CS50
FINAL PROJECT
PROJECTILE TRAJECTORY COMPARISON (WITH DRAG COEFFICIENT AND COEFFICIENT OF RESTITUTION)

OUTCOME

SOFTWARE PROGRAM (WRITTEN IN PYTHON 3.6 USING SPYDER)

The following program asks the user for the values of the mass, initial velocity and initial angle of projection of projectile along with values of drag coefficient and coefficient of restitution in S.I. units. The program then plots the projectiles of normal projectile(without drag), drag projectile and their comparison and also displays the maximum height and range in both the cases and along with the difference between the ranges and maximum heights obtained by the projectiles(assuming the projectile to be a point particle).

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AIM

The aim of the program is to compare the projectile trajectory, maximum height reached and range of a particle in two cases:-

-With Coefficient of Restitution, Without Drag

-With Coefficient of Restitution, With Drag

MATERIALS REQUIRED

Python 3.6(Using Spyder in Anaconda IDE), Pylab directory (for plotting graphs), Math directory(for calculations).

THEORY

When a particle is projected at an angle with an initial velocity, two forces act on it, namely-drag force and gravitational force. The drag force always acts opposite to the velocity of the particle and is proportional to the velocity of the particle at that particular moment.

$$F_{drag} = -k\,(V_x\,i + V_y\,j)$$

where k=drag coefficient

V_x= velocity along x axis at that particular moment

V_y=Velocity along y axis at that particular moment

The gravitational force always acts along the negative y direction and is proportional to the mass of the particle.

$$F_{gravitational} = mg(-j)$$

where m= mass of the particle

$g=9.8m/s^2$ (acceleration due to gravity)

Combining the above equations, the force equation of the particle gives

$$F_{net} = F_{drag} + F_{gravitational}$$

$$F_{net} = [-k(V_x)]i + [-mg-k(V_y)]j$$

which on solving and subsequent integration yields the following equations.

$$Y(t)=(mV_y/k)(1-\exp(-kt/m))-(mg/k)(t-(m/k)(1-\exp(-kt/m)))$$

$$X(t)= (mV_x/k)(1-\exp(-kt/m))$$

where $Y(t)$=y coordinate of particle at time t

$X(t)$=x coordinate of particle at time t

Now when the projectile collides with the ground, it rebounds with a velocity

$$V_y' = e*V_{y,\text{just before hitting the ground}}$$

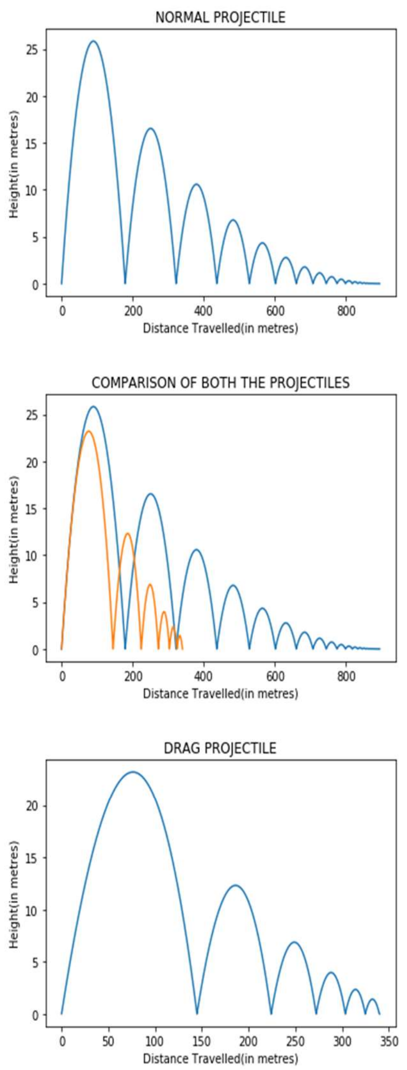
where e=coefficient of restitution.

Using these equations and y axis constraints we plot the trajectories using Pylab.

