MODULE-1(15MAT31)

FOURIER SERIES

1. Expand the function $f(x) = x(2\pi - x)$ in Fourier series over the interval $(0\,,2\pi)$, hence deduce that $(i)\ \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ $\cancel{(ii)\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}}$

2. Expand the function $f(x) = x2\pi - x^2$ in Fourier series over the interval $(0\,,2\pi)$, hence deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. ~~Sketch the graph of $f(x)$~~.

3. Find the Fourier series for the function $f(x) = \frac{\pi - x}{2}$ $in\ 0 < x < 2\pi$

   Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots \ldots\ldots\ldots\ldots$

4. Expand $f(x) = 2x - x^2$ as a Fourier series in $0 \le x \le 2$.
5. Expand $f(x) = e^{-x}$ as a Fourier series in the interval $(-l, l\,)$
6. Express e^x as Fourier series in [-1 , 1].
7. Find a Fourier series to represent $f(x) = x - x^2$ from $-\pi\ to\ \pi$ and deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \ldots\ldots\ldots\ldots$
8. Find the Fourier series for the function $f(x) = x + x^2$ from $x = -\pi\ to\ x = \pi$ and deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots$
9. Expand the function $f(x) = x\ sinx$ as a Fourier series in the interval $-\pi \le x \le \pi$. Hence deduce the following HW

   $(i)\ \frac{\pi}{2} = 1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7}$ $(ii)\ \frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - + \cdots$,
10. Express $f(x) = x + x^2$ as a Fourier series in the interval $(-\pi\,,\pi\,)$. and deduce that, $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots \ldots\ldots\ldots\ldots \infty$
11. Find a Fourier series for the function $f(x) = |x|\ in\ -\pi \le x \le \pi$

    Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots$
12. If $f(x) = |cosx|$, expand $f(x)$ as a fourier series in the interval$(-\pi, \pi)$
13. Expand $f(x) = \sqrt{1 - cosx}\,, 0 < x < 2\pi$ in a Fourier series. Hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots$

14 An alternating current after passing through a half wave rectifier has the form

$$i = \begin{cases} I_0 \sin\theta & for\ 0 \le x \le \pi \\ 0 & for\ \pi \le x \le 2\pi \end{cases}$$

(where I_0 in the maximum current). Express i in a Fourier series.

1. Obtain the Fourier series for the function $f(x) = \begin{cases} -\pi & for\ -\pi < x < 0 \\ x & for\ 0 < x < \pi \end{cases}$

   and hence deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

2. Obtain the Fourier series for the function $f(x) = \begin{cases} \pi x & for\ 0 \le x \le 1 \\ \pi(2-x) & for\ 1 \le x \le 2 \end{cases}$

   and hence deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

3. Find a Fourier series to represent $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ x^2, & 0 \le x \le \pi \end{cases}$

4. Obtain the Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$$

   and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots$

5. Find the Fourier series of the function $f(x) = \begin{cases} -\cos x & (-\pi, 0) \\ \cos x & (0, \pi) \end{cases}$

6. An alternating current after passing through a half wave rectifier has the form

$$i = \begin{cases} I_0 \sin\theta & for\ 0 \le x \le \pi \\ 0 & for\ \pi \le x \le 2\pi \end{cases}$$

   (where I_0 in the maximum current). Express i in a Fourier series.

7. Obtain the Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1 + \frac{4x}{3}, & \frac{-3}{2} \le x \le 0 \\ 1 - \frac{4x}{3}, & 0 \le x \le \frac{3}{2} \end{cases}$$

   and hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots$$

HALF RANGE FOURIER SERIES

1. Obtain the half-range sine series for the function $f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1 \end{cases}$

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2. Find the Fourier cosine series of $f(x) = sin\left[\frac{m\pi}{l}\right]x$,where m is positive integer.

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3. If $f(x) = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \end{cases}$

   Show that $i)\ f(x) = \frac{4}{\pi}\left[sinx - \frac{1}{3^2}sin3x + \frac{1}{5^2}sin5x - \cdots \ldots\ldots\right]$

   $ii)\ f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left[\frac{1}{1^2}cos2x + \frac{1}{3^2}cos6x + \frac{1}{5^2}cos10x - \cdots \ldots\ldots\right]$

4. Find the half range Cosine series for $f(x)$ defined by,

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$$f(x) = \begin{cases} kx & for\ \ 0 \le x \le \frac{l}{2} \\ k(l - x) & for\ \ \frac{l}{2} \le x \le l \end{cases}$$

   And hence obtain the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots$

5. Expand $f(x) = (x - 1)^2$ as half-range cosine series over $0 \le x \le 1$.

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   Hence show that $\pi^2 = 8\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots\ldots\right)$

6. Obtain the half range cosine series for the function $f(x) = sinx\ in\ 0 \le x \le \pi$

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7. Find half range cosine series of $f(x) = 1 - \frac{x}{l}$ in $(0, l)$

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8. Find half range Fourier cosine series of $f(x) = x(l - x)$ in $(0, l)$
9. Find the half range sine series of $f(x) = e^x$ in (0, 1)
10. Obtain the Fourier expansion of $xsinx$ as a cosine series on $(0, \pi)$. Hence show that $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots = \frac{\pi - 2}{4}$

HARMONIC ANALYSIS

1. Compute the Fourier coefficients a_0, a_1, a_2, b_1, b_2 for $f(x)$ tabulated below :

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	30

2. Express y as a Fourier series up to first harmonic given

X	0	60^0	120^0	180^0	240^0	300^0	360^0
y	7.9	7.2	3.6	0.5	0.9	6.8	7.9

3. Obtain the constant term and the first two harmonics in the Fourier series

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
F(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

4. The following table gives the Variating of periodic current over a period

t(sec)	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A(Amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic.

5. Obtain the constant term and the coefficients of $sin\theta$ and $sin2\theta$ in the Fourier expansion of y given the following data

θ^0	0	60	120	180	240	300	360
Y	0	9.2	14.4	17.8	17.3	11.7	0

6. Express y in a Fourier series up to second Harmonics given,

X	0	30	60	90	120	150	180	210	240	270	300	330
Y	1.80	1.10	0.30	0.16	0.50	1.30	2.16	1.25	1.30	1.52	1.76	2.00

7. Obtain the first three coefficients in the Fourier cosine series of $f((x)$ over the interval (0 , 6). Given

X	0	1	2	3	4	5
f(x)	4	8	15	7	6	2