

QUESTION BANK

TRANSFORM CALCULUS, FOURIER SERIES AND NUMERICAL TECHNIQUES (18MAT31)

MODULE-3 FOURIER TRANSFORMS & Z-TRANSFORMS

1. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$
2. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \left[\frac{x \cos x - \sin x}{x^3} \right] \cos\left(\frac{x}{2}\right) dx$
3. Find the Fourier transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{\cos xt}{1+t^2} dt$
4. Find the Fourier sine transform of $e^{-|x|}$ and hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$
5. Obtain the Fourier sine transform of $f(x) = \begin{cases} x, & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 < x < 2 \\ 0 & \text{for } 2 < x \end{cases}$
6. Find the Fourier cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$
7. Find the inverse Fourier transform of e^{-u^2}
8. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, $a > 0$
9. Find the inverse Fourier sine transform of $\frac{e^{-au}}{u}$ and hence obtain the inverse Fourier sine transform of $\frac{1}{u}$
10. Solve the integral equation $\int_0^\infty f(\theta) \cos \alpha \theta d\theta = \begin{cases} 1-\alpha & \text{for } 0 \leq \alpha \leq 1 \\ 0 & \text{for } \alpha > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$
11. Find the Z-transform of $\cosh n\theta$ and $a^n \cosh n\theta$
12. Find the Z-transform of i) $\sin(3n+5)$ ii) $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$ iii) $\cosh\left(\frac{n\pi}{2} + \theta\right)$
13. Find the Z-transform of i) $2n + \sin\left(\frac{n\pi}{4}\right) + 1$ ii) $\frac{n}{3^n} + 2^n n^2 + 4 \cos n\theta + 4^n + 8$
14. Find the inverse Z-transform of $\frac{z^3 - 20z}{(z-2)^3(z-4)}$
15. Find the inverse Z-transform of $\frac{18z^2}{(2z-1)(4z+1)}$
16. Find the inverse Z-transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$
17. Solve the difference equation $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = u_1 = 0$ using Z-transform
18. Solve the difference equation $u_{n+2} - 3u_{n+1} + 2u_n = 2^n$ with $u_0 = 0, u_1 = 1$ using Z-transform
19. Solve the difference equation $y_{n+2} + 2y_{n+1} + y_n = n$ with $u_0 = 0, u_1 = 0$ using Z-transform
20. Solve the difference equation $u_{n+2} - 4u_n = n - 1$ with $u_0 = 1, u_1 = 2$ using Z-transform