OBSERVATIONS

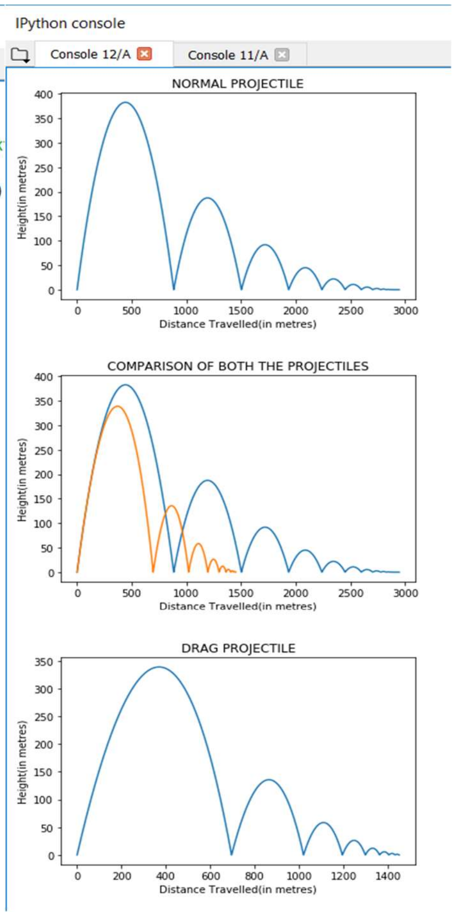
OBSERVATION 1

```
IPython console
Console 12/A    Console 11/A

In [1]: runfile('C:/Users/shash/Desktop/ME122 Project/ME122Project.py', wdir='C:/Users/shash/Desk
The following program illustrates the trajectory followed by a drag projectile
and normal projectile (when coefficient of restitution,drag coefficients,etc values are provided)
and the compares the two trajectories assuming the particle to be a point particle.


Enter the initial angle(in degrees):  30

Enter the initial velocity(in m/s):  45

Enter the value of the coefficient of restitution:  .8

Enter the value of the drag coefficient(in Ns/m):  .3

Enter the value of mass of projectile(in kgs):  4


INITIAL VALUES:
Velocity                  :45.0m/s
Velocity(along x axis)    :38.971m/s
Velocity(along y axis)    :22.5m/s
Coefficient Of Restitution:0.8
Mass of particle          :4.0kg
Drag Coefficient          :0.3Ns/m


RESULT:
Range(Normal Projectile)          :894.707m
Range(Drag Projectile)            :340.053m
Difference between Ranges         :554.653m
Maximum Height(Normal Projectile) :25.829m
Maximum Height(Drag Projectile)   :23.201m
Difference between Maximum Heights:2.628m
```



OBSERVATION 2

```
IPython console
Console 12/A   Console 11/A

In [5]: runfile('C:/Users/shash/Desktop/ME122 Project/ME122Project.py', wdir='C:/Users/shash/Desk
The following program illustrates the trajectory followed by a drag projectile
and normal projectile (when coefficient of restitution,drag coefficients,etc values are provided)
and the compares the two trajectories assuming the particle to be a point particle.


Enter the initial angle(in degrees):  60

Enter the initial velocity(in m/s):  100

Enter the value of the coefficient of restitution:  .7

Enter the value of the drag coefficient(in Ns/m):  .2

Enter the value of mass of projectile(in kgs):  9


INITIAL VALUES:
Velocity                 :100.0m/s
Velocity(along x axis)   :50.0m/s
Velocity(along y axis)   :86.603m/s
Coefficient Of Restitution:0.7
Mass of particle         :9.0kg
Drag Coefficient         :0.2Ns/m


RESULT:
Range(Normal Projectile)         :2945.632m
Range(Drag Projectile)           :1450.933m
Difference between Ranges        :1494.699m
Maximum Height(Normal Projectile) :382.653m
Maximum Height(Drag Projectile)   :338.938m
Difference between Maximum Heights:43.715m
```



RESULT

The given trajectories are plotted and compared and the results given by the program are accurate and also intuitive.

PRECAUTIONS

1) The values of velocity and drag coefficient should be large enough so that the program doesn't break out of drag loop earlier than expected, giving an unexpected output.
2) The horizontal component of velocity(and hence magnitude) should be small enough so that the horizontal component doesn't vanish in mid-air before first collision with the ground.