector is given by $p^{(0)} = (0, 0, 1)$

Since the prob. are desired after three throws we've to find $p^{(3)} = p^{(0)} P^3$

$$P^3 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$

$$\therefore p^{(3)} = p^{(0)} P^3 = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right]$$

$\therefore$ After three throws the prob. that the ball is with A is $\frac{1}{4}$, with B is $\frac{1}{4}$ and with C is $\frac{1}{2}$.