PART-I

Problems on Fourier Series

1. Expand the function $f(x) = x(2\pi - x)$ in Fourier series over the interval $(0, 2\pi)$, hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$
2. Expand the function $f(x) = x2\pi - x^2$ in Fourier series over the interval $(0, 2\pi)$, hence deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. Sketch the graph of $f(x)$
3. Find the Fourier series for the function $f(x) = \frac{\pi - x}{2}$ $in\ 0 < x < 2\pi$

   Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots \ldots \ldots \ldots \ldots$
4. Expand $f(x) = 2x - x^2$ as a Fourier series in $0 \leq x \leq 2$.
5. Expand $f(x) = e^{-x}$ as a Fourier series in the interval $(-l, l)$
6. Find a Fourier series to represent $f(x) = x - x^2$ from $-\pi\ to\ \pi$ and deduce that
   $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots$
7. Find the Fourier series for the function $f(x) = x + x^2$ from $x = -\pi\ to\ x = \pi$ and deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots$
8. Expand the function $f(x) = x\ sinx$ as a Fourier series in the interval $-\pi \leq x \leq \pi$.
9. Express $f(x) = x + x^2$ as a Fourier series in the interval $(-\pi, \pi)$. and deduce that, $\frac{\pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \infty$
10. Find a Fourier series for the function $f(x) = |x|\ in\ -\pi \leq x \leq \pi$

    Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots$
11. If $f(x) = |cosx|$, expand $f(x)$ as a fourier series in the interval$(-\pi, \pi)$

PART II

3. Obtain the Fourier series for the function

$$f(x) = \begin{cases} -\pi & for \quad -\pi < x < 0 \\ x & for \quad 0 < x < \pi \end{cases}$$ and hence deduce that

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

4. If $f(x) = \begin{cases} x, & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$

Show that $i)$ $f(x) = \frac{4}{\pi}\left[sinx - \frac{1}{3^2} sin3x + \frac{1}{5^2} sin5x - \cdots \ldots \ldots\right]$

$ii)$ $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left[\frac{1}{1^2} cos2x + \frac{1}{3^2} cos6x + \frac{1}{5^2} cos10x - \cdots \ldots \ldots\right]$

5. Obtain the Fourier series for the function

$$f(x) = \begin{cases} \pi x & for \quad 0 \leq x \leq 1 \\ \pi(2-x) & for \quad 1 \leq x \leq 2 \end{cases}$$ and hence deduce that

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$$

6. Find a Fourier series to represent $f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ x^2, & 0 \leq x \leq \pi \end{cases}$

7. Obtain the Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$$

and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots$

8. Find the Fourier series of the function $f(x) = \begin{cases} -cosx & (-\pi, 0) \\ cosx & (0, \pi) \end{cases}$

3. An alternating current after passing through a half wave rectifier has the form $i = \begin{cases} I_0 \, sin\theta & for\ 0 \le x \le \pi \\ 0 & for\ \ \pi \le x \le 2\pi \end{cases}$ (where I_0 in the maximum current).

Express i in a Fourier series.

9. Obtain the half-range sine series for the function

$$f(x) = \begin{cases} \frac{1}{4} - x\,, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1 \end{cases}$$

10. Find the Fourier cosine series of $f(x) = sin\left[\frac{m\pi}{l}\right]x$,where m is positive integer.

11. Expand $f(x) = \sqrt{1 - cosx}\,, 0 < x < 2\pi$ in a Fourier series. Hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots$ ..