

QUESTION BANK

TRANSFORM CALCULUS, FOURIER SERIES AND NUMERICAL TECHNIQUES (18MAT31)

MODULE-2

FOURIER SERIES

1. Obtain the Fourier series for the function $f(x) = |x|$ in the interval $-\pi \leq x \leq \pi$ and hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

2. Obtain the Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & \text{for } -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & \text{for } 0 \leq x \leq \pi \end{cases}$ and hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

3. Find the Fourier series to represent the function $f(x)$ given by $f(x) = x$ for $0 \leq x \leq \pi$ and $f(x) = 2\pi - x$

for $\pi \leq x \leq 2\pi$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

4. Obtain the Fourier series for the function $f(x) = 2x - x^2$ in $0 \leq x \leq 2$

5. Obtain the Fourier series for the function $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2 - x), & 1 \leq x \leq 2 \end{cases}$ and hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

6. Obtain the Fourier series for the function $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 < x < 2\pi$ and hence deduce

that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

7. Expand $f(x) = \sqrt{1 - \cos x}$, $0 < x < 2\pi$ in a Fourier series.

8. Obtain Half range cosine series for $f(x) = x(\pi - x)$ in $0 < x < \pi$

9. Find Half range cosine series for $f(x) = (x - 1)^2$ in $0 \leq x \leq 1$

10. Find Half range sine series for $f(x) = e^x$ in $(0, 1)$

11. If $f(x) = \begin{cases} x & \text{in } 0 < x < \frac{\pi}{2} \\ \pi - x & \text{in } \frac{\pi}{2} < x < \pi \end{cases}$ then show that $f(x) = \frac{4}{\pi} \left[\sin x - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \dots \right]$

12. Express y as Fourier series up to first harmonics

x°	0	60	120	180	240	300	360
$y = f(x)$	7.9	7.2	3.6	0.5	0.9	6.8	7.9

- 13 Obtain the constant term and first sine and cosine terms in the expansion of y from the following table

x	0	1	2	3	4	5
$y = f(x)$	9	18	24	28	26	30

14. The following data gives the variation of periodic current over a period

$t(\text{secs})$	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
$A(\text{amp})$	1.98	1.3	1.05	1.3	-0.88	-0.25	1.98

Show that there is a direct current pass of 0.75A in the variable current and obtain the amplitude of first harmonics.

15. Obtain the constant term and the coefficients of $\sin x$ and $\sin 2x$ in the Fourier expansion of y given in the following table

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$y = f(x)$	0	9.2	14.4	17.8	17.3	11.7	0