Third Semester B.E AY 2019-20

TRANSFORM CALCULUS, FOURIER SERIES AND NUMERICAL TECHNIQUES (18MAT31)

QUESTION BANK

MODULE 5: Numerical Solution of Ordinary Differential Equations and Calculus of Variations

1. Using Runge Kutta method solve $y'' = x\left(y'\right)^2 - y^2$ at x=0.2 with x_0 =0, y_0= 1, $\left(y'\right)_0$=0, h=0.2
2. Using Runge Kutta method find the solution at x=0.1 of the D.E $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$ with y(0)=1, $y'(0)$= 0, h=0.1. Also find $y'(0.1)$
3. Using Runge Kutta method solve $y'' = y + xy'$ with y(0)=1, $y'(0)$= 0 to find y(0.2) and $y'(0.2)$
4. Apply Milne's predictor-corrector method to compute y(0.4) given the differential equation $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following table of values. Apply corrector formula once

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y'	1	1.2103	1.4427	1.6990

5. Obtain the solution of $2y'' = 4x + y'$ at x=1.4 by Milne's method given that

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2.3178	2.3178	2.6725	3.0657

Apply the corrector formula once

6. Using Milnes predictor-corrector method obtain the solution at the point x= 0.4 of the D.E $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$ with y(0)=1, y(0.1)=1.03995, y(0.2)=1.138036, y(0.3)=1.29865, $y'(0)$=0.1, $y'(0.1)$=0.6955, $y'(0.2)$=1.258, $y'(0.3)$=1.873. Apply corrector formula once
7. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$
8. Find the extremal of the functional $\int_0^{\pi}\left(y'^2 - y^2 + 4y\cos x\right)dx$, y (0) =0= y (π)
9. Find the curve on which the functional $\int_0^1\left[\left(y'\right)^2 + 12xy\right]dx$ with y (0) =0 and y (1) =1 can be extremized
10. Solve the Euler equation for the functional $\int_{x_0}^{x_1}\left(1 + x^2y'\right)y'\,dx$
11. Find the extremal of the functional $\int_{x_1}^{x_2}\left(y'^2 - y^2 + 2y\sec x\right)dx$
12. Find the extremals of the functional $\int_{x_0}^{x_1}\left(\frac{y'^2}{x^3}\right)dx$
13. Find the curve on which the functional $\int_0^{\pi/2}\left(y'^2 - y^2 + 2xy\right)dx$ with y (0) =0= y (π/2) can be extremized
14. Find the extremal of the functional $\int_{x_1}^{x_2}\left(y'^2 + y^2 + 2ye^x\right)dx$
15. Find the extremal of the functional $\int_0^{\pi/2}\left(y^2 - y'^2 - 2y\sin x\right)dx$ with y (0) =0= y (π/2)
16. Prove that geodesics of a plane are straight lines
17. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a catenary